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PHOTOGRAPH BY GESSFORD

F. R. Low

PRESIDENT 1924

OF

THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS

TRANSACTIONS

VOLUME 46

CLEVELAND MEETING
NEW YORK MEETING
1924

PUBLISHED BY
THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
29 WEST 39TH STREET, NEW YORK
1925

PUBLICATIONS COMMITTEE'S FOREWORD

IN SCOPE and size this volume of TRANSACTIONS marks a return to normal. It reports the Spring and Annual Meetings more adequately than has been possible in preceding years and it includes a larger percentage of the papers presented at those meetings. In selecting the papers the Committee, as always, has been guided by the principle of choosing material of permanent technical value, and the discussions have been carefully edited.

The 1924 TRANSACTIONS departs somewhat from the arrangement which has been followed for some years, in that the list of officers and committees, and the statement of the membership of the Society which has formerly appeared in the front of the book on pages with Roman numeral folios, is now a part of the Society Affairs.

In the index to the volume another change has been made. Spring and Annual Meeting papers printed in *Mechanical Engineering* but not in this volume, Section papers, technical reports, and other important contributions to *Mechanical Engineering* during 1924, are listed on pages immediately following the main index to the volume instead of being included within that index.

This page replaces the introductory note which has always preceded the biographical sketch of the president, and a statement by the Committee which has sometimes been inserted on a loose sheet in the volume.

A. G. CHRISTIE, *Chairman*
J. T. WILKIN
O. G. DALE
R. E. FLANDERS
KENNETH H. CONDIT
Publications Committee.

CONTENTS OF VOLUME 46[1]

CLEVELAND AND NEW YORK MEETINGS

[1] The Society shall not be responsible for statements or opinions advanced in papers or in discussion at meetings of the Society or of its Divisions or Sections, or printed in its publications.

FRED R. LOW

PRESIDENT OF THE SOCIETY FOR 1924

FRED R. LOW was born in Chelsea, Mass., in April, 1860. His education was received in the public schools of that city.

As a boy Mr. Low found a position with the Western Union Telegraph Company in Boston. He learned both telegraphy and stenography and for several years was a court and commercial stenographer. In 1880 he became stenographer to the editor of the Boston *Journal of Commerce.* He remained with this journal until 1888, part of the time as editor of the steam-engineering department. During his years with this paper, a time when attention was beginning to be directed strongly to steam-engine economy, he operated with Frank M. Clark, the Clark & Low Machine Company, and was co-inventor with Mr. Clark of a flue cleaner for vertical boilers, an integrating steam-engine indicator, an elevator control, a rotary engine, etc.

Since 1888 Mr. Low has been editor of *Power.* In this capacity he has attained distinction not only as an editor but as an authority on power-plant subjects. His lectures during the early days of *Power,* delivered before meetings of engineers and published monthly, were of inestimable educational value. More recently his editorials have achieved a permanent place in engineering literature. He is the author of The Power Catechism, The Compound Engine, Condensers, and The Steam Engine Indicator.

Mr. Low has been a member of the Society since 1886 and is one of its most active workers. In 1909 he was chairman of the Gas Power Section which was organized the preceding year and was the first professional division of the Society. He was a vice-president from 1918 to 1920, and president for the year 1923-24. Since 1917 he has been a member of the Boiler Code Committee. He was a member of the organizing committee of the Fuels Division and chairman of its Executive Committee during 1922-1923. From 1912 to 1917 he was a member of the Publications Committee of the Society. Probably his most outstanding contribution to the work of the Society has been in connection with the formulation of its Power Test Codes. He has served as chairman of this committee ever since its organization in 1918. He is also the Society's representative on the American Engineering Council.

Mr. Low is an honorary member of the Institution of Mechanical Engineers of Great Britain, the Marine Engineers Beneficial

Association, and the National Association of Practical Refrigerating Engineers, and a member of the American Association for the Advancement of Science, the National Association of Stationary Engineers, and the Verein Deutscher Ingenieure. At the centennial celebration of Rensselaer Polytechnic Institute in October, 1924, the degree of Doctor of Engineering was conferred upon him.

No. 1919

SOCIETY AFFAIRS

ORGANIZATION AND MEMBERSHIP

ON THE following pages are given the names of those who made up the executive and administrative personnel of the Society, its representatives on joint activities, and a summary of its membership for the year 1924. The personnel of professional committees and divisions, Local Sections officers, and detailed information concerning the organization of the Society was printed in the Year Book for 1924.

OFFICERS AND COUNCIL

PRESIDENT

Fred R. Low.................................New York, N. Y.

VICE-PRESIDENTS

Terms Expire December, 1924

W. S. Finlay, Jr.................................New York, N. Y.
Wm. H. Kenerson.................................Providence, R. I.
Earl F. Scott.................................Atlanta, Ga.

Terms Expire December, 1925

George I. Rockwood.................................Worcester, Mass.
W. J. Sando.................................Milwaukee, Wis.
H. Birchard Taylor.................................Philadelphia, Pa.

MANAGERS

Terms Expire December, 1924

Sherwood F. Jeter.................................Hartford, Conn.
H. P. Liversidge.................................Philadelphia, Pa.
Hollis P. Porter.................................Tulsa, Okla.

Terms Expire December, 1925

A. G. Christie.................................Baltimore, Md.
James H. Herron.................................Cleveland, Ohio
Roy V. Wright.................................New York, N. Y.

Terms Expire December, 1926

E. O. Eastwood.................................Seattle, Wash.
E. R. Fish.................................St. Louis, Mo.
Frank A. Scott.................................Cleveland, Ohio

PAST-PRESIDENTS

M. E. Cooley..................................Ann Arbor, Mich.
Fred J. Miller..................................New York, N. Y.
Edwin S. Carman..................................Cleveland, Ohio
Dexter S. Kimball..................................Ithaca, N. Y.
John Lyle Harrington..................................Kansas City, Mo.

TREASURER

William H. Wiley..................................New York, N. Y.

SECRETARY

Calvin W. Rice..................................New York, N. Y.

STANDING ADMINISTRATIVE COMMITTEES

FINANCE

Erik Oberg (1928), *Chairman and Representative on Council*

H. V. Coes (1924) — A. E. Walden (1926)
M. M. Upson (1925) — Chas. E. Gorton (1927)

Council Representatives: W. S. Finlay, Jr. (1924)
George I. Rockwood (1925)

MEETINGS AND PROGRAM

J. W. Roe (1924), *Chairman and Representative on Council*

L. B. McMillan (1925) — E. Howard Reed (1927)
C. N. Lauer (1926) — R. M. Gates (1928)

PUBLICATIONS

A. G. Christie (1924), *Chairman and Representative on Council*

J. T. Wilkin (1925) — R. E. Flanders (1927)
O. G. Dale (1926) — Kenneth H. Condit (1928)

MEMBERSHIP

S. D. Collett (1924), *Chairman and Representative on Council*

F. A. Waldron (1925) — Charles E. Gorton (1927)
Clyde R. Place (1926) — Hosea Webster (1928)

PROFESSIONAL DIVISIONS

James Partington (1924), *Chairman and Representative on Council*

F. O. Hoagland (1925) — John H. Lawrence (1927)
H. V. Coes (1926) — R. T. Kent (1928)

LOCAL SECTIONS

Thos. L. Wilkinson (1924), *Chairman and Representative on Council*

Charles Penrose (1925) — W. A. Hanley (1927)
James A. Hall (1926) — James D. Cunningham (1928)

CONSTITUTION AND BY-LAWS

E. L. Ohle (1924), *Chairman and Representative on Council*

Arthur L. Rice (1925) — Charles Bromley (1927)
T. E. Coon (1926) — E. E. Howard (1928)

Note: Dates in parentheses denote expiration of term.

AWARDS AND PRIZES

IRA N. HOLLIS (1925), *Chairman and Representative on Council*

H. B. SARGENT (1924) R. H. FERNALD (1927)

R. SANFORD RILEY (1926) L. P. ALFORD (1928)

RELATIONS WITH COLLEGES

WM. H. KENERSON (1926), *Chairman and Representative on Council*

H. G. TYLER (1924) L. C. MARBURG (1927)

D. ROBERT YARNALL (1925) WM. H. KAVANAUGH (1928)

EDUCATION AND TRAINING FOR THE INDUSTRIES

JOHN T. FAIG (1927), *Chairman and Representative on Council*

S. S. EDMANDS (1924) D. C. JACKSON (1926)

IRA N. HOLLIS (1925) R. L. SACKETT (1928)

LIBRARY

HENRY A. LARDNER (1928), *Chairman and Representative on Council*

SYDNEY BEVIN (1924) W. C. WETHERILL (1926)

W. W. MACON (1925) THE SECRETARY

SPECIAL ADMINISTRATIVE COMMITTEES

POLICY OF THE SOCIETY

Appointed 1923 to report on the fundamental policy of the Society, and the financing of its activities

DEXTER S. KIMBALL, *Chairman* W. H. KENERSON

JOHN LYLE HARRINGTON ERIK OBERG

W. S. FINLAY, JR. GEO. A. ORROK

ROY V. WRIGHT

PRESENTATION OF DUES INCREASE

DEXTER S. KIMBALL, *Chairman*

W. S. FINLAY, JR. ROY V. WRIGHT

W. H. KENERSON ERIK OBERG

DEVELOPMENT OF INCOME OF THE SOCIETY FROM BUSINESS ACTIVITIES

CHARLES E. GORTON, *Chairman*

FRED J. MILLER, *Vice-Chairman* R. E. FLANDERS

H. V. COES R. J. S. PIGOTT

GENERAL ORGANIZATION OF THE SOCIETY

W. S. FINLAY, JR., *Chairman*

DEXTER S. KIMBALL JAMES PARTINGTON

INVENTORY OF PROPERTY AND FUNDS OF THE SOCIETY

SHERWOOD F. JETER, *Chairman*

CLARKE FREEMAN W. H. GREUL

NOTE: Dates in parentheses denote expiration of term.

NOMINATING COMMITTEE

The members of the Nominating Committee representing the seven groups into which the membership of the Society has been divided, were elected at the 1923 annual meeting:

GROUP I......WM. R. WEBSTER, BRIDGEPORT, CONN.
R. SANFORD RILEY, WORCESTER, MASS., *Alternate*
GROUP II......KINGSLEY L. MARTIN, NEW YORK
J. H. LAWRENCE, NEW YORK, *Alternate*
GROUP III......CHAS. LOEBER, RICHMOND, VA.
O. P. HOOD, WASHINGTON, D. C., *Alternate*
GROUP IV......R. G. NYE, BUFFALO
JAMES GUTHRIE, CLEVELAND, *Alternate*
GROUP V......WM. M. WHITE, MILWAUKEE
HARRY REID, INDIANAPOLIS, *Alternate*
GROUP VI......E. W. BURBANK, DALLAS
H. R. AUERSWALD, OKLAHOMA, *Alternate*
GROUP VII......BRUCE LLOYD, SAN FRANCISCO
WYNN MEREDITH, SAN FRANCISCO, *Alternate*

LOCAL SECTIONS IN NOMINATING COMMITTEE GROUPS

GROUP I

Green Mountain (Vt.), Boston, Bridgeport, Hartford, Meriden, New Haven, New Britain, Waterbury, Providence, Western Mass., Worcester

GROUP II

Metropolitan (N. Y.), Foreign Members

GROUP III

Atlanta, Baltimore, Birmingham, Carolinas, Central Pa., Chattanooga, Knoxville, Lehigh Valley, Louisville, Memphis, Philadelphia, Plainfield, Savannah, Virginia, Washington, D. C.

GROUP IV

Akron, Buffalo, Cleveland, Columbus, Eastern New York, Erie, Ontario, Pittsburgh, Rochester, Syracuse, Toledo, Utica

GROUP V

Chicago, Cincinnati, Detroit, Indianapolis, Milwaukee, St. Paul, Minneapolis, Tri-Cities

GROUP VI

Colorado, Houston, Kansas City, Mid-Continent, New Orleans, North Texas, St. Louis, Nebraska

GROUP VII

Inland Empire, Los Angeles, Oregon, San Francisco, Utah, Western Washington

HISTORICAL GUILD

J. W. LIEB, *Chairman*

D. S. JACOBUS — REAR-ADMIRAL ROBERT S. GRIFFIN — J. W. ROE

DESIGN OF A.S.M.E. AND MELVILLE MEDALS

JOHN W. LIEB, *Chairman*

W. N. DICKINSON — GEORGE F. KUNZ
W. F. M. GOSS — AMBROSE SWASEY

TELLERS OF ELECTION OF OFFICERS

A. A. ADLER — LEWIS F. LYNE — DAVID B. PORTER

STANDING PROFESSIONAL COMMITTEES

STANDARDIZATION

E. C. Peck (1925), *Chairman and Representative on Council*

William P. Eales (1924)
Cloyd M. Chapman (1926)
C. F. Hirshfeld (1927)
A. M. Houser (1928)

RESEARCH

R. J. Pigott (1928), *Chairman and Representative on Council*

Arthur M. Greene, Jr. (1924)
Fred G. Hechler (1925)
Albert Kingsbury (1926)
D. Robert Yarnall (1927)

POWER TEST CODES

Fred R. Low, *Chairman and Representative on Council*

MAIN COMMITTEE

Term Expires November 30, 1924

C. Harold Berry
Francis Hodgkinson
* D. S. Jacobus
L. F. Moody
Harry F. Smith

Term Expires November 30, 1925

* Fred R. Low
* L. P. Breckenridge
* R. H. Fernald
C. F. Hirshfeld
R. J. S. Pigott

Term Expires November 30, 1926

A. G. Christie
Paul Diserens
C. E. Lucke
E. F. Miller
Geo. A. Orrok

Term Expires November 30, 1927

G. M. Basford
Harte Cooke
E. R. Fish
O. P. Hood
* Wm. F. Uhl

Term Expires November 30, 1928

* N. A. Carle
G. A. Goodenough
L. S. Marks
E. N. Trump
* A. C. Wood

BOILER CODE

(*Special Committee*)

John A. Stevens, *Chairman*
D. S. Jacobus, *Vice-Chairman*
C. W. Obert, *Secretary*
Wm. H. Boehm
N. A. Carle
Frank H. Clark
Francis W. Dean
Thomas E. Durban
E. R. Fish
Elbert C. Fisher
Chas. E. Gorton
Arthur M. Greene, Jr.
Charles L. Huston
S. F. Jeter
Wm. F. Kiesel, Jr.
W. F. MacGregor
Edward F. Miller
M. F. Moore
I. E. Moultrop
C. O. Myers
William B. Reed
H. H. Vaughan
Fred R. Low, *ex-officio*

SAFETY CODES

John W. Upp (1926), *Chairman and Representative on Council*

Carl M. Hansen (1924)
John Price Jackson (1925)
Carl B. Auel (1927)
H. L. Whittemore (1928)

Note: Dates in parentheses denote expiration of term.
* Members of Executive Committee.

PROFESSIONAL CONDUCT

Charles T. Main (1928), *Chairman and Representative on Council*
George I. Rockwood (1924)
Charles L. Newcomb (1926)
Fred J. Miller (1925)
Edward N. Trump (1927)

JOINT ACTIVITIES

IN WHICH THE SOCIETY FORMS A CORPORATE PART

The President and Secretary represent the Society on the Founder Societies' Joint Conference Committee, which directs all matters of joint interest into the proper channels.

AMERICAN ENGINEERING COUNCIL

Comprised of 28 member organizations to represent the engineers of America in matters of public welfare to the engineering and allied technical professions.

Term Expires January, 1925

Fred R. Low, New York, N. Y., *Chairman*
L. P. Alford, New York, N. Y., *Vice-Chairman*
M. E. Cooley, Ann Arbor, Mich.
E. R. Fish, St. Louis, Mo.
A. M. Greene, Jr., Princeton, N. J.
J. L. Harrington, Kansas City, Mo.
Edwin B. Katte, New York, N. Y.
Fred J. Miller, New York, N. Y.
Max Toltz, St. Paul, Minn.

Term Expires January, 1926

F. K. Copeland, Chicago, Ill.
J. T. Faig, Cincinnati, Ohio
R. E. Flanders, Springfield, Vt.
Dexter S. Kimball, Ithaca, N. Y.
W. B. Powell, Buffalo, N. Y.
Wm. Schwanhausser, New York, N. Y.
S. W. Stratton, Cambridge, Mass.
C. C. Thomas, Los Angeles, Cal.
P. F. Walker, Lawrence, Kan.

AMERICAN ENGINEERING STANDARDS COMMITTEE:

E. C. Peck (1924)
C. M. Chapman (*Alternate*) (1924)
F. E. Rogers (1925)
S. G. Flagg, Jr. (1926)

EMPLOYMENT SERVICE BUREAU:

Calvin W. Rice, Secretary A.S.M.E., *Chairman*

ENGINEERING FOUNDATION BOARD:

John H. Barr (1925)
W. F. M. Goss (1926)
Geo. A. Orrok (1927)

JOINT FINANCE COMMITTEE OF THE FOUNDER SOCIETIES:

Erik Oberg
Walter M. McFarland

Note: Dates in parentheses denote expiration of term.

JOHN FRITZ MEDAL BOARD OF AWARD:
Walter M. McFarland (Feb., 1925)
W. F. M. Goss (Feb., 1926)
Fred J. Miller, *Secretary* (Feb., 1927)
Henry B. Sargent (Feb., 1928)

LIBRARY BOARD: See Library Committee, page 9

UNITED ENGINEERING SOCIETY:
W. L. Saunders (1925)
W. F. M. Goss (1926)
James H. Herron (1927)

SOCIETY REPRESENTATION

OTHER ORGANIZATIONS IN WHICH THE SOCIETY IS REPRESENTED BY COURTESY

AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE, Section M, Engineering:
Alex. C. Humphreys
V. E. Muncy

JOSEPH A. HOLMES MEMORIAL BOARD:
Brig-Gen. Wm. A. Bixby

NATIONAL RESEARCH COUNCIL, Division of Engineering:
D. S. Jacobus (June, 1924)
Albert Kingsbury (June, 1925)
R. J. S. Pigott (June, 1926)
Geo. A. Orrok (June, 1927)

SOCIETY FOR THE PROMOTION OF ENGINEERING EDUCATION, Board of Coordination and Investigation:
John Lyle Harrington
Frank A. Scott

WESTERN SOCIETY OF ENGINEERS, Washington Award:
Chas. Russ Richards (June, 1924)
Herbert S. Philbrick (June, 1925)

WORLD POWER CONFERENCE, LONDON, 1924, American Committee:
Fred R. Low
J. W. Lieb
David B. Rushmore
Calvin W. Rice

INTERNATIONAL MANAGEMENT CONGRESS, PRAGUE, CZECHOSLOVAKIA, 1924, American Committee:
Robert T. Kent, *Chairman*

WORLD'S CONGRESS OF ENGINEERS, PHILADELPHIA, 1926:
Ira N. Hollis
D. Robert Yarnall

Note: Dates in parentheses denote expiration of term.

SUMMARY OF MEMBERSHIP BY RESIDENCE

United States and Possessions

Alabama	99	Nevada	2
Arizona	21	New Hampshire	42
Arkansas	9	New Jersey	1154
California	676	New Mexico	4
Canal Zone	5	New York	4331
Colorado	80	North Carolina	83
Connecticut	673	North Dakota	3
Delaware	75	Ohio	1137
District of Columbia	169	Oklahoma	79
Florida	57	Oregon	44
Georgia	129	Pennsylvania	1850
Hawaiian Islands	19	Philippine Islands	26
Idaho	4	Porto Rico	21
Illinois	1042	Rhode Island	151
Indiana	259	South Carolina	35
Iowa	40	South Dakota	6
Kansas	53	Tennessee	93
Kentucky	53	Texas	150
Louisiana	117	Utah	21
Maine	40	Vermont	38
Maryland	206	Virginia	118
Massachusetts	1162	Washington	100
Michigan	514	West Virginia	52
Minnesota	110	Wisconsin	308
Mississippi	10	Wyoming	9
Missouri	298		
Montana	20		
Nebraska	22	Total	15,899

Other Countries

North America

Canada	241	
Newfoundland	2	
Mexico	31	
		274

Central America

Costa Rica	4	
Guatemala	1	
Honduras	3	
		8

West Indies

Cuba	68	
Dominican Republic	4	
Jamaica	1	
Trinidad	1	
		74

South America

Argentina	15	
Bolivia	1	
Brazil	13	

South America (*continued*)		
Chile	21	
Colombia	2	
Peru	5	
Uruguay	2	
Venezuela	3	
		62
Africa		
Egypt	1	
Mauritius	2	
Senegal	1	
Union of S. Africa	11	
		15
Asia		
China	22	
Dutch East Indies	2	
India	24	
Japan	31	
Manchuria	3	
Persia	1	
Syria	1	
		84
Australasia		
Australia	17	
New Zealand	3	
Tasmania	1	
		21
Europe		
Austria	1	
Belgium	2	
Czechoslovakia	4	
Denmark	6	
Finland	3	
France	34	
Germany	19	
Great Britain	87	
Greece	1	
Holland	2	
Italy	6	
Norway	3	
Poland	2	
Roumania	1	
Russia	1	
Spain	7	
Sweden	13	
Switzerland	7	
		199
Total		737

Membership in United States	15,899
Membership in Other Countries	737
Present Address Unknown	30
Total Membership	16,666

Summary of Membership by Grades

Honorary Members	18
Members	7696
Associates	723
Associate-Members	3935
Juniors	4294
Total	16,666

REPORTS OF GENERAL MEETINGS

ONLY the Spring and Annual Meetings of the Society are reported in TRANSACTIONS, and these not in detail. More complete accounts of the general meetings, and reports of the regional and sectional meetings were published during the year in *Mechanical Engineering* and the *A.S.M.E News*.

THE SPRING MEETING

Cleveland, Ohio, May 26 through 29

The program for the Cleveland Spring Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS in interest, scope, and diversity resembled that of an Annual Meeting. The four days from May 26 through 29 were well filled with important technical sessions, excursions, and committee meetings. Technically and industrially it was the most successful Spring Meeting yet held. Nine hundred and fifty members and guests were in attendance and availed themselves of the opportunity to enjoy the attractions offered by the Cleveland Committee.

The arrangements at Cleveland were planned and conducted by a group of sixty Cleveland members under the leadership of the following executive committee: Frank A. Scott, Chairman; James Guthrie, Vice-Chairman; James H. Herron, Second Vice-Chairman; Warner Seely, Secretary; E. G. Bailey, C. H. Baker, W. S. Bidle, E. E. Blundell, A. C. Brown, J. Rowland Brown, E. P. Burrell, T. W. Carlisle, E. S. Carman, J. B. Dillard, H. C. Gammeter, F. J. Lyke, A. G. McKee, L. A. Quayle, F. L. Sessions, F. H. Vose, T. A. Weager, and H. M. Wilson.

The feature session of the meeting, that on the evening of Wednesday, May 28, was devoted to Industrial Preparedness as Insurance Against War, and was attended by more than a thousand people. Remarks by Frank A. Scott, member of Council, former Chairman of the War Industries Board and presiding officer at the session, and Maj-Gen. C. C. Williams, Chief of Ordnance of the U. S. Army, were followed by addresses by Bernard M. Baruch, former Chairman of the War Industries Board, and Col. Dwight F. Davis, Assistant Secretary of War. A moving picture provided by the War Department was also shown. Organizations participating in this meeting included the A.S.C.E., A.I.M.E., A.I.E.E., Army Ordnance Association, Cleveland Engineering Society, Society of American Military Engineers, and American Legion.

A session of outstanding technical interest was the one on Wednesday afternoon, May 28, at which W. L. R. Emmet described the mercury-vapor process. The ball room of the Hotel Cleveland was well filled for this occasion and those who came stayed through the presentation and discussion. As second in interest the session with the American Society for Testing Mate-

rials might be selected. On this occasion a strong group of papers provided by a committee of the A.S.T.M. furnished the basis for three hours of valuable discussion. The Joint Session with the American Society of Refrigerating Engineers was held on Thursday morning, May 29, with an excellent attendance.

On Tuesday simultaneous sessions were held on machine-shop practice, windmill and fan design, materials handling in industrial plants, and power problems of the steel industry. On Wednesday morning the subjects of the sessions were power, measurement of management, aeronautics, and ordnance. A session on design followed that on the mercury-vapor process Wednesday afternoon. In addition to the joint sessions with the A.S.T.M. and A.S.R.E. on Thursday morning, there were two others dealing respectively with recent developments in heavy electric locomotives, and with tooling and gaging for interchangeable manufacture.

The Council held three sessions in Cleveland: Monday morning, Tuesday morning and Tuesday afternoon, with an almost complete attendance of Council members and Chairmen of Standing Committees. The President, Fred R. Low, presided. Announcement was made of the amendment to the By-Laws, effective July 1, 1924, stating that a member whose dues shall remain unpaid for three months after payment becomes due, shall not receive the Society's publications nor be entitled to vote until his dues are paid. Additions to the Rules regarding the termination of Junior Membership were adopted and Milwaukee was selected as the place for the 1925 Spring Meeting. The Council authorized the acceptance of the appointment of the Secretary by the Pan-American Union on the national committee for the Third Pan-American Scientific Congress and the First Engineering Standards Conference held in December in Lima, Peru.[1] It was voted to accept an offer made by George I. Rockwood, Vice-President of the Society, to endow a medal in memory of Alexander L. Holley, a founder and the first presiding officer of the Society. This medal is to be known as the Holley medal and is to be awarded only "in those rare cases where an individual has succeeded by the exercise of his genius and character in powerfully assisting the fortunes of our country or the general engineering progress of the world." Awards will be made by the Council upon the recommendation of the Committee on Awards and Prizes.

Representatives of about twenty-five Local Sections met in conference Monday morning to discuss matters of mutual interest, among these being the question of policy in regard to appropriations for the Local Section activities, and coöperation between

[1] Mr. A. W. Whitney, Chairman of the Engineering Standards Committee, was the official delegate both of the United States and the several engineering and scientific organizations.

Local Sections and Student Branches. At the conclusion of the conference those present joined with the Council and officers of the Society at luncheon.

Monday afternoon, at the Business Meeting, Secretary Rice read the report of the Tellers of Election on the vote amending the Constitution of the Society to increase the dues for Members, Associates, Associate-Members, and those Juniors who retain their Junior status after six years, from fifteen dollars per year to twenty dollars. Of the 7602 votes cast, 176 were defective, 4039 were recorded in favor of the amendment, and 3387 against it. The increased dues therefore went into effect October 1 for those who came under the classification involved and for all joining the Society subsequent to the Cleveland Meeting.

Two Standards were read by title, namely, the first reports of the Sectional Committee on Standardization of Shafting, covering Standard Diameters, Tolerances, and Lengths for Cold-Finished Transmission and Machinery Shafting, and Standard Sizes for Shafting Key Stock.

The selection by the Council of Milwaukee as the place for the 1925 Spring Meeting was ratified.

During the week there was held a meeting of the Boiler Code Committee, which was a joint meeting with the National Board of Boiler and Pressure Vessel Inspectors (the organization of chief inspectors in states and municipalities where the A.S.M.E. Boiler Code is operative). The Boiler Code Committee meeting, which was the regular May interpretation meeting of the Committee, was held on Tuesday, May 27, and by virtue of the attendance of the group of inspectors who are members of the Conference Committee to the Boiler Code Committee, was one of the largest-attended regular meetings ever held by the Committee.

Meetings of the National Board of Boiler and Pressure Vessel Inspectors, which were attended by the Boiler Code Committee members, were held on Wednesday and Thursday, May 28 and 29, and comprised both business and professional sessions devoted to the work of boiler inspection and consideration of problems in connection with the enforcement of the rules in the A.S.M.E. Boiler Construction Code.

The Power Test Codes Committee held a public hearing on Wednesday morning, May 28, on the Test Code for Condensing Apparatus. Geo. A. Orrok, member of the Power Test Codes Main Committee, was in the chair.

On Wednesday afternoon, May 28, the Committee on Application of Formulas of the Materials Handling Division held a public hearing for a discussion of the formulas. James A. Shepard, Chairman of the Committee, presided and pointed out the importance of the subject in developing the fundamental economics of materials handling. A number of comments and suggestions were received.

The Committee on Education and Training for the Industries held a conference on Wednesday afternoon, May 28. John T. Faig, Chairman of the Committee, presided and Dean R. L. Sackett, Dr. Ira N. Hollis, and Prof. S. S. Edmands participated in the discussion.

The conference of the student branches was also held Wednesday afternoon. There were short talks by Dean Dexter S. Kimball, John Lyle Harrington, and Dr. Ira N. Hollis, and reports from representatives of various branches.

Approximately twenty committee meetings took place during the meeting, among which were the Sectional Committee on Bolt, Nut and Rivet Proportions, the Sub-Committee on Wrench-Head Bolts and Nuts, the Power Test Codes Individual Committee on Instruments and Apparatus, the Sub-Committee on Tee Slots of the Sectional Committee on Small Tools and Machine-Tool Elements, the Power Test Codes Individual Committee on Gas Producers, Sub-Committee on Standard Formulas for Design of Transmission Shafting, Special Research Committee on Gaging and Forming of Metals, Special Research Committee on Springs, and the Sectional Committee on Plain Limit gages for General Engineering Work.

In addition many Administrative and Professional Divisions Committees met informally throughout the meeting.

The first entertainment event of the meeting was an excursion to Nela Park on Monday afternoon, May 26. After the visitors had viewed the historical exhibits and demonstrations of home lighting, color lighting, automatic bulb blowing, and lamp manufacture, there was a brief program of sports. Later a buffet supper was served, after which those present were entertained by a glee club made up of Cleveland members, and by several monologues. Dancing followed.

About two hundred people visited the rubber plants at Akron on Tuesday afternoon. The party was divided into four groups as follows: One to the B. F. Goodrich Company where the making of mechanical rubber products was shown; a second to the Goodyear Tire and Rubber Company where balloon manufacture was exhibited; a third to the Firestone Tire and Rubber Company to see the manufacture of all types of tires; and a fourth to Barberton to the plants of the Babcock & Wilcox Company and the Ohio Insulator Company where high-pressure boilers and high-voltage insulators were shown in process of manufacture and testing.

After the inspections trips, parties met at the Hotel Portage in Akron for dinner. R. R. Jones, former Chairman of the Akron Local Section, acted as toastmaster and addresses were given by Fred E. Ayer, Dean of Engineering and Commerce at the Municipal University of Akron; James E. Hartness, President of the American Engineering Council; Dexter S. Kimball, Past-President

of the A.S.M.E.; E. O. Eastwood, member of A.S.M.E. Council from Seattle, Wash.; and H. P. Porter, member of A.S.M.E. Council from Tulsa, Okla.

The National Tube Company entertained about three hundred members of the Society at their Lorain plant on the closing afternoon of the meeting. After luncheon the party was divided into groups to inspect the process of pipe making from the unloading of ore at the docks through the blast furnaces, open-hearth furnaces, slate mills, skelp mills, and weld mills.

During the mornings of the meetings visits were scheduled to the Joseph & Feiss Company, The Industrial Fibre Company, the White Motor Company, the Warner & Swasey Company, the Lees-Bradner Company, and the Lake Shore Plant of the Cleveland Electric Illuminating Company. The City Water Works was visited on Wednesday afternoon. In addition a number of other industrial plants were thrown open to any visitors who cared to inspect them.

There were one hundred and fifteen ladies registered at the meeting and a comprehensive program for them was provided which included shopping trips, visits to clothing manufacturers, a visit to the Cleveland Museum of Art, a theater party, an automobile trip to the residence of Mr. and Mrs. Frank A. Scott, followed by tea at the country club, and a luncheon.

The committee in charge of entertainment for the ladies consisted of Mrs. Frank A. Scott, Chairman, Mrs. Lyman H. Treadway, Mrs. Irving C. Bolton, Mrs. Rollin H. White, Mrs. Alexander C. Brown, Mrs. H. S. Pickands, Mrs. C. W. Lundoff, and Mrs. C. F. Brush, Jr.

PROGRAM

Monday Morning, May 26

Opening of Headquarters and Registration Bureau.
Council Meeting.
Conference of Local Sections' Delegates.

Monday Afternoon

BUSINESS MEETING

Report of Tellers on Constitutional Amendment on Increase in Dues; reading by title of first two reports of the Sectional Committee on Standardization of Shafting.

Tuesday Morning, May 27

SIMULTANEOUS SESSIONS

MACHINE-SHOP PRACTICE

First Report of Sectional Committee on the Standardization and Unification of Screw Threads.
British Machine-Tool Design, W. E. Sykes.
Limiting Cases in Involute Spur Gearing, A. B. Cox.
The Sine Bar as a Universal Planing Gage, R. H. Rausch.

WINDMILL AND FAN DESIGN

Performance of Centrifugal Fans for Electrical Machinery, C. J. Fechheimer.

Wind Power for Farm Electric Plants, F. J. Pancratz.

MATERIALS HANDLING IN INDUSTRIAL PLANTS

Safety Code for Elevators.

Fundamental Economies of Materials Handling, M. L. Begeman.

Material-Handling Problems Encountered in the Assembly of Automobiles, M. R. Denison.

Lumber Handling in an Automobile-Body Plant, B. Nagelvoort and Thomas D. Perry.

POWER PROBLEMS OF STEEL INDUSTRY

Safety Code for Mechanical Power Transmission Apparatus

Power Organization in the Steel Industry, Bryant Bannister and F. M. Van Deventer.

The Generation and Utilization of Steam in the Iron and Steel Industry, John A. Hunter.

The Gas Engine in the Steel Industry, A. C. Danks.

Tuesday Afternoon and Evening

Excursion to Akron and Joint Dinner Meeting with Akron Local Section.

Wednesday Morning, May 28

SIMULTANEOUS SESSIONS

CLEVELAND POWER SESSION

Pulverized Fuel at Cleveland Electric Illuminating Company's Lake Shore Plant, W. H. Aldrich.

Fairmount Pumping Station and Heating Plant, L. A. Quayle.

Heat Losses through Insulating Materials, R. H. Heilman.

MEASUREMENT OF MANAGEMENT

Measurement of the Quality of Product, G. S. Radford.

The Control of Idleness in Industry, W. L. Conrad.

The Manit System for Measuring and Stimulating Labor Effort, Hasbrouck Haynes.

Topical Discussion on the Measurement of the Cost-Accounting Function.

PUBLIC HEARING, POWER TEST CODES

Condensing Apparatus.

AERONAUTICS AND ORDNANCE

Opening Remarks by Major-General C. C. Williams.

Aerial Bombing, Major A. H. Hobley and H. B. Inglis.

The War's Impress on the Steel Industry, A. E. White.

The Rôle of the Engineer in Industrial Mobilization Planning, Captain E. E. MacMorland.

Wednesday Afternoon

MERCURY-VAPOR PROCESS

The Emmet Mercury-Vapor Process, W. L. R. Emmet and L. A. Sheldon.

DESIGN

Temperature and Stress Distribution in Hollow Cylinders, O. G. C. Dahl.

The Protection of Steam-Turbine Disk Wheels from Axial Vibration, Wilfred Campbell.

Mathematical Theory of Dynamic Stresses in Rotating Gear Pinions, Paul Heymans.

EDUCATION AND TRAINING FOR THE INDUSTRIES

Opening Remarks by John T. Faig.

Making Industry Attractive to High-School or College Graduates, R. L. Sackett.

Five-Minute Discussions by Dr. Ira N. Hollis, Prof. Dugald C. Jackson, and Prof. S. S. Edmands.

PUBLIC HEARING, MATERIALS-HANDLING FORMULAS

Presentation of Formulas for Computing Economies of Labor-Saving Equipment.

Report of Committee on Application of the Formulas.

STUDENT BRANCH CONFERENCE

Wednesday Evening

INDUSTRIAL-PREPAREDNESS MEETING

Introductory Remarks by Frank A. Scott, and Major-General C. C. Williams.

Addresses by Bernard M. Baruch and Col. Dwight F. Davis, Assistant Secretary of War.

Moving Pictures provided by War Department.

Thursday Morning, May 29

SIMULTANEOUS SESSIONS

JOINT SESSION WITH AMERICAN SOCIETY FOR TESTING MATERIALS

Industrial Applications of Metals at Various Temperatures, L. W. Spring.

Methods of Testing at Various Temperatures and Their Limitations, V. T. Malcolm.

Available Data on the Properties of Irons and Steels at Various Temperatures, H. J. French and W. A. Tucker.

Available Data on the Properties of Non-Ferrous Metals and Alloys at Various Temperatures, C. Upthegrove and A. E. White.

RECENT DEVELOPMENTS IN HEAVY ELECTRIC LOCOMOTIVES

Recent Developments in Electric Locomotives, N. W. Storer.

Development of the Electric Locomotive, A. H. Armstrong.

JOINT SESSION WITH AMERICAN SOCIETY OF REFRIGERATING ENGINEERS

TEMPERATURE MEASUREMENTS, Percy Nicholls.
RESEARCH IN HEAT TRANSMISSION, Edgar Buckingham.
DEFINITIONS AND NOMENCLATURE IN INSULATION, E. F. Mueller.
HEAT-INSULATION DATA IN THE REFRIGERATING FIELD, Percy Nicholls.

TOOLING AND GAGING FOR INTERCHANGEABLE MANUFACTURE

First Report of Sectional Committee on Standardization of Plain Limit Gages for General Engineering Work.
ANALYSIS OF A MACHINE-SHOP PROBLEM ON A QUANTITY AND FINAL-ECONOMY BASIS, A. L. De Leeuw.
MANUFACTURE OF THE BOLT OF THE SPRINGFIELD RIFLE, Major Earl McFarland.

The Annual Meeting

New York, N. Y., December 1 through 4

Those who attended the Forty-Fifth Annual Meeting, though distributed over many simultaneous events, found innovations in the program which on several occasions brought them all together on a common ground. This spirit of professional good fellowship, bespeaking strongly the possibilities of solidarity in so large an organization, together with a registration of 2174, which exceeded by three the largest previous record, that of 1920, sets new standards for future general meetings of the Society.

The revision of the social program, placing the presidential address and reception on Tuesday evening instead of Monday as formerly, permitted an informal "get-together" for members and guests on Monday evening in the Engineering Societies Building. This occasion and the informal dinner for members at the Hotel Astor Wednesday evening, provided excellent opportunities for renewing old friendships and forming new ones.

On Tuesday and Thursday afternoons a large percentage of those attending the meeting were drawn together by lectures of general appeal, dealing with economic and scientific subjects. On Tuesday, Dr. Julian D. Sears, administrative geologist, U. S. Geological Survey, Washington, D. C., discussed the problems of the American petroleum situation which most require the attention and help of engineers, and on Thursday, Properties of Matter under High Pressure was the subject of a lecture by P. W. Bridgman, professor of physics at Harvard University.

Many members also availed themselves of the opportunity to hear the addresses by Drs. Michael Pupin and W. L. R. Emmet at the celebration of the Carnot Centenary held Thursday evening under the auspices of the Engineering Foundation, this Society, and other coöperating societies and institutions.

Special efforts were made by the Committee on Meetings and Program to have the papers available in advance of the meeting.

Part of the papers appeared in a special Mid-November issue of *Mechanical Engineering* and the remainder were issued in pamphlet form. This method resulted in excellent discussion throughout the meeting, and in practically every session the arrangement of the program gave ample opportunity for adequate discussion.

Among the high lights of the technical program were the valuable address of Assistant Secretary of War Davis at the National Defense Session, the papers of Professor Marks and Dr. Lucke at the Oil and Gas Power Session, the two excellent sessions on Management, at one of which Dr. Taylor's paper on Shop Management, now twenty-one years of age, was re-presented, the important Steam Power Session, and the four sessions of interest to machine-shop men. In this last respect the meeting was the strongest held in a long time, and the contributions, especially those on lubrication, were of great importance. The American Society of Refrigerating Engineers coöperated in an interesting session which brought out valuable discussion. Other technical sessions dealt with oil handling and storing, textile problems, turbo-locomotives, oil burning, mechanical design for safety, aeronautics, hydraulics, and progress in steam research. A general session was held Tuesday morning and included three interesting papers.

A public hearing on Power Test Codes was held Tuesday afternoon, at which the Power Test Codes for Solid Fuels, Gas Producers, and Speed Responsive Governors were presented for criticism. The Committee on Education and Training for the Industries of Non-College Type sponsored a session Wednesday afternoon at which three addresses were made. The Boiler Code Committee held its December meeting during the Annual Meeting. This meeting marked the completion of the work of formulating the Code for Unfired Pressure Vessels which had been in process of preparation for a number of years.

Among the numerous committees which held meetings during Annual Meeting week were the Main Committee on Power Test Codes and a number of its individual Committees; four of the research committees; various standardization committees; and executive committees of professional divisions.

Seventy-five men interested in the technical activities of the Society, met at a Progress Conference Thursday evening, December 4, to hear reports on engineering developments in all of the fields of Society work. Through knowledge gained at this Conference, there will be increased effort to coördinate the activities of the Society.

The important feature of the Business Session held on Wednesday afternoon was the formal presentation of a bust of Admiral George Wallace Melville, Past-President and Honorary Member of the A.S.M.E. In the absence of Walter M. McFarland, Chair-

man of the Special Committee on the Melville Bust, President-Elect Durand read the presentation speech. The bust was purchased by personal friends of Admiral Melville in the Society and is the work of another friend, Samuel Murray, a sculptor of Philadelphia. The Committee for the purchase and presentation of the bust consisted of Alexander C. Humphreys, Asa M. Mattice, Ira N. Hollis, Robert S. Griffin, William D. Hoxie, and Walter McFarland.

The Annual Report of the Council was summarized by Secretary Rice and then President Low announced the award of the Junior Prize to R. H. Heilman, of Pittsburgh, Pa., for his paper on Heat Losses through Insulating Materials. The first Student Prize was awarded to George Stuart Clark, of Stanford University, Cal., for his paper on Determination of the Gasoline Content of Absorption Oils, and was received for Mr. Clark by Mr. Durand. The second Student Prize was awarded jointly to L. J. Franklin and Charles H. Smith of Stanford University, Cal., and was received by Mr. Smith.

The Revised National (American) Standard Fire Hose Coupling Screw Thread and the Proposed Standard for Tolerances and Allowances for Machined Fits in Interchangeable Manufacture were read by title.

The following selection of the Nominating Committee by the Conference of Local Sections Delegates was approved:

C. K. Decherd, Meriden, Conn.; W. R. Webster, Bridgeport, Conn., Alternate
J. J. Nelis, Metropolitan; V. M. Frost, Newark, N. J., Alternate
O. P. Hood, Washington, D. C.; J. G. Hatman, Philadelphia, Alternate
R. R. Jones, Akron; E. G. Bailey, Cleveland, Alternate
Thomas N. Wynne, Indianapolis, Ind.; Arthur L. Rice, Chicago, Alternate
Perley F. Walker, Lawrence, Kan.; W. G. Christy, St. Louis, Mo., Alternate
C. I. Carpenter, Spokane, Wash.; U. B. Hough, Spokane, Wash., Alternate

Two meetings of the Council were held during the meeting. At the first it was voted to bind Volume 45 of Transactions in half morocco, instead of in paper, and at the second, new officers were installed.

The conference of the Local Sections delegates was held the first day of the meeting, at which sixty out of sixty-four sections were represented. Over thirty Student Branches were represented at a conference held Wednesday afternoon.

In his Presidential Address on Tuesday evening, Fred R. Low discussed American power resources and their control.

Following the address, the report of the Tellers of Election of officers was presented by the chairman, Dr. A. A. Adler.

President-Elect Durand was then escorted to the platform by Past-Presidents James Hartness and Dexter S. Kimball. Governor Hartness presented Dr. Durand, who formally accepted his new position. The Society then tendered a reception to the President and President-Elect and the guests of the evening. This was followed by dancing.

The Annual Dinner for members of the Society was held on Wednesday evening, and there has possibly been no Society event in recent years so well received and favorably commented upon. Conrad N. Lauer, President of the Philadelphia Engineers' Club, was toastmaster. Among the guests for the evening were Charles F. Rand, Past-President and representative for the President of the A.I.M.E.; Farley Osgood, President of the A.I.E.E., and Ex-Governor Hartness, President of the A.E.C. Seventy-seven men of those who had become members during the year, thirty who had been members for over thirty-five years, and two of the founders of the Society, Ambrose Swasey and Worcester R. Warner, were among those present. The total attendance was 581. Dean Kimball and Dr. Livingston Farrand, President of Cornell University, were the chief speakers of the evening.

A large number of ladies dined together in an adjoining banquet hall at the Astor and entered the Gallery of the Ball Room to hear the speeches.

The Ladies' Tea and Reception was held Wednesday afternoon, and a number of college reunions took place Thursday evening.

An unusual feature of the Ladies' Program was a luncheon in the Blue Room of the McAlpin Hotel on Tuesday, attended by 160 women. Addresses were made by women who had attended during the summer the World Power Conference in England and the Management Conference in Prague. There was also a Get-Together for the ladies Monday evening, and visits to a number of points of interest in New York and to Pratt Institute in Brooklyn. On the latter trip, Mrs. James W. Nelson entertained the party at luncheon at her home. A large party, the limit that could be entertained at one occasion, inspected the *Olympic* on Thursday afternoon.

The feature excursion of the meeting was the trip to Lakehurst, N. J., on Friday afternoon, December 5, to see the new dirigibles *Los Angeles* and *Shenandoah*. A special train of nine cars carrying 548 people started from New York at 12: 25 P. M. and returned by 7: 00 o'clock. Opportunity was given to inspect the structure of the ships and to view the operating cabins.

During the week visits were made to the Sherman Creek Station of the United Electric Light & Power Company, the oil-engine-driven electric locomotive which was undergoing tests in the Long Island yards, the new Hudson Avenue Station of the

Brooklyn Edison Company, and the Bayway Refinery of the Standard Oil Company of New Jersey.

Prof. C. P. Bliss was general chairman of the sub-committees for the Annual Meeting. The chairmen of these committees were as follows: Reception, J. P. Kottcamp; Information, E. Fezandie; Courtesy, B. C. McClure; Excursions, L. H. Welling; Open House, L. F. Lyne, Jr.; President's Reception, F. A. Scheffler; Dinner, R. V. Wright; and Catering, J. D. Taylor. Mrs. Richard S. Austin was general chairman of the ladies' committees, the chairmen of which were Mrs. R. V. Wright, Acquaintanceship; Mrs. C. T. Owens, Excursions; Mrs. G. L. Knight, Luncheon; and Mrs. G. R. Tuska, Reception and Tea.

The Third National Exposition of Power and Mechanical Engineering, which was held in the Grand Central Palace, New York City, December 1 through December 6, was well attended by members of the Society.

PROGRAM

Monday Morning and Afternoon, December 1

Council Meeting.

Conference of Local Sections' Delegates.

Luncheon of Council, with Section Delegates. Brief addresses by President Low, Mr. Swasey, and President-Elect Durand.

Monday Evening

Open House.

Tuesday Morning, December 2

SIMULTANEOUS SESSIONS

OIL HANDLING AND STORING

The Storage and Handling of Fuel Oil in Industrial Plants, C. G. Sheffield and H. H. Fleming.

RESEARCH IN MACHINE DESIGN AND OPERATION

Comparison of Herbert Pendulum Hardness Tester with Other Hardness Testers, J. O. Keller.

Preliminary Progress Report of the A.S.M.E. Special Research Committee on Metal Springs.

TEXTILE SESSION

The Development of the Spinning Frame, Robert E. Naumburg.

The Engineer's Field in Industrial Economics, Eugene Szepesi.

GENERAL SESSION

The Strength and Proportions of Wheels, Wheel Centers, and Hubs, R. Eksergian.

An Experimental Investigation of Nozzle Efficiency, H. Loring Wirt.

Test of a Prosser-Type Reciprocating Steam Engine, L. V. Ludy.

Tuesday Afternoon

SIMULTANEOUS SESSIONS

JOINT SESSION WITH AMERICAN SOCIETY OF REFRIGERATING ENGINEERS

Some Factors Influencing Friction, Velocity Distribution, and Heat Transmission, for Fluids Flowing inside Pipes, W. H. McAdams.

The Temperatures of Evaporation of Water into Air, W. H. Carrier and D. C. Lindsay.

Water-Cooling-System Efficiency, Victor J. Azbe.

MACHINE-SHOP PRACTICE

The Effect of Inaccuracy of Spacing on the Strength of Gear Teeth, Lloyd J. Franklin and Charles H. Smith.

Mechanical Springs, Joseph Kaye Wood.

Ruling Line Standards, Herbert B. Lewis and C. G. Peters.

TURBO-LOCOMOTIVES

The Zoelly Turbine-Driven Locomotive, H. Zoelly.

The Ramsay Condensing Turbo-Electric Locomotive, George F. Jones and T. Laurence Hale.

PUBLIC HEARING, POWER TEST CODES

Power Test Codes for Solid Fuels, Gas Producers, and Speed-Responsive Governors.

LECTURE

Engineers and the American Petroleum Situation, Dr. Julian D. Sears.

Tuesday Evening

PRESIDENTIAL ADDRESS AND RECEPTION

Power Resources, Present and Prospective, Fred R. Low.

Report of Tellers of Election of Officers.

Introduction of President-Elect Durand.

Reception.

Wednesday Morning, December 3

SIMULTANEOUS SESSIONS

OIL BURNING

Fuel-Oil Burning in the United States Navy, H. G. Donald.

Oil Burning in Industrial-Plant and Central-Station Service, Nathan E. Lewis.

Hazards of Industrial Oil Burning, H. E. Newell.

LUBRICATION

An Investigation of the Critical Bearing Pressures Causing Rupture in Lubricating-Oil Films, Leonard Noel Linsley.

High-Pressure-Bearing Research, Louis Illmer.

A Graphical Study of Journal Lubrication (Part II), H. A. S. Howarth.

NATIONAL DEFENSE

ENGINEERING PROBLEMS OF NATIONAL DEFENSE, Dwight F. Davis.
SOME PROBLEMS IN THE DESIGN OF ORDNANCE, J. B. Rose.
X-RAY EXAMINATION OF METALS AT THE WATERTOWN ARSENAL, WATERTOWN, MASS., T. C. Dickson.
NEW DEVELOPMENTS IN GUN CONSTRUCTION, F. C. Langenberg.

MECHANICAL DESIGN FOR SAFETY

THE HAZARDS OF PULVERIZED-FUEL SYSTEMS, H. E. Newell and Robert Palm.
A PLACE FOR SAFETY, Lewis A. DeBlois.

Wednesday Afternoon, December 3

BUSINESS MEETING

Presentation to the Society of the bust of Rear-Admiral Melville; Annual Report of Council; presentation of awards; reading of proposed engineering standards by title.

STEAM TABLES RESEARCH

REPORT OF EXECUTIVE COMMITTEE, STEAM TABLE FUND, Geo. A. Orrok.
RESEARCH PROGRESS REPORTS, R. V. Kleinschmidt, F. G. Keyes, N. S. Osborne, and H. F. Stimson.
SUMMATION OF RESEARCH RESULTS, Harvey N. Davis.

EDUCATION AND TRAINING FOR THE INDUSTRIES OF NON-COLLEGE TYPE

INDUSTRY'S INTEREST IN INDUSTRIAL TRAINING, Magnus W. Alexander.
INDUSTRIAL EDUCATION—A SUMMARY OF THE WORK OF THE AMERICAN MANAGEMENT ASSOCIATION AND ITS PREDECESSORS, George B. Thomas.
TRAINING FOR INDUSTRY AND THE PUBLIC PROGRAM OF VOCATIONAL EDUCATION, Frank Cushman.
THE NEED FOR DISTRICT ORGANIZATION OF MODERN APPRENTICESHIP, H. A. Frommelt.

CONFERENCE OF STUDENT BRANCHES

LADIES' TEA AND RECEPTION

Wednesday Evening, December 3

ANNUAL DINNER

Thursday Morning, December 4

SIMULTANEOUS SESSIONS

AERONAUTICS

EQUIPMENT USED FOR AERIAL SURVEYING, Ernest Robinson.
AN INTRODUCTION TO THE HELICOPTER, Alexander Klemin.
PRODUCTION AIRPLANES OF METAL, Edmund Burke Carns.

STEAM POWER

Water Treatment for Continuous Steam Production, R. E. Hall.

The Increase in Thermal Efficiency Due to Resuperheating in Steam Turbines, W. E. Blowney and G. B. Warren.

A Review of Recent Applications of Powdered Coal to Steam Boilers, Henry Kreisinger.

Recent Developments in the Burning of Anthracite, W. A. Shoudy and R. C. Denny.

MANAGEMENT

Shop Management, Frederick W. Taylor.

Presentation of Resolutions in Memory of Frank B. Gilbreth.

The Development of a Modern Hosiery Plant, Sanford E. Thompson and H. T. Rollins.

Thursday Afternoon, December 4

SIMULTANEOUS SESSIONS

MANAGEMENT AND MACHINE-SHOP PRACTICE

Production Control, George D. Babcock.

Design, Manufacture, and Production Control of a Standard Machine, Ralph E. Flanders.

OIL AND GAS POWER

Solid-Injection Oil Engines, R. Hildebrand.

Large Oil Engines, with Special Reference to the Double-Acting Two-Cycle Type, Charles Edward Lucke.

Gas Turbines, Lionel S. Marks and M. Danilov.

HYDRAULICS

A Method for the Economic Design of Penstocks, H. L. Doolittle.

Intakes for Power Plants, Robert W. Angus.

LECTURE

Properties of Matter under High Pressure, P. W. Bridgman.

Thursday Evening, December 4

PROGRESS CONFERENCE OF TECHNICAL COMMITTEES OF THE SOCIETY

CARNOT CENTENARY

No. 1920

THE PROTECTION OF STEAM-TURBINE DISK WHEELS FROM AXIAL VIBRATION[1]

By Wilfred Campbell,[2] Schenectady, N. Y.

Non-Member

One of the most important features in the design and manufacture of a steam turbine is the elimination of the possibility of vibration occurring at the various natural frequencies of its disk wheels and buckets. This paper describes an investigation by the General Electric Company of various forms of vibrations and waves which may exist in steam-turbine disk wheels. The dangerous critical speeds that must be guarded against are discussed, together with other minor resonant conditions that it is advisable to avoid. The testing machines used for verification of predicted frequencies and critical speeds are described in detail as well as the different types of tests made. The conclusion gives the procedure, necessary in all cases, for the definite protection of steam-turbine bucket wheels from axial vibration, as justified by several years of successful manufacture.

THE purpose of this paper is to present the main features of the work done by the General Electric Company (of America), which led to the solution of the problem of vibration of turbine disk wheels, and to describe the way in which wheels are designed and tested in order to insure freedom from vibration.

2 The investigation was undertaken in order to account for wheel failures of a peculiar and erratic nature which could not be explained on the basis of high stress alone. The number of failures was small, considering the total number of wheels in operation. These failures were not confined to any single type of machine, but they did show a preference in general for thin wheels of large diameter.

3 That this difficulty has actually been overcome with no major alteration in the turbine is emphatically brought out by results

[1] Prepared with the coöperation of A. L. Kimball, Jr., Assoc. A. S. M. E., and Ernest L. Robinson, Mem. A. S. M. E.

[2] General Electric Company. Deceased, July 7, 1924.

Presented at the Spring Meeting, Cleveland, Ohio, May 26 to 29, 1924, of The American Society of Mechanical Engineers.

obtained by the General Electric Company in the past three years from its use of disk wheels properly designed and tested.

TABLE 1 FOR TURBINES OF OVER 5000 KW. INSTALLED BEFORE MARCH 1, 1924

Number of wheels installed	4399
Number of wheels tested (standing)	3596
Number of wheels rotated in wheel-testing machine	320
Number of tests in wheel-testing machine	405
Number of wheels tested in customer's plants (standing)	1683
Number of machines investigated in customer's plants	291
Number of machines tested under load in customer's plants	24
Number of wheels replaced to avoid possible trouble	497
Number of wheels tuned for vibration	212

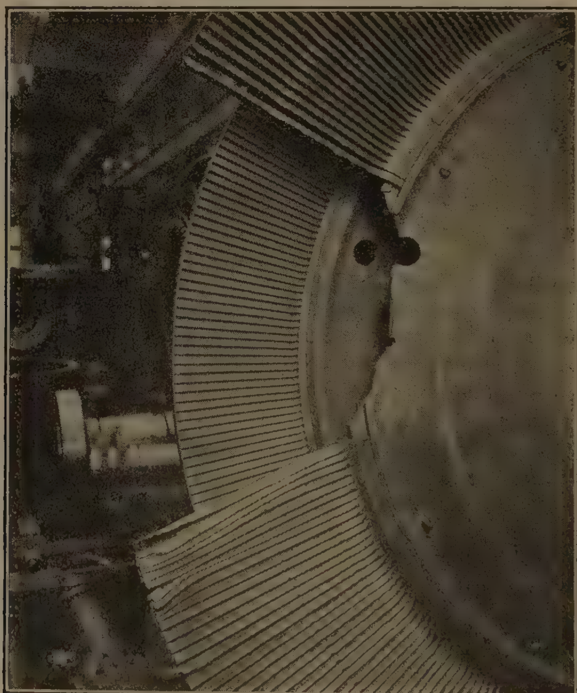

FIG. 1 BROKEN TURBINE BUCKET WHEEL; 9TH STAGE OF 15,000-KW. 1800-R.P.M. 9-STAGE TURBINE

4 Up to the end of 1923 this company had manufactured and installed over 9000 steam turbines aggregating in total generating capacity more than 15,000,000 kw. This investigation is chiefly concerned with large-size machines, that is, of over 5000 kw. capacity. Before the year 1919 the General Electric Company had manufactured and installed in operating plants, a total of 227 turbines of ratings exceeding 5000 kw. each. The total generating capacity represented by these machines was over 2,404,000 kw.

Since the year 1919, when this particular investigation was started, there have been 206 more turbines installed exceeding 5000 kw. each, increasing the total of generating capacity of the larger size turbines to 5,864,500 kw. at the end of 1923.

5 Table 1 shows the magnitude of the investigation, giving a few simple figures as to the number of machines and the number of turbine wheels investigated. The capacity of the testing machines now in operation is sufficient to provide for the testing of 600 wheels annually, under all conditions of speed.

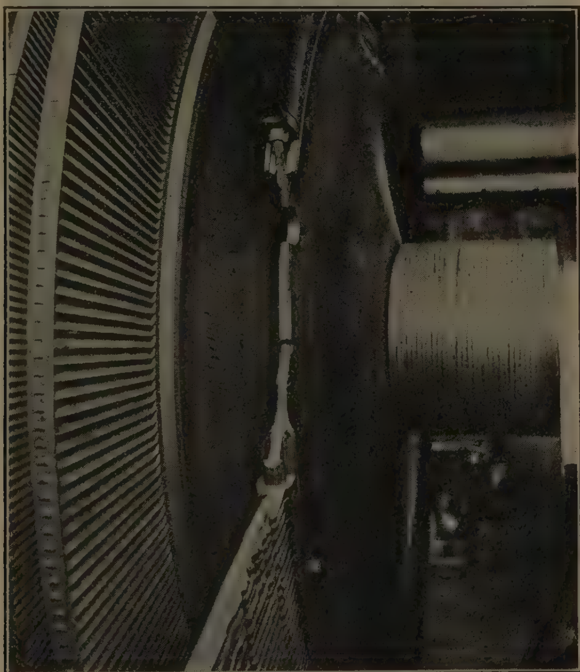

FIG. 2 DETAIL OF FRACTURE SHOWING CHARACTERISTIC FATIGUE FAILURE

6 That the methods developed are effective is attested by the successful elimination of all serious wheel and bucket troubles due to lateral vibration from the operation of recently built turbines.

PART I — HISTORICAL OUTLINE

7 Before discussing in detail the nature of the vibrations to which disk wheels are subject, a brief narrative of the work of investigation will be presented. This seems necessary in order to give a proper perspective of the problem as a whole.

Growth of Capacity

8 The design of turbines with one row of buckets per wheel took place long prior to the entry of America into the war. These designs used higher linear bucket velocities and were produced in unprecedented quantities during the war period. The rapid increases of turbine capacity which took place at the same time

Fig. 3 Fatigue Crack Which Started at a Hole

were in a large degree accomplished by the use of the larger diameters introduced to give the greater bucket speeds, and by using longer buckets. The mechanical possibilities were pushed to the limit. While some improvements in steam conditions came at the same time, the important thing to note is that the real period of increased capacity due to improved thermal processes

occurred later and is still going on whereas, almost at one leap, the early designs were pushed to the limit from the point of view of structural strength.

9 Several principles of design pointed in the direction of light wheels. The maximum wheel stress is at the bore and this could be reduced by using lighter, thinner disks having less centrifugal bursting tendency. In fact, these were cut down in thickness as much as could be without creating a new maximum in the web due to the pull of the buckets. And not only did the desire

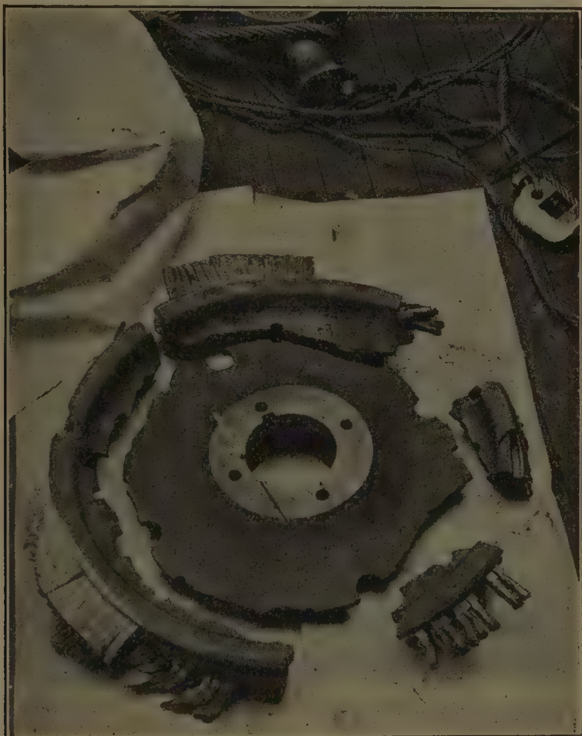

Fig. 4 Small Turbine Wheel Broken by Accidental Overspeeding. 2nd Stage of 500-Kw. 3600-R.p.m. 3-Stage Turbine

for conservative stresses point toward light wheels, but also the desire for a stiff rotor. Heavy wheels are accompanied by lower critical shaft speeds. The unquestioned advantage of a stiff shaft, when possible, also dictated light wheels. These various influences were perfectly natural at the time. The subject of vibration had not been brought to prominence. It was hazy and uncertain and no difficulties had been definitely connected with it. The plain path of reason seemed to be along the lines indicated.

Types of Failures

10 In order to visualize the sort of difficulty which led to the present investigation, it will be well to examine a number of failures.

11 Figs. 1 and 2 show a break which originated in a small tapped hole and passed through a large steam balance hole. This was a vibration fatigue failure. An examination of the fractured surface shows the characteristic central line and progressive curves.

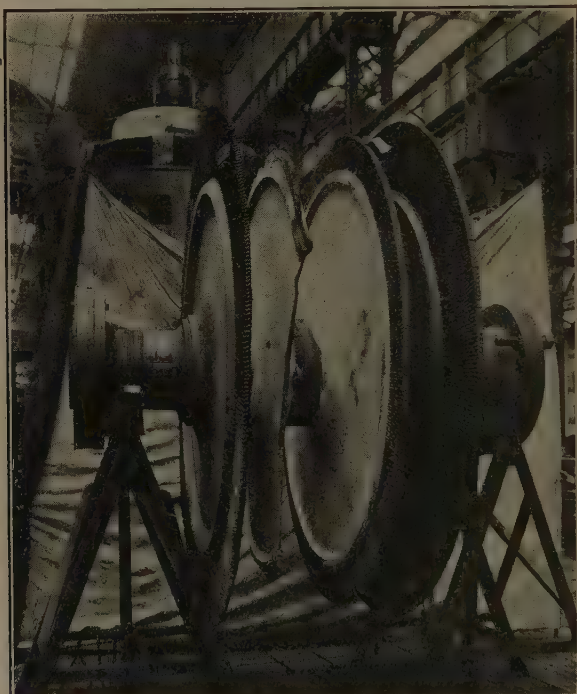

Fig. 5 Fatigue Failure Which Did Not Pass Through Holes. 11th Stage of 30,000-Kw. 1500-R.p.m. 12-Stage Turbine

Fig. 3 shows another crack discovered before complete rupture in the same wheel.

12 Fig. 4 shows a wheel which completely burst. Note that the line of fracture has passed through every one of the holes in the web of the wheel. This was due to accidental overspeed.

13 Figs. 5 and 6 show a typical fatigue fracture which did not originate at a hole.

14 Fig. 7 shows a break in which cracks originated at more than one hole. Figs. 8 and 9 show details of this fracture at each of the two holes. Although Fig. 7 bears a resemblance in its

completeness of failure to Fig. 4, the type of fracture is entirely different and is characteristic of a fatigue failure.

15 In other cases the only damage was loss of buckets. Fig. 10 shows a wheel from which a number of buckets have been broken. In this case the breaks were due to axial vibration.

16 In Fig. 11 various failures occurring in the bucket dovetail are shown. In each case the marking indicates vibration in an axial direction.

Fig. 6 Detail Showing Characteristic Fatigue Failure

17 Fig. 12 shows another class of turbine trouble in which a diaphragm has been scored in two diametrically opposite spots by a rubbing wheel. Fig. 13 is a section of the same diaphragm at the point of greatest rubbing together with a profile of the wheel showing the shape taken by all of the buckets.

18 These examples will serve to illustrate the various types of failure. Most of them were plainly due to vibration resulting in fatigue. A few were clearly the result of accidental overspeed such as Fig. 4. In certain cases the fractures avoided holes, but in general there was a distinct affinity for holes owing undoubtedly to the higher localized stresses around them.

Hole Stresses

19 The first remedial measure used was to give immediate attention to the localized stresses in the neighborhoods of the steam balance holes and other discontinuities in the webs of the wheels.

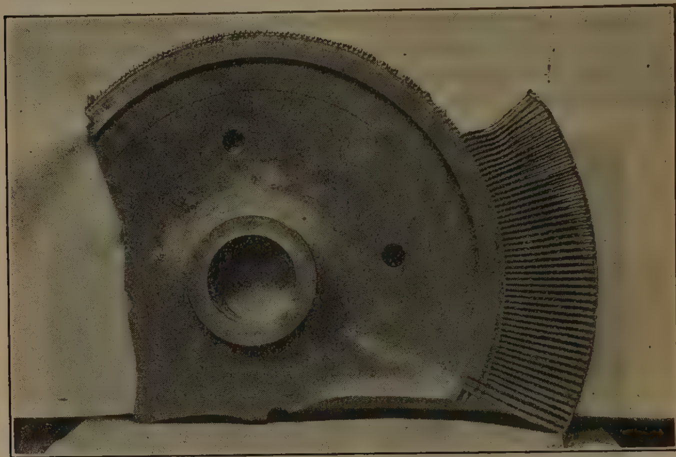

Fig. 7 Turbine Wheel Failure as a Result of Fatigue Bending. 3d Stage of 6000-Hp. Turbine

20 It is not the object of this paper to discuss hole stresses, but it is necessary to note that they become serious only in connection with vibration, by constituting a place for a fatigue crack to start. If a wheel is properly protected from vibration, there will be no repeated stresses at any point. Rather elaborate experiments have shown that the reinforcement of holes will serve to increase

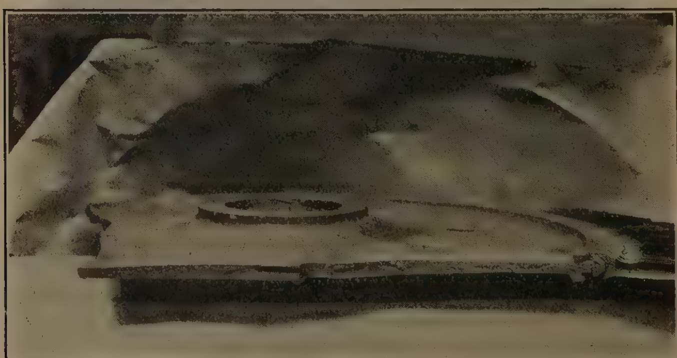

Fig. 8 Detail Showing How the Crack Started at One Hole

the resistance of a wheel to vibration in case it should be necessary to design with the expectation of fatigue stresses. But the greater safety lies in guarding against the stresses themselves by the precautionary measures developed for use in manufacture.

Bucket Lacing

21 The early experiments on the vibration of buckets indicated that a lacing wire paralleling the shroud band and connecting

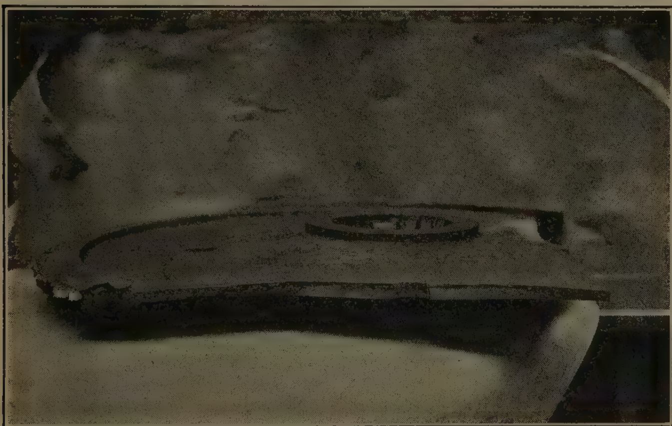

Fig. 9 Detail Showing How the Crack Started at Another Hole

Fig. 10 Roots of Buckets Broken by Fatigue Bending Due to Vibration. 12th Stage of 10,000-Kw. 1500-R.p.m. 12-Stage Turbine

different groups of buckets would remove various secondary vibrations. Although the fundamental period was not much influenced this expedient was actually used in a number of turbines. The

lacing wire, however, failed to remove the cause of the trouble and became itself another hazard. During these experiments a taut wire attached to the end of each bucket raised the frequency, thus illustrating the similar effect due to centrifugal force.

The Idea of Wave Motion

22 About the same time certain types of vibration of standing wheels were investigated by means of sand pictures. The usual

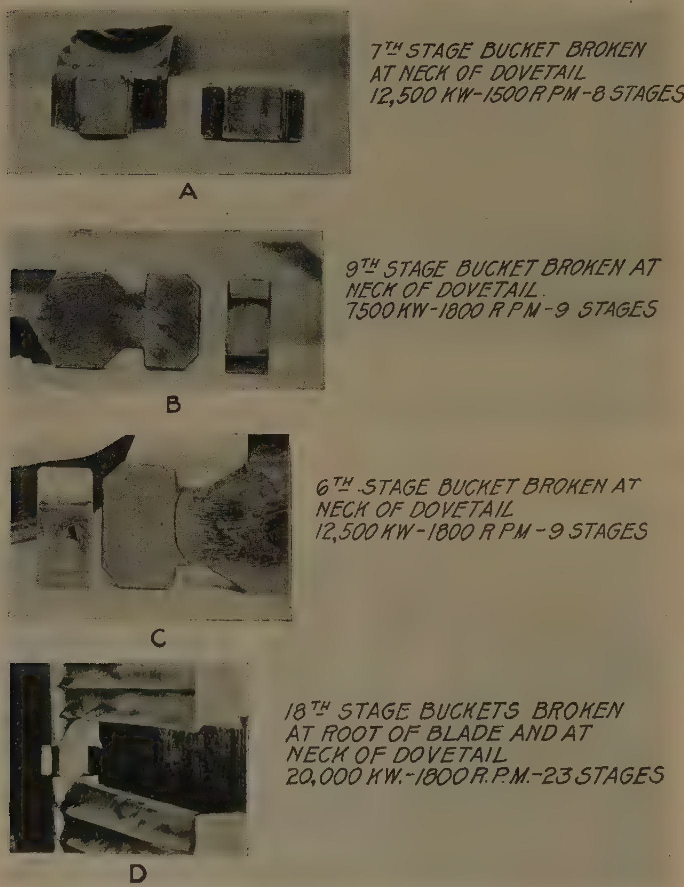

Fig. 11 Details of Bucket Failure Due to Vibration

form was a series of vibrating segments symmetrically arranged about the circumference and extending into the web but separated by radial lines of quiet called nodal radii or nodes. These will be discussed in detail presently.

Windage Theory

23 In connection with the study of strains about holes in wheel webs, a series of india-rubber wheels as shown in Fig. 14 had been made and photographed by means of instantaneous electric sparks while rotating at high speed. In order to produce representative stresses throughout the rubber disks, metal weights were attached about the circumference, shaped to simulate the loading due to turbine bucket blades.

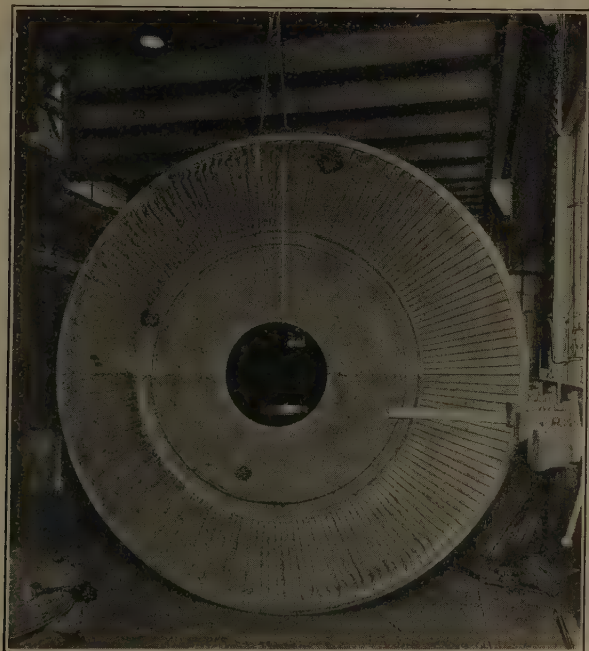

Fig. 12 Turbine Diaphragm Which Has Suffered Rubbing in Two Diametrically Opposite Regions. 17th Stage of 30,000-Kw. 1800-R.p.m. 17-Stage Turbine

24 In running these wheels it was discovered that above certain definite speeds their circumferences developed a form of wave motion as shown in Fig. 15. This was examined by means of an intermittent spark either synchronized with the speed of rotation or adjusted to occur at slightly greater or less frequencies. The shapes of the waves and their rates of progress were thus examined visually, and it was found that these waves progressed around the wheel in the direction of rotation, but at a less speed than the speed of the wheel; that is, relative to the wheel itself the wave was traveling backward, seemingly driven by the windage encountered, like the fluttering of a flag.

25 This gave rise to the so-called windage theory that the waves were developed and driven backward in the wheel by the atmosphere in which it was revolving. Investigation of this theory led to the rotation of paper disks in a vacuum. The wave motions, clearly observable in an atmosphere, disappeared as the atmosphere became rarified. These experiments were immediately extended to exceedingly thin steel disks which were found to behave in a similar manner. Since the waves could not exceed the speed of the actuating wind without encountering resistance, they could only be supported in wheels whose natural frequencies corresponded to waves traveling at less speed than the speed of rotation. It was therefore made a condition of design of turbine wheels that the natural wave speed should exceed the speed of rotation wherever possible. This resulted in a general thickening of all wheels being designed for turbines constructed at that time so as to give them a greater rigidity to withstand the supposedly detrimental effect of the wind action.

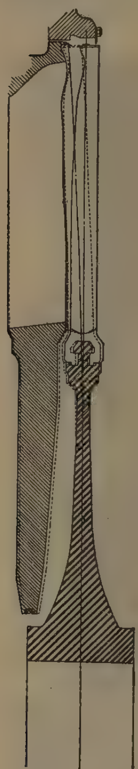

Fig. 13

Showing Depth of Rubbing at Worst Parts of Fig. 12

26 Subsequently, oscillograph coils were placed in two large turbines during operation and complete survey of these machines showed that several stages developed wave phenomena of the same general characteristics. Turbine wheels were thus shown capable of supporting traveling waves. In these turbines the waves appeared only when the turbine carried more than a certain definite load and died out when the load was removed. However, waves were found to be traveling in the wheel in a direction opposite to its rotation and at a higher speed, which could not be explained by the windage theory.

Wave-Phenomena Records Made from a Thin Steel Disk

27 The first oscillograms taken in which a revolving coil was used were made with a thin sheet-metal disk. Fig. 16 is typical of the type of oscillograph record obtained. The upper curve A is taken by a coil stationary in space and opposite the rim of the disk; it shows that there is a wave motion in the disk. The smoother portion of the curve corresponds to the part where the wheel disk is most remote from the coil and the more disturbed portion indicates that the wheel rim is in close proximity. These more disturbed portions, which may be called beats, occur at a

much slower rate than the speed of rotation so that the corresponding wave crest producing them must have progressed around in the disk itself.

28 Curve B is taken by a coil made to revolve with the wheel but mounted on a separate arm. When this coil passed near the supporting pillow block a voltage was induced which made the long narrow lines in the curve, there being one of these for each revolution. The V-shaped points in this record show that a transverse motion is taking place in the wheel disk, resulting in

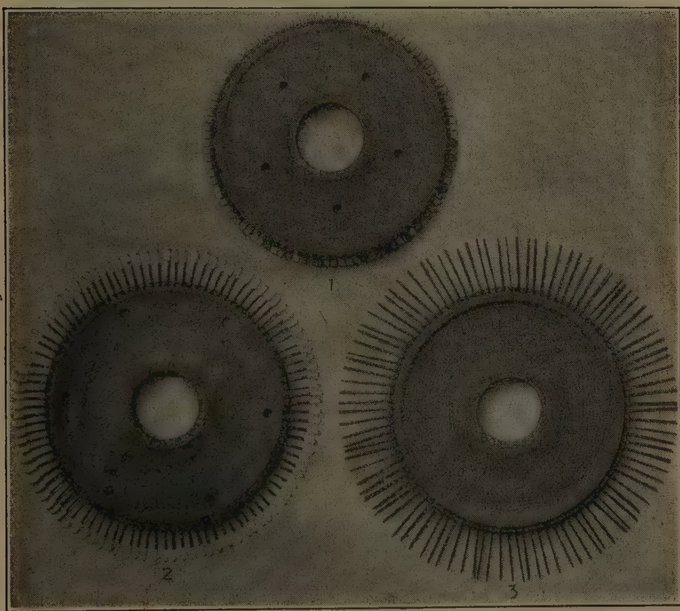

Fig. 14 India-Rubber Wheels Used to Examine the Strains about Holes in Web

change of clearance between the disk and the revolving coil. It can be shown that this is not in any sense synchronous with revolutions but simply records a transverse vibration of the wheel itself.

29 Curve C is taken from the 40-cycle a.c. line and is here used as a timing wave. The small interruptions in this wave are made by a contact on the shaft carrying the model steel disk and they correspond, therefore, to revolutions of the disk.

30 The record shown in Fig. 17 was made by a thin steel disk revolving slowly. This disk was vibrated by means of an alternating-current magnet carried on a rotating arm at the same

speed. The upper record A was obtained by a fixed coil while the lower record B is a 40-cycle timing wave. The irregularities in the record obtained by the stationary coil are caused by the passage of the rotating exciting magnet of which, it will be noted, the electrical frequency is one-half the mechanical frequency. This record, briefly, shows wheel vibration of the 6-node type including both forward and backward waves, which will be explained in the next part of this paper. This record is reproduced in the 5th edition of Stodola's Steam Turbines.

Recognition of Wheel Critical Speeds

31 Subsequently Table 2 was compiled which definitely established the importance of the wave stationary in space, that is, the wave progressing backward in the wheel at the speed at which the wheel rotates forward. This speed is called a *wheel critical speed.*

TABLE 2 SUMMARY OF WHEEL AND BUCKET TROUBLES

Rating	Stage	Year trouble occurred	Trouble	Nodes	Backward speed of wave r.p.s.	Operating speed r.p.s.
35000-1500-20	19	1918	Wheel	4	25.6	25
15000-1800-9	9	1918	Wheel	4	27.9	30
15000-1800-9	9	1919	Wheel	4	29.5	30
15000-1800-7	3	1920	Wheel	6	28.5	30
3000-Variable-4	3	1921	Wheel	6	48.1	48
30000-1500-12	11	1921	Wheel	4	25.2	25
20000-1800-12	10	1917, 1919, 1920	Buckets	8	30.2	30
30000-1800-17	13	1918	Buckets	8	30.1	30
30000-1800-17	17	1918	Buckets	4	32.6	30
45000-1200-21	21	1918	Buckets	4	19.8	20
7500-1500-8	2	1919	Buckets	6	25.4	25
5000-3600-5	2	1919	Buckets	4	57.4	60
30000-1800-17	11	1920	Buckets	8	29.6	30
30000-1800-17	12	1920	Buckets	6	28.8	30
15000-1800-23	21	1920	Dovetail	4	29.5	30
10000-1800-9	8	1921	Buckets	8	30.6	30
35000-1500-22	17	1921	Buckets	6	24.2	25
30000-1500-20	14	1921	Buckets	8	25.9	25

In making a statement of the importance of this phenomenon, it should not be assumed that it is the only type of wheel vibration of a serious nature. This type of vibration has caused by far the largest number of failures, in fact, so large a fraction that breaks caused by other types of vibration may fairly be treated as exceptional.

32 At this time the importance of obtaining test data on full-sized wheels under actual operating conditions was first fully appreciated. This resulted in the design and construction of the first wheel-testing machine.

PART II — EXPOSITION OF THE NATURE AND THEORY OF VIBRATION IN TURBINE WHEELS

33 To illustrate standing vibrations in turbine disk wheels, the following method was used. A turbine wheel was mounted in a horizontal position on a stub shaft. An electromagnet was clamped with its poles close to the edge of the wheel. On passing an alternating current through the coils of this magnet a series of pulls

FIG. 15 PHOTOGRAPH BY AN INSTANTANEOUS SPARK OF AN INDIA-RUBBER WHEEL EXHIBITING WAVE PHENOMENA AT A SPEED OF 650 R.P.M.

was exerted on the wheel tending to deflect it in a direction transverse to the plane of the disk. The frequency of these pulls is twice the frequency of the alternating current used because every complete electric cycle corresponds to two current pulsations in the magnet, and an electromagnet exerts a pull when current flows in either direction through the coil. The alternating-current generator was driven by a variable-speed direct-current motor by means of which the frequency of the magnet pull could be varied over a wide range.

Sand Pictures

34 Sand was scattered over the wheel surface and the frequency of the magnetic pulls was varied until a particular frequency was reached at which the wheel responded. Fig. 18 shows a case where the wheel vibrated in four segments. In this

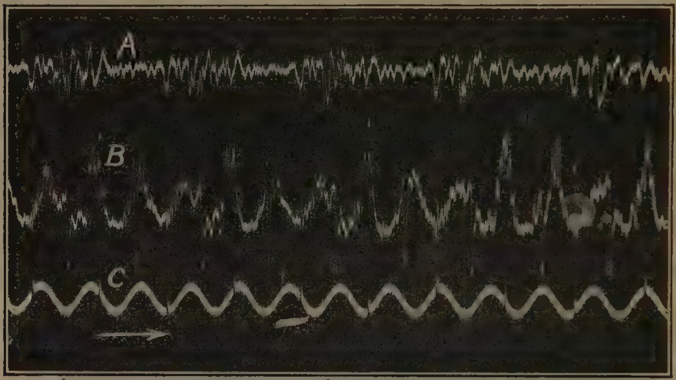

FIG. 16 ONE OF THE FIRST OSCILLOGRAMS RECORDING DISK VIBRATION

(Trace *A* was recorded by a stationary coil, Trace *B* by a coil revolving with the disk, and trace *C* is a 40-cycle timing wave. Model disk wheel of thin sheet metal.)

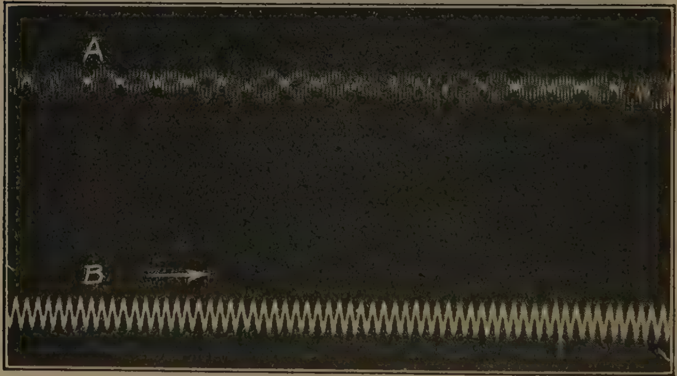

FIG. 17 RECORD OF 6-NODE VIBRATION IN A THIN STEEL DISK EXCITED BY AN A. C. MAGNET REVOLVING WITH THE WHEEL

(Trace *A* was recorded by a stationary coil. Trace *B* is a 40-cycle timing wave.)

vibration each segment springs up and down, scattering the sand over to the quiet or nodal zones where there is no up-and-down motion. If, however, the frequency of the deflecting pulls of the magnet is altered even a very small amount the vibration immediately dies out, although the magnitude of the impulses of the

magnet remains the same as before. On raising the frequency of the magnetic pulls another point is found at which the wheel responds. It vibrates in segments, as before, but with 6 nodal radii or nodes equally spaced around the wheel circumference instead of four.

35 Figs. 19 and 20 illustrate a 6-node vibration and show that its location is not necessarily dependent on the position of a series of symmetrical discontinuities such as the steam balance

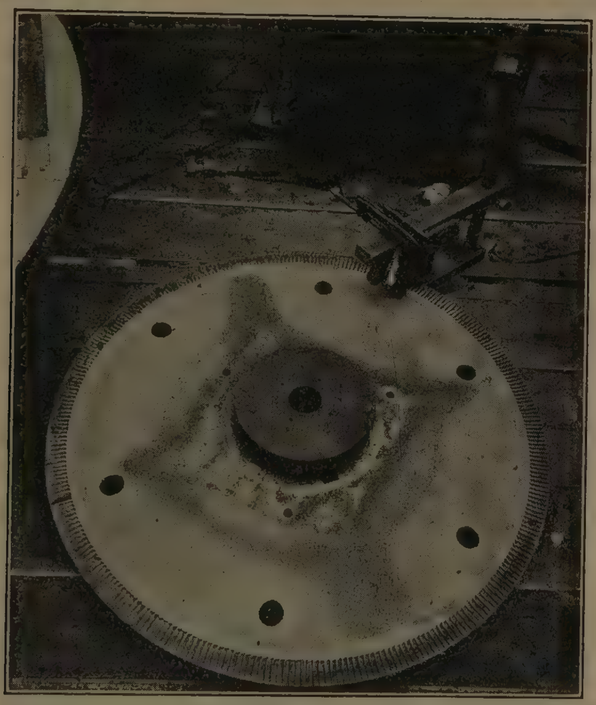

FIG. 18 4-NODE SAND PICTURE MADE BY VIBRATION OF A WHEEL WITH SHORT BUCKETS

holes. Not only does the disk wheel respond when the pull frequency corresponds to four or to six radial nodes, but it may respond readily to frequencies corresponding to 8, 10, 12, or even a larger number of nodes, the number of nodes always being even because for every segment which springs upward during a vibration, the segment next to it on the other side of a nodal line must spring downward. These photographs illustrate the case of a small wheel with short buckets.

36 Figs. 21, 22, 23 and 24 show cases of 4, 6, 8 and 10 nodal vibrations for the case of a disk wheel carrying long buckets, the

total diameter of wheel and buckets being over 8 ft. This wheel was photographed with a layer of paper on the buckets to hold the sand. In the cases of four and six nodes it is seen that the regions of amplitude large enough to move the sand do not extend so deeply into the wheel as in the wheel with short buckets, while in the cases of eight and ten nodes the sand figure is confined to the bucket zone entirely.

37 The following general observations may be made on this type of vibration in which segments around the edge of the wheel

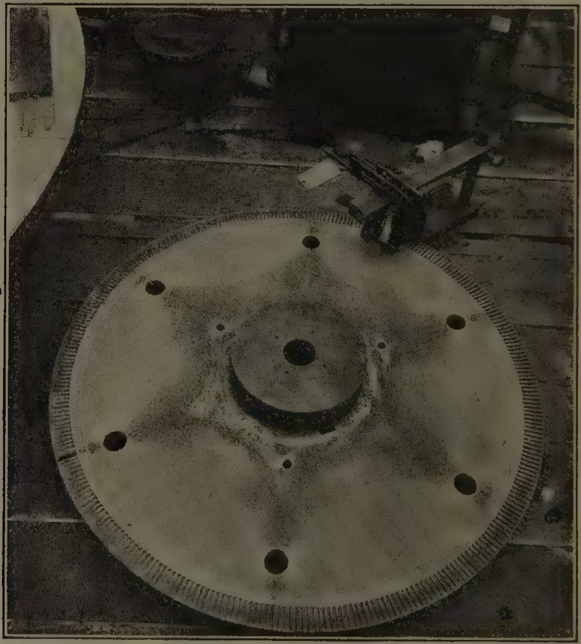

Fig. 19 6-Node Sand Picture Made by Vibration of a Wheel with Short Buckets

spring up and down, being separated from each other by radial nodal lines:

1 Every disk wheel responds readily to vibrations of four, six, eight, etc. radial nodes, each type of vibration having its own characteristic frequency.

2 The higher the number of nodes the higher the frequency of the vibration and the less easily is the vibration excited.

3 The higher the number of nodes the more difficult it is to force the sand figures towards the center of the disk.

4 Both the disk wheel and the buckets vibrate together as a continuous disk and must be treated as a unit in this type of vibration.

38 Vibrations may also take place with two nodes, as will subsequently be discussed. This type exerts a couple on the shaft transverse to its length, while the types described are balanced in their reactions on the shaft.

39 Many other types of vibration exist, including concentric ring nodes and combinations of ring and radial nodes. A hybrid

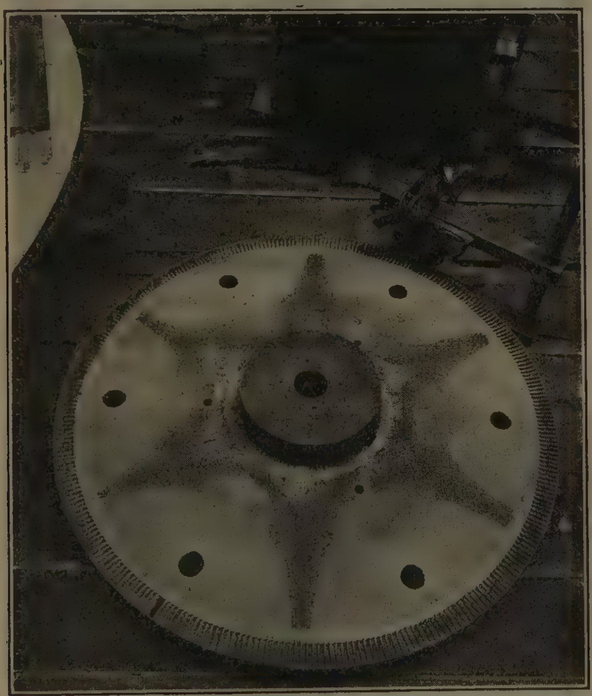

Fig. 20 6-Node Sand Picture Showing Independence of Pattern from Hole Location. Compare with Fig. 19

form resulting from a combination of six- and twelve-node radial types is shown in Fig. 25. These types of vibration are not readily excited and do not enter into this discussion, because they have not been found to be the cause of serious trouble.

Effect of Centrifugal Force on Vibration Frequency

40 After the natural vibration frequencies of a turbine disk wheel when not rotating are determined as described, a question which arises is the effect upon these vibration frequencies of

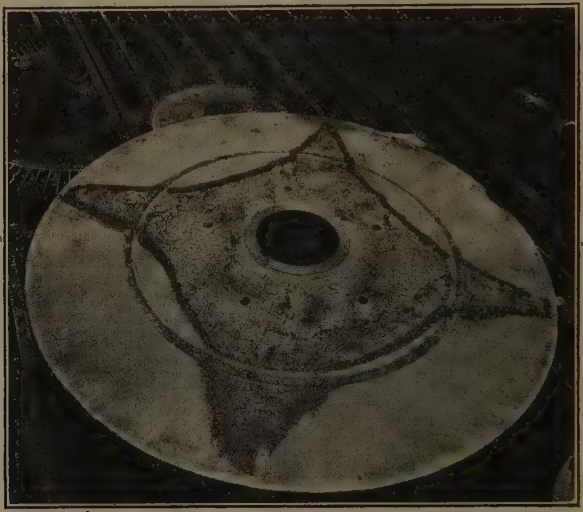

FIG. 21 4-NODE SAND PICTURE MADE BY VIBRATION OF A TURBINE WHEEL WITH LONG BUCKETS COVERED WITH PAPER. THE ACTIVE REGION EXTENDS INTO THE WHEEL

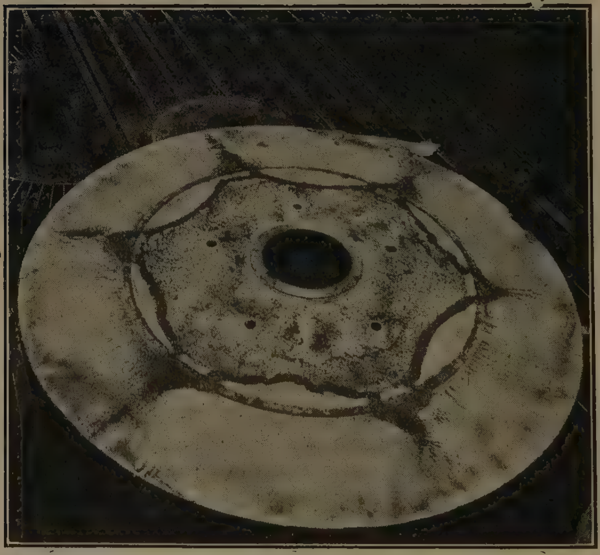

FIG. 22 6-NODE SAND PICTURE MADE BY VIBRATION OF A TURBINE WHEEL WITH LONG BUCKETS COVERED WITH PAPER

FIG. 23 8-NODE SAND PICTURE MADE BY VIBRATION OF A TURBINE WHEEL WITH LONG BUCKETS COVERED WITH PAPER

FIG. 24 10-NODE SAND PICTURE MADE BY VIBRATION OF A TURBINE WHEEL WITH LONG BUCKETS COVERED WITH PAPER. THE ACTIVE REGION IS CONFINED TO THE BUCKETS

rotation of the wheel at high speed. The frequency of a given type of vibration is determined by two factors, (*a*) the stiffness and (*b*) the mass of the vibrating body. The stiffer the body the faster it vibrates, and the more massive it is the slower will it vibrate. Now centrifugal force has no effect on the mass of the wheel, but it has a powerful stiffening effect. This force acting radially outward around the edge of the wheel stiffens it and raises its vibration frequency. This may be compared to the raising of the vibration frequency of a kettle drum by drawing the membrane outward around the edges by the tightening screws. Therefore it may be inferred that centrifugal force raises the natural vibration frequencies of a turbine disk wheel.

41 It is well known that a particle of mass m with an elastic support of such stiffness that a force R_s, required to produce unit

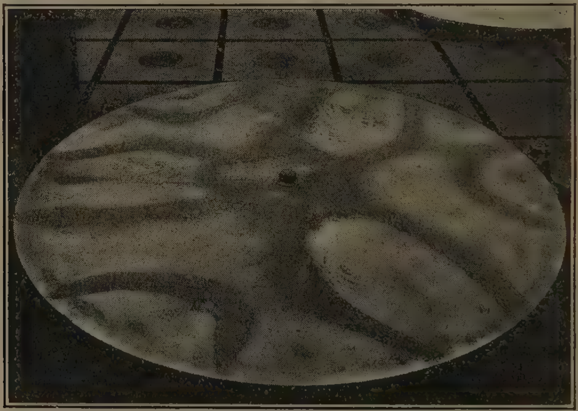

Fig. 25 Complex Sand Picture with 12 Nodes at the Edge and 6 Nodes Near the Center made by Vibration of a Thin Steel Plate. This is a Rare Type of Motion.

deflection, will have a natural frequency of vibration, f_s, expressed by

$$f_s = \frac{1}{2\pi}\sqrt{\frac{R_s}{m}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [1]$$

42 If the same particle is supported in another manner with an elastic factor R_c, its new frequency will be

$$f_c = \frac{1}{2\pi}\sqrt{\frac{R_c}{m}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [2]$$

43 Now when both stiffnesses act at once the frequency will be

$$f_r = \frac{1}{2\pi}\sqrt{\frac{R_s+R_c}{m}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [3]$$

44 Suppose R_s to represent the stiffness furnished by elastic supports and R_c the stiffness contributed by centrifugal effects. Assuming the latter proportional to the square of the speed, N_s, in revolutions per second, this proportionality may be expressed by the use of an arbitrary coefficient B defined by the relation

$$R_c = B(4\pi^2 m N_s^2) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [4]$$

45 Making use of this relation and eliminating R_s by the use of Equation [1] the frequency of the particle, f_r, due to the combined effects of stiffness and rotation may be written

$$f_r = \sqrt{f_s^2 + BN_s^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [5]$$

46 This formula, here derived for the case of a particle, has been justified many hundreds of times for use with a complete turbine bucket wheel by actual measurement as described in later sections of this paper. Stodola[1] arrived at the same conclusion on theoretical grounds.

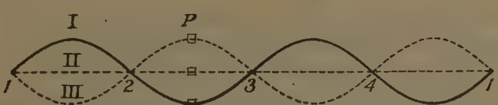

Fig. 26 4-Node Standing Vibration. The Figure Represents the Developed Edge of the Wheel During Three Successive Phases

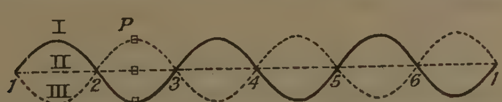

Fig. 27 6-Node Standing Vibration. The Figure Represents the Developed Edge of the Wheel During Three Successive Phases

47 B is the speed coefficient which varies with the design of the wheel and the type of vibration. If the vibrating sectors extend a considerable distance into the wheel so that the deflection curve extends well toward the wheel center, B has a lower value than when most of the bending of the wheel is near its edge, as in the case of a larger number of nodes. The value of the speed coefficient is generally from 2 to 3, and a coefficient as small as unity is rare.

Traveling Waves

48 Thus far disk-wheel vibrations with radial nodes and the effect of centrifugal force on these vibrations have been discussed in some detail. The type of vibration which has been found to be responsible for serious wheel failures will now be taken up. This type of vibration results when, instead of the wheels vibrating in

[1] *Schweitzerische Bauzeitung*, May, 1914.

segments with stationary radial nodes, a wave train travels around the wheel circumference.

49 Before considering traveling waves, a diagrammatic representation of radial nodal vibrations of a turbine wheel will be presented. Fig. 26 represents diagrammatically the edge of a turbine disk wheel, and shows the curves assumed by it when the wheel is vibrating with four nodes. The drawing shows the edge of the wheel developed as though all points along the entire circumference could be seen at once. Evidently the two ends of each curve correspond to the same point on the wheel and are, therefore, numbered identically.

50 Curves I, II, and III show three successive stages one-quarter of a complete period apart. The point on the wheel edge marked P is chosen half-way between nodal points and vibrates through the maximum amplitude. The points 1, 2, 3, etc. remain stationary as they lie in the quiet nodal radii between the vibrat-

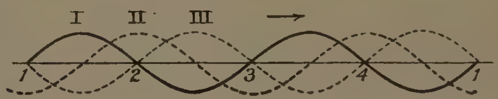

Fig. 28 4-Node Traveling Wave. The Figure Represents the Developed Edge of the Wheel During Three Successive Phases

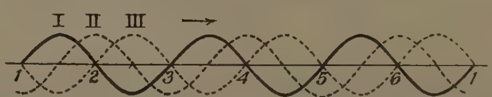

Fig. 29 6-Node Traveling Wave. The Figure Represents the Developed Edge of the Wheel During Three Successive Phases

ing segments. First the wheel edge is bent as shown by the full curve I; one-quarter of a period later the edge becomes straight as shown in curve II, but the portions between nodal points have a rapid motion which carries them over to the maximum deflection in the opposite direction in curve III, one-quarter of a vibration period later, or one-half a period from the initial position. At the end of a full period, the shape evidently is again the same as it was to start with as indicated by the full line curve I. Fig. 27 shows the same sequence for a 6-node vibration.

51 Fig. 28 shows the developed edge of a wheel in a similar manner and represents the case of 4-node traveling waves instead of standing vibrations. The difference between the case of traveling waves and standing vibrations is seen to be that the nodal points 1, 2, 3, etc. move along the edge of the wheel instead of remaining at fixed points.

52 Curve II shows the wheel shape after the nodal points 1, 2, etc. have moved one-quarter of a wave length to the right, and III shows the shape after another one-quarter wave-length

motion where the nodes have moved to the right one-half a wave length in all. At this instant the shape of the wheel is the same as for the case of the standing vibrations previously considered. The difference lies in the motion only. In the case of the standing vibration the nodes are stationary. In the case of the traveling waves the nodes are moving to the right. Fig. 29 shows the same sequence for a six-node vibration.

Comparisons between Standing Vibrations and Traveling Waves

53 The following comparisons may be made between the standing vibrations and the corresponding traveling waves for a given disk wheel:

a In each case there must be an even number of nodes, that is, for every upward portion of the deflection curve there is a corresponding downward portion because of the continuity of the circumference.

b In standing vibrations the nodes are stationary in the wheel; in traveling waves they move around it. In the first case we have true nodes in the sense that they represent parts of the wheel which are always quiet so they may be observed by the eye. In the second case we have traveling nodes; every part of the wheel edge vibrates and no quiet zones can be seen. A rapidly moving traveling wave can only be seen by the eye by means of instantaneous illumination.

c The *frequency* of vibration of every particle along the edge of a given disk wheel is the same *either for a case of standing vibration or for traveling waves,* provided the number of nodes is the same. This important point will presently be explained. A knowledge of it is requisite to the determination of the velocity of a traveling wave from the standing vibration frequency. For instance, turning to Figs. 26 and 28, it is seen that if the vibration frequencies of each point on the rim are the same in each case, the traveling wave must move to the right one whole wave length, while the standing vibration goes through one complete cycle. Thus *the speed of a traveling wave per second equals the number of complete vibrations of the corresponding standing wave per second multiplied by the length of a complete wave.*

d For the standing vibration the *amplitude* of the particles varies along the edge of the disk from zero at the nodal points to the maximum vibration amplitude at points half-way between the nodes. For the traveling waves, all particles around the edge of the wheel vibrate in turn through the same amplitude.

e For a standing vibration all of the particles along the edge of the wheel vibrate in the same time phase, that is, all particles vibrate together so that each comes to rest at the same instant and each has its maximum velocity of motion at the same instant

during the vibratory motion. For traveling waves, however, the particles along the wheel edge do not vibrate in time phase but vibrate one after another in turn, successively coming to rest and successively acquiring their maximum velocity of motion during vibration. Since they all vibrate one after another through

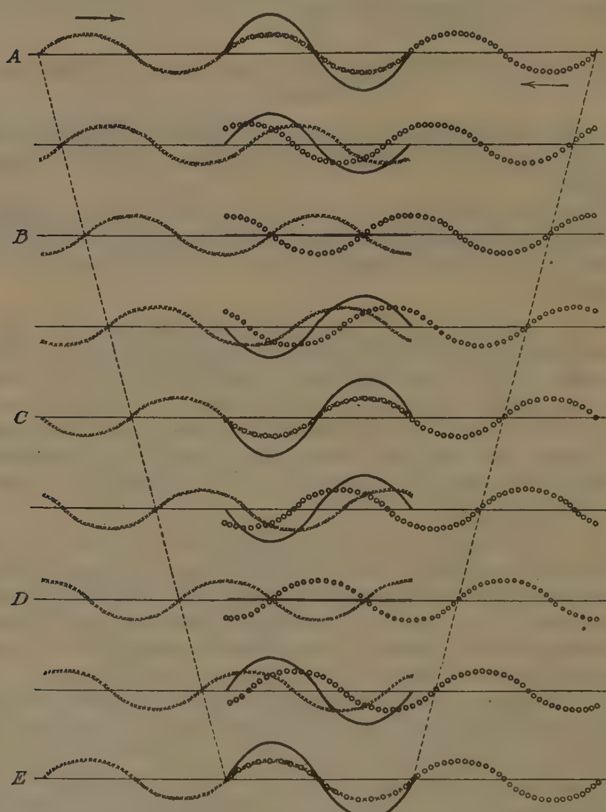

Fig. 30 Composition of Two Equal Waves Traveling in Opposite Directions to Form a Standing Vibration

the same amplitude a wave shape results of constant amplitude traveling around the wheel edge.

54 To sum up the last two paragraphs: for a standing vibration, the particles along the edge of a wheel all vibrate in the same time phase, but their amplitudes vary successively between nodes; for a traveling wave all particles vibrate through the same amplitude, but their time phases vary successively along the wheel edge.

Relations between Standing Vibrations and Traveling Waves

55 A well-established principle of wave motion is that *standing vibrations with stationary nodes result from the superposition of two identical wave trains traveling in opposite directions, each of which has an amplitude equal to half that of the resulting standing vibration.* A familiar illustration of this principle is observed when two stones are thrown on the surface of a pond giving rise to two outspreading wave trains. Midway between the two stones the two identical wave trains approaching from opposite directions are superposed upon each other. There results in this region a series of standing vibrations of the surface of the pond with stationary nodal points between them. The particles of water vibrate up and down with the same frequency for the standing vibrations as for each of the wave trains of which these vibrations are composed.

56 This illustration taken from mechanics has other parallels. The sound vibrations in an organ pipe with fixed nodal points are similarly explained by a combination of oppositely moving sound waves. In long-distance electric transmission lines standing vibrations with fixed nodes between them may also be produced by the combination of two oppositely moving wave trains.

57 A consideration of Fig. 30 will be useful as an illustration. *A, B, C,* and *D* show successive stages of a standing vibration for each quarter of its period, resulting from the superposition of two identical wave trains moving in opposite directions. The crosses represent the wave progressing toward the right, while the circles show a leftward-moving wave of equal amplitude. When the stage *E* is reached the cycle is completed and the deflection curve is the same as at the first stage *A*. In the stage *A* the two oppositely moving waves are exactly superposed upon each other so that they add.

58 When each traveling wave has moved one-quarter of a wave length as shown in *B* they cancel, so there is zero up or down displacement at all points, resulting in the straight line. After the second quarter of a wave length of motion both waves are again superposed so that the displacements add, but the displacements are all opposite to those shown in *A*. In stage *D* the deflections cancel again and in stage *E* after each wave train has moved a complete wave length the deflections add again, giving the original deflection curve.

59 The important point to be understood is that the natural frequency of vibration of the particles is the same for a standing vibration as for a traveling wave. It has already been explained that the frequency of a particle depends only on its mass and a stiffness factor represented by the restoring force per unit of displacement which, for isochronous vibrations, is the same for

each unit of mass throughout the entire structure. The principles of elasticity show that these proportional restoring forces, acting upon the various particles of unit mass, depend on the shape of deformation only. Since the shape is the same for either type of motion, it is seen that the vibration period of each particle is the same in either case.

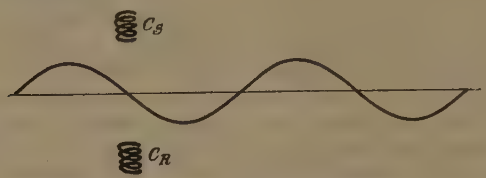

FIG. 31 DEVELOPED EDGE OF A WHEEL CARRYING A 4-NODE WAVE

(C_s is a stationary magnetic exploring coil and C_r a similar coil revolving at the same speed as the wheel.)

60 This explanation shows how the speed of a wave train in a turbine wheel with a particular number of nodes may be calculated from the standing vibration frequency of the wheel when vibrating with the same number of nodes.

WAVE SPEEDS

61 The point to be emphasized is that a wave train of a particular number of wave lengths travels around the edge of a turbine wheel at *one particular characteristic speed only*. Just as a 4-node standing vibration responds at one frequency so also the 4-node wave trains of which the standing vibration is composed must travel around the wheel at one particular speed. So also for wave trains of 6, 8, or 10 nodes, etc. Each has one particular speed with which it must travel in the disk wheel.

DETECTION OF THE PRESENCE OF TRAVELING WAVES IN A REVOLVING DISK

62 Fig. 31 represents the developed edge of a turbine wheel carrying a 4-node wave train, which moves to the right with a certain particular velocity. C_s is a small magnetic coil fixed to the stationary diaphragm so it can register the to-and-fro motions of the wheel in an oscillograph by means of the inductive effect of the wheel as it approaches and recedes from the magnetic coil during vibration. A coil C_r is attached to an arm which is carried around with the revolving disk wheel, so it also can register the vibration frequency of the revolving wheel in an oscillograph.

63 When the wheel is stationary, both coils register the same frequency. Assume that a wave travels around the wheel 25 times a second. The frequency registered by each coil would be $2 \times 25 = 50$ cycles per second, because for the case of four nodes shown the wheel carries two complete waves.

64 Assume now that the wheel is revolving at 10 r.p.s. in the direction in which the wave moves. Since the wave always has a definite speed *in the wheel,* the coil carried around *with the wheel* should register almost the same frequency as before. The frequency would be exactly the same were it not for centrifugal force, the effect of which we have already discussed. The wheel is stiffened by it so that the vibration frequencies are raised and the wave speeds are increased. The effect would not be very great at 10 r.p.s. Assume, for example, that the wave train travels around the wheel 26 times a second instead of 25 times due to this cause. The revolving coil will then register $2 \times 26 = 52$ cycles per second when the wheel is revolving 10 r.p.s.

65 On the other hand, the stationary coil now registers a higher frequency than it registered when the wheel was stationary. The wave train on the wheel is carried forwards by the wheel motion at a speed of 10 r.p.s. besides its natural speed in the wheel of 26 r.p.s., so the wave passes the fixed coil at a speed of $10 + 26 = 36$ r.p.s., and the frequency registered by this coil is $2 \times 36 = 72$, because the wheel carries a train of two waves in the case assumed. Therefore, the forward-traveling wave registers a frequency of 52 cycles per second on the moving coil and 72 cycles per second on the stationary coil, whereas when the wheel was stationary both coils registered 50 cycles per second.

66 Now consider a case where the 4-node wave train is moving backward in the wheel while the wheel is revolving at the same speed of 10 r.p.s. The effect of the centrifugal force is the same as before so the wave must travel *in the wheel* with the same speed of 26 r.p.s. as before, but in the opposite direction. The frequency recorded by the revolving coil is $2 \times 26 = 52$ cycles per second, the same as for the forward-traveling wave, since this coil records the same frequency for the same wave speed whether the wave moves past it forward or backward. The effect upon the frequency recorded by the stationary coil, however, is different. Since this is a backward-traveling wave, the forward motion of the wheel of 10 r.p.s. allows the wave to travel backward past the fixed coil with a speed of only $26 - 10 = 16$ r.p.s. In other words, the forward motion of the wheel subtracts from the backward motion of the wave as measured by the fixed coil. The frequency registered by the fixed coil is, therefore, $2 \times 16 = 32$ cycles per second for the 4-node backward wave train.

67 Again, take the case of this disk wheel carrying both wave trains simultaneously and also revolving at 10 r.p.s. For the forward wave train it will be recalled that the revolving coil registers 52 cycles per second and the fixed coil registers 72 cycles per second, while for the backward moving wave train the revolving coil again registers 52 cycles per second, and the fixed coil registers 32 cycles per second. The revolving coil registers only

one frequency of 52 for either wave train separately or for the combination, but the fixed coil registers 72 for the forward-traveling wave and 32 for the backward-traveling wave, and both 32 and 72 simultaneously for the two waves superposed, that is, for the vibration in the wheel.

68 To sum up Pars. 66 and 67, it may be said that when a disk wheel carrying a standing vibration is revolved so that the radial nodal lines are carried around with the wheel, the frequency recorded by a coil carried around with the wheel slowly rises due

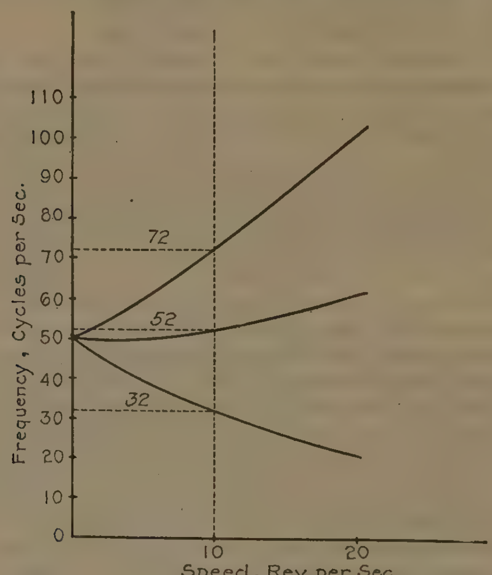

Fig. 32 Frequency-Speed Diagram for 4 Nodes

to centrifugal force as the speed of the wheel increases. The frequency recorded by a coil fixed on the diaphragm, so that it does not revolve with the wheel, registers two frequencies, the higher frequency due to the forward-moving component wave train and the lower frequency of the backward-moving component wave train. These two frequencies diverge more and more as the wheel speed is increased.

Frequency-Speed Diagram

69 These facts are shown graphically in Fig. 32 which gives a diagrammatic representation that has been found to be very useful.

70 The vertical scale represents the frequency registered in an oscillograph by the magnetic coils, and the horizontal scale the rotational speed of the disk. The middle curve gives the variation of frequency with speed as recorded by the revolving coil. The upper and lower curves show the two frequencies registered by the fixed coil, the upper giving the frequency due to the forward component wave and the lower the frequency due to the backward component wave train of the 4-node vibration. When the wheel is at rest the figure shows that both coils register 50. As the speed of the wheel is raised to 10 r.p.s. the revolving-coil frequency rises to 52 and the two frequencies recorded by the fixed coil diverge, the upper rising to 72 and the lower falling to 32.

71 The gradual rise of frequency of the wheel as its speed is increased is expressed by equation [5] previously derived

$$f_r = \sqrt{f_s^2 + BN_s^2}$$

The upper curve shows how the frequency of a forward-moving wave train, as measured at a fixed point, rises relatively to the frequency detected by the revolving coil, because this wave train is carried forwards by the wheel, and is thus passing the fixed coil at a higher speed than it would were the wheel not rotating. This rise in frequency is measured by the number of wave lengths per second that the wave train is carried forward by the wheel rotation which equals the product of the number of waves on the wheel rim, $\frac{1}{2}n$, by the number of revolutions per second of the wheel, N_s. This product $\frac{1}{2}nN_s$ is the frequency in excess of that of the wheel as measured by the revolving coil, that is, in excess of f_r. If H is the higher frequency recorded by the stationary coil and represented by the upper curve, then

$$H = f_r + \tfrac{1}{2}nN_s \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [6]$$

In the same way the lower curve shows how the frequency of the backward-traveling wave as measured at a fixed point is decreased because in this case the wave motion is opposite in direction to the motion of the wheel. Thus if M is the lower frequency recorded by the stationary coil and represented by the lower curve,

$$M = f_r - \tfrac{1}{2}nN_s \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [7]$$

72 If the backward-moving component wave train is absent, only the upper frequency is registered by the fixed coil corresponding to a forward-moving traveling wave. If the forward-moving component wave train is absent, only the lower frequency is recorded due to the backward-traveling wave. The frequency recorded by the revolving coil, however, is always the same whether one or both of the component wave trains exist and in whatever relative amplitudes they exist.

73 It is therefore evident that by the use of two exploring coils as described, one revolving with the wheel, the other being

fixed in space, the presence of a forward- or a backward-traveling wave train or both can be detected.

74 The first observation of traveling waves in a turbine wheel was made by means of fixed oscillograph coils installed in the diaphragms of an operating turbine in 1919. Early in 1920 during

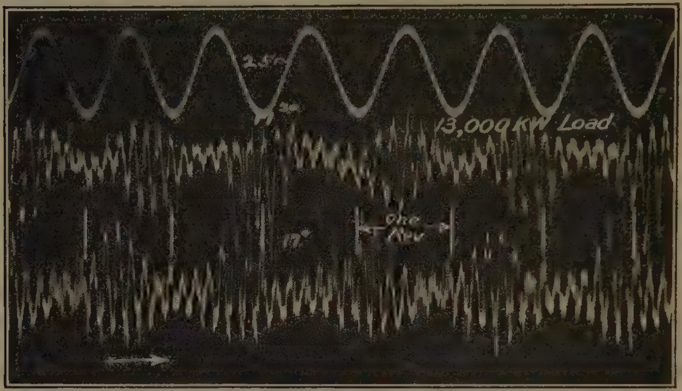

Fig. 33 Oscillograph Records made by 17th Stage of 20,000-Kw. 1500-R.P.M. 23-Stage Turbine

(The upper curve is a 25-cycle timing wave. The other two curves were made by stationary coils 30 deg. apart. This is a case of 6-node forward- and backward-traveling waves.)

the investigation of another turbine, several stages were equipped with two such coils 30 deg. apart. One of these stages yielded the records reproduced in Fig. 33.

Observation of Traveling Waves by Means of Two Fixed Coils 30 Deg. Apart

75 The upper curve of Fig. 33 is produced by the 25-cycle a.c. generator being driven by this turbine. Since the generator has two poles, it revolves once for every cycle, and this a.c. frequency curve marks off the generator revolutions on the film. The two lower oscillograph curves are the records of the two fixed coils, 30 deg. apart. Time is measured to the right. The upper of these two curves is the record of the first of the two fixed coils. The lower curve is the record of the second coil, set in the diaphragm 30 deg. beyond the former so a given point on the disk wheel reaches this coil somewhat later than the first one.

76 The records of these two coils show a close correspondence, both having a high-frequency oscillation which goes through a low-frequency pulsation in amplitude giving the effect of beats. Furthermore the upper curve lies behind the lower one by about one-quarter of the distance between the low-frequency amplitude

pulsations or beats. Since the time recorded by the amplitude pulsations is four times as great as that by which the upper record lies behind the lower, due to the 30 deg. between the coils, the most likely explanation is that there are high spots on the wheel, 4×30 deg., or 120 deg., apart, which cause this amplitude pulsation. This can be made clear from a consideration of Fig. 34.

77 The wheel revolves in a counterclockwise direction as shown by the arrow. The two coils marked 1 and 2 are 30 deg. apart,

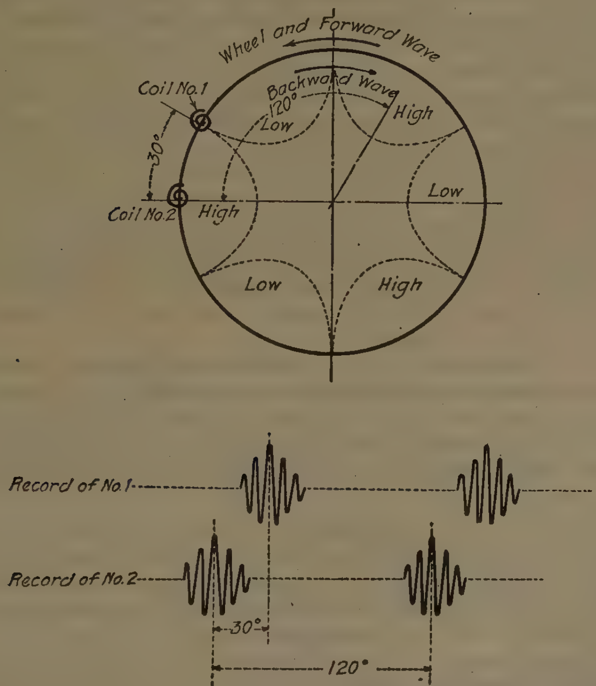

Fig. 34 Diagram Showing 6-Node Forward and Backward Waves Such as are Recorded in Fig. 33

and are fixed in space. The diagrammatic oscillograph record shows the beats recorded by the two coils, No. 1 lagging behind No. 2 by one-quarter of a period. Since it takes four times as much time for successive high spots to reach a given coil as for a given high spot to pass from one coil to the other, these high spots must be 4×30 deg., or 120 deg., apart on the revolving disk. The inference is that these high spots are wave crests corresponding to a train of three waves, 120 deg. apart and that the waves cause the disk wheel to approach and recede from the recording coils periodically. When the wheel is close to a coil

the oscillograph responds strongly, and when it recedes the oscillograph responds less strongly.

78 If there is such a wave train the question now is to determine its direction and speed. The high spots evidently move in a clockwise direction opposite to the wheel rotation, because they reach coil No. 2 before they reach coil No. 1, as shown in Fig. 33 where the record of coil No. 2 shows a time lead over that of coil No. 1. This means that the wave which causes these low period pulsations is traveling in the wheel against the direction of wheel rotation, and that this wave train travels backward even faster than the forward rotational speed of the disk wheel. Therefore, to find the backward speed of this wave train in the wheel it is necessary to add the speed with which it travels backward past the fixed coils to the forward rotational speed of the wheel. From Fig. 33 it is seen that $6\frac{1}{2}$ high spots pass a given coil for nine revolutions of the wheel, or since the wheel revolves 25 times per second, $6\,1/2 \times 25/9 = 18\,1/18$ high spots per second. Since there are three wave lengths on the wheel corresponding to the three high spots 120 deg. apart, $18\,1/18$ high spots per second corresponds to a wave speed past the coils of $18\,1/18 \div 3 = 6\,1/54$ r.p.s. The wheel revolves forwards 25 r.p.s. so the wave speed in the wheel$=25+6\,1/54=31\,1/54$ r.p.s.

79 The conclusion is thus reached that this wheel carries a train of three waves 120 deg. apart (corresponding to 6 nodes) and that this wave train moves backward in the wheel $31\,1/54$ r.p.s. which is $6\,1/54$ r.p.s. faster than the wheel revolves forward, so the wave passes the fixed coils $6\,1/54$ r.p.s. in a backward direction.

80 Thus far there has been no attempt to explain the cause of the superposed higher frequency which shows strongly in the record of Fig. 33. This higher frequency is found to have $60\,1/2$ periods while the wheel revolves 9 times, or a frequency of $60\,1/2 \times 25/9 = 168\,1/18$ cycles per second as measured on the fixed coils, since the disk wheel revolves 25 times a second. This frequency is due to a forward-moving wave train of exactly the same type as the backward-moving wave train, that is, a train of three waves 120 deg. apart, or a 6-node wave train. If such a wave train registers a frequency of $168\,1/18$ cycles per second on the fixed coils, its speed past these coils in r.p.s. must be $168\,1/18 \div 3 = 56\,1/54$ r.p.s., because there are three wave lengths on the wheel rim. Since this wave train is assumed to move forward and the wheel is also moving forward, its speed *in the wheel* must be less than that registered by the fixed coil by an amount equal to the wheel speed, that is, 25 r.p.s., because the wave train is carried forwards by the wheel rotation. This gives a wave speed in the wheel of $31\,1/54$ r.p.s. But this is exactly the characteristic speed of a train of three waves as it checks with the speed of the backward-traveling wave train of this type

already found. Par. 61 showed that a given type of wave has a definite speed in a disk wheel which revolves at a given speed, and that this wave speed is the same whether the wave train travels forward or backward in the wheel. This coincidence is therefore striking evidence of the truth of the statement that the high frequency registered was caused by a forward-moving wave train of six nodes, that is, of the same type as the backward-moving wave train.

81 There is further evidence that the wave train recording the higher frequency is moving forward and the wave crests are

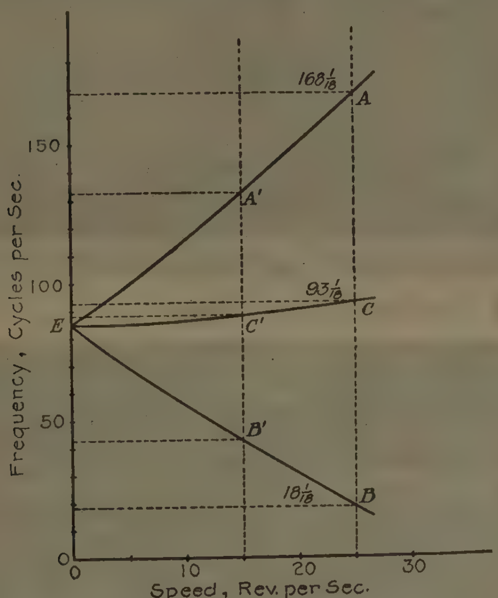

Fig. 35 Frequency-Speed Diagram for 6 Nodes. Points A, B and C Were Determined from the Record in Fig. 33

120 deg. apart as in the case of the backward waves. From a close examination of the film record of Fig. 33 it will be seen that the higher frequency of coil No. 1 leads that of the lower record by a fraction of a period. (The polarity of the two coils happens to be opposite in this record.) This means that the disturbance producing this harmonic moves *forward* because it reaches coil No. 1 before it reaches coil No. 2. Furthermore this lead is as before about one-quarter of a complete period. This again corresponds to waves which are 4×30 deg., or 120 deg. apart.

82 As to the relative amplitudes of these two wave trains, a casual inspection of the film Fig. 33 might lead one to believe that the amplitude of the forward wave train producing the

higher frequency was as great as that of the backward wave train. It is necessary to keep in mind, however, that in an oscillograph record the amplitude is dependent on the induced voltage which in turn depends on both the amplitude and the frequency of the vibration so that higher frequencies have amplitudes recorded

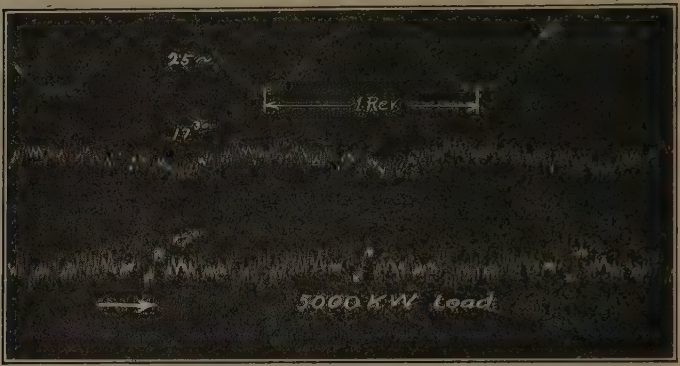

FIG. 36 OSCILLOGRAPH RECORDS MADE BY 17TH STAGE OF 20,000-KW. 1500-R.P.M. 23-STAGE TURBINE, 5000-KW. LOAD. THIS IS THE "AUTOGRAPH" OF A SMOOTHLY RUNNING WHEEL

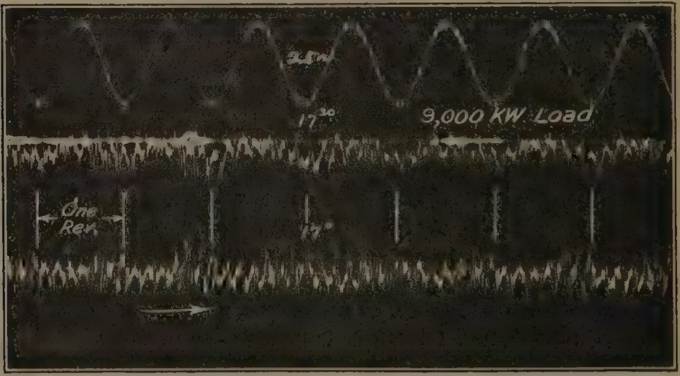

FIG. 37 OSCILLOGRAPH RECORDS MADE BY 17TH STAGE OF 20,000-KW. 1500-R.P.M. 23-STAGE TURBINE, 9000-KW. LOAD. WAVE MOTION HAS NOT YET DEVELOPED

which are magnified in proportion to the increase of frequency. For instance, since the higher frequency is about nine times as great as the lower in Fig. 33, the higher frequency would be expected to be amplified about nine times as much as it should be compared with the lower frequency recorded. In all probability the amplitude of the backward-traveling wave is greater than that of the forward wave. There are other reasons for believing this, to be considered later.

83 The film record of Fig. 33 which has just been analyzed is the one which is reproduced on page 916 of the fifth edition of Stodola's book on Steam and Gas Turbines.

84 Fig. 35 shows the frequency speed diagram for this wheel for six nodes. A and B correspond to the high and low frequencies recorded by the film shown in Fig. 33, i.e., 168 1/18 and 18 1/18

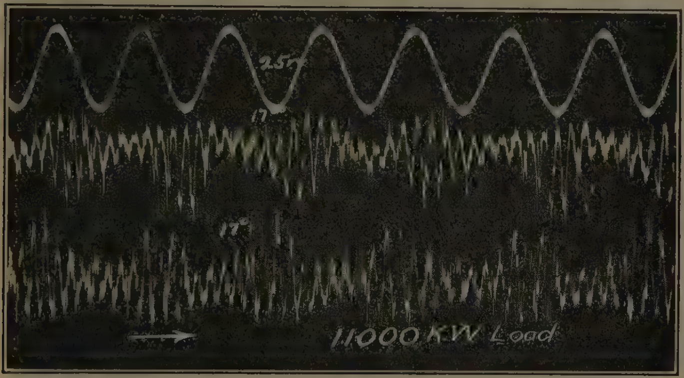

Fig. 38 Oscillograph Records made by 17th Stage of 20,000-Kw. 1500-R.p.m. 23-Stage Turbine, 11,000-Kw. Load. Wave Motion is Fully Developed. Compare with Fig. 33

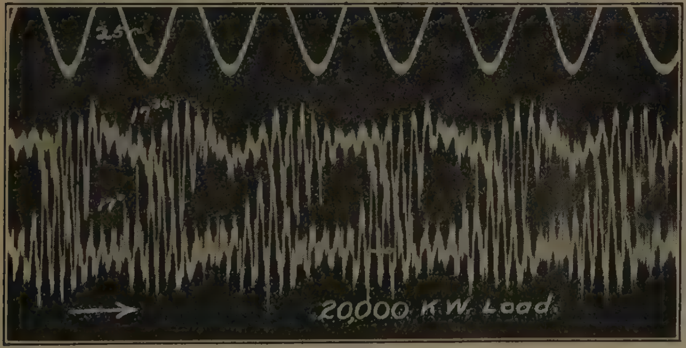

Fig. 39 Oscillograph Records made by 17th Stage of 20,000-Kw. 1500-R.p.m. 23-Stage Turbine, 20,000-Kw. Load. Wave Motion Still Maintained

cycles per second or to the wave speeds 56 1/54 and 6 1/54 r.p.s., because in the case of six nodes the wheel carries three complete waves. C equals the value of f_r, the frequency of the wheel itself rotating at the normal running speed of 25 r.p.s. This may be calculated by formulas derived from Equations [6] and [7].

$$f_r = H - \tfrac{1}{2}nN_s \quad \ldots\ldots\ldots \quad [8]$$

$$f_r = M + \tfrac{1}{2}nN_s \quad \ldots\ldots\ldots \quad [9]$$

Thus $$f_r = 168\ 1/18 - 75 = 93\ 1/18$$
$$f_r = 18\ 1/18 + 75 = 93\ 1/18$$

85 Suppose the 6-node wave train should still persist with the speed of the disk wheel whose 6-node characteristics are shown in Fig. 35 reduced from 25 r.p.s. to 15 r.p.s. Then the fixed coil would record the frequencies A' and B' instead of A and B. If there were a revolving coil, it would record the frequency C' instead of C. If the wheel were brought to rest with the wave still persisting, both coils would record the same frequency or the standing frequency E for six nodes. It can therefore be seen that if the standing frequency E is measured for a given number of nodes such a diagram can be constructed to a fair approximation,

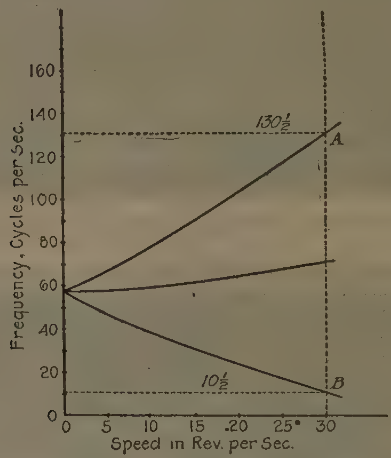

FIG. 40 FREQUENCY-SPEED DIAGRAM FOR 4 NODES. 17TH STAGE OF 15,000-KW. 1800-R.P.M. 23-STAGE TURBINE

(Points A and B were determined from the record in Fig. 41.)

because an approximate value of the speed coefficient can be assumed. For dependable results, however, rotational tests are necessary with the wheel-testing machine described later. The frequency-speed diagram may have on it curves for all of the usual types of radial nodal vibrations as, for instance, 4, 6, 8, and 10 nodes.

86 Fig. 33, the record just discussed, is the record of the 17th stage of a 23-stage, 20,000-kw. turbine. This record was taken while the machine was carrying 13,000 kw. load. Fig. 36 shows a record of the same wheel, but with only 5000 kw. load. No vibration phenomena developed at this load. The jagged and irregular record repeats exactly for each revolution. It may be regarded as the wheel autograph, and due to slight irregularities in the rim opposite which the coils are placed. These are magnified

because of the high speed. Fig. 37 shows where the load has been raised to 9000 kw. Not until the load reaches 11,000 kw. as shown in Fig. 38 do the vibration phenomena distinctly develop. Fig. 39 shows the record at a 20,000-kw. load or full load. The

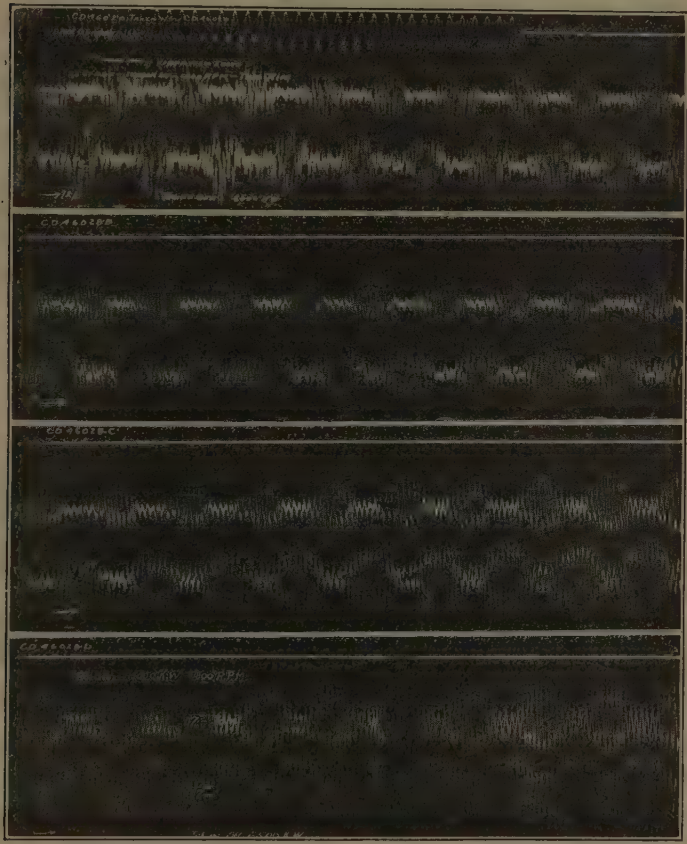

Fig. 41 Oscillograph Records made by 17th Stage of 15,000-Kw. 1800-R.p.m. 23-Stage Turbine

(The upper curve is the 60-cycle line current. The other two curves were made by stationary coils 90 deg. apart. 4-node forward- and backward-traveling waves are indicated.)

wave phenomena when once developed appear to remain about the same up to 20,000 kw. load.

Observation of Traveling Waves by Means of Two Fixed Coils 90 Deg. Apart

87 Fig. 40 gives the frequency-speed diagram for the 17th stage of a 23-stage, 1800-r.p.m., 15,000-kw. turbine, where wave

motion in a wheel was detected by means of two fixed coils on the diaphragm. In this case the coils were 90 deg. apart. Fig. 41 shows the record from which the diagram was made. This case differs from the one previously described in that the higher frequency recorded due to the forward-moving wave develops only after the load is removed. The removal of load is shown by the upper curve which registers the electrical frequency of the generator becoming a straight line, remembering that time is measured to the right. The higher frequency develops in about one second of time, corresponding to 60 a.c. cycles of the upper curve. As this is a 4-pole machine two a.c. cycles correspond to one revolution, and 30 r.p.s. is the running speed of the machine.

88 This is known to be a case of four nodes or of two waves 180 deg. apart, because from the record it appears that it takes half as long for a high spot to go from one coil to another as for

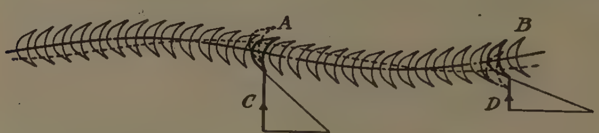

FIG. 42 THE CHANGE OF ANGLE OR "FEATHERING" OF A BUCKET DURING THE PASSAGE OF A TRAVELING WAVE PERMITS THE MAINTENANCE OF THE WAVE BY MEANS OF ENERGY ABSORBED FROM THE STEAM

two successive high spots to pass one coil. The high spots are twice as far apart as the coils, that is, 2×90 deg. $= 180$ deg.

89 Another good check is obtained by the use of Equations [6] and [7] derived in connection with the frequency-speed diagram.

$$H = f_r + \tfrac{1}{2}nN_s$$

$$M = f_r - \tfrac{1}{2}nN_s$$

Subtracting,

$$H - M = nN_s, \text{ or } n = \frac{H-M}{N_s} \quad . \; . \; . \; . \; . \; . \; . \quad [10]$$

90 From the frequency-speed diagram shown in Fig. 35 corresponding to the film of Fig. 33 previously discussed, $H = 168\ 1/18$ cycles per sec., $M = 18\ 1/18$ cycles per sec., and $N_s = 25$ r.p.s.

91 Thus from Equation [10]

$$n = \frac{H-M}{N_s} = \frac{168\ 1/18 - 18\ 1/18}{25} = 6 \text{ nodes}$$

92 For the case shown in Figs. 40 and 41 an exact analysis is difficult because where the higher frequency comes out clearly, the machine has doubtless increased slightly in speed, due to the sudden dropping of the 6500-kw. load. There can be no doubt,

however, that the following interpretation is very close to the truth.

$H = 130\frac{1}{2}$ cycles per sec., $M = 10\frac{1}{2}$ cycles per sec., $N_s = 30$ r.p.s.

$$\frac{H-M}{N_s} = \frac{130\frac{1}{2}-10\frac{1}{2}}{30} = 4 \text{ nodes}$$

"Feathering" Action of Buckets a Possible Cause of Traveling Waves

93 Thus far nothing has been said about the cause of vibration in the two cases just described. Further study brought out the fact that waves of this sort rarely occur in turbine disk wheels and the phenomenon is confined to unusually thin types of wheels

Fig. 43 A Traveling Wave in a Model Wheel Kept in Motion by High-Velocity Air from Model Diaphragm Nozzles

in which waves are easily built up. After these films were analyzed, a satisfactory explanation of the cause was sought. Referring to Fig. 42 in which the circumference of the wheel disk is formed into a wave shape, the relative angular twisting of the two buckets A and B will be somewhat as shown. It will be seen that the axial component from the energy left in the steam at the point of leaving the buckets as shown at C and D will be greater at C than at D, both reactions being vertically upwards in the figure.

94 Assuming the wheel to be stationary in space and the wave form to move toward the right in the wheel to a new position as indicated by the dotted line, it will be seen that the buckets A and B are moved to the positions shown dotted. This means that the force C, which is larger than D, is operating on

the bucket A in the direction that A is moving, due to the wave transition. At the same instant the force D is opposing the motion of bucket B moving in a downward direction due to the wave transition. However, since the forces acting in the direction of bucket motion, as at C, are greater than the forces opposing the bucket motion, as at D, it is clear that energy is added to the maintenance of the wave form after it has been initiated.

95 Fig. 43 was taken from an apparatus specially designed to illustrate the action above described. A small thin sheet-metal disk about 18 in. in diameter was supplied with model turbine buckets around the circumference. This was carried on a shaft mounted in bearings with a small prony brake fitted to the same shaft. A model diaphragm with uniform nozzle openings around the entire circumference was secured to an air-tight box and placed in position to drive the wheel. Air pressure at 5 lb. per sq. in. could be supplied to this diaphragm. The following demonstration was made:

1 The prony brake was secured to prevent rotation and the air turned on so that it passed through the nozzles. No vibrational effect was noted on the wheel.

2 The wheel was struck with a piece of wood and was found to shiver and finally come to rest.

3 The wheel disk was rubbed with a stick quickly in a circumferential direction and in the direction in which it was designed to run, but still it was found to return quickly to rest.

4 Upon rubbing the wheel in a similar manner, but in a direction opposite to that in which it should rotate, a wave shape of large amplitude, traveling in a backward direction, was developed, which was maintained as long as the air pressure was applied.

96 Thus it was proved that, in the case of the model, a backward-traveling wave could be maintained after once being initiated, simply by the passage of the air current through the buckets in the usual manner. The reason just given for the maintenance of the backward wave due to this action can also be used to prove that the forward wave would be damped, as in this case the larger force C would be working against the motion of the bucket. This explains why a forward wave was not maintained when the wheel was rubbed in the forward direction.

97 After the backward-traveling wave was set up in the manner already described, the prony brake was gradually released, allowing the wheel to rotate and gradually pick up speed. The wave shape was still maintained but the velocity of the wave relative to space became slower and slower as the wheel accelerated until finally the wave shape stood stationary in space. An amplitude large enough to rub the diaphragm in spots could be built up by slightly increasing the air pressure.

VIBRATIONS DUE TO FORCES OF RESONANT FREQUENCIES

98 A rotating turbine disk wheel may readily be made to vibrate with the application of an alternating force corresponding to any one of its resonant frequencies. Fig. 44 shows the frequency-speed diagram of a wheel covering the cases of 4-, 6-, 8-, and 10-node vibration frequencies. Higher vibration frequencies exist, but they are not so important since they are not so readily excited as the lower frequencies. Suppose the wheel to be revolving at a speed of 30 r.p.s. If an alternating transverse force be applied to the rim of this wheel by means of an a.c. magnet fixed in space, the wheel should vibrate in response to any one of 8 different

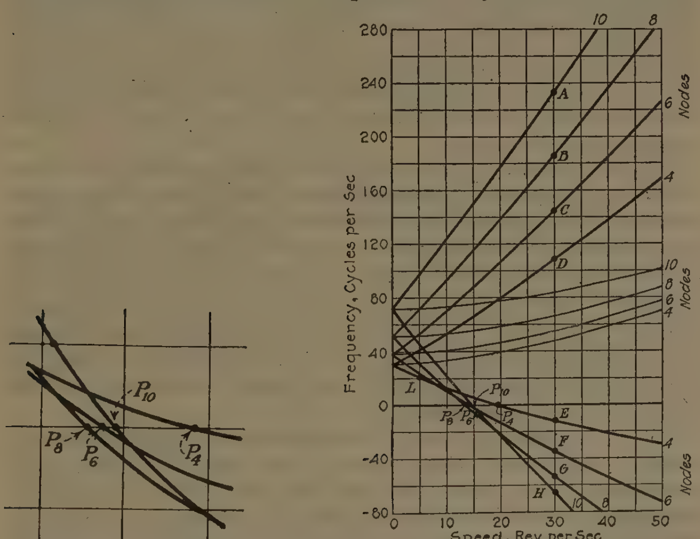

FIG. 44 FREQUENCY-SPEED DIAGRAM FOR 4, 6, 8, AND 10 NODES, 23D STAGE OF 15,000-KW. 1800-R.P.M. 23-STAGE TURBINE

(Points *A, B, C, D, E, F, G,* and *H* show frequencies which, should they occur, would provoke resonant vibration. P_8, P_6, P_{10}, and P_4 (shown in detail in sketch at the left) are the wheel critical speeds in the order in which they occur for this wheel.)

frequencies *A, B, C, D, E, F, G,* and *H,* corresponding to the speed of revolution of the wheel which, in this case, was assumed to be 30 r.p.s. They are the very frequencies which can be observed for 4-, 6-, 8-, and 10-node vibrations by an oscillograph coil which is fixed opposite the rim of the wheel in the same way as the a.c. magnet is fixed. They are the *resonant* frequencies of the wheel for 4, 6, 8 and 10 nodes for a pulsating force which acts transversely from a point fixed in space opposite the wheel. It is on this account that only the two curves of the frequency-speed diagram are considered which correspond to a fixed coil in determining these frequencies, namely, the upper and lower curves

corresponding to frequencies arising from forward and backward wave trains. The middle curves represent frequencies which would be observed by a coil carried around with the wheel and have no connection with a fixed coil or a fixed pulsating force.

99 The points to fix clearly in mind are: (*a*) that a pulsating force of the right frequency applied at a fixed point to a disk wheel revolving at a definite speed may excite wheel vibrations corresponding to any one of a series of resonant frequencies such as *A*, *B*, *C*, *D*, *E*, *F*, *G*, and *H*, as shown in Fig. 44; and (*b*) that these frequencies in each case can correspond to only a single train of traveling waves. For instance, an applied alternating force corresponding to the point *D* is seen from the diagram to have a frequency of 108 cycles per second. Such a pulsating force excites a forward-moving wave train of two waves corresponding to four nodes. This wave is readily built up to a large amplitude. It is not a vibration with radial nodes fixed in the wheel but a train of waves, two in this case.

100 If a force of a frequency of 12 cycles per second, corresponding to the point *E*, were applied to the wheel, the backward-moving 4-node wave train would build up. The negative frequencies simply mean that the frequency is due to the wave train moving backward in the wheel more slowly than the wheel rotates forwards, so the backward wave actually moves forward past a fixed coil and gives a frequency of 12 cycles per second in this case. In the same way pulsating forces of frequencies corresponding to any point on the diagram will develop the wave train for which that point corresponds.

101 For instance, a force of a frequency of 21 cycles per second applied to this wheel when it is rotating at a speed of 5 r.p.s. corresponds to the point *L* on the diagram. This is a point of resonance for a 4-node wave train moving both backward in the wheel and backward in space. The question now arises: To what do the points of intersection (such as P_4 on the 4-node curve) of these curves with the zero frequency line correspond? Here is a wave moving backward in the wheel which registers zero frequency on a coil fixed in space. In other words, a wave train is standing stationary in space because it moves backward in the wheel at the same speed that the wheel moves forward. These waves, although moving in the wheel, are stationary to an observer. This is the most important case of bucket-wheel wave motion, because practically all serious wheel failures have been definitely proved to be the result of this particular type of vibration. This particular condition in turbine wheels is discussed in greater detail in the next section.

102 In this section of the paper, two general causes of wheel vibration have been considered: (*a*) a feathering action of the steam on the buckets which may maintain waves; and (*b*) pulsating forces which may cause various resonance responses of

the wheel. The first type of vibration is eliminated by the use of wheels of sufficient thickness so that the energy dissipation during vibration is too great to permit a building up of such a vibration. The second type of vibration is eliminated by building the disk wheels so that resonant frequencies are removed so far from disturbing frequencies in the turbine that vibrations do not occur.

Stationary Waves and Critical Speeds of Wheels

103 It has already been mentioned that the type of vibration responsible for practically all serious wheel failures consists of a train of backward-traveling waves whose backward speed in the wheel exactly equals the forward speed of rotation of the wheel. This results in waves which are stationary in space. Evidently a fixed coil would register no frequency for a stationary wave train; so the presence of such a wave train alone cannot be detected by means of a single fixed coil.

104 Referring to Fig. 44, the line of zero frequency intersects the 4-, 6-, 8-, and 10-node curves at P_4, P_6, P_8, and P_{10}, respectively. When the wheel is running at the speed of 19.3 r.p.s. corresponding to P_4, the wheel speed and the 4-node wave speed in the wheel are the same so that a stationary wave corresponding to a backward-traveling wave train of four nodes may easily be built up. A small amount of energy may build up a resonant wave train. A wave train made to travel at any other than its natural speed would be a type of forced vibration and has been found to require great force to maintain it. Every turbine wheel has a series of particular speeds for which the speeds of wave trains of 4, 6, 8, 10 nodes, etc. corresponding to 2, 3, 4, and 5 waves, etc. are equal to the speed of the wheel.

105 These particular speeds of a turbine disk wheel are called *wheel critical speeds,* because it is found that when a turbine wheel is running at any one of these speeds, it is possible for a wave train of large amplitude to develop which may cause the wheel to fail. In other words, turbine wheels are liable to develop stationary waves, but stationary waves can only occur at particular speeds of the wheel, called critical speeds, one for each type of wave train. P_4, P_6, etc., on Fig. 44 are some of the critical speeds for this particular wheel. Others exist for higher numbers of nodes, but these have not been found to be so serious.

Probable Cause of Stationary Waves

106 Thus far little has been said about the cause of the development of stationary wave trains at critical wheel speeds. That turbine wheels may develop this particular type of vibration with comparative ease is beyond question, because these wave trains may develop during test *without the application of any special external force.* When once the nature of these stationary

waves is well understood, however, the cause of their development is not difficult to assign.

107 It has been stated that traveling waves could be excited by a pulsating force fixed in space, having a frequency corresponding to the speed of the waves past it. When the wave train is stationary in space, the pulsation of the force plainly is no longer required. In other words, a spot of extra pressure is sufficient to maintain a stationary wave. It is found that the application of a fixed force of only a few pounds, such as a small direct-current magnet or a small steam jet, to the side of a turbine wheel causes it to respond strongly at a whole series of critical speeds, even up to critical speeds corresponding to 16- or 18-node wave trains of eight or nine wave lengths on the wheel, and it responds only at critical speeds. It can be shown by calculation that such a force of only two or three pounds may give a continuous supply of energy to a standing wave train, and that this energy, despite the small force, may easily amount to 40 or 50 watts.

108 Tests on the energy necessary to maintain vibrations of various amplitudes in turbine disk wheels show that this amount of energy may readily maintain a wave train of four or six nodes, of amplitude sufficient to cause serious trouble. Furthermore, small amplitudes which can be detected by oscillograph coils during test may require no more than a few watts of energy to maintain them.

109 This statement seems amazing in connection with a steel turbine disk wheel, but it has been definitely proved by tests. When we consider a wheel operating in a turbine under the action of steam, it hardly needs to be said that the slightest irregularity in the nozzles might result in a transverse steam force on the wheel a few pounds greater from some nozzles than from others. Only a few pounds difference is sufficient to cause serious trouble in a wheel running at a critical speed. Therefore, wheels must be so designed that they do not operate at any of their critical speeds, as there is always a possibility of such small forces being present in turbines.

Importance of Critical Speed

110 Clear evidence of stationary waves has been observed in certain cases of failure and also in some cases where failure has not resulted. Scorings were produced on the diaphragm in spots, 180 deg. apart or 120 deg. apart, by the neighboring wheel, the high spots of the stationary wave actually rubbing the diaphragm and producing marks. Figs. 12 and 13 show a case where the 17th-stage diaphragm of a 30,000-kw. turbine is badly scored in two spots 180 deg. apart. These were doubtless caused by a 4-node stationary wave. Fig. 45 shows the 11th-stage diaphragm of a 30,000-kw. turbine, on which three scorings appear due to a 6-node stationary wave train.

111 In several cases similar rubbing has occurred on the diaphragms on each side of the wheel, there being an equal number of spots cut on each side, the spots on the exit side alternating with those on the entrance side. In these cases of local diaphragm rubbing, the wheel has rubbed around its entire circumference.

112 While such occurrences point toward standing waves as being a possible cause of trouble, the most convincing evidence of all as to the importance of critical speeds is given in Table 2. Nearly every case of wheel and bucket troubles on record shows

Fig. 45 11th-Stage Diaphragm of 30,000-Kw. 1800-R.p.m. 17-Stage Turbine Showing Scorings Produced by Rubbing of the Shroud Band of the Neighboring Wheel at Three Equidistant Spots 120 Deg. Apart Just Outside the Nozzles, Due to a 6-Node Stationary Wave at Critical Speed

a coincidence between some particular wave speed in the bucket wheel and running speed. Furthermore, many tests on actual turbine wheels in the vibration-testing machines have confirmed the truth of this conclusion.

Vibrations from the View-Point of Energy

113 The determining factor in the building up and maintenance of vibrations and waves is the relation between energy supply and dissipation. Given a certain supply of energy tending

to build up waves and a small amount of dissipation of the wave energy per unit of amplitude, the amplitude of the wave will be large. Given a large amount of dissipation of energy, the wave amplitude will be correspondingly small.

114 Two ways of preventing vibrations of waves therefore appear: (*a*) Decrease the possibility of absorption of energy in wave production; (*b*) Increase the dissipation of the wave energy.

115 Both methods are used in the production of disk wheels. The first is used by so adjusting the wheel vibration frequencies that critical speeds are avoided. Critical speeds, looked at from the energy view-point, are simply speeds at which the wheel easily

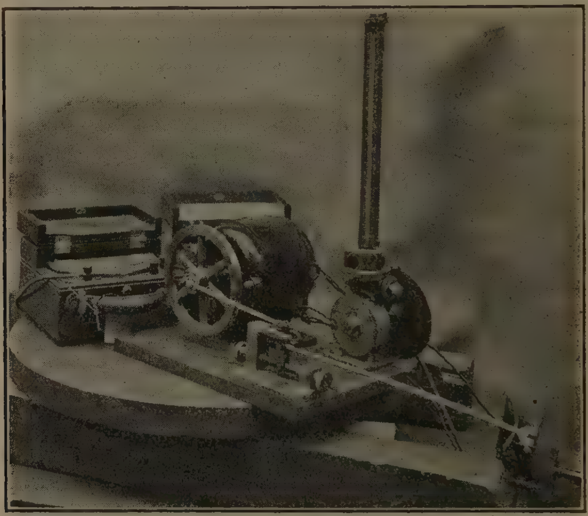

FIG. 46 MECHANICAL VIBRATOR USED TO MEASURE FREQUENCY AND ENERGY REQUIRED TO MAINTAIN VIBRATION OF TURBINE DISK WHEELS

absorbs wave energy. The second method is used in the increased thickness of disk wheels, whereby the dissipation of vibration energy may be greatly increased.

116 When a wave train of constant amplitude is maintained in a disk wheel, the energy dissipated must equal the energy supplied, otherwise the amplitude would not remain constant. The appreciation of this fact makes it possible to find the actual supply of energy necessary to maintain a given wave train by a measurement of the dissipation. The energy supply in maintaining waves in a disk wheel cannot be directly measured as it comes from forces in the turbine during operation about which little quantitative information is available. The energy dissipation, however, may be directly measured by a test on the wheel outside of the turbine.

Measurement of Rate of Energy Dissipation in a Vibrating Disk Wheel

117 The rate of energy dissipation was first measured by means of the apparatus shown in Fig. 46. A periodic force was applied transversely to the edge of the turbine wheel through the rod indirectly connected to the crankpin of the motor-driven flywheel. The speed of the motor was adjusted until the turbine wheel vibrated in resonance. The energy input to the motor is the sum of the energy required to maintain vibration plus all of the losses. This amount of energy was found to be about 80 watts for a

Fig. 47 Balance for Measuring the Energy Dissipated by the Vibration of a Turbine Wheel

125-in. diameter turbine wheel vibrating in four nodes through a total amplitude of 3/8 in. Therefore the energy dissipated in the wheel cannot be over 80 watts, and if corrections are made for losses it appears to be of the order of magnitude of 50 watts.

118 In order to confirm these measurements another method was tried in which the vibration energy was supplied to the wheel by means of an electromagnet suspended from the end of a balance beam as shown in Fig. 47. The frequency of the alternating current supplied to the magnet was adjusted until the wheel vibrated in resonance. When the maximum possible amplitude of vibration, for a given amount of excitation of the magnet was obtained, the *average* pull on the magnet was weighed directly. Assuming that the pull varies according to a sine-squared law from zero to a maximum of twice the average value (that is, neglecting the

effect of variation of the air gap) the rate of energy supply to the wheel can be computed from the frequency and amplitude of motion.

119 The advantage of this method of test is that practically all losses in the apparatus used to make the measurement are eliminated. The energy supplied is calculated directly from the product of the applied force and the motion of the wheel edge at the point of application of the force, integrated over the required number of vibration cycles. The only source of external loss is through the pedestal on which the wheel is supported. It was concluded that this was small, because, as far as could be measured, for several different types of pedestal the wheel dissipated the same amount of energy.

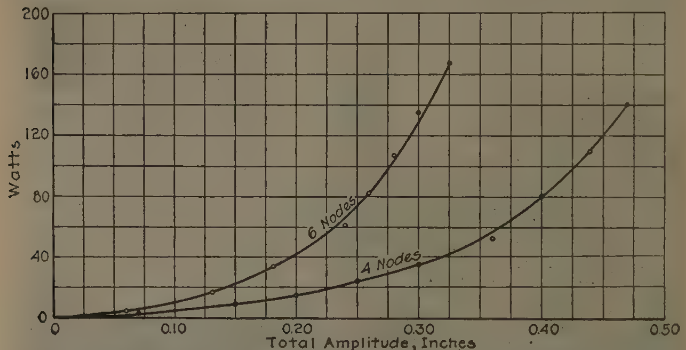

FIG. 48 CURVES SHOWING RELATION BETWEEN ENERGY DISSIPATED AND TOTAL AMPLITUDE OF WAVE TRAIN AT SHROUD BAND

120 Fig. 48 shows curves of energy dissipation for 4- and for 6-node wave trains for a medium-thin last-stage disk wheel. The energy dissipation is seen to increase about as the square of the increase of amplitude of vibration for small amplitudes and at a more rapid rate for larger amplitudes. The energy dissipated by the air friction is comparatively small. The larger part of this energy is dissipated by internal friction within the steel itself due to the repeated bending.

121 The curves show how very small an amount of energy is required to maintain small amplitudes of vibration. For a total amplitude of 1/10 in. at the bucket tips, four watts will maintain a 4-node wave train, and ten watts will maintain a 6-node wave train at this amplitude. For a 1/4-in. amplitude, however, 24 watts is required for the 4-node wave train, and 73 watts for the 6-node wave train.

Calculated Rate of Supply of Energy in Maintaining Waves in Turbine Wheels

122 The rate of supply of energy to turbine wheels in the maintaining of waves has been calculated for waves built up by the feathering action of steam on the wave forms in the buckets, described elsewhere, and for a stationary wave train built up by a spot of extra transverse pressure. These calculations show that for waves built up by bucket feathering the energy supply varies as the square of the amplitude, but for waves built up by a fixed pressure spot, which comes into action at critical speeds, the energy supply varies directly as the amplitude. The latter effects are consequently relatively large at small amplitudes as shown in Fig. 49 where curve II represents a larger energy supply than

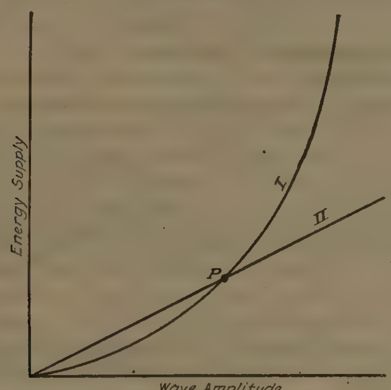

Fig. 49 Relation Between Energy Supply and Amplitude of Wave

(Curve I is for a traveling wave, while curve II is for a stationary wave. From the shapes of the curves it is seen that the latter may *originate* more easily.)

curve I for amplitudes below that corresponding to the point of intersection of the curves at P. According to this analysis a reason why stationary wave trains developed by pressure spots are more serious than traveling waves due to feathering is that the feathering cannot come into effective action until the wave train has reached a certain amplitude. To initiate a wave train the transverse pressure spot is the more effective.

123 Since stationary wave trains have been found to be the primary cause of turbine wheel failures, and since the building up of a stationary wave train is believed to be due to the second of the two causes of wave development presented, an indication of the manner in which a fixed force may supply energy continuously to a stationary wave train will be presented.

124 Fig. 50 represents the developed edge of a turbine wheel carrying a 4-node wave train stationary in space. The wheel edge is shown moving to the right with a velocity V and the wave is

traveling to the left in the wheel with the same velocity, so that it stands stationary in space. Assume a fixed force F acting at a nodal point as shown. At this point a particle P has a component of motion in the direction of this fixed force equal to V_F in the figure. The product of this velocity and the force F gives the rate at which work is done on the particle at this instant. Since the force is fixed in space it acts first on one particle and then on the next as the edge of the disk wheel moves along, but it does work continuously at this rate. This is therefore the rate at which work is supplied to the wheel in building and maintaining the wave train. It is easily shown for a sine wave that the value of this transverse velocity component is

$$V_F = \frac{2\pi V y_0}{\lambda} \quad \text{. . . . . [11]}$$

where y_0 = half amplitude of the wave
λ = wave length
V = velocity of the wheel edge or of the wave.

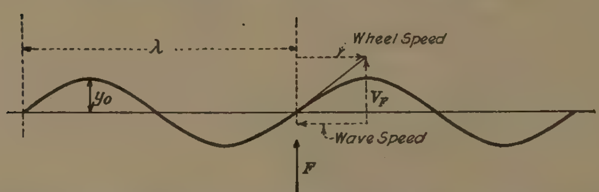

Fig. 50 Developed Edge of a Rotating Wheel Carrying a Wave, Stationary in Space. The Fixed Force F Supplies the Energy that Maintains the Wave

Multiplying this velocity component by the force F, the work per second supplied to the wave is

$$W = \frac{2\pi V y_0 F}{\lambda} \quad \text{. . . . . [12]}$$

In the tests described it was found that when $y_0 = \frac{1}{4}$ in., W may be about 25 watts. Taking average values for V and for λ, it is found from Equation [12] that a force F of only a few pounds may supply the required 25 watts to the wheel.

125 This equation assumes the force to be concentrated at a point. If the force is distributed over half a wave length, the total force required will be somewhat larger.

Utility and Limitations of Theoretical Design

126 As will appear subsequently, the actual process of design, aside from wheels of totally new characteristics, is a process of comparison with a carefully correlated catalog of all previous similar wheels. Such a process, checked by test, is thoroughly

logical, and proves to be satisfactory. In fact, for reasons which it is the intention of this section to make clear, comparative selection and test is the only satisfactory means of design known at this time. Such a method is comprehensive.

127 Even now shapes proposed by theoretical means come within the scope of the testing organization for verification. And new shapes in which theoretical analysis is most needed are, unfortunately, by that fact most subject to variation from theory and, therefore, especially need to be checked. Similarly the wheels of ordinary design which need least to be checked are most susceptible of calculation. It is entirely conceivable that at some future time material specifications can be made sufficiently rigid, machine work sufficiently exact, and methods of assembly so precise that every wheel can be constructed under conditions of uniformity sufficient for a general dependence on theory. At the present time it is unsafe to accept the successful calculation of nine out of ten wheels as evidence that the tenth wheel also will be correct.

128 It seems unnecessary to give here in full the elaborate theories developed during these investigations, but no adequate idea of the importance of actually testing wheels can be had without a thorough understanding of the scope of the various means of design tried. Space must be given, therefore, to a description of the theory and the tests made in its verification.

129 The simplest elastic system considered was a simple cantilever bar or beam of uniform cross-section, similar to a vibrating reed. This system is capable of a complete analytic treatment, from which the shape during vibration and the exact frequency can be obtained. The latter is what is of interest here and it may be written

$$F=\frac{1}{0.569\pi}\sqrt{g\frac{EI}{wl^4}}=\frac{3.18t}{l^2}\sqrt{\frac{E}{\delta}} \quad . \; . \; . \; . \quad [13]$$

in which

F = frequency, cycles per sec.
g = gravity, in. per sec. per sec.
E = Young's modulus, lb. per sq. in.
I = moment of inertia of cross-section, in.^{4}
w = weight, lb. per in. length of bar
l = length, in.
t = thickness of rectangular bar, in.
δ = density of steel, lb. per cu. in.

130 In making a verification of this formula the dimensions and weight of the bar may be easily measured. For the same material the frequency should vary directly as the thickness and inversely as the square of the length. Fig. 51, curves A, B, and C show a series of three tests made in an unsatisfactory attempt to prove this simple rule. All bars were of steel, but no particular

attention was paid to getting the same stock. The thickest bar B actually shows a progressive deviation and suggests a variation with $l^{1.9}$ instead of l^2. The last test C was made with two bars balanced like a fork, and while this test is better, it was not considered satisfactory for such a simple rule. Therefore, the test represented by curve D, Fig. 51, was made with a $\frac{1}{2}$-in. square bar clamped directly to the heavy cast-iron floor of the shop. The modulus of elasticity was separately determined by a deflection test as a simple beam. The plotted points show agreement with the theoretical line over a 10 to 1 range of frequency varia-

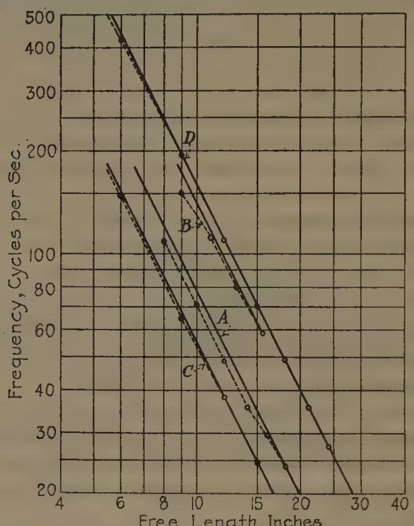

FIG. 51 VIBRATION OF STEEL BARS FIXED AT ONE END

(Full lines show theoretical variation of frequency. Broken lines show deviations in test.)

Bar A is 1.49 in. by 0.245 in.
Bar B is $\frac{7}{16}$ in. square.
Bar C is a 2-bar adjustable fork 0.86 in. by 0.180 in.
Bar D is $\frac{1}{2}$ in. square.

tion, but even with this clamp the agreement starts to fall off at frequencies of about 300 cycles per second.

131 These simple tests have been described thus at length in order to show that laboratory care is necessary in the simplest cases in order to secure frequencies within one or two per cent of theoretical expectations.

132 Turning next to the question of disk wheels, the theory of vibration will be described. In previous sections it has been made clear that, so far as the frequency of various particles of the disk is concerned, it makes no difference whether the disk is subject to traveling waves or whether it vibrates with fixed nodes. In either case the natural frequency is the same, and all that is

essential is a solution capable of giving the frequency of vibration in each of the various natural types.

133 The problem of vibrating flat plates is thoroughly discussed by Lord Rayleigh,[1] but the complexity of a turbine wheel is somewhat beyond the formulas developed. Stodola[2] gives the formula for a flat circular plate based upon Kirchhoff's contribution to *Crelles Journal* in 1850, as

$$F=c\,\frac{t}{r^2}\sqrt{\frac{E}{\delta}} \quad \ldots\ldots\ldots\ldots \quad [14]$$

in which r=radius of plate, in.

c=a constant depending on the nodal configuration and Poisson's ratio.

134 Stodola then goes on to examine at some length a method attributed to Ritz[3] in which the principle of least work is employed. Finally he selects a principle noted, in passing, by Rayleigh,[4] which proves to be the most practicable for use so far found.

135 Rayleigh's principle is that "The period calculated from any hypothetical type cannot exceed that belonging to the gravest normal type". (Section 89.)

136 The indicated method of calculation is to assume any hypothetical shape of deformation and calculate its true potential energy. Equate this to the maximum kinetic energy which the system would have if it could vibrate in the hypothetical shape, and the period so calculated cannot exceed the gravest natural period. Plainly the assumed shape which gives the longest period is nearest to the true condition.

137 In applying this method Stodola represents the wheel by a hyperbolic profile and makes ingenious allowances for the stiffness of the wheel rim and the weight of attached buckets. The hypothetical shape is sinusoidal about the circumference and along a radius it is an exponential curve. The potential and kinetic energies are formulated and the frequency of vibration expressed in terms of the above quantities and made a function of the exponent of the radial-deflection curve. Then by a differentiation the exponent leading to the minimum frequency (or gravest period) is selected. All of these steps are well justified on a theoretical basis. Rayleigh shows that there can be a considerable variation of the hypothetical from the true shape without greatly affecting the result. The exponential shape is, in itself, a reasonable approximation, but the Stodola method selects not only the

[1] The Theory of Sound, 1894.

[2] Ueber die Schwingungen von Dampfturbinen-Laufrädern. *Schweizerische Bauzeitung,* May 2, 1914.

[3] *Crelles Journal,* 1908.

[4] The Theory of Sound.

exponential shape but uses that exponent best suited to the case in hand.

138 In reviewing the various points of attack another plan may be mentioned which has not yet been worked out for application although its very simplicity commends it if it were as easy to construct the deflection curve for a disk wheel as for a rotor shaft. Since the idea of natural vibration supposes that the motion of all particles is both harmonic and isochronous, it follows that in the case of every particle, irrespective of displacement or time, the same ratio exists between weight and force per unit displacement, because all particles have the same period. In other words, if the material is all alike and all particles are of the same weight, the force per unit displacement is the same

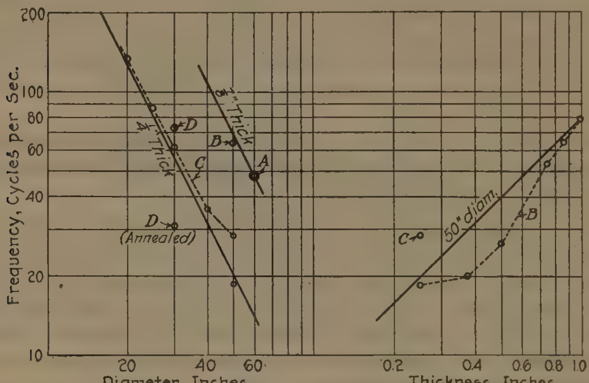

FIG. 52 4-NODE FREQUENCY OF FLAT CIRCULAR PLATES

(Full lines indicate theoretical relations. Broken lines show deviations in test. *A*, *B*, *C*, and *D* indicate the four different pieces of steel used.)

throughout the system. If the system is a shaft of uniform section or a plate of uniform thickness, the shape taken will always be such that at every point the unbalanced force (shear) on an element of length or surface will be strictly proportional to the deflection. In the case of a shaft or a reed where a deflection curve can be easily constructed by graphic processes, the indicated procedure is to assume a loading, construct the corresponding deflection curve and then alter the loading, and repeat until, by successive approximations, agreement of proportionality is secured between deflection and load.

139 Then the relation between change of shears in any part of the length, deflection and weight of the same section determines the frequency of the system. The method would be identical with that used in calculating the critical speed of rotation of a shaft. In the case of a vibrating disk wheel the shape within the rim

is such as would be taken under a loading per unit area proportional to the product of deflection and thickness.

140 The Rayleigh principle, mentioned above, has been developed along three different lines by the General Electric Com-

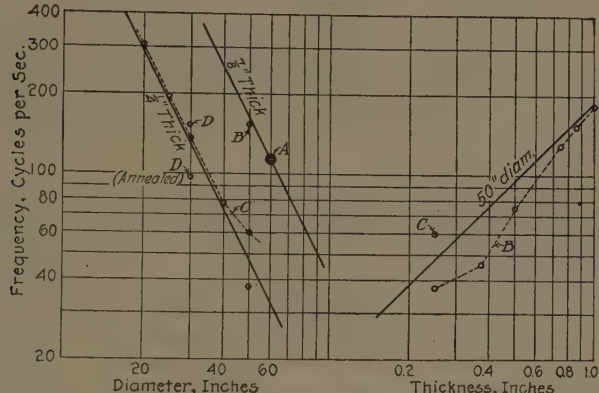

FIG. 53 6-NODE FREQUENCY OF FLAT CIRCULAR PLATES

(Full lines indicate theoretical relations. Broken lines show deviations in test. *A*, *B*, *C*, and *D* indicate the four different pieces of steel used.)

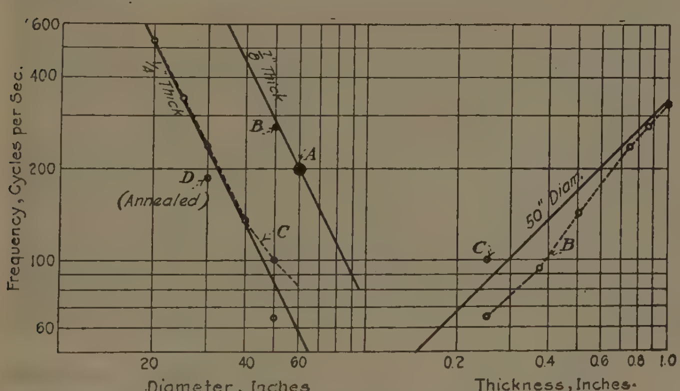

FIG. 54 8-NODE FREQUENCY OF FLAT CIRCULAR PLATES

(Full lines indicate theoretical relations. Broken lines show deviations in test. *A*, *B*, *C*, and *D* indicate the four different pieces of steel used.)

pany. The first frequency calculations were made by adapting the formulas of Stodola directly to the wheels in question. Charts of coefficients, tables, and printed forms were provided, and some hundreds of wheels were calculated in this way with good success. Usually wheels of nearly hyperbolic profile with standard rims and one row of short buckets or medium-length buckets were

easily and quickly computed by this method. There were, however, many important wheels with long buckets which failed to check the calculated frequencies. The protection of every wheel without exception was necessary. The difficulties were sought in variation of the actual profile from the assumed hyperbolic shape, in the action of the rim, in the bending of the bucket, and in the consideration of the bucket dovetail, and the wheel hub. These are the reasons that account for subsequent developments of calculation methods. That other troubles of a more serious nature were also accountable for the discrepancies will presently be shown.

141 In order to take account of the various items to which the troubles of analytical calculation were attributed, two other methods were developed in one of which the Rayleigh principle was applied graphically. By this method the true wheel shape is plotted and a deflection curve assumed without restrictions as to type. The wheel profile is divided into a series of sections and the kinetic and potential energies of each section are calculated and the frequency found by equating the summations. Of course a variety of deflection curves have to be examined until that giving the minimum frequency is found. Although standard calculation forms are used, the method is laborious as compared with analytic work, but it is comprehensive and sound in principle and with similar assumptions and limitations the two methods give exactly the same results. A limited variety of wheels, which, however, includes standard types, can be satisfactorily calculated by the analytic method. But wheels of limiting designs or peculiar contour require the more elaborate method.

142 The third method used the Rayleigh principle as developed by Stodola, but faced the fact that wheels are not necessarily hyperbolic in shape. This method was adapted at the outset for tabular calculation in order that a wheel of any profile might be handled. In various other details, the process of application differed slightly. These various methods have been applied and in many cases give satisfactory results. The important thing left is to find out what is the matter with the wheels for which the theory is inadequate.

143 In the case of the cantilever bar it was found very difficult to secure tight clamping. From bars the investigation was extended to flat circular plates. Kirchhoff's formula has already been cited, showing that for uniform material the frequency should vary in direct proportion to the thickness and in inverse proportion to the square of the radius. A series of tests was made on four plates to verify this rule.

144 The results are shown in Figs. 52, 53 and 54 for four, six, and eight nodes, respectively. The letters A, B, C, and D, are used to designate the four pieces of metal used. The left-hand curves show the variation of frequency due to change in diameter

for $\frac{7}{8}$ in. and $\frac{1}{4}$ in. thickness. The right-hand curves show the variation in frequency due to change of thickness for disks 50 in. in diameter. The *A* plate was tested but once and, as it agreed with the calculation, it is plotted as a master point from which the full lines representing theoretical relations are drawn. The *B* plate was 50 in. in diameter, and it was tested with thicknesses of 1 in., $\frac{7}{8}$, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{3}{8}$ and $\frac{1}{4}$-in. Note that when 1 in. thick this wheel vibrated in substantial agreement with theory, but when $\frac{1}{2}$ in. thick the 4-node frequency was too low by 33 per cent. Note also that the *C* plate when 50 in. in diameter had a 4-node frequency more than 40 per cent too high, but as the diameter was subsequently reduced to 40, 30, 25, and 20 in. the agreement became better. It is also noteworthy that the agreement is better, the larger the number of nodes. Now note the plate *D*, 30 in. in diameter and $\frac{1}{4}$ in. thick. The expected 4-node frequency was 56. The measured value was 73, which is 30 per cent too high. This plate was subsequently "annealed" but the effect of this heat treatment was to reduce the 4-node frequency to 31, or less than half of its previous value, and 45 per cent too small as compared with expectations.

145 It had been noted that the same piece of metal — when altered in thickness or diameter — varied widely in the amount of its divergence from theory, far more widely then could be accounted for by variation in elastic modulus or density. It seemed probable that the condition of initial internal stress in the plate was changed by the removal of certain stressed portions of the material. It was difficult to account for the variations otherwise. Widely varying methods of support had failed to have appreciable effects. After the *D* plate had been put through a heat treatment intended to anneal it so as to remove internal stress, with the result that its frequency was reduced more than half, it appeared necessary to accept the idea of internal stress to explain the fluctuations. The easiest way to vary the condition of internal stress was to vary the temperature distribution.

146 A momentary application of a gas flame to the center of the disk *D* changed the frequency from 31 to 65, more than double, this frequency being measured while there was a temperature gradient along radial lines. The resulting internal stresses had the expected effect on the frequency which dropped as the temperature became more uniformly distributed throughout the disk. It is comforting to note that the fluctuations shown in the test plates of Figs. 52, 53 and 54 are extreme and appear much less severe in thicker plates. Most turbine wheels are of such proportions as to be less subject to variation of frequency on account of internal stress.

147 This series of simple experiments furnishes easily-understood evidence that one of the factors of prime importance

influencing the natural frequency of turbine wheel vibration is the exact condition of internal stress. It is also plain that on this account very refined methods of material treatment, machining, and assembly must be developed so that perfect uniformity can be assured before the practice of keeping a strict watch of every wheel for fluctuations from expected frequencies can be superseded. A very considerable amount of information has already

FIG. 55 SMOKED-GLASS APPARATUS FOR MEASURING AMPLITUDE OF VIBRATION OF A TURBINE DISK WHEEL

been cataloged in explanation of frequency variations by the test methods in use.

MEASUREMENT OF DEFLECTION SHAPE DURING VIBRATION

148 The determination of stresses due to vibration in a vibrating disk wheel is easily found from the deflection shape of the wheel. Once the shape is known, the stresses can be calculated by well-established formulas. It will be remembered that the

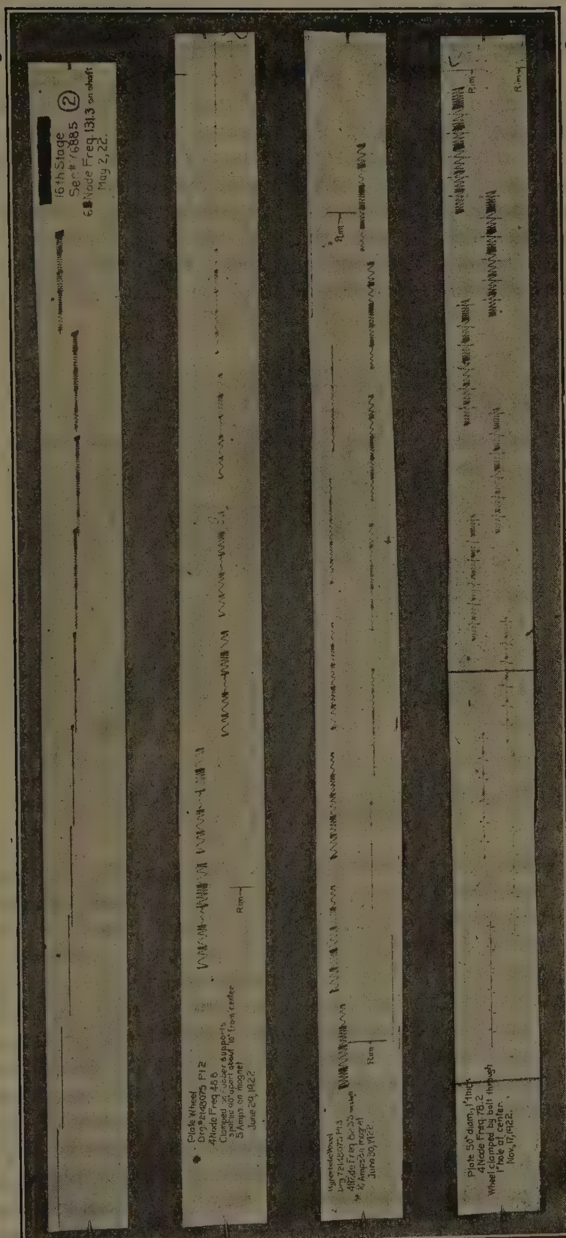

FIG. 56 SMOKED-GLASS RECORDS OF AMPLITUDE OF VIBRATION OF TURBINE DISK WHEELS

(The full-sized record is measured with an optical micrometer.)

determination of frequency by the methods described also implies a knowledge of the deflection shape from which the potential and kinetic energies are calculated. The deflection shape is, therefore, vital to a knowledge of the problem.

149 In order to make actual measurements of the radial shape during vibration the apparatus illustrated in Fig. 55 was constructed. A frame fitted with guides in which a long plate glass could be inserted was placed on a radial line with the glass perpendicular to the surface of the plate, diametrically opposite to the magnet in order to secure the record mid-way between nodal radii. The frame was rigidly supported independently of the wheel at each end. Phosphor-bronze indicators, each carrying

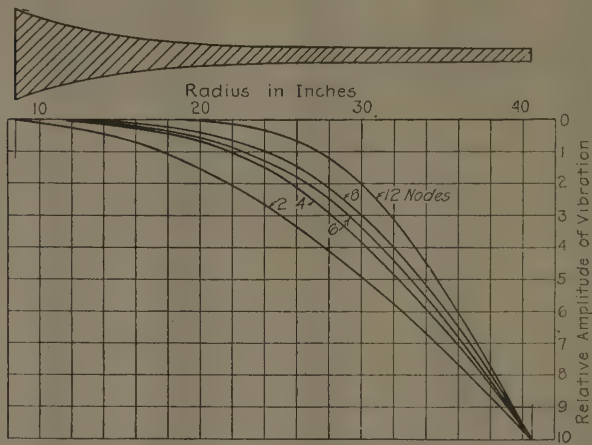

FIG. 57 MEASURED DEFLECTION CURVES FOR A DISK OF TRUE HYPERBOLIC PROFILE

a needle-point stylus, were rigidly fastened to the wheel with a mixture of resin and beeswax. The glass was coated with lampblack. The needle contact was controlled by a tangent screw at the outer end of the frame. While the wheel was vibrating, the plate was pushed radially and each needle records a wavy curve. A stop plug makes it possible to take four to six records on a single glass, each record containing five to ten waves for each needle. Several series of corresponding waves are selected and the amplitudes at the various needles are measured directly from the smoked glass by means of an optical micrometer. Fig. 56 shows photographic prints made from the smoked glass records. Fig. 57 shows a series of deflection curves for various nodal configurations made on a special wheel of truly hyperbolic profile. Fig. 58 shows the curves measured for the *B*-plate wheel referred to before when 50 in. in diameter by 1 in. thick. In addition it shows the

curves selected by the Stodola process for calculation. It will be noted that, while not exact in shape, the agreement is very good, in fact as far as the frequency calculation goes, very little difference can be distinguished.

150 The calculation of stress requires a better knowledge of the deflection shape than does the calculation of frequency. The smoked-glass method has been of especial value in furnishing a process for measuring deflection and thus indicating stress. When the deflection shape is known the stresses are given by the formulas for a bent plate deformed in two directions. It is not always possible to determine from resonant frequencies what type of vibration has caused a failure because more than one type may have a frequency not far from running speed. In case of certain

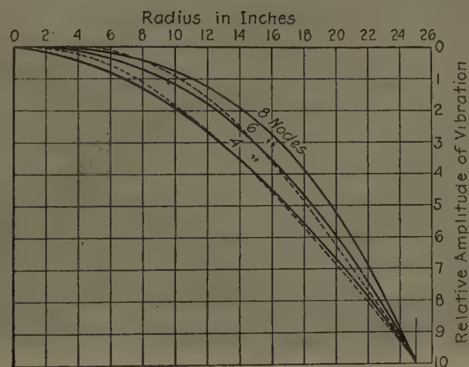

FIG. 58 COMPARISON OF CALCULATED AND MEASURED DEFLECTION CURVES FOR PLATE *B*, 50 IN. DIAM. x 1 IN. THICK

(Full lines calculated by formula $y = aR^2$. Broken lines show measured amplitudes.)

failures a stress analysis can sometimes show definitely that only one of the nearly resonant types could possibly have produced the failure.

151 The utility of stress analysis presupposes a complete knowledge of the strength of the material under conditions of vibration. The subject of fatigue stresses has been the object of many investigations, but usually devoted to completely reversed stresses. At the especial request of the General Electric Company, the University of Illinois Engineering Experiment Station undertook to investigate the case of repeated bending superposed on tension such as exists in a rotating turbine wheel. A variety of heat treatments of turbine steels have been investigated under these conditions and the first results of these experiments are described in Chapter IV of Bulletin 136 of the Engineering Experiment Station of the University of Illinois.

PART III — METHODS OF DESIGN AND TESTING FOR THE PROTECTION OF TURBINE BUCKET WHEELS FROM AXIAL VIBRATION

152 The data obtained in the previous building and testing of wheels are utilized in the design of wheels having similar dimensions and forms. In the case of wheels already in service the speed coefficient may be used with considerable confidence in calculating critical speeds when the frequency of vibration of the wheel at rest is known. It is highly desirable, however, to check

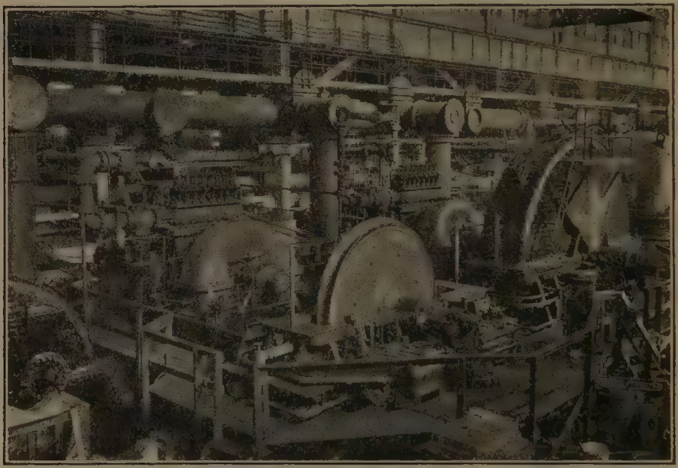

Fig. 59 Testing Machines for Determining the Vibration Characteristics of Turbine Wheels under Running Conditions

(The cover is removed from No. 1 machine to show test wheel within.)

results whenever possible by rotating the wheel in a wheel-testing machine.

Turbine Wheel-Testing Machine

153 Fig. 59 is a photograph of the testing laboratory showing two complete wheel-testing machines. The smaller machine in the foreground has the cover removed. Fig. 60 is a photograph of the smaller machine with the cover on ready for test. This machine comprises a sort of bombproof chamber within which the wheel to be tested is operated. The upper half of the casing of this testing chamber is semicircular in shape, to constitute a hood over the wheel under test. This member is made of cast steel 8 in. thick for protection in case of accident to the test wheel.

154 The machine consists of a steam chamber in which the test wheel is mounted on the shaft alongside a heavy disk or

wheel as appears more clearly in Fig. 61. This shaft carrying the two wheels may be rotated at any required speed by means of a steam turbine. One pipe supplies steam to the casing when necessary while another pipe is utilized to convey the exhaust steam to the vacuum pump and condenser. An absolute pressure of about 4 lb. per sq. in. is maintained in the casing during standard tests.

155 The purpose of the steam in the wheel chamber circulating during the test is to keep the wheel relatively cool. If turbine wheels are rotated at high speed while surrounded by air at atmos-

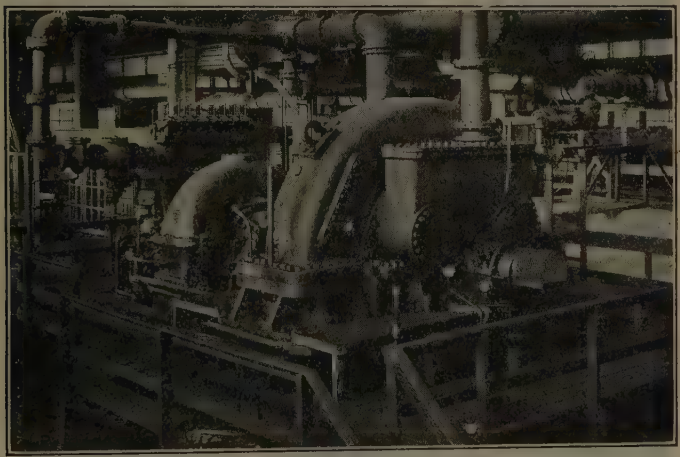

Fig. 60 No. 1 Wheel-Testing Machine Assembled with Cover on Ready for Test

pheric pressure so much heat is generated by the windage that the temperature of the wheel rises unduly. By circulating steam through the casing to the vacuum pump and condenser, the heating energy realized by rotation of the wheel is removed and the temperature of the wheel is maintained at the desired value.

Use of the Oscillograph

156 In order to observe or record wave motions or vibrations which may occur in the wheel under test, a standard oscillograph is used, together with two sets of exploring coils suitably located within the wheel-testing machine. One set is stationary with respect to the wheel to be tested while the others rotate with the wheel. These exploring coils transmit to the oscillograph, which records them, electrical indications of the movement of the turbine wheel. The stationary coils indicate the movements of the wheel rim towards and away from the exploring coil as the wheel passes by the coil. The movable coil which rotates with the wheel,

on the other hand, records only the lateral motion of one given point in the wheel circumference. The records from these two coils disclose the nature of the wave phenomena developed in the wheel.

157 Exploring coils, located within the casing of the wheel-testing machine, are made steamproof by complete enclosure in a metal casing. Metal-cased wire is used for the electrical connections to the coils.

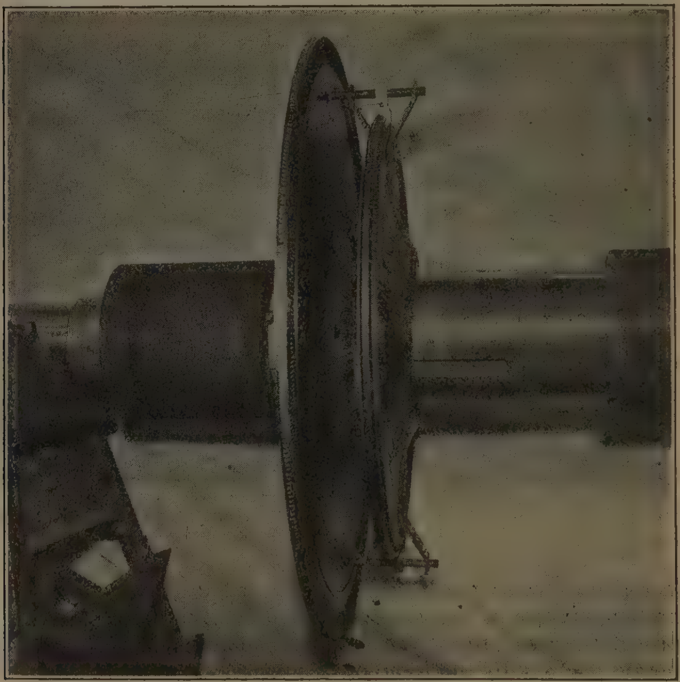

Fig. 61 Testing-Machine Shaft Showing Exploring Coils Carried on an Adjacent and Relatively Stiff Wheel

158 The rotating coils are carried in a tubular member fastened on the periphery of the relatively stiff coil-carrying wheel, always used in testing, whose vibration characteristics are so well known as not to be confused with those of the wheel under test. The tubular member is held parallel to the shaft and is adjusted in order to obtain the desired air gap between the coil and the test wheel. The metal-cased conductor from this coil is carried down the side of the coil-carrying wheel, brought out through the end of the hollow shaft, and connected to a collector ring. Another coil is placed 180 deg. away. This is used as a reserve and it serves also to maintain balance.

159 In some cases the wheel to be tested is of such dimensions that the vibration data are obtained from a point in the bucket region. In this case it is customary to silver-solder a small armature between two buckets in front of the rotating coil. Ordinarily, the air gap is between $\frac{1}{8}$ and $\frac{3}{8}$ in. on the large wheels and sometimes less on very small wheels.

160 The fixed exploring coil, already referred to, is suitably supported within the casing adjacent to the wheel rim, the metal-cased wire connection from this coil being brought outside the casing where electrical connection is made with the oscillograph.

Electrical Connections

161 The electrical connections of the various exploring coils, the electrical circuits of the amplifiers used, and of the oscillograph and of the exciting magnet used in test are shown diagrammatically in Fig. 62. The connections from the rotating coils lead out through the shaft and are connected with slip rings, the brushes from which lead to the primary of a transformer, a suitable source of direct current being connected in series. The development of brushes and rings which would satisfactorily collect the minute currents for the oscillograph was one of the many lesser achievements in the construction of this apparatus.

162 The stationary coil is connected through a switch to the primary of another transformer. The secondary windings from these transformers are connected respectively to suitable amplifier devices for magnifying the current fluctuation produced through the action of the test wheel on the exploring coils.

163 It will be understood that the current in one of these exploring coils, coming from the battery, develops a magnetic field, whose magnitude varies in accordance with the variation in the air gap between the adjacent parts of the wheel and the magnetic coil. In the case of the rotating coil the lateral vibration of the adjacent part of the wheel produces a change in magnetic reluctance. In the case of the stationary coil the change in magnetic reluctance is produced by variations in the distance of the wheel rim as it sweeps by the stationary coil. These variations in distance are due partly to irregularities in the structure of the wheel itself, and in case of wave phenomena, to lateral deflections of the wheel.

164 The current induced in the transformer secondary windings is amplified by vacuum tubes. This amplified fluctuating current is then led through another transformer, the secondaries of which lead to the oscillograph vibrators.

165 The oscillographs, indicated diagrammatically in the upper portion of Fig. 62 are standard instruments for producing and for recording images representing the fluctuations from instant to instant of electric currents. These images may be produced by

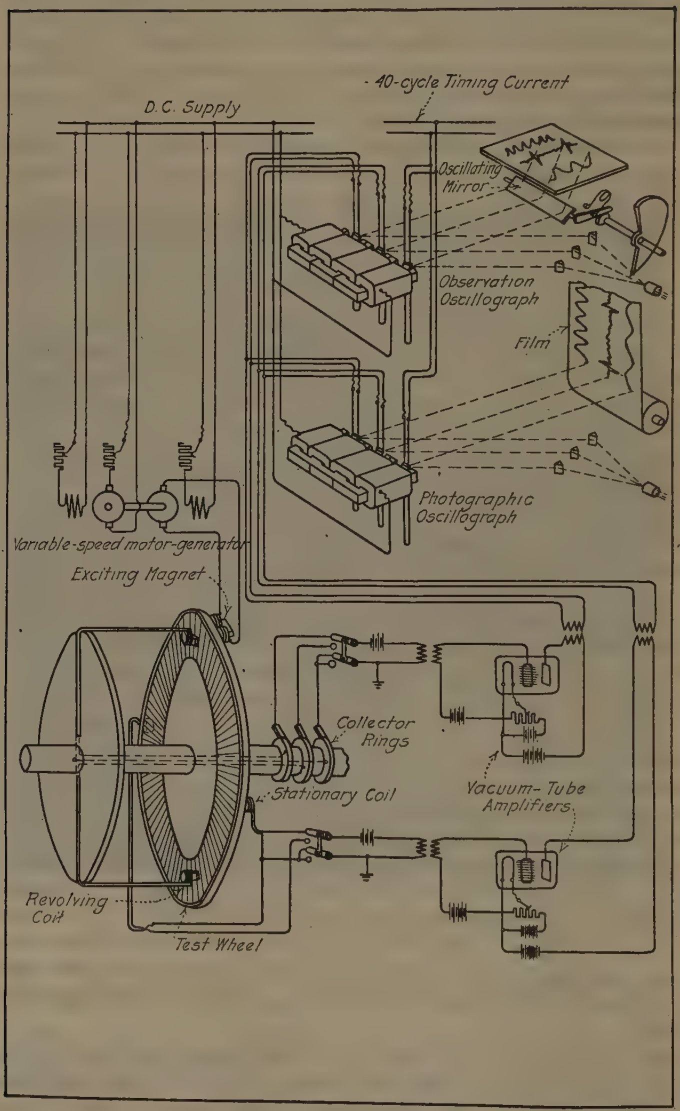

FIG. 62 ELECTRICAL CONNECTIONS FOR TESTING MACHINE

(Both revolving and stationary exploring coils are connected through vacuum-tube amplifiers to the observation and photographic oscillograph instruments. The exciting magnet receives current from a variable-speed motor-generator set.)

the trace of a point of light upon a ground glass or in a mirror, or images may be recorded permanently on a photographic film.

166 The fluctuating current is led to the bifilar suspension armature which carries a small mirror. This armature, which is located in a strong magnetic field, oscillates in proportion to the fluctuations in the current. Light from an arc lamp is transmitted through a lens and prisms to the mirror, and is reflected in turn to a suitable receiving surface. As used at present, this consists of an oscillating mirror, which is arranged to oscillate about a horizontal axis. The frequency of these oscillations is directly proportional to the speed of rotation of the test wheel. When this mirror is oscillated the wave forms are rendered visible, by reflection on a ground-glass receiving screen. The pivoted mirror is caused to oscillate by means of an arm which is held by a spring in contact with a cam. This cam is driven by a synchronous motor connected with the cam shaft. The synchronous motor receives the current from a small alternator driven directly by the shaft carrying the test wheel, the result being that the wave motions appear to be stationary instead of progressing across the field of vision.

167 The illumination of the screen is interrupted periodically by a shutter attached to the camshaft. During this dark period the cam returns the mirror to its initial position, and the light is then allowed to illuminate the screen again. Actually, only a spot of light is reflected on the screen, but owing to the rapid rotation of the camshaft and the phenomenon known as the " persistence of vision " the spot appears as a complete, more or less wavy line.

168 Three bifilar circuits carrying oscillating mirrors are used in the oscillograph, and are connected respectively to the coil circuits, whose wave forms it is desired either to observe or record. For reasons which will be explained, the oscillograph just referred to is used only for purposes of observation. One circuit receives its current indirectly from the rotating coil, another from the stationary coil and the third circuit receives its current from a source of 40-cycle alternating current, so that the indications produced from the operation of this latter member of the oscillograph serve as a time standard for the waves produced by the other two circuits.

169 A similar oscillograph, provided for the taking of photograph films, has all its circuits connected in multiple with those of the observation oscillograph just described, so that both receive currents of the same character. Inasmuch as the wave phenomena in the turbine wheel are transitory, the presence or absence of these wave phenomena is observed by inspection of the reflections from the oscillating mirror. When, however, the operator observes in the mirror of the observation oscillograph the occurrence of any particular wave phenomenon which it is desired to record, then at the desired instant, he signals an assistant. The latter

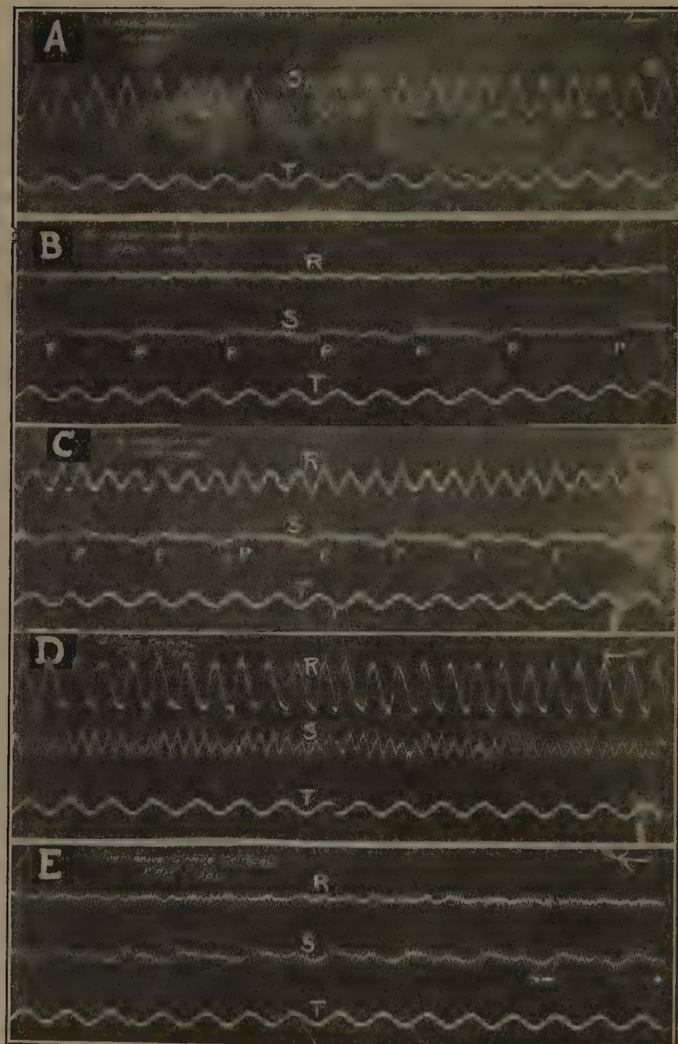

FIG. 63 TYPICAL OSCILLOGRAPH RECORDS

T is a 40-cycle timing wave
S is the record made by the stationary coil
R is the record made by the revolving coil
A — Wheel stationary, 6-node vibration, 55.2 cycles per sec.
B — 1140 r.p.m. The stationary coil *S* gives the wheel "autograph." No vibration is indicated.
C — 1250 r.p.m., 6-node critical speed. The revolving coil *R* shows the backward wave in the wheel. The stationary coil *S* still shows the wheel "autograph" since the wave is stationary in space.
D — 1250 r.p.m., 6-node critical speed. A moment later than *C*. The stationary coil *S* shows the development of the forward wave train at double the frequency shown by the backward wave train in the revolving coil record *R*.
E — 1380 r.p.m. Wave phenomena have vanished. Compare with *B*.

thereupon operates the shutter of the recording oscillograph, causing an exposure to be made upon the film, which is immediately developed in the usual manner.

170 For the purpose of causing at will a vibration of the test wheel, a well-insulated electromagnet is located opposite the periphery of the wheel. This is usually located at the wheel rim in such a manner that the magnetic attraction will be in a direction parallel to the shaft. Electrical connections for this magnet are brought out from the shell and the alternating current is supplied from a variable-speed motor-generator set. Suitable rheostats enable frequencies varying through wide limits to be obtained. An electrical tachometer is used for measuring the speed of the motor-generator set. This is graduated in terms of frequency of the alternating current generated.

171 For the efficient use of the apparatus and the best organization of the work it has been found desirable to have two complete testing machines in use. While one is connected to the instrument chamber for the taking of records, the other machine is open or being assembled with the next wheel. All of the instruments for observation and control are grouped in the instrument room, including both the observation oscillograph and the photographic oscillograph. The necessary tachometers, control switches, rheostats, and the speaking tube to the turbine operator are within reach of the man in charge whose post is at the observation oscillograph. The developing room is immediately adjacent and equipped with all facilities for the quick development of records so that the operator can see, while the wheel remains at speed, whether his films successfully record the desired phenomena.

172 In testing a turbine wheel it is convenient first to determine the natural periods of vibration of the wheel when at rest, or standing still. This is often done with the wheel placed in the wheel-testing machine, previous to making the rotation test. The electromagnet already described, placed close to the rim of the wheel, receives the alternating current, thereby producing a pulsating magnetic pull on the wheel rim. Inasmuch as the magnet exerts a pull for each half wave of the alternating current the pulsating attraction exerted upon the wheel is numerically double that of the frequency of the alternating current. The nodal points on the wheel circumference are determined by observation and touch. Oscillograph records also corresponding to each particular number of nodes are taken and recorded.

Oscillograph Films

173 Fig. 63 gives reproductions of oscillograph records of a wheel tested in the wheel-testing machine. At A is the record taken to determine the standing frequency. The sine curve T is the timing wave corresponding to the alternating current from supply mains. The curve S is the record of the stationary exploring coil.

The vibration of the wheel as disclosed by this curve is caused by the action of the alternating-current magnet. When the frequency of the current in this magnet has been brought up to a value corresponding to one of the nodal frequencies of the wheel, a relatively large vibration of the wheel ensues, as is recorded on the film at S.

174 If now the test wheel be set in rotation, the current in the alternating-current magnet having been discontinued, the stationary exploring coil produces indications in the oscillograph even though no critical speed be obtained at which wave phenomena can devolop.

175 Indications of this character are shown in the film record reproduced at B. Here, as before, T is a timing wave. The irregular line S is the record of the stationary coil. This represents the wheel running rigidly and entirely free from wave phenomena. It will be noted that various peculiarities in the line repeat themselves at regular intervals, as, for example, at the points P. The distance between like points P represents the time of one revolution of the wheel. By comparison of the timing wave T which in this case was that of an alternating current of 40 cycles, the speed of rotation of the wheel can be accurately determined from the film, and may be checked up with the speed of the turbine driving the test wheel. Thus the line S, when its irregularities regularly repeat themselves, may well be referred to as the autograph of the wheel because it is different for every wheel tested.

176 The record at R on the oscillograph film B is that of the rotating coil. It will be noted that this record shows numerous fluctuations of very small amplitude. These are so small as to be negligible and show that there are no wave phenomena now present in the wheel.

177 Upon increasing the speed of the rotation of the wheel a point is finally reached where a wave train is markedly developed. This is clearly shown in film C. It will be observed that the record of the revolving coil is now in the form of a very marked wave, while the record of the stationary coil is substantially unchanged. This indicates the presence of a traveling wave of considerable magnitude in the turbine wheel. As the wave travels past the rotating coil the adjacent portion of the wheel oscillates backward and forward so as to produce the wave record R. It will now be noted that in the space of one revolution, as indicated by the distance between the points P, there occur three wave crests on each side of the zero line in the record of the rotating coil. Since there are three wave crests on each side of the wheel, making a total of six, this record indicates the presence of a 6-node wave train. Furthermore, the fact that the stationary coil gave the same record as in B indicates that the wave train in the wheel had no apparent effect on the stationary coil. It follows, therefore, that

the waves were stationary in space, and that the test wheel was running at a critical speed corresponding to a 6-node wave train.

178 The wheel was then permitted to run at this speed for a short time whereupon the oscillograph disclosed certain changes taking place in the wave phenomena, as shown in *D*. It will be noted here that the wave motion, as shown by the rotating coil record *R*, has become much more marked, and furthermore, that the stationary coil *S* has lost its original character and shows a series of waves, which upon examination will be found to be just twice as many in number as those now appearing in the rotating coil record *R*. This double frequency wave *S* represents a wave traveling forward in the wheel. The fact that the frequency shown on curve *S* is exactly twice the frequency of the curve R, confirms the fact that we are here dealing with a critical-speed phenomenon in which the backward-traveling wave is stationary in space and which has a forward wave superposed, upon it.

179 These speeds have been given the name "critical speeds" because nearly all vibration accidents have been associated with this condition, and also to distinguish them from other known resonant conditions.

180 At *E* is illustrated what happens upon slightly raising the speed of the turbine wheel over that corresponding to the record produced in *D*. It will be noted that the fixed coil record *S*, and the rotating coil *R* have returned practically to the condition exhibited in *B*, wherein the speed of the wheel was slightly below the critical speed.

181 One of the principal reasons for making rotation tests on the turbine wheel itself is that the various frequencies corresponding to vibration with radial nodes depend on the speed of rotation. It has been explained that the natural periods of vibration of the wheels increase as the wheel is set in rotation. This is readily shown by setting a wheel in resonant vibration with a definite number of nodes, by the use of the alternating current magnet with the wheel at rest, and then, while the wheel is in violent vibration, setting it in rotation. When the speed is sufficiently increased, vibration dies out. Upon decreasing the frequency of the magnet a sufficient amount, the wheel will again vibrate and a backward-traveling wave will develop having the same number of nodes as before. Furthermore, a similar but forward-traveling wave may be produced by sufficiently increasing the frequency of the magnet. When, therefore, for a given speed of rotation a certain frequency of impulse is applied to the wheel, less than that which would set the wheel into vibration if the wheel were at rest, a backward-traveling wave in the wheel may be induced, while if a certain higher frequency were applied, a wave traveling forward in the wheel at the same speed in the wheel as the backward-traveling wave, may be induced. These relationships have been explained in connection with the frequency-speed diagrams,

Figs. 32 and 35, and the unprovoked occurrence together with the recording of such waves has been noted.

Demonstration of a Single Wave in the Testing Machine

182 The definite excitation of traveling waves under other than critical conditions is accomplished in the wheel-testing machine

Fig. 64 Coil-Carrying Wheel Equipped with Three Coils Spaced 30 Deg. Apart for Detailed Analysis of Wave Motion

by means of the alternating-current magnet placed within the shell opposite the periphery of the wheel. The existence and direction of motion of such traveling waves in response to artificial excitation have been demonstrated further by a special series of tests using three oscillograph coils placed at intervals of 30 deg. on the carrier wheel as shown in Figs. 64 and 65.

183 In order to interpret the films properly it is necessary to note that the wave crest of the oscillograph record is normally

displaced one-quarter cycle from the crest of the wave in the wheel. Although each cycle on the film represents one vibration cycle, the actual motion of the wheel is recorded only indirectly by the film. The oscillograph records the induced voltage (with a negligible electrical lag) and the voltage reaches its maximum when the change of air gap is occurring most rapidly, that is, when the wheel is in its neutral position. Similarly, when the wheel pauses at its extreme position there is an instant of zero voltage

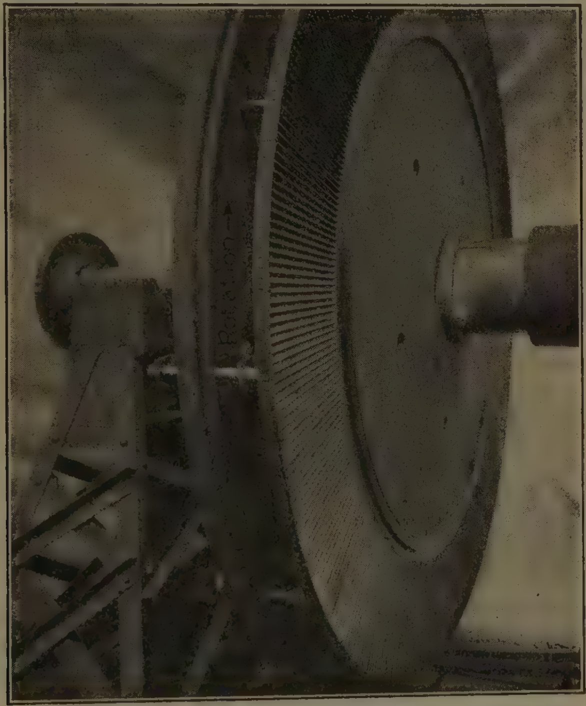

FIG. 65 ASSEMBLY OF TEST WHEEL WITH THREE REVOLVING EXPLORING COILS

during which the motion and induced voltage reverse their directions. Fig. 66 illustrates these relations. In all three coils the polarity was the same and also in each case the convention shown in Fig. 66 applies, namely, when the oscillograph record slopes downward to the right the wheel has a plus deformation and when it slopes upward to the right the deflection is minus.

184 The frequency-speed diagram for the wheel shown in Fig. 65 is given in Fig. 67. The curves for revolving coil frequencies and forward and backward wave frequencies as measured by a fixed coil are given for 4, 6, and 8 nodes.

185 At a speed of 5 r.p.s. a backward wave would register on a fixed coil a frequency of 50.5 cycles per second as shown at *A*, Fig. 67. The test shows the development of such a wave. The wheel was brought up to a speed of 5 r.p.s. and the alternating-current magnet frequency was made 50.5. A record represented by *B*, Fig. 68 was taken. The numbering of the coils is shown at *A*, No. 1 being the leading coil in every case. On the records of the coils, numbered 1, 2, and 3 to correspond with the diagram at *A*, is shown a deformity of the sine wave caused by the stationary alternating-current exciting magnet as each coil in turn passed it. This deformity serves to indicate the velocity of the wheel relative to that of the wave. The projections of the deformity on the zero axis are joined by a line marked " wheel slope ". The horizontal projection of this line represents the time taken by the

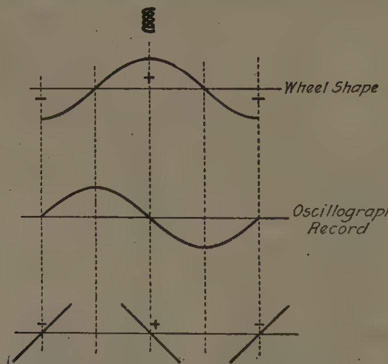

FIG. 66 THE OSCILLOGRAPH RECORD DIFFERS 90 DEG. FROM THE WHEEL MOTION. THE CONVENTION INDICATED ENABLES THE FILMS TO BE ANALYZED

wheel to rotate the 60 deg. separating coil No. 1 from coil No. 3. The actual speed of rotation given by the electric tachometer thus permits the calibration of the horizontal or time scale of the film and gives the frequency of the disk as recorded by the revolving coils as 60.5 as shown at *B*, Fig. 67.

186 The determinations of the direction and speed of the wave travel and the shape of the wave are made as follows: Since the coils are 30 deg. apart, the three traces on the record at *B*, Fig. 68, represent the motion of the edge of the wheel from its neutral position due to the wave motion at those three points on the circumference at any particular instant of time. Utilizing the convention explained in Fig. 66, the shape of the wheel at any instant is shown by the intersection of any vertical line with the three wave records. A series of vertical lines are drawn representing successive instants of time *a* to *j*. For each instant, a separate diagram is made at *C* and the corresponding deformation at each coil is noted.

187 An examination of Fig. 68, *a* to *j*, shows that a 4-node figure is definitely determined and at the same time the direction of rotation of the wave with respect to the three rotating coils is indicated. In this case it will be seen to be in a direction opposite to the rotation of the wheel itself. In order to determine the velocity of the wave, the points corresponding to the intersections of the zero line by the wave records of the three coils may be joined with a straight line as was done in the case of the wheel. This line is marked "wave slope" and the horizontal projection indicates the time taken for the wave to traverse 60 deg. or ⅙ of a revolution. It is plain from Fig. 68 that the wave, which is

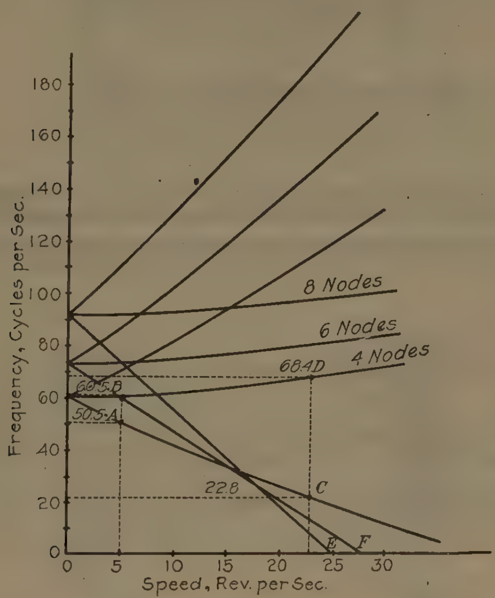

FIG. 67 FREQUENCY-SPEED DIAGRAM FOR 3-COIL TEST

traveling backward in the wheel, moves 60 deg. in a much shorter time than the wheel itself and hence the wave must actually be progressing backward in space.

188 A second test is illustrated in Fig. 69. The speed was brought up to 22.8 r.p.s. as shown in Fig. 67 at *C*. At this condition it will be noted that the frequency of the 4-node backward wave relative to a fixed point is exactly equal to the speed of rotation in r.p.s. This is called a minor resonant speed for 4 nodes, a condition which will be discussed presently. An examination of the records in the same way as described for Fig. 68 shows a 4-node backward wave with a speed of rotation greater than the wheel speed so that the wave is progressing backward in space. The

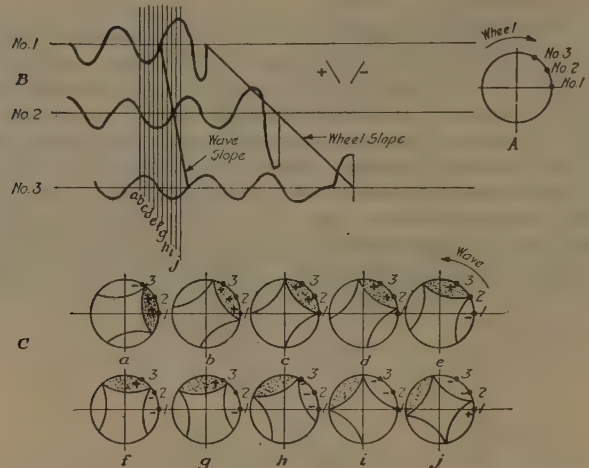

FIG. 68 4-NODE BACKWARD WAVE AT 300 R.P.M. MAINTAINED BY A STATIONARY A. C. MAGNET, FREQUENCY 50.5 (SEE A, FIG. 67)

(The disk frequency is 60.5. The wave velocity exceeds the wheel velocity and therefore the wave travels backward in space as well as in the wheel.)

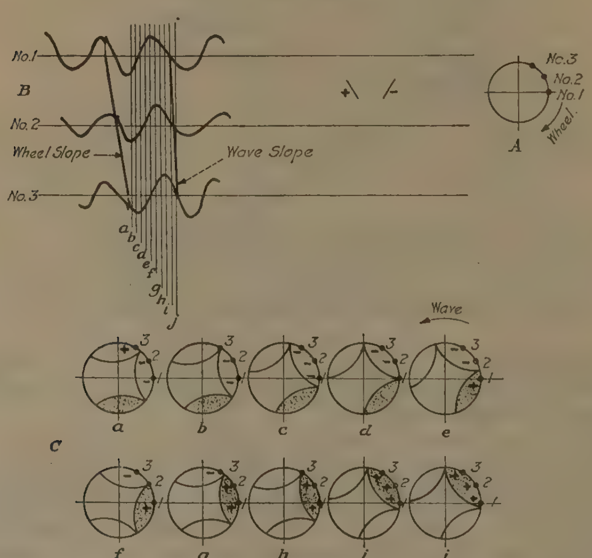

FIG. 69 4-NODE BACKWARD WAVE AT 1370 R.P.M., THE FIRST MINOR RESONANCE (SEE C, FIG. 67)

(The wave velocity exceeds the wheel velocity and therefore the wave travels backward in space as well as in the wheel.)

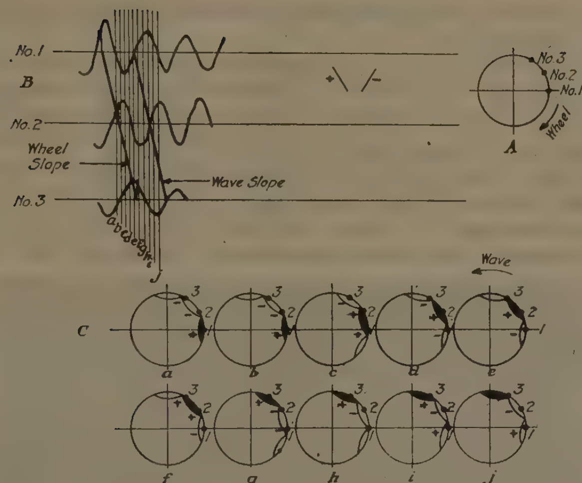

FIG. 70 8-NODE CRITICAL SPEED AT 1480 R.P.M. (SEE *E*, FIG. 67)

(The backward wave velocity in the wheel equals the forward wheel velocity, so that the wave is stationary in space.)

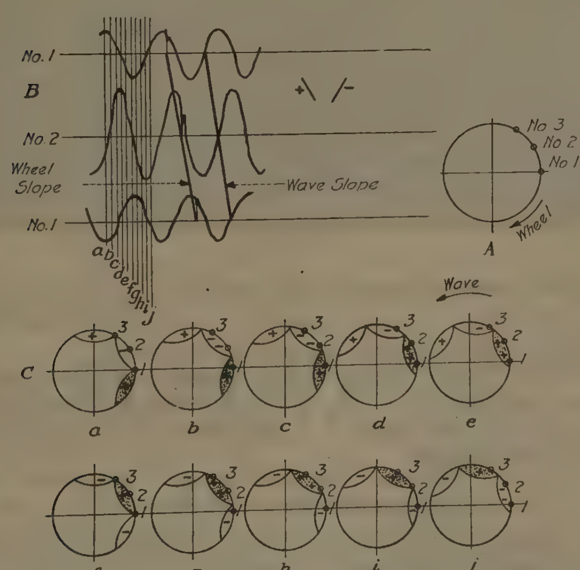

FIG. 71 6-NODE CRITICAL SPEED AT 1660 R.P.M. (SEE *F*, FIG. 67)

(The backward wave velocity in the wheel equals the forward wheel velocity, so that the wave is stationary in space.)

frequency of the wheel relative to the rotating coils is 68.4 as shown at D, Fig. 67.

189 Fig. 70 represents the condition observed with the wheel rotating at 24.7 r.p.s. The phenomenon recorded corresponds to the point E, Fig. 67. In this case the wheel and the wave slopes have exactly the same angle so that the velocity of the wave equals that of the wheel. The analysis shows the wave to be backward relative to the wheel, and therefore in this case the wave was stationary in space. This as an 8-node critical speed. Each particle of the wheel circumference traversing this wave passes through four high spots on each side of the wheel and would thus, in case of contact with the diaphragm, result in rub-

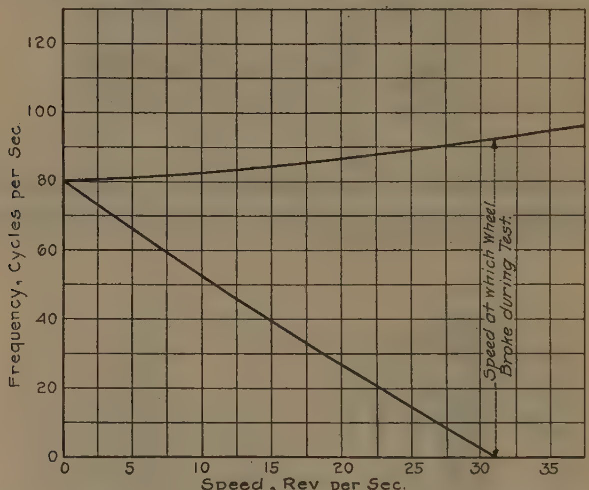

Fig. 72 Frequency-Speed Diagram for 6 Nodes for Wheel Shown in Fig. 73

bing at 4 equidistant spots. Should rubbing occur on the opposite side of the wheel, it would result in 4 equidistant spots intermediate to the spots on the first side of the wheel.

190 Fig. 71 illustrates a 6-node critical speed occurring at 27.7 r.p.s. as shown at F, Fig. 67. In this case if rubbing should occur it would take place at 3 equidistant spots on each side, with alternate spacing.

Confirmation of Breakages

191 It has been the custom, since the first vibration troubles were successfully explained, to make tests in all the cases of serious wheel or bucket trouble. Either a duplicate wheel is made or, if in a suitable condition, the wheel that suffered the injury, is itself used. The first test made in this manner was of the 17th

stage wheel of a 20,000-kw., 1500-r.p.m., 23-stage turbine which developed a traveling wave, as determined by oscillograph record, while under load in the operating turbine.

192 By revolving this wheel in the wheel-testing machine and applying the alternating-current magnet at its rim, the fundamental relations between the applied force and the responding vibration were determined. Using this information together with that obtained from the oscillograms taken under load conditions it was possible to construct Fig. 72. From this diagram a prediction could be made that at a speed of about 31 r.p.s. a contin-

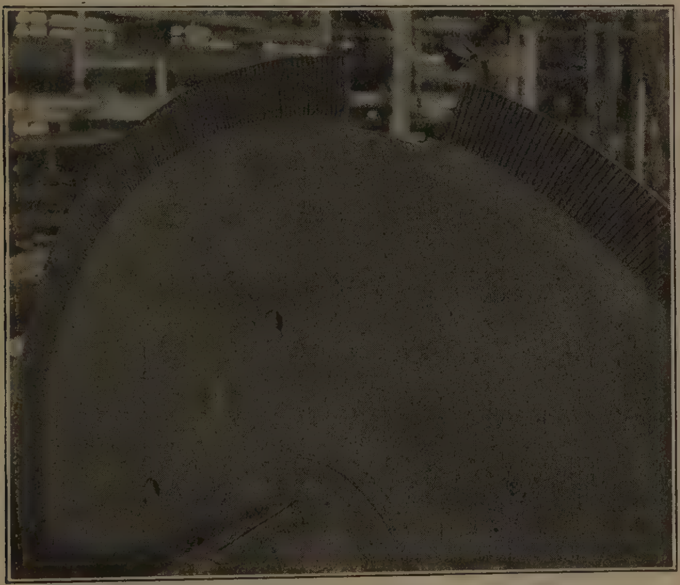

Fig. 73 Turbine Wheel Broken in Testing Machine by Rotation at 6-Node Critical Speed. 17th Stage of 20,000-Kw. 1500-R.p.m. 23-Stage Turbine

uous force applied at a fixed point in space should throw the wheel into vibration with a 6-node stationary wave.

193 This prediction was tested as follows: With no current on the magnet the wheel speed was brought up through the predicted danger point and nothing was observed. The wheel was then slowed down somewhat below that point and the electromagnet was energized by direct current furnishing a fixed force. The speed was then raised almost to the predicted point, when the wheel developed a violent lateral vibration, rubbing off the pole piece of the exciting magnet and very soon tearing out the buckets from the wheel rim as shown in Fig. 73, a striking corroboration

of the prediction and an indication of the seriousness of operating at a critical speed.

194 The 11th-stage wheel of a 12-stage double-flow turbine in a 30,000-kw., 1500-r.p.m. machine had failed in a manner illustrated in Fig. 5. A duplicate wheel in the same turbine was

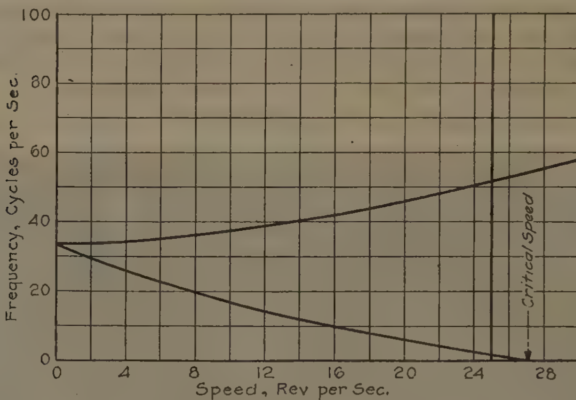

FIG. 74 FREQUENCY-SPEED DIAGRAM FOR 4 NODES. 11TH STAGE OF 30,000-KW. 1500-R.P.M. 12-STAGE TURBINE

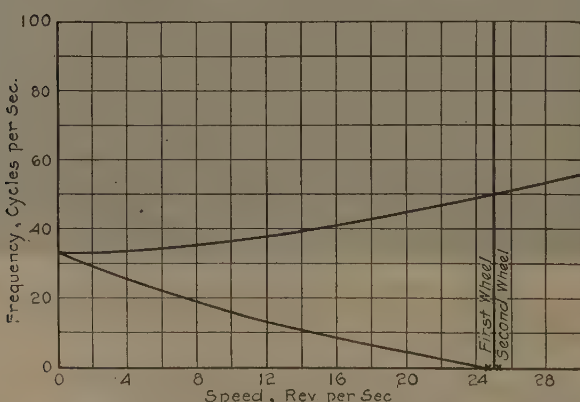

FIG. 75 FREQUENCY-SPEED DIAGRAM FOR 4 NODES. 11TH STAGE OF 30,000-KW. 1500-R.P.M. 12-STAGE TURBINE

used for the purpose of making rotation tests. Both wheels had been in service and operated several years. Fig. 74 was made from data obtained from a test of the duplicate wheel. A 4-node critical speed due to a wave stationary in space is indicated at 27 r.p.s. just above the running speed of 25 r.p.s. It is not unlikely that the wheel that failed differed slightly, due to minor variations in machining, from the duplicate which was tested. Such a variation

might very probably bring its actual wave speed equal to running speed.

195 The wheel in the testing machine, when operated at the speed of the 4-node wave stationary in space, developed heavy vibration and threw off a piece of shroud band which previously had been slightly damaged in the accident to the broken wheel. It was observed that the diaphragm opposite the wheel in service which had failed, had been deeply cut by the shroud band of the wheel, whereas the wheel used in test had no such contact with the diaphragm.

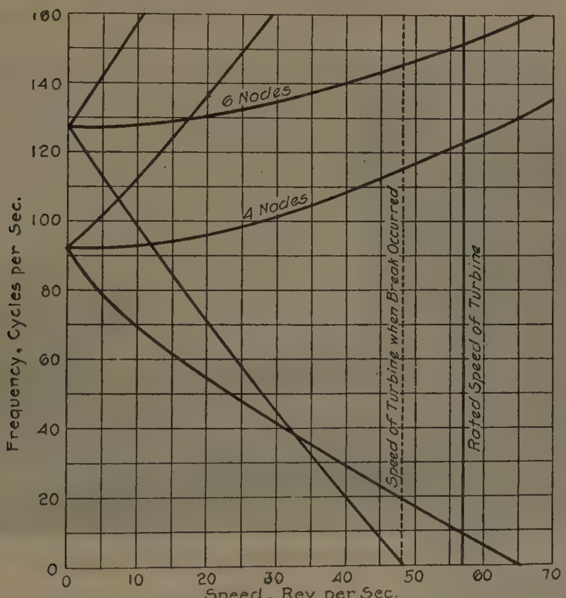

FIG. 76 FREQUENCY-SPEED DIAGRAM FOR 3D STAGE OF 6000-HP. MARINE TURBINE

196 The natural frequencies taken on another pair of duplicate wheels and also of the same dimensions as the former wheel yielded results from which Fig. 75 is plotted. Here, close coincidence with the operating speed is indicated. That trouble had not occurred with these wheels is attributed to the absence of sufficient exciting force.

197 Still another duplicate wheel in another station had been known repeatedly to rub the diaphragm. All wheels of this type were immediately replaced as soon as the cause of the accident to the broken wheel became known.

198 Another typical case was that of a third-stage wheel of a 6000-hp. turbine used for ship propulsion. Fig. 7 is a portion

of the broken wheel showing two breaks which occurred through steam pressure equalizing holes. Figs. 8 and 9 show the development of fatigue fractures which originated, in each case, at each side of a hole. Previous to the failure of the wheel under investigation, the ship had been continuously cruising with a propeller speed of 84 r.p.m. owing to foggy weather. At the time of the accident the tachometer showed 84 r.p.m. of the propeller, corresponding to a turbine speed of 48 r.p.s.

199 The vibration tests were carried out on a duplicate wheel made from the same drawing as the broken one. This wheel was placed upon its own shaft in order to determine the standing frequencies. It was afterward run in the wheel-testing machine to

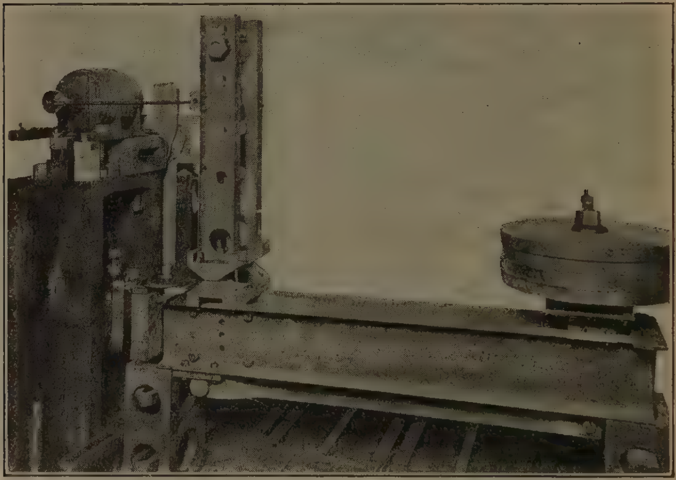

Fig. 77 Turbine-Bucket Vibration Machine

determine the effect of the centrifugal force. These results are plotted on Fig. 76 which shows the 4- and 6-node frequencies. The 6-node critical speed occurred at 48 r.p.s., which was just the speed at which failure actually took place, namely, 84 r.p.m. of the propeller and 48 r.p.s. of the turbine.

200 At this speed, corresponding to the 6-node critical speed of the wheel, the stationary wave could be maintained so that, if this speed were held constant for a considerable time, the wave could develop a large amplitude of vibration and the wheel undergo a series of repeated, or fatigue, stresses which would eventually cause failure. A substitute wheel, designed to replace the broken wheel, was increased in thickness 100 per cent, so that the 4-node critical speed was raised to a point 92 per cent and the 6-node speed to a point approximately 50 per cent above the normal running speed.

Fatigue Cracks in Bucket Dovetails

201 The breaking of bucket dovetails on the last stage of a 30,000-kw., 1800-r.p.m., 17-stage turbine was attributed to wave phenomena, in which the speed of the backward wave was equal to that of the speed of rotation. The nature of the break indicated that axial motion of the bucket system had been occurring for a considerable time before rupture took place.

202 The break occurred at the dovetail and since the vibrational stresses were superposed upon the centrifugal stresses, it was realized that a bucket, to be broken in a similar manner, should be subjected to both kinds of stress. In order to accomplish this, a special machine shown in Fig. 77 was devised, in which it was possible to apply the tension load to the bucket dovetail which it would have in a wheel running at normal speed. The vertical member shown at the left hand end is allowed to rock around the metal ribbons at its base. The heavy weights together with the lever shown below the base create the necessary tension force on the specimen. The upper end of the bucket is secured to the top end of the vertical member and the section of the wheel at the lower end is secured to the block sliding in the base. This block is then pulled down by the lever already referred to. Fig. 78 illustrates the bucket and wheel section assembled in the vertical rocking member. This arrangement allows lateral motion of the bucket and wheel with a minimum variation of tension during the swing. The vertical rocking member is vibrated by means of a motor-driven eccentric to which a revolution counter is attached.

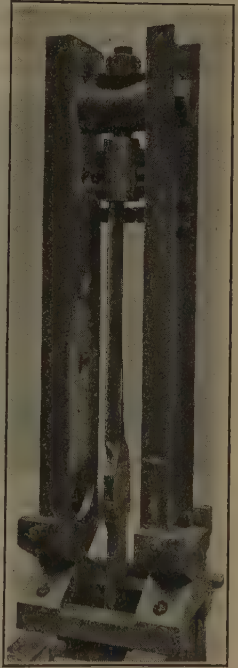

Fig. 78 Detailed View of Bucket in Vibrating Machine

203 Fig. 79 shows at *B* and *C* breakages at the lower tangs of two buckets that occurred in service after two months' operation. The specimen shown at *A* was a duplicate bucket fitted together with a cross-section of the wheel machiried to the same width as the bucket. A smooth crack will be noticed in the lower left-hand tang of the bucket. This was produced by means of the apparatus explained, after six million repetitions of the stress.

204 In the case of the wheel breaking in service no rubbing had been encountered. The amplitude of motion used in the test

machine was equal to one-half the clearance between the shroud band and diaphragm.

205 Upon separating the pieces shown at *A* the surface conditions of the fracture were found to be of the same nature as in the case of the service wheel.

Effect of Forced Rubbing on Vibration at Speeds Other than Wheel Critical Speed

206 Since the efficiency of a steam turbine depends to a certain extent upon the clearance between the discharge edges of the nozzles and the entrance edges of the buckets, a test was made to show the effect of rubbing of the wheels on the diaphragm in producing vibration in the wheels.

Fig. 79 Broken Turbine-Bucket Dovetails. 17th Stage of 30,000-Kw. 1800-R.p.m. 17-Stage Turbine

(The break at *A* was produced in the vibrating machine, Fig. 77. Breaks *B* and *C* occurred in service.)

207 The object was to determine the possibility of causing and maintaining lateral vibration in a steam-turbine wheel by exerting pressure at the rim by means of a rubbing block and also with a steam nozzle. The tests were made on a large-diameter wheel, shown in Fig. 80, of unusual flexibility. The wheel had been in service (3d stage) in a 20,000-kw., 1200-r.p.m., 9-stage turbine.

208 The wheel-testing machine was used and a brake shoe was arranged for the purpose of causing contact. This method of applying pressure would be somewhat similar to the worst condition that could occur in a turbine, namely, rubbing at one place. The brake shoe and method of applying pressure is illustrated in Fig. 81. A rope was attached to the external lever and extended to the instrument room. Rubbing pressure could be brought about at will during observation of wheel vibration. Visual observation

was made on the ground-glass screen of the oscillograph. When the revolving recording coil showed either vibration or erratic scratching the shoe was quickly released and the wheel permitted to run free. This was repeated for various speeds of the wheel. Fig. 82 is a diagram of the complete information obtained from this test. Standing frequencies were obtained in the usual manner, films recording critical and minor resonant speeds (which will be discussed presently) are indicated by stars. As an alternative to the rubbing shoe a steam nozzle projecting high-pressure steam in an axial direction could be substituted.

209 It was noticed that when running at a minor resonant speed the wheel forms a definite wave due to the shoe contact but when not at such a speed the record is very erratic, showing that no definite form of vibration took place. Fig. 83 shows the record made during the rubbing period at other than a critical speed. It was impossible to push the wheel into a definite state of vibration even though a large pressure was exerted.

210 When the shoe was released the wheel usually assumed a 4-node vibration even though it previously had been vibrating in a more complex type. The 4-node type is the one most frequently assumed when a standing wheel is struck a single blow. After the 4-node vibration died out the wheel always ran true again except when running at some critical or minor resonant speed. Figs. 83, 84, and 85, show the wheel in the three states previously referred to at other than a critical or resonant speed, Fig. 83 being made with the shoe in contact, Fig. 84 just after release, and Fig. 85 about a minute later.

211 No tests were made with the rubbing bar at critical speeds, the steam jet showing that amplitudes could be built up to a large amount at these speeds. The wheel when examined after test had the rim blued by heat developed from friction.

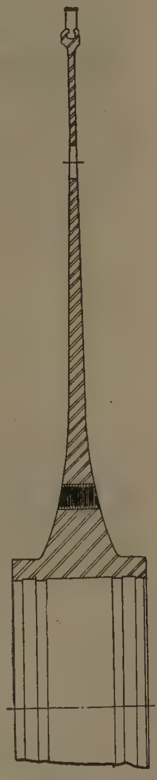

Fig. 80
Turbine Wheel
Used in
Rubbing Test

Effect of Temperature on Frequency of Turbine Wheels

212 Fig. 86 shows the lowering of frequency due to differential temperature. These tests were made in the wheel-testing machine, a gas flame being projected on the rim of the wheel during slow rotation. Thermocouples placed at the rim and the hub, and connected to the collector rings at the end of the shaft, were used

for the purpose of measuring the temperature at these two points. It is seen that there are only slight changes in the 2- and 4-node frequencies, due to a temperature difference of 200 deg. fahr., but greater differences are noticeable with six and eight nodes. These temperature differences are only possible momentarily in service

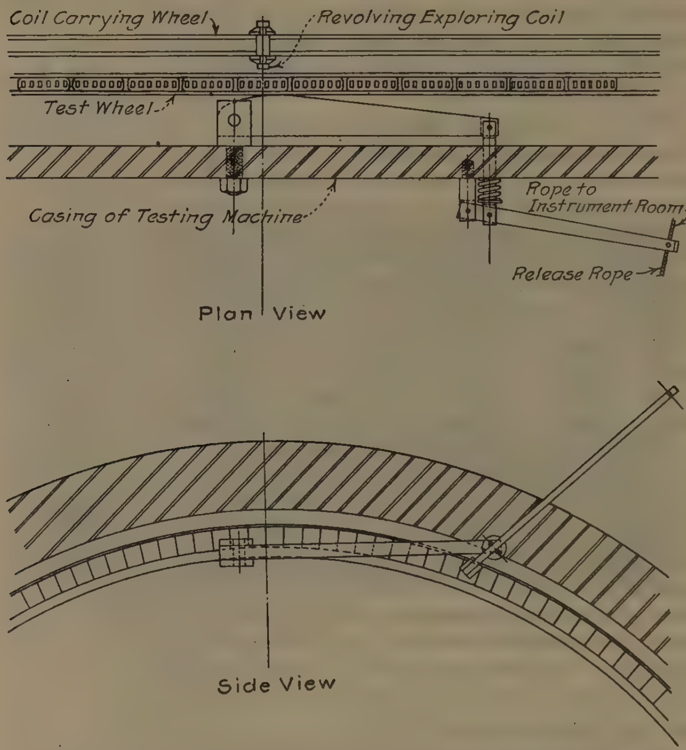

FIG. 81 DETAIL OF RUBBING BAR USED IN RUBBING TEST

and wheels at the hottest end of the turbine are usually the only ones affected in this way.

213 Actual measurements of temperature differences in operating turbines between the inner and outer diameters of the diaphragm have not shown anything like these temperature differences. However, it is believed that sufficient margins for variations of this character have been allowed for in the safety limits adopted which prescribe appropriate margins from normal operating speeds for each of the critical speeds.

Effect of Bucket Tightness on Frequency and Critical Speed

214 The method of fastening buckets to turbine wheels has a very important bearing on frequencies obtained under running

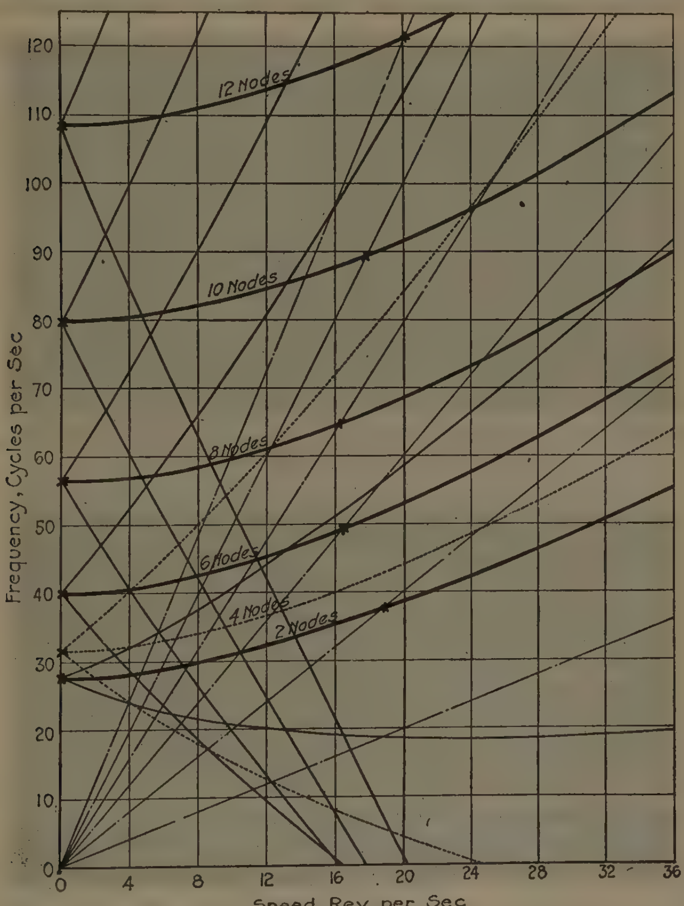

Fig. 82 Frequency-Speed Diagram for Wheel Used in Rubbing Test

conditions. This effect has been very definitely shown from many tests of which a few typical examples are given.

215 Several wheels were built with bucket dovetails of the type to be inserted in a grooved rim. An attempt was made to obtain a tight fit by initial tension in the neck of the bucket instead of forcing the dovetail head into the groove, which is

the common practice. Extra precautions were taken in an attempt to have an initial stress on the narrow neck section of the bucket sufficient to keep this fit tight under running conditions. Fig. 87 shows the various standing frequencies. Test 1 was made on the wheel as originally assembled. Test 2 gives the standing fre-

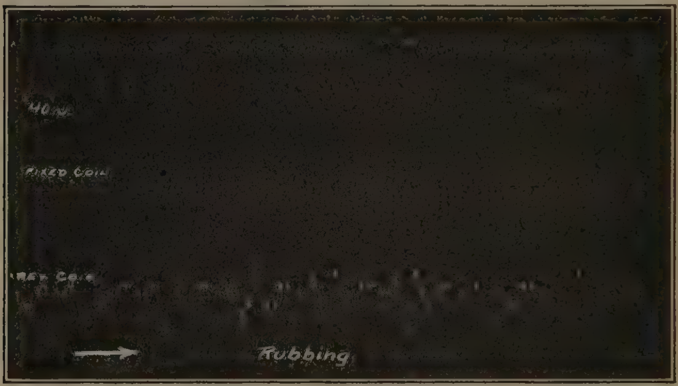

FIG. 83 OSCILLOGRAPH RECORD MADE DURING RUBBING TEST. RUBBING BAR IN CONTACT

FIG. 84 OSCILLOGRAPH RECORD MADE DURING RUBBING TEST. RUBBING BAR JUST RELEASED

quencies after light calking of the bucket and Test 3 after heavy calking at the same point. It will be noted that between Test 1 and Test 2 a considerable change in the 2-node frequency occurred, much smaller differences occurring with the other types of vibration. Fig. 88 gives the critical speeds for 8, 10, and 12 nodes measured by running Tests 1, 2, and 3 of the wheel under the same conditions, respectively, as shown in Fig. 87. It will be

noted that the calking caused very large changes in these critical speeds.

216 Apparently, in spite of the attempts to insure tightness of the dovetail under running conditions, this was not obtained without the calking operation. The usual type of dovetail, in which the fit is obtained by forcing the dovetail in the groove, is much more consistent in this respect, the first type being unreliable in use because there is no assurance that a tight fit can be maintained.

217 Tests on the standard-type dovetail, which is forced into the groove, revealed the fact that, while a very considerable change in the standing frequency was brought about by removing the buckets and replacing them loosely, the running tests showed very little change in the critical speeds in the two cases. In the

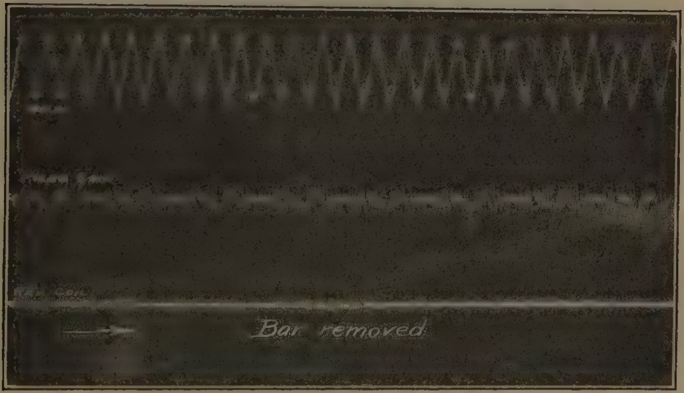

Fig. 85 Oscillograph Record Made During Rubbing Test. A Moment After Release of the Rubbing Bar

standing condition the stiffness due to the dovetail fit had been changed but in both cases during running conditions the centrifugal forces insured a tight fit regardless of whether or not the buckets had been assembled loosely in the dovetail.

218 A somewhat similar case occurred with the type of dovetail in which the bucket straddles the rim. In this type of fastening calking is resorted to along the wheel rim for the purpose of insuring tightness. Table 3 gives the effect on the frequency and critical speed of changes in the tightness of the dovetail. Columns 1, 2 and 3 give the standing frequencies, while columns *A, B* and *C* give the critical speeds recorded during running tests. Columns 1 and *A* represent the original assembly after the buckets had been assembled with relatively heavy calking. Columns 2 and *B* give the results after the buckets had been removed and replaced without recalking. Columns 3 and *C* give the same kind of information after the wheel had been recalked.

219 It will be noted that both the frequencies and the critical speeds were lowered by removing the buckets from the wheel and reassembling, but both the standing frequencies and the critical speeds were raised to practically the normal conditions upon recalking.

TABLE 3 EFFECT OF BUCKET TIGHTNESS ON FREQUENCY AND CRITICAL SPEED

	Standing Frequencies			Running Critical Speeds		
	Original assembly	Loose buckets	Buckets recalked	Original assembly	Loose buckets	Buckets recalked
Nodes	1	2	3	*A*	*B*	*C*
4	48	45.2	47.4	34.5	33.7	34.4
6	56	52.4	55.2	21.6	20.8	21.6
8	68	63.2	67.0	18.4	17.9	18.3
10	78	72.8	77	16.6	16.0	16.6
12	87	80.8	86.8	15	14.3	15.2

Condition for Two-Node Resonance

220 Little has been said so far with regard to the 2-node resonant speed. One reason is that it has rarely been found to have

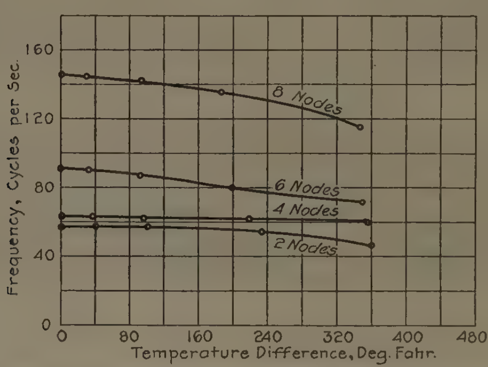

Fig. 86 Effect of Differential Temperature on Frequency (Hot Rim and Cold Hub)

caused trouble. A second reason is that a 2-node wave in a rotating turbine wheel has never been known to be stationary in space. Therefore the same identical causes that produce standing waves with a larger number of nodes cannot be expected to act in the case of the 2-node wave trains.

221 Two-node vibrations differ from all other radial nodal vibrations in that the forces involved form a couple that is opposed by a corresponding couple in the shaft, whereas in the case of 4, 6, 8, etc. nodes all forces are balanced within the disk. Thus an oscillating motion of one end of the shaft would be expected to cause a 2-node vibration if the frequency of the applied force were equal to the resonant frequency of the disk. This is found to be actually the case when a small unbalanced motor is bolted to the

end of the shaft with the armature at right angles to the wheel shaft.

222 The 2-node frequency is affected by the centrifugal force in a manner similar to that of other types of vibration. Thus the frequency when running, according to Equation [5] is

$$f_r = \sqrt{f_s^2 + BN_s^2}$$

223 If it were possible for the 2-node wave to be stationary in space, this would occur when

$$f_r = \tfrac{1}{2}nN_s$$

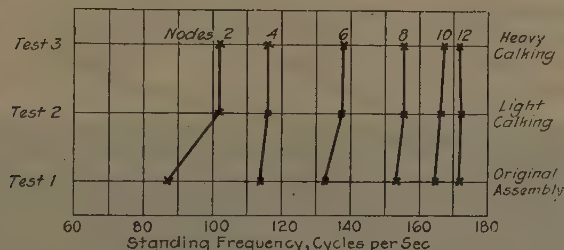

FIG. 87 EFFECT OF DOVETAIL TIGHTNESS UPON STANDING FREQUENCY

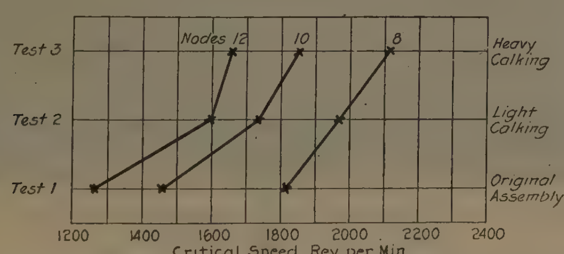

FIG. 88 EFFECT OF DOVETAIL TIGHTNESS UPON CRITICAL SPEED

224 Substituting this in the above formula the critical speed would be

$$N_s = \sqrt{\frac{f_s^2}{(\frac{1}{2}n)^2 - B}} \quad . \; . \; . \; . \; . \; . \; . \; . \quad [15]$$

225 When B is greater than unity, and n equals 2, it can be seen that N_s, the critical speed, would be imaginary. Since B is always found to be above unity except under very unusual conditions, it is therefore believed that 2-node wave trains stationary in space do not occur.

226 However, in the wheel-testing machine 2-node records have been made at what is called a 2-node minor resonant speed. Referring to Fig. 89, which is a frequency-speed diagram for two nodes, it will be seen that there is a diagonal line drawn from the zero corner and passing through all points where the frequency

is equal to the speed of rotation. This intersects at point x the backward-wave frequency line which, it will be recalled, is also the resonant frequency line for an applied impulse, fixed in space, such as the exciting magnet would give. It is at this speed that

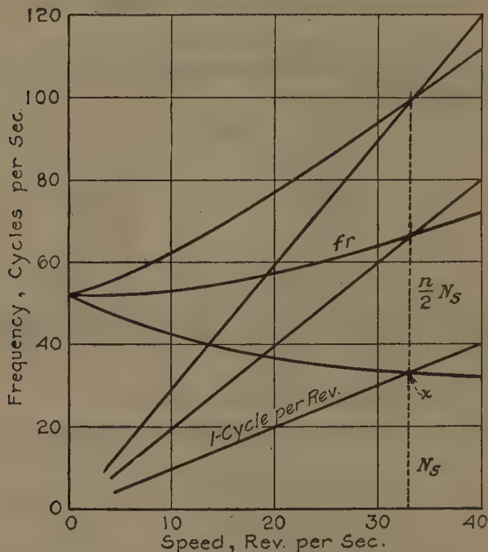

FIG. 89 FREQUENCY-SPEED DIAGRAM FOR 2 NODES. THE POINT x LOCATES THE FIRST MINOR RESONANT SPEED

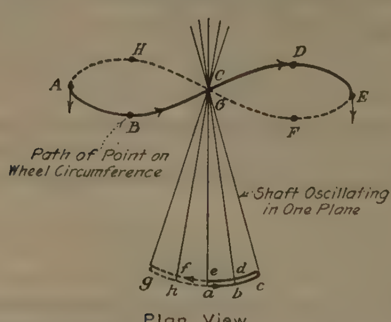

FIG. 90 DIAGRAM SHOWING RELATION BETWEEN DYNAMIC UNBALANCE AND 2-NODE VIBRATION

the periodic effects of shaft unbalance may excite a 2-node wheel vibration.

227 It will also be seen that a turbine wheel is in a resonant condition when the natural 2-node frequency for a particular speed, as recorded by a revolving coil, is just twice the revolutions per second of the wheel itself, so that it results in a wave being

set up with a wave velocity equal to twice that of the wheel itself. It is possible that a simple vibration shape, (i.e., fixed nodes in the disk) could be maintained if the wheel were run in a perfect vacuum, but under other conditions the high velocity component wave would receive a great amount of damping, probably greatly reducing the amplitude, while not so affecting the backward-traveling wave. The high velocity wave alone would travel with three times the wheel velocity while the backward wave would have a velocity in space equal to that of the wheel but in an opposite direction.

228 The path of a bucket during shaft vibration maintained by dynamic unbalance, which might excite a 2-node resonant vibration, is shown in Fig. 90. Dynamic unbalance tends to make the shaft oscillate mostly in one plane, the horizontal stiffness of

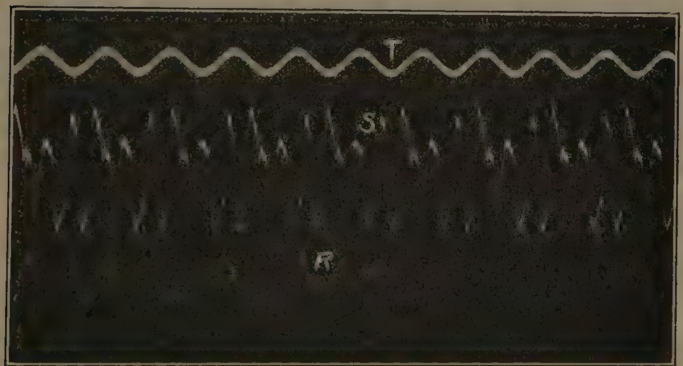

FIG. 91 OSCILLOGRAPH RECORD SHOWING 2-NODE FORWARD WAVE AT FIRST MINOR RESONANT SPEED

the bearing supports usually being less than the vertical stiffness. In the neutral position it exerts no force on the bucket. When it swings from *a* to *c* as indicated, it pulls the bucket on the path *ABC*. Swinging back along *c, d, e, f, g,* the shaft causes the bucket to travel along *C, D, E, F, G,* and on the return swing *g, h, a,* the bucket travels along *G, H, A*. It is evident therefore that during one oscillation, i.e. one revolution, of the shaft the bucket goes through two complete cycles. Thus the transverse frequency of the bucket due to unbalance is twice that of the revolutions per second of the wheel itself, which checks with the revolving coil frequency for a 2-node resonant condition discussed in the previous paragraph.

RECORD OF TWO-NODE FIRST MINOR RESONANT SPEED

229 Fig. 91 is a standard oscillogram taken on a wheel in which excessive dynamic unbalance was intentionally applied to obtain

a 2-node resonant speed. The curve T is a 40-cycle timing wave. The curve S was registered by the coil stationary in space. The curve R is that registered by a revolving coil.

230 During each revolution the revolving coil passed through the influence of the stationary coil causing the disturbances noticed on the revolving coil record. Comparison of the spacing between these disturbances with the timing wave indicates the speed of rotation. It will be noted that while the revolving coil registers exactly two cycles per revolution the stationary coil registers three cycles per revolution. This indicates a 2-node forward wave which was developed in this special case.

231 From the ratio of these frequencies per revolution information is obtained of what is going on at the time the oscillogram is taken. A backward wave would have registered two cycles per revolution of the revolving coil and one cycle per revolution on the stationary coil. In this case caution must be exercised to note that the one cycle per revolution would develop at the same time as the two cycles per revolution on the revolving coil, since the ordinary autograph of a non-vibrating wheel might appear to register one per revolution at any speed, whereas a true backward wave would appear only at the instant of the vibration developing.

Minor Resonant Speeds

232 It is possible to excite and record minor resonant speeds for other nodal systems.

233 From a consideration of the equation of the 2-node minor resonant speed as well as the major resonant speed called critical speed, it can be shown for two nodes (and it also is true for other nodes) that some of the resonant conditions do not occur. The critical speed, if one exists, occurs at a speed given by Equation [15]

$$N_s = \frac{f_s}{\sqrt{(\frac{1}{2}n)^2 - B}}$$

For 2 nodes $(\frac{1}{2}n)^2 = 1$ and the radical becomes $\sqrt{1-B}$. Now in the case of turbine wheels if $B > 1$, N_s is imaginary, namely, the 2-node backward-wave frequency line never intersects the zero line. There is, then, no critical speed for 2-node vibration. The danger from two nodes, if present, would be expected to occur at the minor resonant speed. The equations for first minor resonant speeds for any system are found as follows:

234 Referring to Fig. 92 it will be seen that the running frequency for a minor resonant speed is

$$f_r = N_s + \tfrac{1}{2}nN_s$$

Substituting in Equation [5]

$$N_s + \tfrac{1}{2}nN_s = \sqrt{f_s^2 + BN_s^2}$$

Solving for N_s

$$N_s = \frac{f_s}{\sqrt{(1+\frac{1}{2}n)^2 - B}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [16]$$

The equations for determining the first *sub-minor* resonant speed, indicated as the speed where the backward wave frequency line intersects the 1-cycle per revolution line below the zero frequency line is derived as follows:

235 Referring to Fig. 92,

$$\tfrac{1}{2}nN_s - N_s = \sqrt{f_s^2 + BN_s^2}$$

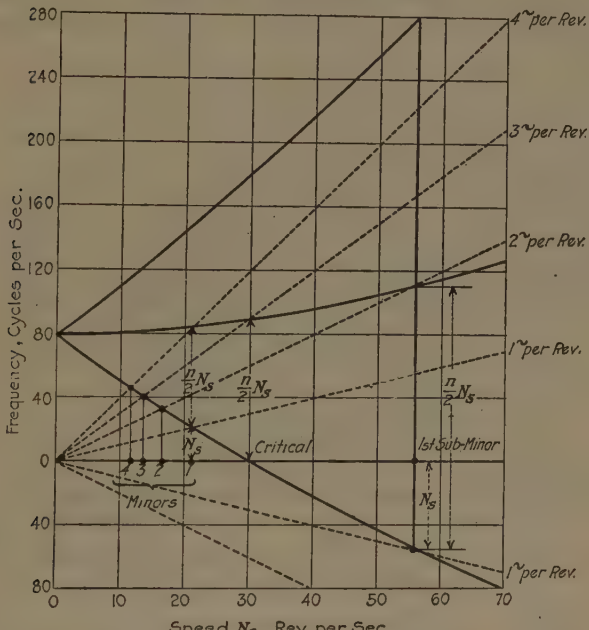

FIG. 92 FREQUENCY-SPEED DIAGRAM SHOWING MINOR, CRITICAL, AND SUB-MINOR RESONANT SPEEDS

The first sub-minor resonant speed then equals

$$N_s = \frac{f_s}{\sqrt{(\frac{1}{2}n - 1)^2 - B}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [17]$$

The second and third minor and sub-minor resonant speeds can be determined in a similar manner, the point of intersection in these cases being the 2-cycle and 3-cycle per revolution lines, respectively.

236 From the formulas in Table 4 the various resonant speeds can be calculated when the standing frequency f_s and the speed coefficient B are known for the particular number of nodes n.

TABLE 4 FORMULAS FOR RESONANT SPEEDS

		Nodes 2	4	6	8	10
3d Minor Resonance	$N_{s_3} = \dfrac{f_s}{\sqrt{\left(3+\frac{n}{2}\right)^2 - B}}$	$\dfrac{f_s}{\sqrt{16-B}}$	$\dfrac{f_s}{\sqrt{25-B}}$	$\dfrac{f_s}{\sqrt{36-B}}$	$\dfrac{f_s}{\sqrt{49-B}}$	$\dfrac{f_s}{\sqrt{64-B}}$
2nd Minor Resonance	$N_{s_2} = \dfrac{f_s}{\sqrt{\left(2+\frac{n}{2}\right)^2 - B}}$	$\dfrac{f_s}{\sqrt{9-B}}$	$\dfrac{f_s}{\sqrt{16-B}}$	$\dfrac{f_s}{\sqrt{25-B}}$	$\dfrac{f_s}{\sqrt{36-B}}$	$\dfrac{f_s}{\sqrt{49-B}}$
1st Minor Resonance	$N_{s_1} = \dfrac{f_s}{\sqrt{\left(1+\frac{n}{2}\right)^2 - B}}$	$\dfrac{f_s}{\sqrt{4-B}}$	$\dfrac{f_s}{\sqrt{9-B}}$	$\dfrac{f_s}{\sqrt{16-B}}$	$\dfrac{f_s}{\sqrt{25-B}}$	$\dfrac{f_s}{\sqrt{36-B}}$
Critical Speed	$N_s = \dfrac{f_s}{\sqrt{\left(\frac{n}{2}\right)^2 - B}}$		$\dfrac{f_s}{\sqrt{4-B}}$	$\dfrac{f_s}{\sqrt{9-B}}$	$\dfrac{f_s}{\sqrt{16-B}}$	$\dfrac{f_s}{\sqrt{25-B}}$
1st Sub-Minor Resonance	$N_{sA} = \dfrac{f_s}{\sqrt{\left(\frac{n}{2}-1\right)^2 - B}}$			$\dfrac{f_s}{\sqrt{4-B}}$	$\dfrac{f_s}{\sqrt{9-B}}$	$\dfrac{f_s}{\sqrt{16-B}}$
2nd Sub-Minor Resonance	$N_{sB} = \dfrac{f_s}{\sqrt{\left(\frac{n}{2}-2\right)^2 - B}}$				$\dfrac{f_s}{\sqrt{4-B}}$	$\dfrac{f_s}{\sqrt{9-B}}$
3d Sub-Minor Resonance	$N_{sC} = \dfrac{f_s}{\sqrt{\left(\frac{n}{2}-3\right)^2 - B}}$					$\dfrac{f_s}{\sqrt{4-B}}$

237 From the similarity of these formulas it will be noted that assuming the standing frequencies and speed coefficients to be constant, a speed which is critical for one nodal type is resonant as a first minor for the next smaller number of nodes and as a first sub-minor for the next larger number of nodes. Thus in order to appreciate this similarity, suppose that the standing frequencies and speed coefficients are the same. It will be seen that the 6-node critical speed is also resonant for a 4-node first minor and a 2-node second minor on the one hand, and also for an 8-node first sub-minor and a 10-node second sub-minor on the other hand.

238 From the fact that the speed coefficient B is always greater than unity it is easily shown that there are no 2-node critical or

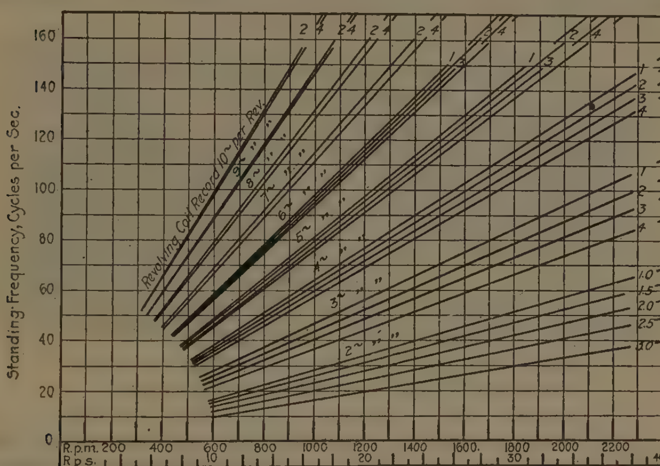

FIG. 93 THE CRITICAL-SPEED CHART. THIS IS USED ALSO FOR THE RAPID DETERMINATION OF MINOR RESONANT SPEEDS

sub-minor resonances, nor any 4-node sub-minor resonances, etc., as indicated by the blank spaces in the table.

239 Fig. 93 is used for the rapid solution of all these formulas. The vertical scale represents the standing frequency f_s in cycles per second. The horizontal scale marked critical speed both in r.p.m. and r.p.s. is also used to determine the other resonant speeds. The diagonal lines, which are grouped for the different nodal systems, are also marked for various values of the speed coefficient B. The lowest group, marked two cycles per revolution, is made first for the 4-node critical speed, the two cycles per revolution meaning that two complete cycles would be recorded on the revolving coil in the wheel-testing machine. The three cycles per revolution group is for the 6-node critical speed, the four cycles per revolution for eight nodes, and so on. However, as

already referred to, since the 2-node first minor resonance is represented by the same formula as the 4-node critical speed, the lowest group is also used for determining the 2-node minor resonant speed. Similarly, the three cycles per revolution group is used for the 4-node first minor and the four cycles per revolution group for the 6-node first minor, and so on. The group below any particular group of diagonal lines is used to determine the sub-minor critical speed for the same particular number of nodes.

240 For example, assuming a wheel with a 4-node standing frequency equal to 50 cycles per second, and with the speed coefficient 2, the major resonant or critical speed will then be at 35.5 r.p.s. For 6 nodes with standing frequency at 57.0 the critical speed would occur at 21.5 r.p.s. For 8 nodes at 75 and the speed coefficient of 2 the critical speed would be at 20 r.p.s.

TABLE 5 FOR INTERPRETATION OF FILMS

(See Explanation in Par. 242.)

Nodes	3rd Minor	2nd Minor	1st Minor	Critical	1st Sub-Min.	2nd Sub-Min.	3rd Sub-Min.	Nodes
2	4 / 5 / 3	3 / 4 / 2	2 / 3 / 1					2
4	5 / 7 / 3	4 / 6 / 2	3 / 5 / 1	2 / 4 / 0				4
6	6 / 9 / 3	5 / 8 / 2	4 / 7 / 1	3 / 6 / 0	2 / 5 / 1			6
8	7 / 11 / 3	6 / 10 / 2	5 / 9 / 1	4 / 8 / 0	3 / 7 / 1	2 / 6 / 2		8
10	8 / 13 / 3	7 / 12 / 2	6 / 11 / 1	5 / 10 / 0	4 / 9 / 1	3 / 8 / 2	2 / 7 / 3	10

KEY. Numbers are cycles per revolution.

Revolving Coil Record | Stationary Coil (Forward Wave) | Stationary Coil (Backward Wave)

Example: A combination of 3 cycles per revolution on the Revolving coil and 5 on the Stationary coil represents a 4 Node 1st Minor, forward wave.

241 The corresponding first minors will be found in the next higher groups than the critical and the first sub-minors in the next lower group. For example, consider another case where the 6-node standing frequency is 50 cycles per second with a speed coefficient of 2. The 6-node first minor resonant speed will then be 13.4 r.p.s., the 6-node critical speed would be 18.9 r.p.s., and the 6-node first sub-minor resonant speed would be 35.5 r.p.s.

Interpretation of Films

242 Table 5 contains the information necessary to determine quickly the particular type of possible resonant speed occurring at the time of making the oscillograms during test in the wheel-testing machine. The number in the upper left-hand corner represents the number of cycles per revolution recorded on the revolving coil. In the lower right-hand corner are two numbers; the upper is that recorded on the stationary coil if a forward wave in the wheel is occurring, while the lower number is that on the same coil registered by a backward-traveling wave in the wheel. The backward wave in all minors always travels in a direction in space

opposite to that of the wheel itself while in all sub-minors it travels in space at a slower speed but in the same direction as the wheel. As an additional precaution, the standing frequency is computed from the running frequency and an estimated speed coefficient. Agreement with the measured standing frequency then forms a check on the particular type of resonant speed recorded and removes any ambiguity of the table.

243 Although little trouble is attributed to minor resonant speeds in general, the subject has been gone into here for the purpose of showing the reasoning used in locating a critical speed far above running speed when it is impossible safely to run the wheel so far above its operating speed. Minor resonant speeds are thus used to get the information necessary for the accurate

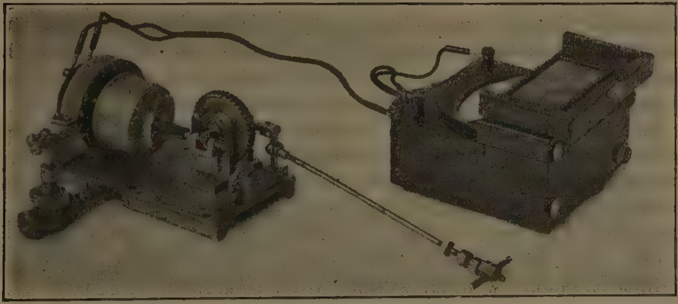

Fig. 94 Portable Compressed-Air-Driven Vibrator for Determining Standing Frequencies of Turbine Wheels

(Note the worm-gear speed reduction for electric tachometer.)

determination of critical speeds when such speeds cannot safely be attained.

Tests of Wheels Already Installed and Operating

244 At an early stage of the investigation and as soon as sufficient evidence had been established as to the principal cause for turbine disk troubles, a program was worked out in order immediately to determine the condition of turbines already in operation. This work was taken up energetically, the machines were opened, and the natural periods of the wheels obtained.

245 For convenience in vibrating the wheels of turbines already installed, portable vibrating machines were used. The first device was operated electrically and was similar to that shown in Fig. 46.

246 A much more convenient device was later devised to be operated by compressed air. Fig. 94 shows this complete. The long connecting rod is provided with a clamping screw for temporary connection to the wheel shroud band. The opposite end of the rod is provided with a clevis, the lugs of which straddle a vertical lever. The lower end of the lever extends into a small

block which is restrained by adjustable laminated springs. The upper end of this lever is connected by a crank to a ball-bearing eccentric shaft, which carries a ratchet-type wheel. Under some conditions the block is vibrated by the lever and at other times the block remains stationary, or practically so and forms a pivotal support for the lever as will be seen later.

247 The ratchet wheel serves as an air turbine and is driven by compressed air from a nozzle. Changes of speed are brought about by regulation of the air valve.

248 The driving shaft is carried in ball bearings. One end of the shaft has a worm, which meshes with a worm wheel, driving an electric generator forming part of an electric tachometer. A flexible rubber coupling is provided between the worm-wheel shaft and the generator. The generator is clamped in felt to absorb vibration. Current from the generator is supplied to a voltmeter which is calibrated in terms of r.p.m., and by means of which the speed of the driving motor, and hence the frequency of vibrations imparted to the turbine wheel, can be ascertained.

249 The operation of the vibrator is as follows: After supporting the device in such a manner as to provide for attaching the connecting rod to the shroud band of the turbine wheel, compressed air is admitted to the motor, causing rotation and actuating the vibrator or lever. At the start the tension of the leaf springs is purposely made comparatively light. The adjustment of the springs is made by moving the spring supporting block to and from the lever block. The lever vibrates about the pivotal connection to the connecting rod as an axis. Until a speed corresponding to a natural resonance of the turbine wheel is reached the connecting-rod pivot of the lever continues to act as a fulcrum. However, when a resonant speed is reached, an amplitude is built up in the bucket wheel allowing this pivotal point to vibrate back and forth. At the same time, the lower end of the lever quickly ceases to vibrate, and practically all the energy is transferred through the connecting rod.

250 The operator either by sense of touch or observation determines the number of nodal points practically at rest in the circumference of the wheel and this, together with the frequency of the applied force, is recorded.

251 It is often necessary, in order to build up a sufficiently large amplitude to obtain trustworthy observations, to increase the applied air pressure after having somewhat increased the spring tension. As the mass of the turbine wheel to be vibrated is great, and the energy of the vibrator is small, it is difficult to change the speed from one natural frequency of the wheel to another by merely changing the air supply to the motor, and therefore a method commonly employed is to release the spring tension on the block until such time as the turbine wheel comes to rest or practically so. The motor is then operated at a gradually increasing

speed, and at the same time the tension is increased somewhat on the spring; the increase of speed is arrested automatically when the next higher natural frequency is reached. The former procedure is then gone through again and this process is repeated until all of the required frequencies have been determined.

252 From the information thus obtained curves are made for each machine showing at what speed it might be possible for each individual wheel to give trouble. Over 1600 wheels were vibrated in this manner and reference to Table 1 will give an idea of the scope of this work.

Safe Design

253 To insure safe operation of turbine wheels, it is essential that all the wheels in a turbine rotor without exception shall be

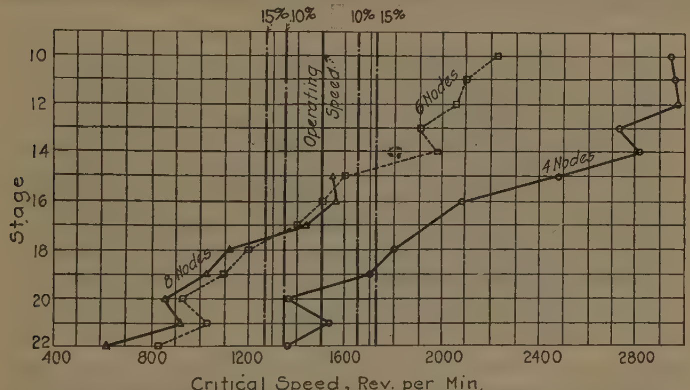

Fig. 95 Critical-Speed Diagram for an Old-Design 22-Stage Turbine

(Stages 15, 16, 17, 19, 20, 21, 22 all have critical speeds separated from running speed by insufficient margins.)

free from critical speeds within dangerous proximity to the operating speed of the rotor.

254 The chart of Fig. 93 is used for the rapid determination of the critical speed when the standing frequency for each particular nodal system is known together with a satisfactory speed coefficient B. After the various critical speeds have been determined in this manner, they are plotted on curves as shown in Fig. 95.

255 This diagram is for a 22-stage steam turbine. The horizontal scale represents revolutions per minute of the turbine rotor. The vertical scale represents wheels in the different stages of the rotor. In the diagram, the 13 largest wheels are the only ones indicated, the critical speeds corresponding to 4, 6 and 8 nodes being shown for each. These critical speeds are indicated in the

diagram by circles, squares and triangles respectively. It will be observed that some critical speeds occur close to the operating speed. Thus the wheels of stages 15, 16 and 17 all have critical speeds corresponding both to six and eight nodes occurring close to the operating speed, whereas stages 19, 20, 21, and 22 all have critical speeds corresponding to 4 nodes close to the operating speed. This diagram serves to indicate those stages whose critical speeds occur so close to running speed that, on account of the known variations of running speed, breadth of resonance, etc., the danger of stationary wave development cannot definitely be said not to exist. Figs. 96 and 97 are for machines especially designed to avoid wave development. It will be observed that there are no critical speeds in either case within 15 per cent of the operating speed. In Fig. 96 stages 16 and 17 have critical speeds below the operating speed, but Fig. 97 shows all critical speeds above the

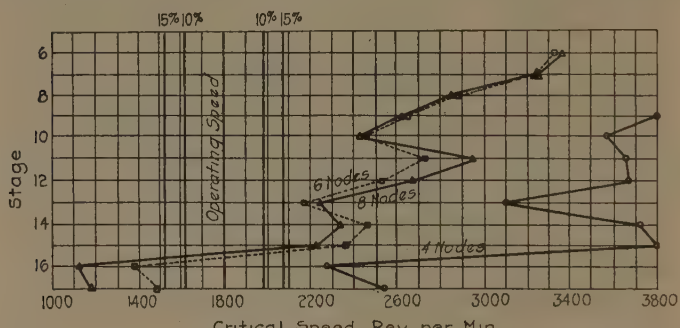

Fig. 96 Critical-Speed Diagram for 17-Stage Turbine

(This machine is designed to avoid development of vibration. All critical speeds fall outside the prescribed margins.)

operating speed. The latter case is of course preferable where possible of attainment. In the case of stages 16 and 17 of Fig. 96 it was not possible to design wheels having 6 or 8 nodes above the operating speed, therefore care is necessary in the design to provide as large an interval as possible between the 4 and 6 node critical speeds.

Safe Limits

256 In determining the safe limits which should exist between the normal operating speed of the rotor and the speeds at which dangerous wave phenomena develop a number of considerations are involved. In the first place a certain broadness of resonance exists, throughout which vibration may develop. That is to say, wave phenomena do not occur with precise mathematical accuracy at exactly a particular speed, but may occur at speeds slightly above or slightly below the speed corresponding to the condition

of maximum resonance. Special tests have shown that this broadness of resonance amounts to about two per cent above and below the critical speed.

257 Another factor, to be considered in determining safe limits between the operating speed and the calculated critical speed, is the uncertainty in the numerical speed coefficient B. This coefficient serves to determine the running frequency of the turbine wheel when the actual standing frequency is known. During the process of design, the errors which may occur in applying this coefficient to wheels similar to those that have been tested must also be allowed for. Another consideration is the possible variation of the line frequency on which the turbine is to be operated. Temperature variations must also be covered.

258 For these effects 10 per cent above and below the operating speed is recommended for six and eight nodes, and 15 per cent

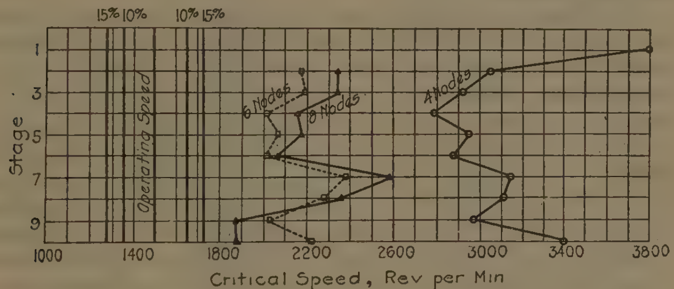

Fig. 97 Critical-Speed Diagram for 10-Stage Turbine

(This machine is designed to avoid development of vibration. All critical speeds are above the prescribed margins.)

for the 4-node condition. These limits have been rigidly adhered to for some time, with apparently sufficient justification. Furthermore wheels are not passed for use that have 10-node, 12-node, etc., critical speeds near the operating speed. Such wheels are corrected by tuning. The same precautions are also taken for possible minor resonant speeds.

Tuning

259 Tuning is resorted to where necessary for the purpose of altering the natural frequencies and is carried out as follows:

260 In proportioning or in *tuning* a wheel to obtain the desired margins between the critical speeds of the wheel and the normal operating speed, the relations between the stiffness and the mass of the parts of the wheel must be made such as to give the desired natural frequencies of vibration in the wheel. In general it may be stated that in a vibrating system, such as a turbine disk, the frequencies of vibration increase with the stiffness of the wheel

but decrease with the increase in mass of the wheel. This may be inferred from Equation [1] for a particle

$$f_s = \frac{1}{2\pi}\sqrt{\frac{R_s}{m}}$$

where R_s equals stiffness and m equals the mass of the vibrating particle. In designing the wheel this relation is borne in mind, since nodal frequencies of different numbers of nodes correspond to bending actions or flexures extending into the body of the wheel toward its center to greater or less degrees. Depending on the number of nodes, changes in the wheel between stiffness and mass may be expected to affect to different degrees, frequencies corresponding to different numbers of nodes. In order, therefore, for the wheel to be free from dangerous critical speeds, a proper coördination between the stiffness and mass of different portions of the wheel should exist.

261 Whenever there is reason to suppose that any wheel may have a critical speed within the limits set, the completely bucketed wheel is first vibrated off of the shaft and the probable critical speeds thereby determined. For the purpose of noting nodal segments during vibration, the wheel may be covered with a thin sheet of water. When the wheel is in vibration, the surface of the water exhibits rough or ripple areas conforming to vibration areas of the wheel. Water is preferred to sand for the purpose of getting immediate results. If a calculation made with a value of the speed coefficient determined for wheels of similar shape in the wheel-testing machine indicates a critical speed within the danger limits, the wheel is then placed upon its own shaft and vibrated again to determine more exactly the critical speeds as altered by the wheel fit on the shaft. Should the previous estimates be verified, then tuning is resorted to.

262 The tuning or reproportioning of the wheel may be done in a variety of ways depending upon the number of nodes and the frequency which it is desired to change. The previous experience and recorded data are found exceedingly helpful in facilitating this work.

263 Suppose, for example, the case of a 22nd-stage wheel with buckets 28 in. long and a disk diameter of 7½ ft., which upon being vibrated is found to have the 4-node critical speed approximately 8 per cent above the operating speed, and the critical speeds corresponding to six and eight nodes well outside the lower 10 per cent limit for these critical speeds. In this case it is necessary to raise the 4-node frequency to 15 per cent or more above the operating speed. This is done by removing the necessary amount of weight from the buckets. Temporarily attaching known weights to the shroud band of the wheel approximately indicates the effect of bucket weight on the frequencies of the wheel. From these data the necessary amount of weight to be removed from the

buckets is determined. Of course it is necessary to remove the buckets from the wheel in order to machine off metal which is taken from the backs of the buckets. Removal of weight from the outer or convex portion of the buckets is illustrated in Fig. 98 as indicated by the dotted lines. This remedy however is very rarely resorted to, as attempts are always made to insure a 4-node critical speed occurring well above the 15 per cent limit.

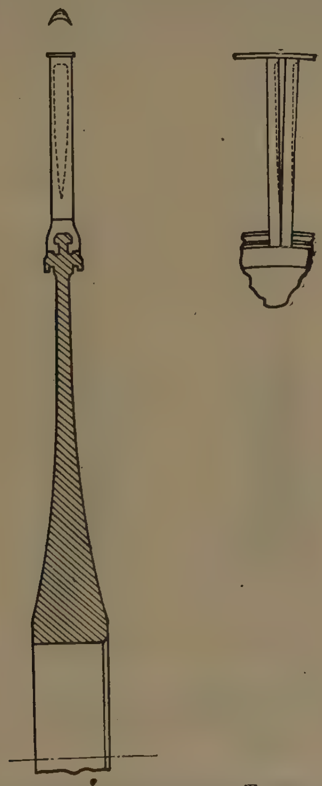

Fig. 98 Methods of Tuning

(Frequency may be raised by removing weight from the buckets as shown.)

264 Removing weight from the buckets in this manner results in a reduction of the mass without any appreciable effect on the stiffness, resulting in raising the frequency of the complete wheel.

265 As another illustration, consider a wheel with buckets 11½ in. long and with a disk diameter of 6 ft., but let it be supposed that the vibration tests show the existence of the 6-node critical speed about 3 per cent above the operating speed and the 8-node critical speed about 4 per cent below the operating speed, the

4-node critical speed occurring about 25 per cent above the operating speed. In this case, the 6-node and 8-node critical speeds may be lowered by removal of material from the wheel at a suitable point, but this must be done in such a way that the 4-node critical speed will not at the same time be lowered so far as to come within the danger limit of 15 per cent. This means that the 6-node critical speed must be brought to a point at least 10 per cent below the operating speed, the 8-node critical speed lowered from 4 per cent to 10 per cent below the running speed,

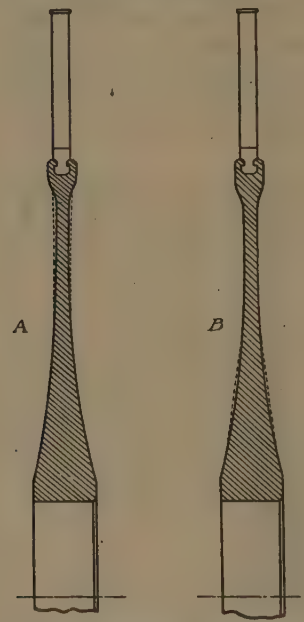

FIG. 99 METHODS OF TUNING

(Removal of weight from the outer web, as shown at *A*, affects mainly the 6- and 8-node frequencies. Removal of weight from the inner web, as shown at *B*, affects mainly the 4-node frequency.)

and all of these without lowering the 4-node critical speed by more than 10 per cent. In a case like this, great care must be taken to cut the wheel in the location which will most affect the critical speed which it is desired to change. Thus in the one under consideration, alterations in the web of the wheel next to the rim circumference have the greatest effect on the 8-node and 6-node frequency, while in the zone nearest the center of the wheel the removal of the material from the web has its greatest effect on the 4-node frequency.

266 For adjusting the 6- and 8-node frequency the wheel is then placed on the boring mill and slightly machined in the zone

extending from 6 to 8 in. inwardly from the rim. The wheel is then vibrated and the change in frequency noted. The operations are repeated until the desired frequencies are obtained. It is not passed however until it has received a running test in the wheel-testing machine or has shown by tests on its own shaft that the desired results have been obtained.

267 Fig. 99 at *A* is a profile of the turbine wheel. Dotted lines indicate where metal has been removed from the outside portion of the web of the wheel. The removal of metal from this region affects mostly the frequencies corresponding to six or eight nodes but has little effect upon the frequency corresponding to four nodes. Fig. 99 at *B* shows by dotted lines the location having most effect on the 4-node frequency.

Summary of Procedure for the Production of a Turbine Wheel

268 The principles and theory of bucket-wheel vibration and the methods of testing have been explained and described. In conclusion it remains to summarize the protective measures utilized in the ordinary process of production in order to assure the safety of turbine wheels.

269 The first step is to design the bucket in accordance with the thermodynamic requirements, select the dovetail suited to the bucket and thus fix upon the design of wheel rim.

270 The second step is to select the general dimensions and contour of the wheel with a view to vibration characteristics. For this, reference is made to records of wheels of like bucket design and diameter. There are two complete catalogs showing the complete vibration characteristics of all wheels so far tested. In one catalog the wheels are listed according to dimension and this is used for design purposes. In the other the wheels are catalogued by machines and this is useful in tracing the record of any particular turbine or wheel.

271 The third step is the calculation of the stresses due to centrifugal force and the completion of the details in a satisfactory manner.

272 The fourth step is a review of the resulting design on two bases. The stresses must be satisfactory and the wheel design must suit the space and clearance requirements of the turbine assembly. Either one of these check inspections may serve to condemn the design and require a new start from the beginning.

273 The fifth step is the selection and assignment of a forging. It should be understood that every forging is examined for its strength properties by means of coupons and for its uniformity by a magnetic survey of its entire structure. A complete record of every forging is kept by serial number, and the assignment of a forging is a separate step in the process. Any deviation in the

shop, from drawing, however slight, that does not result in the destruction of the forging, is placed on record by a new drawing.

274 The sixth step is the standing vibration survey of the finished wheel to make sure that it does not deviate more than an allowable amount from the original expectations. If all the resonant speeds, including both the critical and the various minor resonant speeds do not have the required margins from running speed, tuning is resorted to. Generally one or two trials are sufficient to obtain by tuning the required vibration characteristics but occasionally a wheel has to be altered five or six times. This process of course, necessitates a revision of the drawing and a corresponding alteration of all the records involved.

275 In some cases the standing vibration test confirms the predictions from the catalogs of similar wheels for which satisfactory speed coefficients have been determined in the wheel-testing machine. Such wheels may be accepted as having satisfactory margins on the basis of the standing vibration test alone. But in the majority of all cases each wheel receives a complete running test in a wheel-testing machine.

276 The seventh step is the complete running vibration test in the wheel-testing machine. From 30 to 60 oscillograph films are taken showing exactly all phases of the wheel's behavior under running conditions. These films are all examined and those of value are indexed and filed. About 18,000 such records have been taken, and, of this great number, about one-third are preserved and available for quick reference at the present time.

277 The final result is a wheel which is either satisfactory and thoroughly protected from the possibility of resonant vibration, or else the wheel is rejected and a new design started through the necessary seven steps toward a final acceptance.

278 It will be readily appreciated that this thoroughness of examination and verification of predicted properties is at the basis of the remarkable freedom from vibrational troubles exhibited in the behavior of recently built turbines.

DISCUSSION

S. Timoshenko.[1] The problem of the turbine-disk vibration is of great practical importance. The first analytical investigation on this subject was by Stodola, followed by the work of Lamb and Southwell.[2] These authors had been previously attracted to this subject by the experiments of Stoney[3] on a rotating india-rubber

[1] Westinghouse Research Laboratory, East Pittsburgh, Pa.
[2] Proc. Royal Society, vol. 99, p. 272, and vol. 101, p. 133.
[3] Institute of Technology, Manchester, England.

disk. In the same year Bauman discussed different types of vibration such as

a Umbrella-shaped vibration
b Segmental vibration with nodal diameters stationary in space
c Segmental vibrations with nodal diameters rotating with shaft.

Lamb, in the work previously mentioned, discussed analytically all these types of vibration for the simple case of a disk having constant thickness.

The principal results described in the present paper can be best summarized as follows: The importance of obtaining vibration data on full-sized wheels under actual operating conditions has been fully appreciated by the author.

This method of procedure was essential. The exploring electric coil used in this machine in conjunction with the oscillograph proved to be a helpful device. By its use different types of vibration have been studied and satisfactory explanation given of their causes. The author shows that the type of vibration responsible for practically all serious wheel failures consists of a train of backward-traveling waves whose backward speed in the wheel exactly equals the forward speed of wheel rotation. This again emphasizes the importance of knowing critical speed.

In this respect it is important to observe that in calculating the critical speed, Rayleigh's approximate method[1] and its further development given by W. Ritz[2] is used in the paper. In applying this method the potential and kinetic energies must be calculated on the basis of an assumed form of deflection. In such manner the system under consideration is transformed into a system with only one degree of freedom. The frequency obtained in this manner will be the upper limit to the true value. While this method of calculation is usually accurate enough in the case of vibrating bars, it is not sufficiently accurate for plates such as disks where more complicated conditions at the edge occur. A calculation giving a better approximation then becomes necessary. The expression for the deflection must be taken in such a manner as to satisfy the conditions at the edge so far as the deflection and the slope are concerned. It must contain also a number of parameters whose magnitude must be determined in such a manner as to make the frequency a minimum. By increasing the number of these parameters, and by taking into consideration edge conditions while choosing the deflection, the accuracy in determining the frequency can be increased.

The method of calculating frequencies outlined in Pars. 138 and 139 is analogous to that of calculating critical loads for compressed

[1] Theory of Sound, pars. 88, 89.
[2] *Annalen der Physik,* vol. 28, p. 797.

columns, proposed by Vianello. It is accurate enough for bars and shafts, but satisfactory results cannot be expected for plates. A simple graphical method does not exist for calculating deflections of plates. Some simplification of the solution can be obtained by using the method of Marcus,[1] which is analogous to the Mohr method of graphical determination of the deflection curve for a bar. Instead of a funicular curve in the Mohr method, the deflection of a flexible membrane must be studied.

The effect of inertia forces on the frequency of vibration of a disk wheel mentioned in Pars. 41 to 43 and explained in an elementary way was studied by R. Southwell[2] and an interesting conclusion obtained. If f_s denotes the frequency of a disk due to stiffness furnished by the elastic property of the disk and f_c the frequency due to the stiffness contributed by centrifugal effects, then a lower limit for frequency f_r of the disk at rotation will be obtained from the equation

$$f_r^2 = f_s^2 + f_c^2$$

This result in conjunction with that obtained by Rayleigh's method makes it possible to establish the accuracy of approximate calculations.

In considering the traveling waves in Par. 48 et seq., an analytical expression for these waves, in addition to their diagrammatic representation, would be useful for explaining this phenomenon. Let $R \cos pt \cos n\theta$ represent standing waves of the segmental type with n nodal diameters. Taking the same waves, but of different phase as in the form $R \sin pt \sin n\theta$ and combining these two types, the traveling waves represented by $R \cos (pt \pm n\theta)$ will be obtained. The corresponding angular wave velocities are $\pm p/n$. Adding to this ω, the angular velocity of the wheel, the angular velocities in space for the forward- and backward-traveling waves will be $\omega + p/n$ and $\omega - p/n$.

It thus can be shown that segmental waves, stationary in space, are produced by combining segmental waves with nodal diameters rotating with the shaft.

The experiments on the dissipation of energy by internal friction are interesting. Little is known on this subject[3] and further study can give important results. In experimenting with simple types of vibration, a study can be made of the manner in which internal friction depends on the frequency and on the magnitude of the stresses.

In regard to the method of determining stresses, it is difficult to agree that the calculation of stresses due to vibration in a

[1] *Armierter Beton,* 1919.

[2] Ibid.

[3] See paper by Honda and Konno, *Phil. Mag.,* vol. 42 (1921), p. 115; also *Zeitschr. f. Angew. Math. und Mech.,* vol. 4 (1924), p. 124.

vibrating disk wheel from the deflection shape of the wheel can be made with sufficient accuracy. It is known that approximate methods for calculating the deflection of plates such as the Rayleigh-Ritz method are sufficiently accurate, so far as the magnitude of the deflection is concerned. For determining the stresses the second and third derivatives of the deflection curve are necessary, and the accuracy with which these derivatives can be obtained from an approximate deflection shape is usually insufficient for practical applications. In order to obtain satisfactory results, direct measurements of variation of distances between the points on the surface of the vibrating wheel are necessary.

In studying vibration of a circular plate a discrepancy between theory and experiment was found. It should be observed that in this study the amplitude of vibrations and the manner of clamping the edge are not given. If the amplitude of the vibrations is not small in comparison with the thickness of the plate, then higher frequencies than those calculated by the Kirchhoff formula must be expected. This increase of frequency will be more likely to appear in the cases of thinner plates and of lower types of vibration.

H. F. Moore.[1] The paper is not only a successful attempt to meet a very practical problem in machine design, but an expedition into the almost totally unexplored field of the dynamics of stress and strain. Practically all the common formulas and methods used in studying the mechanics of materials are based on the laws of statics. In considering stresses and strains in high-speed machinery we should consider the effect of waves of stress, with crests, troughs, and maximum and minimum values. The present paper is an interesting and probably an important contribution to that field of knowledge.

The author states that the practical problem presented was to devise such means that axial vibration would not occur. It may be pointed out that the prevention of localized regions of high stress in a wheel is a secondary method of increasing safety by increasing the resistance to fatigue breakdown should vibration occur. Reference has been made to the investigation of the fatigue of metals carried on at the University of Illinois under the auspices of the National Research Council, Engineering Foundation, and various manufacturing companies. Among the conclusions reached in that investigation are two which bear on this point:

1 A rough-machined surface tends to create regions of high stress at the bottom of tool marks. The resistance to fatigue failure of a machine part with a rough-machined surface may be as much as 15 per cent less than that

[1] Research Professor of Engineering Materials, University of Illinois, Urbana, Ill. Mem. A.S.M.E.

of a machine part of the same material and dimensions with a smooth-finished surface.

2 Holes in the surface may double the localized stress at their edges. It is to be noted that heavy localized stress is not of great importance when the action of a steady stress, such as that caused by centrifugal force, is considered, but that heavy localized *repeated* stress may cause a fatigue crack to start and spread in the metal.

PAUL HEYMANS.[1] The object of this discussion is (1) to call attention to existing stress analyses related to certain of the problems of the present paper; and (2) to emphasize and more closely define some points of theoretical importance bearing upon the interpretation of the experimental data obtained by the author.

Ruptures of the Disks. Examining the types of failures of the turbine disks presented in the first part of the paper, one is impressed by the fact that, although in certain cases the fractures avoided the holes existing in the disks, the majority of ruptures started and passed through the holes *and through very definite points of the holes.* This brings attention to the more general question of the disturbances caused in different types of stress distributions by the presence of circular or other types of internal discontinuities.

A. Leon[2] has given an analytical solution for the stress distribution around a circular hole in an infinitely extending plate. Calling $\widehat{rr}$ and $\widehat{\theta\theta}$ the radial and tangential stresses, the elastic state is given at any point r, ϕ:

1 If the plate is put under a uniform pull in a direction parallel to the x-axis (Fig. 100), p being the mean longitudinal stress, by:

$$\widehat{rr} = \frac{p}{2}\left(1 - \frac{r_0^2}{r^2}\right)\left[2 - 3\frac{r_0^2}{r^2} - 2\left(1 - 3\frac{r_0^2}{r^2}\right)\sin^2\phi\right]$$

$$\widehat{\theta\theta} = \frac{p}{2}\left[\left(1 - 3\frac{r_0^2}{r^2}\right)\frac{r_0^2}{r^2} + 2\left(1 + 3\frac{r_0^4}{r^4}\right)\sin^2\phi\right]$$

2 If the plate is put under uniform pull in all directions, p being the mean stress, by:

$$\widehat{rr} = p\left(1 - \frac{r_0^2}{r^2}\right)$$

$$\widehat{\theta\theta} = p\left(1 + \frac{r_0^2}{r^2}\right)$$

It readily results from the above solution that:

[1] Assistant Professor of Theoretical Physics and Photoelasticity, Massachusetts Institute of Technology, Cambridge, Mass. Assoc-Mem. A. S. M. E.

[2] Alfons Leon, Uber die Störungen, die in elastischen Körpern durch Bohrungen und Bläschen entstehen.—*Oesterreichische Wochenschrift für Oeffentlichen Baudienst,* 1909.

1 In infinitely extending plates, uniformly pulled in one direction, the stress is a maximum for $\phi = 90$ deg. and $r = r_0$, i.e., at the boundary of the hole, *at the extremity of a diameter normal to the direction of pull,* and is equal to three times the mean stress. For $\phi = 0$ deg. and $r = r_0$, the stress is a compression and is equal to the mean stress.

2 In infinitely extending plates, uniformly pulled in all directions, the stress is a maximum for $r = r_0$, i. e., at the boundary of the hole, and is equal to twice the mean stress.

In finite members, such as a bar of rectangular cross-section which is submitted to a uniform longitudinal pull, the maximum stress at the boundary of the hole should be equal to three times the mean stress if the ratio of the diameter of the hole to the width of the plate is small enough. Fig. 101, which represents the photoelastic analysis,[1] shows that the actual stresses approach the theoretical values as the ratio specified above decreases. The solid lines represent the stresses measured, whereas the dotted lines are the analytical results for an infinitely extending plate.

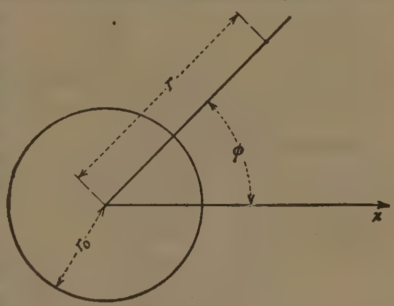

FIG. 100

No investigations, analytical or by the photoelastic method, of the stresses around discontinuities in rotating disks have been made, although they might be highly interesting in the light of the data obtained by the author. However, if the diameter of the hole be small as compared to the radius of the disk, and the hole be far enough from the center and from the rim of the wheel, the case of a plate under uniform unidirectional pull may illustrate qualitatively the disturbance, under centrifugal action, on the stress distribution, which without the presence of the hole would be nearly uniform in the region considered.

It will be noticed that the ruptures run or start at the holes at the two ends of diameters very approximately coinciding with the normal to the direction of the centrifugal force. This accords with the location of the points of maximum stress in the case of the pulled plate to which reference has been made. It might be estimated, although a verification by photoelastic analysis would be more satisfactory, that the hole introduces a maximum stress

[1] E. G. Coker, Trans., Instn. Engrs. & Shipbuilders in Scotland, Dec., 1919. Paul Heymans, Bull. Soc. Belge Ing. & Ind., Aug., 1921.

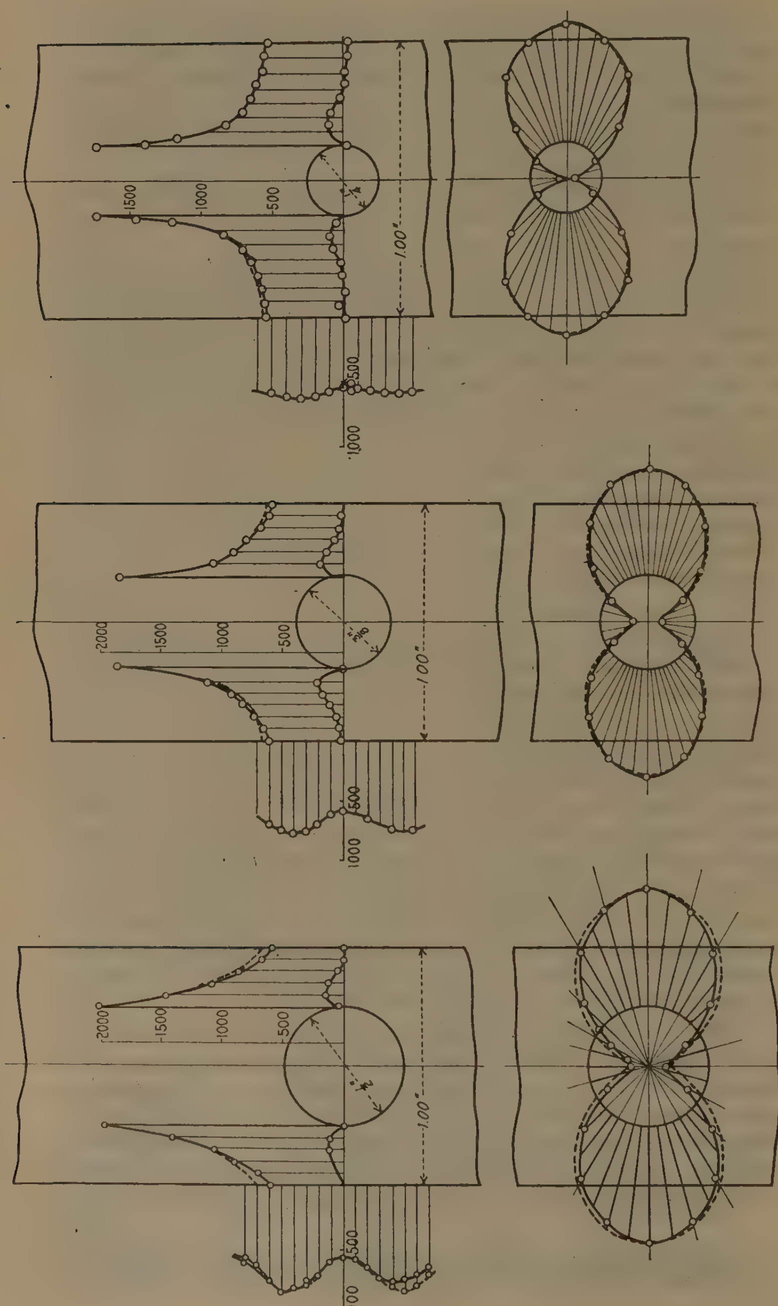

Fig. 101 Photoelastic Analyses of Stresses at Boundaries of Holes

equal to three times the stress which would exist without the presence of the hole.

The ruptures of the wheels emphasize the existence of highly localized stresses around the holes, and it may therefore be worth while to say that full information concerning the effect of these discontinuities can be obtained by photoelastic analysis. If a celluloid disk wheel be rotated and a stationary photoelastic image of the stressed wheel at any desired speed between 300 and 3000 r.p.m. be obtained by means of a timed electric spark, such as has been developed for other photoelastic investigations,[1] the stress distributions can be completely determined.

Ruptures of the Buckets. The ruptures of the buckets, as described in the paper, call attention to the concentration of stresses

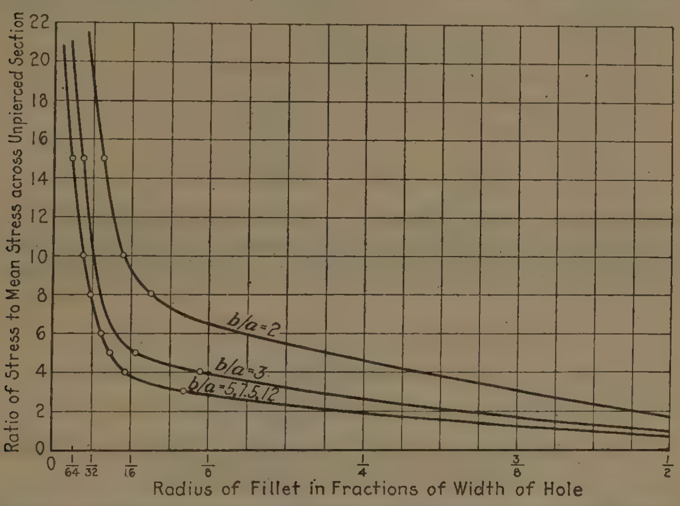

FIG. 102 EFFECT OF RADIUS OF FILLET ON STRESS IN PLATES WITH SQUARE OPENINGS

around external variations of profiles, such as exist in these buckets. It is known that when in the stressed member the radius of curvature at a variation of profile decreases, the intensity of the disturbance on the stress distribution increases; that is, the maximum stress increases. This increase of the maximum stress is well illustrated by the following photoelastic investigation.[2] The stresses around the corners of a square opening in a bar under uniform

[1] P. Heymans and A. L. Kimball, Jr., Stress Distribution in Rotating Gear Pinions as Determined by the Photoelastic Method. *Mechanical Engineering,* March, 1924.

[2] P. E. Pihl and O. D. Colvin, Jr., Investigation by the Photoelastic Method of the Distribution of Stresses around Various Square Openings in a Flat Plate. Thesis for the degree of Master of Science, presented at Massachusetts Institute of Technology, June, 1924.

longitudinal pull were measured with decreasing radii at the fillets in the corners. The increase in maximum stress with curvature at the corners is given by Fig. 102. Fig. 103 shows a similar investigation with a V-shaped lateral notch, with decreasing radii of curvature at the bottom of the V-shaped groove.[1]

The problem in turbine design should be to meet the kinetic conditions of profile of the buckets with the optimum curvatures avoiding stress concentrations.

Points Bearing on Interpretation of Experimental Data

The object of the second part of this discussion is to emphasize and more closely define some points of theoretical importance

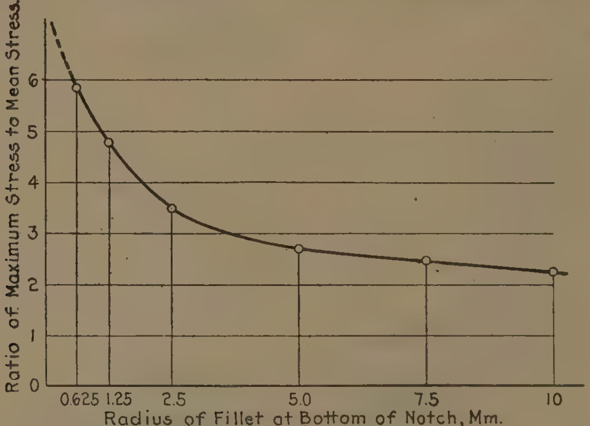

Fig. 103 Effect of Radius of Curvature at Bottom of V-Shaped Lateral Notch on Stresses in Plates

bearing upon the interpretation of certain experimental data presented in this paper.

Tests by Means of Models Used on Account of Their Great Deflections. The author of the paper reports a series of tests made on india-rubber wheels and very thin steel disks which were used on account of their great deflections. Attention must be called to the erroneousness of such tests when intended to yield information regarding materials or designs where those great deflections do not occur.

Let us write Lamé's general stress-strain relations:[2]

[1] Paul Heymans, La Détermination par la Photo-Elasticimetrie des Surtensions dues à Certaines Discontinuités. Mémoires, Académie Royale de Belgique, 2me. Série, t. VI, 1921.

[2] G. Lamé, Leçons sur la Théorie Mathématique de l'Elasticité des Corps Solides, 1852. A. E. H. Love, A Treatise on the Mathematical Theory of Elasticity, 3d ed., p. 97.

$$
\begin{aligned}
X_x &= a_1 e_{xx} + b_1 e_{yy} + c_1 e_{zz} + d_1 e_{yz} + e_1 e_{zx} + f_1 e_{xy} \\
Y_y &= a_2 e_{xx} + b_2 e_{yy} + c_2 e_{zz} + d_2 e_{yz} + e_2 e_{zx} + f_2 e_{xy} \\
Z_z &= a_3 e_{xx} + b_3 e_{yy} + c_3 e_{zz} + d_3 e_{yz} + e_3 e_{zx} + f_3 e_{xy} \\
Y_z &= a_4 e_{xx} + b_4 e_{yy} + c_4 e_{zz} + d_4 e_{yz} + e_4 e_{zx} + f_4 e_{xy} \\
Z_x &= a_5 e_{xx} + b_5 e_{yy} + c_5 e_{zz} + d_5 e_{yz} + e_5 e_{zx} + f_5 e_{xy} \\
X_y &= a_6 e_{xx} + b_6 e_{yy} + c_6 e_{zz} + d_6 e_{yz} + e_6 e_{zx} + f_6 e_{xy}
\end{aligned}
$$

These equations assume the generalized Hooke's Law that each of the six components of stress at any point of the stressed body is a linear function of the six components of strain at that point. It is seen that in this three-directional isotropic body the stress distribution is determined by thirty-six elastic constants. By introducing the conditions of isotropy in *all* directions the above equations reduce to the following:

$$
\begin{aligned}
X_x &= (c_1 + 2d_4) e_{xx} + c_1 e_{yy} + c_1 e_{zz} \\
Y_y &= c_1 e_{xx} + (c_1 + 2d_4) e_{yy} + c_1 e_{zz} \\
Z_z &= c_1 e_{xx} + c_1 e_{yy} + (c_1 + 2d_4) e_{zz} \\
Y_z &= d_4 e_{yz} \\
Z_x &= d_4 e_{zx} \\
X_y &= d_4 e_{xy}
\end{aligned}
$$

Or, introducing Lamé's elastic constants λ and μ, and assuming that the cubical dilatation Δ is equal to the sum of the three linear dilatations e_{xx}, e_{yy}, e_{zz}, these equations take the well-known form:[1]

$$
\begin{aligned}
X_x &= \lambda\Delta + 2\mu e_{xx} \\
Y_y &= \lambda\Delta + 2\mu e_{yy} \\
Z_z &= \lambda\Delta + 2\mu e_{zz} \\
Y_z &= \mu e_{yz} \\
Z_x &= \mu e_{zx} \\
X_y &= \mu e_{xy}
\end{aligned}
$$

or:

$$
\begin{aligned}
e_{xx} &= E^{-1}[X_x - \sigma(Y_y + Z_z)] \\
e_{yy} &= E^{-1}[Y_y - \sigma(X_x + Z_z)] \\
e_{zz} &= E^{-1}[Z_z - \sigma(X_x + Y_y)]
\end{aligned}
$$

where Young's modulus E is substituted for $\dfrac{\mu(3\lambda + 2\mu)}{\lambda + \mu}$ and Poisson's ratio σ for $\dfrac{\lambda}{2(\lambda + \mu)}$.

These equations are the fundamental equations of the theory of elasticity and all theories derived therefrom. They postulate:

1 Isotropy
2 Hooke's law of linear proportionality between stress and strain

[1] A. E. H. Love, *loc. cit.*, p. 100.

3 Deformations, sufficiently small so that the square of any of the three principal dilatations or the double product of any of two of them is negligible compared with any one of them.

These relations express the stress and strain distributions for those and *only* those bodies which satisfy the three postulates. It results directly from these equations that, other things being equal, the stress and strain distributions will be the same in all such bodies; they will be different in those for which the three postulates are not satisfied. *Elastic similarity rests upon these three postulates.* When the model departs from any of these three fundamental conditions it is doubtful, in the majority of cases, if the data obtained can be *interpreted* so as to throw light upon the behavior of the structure itself, and it is certainly erroneous to transfer directly the results from the model to the structure.

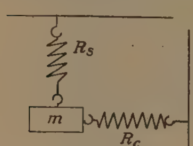

FIG. 104

Effect of Centrifugal Force on the Transverse Vibration Frequency (Pars. 41, 42, 43). Let us consider, as suggested by the author (Pars. 41 and 42), a particle m supported by two different elastic connections R_s and R_c at an angle to each other as shown in Fig. 104. The frequencies due to each of these elastic connections, will be, as stated in the paper:

$$f_s = \frac{1}{2\pi}\sqrt{\frac{R_s}{m}} \quad \ldots\ldots\ldots\ldots \quad [1]$$

$$f_c = \frac{1}{2\pi}\sqrt{\frac{R_c}{m}} \quad \ldots\ldots\ldots\ldots \quad [2]$$

These two frequencies are only susceptible of direct addition, leading to a resulting frequency

$$f_r = \frac{1}{2\pi}\sqrt{\frac{R_s+R_c}{m}} \quad \ldots\ldots\ldots \quad [3]$$

if they affect the same degree of freedom of the system, i. e., if the elastic connections bear upon the same independent coördinate. In Fig. 104 this is not the case. It must be noted that in the case described in the paper one elastic connection corresponds to the transverse motion of the disk, whereas the centrifugal action primarily caused radial stress and strain. The two stiffnesses are *not* susceptible of direct addition. It must therefore be borne in mind, as the author of the paper seems to be well aware, that Equation [3] is only justified, as far as the centrifugal action is concerned, when R_c represents the change in the transverse stiffness caused by the centrifugal force.

It is proposed, moreover, that the variation of "transverse stiffness" or the disk at rest and the rotating disk be analyzed as follows:

Let e_{zz} be the transverse deformation per unit thickness at any point of the disk. The deformation e_{zz} is related to the radial, tangential and transverse stress $\widehat{rr}$, $\widehat{\theta\theta}$ and $\widehat{ss}$ by the following classical stress-strain relations:

$$e_{zz} = E^{-1}[\widehat{ss} - \sigma(\widehat{rr} + \widehat{\theta\theta})]$$

where E and σ are respectively Young's modulus and Poisson's ratio.

For $\widehat{rr}$ and $\widehat{\theta\theta}$ equal to zero (disk at rest) the transverse stress will at any point produce a deformation equal to $[E^{-1}\widehat{ss}]$. The radial stress $\widehat{rr}$ and $\widehat{\theta\theta}$ (rotating disk) due to centrifugal action, and torque (and inertia in the transient state) will decrease this deformation by $\left[\frac{\sigma}{E}\widehat{rr}\right]$ and $\left[\frac{\sigma}{E}\widehat{\theta\theta}\right]$. Let $(\widehat{ss})_1$ be the stress producing unit deflection e_{zz} when $\widehat{rr} = \widehat{\theta\theta} = 0$. The stress $(\widehat{ss})_2$ necessary to produce this same deflection when $\widehat{rr}$ and $\widehat{\theta\theta}$ are different from zero, becomes:

$$(\widehat{ss})_2 = [(\widehat{ss})_1 + \sigma(\widehat{rr} + \widehat{\theta\theta})]$$

This relation expresses the increase in "transverse stiffness."

It is consequently seen that the "transverse stiffness" increases *not only with the centrifugal force, but also with the torque.* The frequency f_r (Equations [3] and [5] of the paper) should therefore include centrifugal action and torque. It would be interesting to know if such variations of the frequency with the torque were observed and if not, what explanation would be suggested.

Relations between Standing Vibrations and Traveling Waves. The statement made in Par. 59 is not free from objection. Whereas it is true that the frequency of a particle, part of a *discontinuous* medium, depends only on its mass and a stiffness factor, this property does not hold for continuous or pseudo-continuous media; that is, the frequency is *not* the same in such a medium for each unit of mass throughout the entire structure. This fact is demonstrated in the writer's paper entitled Mathematical Theory of Dynamic Stresses in Rotating Gear Pinions.[1]

Test on a Simple Vibrating Cantilever Bar (Pars. 129-131). The author of the paper refers to the theory of a simple vibrating cantilever bar and shows in Fig. 51 the calculated and observed frequencies of vibration of different tests. He finds that the observed frequencies check only within one or two per cent of the frequencies which he is led to expect theoretically. The writer is

[1] *Mechanical Engineering,* vol. 46, 1924, p. 583.

of the opinion that this lack of check may be due to the fact that the author's theoretical expectations are not entirely accurate. Indeed, following the classical theory, the differential equation of transverse vibratory motion of a beam of uniform rectangular cross-section is:

$$\varepsilon \frac{\partial^2 z}{\partial t^2} + \frac{Eh^2}{12} \frac{\partial^4 z}{\partial x^4} = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \quad [18]$$

where:

z = transverse displacement
ε = density
E = Young's modulus
h = height of the beam
x = longitudinal coördinate.

If the beam is "encastré" at one end and free at the other, then:

$$(z)_{x=0} = 0, \; \left(\frac{\partial z}{\partial x}\right)_{x=0} = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \quad [19]$$

$$\left(\frac{\partial^2 z}{\partial x^2}\right)_{x=l} = 0, \; \left(\frac{\partial^3 z}{\partial x^3}\right)_{x=l} = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \quad [20]$$

These two last conditions [20] express that at the free end the beam has neither curvature nor variation of curvature.

For simplicity assume that $l = 1$. The Equations [20] become:

$$\left(\frac{\partial^2 z}{\partial x^2}\right)_{x=1} = 0, \; \left(\frac{\partial^3 z}{\partial x^3}\right)_{x=1} = 0 \quad . \; . \; . \; . \; . \; . \quad [21]$$

For integrating Equation [18], we can write, in the classical manner, as solutions:

$$z = \phi(x) \cos nt \; \text{ or } \; z = \phi(x) \sin nt \quad . \; . \; . \; . \; . \quad [22]$$

where n is to be determined.

Equations [22] and their derivatives introduced in [18], [19] and [20] give the following relations which determine the function ϕ:

$$\left.\begin{array}{ll} (a) \; \dfrac{d^4\phi}{dx^4} - \lambda\phi\,(x) = 0 & \text{where } \lambda = \dfrac{12\varepsilon n^2}{Eh^2} \\ (b) \; \phi\,(0) = \phi'\,(0) = 0 & \\ (c) \; \phi''(1) = \phi'''(1) = 0 & \end{array}\right\} \quad . \; . \quad [23]$$

Equation [23(a)] is a linear and homogeneous differential equation with constant coefficients, whose solution is of the form

$$\phi(x) = e^{\tau x} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [24]$$

where τ is determined by introducing solution [24] into the primitive Equation [23(a)]

$$\tau^4 - \lambda = 0$$

whence $\tau = \pm\sqrt{\pm\sqrt{\lambda}}$

or
$$\tau_1 = +\sqrt[4]{\lambda}$$
$$\tau_2 = -\sqrt[4]{\lambda}$$
$$\left.\begin{aligned}\tau_3 &= +i\sqrt[4]{\lambda}\\ \tau_4 &= -i\sqrt[4]{\lambda}\end{aligned}\right\}\text{where } i = \sqrt{-1}$$

Putting $s = +\sqrt[4]{\lambda}$, the solutions of Equation [24] are:

$$\left.\begin{aligned}\phi_1 &= e^{+sx}\\ \phi_2 &= e^{-sx}\\ \phi_3 &= e^{+isx}\\ \phi_4 &= e^{-isx}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad [25]$$

Equation [23(*b*)] being linear, any linear combination of the solutions [25] will also be solutions of the primitive equation. The following combinations represent the real solutions:

$$\frac{\phi_1+\phi_2}{2}, \quad \frac{\phi_1-\phi_2}{2}, \quad \frac{\phi_3+\phi_4}{2}, \quad \frac{\phi_3-\phi_4}{2}$$

Indeed

$$\frac{\phi_1+\phi_2}{2} = \frac{e^{sx}+e^{-sx}}{2} = \cosh(sx)$$
$$\frac{\phi_1-\phi_2}{2} = \frac{e^{sx}-e^{-sx}}{2} = \sinh(sx)$$
$$\frac{\phi_3+\phi_4}{2} = \frac{e^{isx}+e^{-isx}}{2} = \cos(sx)$$
$$\frac{\phi_3-\phi_4}{2} = \frac{e^{isx}-e^{-isx}}{2} = \sin(sx)$$

Hence we can write:

$$\phi_1(x) = \cosh(sx)$$
$$\phi_2(x) = \sinh(sx)$$
$$\phi_3(x) = \cos(sx)$$
$$\phi_4(x) = \sin(sx)$$

and the general solution for ϕ is

$$\phi(x) = A\cosh(\sqrt[4]{\lambda}x) + B\sinh(\sqrt[4]{\lambda}x) + C\cos(\sqrt[4]{\lambda}x) + D\sin(\sqrt[4]{\lambda}x) \quad \ldots\ldots\ldots\ldots \quad [26]$$

where the values of A, B, C, D and λ satisfy the boundary conditions [23(*b*)] and [23(*c*)].

These boundary conditions introduced in Equation [26] yield the following condition equation:

$$\cosh s \cos s = -1$$

The roots of the transcendental equation $\cosh s \cos s = \pm 1$ are given in Jahnke-Emde:[1]

$$x_k = \tfrac{1}{2}(2k\pm1)\pi - (-1)^k\alpha_k$$

[1] E. Jahnke-F. Emde, Funktionentafeln mit Formeln und Kurven, 1923, p. 3.

where $\alpha_k = \frac{2}{a} \pm (-1)^k \frac{4}{a^2} + \frac{34}{3a^3} \pm (-1)^k \frac{112}{3a^4} + \ldots$, and

$$a = e^{(2k+1)}\frac{\pi}{2}$$

The roots of the equation $\cosh s \cos s = -1$ consequently are: $s_1 = 1.8751$, $s_2 = 4.6941$, $s_3 = 7.8548$, $s_4 = 10.9955$, and where $k > 4$, the values of s can be obtained with satisfactory approximation from the expression:

$$s_k = \tfrac{1}{2}(2k-1)\pi$$

These values of s represent, according to the relation $s = \sqrt[4]{\lambda}$, the fourth power of λ. From the relation

$$\lambda = \frac{12\varepsilon n^2}{Eh^2}$$

we thereby obtain the values of n:

$$n_k = \sqrt{\frac{Eh^2}{12\varepsilon}\lambda_k} = s_k{}^2\sqrt{\frac{Eh^2}{12\varepsilon}}$$

This gives as a general solution of Equation [18] with the condition Equations [19] and [21]:

$$z = \sum_k^{k=1,2,\ldots,\infty} \phi_k(x)\,(\alpha_k \cos n_k t + \beta_k \sin n_k t) \quad . \quad . \quad . \quad [27]$$

where α_k, and β_k are constants which can be derived from the conditions of the system at the origin or at any other time.

It is thereby seen that the kinetic equation of motion assumed by the author of the paper is only a first approximation, the complete solution being given by Equation [27] where the values of the frequencies of the component oscillations are:

$$n_1 = \sqrt{\frac{Eh^2}{12\varepsilon}}\; 1.8751^2$$

$$n_2 = \sqrt{\frac{Eh^2}{12\varepsilon}}\; 4.6941^2$$

$$n_3 = \sqrt{\frac{Eh^2}{12\varepsilon}}\; 7.8548^2$$

$$n_4 = \sqrt{\frac{Eh^2}{12\varepsilon}}\; 10.9955^2$$

$$\vdots$$

$$n_k = \sqrt{\frac{Eh^2}{12\varepsilon}}\; \tfrac{1}{2}(2k-1)^2\pi^2$$

E. D. Dickinson.[1] When the potential possibilities of the steam turbine came to be realized, there was a demand for sizes heretofore not considered practical or commercial. To supply this demand and to obtain simultaneously better efficiencies called for new standards of design. Mechanical design will be sound only when the fundamentals are understood. The paper under discussion shows, in a most logical and thoroughly analytical manner, how the explanation has been found for heretofore inexplainable failures in turbine wheels. With proper application of this additional knowledge, the likelihood of failure has been reduced to the minimum.

Referring to Table 1, it is the writer's understanding that the number of wheels in every case refers to single-row wheels and not to wheels with relatively wide rims for two or more rows of buckets. This table refers to turbines of over 5000 kw. capacity. The majority of turbines of 5000 kw. and smaller operate at higher speeds and are fitted with stiffer wheels of smaller diameter. There had been no indication of vibration of these smaller wheels. However, an exhaustive investigation was undertaken of all single-row turbine wheels as shown in Table 6.

TABLE 6 FOR TURBINES UNDER 5000 KW. INSTALLED BEFORE NOVEMBER, 1923

No. of wheels installed	Single Row	3463
	Double Row	1307
	Total	4770
No. of wheels tested (standing)		170
No. of wheels rotated in wheel-testing machine		10
No. of tests in wheel-testing machine		15
No. of wheels tested in customers' plants (standing)		39
No. of machines investigated in customers' plants		11
No. of machines tested under load		1
No. of wheels replaced to avoid possible trouble		8
No. of wheels tuned for vibration		8

Referring to Item 5, Table 2, and to Pars. 198 and 199: While it is true that this wheel let go at approximately 2800 r.p.m. which is 48 r.p.s. and at this speed the wheel showed six nodes, it is also true that the turbine in all probability operated for a while at 3900 r.p.m. or 65 r.p.s. At this speed the wheel would be subject to 4-node vibration. It is therefore probable that the fracture was started while operating at the higher speed and was the direct result of 4-node vibration. A crack once started, it is to be expected that the wheel would ultimately fail. Vibration at 2800 r.p.m. would hasten the progress of the fracture. This is the only case on record of the failure of one of the smaller wheels which might be attributed to lateral vibration. The amplitude of the 6-node vibration of the smaller wheels is so minute that it

[1] Designing Engineer, Turbine Engineering Department, River Works, General Electric Company, West Lynn, Mass. Mem. A. S. M. E.

does not constitute an element of danger. In Par. 124 reference is made to a value of y_0 of $\frac{1}{4}$ in., which means an amplitude of vibration of approximately $\frac{1}{2}$ in. This refers to a wheel of relatively large diameter. In the smaller-diameter wheels when vibration tests were conducted with a magnet similar to that used on the large wheels, it was in many instances difficult to get an amplitude of vibration sufficient to give positive indications with the oscillograph. In referring to this it is not the intention to give the impression that small wheels are safer than large, but merely to bring out the fact that when designed for strength, they are inherently stiffer. Exhaustive investigations showed clearly that 6-node vibration of the smaller wheels contained no element of danger.

The smaller diameter wheels designed for 3600 r.p.m. have a 4-node major well above the running speed. Tests on a large number of wheels are very consistent. It is present practice to test two wheels of every new design as brought out.

The relative advantages and disadvantages of the so-called stiff shaft referred to in Par. 9 might well be the subject of another paper. Stiff shafts have the first critical above the running speed. There are in commercial service several thousand turbines with shafts having the first critical below the operating speed. These turbines have given most excellent account of themselves on the score of mechanical operation.

It is impossible to overestimate the value of the author's contribution. With thorough knowledge of the precautions that must be taken in a design, it is possible to minimize the liabilities of failure. So long as some of the failures referred to by the author were still unexplained, it is easily understood why many people considered the operation of turbines, and especially the larger ones, as hazardous. At the present time there is no more hazard in the operation of large steam turbines designed in accordance with the best practice than there is in the operation of steam boilers, oil engines, or any other piece of apparatus in which many of the component parts are subject to high stresses.

Roger D. De Wolf.[1] It seems to be a question as to whether it is necessary to have an exact equality of speed between the traveling wave and the speed of rotation. In Table 2, Item 2, there is a difference of 7 per cent between the backward speed of the wave and the operating speed of the wheel. Item 4 of the same table shows a difference of 5 per cent. In both cases the wheel broke. In Item 8, the second case of bucket failure, the difference in speed of the wave and the wheel was 8.7 per cent. These figures would indicate either that it is not necessary for the backward speed of

[1] Chief Operating Engineer, Rochester Gas & Electric Corporation, Rochester, N. Y. Mem. A. S. M. E.

the rotating wave to be exactly the same as the speed of rotation, or that the company is operating the machine for a considerable length of time at speeds different from that at which it was supposed to be operated.

In addition to the vibrations that may be set up at the critical speed, the author treats of other vibrations that may be set up at minor resonant speeds. The writer understands that these vibrations would not be stationary in space, and would inquire if they continue after being set up, and if they are of such amplitude as to engender serious fatigue in the metal of the wheel.

Closure.[1] Dr. Timoshenko apparently fails to appreciate that a high degree of accuracy in the theoretical calculation of turbine-disk vibrations is futile as regards actual wheels. The author tried to make it clear that there is no lack of satisfactory theory and that, in the laboratory, both reeds and disks can be made to agree with theory. The paper has not been concerned with the shapes of wheel that can be accurately calculated, but has rather devoted itself to an exposition of the difficulties encountered in actual manufacture and a discussion of the divergences from theory which have never been appreciated before.

For instance, there are many variable factors in the problem of frequency calculation which are not susceptible of theoretical expression, such as the state of internal stress and the tightness of bucket fit, which have important effects upon the vibration frequency and which, if neglected, vitiate the results of the most exact theoretical calculations.

Southwell's conclusion that $f_r^2 = f_s^2 + f_c^2$, mentioned by Dr. Timoshenko, is interesting in view of the fact that, as noted in the paper, it is approximately confirmed in the case of many wheels tested.

As regards the uncertainty of stress determination, Dr. Timoshenko fails to realize the difficulty of determining in a turbine the actual maximum deflection itself. In comparison with this difficulty the determination of the shape of the deflection curve and its necessary derivatives is a mere detail. However, in a turbine the amplitude cannot exceed the clearance, and this *limit* alone permits useful calculations to be made even if a higher degree of accuracy might be desired.

Dr. Timoshenko's closing remarks are obviously due to haste incident to the short time available for consideration of the paper. The discrepancies in the case of circular plates became large for

[1] Owing to Mr. Campbell's untimely death very shortly after the presentation of this paper, the closure is prepared entirely by Messrs. Kimball and Robinson, but includes the remarks made by Mr. Campbell at the meeting in answer to Mr. DeWolf. The short time allowed at the meeting prevented the author from replying in person to any other discussion at that time.

thin plates or plates of large diameter. Those of moderate proportions, as shown in the plottings, agreed with theory. In view of the many photographs reproduced in the paper, the author hardly thought it necessary to state explicitly that in all cases the disks were mounted at the center as in a turbine and were entirely free at the edge. The actual amplitudes in sample cases may be scaled from the smoked-glass curves presented in Fig. 56 and by reference to Figs. 57 and 58. As a matter of fact, no significant variations of frequency have been observed with amplitudes so great as to be accompanied by bending stresses equal to the elastic limit.

Professor Moore has noted that the actual problem was to devise such means that axial vibration would not occur. He notes two important conclusions from his investigation of the fatique of metals. Although the present paper shows how dangerous vibrations are guarded against, still it is a fact that the stresses at steam balance holes in wheel webs are carefully limited and in addition the region about such holes is well polished to remove tool marks.

Dr. Heymans, in his contribution, first discusses the nature of the stress about a circular hole in an infinitely extending plate, giving the formulas for the stress distribution. It is interesting to note that these are the standard formulas used by the General Electric Company for calculations of stresses about holes in turbine wheels. It is pointed out that "ruptures run or start at the holes at the two ends of diameters very approximately coinciding with the normal to the direction of the centrifugal force." This fails to recognize that the centrifugal force, although radial often produces tangential stresses which are *greater* than the radial. As a matter of fact, the directions of the ruptures are determined by the superposition of the vibrational stresses upon the centrifugal stresses and by using the Goodman diagram for repeated stress other than reversed stress as determined by Professor Moore for the Joint Investigation on the Fatigue of Metals.

As regards the use of the photoelastic method of analysis for local stresses in internal angles and fillets, it can be said that the entire line of standard dovetails used by the General Electric Company has been analyzed by this method in comparison with a large variety of special dovetail designs, and shown to be capable of very little improvement.

The stress-strain formulas presented by Dr. Heymans are the standard ones which, as he notes, may be found in any work on the theory of elasticity. They do not apply to rubber models unless the strains are comparatively small. In the stress experiments on rubber models cited, the results give an approximate idea of the stress distribution, as was verified from comparisons with values calculated by the elastic theory using the same formulas mentioned by Dr. Heymans for stresses about a circular hole.

As regards the vibrations of thin disks even though the deflections are seemingly large, the classical theory of elasticity gives good results. A few simple experiments with vibrating disks and vibrating reeds show no perceptible changes of frequency with amplitude, as should be the case if the vibrational strains were large enough to make the standard elastic theory invalid.

Regarding the reference to Pars. 41 and 42, the two elastic connections R_s and R_c are not to be regarded as so applied as to give the vibrating mass two degrees of freedom as shown in Fig. 104. There is perhaps an ambiguity in the first line of Par. 42 which led to a misunderstanding of this point, but the way the two elastic constants R_s and R_c are combined in calculating the new frequency f_r shows that only one degree of freedom is concerned, as Dr. Heymans himself concedes.

Regarding the remarks on "transverse stiffness," the use of this expression is evidently misunderstood. "Transverse stiffness" as used in the paper means resistance to transverse bending and not resistance to transverse elastic strains as set forth in Dr. Heymans' discussion in the two paragraphs beginning with "Let e_{zz} be the . . . etc." Furthermore, the transverse stress *ss* mentioned in these paragraphs is purely hypothetical, as no such stress exists in the ordinary theory of disk-wheel stresses which treats the problem purely as a case of plane stress. In view of the misunderstanding as to transverse stiffness, the remarks on the effect of torque upon transverse stiffness do not apply.

In his objection to Par. 59 Dr. Heymans states that the frequency of a vibrating body is *not* the same for each unit of mass throughout the entire structure. While this statement is, mathematically speaking, true in the case of actual turbine wheels, one mode of vibration predominates to the practical exclusion of all others, as simple tests show, such as the sand pictures of Figs. 18 to 24 in the paper.

Referring to Dr. Heymans' remarks on a simple cantilever bar, he is incorrect in his statement that "the author's theoretical expectations are not entirely accurate." The classical theory which he presents is exactly that used by the author, and his and the author's formulas (Par. 129) are identical for the fundamental mode of vibration, except that Dr. Heymans assumes the bar to be of unit length, and has expressed his frequencies in radians per second instead of in cycles per second. The slight variations of the experiments on bars from theory are erratic and are the result of the difficulty of exactly realizing in the experiments the clamping conditions required by theory, and are not due to faulty theory. It has not been the intention of this paper to discuss at length the vibration of reeds and bars, but in view of the discussion it may be noted that experiments show it equally possible to have a pure vibration at the higher frequency of what Dr. Heymans calls the "second component oscillation," and to have

this independently of the first or fundamental type. Dr. Heymans' formulas merely show the possibility of the various independent types, but his contention that they necessarily all exist at once, while mathematically true, does not hold in practical cases.

The information presented by Mr. Dickinson in regard to small sizes of turbines is a valuable contribution.

Mr. DeWolf's question as to the difference between the speed of the backward-traveling wave and the speed of rotation of the wheel, is covered by what is called the "broadness of resonance." While vibration can be excited somewhat off the actual coincidence, tests show that vibrations do not build up unless the running speed is within a margin of two per cent of the wheel critical speed. The wave speeds given in Table 2, to which Mr. DeWolf refers, were made up from various sources and, especially in these early cases, had to be estimated by calculation or reference to similar designs because no actual records of the wheel characteristics were on file. The greater differences of this table are thus due to error of calculation or estimate, and not to excessive broadness of resonance.

In regard to minor resonant speeds, it may be said that waves which are not stationary in space have been produced in the wheel-testing machine, and that whereas a fixed-pressure spot will hold a stationary wave, an unbalance will give a minor resonance. If the unbalance moves the shaft in a horizontal plane with one exciting impression per revolution, and if the wave was started and happened to be of the correct velocity so that this one pressure per revolution was in resonance with it as it passed the particular plane of unbalance, the vibration could be excited. This is done in the wheel-testing machine by attaching a large piece of iron to the shaft, which causes one end of the shaft to move horizontally since the horizontal stiffness of the bearing is much less than the vertical stiffness. Although the possibility of the maintenance of such waves by unbalance has been thus demonstrated, it has never been possible to cause minor vibrations of serious amplitude. Minor resonant vibrations are associated with rough running, and the possibility of such resonance is not considered serious in machines in good operating condition.

No. 1921

TEMPERATURE AND STRESS DISTRIBUTION IN HOLLOW CYLINDERS

APPLICATIONS TO BOILER TUBES, CYLINDER LINERS, AND ORDNANCE

By O. G. C. Dahl,[1] Cambridge, Mass.
Non-Member

This paper contains an analysis of temperature and stress distribution in hollow cylinders. The results should be applicable to tubes, cylinder liners and ordnance.

Whenever a difference in temperature exists between the two walls of a hollow cylinder, stresses are set up. The magnitude of the temperature stresses depends upon the magnitude of the temperature difference and upon its variation with time. The resultant stresses to which a cylinder is subjected are the sum of the temperature stresses and the stresses due to forces and pressures.

The analysis has been limited to the case where the distribution of temperature is symmetrical with respect to the axis of the cylinder and also independent of distance parallel to the axis. In other words, the temperature at any point is assumed to be a function of radius and time only.

Complete solutions are presented for the case where the temperature difference between the two walls is constant. Steady-state solutions are presented for cases where the temperature difference varies sinusoidally with time, where it consists of a sinusoidally varying temperature superimposed upon a constant temperature, and where it undergoes any cyclic variation expressible as a Fourier series.

Numerical examples of computing the stresses in boiler tubes and in the cylinder liner of a Diesel engine are given and the results discussed.

IT IS a well-known fact that stresses may be set up when differences in temperature exist between points in a piece of metal, depending upon whether or not it is free to expand. If the piece of metal under consideration constitutes a part of an engine, it is very likely that its capability of expanding is limited. It may perhaps expand freely in one direction while it is prevented from doing so in other directions. In such cases stresses due to temperature will be produced in addition to the stresses due to forces or pressures.

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2 In many cases the temperature stresses are insignificant. In other cases they may be of a magnitude comparable to that of stresses caused by forces or pressures. Prediction of temperature stresses, therefore, is of importance in such cases.

3 It is evident that, in order to calculate stresses caused by a certain temperature distribution, this temperature distribution must be known. Hence the first part of the problem is to determine the temperatures at the instant at which the stresses are desired; the second part is to compute the stresses themselves. Both may very readily become problems of considerable difficulty, depending upon the shape of the machine part for which the computations are made and also to a large extent upon the temperature condition. It very rapidly becomes necessary to impose limiting assumptions in order to keep the problems within the reach of present-time mathematics.

4 It is the purpose of this paper to treat the comparatively simple case of distribution of temperature and stresses in hollow cylinders for certain limited conditions. The results should be applicable to hollow tubes, cylinder liners, and ordnance.

Temperature Distribution

5 The stresses at a fixed instant depend upon the temperature distribution at that particular instant. This implies that if the temperature at any point is a function of time it is necessary to know this function in order to arrive at a value for the stresses. The stresses will also vary with time in such cases.

6 The variation with time may be of either a transient or periodic nature. The former involves the *formation* of the temperature gradient, while the latter is merely a cyclic variation in an *already established* temperature distribution and is therefore a "steady state" phenomenon which in most cases can be almost as easily handled as if the distribution were a function of *dimensions* only and *entirely independent* of time.

7 The general equation for flow of heat in a homogeneous material is the well-known Fourier equation which in rectangular coördinates may be written[1]:

$$\nabla^2\theta = h^2 \frac{\partial \theta}{\partial t} \quad \ldots\ldots\ldots \quad [1]$$

where

$$\nabla^2\theta = \frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2} \quad \ldots\ldots \quad [2]$$

8 In polar coördinates Fourier's equation becomes:

$$\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{1}{r^2}\frac{\partial^2\theta}{\partial\phi^2} + \frac{\partial^2\theta}{\partial z^2} = h^2\frac{\partial\theta}{\partial t} \quad \ldots \quad [3]$$

[1] Consult Appendix No. 3 for the meaning of the symbols used.

Obviously, in considering temperature distributions in *cylinders* Equation [3] should be used.

9 Throughout this paper the following *limitations* will be imposed:

$$\frac{\partial^2\theta}{\partial\phi^2}=0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; [4]$$

$$\frac{\partial^2\theta}{\partial z^2}=0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; [5]$$

10 Equations [4] and [5] express the fact that the distribution of temperature is symmetrical with respect to the axis of the cylinder and also independent of distances parallel to the axis. In other words, the temperature at any point will be a function of radius and time only.

11 The temperature difference which is effective in establishing a temperature gradient through the cylinder is consequently the instantaneous difference in temperature between its inside and outside walls. It is therefore permissible to express the temperatures in degrees *above* the actual temperature of one of the walls. The temperature of this wall, for instance the outside, may then be considered zero when solving the differential equations and the temperature of the inside wall taken as the difference between the *actual* temperatures of the two walls.

12 It has been stated above that the polar-coördinate differential equation should be used. In *thin* cylinders, however, where the area through which the heat flows radially ($2\pi r$ per unit axial length) does not change appreciably from the inner to the outer wall, it is sufficiently accurate to use rectangular-coördinate equations. This is equivalent to neglecting the term

$$\frac{1}{r}\frac{\partial\theta}{\partial r}$$

in Equation [3].

13 Fig. 1 illustrates the degree of approximation involved when this term is neglected. The curve shows the maximum percentage error introduced in the *steady-state* distribution versus ratio $\left(\frac{r_2}{r_1}\right)$ between outer and inner radii of the cylinder.

(*a*) *Thin Cylinders*[1]

14 The simplified differential equation is obtained from [3] by neglecting the term $\frac{1}{r}\frac{\partial\theta}{\partial r}$ and imposing the limitations expressed by Equations [4] and [5]. Hence

$$\frac{\partial^2\theta}{\partial r^2}=h^2\frac{\partial\theta}{\partial t} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; [6]$$

[1] See Appendix No. 1 for details of solutions.

15 1—*Constant Temperature Difference.* When the difference between the temperatures of the inner and outer wall is a *constant*=θ_1 the solution of [6] becomes

$$\theta=\theta_1\left[1-\frac{r-r_1}{r_2-r_1}\right]-\frac{2\theta_1}{\pi}\sum_{n=1}^{n=\infty}\frac{1}{n}\varepsilon^{-\frac{n^2\pi^2}{h^2(r_2-r_1)^2}t}\sin\frac{n\pi(r-r_1)}{r_2-r_1}\quad.\ [7]$$

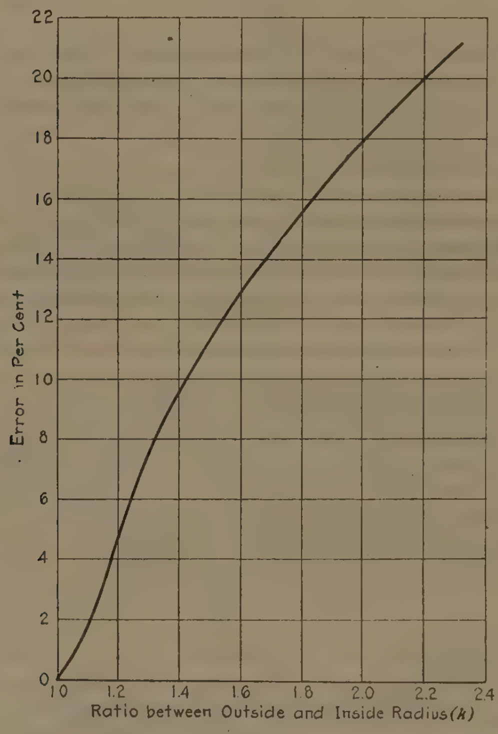

FIG. 1 ERROR IN TEMPERATURE DISTRIBUTION BY APPROXIMATE FORMULA

(This curve gives the maximum percentage error in the steady-state temperature distribution through the cylinder wall when the "thin-cylinder formula" is used in place of the correct "thick-cylinder formula".)

16 The temperature distribution given by Equation [7] is plotted in Fig. 2 for $t=0$ and $t=\infty$. When $t=\infty$ the last term of [7] vanishes; the first term therefore represents the *steady-state* distribution and is a straight line:

$$\theta=\theta_1\left[1-\frac{r-r_1}{r_2-r_1}\right]\quad.\ .\ .\ .\ .\ .\ .\ .\ [8]$$

17 When $t=0$ the second or transient term equals the steady-state term and the temperature is zero at all points where $r>r_1$.

18 2—*Sinusoidally Varying Temperature Difference.* Assuming that the temperature difference θ_1 undergoes a sinusoidal variation with time, i.e., may be expressed as

$$\theta_1 \sin \omega t = \theta_1 \sin 2\pi f t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [9]$$

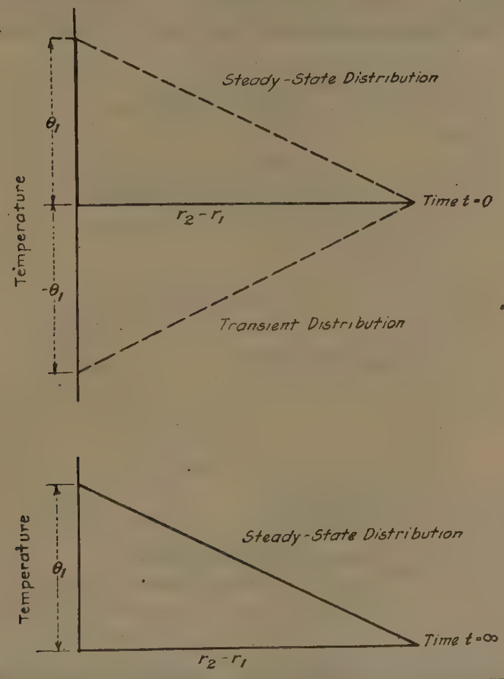

Fig. 2 Temperature Distribution through Cylinder Wall for Constant Temperature Difference

the plane-vector solution of Equation [6] becomes

$$\dot{\theta} = \frac{\sinh \alpha(r_2-r)}{\sinh \alpha(r_2-r_1)} \dot{\theta}_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [10]$$

where

$$\alpha = \sqrt{j\omega h^2} = h\sqrt{\omega}\underline{/45°} \quad . \quad . \quad . \quad . \quad . \quad . \quad [11]$$

19 As seen, α is a semi-imaginary quantity. Equation [10] therefore involves hyperbolic sines of semi-imaginary arguments.[1] The hyperbolic sines will in general be complex quantities; hence

[1] Excellent tables of such functions are included in A. E. Kennelly's Tables of Complex Hyperbolic and Circular Functions.

their ratio will also in general be a complex quantity and may be written

$$\frac{\sinh \alpha(r_2-r)}{\sinh \alpha(r_2-r_1)}=\rho/\phi \quad . \; . \; . \; . \; . \; . \; . \; [12]$$

Hence

$$\dot{\theta}=\rho\dot{\theta}_1/\phi \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; [13]$$

20 Equation [13], which still is a plane-vector equation, indicates that the temperature at any radius r varies sinusoidally

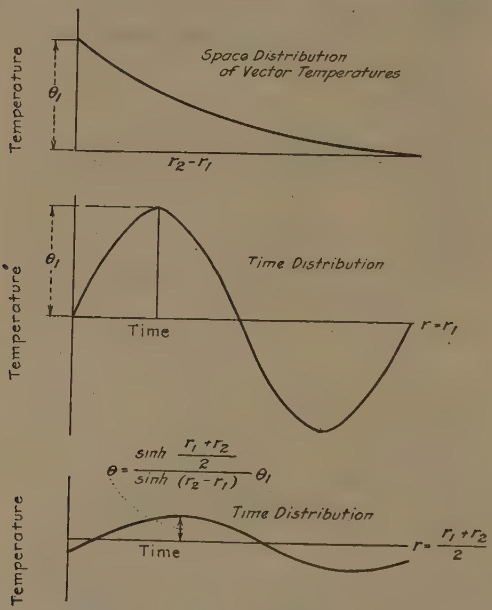

FIG. 3 STEADY-STATE TEMPERATURE DISTRIBUTION FOR SINUSOIDALLY VARYING TEMPERATURE DIFFERENCE

with time but is phase-displaced ϕ degrees[1] from the sinusoidally varying temperature θ_1.

21 The instantaneous steady-state value of θ at any time t is therefore

$$\theta=\rho\theta_1 \sin(\omega t+\phi) \quad . \; . \; . \; . \; . \; . \; . \; [14]$$

or

$$\theta=\left|\frac{\sinh \alpha(r_2-r)}{\sinh \alpha(r_2-r_1)}\right| \theta_1 \sin(\omega t+\phi) \quad . \; . \; . \; . \; [15]$$

where

$$\left|\frac{\sinh \alpha(r_2-r)}{\sinh \alpha(r_2-r_1)}\right|$$

means the *numerical* value ρ of the ratio of the hyperbolic sines.

[1] *Circular* degrees, not *temperature* degrees.

22 Fig. 3 illustrates the space distribution of vector temperatures through the cylinder wall and also the time distribution of the temperatures when $r=r_1$ and $r=\frac{r_2+r_1}{2}$.

23 3—*Sinusoidally Varying Temperature Superimposed upon Constant Temperature.* If the temperature difference θ_1 is com-

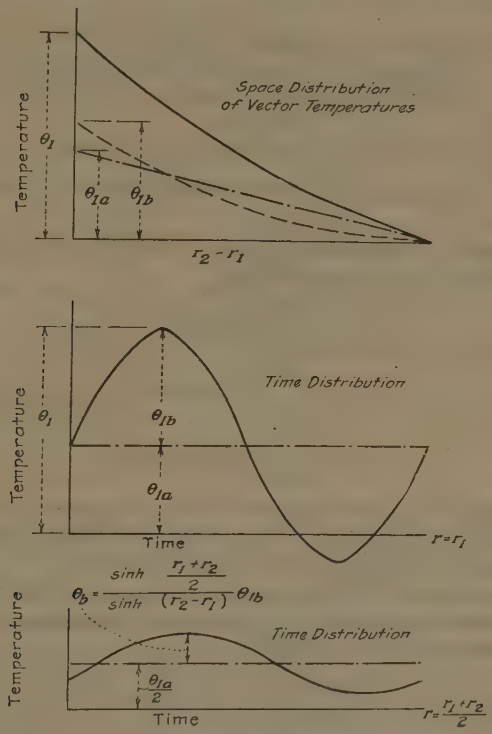

FIG. 4 STEADY-STATE TEMPERATURE DISTRIBUTION FOR SINUSOIDALLY VARYING TEMPERATURE DIFFERENCE SUPERIMPOSED UPON CONSTANT TEMPERATURE DIFFERENCE

posed of a sinusoidally varying temperature superimposed upon a constant temperature difference, i.e.,

$$\theta_1=\theta_{1a}+\theta_{1b}\sin\omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [16]$$

the steady-state distribution is obtained by adding solutions [8] and [10] or solutions [8] and [15]. Fig. 4 illustrates the temperature distribution in this case.

24 4—*Any Periodic Temperature Variation.* If the temperature difference θ_1 undergoes any periodic variation with time, i.e.,

$$\theta_1=F(t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [17]$$

$F(t)$ may be expressed as a Fourier series. Hence θ_1 may be written

$$\theta_1 = A_0 + A_1 \sin \omega t + A_2 \sin 2\omega t + A_3 \sin 3\omega t + \ldots \\ + B_1 \cos \omega t + B_2 \cos 2\omega t + B_3 \cos 3\omega t + \ldots \quad [18]$$

25 If the periodic function $F(t)$ is symmetrical with respect to the time axis, the constant term and the even terms vanish, giving

$$\left.\begin{aligned} \theta_1 &= A_1 \sin \omega t + A_3 \sin 3\omega t + A_5 \sin 5\omega t + \ldots \\ &\quad + B_1 \cos \omega t + B_3 \cos 3\omega t + B_5 \cos 5\omega t + \ldots \\ &= \theta_1' \sin (\omega t + \beta') + \theta_1''' \sin (3\omega t + \beta''') + \ldots \end{aligned}\right\} \quad [19]$$

where

$$\left.\begin{aligned} \theta_1' &= \sqrt{A_1^2 + B_1^2}, \quad \theta_1''' = \sqrt{A_3^2 + B_3^2} \text{ etc.} \\ \beta' &= \arctan \frac{B_1}{A_1}, \quad \beta''' = \arctan \frac{B_3}{A_3} \text{ etc.} \end{aligned}\right\} \quad [19a]$$

26 When $F(t)$ is known, the Fourier coefficients are determined in the ordinary way by computation.

$$A_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} F(t) dt \quad [20]$$

$$A_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(t) \sin nt \, dt \quad [21]$$

$$B_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(t) \cos nt \, dt \quad [22]$$

27 When $F(t)$ is not capable of being expressed in mathematical form but a plot of it is at hand, the Fourier coefficients are obtained by analyzing the plotted curve.[1]

28 Assume that θ_1 is expressible by Equation [19]. Treating each harmonic separately, the harmonic temperatures at any radius r become (Equation [10]):

$$\dot{\theta}' = \frac{\sinh \alpha (r_2 - r)}{\sinh \alpha (r_2 - r_1)} \dot{\theta}_1' = \rho' \dot{\theta}_1' / \underline{\phi'} \quad [23]$$

$$\dot{\theta}''' = \frac{\sinh \sqrt{3}\alpha (r_2 - r)}{\sinh \sqrt{3}\alpha (r_2 - r_1)} \dot{\theta}_1''' = \rho''' \dot{\theta}_1''' / \underline{\phi'''} \quad [24]$$

$$\dot{\theta}^{v} = \frac{\sinh \sqrt{5}\alpha (r_2 - r)}{\sinh \sqrt{5}\alpha (r_2 - r_1)} \dot{\theta}_1^{v} = \rho^{v} \dot{\theta}_1^{v} / \underline{\phi^{v}} \quad [25]$$

29 The instantaneous steady-state value of θ at any time t is (Equations [15] and [19]):

$$\theta = \rho' \theta_1' \sin (\omega t + \beta' + \phi') + \rho''' \theta_1''' \sin (3\omega t + \beta''' + \phi''') \\ + \rho^{v} \theta_1^{v} \sin (5\omega t + \beta^{v} + \phi^{v}) \quad [26]$$

(*b*) *Thick Cylinders*

[1] See for instance, J. Lipka: Graphical and Mechanical Computations.

30 The differential equation for this case is obtained from Equations [3], [4] and [5]:

$$\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}=h^2\frac{\partial\theta}{\partial t} \quad . \; . \; . \; . \; . \; . \; . \quad [27]$$

31 When θ_1 is constant the complete solution of this equation is:

$$\theta=\frac{\theta_1}{\log\frac{r_2}{r_1}}\log\frac{r_2}{r}-\sum_{k=1}^{k=\infty}A_k\left[J_0(\mu_k r)-\frac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)}N_0(\mu_k r)\right]\varepsilon^{-\frac{\mu_k^2}{h^2}t} \quad . \quad [28]$$

32 The μ_k's are roots of the equation

$$J_0(\mu r_1)-\frac{J_0(\mu r_2)}{N_0(\mu r_2)}N_0(\mu r_1)=0 \quad . \; . \; . \; . \; . \quad [29]$$

33 The coefficients A_k are obtained from

$$A_k=\frac{A_0}{\mu_k}r_1\log\frac{r_2}{r_1}\left[\frac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)}N_1(\mu_k r_1)-J_1(\mu_k r_1)\right] \quad . \quad [30]$$

when A_0 is given by:

$$A_0=\frac{\dfrac{2\theta_1}{\log\frac{r_2}{r_1}}}{r_2^2\left[\frac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)}N_1(\mu_k r_2)-J_1(\mu_k r_2)\right]^2-r_1^2\left[\frac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)}N_1(\mu_k r_1)-J_1(\mu_k r_1)\right]^2} \quad [31]$$

34 In Equations [28] to [31], inclusive, the symbol J designates a cylinder function (Bessel function) of the first kind and N designates a cylinder function (in this case a Neumann type of Bessel function[1]) of the second kind. The subscripts attached to the symbols indicate the order of the function.

35 We are concerned only with the real roots of Equation [29], hence numerical computations will involve Bessel functions of real arguments. Very complete tables and curves of such functions will be found in the tables referred to in the footnote[1] below.

36 Fig. 5 gives the value of the functions $J_0(x)$ and $J_0'(x)=-J_1(x)$ and Fig. 6 the value of the functions $N_0(x)$ and $N_0'(x)=$

[1] This type is chosen instead of the more frequently encountered Bessel function of the second kind K merely because the tables of cylinder functions to which the author had access (Jahnke und Emde: Funktionentafeln mit Formeln und Kurven) have complete tables of N_0 and N_1, while tables of the K functions are lacking. The two types of functions merely differ by a constant factor, viz. $\frac{\pi}{2}(N_n(x))=-K_n(x)$.

$-N_1(x)$ for values of x from 0 to 10. Fig. 7 gives the first six roots $(k-1)\mu$ of the equation

$$J_0(\mu)-\frac{J_0(k\mu)}{N_0(k\mu)}N_0(\mu)=0 \quad . \; . \; . \; . \; . \; [32]$$

They are plotted versus $k=\frac{r_2}{r_1}$.

37 When the roots are given in this form $[(k-1)\mu]$, they are coincident with the roots of Equation [29].

38 The temperature distribution given by Equation [28] is plotted in Fig. 8 for $t=0$ and $t=\infty$. When $t=\infty$ the second term

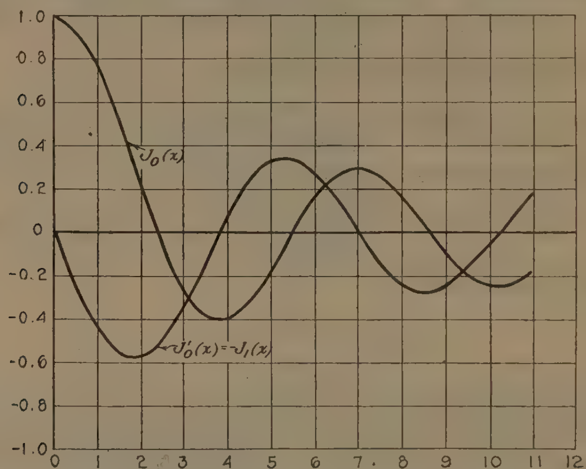

Fig. 5 Bessel Functions of the First Kind, Zeroeth and First Order

vanishes and the first term represents the *steady-state* distribution, which is logarithmic and given by

$$\theta=\frac{\theta_1}{\log\frac{r_2}{r_1}}\log\frac{r_2}{r} \quad . \; . \; . \; . \; . \; . \; . \; . \; [33]$$

39 When $t=0$ the second or *transient* term equals the steady-state term and the temperature is zero at all points where $r>r_1$.

40 2—*Sinusoidally Varying Temperature.* Assuming the temperature difference θ_1 to vary sinusoidally with time as expressed by Equation [9], the plane-vector solution of Equation [27] becomes

$$\dot{\theta}=\frac{J_0(qr)N_0(qr_2)-J_0(qr_2)N_0(qr)}{J_0(qr_1)N_0(qr_2)-J_0(qr_2)N_0(qr_1)}\dot{\theta}_1 \quad . \; . \; . \; [34]$$

$$q=\sqrt{-j\omega h^2}=h\sqrt{\omega}\diagdown 45^\circ \quad . \; . \; . \; . \; . \; [35]$$

q is thus a semi-imaginary quantity as is the quantity α in Equation [11]. The numerical value of the two quantities is the same, but in α the imaginary part is positive, in q it is negative. The relation between the two is given by

$$q = -j\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [36]$$

41 Bessel functions of semi-imaginary arguments are in general complex quantities and they may be obtained in almost any table of Bessel functions.[1]

42 The same discussion as was presented in connection with Equations [10] and [11] applies to Equations [33] and [34]

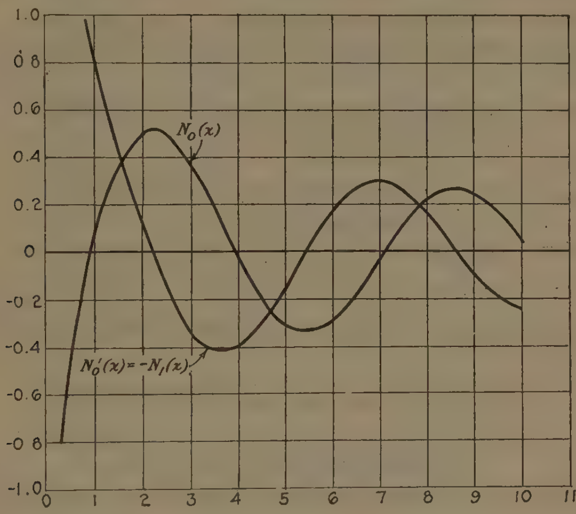

Fig. 6 Bessel Functions of the Second Kind (Neumann's Type), Zeroeth and First Order

and will not be repeated. The instantaneous steady-state value of θ at any time t and radius r is therefore

$$\theta = \left| \frac{J_0(qr)N_0(qr_2) - J_0(qr_2)N_0(qr)}{J_0(qr_1)N_0(qr_2) - J_0(qr_2)N_0(qr_1)} \right| \theta_1 \sin(\omega t + \phi) \quad . \quad [37]$$

where the first part designates the *numerical* value of the Bessel functions and ϕ the phase displacement[2] between the sinusoidally varying temperatures θ_1 and θ. Fig. 3 may be taken as an illustration of the temperature distribution in this case.

43 3 — *Sinusoidally Varying Temperature Superimposed Upon Constant Temperature.* If the temperature difference θ_1 is com-

[1] For instance, Jahnke und Emde: Funktionentafeln mit Formeln und Kurven.

[2] In circular degrees.

posed of a sinusoidally varying temperature superimposed upon a constant temperature as expressed by Equation [**16**], the steady-state distribution is obtained by adding solutions [**33**] and [**34**] or solutions [**33**] and [**37**]. Fig. 4 illustrates the temperature distribution in this case.

44 4—*Any Periodic Temperature Variation.* If the temperature difference θ_1 undergoes any periodic variation with time which is capable of being expressed as a Fourier series, the method previously discussed in connection with Equations [**17**] to [**26**], inclusive, may be employed.

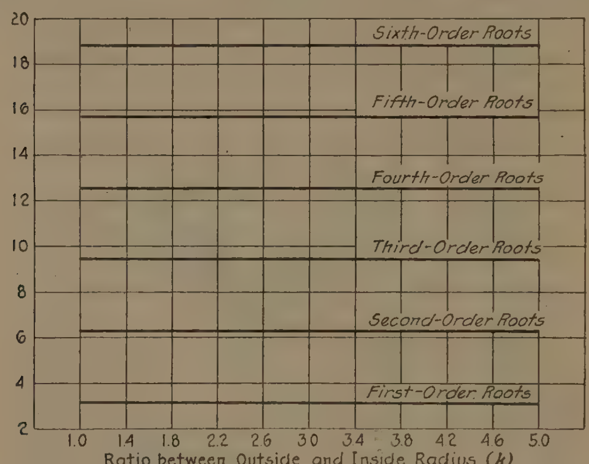

FIG. 7 ROOTS $(k-1)x$ OF THE EQUATION $J_0(x)N_0(kx)-J_0(kx)N_0(x)=0$

45 Treating each harmonic separately and assuming that θ_1 is expressible by Equation [**19**], the plane-vector temperatures at any radius r become (Equation [**34**]):

$$\dot{\theta}'=\frac{J_0(qr)N_0(qr_2)-J_0(qr_2)N_0(qr)}{J_0(qr_1)N_0(qr_2)-J_0(qr_2)N_0(qr_1)}\dot{\theta}_1'=\rho'\dot{\theta}_1'/\phi' \quad . \quad . \quad . \quad [38]$$

$$\dot{\theta}'''=\frac{J_0(\sqrt{3}qr)N_0(\sqrt{3}qr_2)-J_0(\sqrt{3}qr_2)N_0(\sqrt{3}qr)}{J_0(\sqrt{3}qr_1)N_0(\sqrt{3}qr_2)-J_0(\sqrt{3}qr_2)N_0(\sqrt{3}qr_1)}\dot{\theta}_1'''$$
$$=\rho'''\dot{\theta}_1'''/\phi \quad . \quad [39]$$

$$\dot{\theta}^{v}=\frac{J_0(\sqrt{5}qr)N_0(\sqrt{5}qr_2)-J_0(\sqrt{5}qr_2)N_0(\sqrt{5}qr)}{J_0(\sqrt{5}qr_1)N_0(\sqrt{5}qr_2)-J_0(\sqrt{5}qr_2)N_0(\sqrt{5}qr_1)}\dot{\theta}_1^{v}$$
$$=\rho^{v}\dot{\theta}_1^{v}/\phi^{v}. \quad . \quad [40]$$

46 The instantaneous steady-state value of θ at any time t and radius r is (Equations [**19**] and [**37**]):

$$\theta = \rho'\theta_1' \sin(\omega t + \beta' + \phi') + \rho'''\theta_1''' \sin(3\omega t + \beta''' + \phi''') + \rho^{v}\theta_1^{v} \sin(5\omega t + \beta^{v} + \phi^{v}) \quad \ldots \ldots \text{[41]}$$

Stress Analysis

47 When the temperature distribution through the wall of the cylinder is known at a certain instant the corresponding

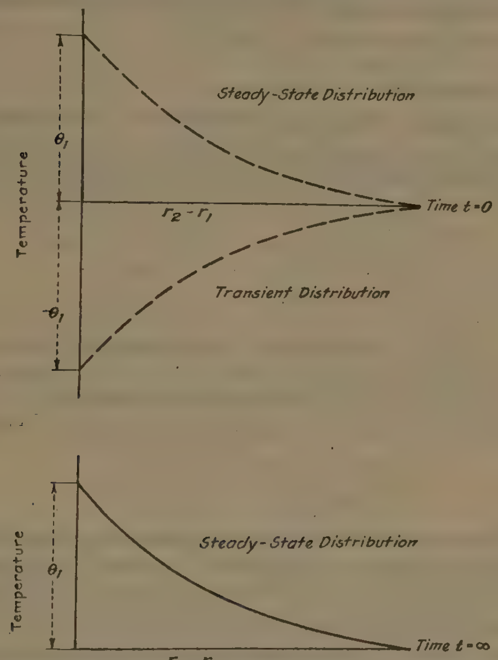

Fig. 8 Temperature Distribution through Wall of Thick Cylinder for Constant Temperature Difference

stresses set up can be computed by the following formulas[1]:

$$f_r = aE\left[-\frac{1}{r^2}\int_{r_1}^{r}\theta r dr + \left(1 - \frac{r_1^2}{r^2}\right)\frac{\int_{r_1}^{r_2}\theta r dr}{r_2^2 - r_1^2}\right] \quad \ldots \text{[42]}$$

$$f_t = aE\left[\frac{1}{r^2}\int_{r_1}^{r}\theta r dr - \theta + \left(1 + \frac{r_1^2}{r^2}\right)\frac{\int_{r_1}^{r_2}\theta r dr}{r_2^2 - r_1^2}\right] \quad \ldots \text{[43]}$$

48 Equation [42] gives the radial and Equation [43] the tangential stress intensity at any radius r. It has been assumed

[1] See Appendix No. 2 for derivation of formulas.

that the cylinder is free to expand axially so that no direct temperature stresses will be set up in this direction.[1]

49 θ in the two equations is a function of r, i.e.,

$$\theta = F(r) \quad \text{[44]}$$

When the function $rF(r)$ is integrable, the computation of the stress intensities is a simple matter. When $rF(r)$ is not directly integrable, the evaluation of the integrals will have to be performed by graphical or approximate methods and the calculation of the stresses becomes more involved.

A — Thin Cylinders

50 1 — *Constant Temperature Difference.* The *steady-state* distribution of stress intensities is obtained by substituting Equation [8] in Equations [42] and [43] and integrating. This gives:

$$f_r = \frac{aE\theta_1}{3(r_2-r_1)}\left[r - \frac{r_1^3}{r^2} - \left(1 - \frac{r_1^2}{r^2}\right)\frac{r_2^3-r_1^3}{r_2^2-r_1^2}\right] \quad \text{[45]}$$

$$f_t = \frac{aE\theta_1}{3(r_2-r_1)}\left[2r + \frac{r_1^3}{r^2} - \left(1 + \frac{r_1^2}{r^2}\right)\frac{r_2^3-r_1^3}{r_2^2-r_1^2}\right] \quad \text{[46]}$$

51 Obviously, at the inside and outside surfaces of the cylinder the radial temperature stress must be zero. In Equation [45] f_r becomes zero when r is made equal to r_1 or r_2.

52 Maximum radial stress intensity will occur at a radius

$$r = \sqrt[3]{\frac{2r_1^2r_2^2}{r_1+r_2}} = r_1\sqrt[3]{\frac{2k^2}{k+1}} \quad \text{[47]}$$

The radial stresses will always be compressive (f_r from Equation [45] is always negative) when θ_1 is positive. θ_1 is obviously positive when the inside wall of the cylinder is hotter than the outside wall. When θ_1 is negative, viz., the outer surface is hotter than the inner, the radial stresses will be tensile stresses.

53 The tangential stress intensities at the inside ($r=r_1$) and the outside surface ($r=r_2$) of the cylinder are given by Equations [48] and [49], respectively.

$$\left.\begin{aligned} f_{t1} &= \frac{aE\theta_1}{3(r_2-r_1)}\left[3r_1 - 2\frac{r_2^3-r_1^3}{r_2^2-r_1^2}\right] \\ &= \frac{aE\theta}{3(k-1)}\left[3 - 2\frac{k^3-1}{k^2-1}\right] \end{aligned}\right\} \quad \text{[48]}$$

$$\left.\begin{aligned} f_{t2} &= \frac{aE\theta_1}{3(r_2-r_1)}\left[2r_2 + \frac{r_1^3}{r_2^2} - \left(1 + \frac{r_1^2}{r_2^2}\right)\frac{r_2^3-r_1^3}{r_2^2-r_1^2}\right] \\ &= \frac{aE\theta_1}{3(k-1)}\left[2k + \frac{1}{k^2} - \left(1 + \frac{1}{k^2}\right)\frac{k^3-1}{k^2-1}\right] \end{aligned}\right\} \quad \text{[49]}$$

[1] See footnote referred to in par. 164.

54 Evidently f_{t1} as obtained by Equation [48] is negative, indicating that the tangential stress intensity at this point is compressive. Equation [49] shows that f_{t2} always will be positive; in other words, the tangential stresses at the outer surface are tensile stresses. This is on the assumption that the temperature gradient is positive (θ_1 positive). When the temperature gradient is negative, the outer part of the cylinder will be subjected to tangential compression, the inner part to tangential tension. Somewhere in the cylinder wall between r_1 and r_2 the tangential stress must therefore be zero.

55 These conclusions may also be substantiated by plain reasoning, bearing in mind the physical aspects of the phenomena taking place when a temperature gradient is established through the wall.

56 The radius at which the tangential stress intensity goes through zero is given by the positive real root of the equation:

$$\left.\begin{aligned} & r^3-\frac{r_2^3-r_1^3}{2(r_2^2-r_1^2)}r+\frac{r_2^2r_1^2}{2(r_2-r_1)}=0 \\ \text{or} \quad & \\ & r^3-\frac{(k^3-1)r_1}{2(k^2-1)}r+\frac{k^2r_1}{2(k+1)}=0 \end{aligned}\right\} \quad \ldots \ldots [50]$$

57 The mathematical maximum of Equation [46] occurs at a point:

$$r=\sqrt[3]{-\frac{r_1^2r_2^2}{r_1+r_2}}=r_1\sqrt[3]{-\frac{k^2}{k+1}} \quad \ldots \ldots [51]$$

This maximum has evidently no physical significance. The *maximum tangential stresses* (tensile and compressive, respectively) therefore occur at the *outside and inside cylinder walls.*

58 The distribution of stress intensities during the period when the temperature gradient is being established may be found by adding the stresses due to the transient part (the second, negative part) of Equation [7] to the steady-state stress distribution given by Equations [45] and [46]. The transient stress distribution is obtained by evaluating Equations [42] and [43], substituting for θ the function representing the transient temperature distribution. This gives

$$f_r'=\frac{2aE\theta_1}{\pi^2}\sum_{n=1}^{n=\infty}\frac{1}{n^2}\varepsilon^{-\frac{n^2\pi^2}{h^2(r_2-r_1)^2}t}$$
$$\left[-\frac{r_2-r_1}{r^2}\left(r\cos\frac{n\pi(r-r_1)}{r_2-r_1}-\frac{r_2-r_1}{n\pi}\sin\frac{n\pi(r-r_1)}{r_2-r_1}-r_1\right)\right.$$
$$\left.+\left(1-\frac{r_1^2}{r^2}\right)\frac{r_2\cos n\pi-r_1}{r_2+r_1}\right] \quad \ldots \ldots \ldots \ldots [52]$$

$$f_t' = \frac{2aE\theta_1}{\pi} \sum_{n=1}^{n=\infty} \frac{1}{n} \varepsilon^{-\frac{n^2\pi^2}{h^2(r_2-r_1)^2}t}$$

$$\left[\frac{(r_2-r_1)}{n\pi r^2}\left(r \cos \frac{n\pi(r-r_1)}{r_2-r_1} - r_1\right) - \left(\frac{r_2^2-r_1^2}{n^2\pi^2 r^2} - 1\right)\sin \frac{n\pi(r-r_1)}{r_2-r_1}\right.$$

$$\left. + \left(1+\frac{r_1^2}{r^2}\right)\frac{r_2 \cos n\pi - r_1}{n\pi(r_2+r_1)}\right] \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ [53]$$

59 When $r=r_1$ or $r=r_2$, f_r' from Equation [52] becomes zero, which of course is a physical necessity. This holds true for all values of t.

60 When $t=0$, i.e., at the instant the temperature difference θ_1 is applied, it is of great interest to calculate the tangential stress intensity at the inside and outside walls of the cylinder. The transient part of these stresses is obtained by solving Equation [53] for the given conditions. It appears that the transient tangential stress intensities at the two cylinder surfaces are of equal magnitude. They will always be compressive when θ_1 is positive and are given by:

$$f'_{t1} = f'_{t2} = -\frac{aE\theta_1(r_2+2r_1)}{3(r_2+r_1)} = -\frac{aE\theta_1(k+2)}{3(k+1)} \quad . \ . \ [54]$$

61 The actual tangential stress intensities at this instant at the two cylinder surfaces are obviously obtained by adding [48] and [54], and [49] and [54], respectively. Hence

$$f_{t1} = -aE\theta_1 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ [55]$$

$$f_{t2} = 0 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ [56]$$

62 These two equations reveal interesting and important facts. Assuming positive temperature difference θ_1 (the inner surface hotter than the outer), Equation [55] shows that the *maximum possible temperature* stress (compression) occurs at the inside wall at the instant the temperature difference is applied, i.e., at the instant the temperature gradient commences to build up.

63 Equation [56] shows that the tangential stress intensity at the outside of the cylinder is zero at the same instant. This is due to the fact that no heat has as yet diffused into the metal.

64 After a short interval of time the transient terms in the stress equations vanish on account of the exponential damping factor, and the steady-state distribution of stress persists as given by Equations [45] and [46].

65 2—*Sinusoidally Varying Temperature.* The steady-state distribution of stress is obtained by substituting Equation [10] in Equations [42] and [43] and integrating. This gives the plane-vector stress intensities as follows:

$$\dot{f}_r=\frac{aE\dot{\theta}_1}{\alpha}\left[\frac{1}{r^2}\left(\frac{\sinh\alpha(r_2-r)}{\alpha\sinh\alpha(r_2-r_1)}\right.\right.$$
$$\left.+r\frac{\cosh\alpha(r_2-r)}{\sinh\alpha(r_2-r_1)}-r_1\coth\alpha(r_2-r_1)-\frac{1}{\alpha}\right)$$
$$\left.+\frac{1-\frac{r_1^2}{r^2}}{r_2^2-r_1^2}\left(r_1\coth\alpha(r_2-r_1)-\frac{r_2}{\sinh\alpha(r_2-r_1)}+\frac{1}{\alpha}\right)\right] \quad [57]$$

$$\dot{f}_t=\frac{aE\dot{\theta}_1}{\alpha}\left[-\frac{1}{r^2}\left(\frac{\sinh\alpha(r_2-r)}{\alpha\sinh\alpha(r_2-r_1)}+r\frac{\cosh\alpha(r_2-r)}{\sinh\alpha(r_2-r_1)}\right.\right.$$
$$\left.-r_1\coth\alpha(r_2-r_1)-\frac{1}{\alpha}\right)-\frac{\alpha\sinh\alpha(r_2-r)}{\sinh\alpha(r_2-r_1)}$$
$$\left.+\frac{1+\frac{r_1^2}{r^2}}{r_2^2-r_1^2}\left(r_1\coth\alpha(r_2-r_1)-\frac{r_2}{\sinh\alpha(r_2-r_1)}+\frac{1}{\alpha}\right)\right] \quad [58]$$

66 The expressions in the main brackets of Equations [57] and [58] are complex (or plane-vector) functions of r. Both the magnitude and angle of these functions when they are expressed in polar form will vary when r is changed. Equations [57] and [58] may be written:

$$\dot{f}_r=aE\dot{\theta}_1 p_r/\underline{\psi_r} \quad . . . . [59]$$
$$\dot{f}_t=aE\dot{\theta}_1 p_t/\underline{\psi_t} \quad . . . . [60]$$

where p_r, p_t, ψ_r, and ψ_t are functions of the radius r.

67 The actual stress intensities at any time t are consequently given by:

$$f_r=aEp_r\theta_1\sin(\omega t+\psi_r) \quad . . . . [61]$$
$$f_t=aEp_t\theta_1\sin(\omega t+\psi_t) \quad . . . . [62]$$

68 As seen from Equations [61] and [62], the stress intensities undergo a sinusoidal variation with time. At $r=r_1$ and $r=r_2$ Equation [57] gives $\dot{f}_r=0$. At other points in the wall the radial stress intensity will have a *definite maximum* value and will vary periodically between tension and compression.

69 The tangential stress intensity will remain zero at a fixed layer in the wall. At other points it will vary periodically between tension and compression and at each point between fixed maximum limits. The largest tangential stresses will be reached at the inner and outer surfaces. The vector expressions for these stresses ($r=r_1$ and $r=r_2$) are respectively:

$$\dot{f}_{t1}=\frac{2aE\dot{\theta}_1}{\alpha(r_2^2-r_1^2)}\left[\frac{r_1\cosh\alpha(r_2-r_1)-r_2}{\sinh\alpha(r_2-r_1)}+\frac{1}{\alpha}-\frac{\alpha}{2}(r_2^2-r_1^2)\right] \quad [63]$$

$$f_{t2}=\frac{2aE\dot{\theta}_1}{\alpha(r_2^2-r_1^2)}\left[\frac{r_1\cosh\alpha(r_2-r_1)-r_2}{\sinh\alpha(r_2-r_1)}+\frac{1}{\alpha}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad [64]$$

70 The actual stresses at $r=r_1$ and $r=r_2$ may be written:

$$f_{t1}=\frac{2aE}{r_2^2-r_1^2}p_{t1}\theta_1\sin(\omega t+\psi_{t1}) \quad . \quad . \quad . \quad . \quad [65]$$

$$f_{t2}=\frac{2aE}{r_2^2-r_1^2}p_{t2}\theta_1\sin(\omega t+\psi_{t2}) \quad . \quad . \quad . \quad . \quad [66]$$

71 3—*Sinusoidally Varying Temperature Superimposed upon Constant Temperature.* The stresses in the steady state may obviously in this case be found at any radius r and time t by adding Equations [45] and [61], and [46] and [62]. This will give the radial and tangential stress intensities, respectively.

72 4—*Any Periodic Temperature Variation.* If the temperature difference θ_1 varies periodically in such a manner that it may be expressed by a Fourier series, the stresses set up can be found by treating each harmonic separately. Assume temperature conditions as described and treated in connection with Equations [19] to [26], inclusive. Applying Equations [57] and [58], the coefficients p_r and p_t and the angles ψ_r and ψ_t are determined for each harmonic. The stress intensities at any time t are then given by:

$$\begin{aligned} f_r=aE[&p_r'\theta_1'\sin(\omega t+\beta'+\psi_r') \\ &+p_r'''\theta_1'''\sin(3\omega t+\beta'''+\psi_r''') \\ &+p_r^{\mathrm{v}}\theta_1^{\mathrm{v}}\sin(5\omega t+\beta^{\mathrm{v}}+\psi_r^{\mathrm{v}})] \quad . \quad . \quad . \quad . \quad . \quad [67] \end{aligned}$$

$$\begin{aligned} f_t=aE[&p_t'\theta_1'\sin(\omega t+\beta'+\psi_t') \\ &+p_t'''\theta_1'''\sin(3\omega t+\beta'''+\psi_t''') \\ &+p_t^{\mathrm{v}}\theta_1^{\mathrm{v}}\sin(5\omega t+\beta^{\mathrm{v}}+\psi_t^{\mathrm{v}})] \quad . \quad . \quad . \quad . \quad . \quad [68] \end{aligned}$$

73 As before, the radial stress intensity for all values of t will be zero for points at the inside and outside walls of the cylinder. At the same points the maximum tangential stresses will occur periodically.

74 When the Fourier series representing the temperature difference θ_1 includes a constant term, the total stresses are evidently equal to the stresses due to this constant temperature difference (as given by Equations [45] and [46]) in addition to the stresses found by equations of the [67] and [68] type.

B — Thick Cylinders

75 1—*Constant Temperature Difference.* The steady-state distribution of stress intensities is obtained by substituting Equation [33] in Equations [42] and [43] and integrating. This gives:

$$f_r = \frac{aE\theta_1}{2\log\frac{r_2}{r_1}}\left[-\log\frac{r_2}{r}+\frac{r_1^2}{r^2}\log\frac{r_2}{r_1}-\left(1-\frac{r_1^2}{r^2}\right)\frac{r_1^2}{r_2^2-r_1^2}\log\frac{r_2}{r_1}\right] \quad [69]$$

$$f_t = \frac{aE\theta_1}{2\log\frac{r_2}{r_1}}\left[1-\log\frac{r_2}{r}-\frac{r_1^2}{r^2}\log\frac{r_2}{r_1}-\left(1+\frac{r_1^2}{r^2}\right)\frac{r_1^2}{r_2^2-r_1^2}\log\frac{r_2}{r_1}\right] \quad [70]$$

76 Equation [69] gives $f_r=0$ when $r=r_1$ and $r=r_2$. Maximum tangential stresses occur as in the thin cylinder at the inside and outside walls and are:

$$f_{t1} = \frac{aE\theta_1}{2\log\frac{r_2}{r_1}}\left[1-\frac{2r_2^2}{r_2^2-r_1^2}\log\frac{r_2}{r_1}\right] = \frac{aE\theta_1}{2\log k}\left[1-\frac{2k^2\log k}{k^2-1}\right] \quad [71]$$

$$f_{t2} = \frac{aE\theta_1}{2\log\frac{r_2}{r_1}}\left[1-\frac{2r_1^2}{r_2^2-r_1^2}\log\frac{r_2}{r_1}\right] = \frac{aE\theta_1}{2\log k}\left[1-\frac{2\log k}{k^2-1}\right] \quad [72]$$

77 What was said in discussing the steady-state stress distribution in thin cylinders is valid also for thick cylinders and will not be repeated here.

78 The distribution of stresses during the period when the temperature gradient is being formed may be found by the method outlined in the discussion of thin cylinders, i.e., by adding the stresses due to the transient part of Equation [28] to the steady-state stress distribution given by Equations [69] and [70]. The transient stress distribution is obtained by evaluating Equations [42] and [43], substituting for θ the function representing the transient temperature distribution. This gives for the transient stresses:

$$f_r' = aE\sum_{k=1}^{k=\infty}\frac{A_k}{\mu_k}\varepsilon^{-\frac{\mu_k^2}{h^2}t}\left\{\frac{1}{r^2}\left[rJ_1(\mu_k r)-r_1J_1(\mu_k r_1)+\frac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)}\Big(rN_1(\mu_k r)-r_1N_1(\mu_k r_1)\Big)\right]-\frac{1-\frac{r_1^2}{r^2}}{r_2^2-r_1^2}\left[r_2J_1(\mu_k r_2)-r_1J_1(\mu_k r_1)+\frac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)}\Big(r_2N_1(\mu_k r_2)-r_1N_1(\mu_k r_1)\Big)\right]\right\} \quad [73]$$

$$f_t' = aE \sum_{k=1}^{k=\infty} \frac{A_k}{\mu_k} \varepsilon^{-\frac{\mu_k^2}{h^2}t} \left\{ -\frac{1}{r^2}\left[rJ_1(\mu_k r) - r_1 J_1(\mu_k r_1) + \frac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)}\Big(rN_1(\mu_k r) - r_1 N_1(\mu_k r_1)\Big)\right] + \mu_k\left[J_0(\mu_k r) - \frac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)} N_0(\mu_k r)\right] - \frac{1+\frac{r_1^2}{r^2}}{r_2^2 - r_1^2}\left[r_2 J_1(\mu_k r_2) - r_1 J_1(\mu_k r_1) + \frac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)}\Big(r_2 N_1(\mu_k r_2) - r_1 N_1(\mu_k r_1)\Big)\right]\right\} \quad . \; . \; . \; [74]$$

79 At the inside and outside walls of the cylinder f_r' is zero for all values of t.

80 The transient part of the tangential stress intensity when $t=0$ is the same at the outside and inside of the cylinder (compression when θ_1 is positive), namely,

$$f_{t1}' = f_{t2}' = \frac{2aE}{(r_2^2 - r_1^2)} \sum_1^{\infty} -\frac{A_k}{\mu_k}\left[r_2 J_1(\mu_k r_2) - r_1 J_1(\mu_k r_1) + \frac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)}\Big(r_2 N_1(\mu_k r_2) - r_1 N_1(\mu_k r_1)\Big)\right] \quad . \; . \; . \; [75]$$

81 The total initial tangential stresses at $r=r_1$ and $r=r_2$ are hence:

$$f_{t1} = \text{Equation [71]} + \text{Equation [75]} \quad . \; . \; . \; [76]$$

$$f_{t2} = \text{Equation [72]} + \text{Equation [75]} \quad . \; . \; . \; [77]$$

82 The stress intensity at the inner surface f_{t1} (Equation [76]) represents the maximum possible stress for a given temperature difference θ_1. When the latter is positive (the inner wall hottest) this maximum stress is compressive; when θ_1 is negative the maximum stress is tensile.

83 2—*Sinusoidally Varying Temperature*. The steady-state distribution is obtained by substituting Equation [34] in Equations [42] and [43] and integrating. The plane-vector stress intensities are then given by:

$$f_r = \frac{aE\theta_1}{q[J_0(qr_1)N_0(qr_2) - J_0(qr_2)N_0(qr_1)]} \left\{ -\frac{1}{r}\left[\Big(rJ_1(qr) - r_1 J_1(qr_1)\Big) N_0(qr_2) - \Big(rN_1(qr) - r_1 N_1(qr_1)\Big) J_0(qr_2)\right] + \frac{1-\frac{r_1^2}{r^2}}{r_2^2 - r_1^2}\left[\Big(r_2 J_1(qr_2) - r_1 J_1(qr_1)\Big) N_0(qr_2) - \Big(r_2 N_1(qr_2) - r_1 N_1(qr_1)\Big) J_0(qr_2)\right]\right\} \quad . \; . \; . \; . \; . \; . \; [78]$$

$$\dot{f}_t = \frac{aE\dot{\theta}_1}{q[J_0(qr_1)N_0(qr_2)-J_0(qr_2)N_0(qr_1)]}$$
$$\left\{\frac{1}{r^2}\left[\left(rJ_1(qr)-r_1J_1(qr_1)\right)N_0(qr_2)\right.\right.$$
$$\left.-\left(rN_1(qr)-r_1N_1(qr_1)\right)J_0(qr_2)\right]$$
$$-q[J_0(qr)N_0(qr_2)-J_0(qr_2)N_0(qr)]$$
$$+\frac{1+\frac{r_1^2}{r^2}}{r_2^2-r_1^2}\left[\left(r_2J_1(qr_2)-r_1J_1(qr_1)\right)N_0(qr_2)\right.$$
$$\left.\left.-\left(r_2N_1(qr_2)-r_1N_1(qr_1)\right)J_0(qr_2)\right]\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad [79]$$

84 Equations [78] and [79] may be written:

$$\dot{f}_r = aE\dot{\theta}_1 p_r/\underline{\psi_r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [80]$$
$$\dot{f}_t = aE\dot{\theta}_1 p_t/\underline{\psi_t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [81]$$

where p_r, p_t, ψ_r and ψ_t are functions of r. These equations are identical in form to Equations [59] and [60].

85 The actual stress intensities at any time t are given by:

$$f_r = aEp_r\theta_1 \sin(\omega t+\psi_r) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [82]$$
$$f_t = aEp_t\theta_1 \sin(\omega t+\psi_t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [83]$$

These equations are similar to those for the thin cylinder. No additional discussion of the stresses is therefore necessary.

86 When $r=r_1$ and $r=r_2$ the radial stresses become zero. The tangential stresses are given by:

$$\dot{f}_{t1} = \dot{f}_{t2} - aE\dot{\theta}_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [84]$$

$$\dot{f}_{t2} = \frac{2aE\dot{\theta}_1}{q[J_0(qr_1)N_0(qr_2)-J_0(qr_2)N_0(qr_1)][r_2^2-r_1^2]}$$
$$\left[\left(r_2J_1(qr_2)-r_1J_1(qr_1)\right)N_0(qr_2)\right.$$
$$\left.-\left(r_2N_1(qr_2)-r_1N_1(qr_1)\right)J_0(qr_2)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [85]$$

87 The actual stresses at $r=r_1$ and $r=r_2$ may be written

$$f_{t1} = \frac{2aE}{r_2^2-r_1^2} p_{t1}\theta_1 \sin(\omega t+\psi_{t1}) \quad . \quad . \quad . \quad . \quad [86]$$
$$f_{t2} = \frac{2aE}{r_2^2-r_1^2} p_{t2}\theta_1 \sin(\omega t+\psi_{t2}) \quad . \quad . \quad . \quad . \quad [87]$$

It is easily seen from Equations [84] and [85] what terms are to be included in the coefficients p_{t1} and p_{t2}.

88 3 — *Sinusoidally Varying Temperature Superimposed upon Constant Temperature.* The steady-state stresses may in this case be found by adding Equations [69] and [82], and [70] and [83].

89 4 — *Any Periodic Temperature Variation.* Referring to the discussion of this case in connection with thin cylinders, and assuming the same temperature conditions, the coefficients p_r and p_t and the angles ψ_r and ψ_t are determined for each harmonic by applying Equations [78] and [79]. The stress intensities at any time t are then given by:

$$\begin{aligned} f_r = aE[&p_r'\theta_1' \sin(\omega t + \beta' + \psi_r') \\ &+ p_r'''\theta_1''' \sin(3\omega t + \beta''' + \psi_r''') \\ &+ p_r^{v}\theta_1^{v} \sin(5\omega t + \beta^{v} + \psi_r^{v})] \end{aligned} \quad \ldots\ldots [88]$$

$$\begin{aligned} f_t = aE[&p_t'\theta_1' \sin(\omega t + \beta' + \psi_t') \\ &+ p_t'''\theta_1''' \sin(3\omega t + \beta''' + \psi_t''') \\ &+ p_t^{v}\theta_1^{v} \sin(5\omega t + \beta^{v} + \psi_t^{v})] \end{aligned} \quad \ldots\ldots [89]$$

APPLICATIONS

1 — *Water-Tube Boiler*

90 The combined tangential stress intensities at the outside and inside walls of the tubes in a water-tube boiler will be calculated.

Data:

Box header type of water-tube boiler
Horsepower $= 250$
Pressure $= 175$ lb. per sq. in.
External diameter of tubes $= 3\frac{1}{2}$ in.
Wall thickness of tubes $= 0.12$ in.
Modulus of elasticity $E = 30 \times 10^6$ lb. per sq. in.
Poisson's ratio $\frac{1}{m} = \frac{1}{3} = 0.333$
Coefficient of linear expansion $a = 11.6 \times 10^{-6}$ in. per in. per deg. cent.

Assumption

Temperature difference between outside and inside walls of hottest tube $= 100$ deg. cent.[1]

91 The formula for the tangential stresses due to the internal pressure is:

$$f_t = \frac{p_1}{k^2 - 1}\left[\left(\frac{r_2}{r}\right)^2 + 1\right] \quad \ldots\ldots [90]$$

and as

$$k = \frac{r_2}{r_1} = \frac{3.5}{3.26} = 1.074$$

$$f_t = \frac{175}{1.074^2 - 1}\left[\left(\frac{r_2}{r}\right)^2 + 1\right] = 1136\left[\left(\frac{r_2}{r}\right)^2 + 1\right]$$

[1] This temperature difference obviously depends upon the location of the tube in the combustion chamber and also to a very large extent upon how free it is from scale.

giving

$$f_{t1}=2450 \text{ lb. per sq. in. (tension)}$$
$$f_{t2}=2270 \text{ lb. per sq. in. (tension)}$$

Using Equations [71] and [72] the temperature stresses become:

$$f_{t1}=\frac{11.6\times30\times\theta_1}{2\log 1.074}\left[1-\frac{2\times1.074^2\times\log 1.074}{1.074^2-1}\right]=-175\theta_1$$

$$f_{t2}=\frac{11.6\times30\times\theta_1}{2\log 1.074}\left[1-\frac{2\log 1.074}{1.074^2-1}\right]=+175\theta_1$$

Using $\theta_1=-100$ deg. cent. gives for the temperature stresses:

$$f_{t1}=\quad 17{,}500 \text{ lb. per sq. in. (tension)}$$
$$f_{t2}=-17{,}500 \text{ lb. per sq. in. (compression)}$$

The total stresses set up at the two walls are consequently

$$f_{t1}=\quad 17{,}500+2{,}450=\quad 19{,}950 \text{ lb. per sq. in. (tension)}$$
$$f_{t2}=-17{,}500+2{,}270=-15{,}230 \text{ lb. per sq. in. (compression)}$$

92 This example brings out the fact that in the tubes of a water-tube boiler the largest pressure stress is further, and quite considerably, increased by the temperature stress.

2 — *Cylinder Liner of Internal-Combustion Engine.*

93 The tangential stress intensities will be computed when the temperature difference between the two walls is constant and also when it varies according to an experimental (or assumed) curve. Finally, the initial stresses will be computed.

Data:

2-cycle Diesel engine (120 r.p.m.)
Cast-iron cylinder liner
Maximum pressure $=500$ lb. per sq. in.
External radius $=11\frac{1}{2}$ in.
Internal radius $=10$ in.
Modulus of elasticity $E=14\times10^6$ lb. per sq. in.
Poisson's ratio $\frac{1}{m}=\frac{1}{5}$
Coefficient of linear expansion $a=10.2\times10^{-6}$ in. per in. per deg. cent.
Specific heat $c=0.113$ gram-cal. per gram per deg. cent.
Specific density $\delta=7.60$ grams per cu. cm.
Specific conductivity $\lambda=0.14$ gram-cal. per cm. cube per deg. cent. per sec.

94 (*a*) *Constant Temperature Difference.* Assume constant temperature difference $\theta_1=40$ deg. cent. Since the ratio between the outside and inside radii is

$$k=\frac{11.5}{10}=1.15$$

Fig. 1 shows that only about 2.8 per cent error is to be expected in the steady-state temperature distribution by applying the thin-cylinder formula.

95 The distribution of tangential pressure stresses has been computed using Equation [90]. In order to demonstrate that the

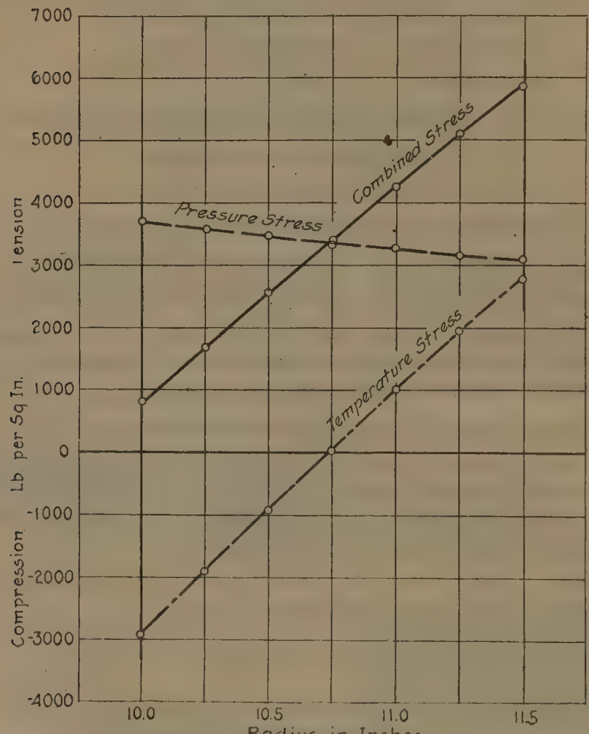

FIG. 9 STRESS DISTRIBUTION IN WALL OF CYLINDER LINER

radial temperature stresses are negligible, the *maximum* radial stress intensity will be computed. Maximum f_r occurs at a radius (Equation [47])

$$r=10\sqrt[3]{\frac{2\times 1.322}{2.15}}=10.71 \text{ in.}$$

Equation [45] gives when $r=10.71$ in.

$$\text{Max. } f_r=-114 \text{ lb. per sq. in. (compression)}$$

96 The distribution of tangential temperature stresses has been computed from Equation [46]. These stresses, as well as the

pressure stresses and the combined stresses, are given in the following table:

Radius in.	Pressure stress, lb. per sq. in.	Temperature stress, lb. per sq. in.	Combined stress, lb. per sq. in.
10.00	3610	— 2920	690
10.25	3510	— 1890	1620
10.50	3420	— 950	2470
10.75	3330	25	3360
11.00	3250	1000	4250
11.25	3180	1930	5110
11.50	3110	2780	5890

97 Since the pressure in the cylinder varies from a maximum (in this case 500 lb. per sq. in.) to a minimum (almost atmospheric

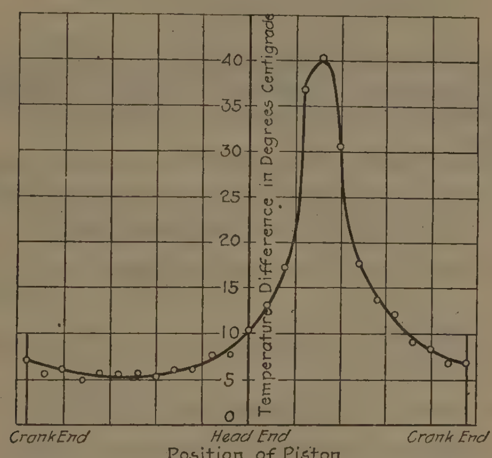

FIG. 10 TIME DISTRIBUTION OF TEMPERATURE DIFFERENCE BETWEEN THE INSIDE AND OUTSIDE WALLS OF CYLINDER LINER DURING EACH CYCLE

(A line through the indicated points gives the Fourier series' representation of the drawn curve.)

pressure) during one revolution of the crankshaft, the pressure stresses and consequently the combined stresses will undergo a cyclic variation even though the temperature stresses are constant as assumed in this example.

98 Fig. 9 illustrates the stress distribution through the cylinder wall at the instant of maximum gas pressure.

99 (*b*) *Cyclically Varying Temperature Difference.* The case just considered where the difference between the temperatures of the two cylinder walls is constant is evidently extremely hypothetical. The temperature difference must of necessity undergo some kind of cyclic variation.

100 The time-distribution curve in Fig. 10 will be assumed to hold. This curve is in approximate accordance with data given

by Dr. Kurt Neumann.[1] As seen, the curve has a pronounced peak during the combustion period. The *maximum* temperature difference is assumed to be 40 deg. cent. or equal to the *constant* temperature difference used in the previous example.

101 Representing the curve by a Fourier's series including all harmonics up to the eleventh, its equation becomes[2]

$$\begin{aligned}\theta_1 = 12.10 &- 10.64\sin(\omega t + 28^\circ.0) + 6.41\sin(2\omega t - 28^\circ.7)\\ &- 4.33\sin(3\omega t - 83^\circ.9) - 3.08\sin(4\omega t + 40^\circ.5)\\ &+ 2.31\sin(5\omega t - 14^\circ.8) - 1.65\sin(6\omega t - 72^\circ.3)\\ &- 1.06\sin(7\omega t + 52^\circ.7) + 0.62\sin(8\omega t + 17^\circ.0)\\ &- 0.42\sin(9\omega t + 6^\circ.7) + 0.52\sin 10\omega t\\ &- 0.76\sin(11\omega t - 44^\circ.5)\end{aligned}$$

A curve through the points (small circles) laid off on Fig. 10 will represent this equation. As seen, the degree of coincidence with the actual temperature curve is entirely satisfactory for all practical purposes.

102 Since the speed of the engine is 120 r.p.m., the fundamental frequency in the Fourier series is 2 cycles per second. Hence

$$\omega = 2\pi f = 4\pi = 12.56 \text{ radians per sec.}$$

103 From the given data the quantity h $[=\sqrt{(c\delta/\lambda)}]$ becomes:

$$h = \sqrt{\frac{0.113\times 7.60}{0.14}} = 2.48 \text{ sec.}^{\frac{1}{2}} \text{ per cm.} = 7.44 \text{ sec.}^{\frac{1}{2}} \text{ per inch}$$

104 The quantity α is obtained from Equation [**11**]:

$$\alpha = 7.44\sqrt{12.56}/\underline{45^\circ} = 26.4/\underline{45^\circ} \text{ hyps per inch}$$

105 The tangential stress intensities will be calculated at the outside and inside walls of the cylinder. This involves computing the stress due to the constant temperature term in the Fourier series and evaluating the harmonic stresses by application of Equations [**63**] and [**64**] to each harmonic separately and putting

[1] Kurt Neumann: Untersuchungen an der Dieselmaschine, Forschungsarbeiten, Heft 225. Among other things Dr. Neumann succeeds in determining with, presumably, reasonable accuracy the actual temperature of the two cylinder walls for various positions of the piston. He obtains his results by tests of a 50-hp. Diesel engine in connection with extensive calculations.

The coming fall Mr. Gabriel Smith will conduct tests at the Massachusetts Institute of Technology for the purpose of determining piston and liner temperatures of a Mietz and Weiss 2-cycle kerosene engine. *Direct measurement* of all temperatures involved will be attempted. It would indeed be of value if Mr. Smith could check the general shape of temperature curves based on Dr. Neumann's data.

[2] The particular schedule method used for this analysis is given by H. O. Taylor in the *Physical Review*, October 1915.

the results for each harmonic into the form given by Equations [65] and [66].

106 Applying Equations [48] and [49], the stresses due to the constant temperature difference of 12.10 deg. cent. are obtained as:

$$\text{Const. } f_{t1} = -880 \text{ lb. per sq. in. (compression)}$$
$$\text{Const. } f_{t2} = 840 \text{ lb. per sq. in. (tension)}$$

107 The *smallest* hyperbolic sines and cosines in Equations [63] and [64] will occur when the fundamental is considered, viz.,

$$\sinh \alpha(r_2 - r_1) = \sinh(39.6\underline{/45^\circ})$$
$$\cosh \alpha(r_2 - r_1) = \cosh(39.6\underline{/45^\circ})$$

108 By consulting a table of hyperbolic functions of semi-imaginary arguments it will be found that whenever the magnitude of the argument exceeds $6.0\underline{/45^\circ}$ the values of sinh and cosh coincide. Furthermore, these values rapidly grow very large; at about $21\underline{/45^\circ}$, for instance, they become approximately 1,000,000.

109 Obviously then, in our case (see Equations [63] and [64]) the ratio of cosh to sinh (or coth) is unity, while the ratio of r_2 to sinh is negligibly small. Hence Equations [63] and [64] may be written in a simplified form:

$$f_{t1} = \frac{2aE\dot{\theta}_1}{r_2^2 - r_1^2}\left[\frac{r_1}{\alpha} + \frac{1}{\alpha^2} - \frac{r_2^2 - r_1^2}{2}\right] = \frac{2aE\dot{\theta}_1}{r_2^2 - r_1^2} p_{t1} \quad . \quad [91]$$

$$f_{t2} = \frac{2aE\dot{\theta}_1}{r_2^2 - r_1^2}\left[\frac{r_1}{\alpha} + \frac{1}{\alpha^2}\right] = \frac{2aE\dot{\theta}_1}{r_2^2 - r_1^2} p_{t2} \quad . \; . \; . \; . \; . \quad [91a]$$

110 The quantity corresponding to α is for the second harmonic $\sqrt{2}\alpha$, for the third $\sqrt{3}\alpha$, etc. For the fundamental we have:

$$\frac{r_1}{\alpha} = \frac{10}{26.4\underline{/45^\circ}} = 0.379\overline{\diagdown 45^\circ} = 0.268 - j0.268$$

$$\frac{1}{\alpha^2} = \frac{1}{26.4^2\underline{/90^\circ}} = 0.001436\overline{\diagdown 90^\circ} = -j0.001436$$

The values of $\frac{r_1}{\alpha}$, $\frac{1}{\alpha^2}$, p_{t1} and p_{t2} for all harmonics are given in Table 1.

111 The fundamental term, for instance, of the stress intensity at the inside wall is then found by:

$$f'_{t1} = -8.85 \times 10.64 \times 15.86 \sin(\omega t + 28°.0 - 179°.0)$$
$$= -1494 \sin(\omega t - 151°.0) \text{ lb. per sq. in.}$$

112 Determining the other harmonic stresses in the same manner and adding the stress due to the constant temperature

term gives for the stress intensities at the two surfaces of the cylinder:

$$
\begin{aligned}
f_{t1} = {} & -880 - 1494 \sin(\omega t - 151^\circ.0) + 904 \sin(2\omega t + 152^\circ.0) \\
& - 613 \sin(3\omega t - 96^\circ.7) - 436 \sin(4\omega t - 139^\circ.0) \\
& + 328 \sin(5\omega t + 165^\circ.6) - 234 \sin(6\omega t + 108^\circ.1) \\
& - 150.4 \sin(7\omega t - 126^\circ.9) + 88.0 \sin(8\omega t + 162^\circ.7) \\
& - 59.6 \sin(9\omega t - 173^\circ.0) + 73.8 \sin(10\omega t - 179^\circ.7) \\
& - 120.0 \sin(11\omega t + 135^\circ.8) \text{ lb. per sq. in.}
\end{aligned}
$$

$$
\begin{aligned}
f_{t2} = {} & 840 - 35.8 \sin(\omega t - 17^\circ.2) + 15.5 \sin(2\omega t - 73^\circ.8) \\
& - 8.44 \sin(3\omega t - 129^\circ.0) - 5.18 \sin(4\omega t - 4^\circ.6) \\
& + 3.48 \sin(5\omega t - 59^\circ.9) - 2.26 \sin(6\omega t - 117^\circ.4) \\
& - 1.34 \sin(7\omega t + 7^\circ.6) - 0.74 \sin(8\omega t - 28^\circ.1) \\
& - 0.47 \sin(9\omega t - 38^\circ.3) + 0.55 \sin(10\omega t - 45^\circ.0) \\
& - 0.77 \sin(11\omega t - 89^\circ.5) \text{ lb. per sq. in.}
\end{aligned}
$$

113 The stress intensities given by these two equations are plotted in Fig. 11. The curves show that the temperature stress

TABLE 1 VALUES OF $\frac{r_1}{a}$, $\frac{1}{a^2}$, p_{t1} AND p_{t2} FOR ALL HARMONICS

Harmonic	$\frac{r_1}{a}$	$\frac{1}{a^2}$	P_{t1} Rect.	P_{t1} Polar	P_{t2} Rect.	P_{t2} Polar
1	$0.268 - j0.268$	$-j0.0014$	$-15.86 - j0.2694$	$15.86\overline{\backslash 179^\circ.0}$	$0.268 - j0.2694$	$0.380\overline{\backslash 45^\circ.2}$
2	$0.189 - j0.189$	$-j0.0007$	$-15.44 - j0.1897$	$15.94\overline{\backslash 179^\circ.3}$	$0.189 - j0.1897$	$0.268\overline{\backslash 45^\circ.1}$
3	$0.155 - j0.155$	$-j0.0005$	$-15.97 - j0.1555$	$15.97\overline{\backslash 179^\circ.4}$	$0.155 - j0.1555$	$0.220\overline{\backslash 45^\circ.1}$
4	$0.134 - j0.134$	$-j0.0004$	$-16.00 - j0.1344$	$16.00\overline{\backslash 179^\circ.5}$	$0.134 - j0.1344$	$0.190\overline{\backslash 45^\circ.1}$
5	$0.120 - j0.120$	$-j0.0003$	$-16.01 - j0.1203$	$16.01\overline{\backslash 179^\circ.6}$	$0.120 - j0.1203$	$0.170\overline{\backslash 45^\circ.1}$
6	$0.109 - j0.109$	$-j0.0002$	$-16.02 - j0.1092$	$16.02\overline{\backslash 179^\circ.6}$	$0.109 - j0.1092$	$0.155\overline{\backslash 45^\circ.1}$
7	$0.101 - j0.101$	$-j0.0002$	$-16.03 - j0.1012$	$16.03\overline{\backslash 179^\circ.6}$	$0.101 - j0.1012$	$0.143\overline{\backslash 45^\circ.1}$
8	$0.095 - j0.095$	$-j0.0002$	$-16.03 - j0.0952$	$16.03\overline{\backslash 179^\circ.7}$	$0.095 - j0.0952$	$0.135\overline{\backslash 45^\circ.0}$
9	$0.089 - j0.089$	$-j0.0002$	$-16.04 - j0.0892$	$16.04\overline{\backslash 179^\circ.7}$	$0.089 - j0.0892$	$0.126\overline{\backslash 45^\circ.0}$
10	$0.085 - j0.085$	$-j0.0001$	$-16.04 - j0.0851$	$16.04\overline{\backslash 179^\circ.7}$	$0.085 - j0.0851$	$0.120\overline{\backslash 45^\circ.0}$
11	$0.081 - j0.081$	$-j0.0001$	$-16.05 - j0.0811$	$16.05\overline{\backslash 179^\circ.7}$	$0.081 - j0.0811$	$0.115\overline{\backslash 45^\circ.0}$

$$\frac{2aE}{r_2^2 - r_1^2} = \frac{2 \times 10.2 \times 14}{32.25} = 8.85$$

at the inner wall fluctuates violently during each revolution of the crankshaft. It goes down to zero and shoots up to a peaked maximum of about 4900 lb. per sq. in. (compression). The temperature stress at the outside wall, on the contrary, is an almost constant tension of about 850 lb. per sq. in.

114 This is due to the fact that the harmonically varying heat waves and corresponding temperatures in passing through the wall are damped and phase displaced in such a manner that they do not give rise to any appreciable stress at the outer wall. The stress intensity here is therefore practically that caused by the constant temperature difference.

115 The dotted horizontal lines in Fig. 11 represent the stresses calculated in the preceding example, i.e., when a constant temperature difference equal to the peaked maximum in the present case exists between the walls.

116 Comparison of the two sets of curves is interesting. Although the maximum temperature difference is the same, the stress intensity at the inside wall is considerably greater when the temperature difference varies during the cycle. The tension at the outside walls, however, is much smaller for this case.

117 The curves serve to illustrate how intimately the stress distribution and hence the maximum stresses are linked with the temperature conditions. In order to estimate temperature stresses correctly the time distribution of the temperature difference between the two walls must be known with reasonable certainty. It is therefore of vital importance that more tests on engines of

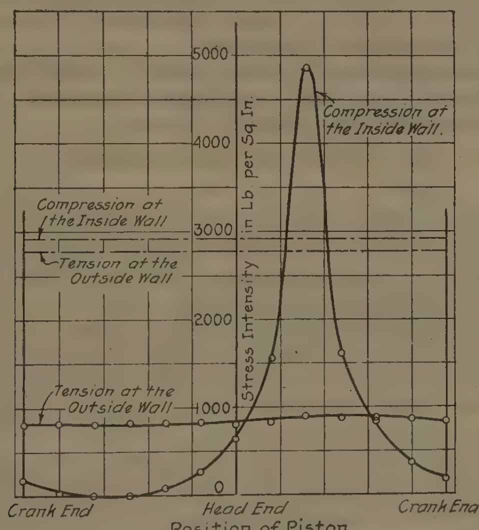

FIG. 11 TIME DISTRIBUTION OF TEMPERATURE STRESSES IN WALL OF CYLINDER LINER DURING EACH CYCLE

(The full-line curves hold for the temperature condition shown in Fig. 10. Dotted curves hold for the condition where the temperature difference is constant.)

different types and sizes be performed so that more data on actual temperatures of liners, pistons, cylinder heads, etc. may be gathered and used for reference by designers.

118 The curves in Fig. 11 indicate temperature stresses only. In order to get the combined stresses due to both pressure and temperature, the pressure stress at each instant of the cycle should be added to the temperature stress. This involves knowledge of the pressure variation during the cycle, but introduces no new difficulties as the pressure for different positions of the piston can easily be estimated by well-known methods.

119 The pressure stresses for maximum pressure were calculated for this cylinder in the preceding example. Assuming that maximum temperature and maximum pressure occur simul-

taneously (which is approximately true), this particular case reveals the rather interesting fact that the stress at the inside cylinder surface will be compressive at the instant of maximum gas pressure. This is caused by the predominance of the temperature stress over the pressure stress at that instant.

120 (*c*) *Initial Stresses.* It was stated in connection with Equations [55] and [56] that when the difference between the temperatures of the two cylinder walls (θ_1) is constant and positive, the maximum possible temperature stress will occur at the inside wall at the instant the temperature difference is applied.

121 The stresses at this instant may be called *initial stresses.* In an internal-combustion engine the initial stresses will occur when the first explosion takes place.

122 Assuming, as in the first example, a constant temperature difference of 40 deg. cent., the initial temperature stresses as found by Equations [55] and [56] become:

$$f_{t1} = -5710 \text{ lb. per sq. in. (compression)}$$
$$f_{t2} = 0$$

123 Comparison of the first of these figures with the steady-state temperature stress for this case shows that the compressive stress intensity at the inner surface of the wall initially is 2790 lb. per sq. in. larger than the steady-state value.

124 Comparing the same figure with the steady-state temperature stress for the case where the temperature difference varies according to the peaked curve, Fig. 10, shows that the initial stress at the inside of the wall is about 800 lb. per sq. in. larger than the maximum steady-state value.

125 The initial stresses just computed were obtained on the assumption that the initial temperature difference was equal to the steady-state temperature difference (40 deg. cent.). Actually this is far from correct. The truth is that the initial temperature difference may be considerably larger.

126 In the steady state with a steady flow of heat through the cylinder wall three distinct temperature gradients exist as the heat flows from the hot gases to the cooling water in the jacket: First, the gradient through the gas film which clings to the inner cylinder surface; second, the gradient through the iron wall which is effective in establishing the stresses; and third, the gradient through the water film which clings to the outer surface of the cylinder.

127 The thermal conductivity of the gas and water films is much lower than that of the iron. The conductivity of the gas film is particularly low. The main part, therefore, of the temperature drop between the gas and the cooling water takes place through the gas film, the next largest part through the water film, and the smallest drop occurs through the iron wall itself. The inside wall is consequently considerably cooler than the gases

in the cylinder, while the outside wall is a good deal hotter than the cooling water.[1]

128 Assume the engine to have been idle for some time and assume the entire cylinder liner to have the same temperature as the cooling water. The engine is now started. When the first explosion takes place, heat flows outward and a high temperature equal to the temperature of the hot gases minus the drop through the gas film is suddenly applied to the inner wall. The outer wall, however, is still at cooling water temperature. The initial temperature difference between the two walls is therefore equal to the actual temperature initially applied to the inner wall minus the temperature of the cooling water.

129 After the heat has penetrated the wall and commences to flow out into the cooling water the temperature gradient through the water film is established, increasing the actual temperature of the outer wall. The difference in temperature between the two walls is hence successively decreased until steady-state conditions persist.

130 This analysis shows that the initial temperature difference without doubt must be larger than the steady-state difference. But how much larger is it?

131 If the first explosion is normal, a normal amount of heat will be developed in the cylinder and the temperature of the gases will be normal. Under these circumstances it is, presumably, fair to suppose that the temperature initially applied to the inner cylinder wall is approximately equal to its actual temperature during normal operation.

132 The above statement is correct only on the assumption that the initial ratio between the amounts of heat which are dissipated through the principal leakage paths (liner, piston, cylinder head) is the same as for normal operation.

133 In the case of a piston without water or oil cooling, and also to some (although a very small) extent in the case of a cooled piston, the amount of heat which flows through the piston will depend upon the temperature in the crankcase. If this temperature initially is lower than during normal operation, more heat will initially flow in this direction. This means that less heat will tend to flow out radially through the liner. A smaller amount of heat causes a smaller temperature drop through the gas film, and a higher temperature is applied to the inner surface of the liner.

134 If, on the contrary, for some reason less heat than normal is initially dissipated through the piston, more heat will flow through the liner. The drop through the gas film will increase,

[1] Dr. Neumann in his paper previously referred to gives about 120 deg. cent. as the *actual* maximum temperature of the inside cylinder wall of his 50-hp. engine.

causing a reduction in the temperature applied to the inner surface of the liner.

135 Whether or not the temperature in the crankcase during normal operation will deviate appreciably from the initial temperature depends primarily upon the system used for scavenging. If the crankcase compression system is used, the temperature will not change materially because new cool scavenging air is let in once during each revolution. If, on the other hand, the scavenging is not performed in this manner, the initial air in the crankcase will remain there and successively heat up. In such an engine, therefore, the initial flow of heat through the piston may be somewhat greater than normal.

136 It has hitherto been assumed that the first explosion is normal. In starting an engine it is rather probable that it will be subnormal and hence develop less heat and a lower temperature. In such cases the temperature initially applied to the inside wall will be lowered. Cases may also be conceived, however, where the first explosion is supernormal. This would evidently increase the initially applied temperature.

137 Assume for the engine here considered that the initial temperature applied to the inside cylinder wall is 120 deg. cent. and the temperature of cooling water 20 deg. cent. The initial temperature difference between the inner and outer wall is hence 100 deg. cent. Using this value for θ_1 in Equations [55] and [56] the initial temperature stresses set up become:

$$f_{t1} = -14{,}280 \text{ lb. per sq. in. (compression)}$$
$$f_{t2} = 0$$

138 The compression at the inside wall is in this case quite appreciable. It is, however, actually reduced somewhat by the tension due to the gas pressure. It is fortunate that the pressure and temperature stresses are of opposite sign at the inner wall where both are maximum. It is a protection, indeed, in the case of abnormal explosions during which both pressure and temperature may reach abnormally high values.

139 To be exact, the method used in obtaining the initial stresses is valid only when the temperature difference between the two walls is instantaneously applied; in other words, when it jumps from zero and right up to its final value. This is not quite in accordance with the phenomenon as it actually takes place in the cylinder. A glance at Fig. 10, however, shows that when the explosion occurs, the temperature very rapidly jumps to its maximum value as indicated by the peak. Since the rise takes place in an extremely short interval of time, it is permissible as a first approximation to consider it as instantaneous and to compute the initial stresses by the formulas developed on this basis. The error introduced will be insignificant.

3 — *Piece of Ordnance*

140 The maximum gas pressure developed in a piece of heavy ordnance is about 2500 atmos. and the maximum temperature may be about 2200 deg. cent.

141 The pressure stresses can easily be computed by well-developed and reliable methods. Since the temperature of the gases is very high it will undoubtedly give rise to appreciable temperature stresses. It would therefore be of considerable interest to investigate the effect of the temperature on the distribution of stress through the material of a cannon, particularly when the latter is built up of a central tube and one or more external cylinders:

142 Such a problem may be approximately solved by assuming that the maximum temperature difference between the inside and outside surfaces of the gun when a shot is fired is *instantaneously applied*. This assumption should introduce but little error since the time it takes for the temperature to reach its maximum is extremely short.

143 The complete distribution of temperature stresses through the material may then be computed using Equations ([71] +[73]) and ([72]+[74]) by substituting $t=0$. Obviously the actual numerical solution of this problem would be very laborious.

144 If merely the temperature stresses at the inner and outer surfaces are desired, Equations [76] and [77] may be used.

145 The same difficulty as in the previous examples arises also in this case: What is the actual temperature of the inside surface? It will evidently be lower than the gas temperature, but how much lower? Presumably, on acount of the turbulence created by the violent explosion of the charge, the contact between the hot gases and the steel is comparatively good. This means that the temperature drop through the gas film is smaller than if the hot gases had been in rest.

146 The author possesses no data pertaining to actual temperatures of the material in a gun, so no illustrative figures can be given.

Summary

147 Temperature and stress distribution in hollow cylinders for certain specified conditions have been analyzed, and formulas derived.

148 Thin and thick cylinders have been treated separately. The thin-cylinder formulas are in some cases simpler, but they involve a slight approximation.

149 Complete solutions have been obtained for the case where the temperature difference between the two walls is constant. Steady-state solutions have been obtained for cases where the temperature difference varies sinusoidally with time, where it

consists of a sinusoidally varying temperature superimposed upon a constant temperature, and where it undergoes any cyclic variation expressible as a Fourier series.

150 The case involving a sinusoidally varying temperature difference is of minor importance *per se* but is of fundamental importance as a step toward the solution of the case where the Fourier series representation of the cyclic temperature variation is employed.

151 The complete solutions for the constant-temperature case lend themselves to the evaluation of what has been termed the initial stresses, i.e., the stresses set up at the first instant when a temperature difference is suddenly applied. As the initial stresses may be large, their correct determination is of importance.

152 It has been emphasized that knowledge of the actual temperature conditions is necessary if the stresses are to be correctly determined. More experimental data on wall temperatures in Diesel-engine cylinders, boiler tubes, etc., on the bases of which intelligent estimates may be made, would therefore be of great value.

APPENDIX NO. 1

DERIVATION OF FORMULAS FOR TEMPERATURE DISTRIBUTION

153 (*a*) *Thin Cylinders.* Differential equation (see Equation [6]):

$$\frac{\partial^2 \theta}{\partial r^2} = h^2 \frac{\partial \theta}{\partial t} \quad \text{[92]}$$

This equation, as well as the general differential equation holding for temperature distribution in thick cylinders, will be solved by the ordinary methods usually applied in solving partial differential equations.

154 Assume the solution $\theta = RT$, where R is a function of r alone and T is a function of t alone. The differential equation then splits up into two parts:

$$\frac{\partial T}{\partial t} + \frac{\mu^2}{h^2} T = 0 \quad \text{[93]}$$

$$\frac{\partial^2 R}{\partial r^2} + \mu^2 R = 0 \quad \text{[94]}$$

μ being an undetermined constant. The solution of [93] is

$$T = A_1 \epsilon^{-\frac{\mu^2}{h^2} t} \quad \text{[95]}$$

and the solution of [94] is

$$R = A_2 \sin \mu r + A_3 \cos \mu r \quad \text{[96]}$$

155 Any number of solutions of the form RT satisfy the differential equation. A particular solution of [92] is the steady-state solution, (i. e., for $\frac{\partial \theta}{\partial t} = 0$) which evidently is

$$\theta = C \left[1 - \frac{r - r_1}{r_2 - r_1} \right] \quad \text{[97]}$$

The general solution is therefore:

$$\theta = C\left[1-\frac{r-r_1}{r_2-r_1}\right]-\sum_1^{\infty}[A \sin \mu r+B \cos \mu r]\ \epsilon^{-\frac{\mu^2}{h^2}t} \quad . \quad . \quad .[98]$$

156 Equation [98] is the desired solution when it is made to conform with initial and boundary conditions. These are

Initial Conditions ($t=0$):	Boundary Conditions (any value of t):
$\theta=\theta_1$ when $r=r_1$	$\theta=\theta_1$ when $r=r_1$
$\theta=0$ when $r>r_1$	$\theta=0$ when $r=r_2$

Hence $C=\theta_1$, $B=0$ and the A's must be the coefficients in a Fourier series representing the function

$$\theta_1\left[1-\frac{r-r_1}{r_2-r_1}\right]$$

between r_1 and r_2. Evaluating these coefficients by the usual methods gives for the solution:

$$\theta=\theta_1\left[1-\frac{r-r_1}{r_2-r_1}\right]-\frac{2\theta_1}{\pi}\sum_{n=1}^{n=\infty}\frac{1}{n}\ \epsilon^{-\frac{n^2\pi^2}{h^2(r_2-r_1)^2}t} \sin\frac{n\pi(r-r_1)}{r_2-r_1} \quad .[99]$$

157 When the temperature varies sinusoidally, the differential equation [92] becomes

$$\frac{\partial^2\theta}{\partial r^2}=h^2\theta\frac{\partial}{\partial t}(\sin \omega t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .[100]$$

Introducing the plane-vector conception of sinusoidally varying quantities Equation [100] may in the *steady state* be written:

$$\frac{\partial^2\dot\theta}{\partial r^2}=j\omega h^2\dot\theta=a^2\dot\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .[101]$$

which again for the sake of simplicity in determining the constants of integration may be written:

$$\frac{\partial^2\dot\theta}{\partial(r_2-r)^2}=a^2\dot\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .[102]$$

158 The solution of [102] is

$$\dot\theta=A \cosh a(r_2-r)+B \sinh a(r_2-r) \quad . \quad . \quad . \quad . \quad [103]$$

When

$$r=r_1 \qquad \dot\theta=\dot\theta_1$$
$$r=r_2 \qquad \dot\theta=0$$

Hence

$$B=\frac{\dot\theta_1}{\sinh a(r_2-r_1)} \text{ and } A=0.$$

The solution is therefore

$$\dot\theta=\frac{\sinh a(r_2-r)}{\sinh a(r_2-r_1)}\dot\theta_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .[104]$$

159 (*b*) *Thick Cylinders.* Differential equation (see Equation [27]):

$$\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}=h^2\frac{\partial\theta}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .[105]$$

Assume the solution $\theta=RT$, where R is a function of r alone and T is a function of t alone. Equation [105] then splits up into two parts:

$$\frac{\partial T}{\partial t}+\frac{\mu^2}{h^2}T=0 \quad [106]$$

$$\frac{\partial^2 R}{\partial r^2}+\frac{1}{r}\frac{\partial R}{\partial r}+\mu^2 R=0 \quad [107]$$

The solution of [106] is

$$T=A_1\,\epsilon^{-\frac{\mu^2}{h^2}t} \quad [108]$$

the solution of [107] is

$$R=A_2J_0(\mu r)+A_3N_0(\mu r) \quad [109]$$

and the steady-state solution is

$$\theta=C\log r+D \quad [110]$$

The general solution is therefore

$$\theta=C\log r+D-\sum_1^{\infty}[AJ_0(\mu r)+BN_0(\mu r)]\epsilon^{-\frac{\mu^2}{h^2}t} \quad [111]$$

160 The initial and boundary conditions are exactly as for the thin cylinder, hence

$$C=\frac{\theta_1}{\log\frac{r_1}{r_2}};\ D=-C\log r_2;\ \text{and } B=-A\frac{J_0(\mu r_2)}{N_0(\mu r_2)}$$

161 Equation [111] may now be written

$$\theta=\frac{\theta_1}{\log\frac{r_2}{r_1}}\log\frac{r_2}{r_1}-\sum_1^{\infty}A\left[J_0(\mu r)-\frac{J_0(\mu r_2)}{N_0(\mu r_2)}N_0(\mu r)\right]\epsilon^{-\frac{\mu^2}{h^2}t} \quad [112]$$

The initial conditions require that the transient (second) term shall vanish when $t=0$ and $r=r_1$. Hence the μ's must be roots of the equation

$$J_0(\mu r_1)-\frac{J_0(\mu r_2)}{N_0(\mu r_2)}N_0(\mu r_1)=0 \quad [113]$$

The general solution is therefore

$$\theta=\frac{\theta_1}{\log\frac{r_2}{r_1}}\log\frac{r_2}{r}-\sum_{k=1}^{k=\infty}A_k\left[J_0(\mu_k r)-\frac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)}N_0(\mu_k r)\right]\epsilon^{-\frac{\mu_k^2}{h^2}t} \quad [114]$$

162 Obviously, the initial conditions also require that whenever $r>r_1$, the summation of the Bessel functions must equal the steady-state term. Determining the coefficients A_k on this basis gives[1]:

$$A_k=A_0\int_{r_1}^{r_2}r\log\frac{r_2}{r}\left[J_0(\mu_k r)-\frac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)}N_0(\mu_k r)\right]dr$$

$$=\frac{A_0}{\mu_k}r_1\log\frac{r_2}{r_1}\left[\frac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)}N_1(\mu_k r_1)-J_1(\mu_k r_1)\right] \quad [115]$$

[1] Byerly: Fourier's Series and Spherical, Cylindrical and Ellipsoidal Harmonics, Art. 127.

where A_0 is given by

$$A_0=\frac{\dfrac{2\theta_1}{\log\dfrac{r_2}{r_1}}}{r_2^2\left[\dfrac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)}N_1(\mu_k r_2)-J_1(\mu_k r_2)\right]^2-r_1^2\left[\dfrac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)}N_1(\mu_k r_1)-J_1(\mu_k r_1)\right]} \quad .[116]$$

hence

$$A_k=\frac{2\theta_1\dfrac{r_1}{\mu_k}\left[\dfrac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)}N_1(\mu_k r_1)-J_1(\mu_k r_1)\right]}{r_2^2\left[\dfrac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)}N_1(\mu_k r_2)-J_1(\mu_k r_2)\right]^2-r_1^2\left[\dfrac{J_0(\mu_k r_2)}{N_0(\mu_k r_2)}N_1(\mu_k r_1)-J_1(\mu_k r_1)\right]^2} \quad .[117]$$

163 When the temperature varies sinusoidally the differential equation [105] becomes

$$\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}=h^2\theta\frac{\partial}{\partial t}(\sin\omega t) \quad \ldots\ldots .[118]$$

which in plane-vector form may be written

$$\frac{\partial^2\dot\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\dot\theta}{\partial r}=j\omega h^2\dot\theta=-q^2\dot\theta \quad \ldots\ldots .[119]$$

The solution of [119] is:

$$\dot\theta=AJ_0(qr)+BN_0(qr) \quad \ldots\ldots [120]$$

Determining the coefficients by fitting the equation to the boundary condition gives

$$A=\frac{N_0(qr_2)}{J_0(qr_1)N_0(qr_2)-J_0(qr_2)N_0(qr_1)}\dot\theta_1$$

$$B=-A\frac{J_0(qr_2)}{N_0(qr_2)}$$

The desired solution is therefore

$$\dot\theta=\frac{J_0(qr)N_0(qr_2)-J_0(qr_2)N_0(qr)}{J_0(qr_1)N_0(qr_2)-J_0(qr_2)N_0(qr_1)}\dot\theta_1 \quad \ldots .[121]$$

APPENDIX NO. 2

DERIVATION OF THE GENERAL FORMULAS FOR TEMPERATURE STRESSES

164 Assuming the cylinder to be free to expand axially so that no direct stress can be set up in this direction, the radial and tangential stress intensities expressed in terms of the radial and tangential strains are given by the following well-known equations[1]:

$$f_r=\frac{mE}{m^2-1}(me_r+e_t). \quad \ldots\ldots .[122]$$

$$f_t=\frac{mE}{m^2-1}(me_t+e_r). \quad \ldots\ldots .[123]$$

[1] See Note following Par. 170.

165 Consider a differential element of dimensions as indicated in Fig. 12 and of unit axial length. If u designates the change in length of the radius r due to the distortion of the cylinder caused by stresses and heating, the total radial and tangential strains become

$$e_r = \frac{du}{dr} - a\theta \quad \text{[124]}$$

$$e_t = \frac{2\pi(u+r) - 2\pi r}{2\pi r} - a\theta = \frac{u}{r} - a\theta \quad \text{[125]}$$

The term $a\theta$ represents the specific linear expansion of the element in any direction due to its increased temperature. θ is the temperature

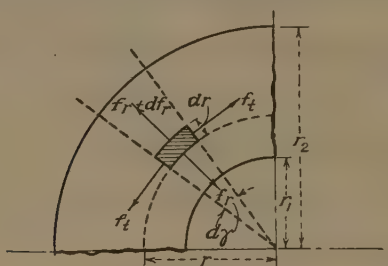

Fig. 12

at any radius above the temperature at the outside of the cylinder.

166 The fact that the particle under consideration (Fig. 12) is in equilibrium may be expressed by

$$(f_r + df_r)(r+dr)d\gamma - f_r r d\gamma - 2f_t dr \sin\frac{d\gamma}{2} = 0 \quad \text{[126]}$$

which reduces to

$$\frac{f_r - f_t}{r} + \frac{df_r}{dr} = 0 \quad \text{[127]}$$

or

$$\frac{d}{dr}(rf_r) - f_t = 0 \quad \text{[128]}$$

167 Substituting [124] and [125] in [122] and [123] gives:

$$f_r = \frac{mE}{m^2-1}\left[m\frac{du}{dr} + \frac{u}{r} - (m+1)a\theta\right] \quad \text{[129]}$$

$$f_t = \frac{mE}{m^2-1}\left[\frac{du}{dr} + m\frac{u}{r} - (m+1)a\theta\right] \quad \text{[130]}$$

From [128], [129], and [130] is obtained

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = \frac{(m+1)a}{m}\frac{d\theta}{dr} \quad \text{[131]}$$

which may be written

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(ru)\right] = \frac{(m+1)a}{m}\frac{d\theta}{dr} \quad \text{[132]}$$

Integrating twice gives

$$u = \frac{(m+1)a}{mr}\int \theta r dr + C_1 r + \frac{C_2}{r} \quad \text{[133]}$$

168 Equations [129] and [130] in connection with [133] give

$$f_r=\frac{mE}{m^2-1}\left[-\frac{(m^2-1)a}{mr^2}\int\theta r dr+(m+1)C_1-(m-1)\frac{C_2}{r^2}\right] \quad \ldots \quad [134]$$

$$f_t=\frac{mE}{m^2-1}\left[\frac{(m^2-1)a}{mr^2}\int\theta r dr-\frac{(m^2-1)a}{m}\theta+(m+1)C_1+(m-1)\frac{C_2}{r^2}\right] \quad [135]$$

169 The radial stress intensity must obviously be zero at the surfaces of the cylinder. On this basis the constants of integration C_1 and C_2 are easily determined. It facilitates their evaluation to consider the integral in [134] and [135] as *definite* with respect to a lower limit r_1. We have

$$r=r_1 \quad r=r_2 \quad f_r=0$$

For the first of these conditions

$$\int_{r_1}\theta r dr=\int_{r_1}^{r_1}\theta r dr=0$$

and for the second,

$$\int_{r_1}\theta r dr=\int_{r_1}^{r_2}\theta r dr$$

The constants of integration then become

$$C_1=\frac{(m-1)a}{m}\times\frac{\int_{r_1}^{r_2}\theta r dr}{r_2^2-r_1^2} \quad \ldots \quad [136]$$

$$C_2=\frac{(m+1)a}{m}\times\frac{r_1^2\int_{r_1}^{r_2}\theta r dr}{r_2^2-r_1^2} \quad \ldots \quad [137]$$

Substituting the constants in the two stress equations gives

$$f_r=aE\left[-\frac{1}{r^2}\int_{r_1}^{r}\theta r dr+\left(1-\frac{r_1^2}{r^2}\right)\frac{\int_{r_1}^{r_2}\theta r dr}{r_2^2-r_1^2}\right] \quad \ldots \quad [138]$$

$$f_t=aE\left[\frac{1}{r^2}\int_{r_1}^{r}\theta r dr-\theta+\left(1+\frac{r_1^2}{r^2}\right)\frac{\int_{r_1}^{r_2}\theta r dr}{r_2^2-r_1^2}\right] \quad \ldots \quad [139]$$

These two equations are perfectly general and hold for any temperature distribution through the cylinder wall.

170 It was stated that the temperature (θ) should be taken with reference to the temperature at the outside wall; it is immaterial what this reference temperature actually is. This is due to the fact that a constant temperature distribution through the wall will cause no stresses. It may be verified by considering the temperatures above zero or above an "accepted no-stress temperature," i. e., by using $(\theta_0+\theta)$ instead of merely θ. It will always be found that the terms involving θ_0 cancel out.

NOTE.—Strictly both radial, tangential, axial, and shearing stresses should be considered. There will be shear on planes where radial and

axial stresses act, while no shearing stress will be set up on any plane subjected to tangential stress.

The complete stress-strain equations then become

$$f_r = \frac{mE}{(m-1)(m-2)}[(m-1)e_r + e_t + e_z] \quad . \quad . \quad . \quad . [122a]$$

$$f_t = \frac{mE}{(m-1)(m-2)}[(m-1)e_t + e_r + e_z] \quad . \quad . \quad . \quad . [122b]$$

$$f_z = \frac{mE}{(m-1)(m-2)}[(m-1)e_z + e_r + e_t] \quad . \quad . \quad . \quad . [122c]$$

$$f_s = G\nu = \frac{1}{2}\frac{mE}{m+1}\nu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . [122d]$$

Here f_z and e_z designate axial stress intensity and axial strain, respectively, while f_s is the shearing stress intensity, G the modulus of rigidity, and ν the shearing strain.

$$\nu = \frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}$$

and

$$e_z = \frac{\partial v}{\partial z} - a\theta$$

where v is the change in length of any axial coördinate z.

The equations of equilibrium of a particle in connection with Equations [122a, b, c and d] give the following two differential equations (the first corresponds to Equation [131]):

$$\frac{\partial^2 u}{\partial z^2} + 2\frac{m-1}{m-2}\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru)\right] + \frac{m}{m-2}\frac{\partial^2 v}{\partial r \partial z} - 2\frac{m+1}{m-2}a\frac{\partial \theta}{\partial r} = 0 \; . [131a]$$

$$\frac{\partial}{\partial r}\left[r\frac{\partial v}{\partial r}\right] + 2\frac{m-1}{m-2}r\frac{\partial^2 v}{\partial z^2} + \frac{m}{m-2}\frac{\partial^2}{\partial r \partial z}(ru) = 0 \quad . \quad . \quad . [131b]$$

If these equations could be solved and the results made to satisfy the boundary conditions of the cylinder an exact solution of the problem would be obtained. There is, however, little hope of our being able to arive at such a solution. Approximations, therefore, have to be made.

If the cylinder is *infinitely* long the stress distribution in the middle portion of the same can be correctly determined. Since the temperature distribution is independent of z (distance axially) the same stress condition must exist in each cross-section. This is possible only when u is independent of z, and v is independent of r, i.e., when both $\frac{\partial u}{\partial z}$ and $\frac{\partial v}{\partial r}$ vanish. Equations [131a] and [131b] then reduce to

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(ur)\right] - \frac{m+1}{m-1}a\frac{d\theta}{adr} = 0 \quad . \quad . \quad . \quad . [131c]$$

$$\frac{d^2 v}{dz^2} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . [131d]$$

which can be easily solved. This has been done by Dr. Ing. Rudolph Lorenz (Temperaturspannungen in Hohlzylindern, *Zeitschrift des Vereines deutscher Ingenieure,* 1907, p. 743). The solutions show that in the middle part of an infinite cylinder radial, tangential, and axial stresses are set up, while no shearing stresses exist.

The cylinders, however, which are encountered in practice are far from being infinitely long. The solutions for an infinite cylinder are

therefore merely approximations when applied to an actual finite cylinder. Since in such a cylinder the axial stress and the shear must be zero at the end surfaces and since the exact solution cannot be obtained, the author of the present paper believes the truth is equally well and perhaps even better approximated by ***neglecting the axial stress and the shear entirely.***

The author feels so much more justified in doing this as a recognized authority as Stodola treats cylindrical disks without even mentioning axial stresses and shear. (A. Stodola: Die Dampfturbinen, Art. 82.)

Fuller and Johnston in analyzing the pressure stresses in thick cylinders also consider radial and tangential stresses without taking axial and shearing stresses into account, and base the derivation of their formulas on equations [122] and [123]. (Fuller and Johnston: Applied Mechanics, vol. II, p. 476.)

APPENDIX NO. 3

LIST OF SYMBOLS

A, B, C and D with or without subscripts are numerical constants
E modulus of elasticity
F indicates a function
J_0 and J_1 indicate a zeroeth and first-order Bessel function of the first kind
N_0 and N_1 indicate a zeroeth and first-order Bessel function (Neumann type) of the second kind
R indicates a function of r, radius, only
T indicates a function of t, time, only
a coefficient of linear expansion
c specific heat
e_r, e_t radial and tangential strains
f frequency
f_r, f_t radial and tangential stress intensities
f'_r, f'_t transient part of the radial and tangential stress intensities. Primes are also used to indicate the order of the harmonic when harmonic stresses are considered
$h = \sqrt{\frac{c\delta}{\lambda}}$
$j = \sqrt{-1}$ It also represents a symbolic operator indicating a plane-vector in quadrature with the axis of reference
$k = \frac{r_2}{r_1}$ ratio of outer and inner radius of cylinder. Also used to indicate order of roots and coefficients in one of the solutions involving Bessel functions
$\frac{1}{m}$ Poisson's ratio
n numerical constant
p_r, p_t numerical coefficients used when sinusoidally varying temperatures and stresses are considered.
$q = h\sqrt{\omega}\ \diagdown 45°$
r any radius
r_1, r_2 inside and outside radii
t time
u change in length of the radius r due to distortion
$\alpha = h\sqrt{\omega}/45°$
$\beta =$ phase displacement of a harmonic from zero of the complex wave in a Fourier series representing θ_1

γ angle between two radii
δ specific density
$\epsilon=$ 2.7183 (Napierian base)
θ temperature at radius r above temperature of outside wall
λ specific conductivity
μ numerical constant
μ_k k'th order root of the equation

$$J_0(\mu r_1)-\frac{J_0(\mu r_2)}{N_0(\mu r_2)}N_0(\mu r_1)=0$$

ρ numerical coefficient used when sinusoidally varying temperatures are considered
ϕ phase displacement between any harmonic in θ and the corresponding harmonic in θ_1
ψ_r, ψ_t phase displacement between any harmonic in the radial and tangential stress intensities at any radius r and the corresponding harmonic in θ_1
$\omega=2\pi f$ angular velocity of rotating plane-vector

Subscripts 1 and 2 refer a quantity to the inside and outside cylinder surfaces, respectively

Primes attached to a harmonically varying quantity indicate the order of the harmonic.

DISCUSSION

R. Eksergian.[1] The importance of temperature stresses has been more or less recognized, as in the treatment of the elastic arch, etc., and especially in shrinkage stresses in castings and in temperature stresses in elementary machine parts. Except in a very few cases of simple geometrical configurations, the entire analysis must be extremely qualitative, and only the roughest approximations can be hoped to be realized. This paper treats of the stress variation due to temperature variations in a simple geometrical configuration, a hollow cylinder, wherein a mathematical analysis is possible. Its real value is more in the application of detail analysis based on well-known physical methods and analysis to a definite engineering structure, rather than on the final solution of the problem attacked, which would require further experimental verification. The problem of temperature stresses has been very lightly touched upon, and a mathematical analysis can be applied to a greater extent to approximated simple geometrical forms wherein the detail methods of solutions and interpretation shown in this paper will be of great service in similar work.

Two fundamental physical equations form the basis for the solution of this problem. It was necessary to find the temperature at any point, both as a function of the time and geometrical displacement, and then to substitute the temperature values in

[1] Engineer, The Baldwin Locomotive Works, Philadelphia, Pa. Mem. A.S.M.E.

the elastic stress equation. For the determination of temperature, the Laplacian function or Fourier's equation of heat was obviously used for this problem in the form of cylindrical coördinates. The use of the Laplacian function is fundamental in all flow problems and an exactly similar equation to Fourier's equation of heat flow is obtained for the distribution of electrostatic and magnetic flux in the propagation of eddy currents in a metal, in the flow of water, etc. It is of interest to note that an exactly similar analysis is applied in the solution of the distribution of flux density and current intensity in the interior of metals, etc. Therefore, certain points in the particular solutions of this paper will have a more nearly universal value in the domain of the general application of the Laplacian function to flow distribution problems. The elastic stress temperature equations have been developed and applied to particular solutions by Duhamel, Neumann, Lord Kelvin, and others. An analysis to a similar engineering problem was worked out by Stodola in his discussion of the temperature effects on the stress of a rotating disk. It is believed, however, that the stress-temperature equations given in this paper are of a somewhat more generalized form for cylindrical coördinates than those given by Stodola.

The first basic equation, that is, the Laplacian or Fourier heat-conductivity formula, is readily established on the basis of a constant specific heat at constant pressure. That is, if Q is the heat flow per second, θ the temperature, and C_p the specific heat of the metal itself,

$$\frac{\partial Q}{\partial x} dx(dy \cdot dz) + \frac{\partial Q}{\partial y} dy(dxdz) + \frac{\partial Q}{\partial z} dz(dydx) = C_p \frac{\partial \theta}{\partial t} dx \cdot dy \cdot dz$$

where

$$Q = \lambda A \frac{\partial \theta}{\partial S}$$

A = Area of section of flow
S = Displacement in direction of flow

hence

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} = h^2 \frac{\partial \theta}{\partial t}$$

where

$$h^2 = \frac{C_p}{\lambda}$$

λ being the heat conductivity of the metal. Transferring this to cylindrical coördinates we have the first basic equation used in this analysis. For very thin cylinders the author shows that the term $\frac{1}{r}\frac{\partial \theta}{\partial t}$ can be neglected, whereas for thick cylinders it must

be included as would be expected. Hence the basic equations for the determination of temperature are

1 For thin cylinders, $\frac{\partial^2 \theta}{\partial r^2} = h^2 \frac{\partial \theta}{\partial t}$ . . . . [6]

2 For thick cylinders, $\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} = h^2 \frac{\partial \theta}{\partial t}$ . . . . . [27]

which are special forms of Fourier's heat conductivity, assuming the flow of heat to be entirely radial. The assumption of no longitudinal flow is obviously due to symmetry and the conditions of this problem. The author then solves these equations for the following temperature conditions:

1 Constant temperature difference under a steady and transient state, that is, when $T = \infty$ and $T = 0$
2 Sinusoidally varying temperature difference
3 Sinusoidal varying temperature superimposed on constant temperature
4 Any periodic temperature difference which involves the solution of a Fourier series.

In a thin cylinder for the steady state, that is, when $T = \infty$, the temperature gradient is constant, while for a transient state when $T = 0$, the temperature gradient is zero, that is, there is no heat flow. With a sinusoidal varying temperature, the temperature drops exponentially in space. That is, the amplitudes of the sinusoidal time variations at any point vary from point to point along a descending exponential curve. The solution of the equation further shows a time lag along the space of distribution. That is, heat flow of a periodic nature manifests properties analogous to inertia in mechanics or inductance in electricity. This is due to the fact that heat is retained by the volume of metal itself in addition to its conduction properties, and is exactly similar to the inertia or inductive effects of an electric current due to the transfer of magnetic energy to and from the volume of the space or magnetic field surrounding the wire. This explains the physical cause of the time lag in Fig. 3. When the thick cylinder is considered the solution of [27] becomes complicated, requiring the use of Bessel's functions. For the condition of constant temperature difference we have a curve in place of the straight line variation through the cylinder as in the thin cylinder.

The second part of the problem involves the use of the elastic stress-temperature equations. This is based on the equations,

Temperature Strain + Stress Strain = Geometrical Strain

$$a\theta + e_r = \frac{\partial u}{\partial r} \text{ in a radial direction}$$

$$a\theta + e_t = \frac{u}{r} \text{ in a tangential direction}$$

where a equals the coefficient of linear expansion of the metal. Now the stress strains are connected up with their stress by the well-known equations,

$$f_r = \frac{mE}{m^2-1} \ (me_r + e_t)$$

$$f_t = \frac{mE}{m^2-1} \ (me_t + e_r)$$

Combining these with the equilibrium equations for a differential particle, we have,

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = \frac{m+1}{m} a \ \frac{d\theta}{dt} \quad . \ . \ . \ . \ [131]$$

This is a special case of the more general differential equation given by Stodola for variable thickness disk wheels. Stodola's equation is,

$$\frac{d^2u}{dr^2} + \left(\frac{d(\log y)}{dr} + \frac{1}{r}\right)\frac{\partial u}{\partial r} + \left(\frac{1}{mr}\frac{d(\log y)}{dr} - \frac{1}{r^2}\right)u$$
$$- \frac{m+1}{m} a \frac{d\theta}{dt} - \frac{m+1}{m} a\,t\,\frac{d(\log y)}{dr} + Ar = 0. \quad . \ [131']$$

For a constant thickness, y is a constant, and therefore $\frac{d(\log y)}{dr} = 0$ and the term $A = \left[1 - \left(\frac{1}{m}\right)^2\right] \frac{u\omega^2}{E}$ is zero for a stationary disk or cylinder. Therefore, the differential temperature stress equation used by the author is identically that given by Stodola. The differential equation [131] is the basis of the stress strain analysis, the remaining work being the integration of this equation and then the substitution of the temperature values for the several solutions of the temperature equations [6] and [27].

The real contribution in this paper is, therefore, the particular solutions and their interpretations, of the temperature and stress-temperature differential equations for the various temperature conditions and their variations for a hollow cylinder. In the practical application to a boiler tube a temperature difference of 100 deg. cent. is used. This corresponds to a heat flow of $\frac{26.2 \times 180}{0.12} = 39{,}300$ B.t.u. per sq. ft., or to an equivalent evaporation at 175 lb. per sq. in. pressure of $\frac{39{,}300}{853} = 46$ lb. per sq. ft. This is quite possible to be realized for firebox tubes and the computations show high stress and the importance of the consideration of temperature stresses. The author is to be congratulated on the solution of a difficult problem, on the presentation of a well-balanced solution and explanation of the solutions involved, and finally on opening up a general procedure of math-

ematical attack for similar problems pertaining to stress due to temperature. It is hoped, however, that the author or others will check experimentally the validity of the equations and their solutions, since the question of varying specific heat of the metal, the change of elastic constants, etc., as a function of the temperature, and the maximum metal surface temperatures within the gas film may considerably modify the results.

S. Timoshenko.[1] In studying the thermal stresses in a cylinder symmetrically heated, there are two simple cases: i.e., a very long cylinder, and a very thin disk. In both cases the problem is a two-dimensional one. The author of the paper believes the truth is equally well and perhaps even better approximated by neglecting the axial stress and the shear entirely. On this basis he considers the stresses in a thin disk only and it is difficult to agree with the conclusions which have been drawn.

It is known from the application of the general principle of Saint Venant that the disturbance in stress distribution expected at the ends does not spread very far into the cylinder. In the case of a hollow cylinder a distance two or three thicknesses from the ends is sufficient for establishing a stress distribution very near to that taking place in an infinitely long cylinder. This is the reason why the solution for a long cylinder must be used in studying the question of thermal stresses in hollow cylinders.

There is another reason for this, namely, in actual cases not only does a variation in temperature occur in a radial direction but also in an axial direction. Due to this fact bending of the wall takes place. The corresponding bending stresses must be combined with the axial stresses calculated for a two-dimensional problem on the basis of a very long cylinder.

It is surprising that the author, in order to justify his method or procedure, mentioned the books of Stodola and of Fuller and Johnson. In the consideration by Stodola, the problem of stress distribution in a thin disk is discussed and it is permissible to neglect the axial stresses since they will be very small.

In the second case a thick cylinder under internal pressure is considered. Under such conditions the sum of two principal stresses is constant and it is not necessary to consider the axial stresses in order to get a two-dimensional problem.

The author in his paper limited himself to the study of stress distribution in a two-dimensional problem. This has been sufficiently well analyzed before in scientific and technical literature. For practical application we need a further study of the problem and the writer is of the opinion that something can be done.

The problem of temperature distribution can be solved, for instance, in a more general case than that considered in the paper,

[1] Westinghouse Research Laboratory, East Pittsburgh, Pa.

namely, for those cases where the body has a form of rotation. The general equation in this case coincides with that for the twist of a shaft of variable cross-section. This enables us to use the graphical and numerical methods developed for the solution of torsion problems. This was done in a paper by Eichelberg, entitled Temperaturverlauf und Wärmespannungen in Verbrennungsmotoren.[1]

The problem of the bending stresses produced in the wall of a hollow cylinder by variation in temperature along the axis of the cylinder is also of interest and can be studied analytically in the case of thin cylindrical tubes. Some cases were considered by the writer. It was found that these stresses were very high and for such cases as, for instance, Diesel-engine cylinders, their consideration is necessary.

The real problem on thermal stresses in cylinders is complicated and the writer believes that further progress in the solution of it will be possible only on the basis of careful experiments on the distribution of temperature. Only by using analysis in combination with such experimental data can a satisfactory solution of the problem be expected.

[Editor's Note. Dr. Timoshenko also called attention to a few inaccuracies in the paper as it was presented at the meeting. Acting upon his suggestions the author subsequently revised the paper for publication in the Transactions. The revisions included corrections in Equations [**54**], [**55**], and [**56**], p. 176, caused by the dropping of a factor of two in summing the infinite series in Equation [**53**]; the substitution of the value $a=11.6\times10^{-6}$ for the coefficient of linear expansion in Par. 90, p. 182, instead of $a=9.3\times10^{-6}$; and the use of the Lamé formula for tangential pressure stresses in Par. 91 instead of the Reinhardt formula. All examples and tables throughout the paper affected by these changes have been corrected.]

The Author. The validity of the derived equations for temperature stresses in hollow cylinders for the stated temperature conditions depends on the degree of approximation involved in treating the problem on a two-dimensional basis. When preparing the paper the author was fully aware of this fact, as briefly discussed at the end of Appendix No. 2.

An entirely rigorous three-dimensional analysis is not possible. We have the choice, then, between considering a section of an infinite cylinder at a sufficient distance from the ends and considering the problem as two-dimensional.

Dr. Timoshenko criticises the latter procedure which was used in the paper. The author agrees that it would in general be preferable to consider the cylinder as being infinitely long. There is evidently considerably more justification for treating a disk or a

[1] *Forschungsarbeiten*, Heft 263, 1923.

short cylinder two-dimensionally than there is for treating a comparatively long cylinder in the same manner. This fact, therefore, imposes limitations on the formulas arrived at in the paper.

As also mentioned in Appendix No. 2, Dr. Ing. Rudolf Lorenz obtains the temperature stresses in an infinite cylinder for a constant temperature difference between the two surfaces. Using the procedure in the present paper it would, presumably, not be difficult to work out the solutions for the infinite cylinder, also for the case where the temperature difference undergoes a periodic variation. In the author's opinion this case is of considerable practical importance. It is believed that the solutions for the thin cylinder still would be obtained in terms of hyperbolic functions of semi-imaginary arguments which can very easily be handled. The author proposes to look into this matter.

When Dr. Timoshenko states that the problem of stresses in a long cylinder subjected to an internal pressure reduces to a two-dimensional one because the sum of two principal stresses is constant, the author must disagree. This problem is also fundamentally three-dimensional, but becomes two-dimensional on the *assumption* that all shearing stresses are zero and the axial strain constant. The fact that the sum of two principal stresses is constant is a *consequence* of the two-dimensional problem.

The interesting paper by Dr. Ing. Gustav Eichelberg referred to by Dr. Timoshenko had not appeared when the present paper was prepared, and the author has but recently seen it. Dr. Eichelberg's treatment of temperature stresses in cylinders is extremely interesting and very complete. It is believed, however, that certain of his solutions should be accepted with reservation.

Both Mr. Eksergian and Dr. Timoshenko point out that experimental work is required for verification of analytical methods. This fact was also mentioned by the author. The problem of thermal stresses is a most difficult one, and all physical factors cannot possibly be taken into account in a mathematical treatment. Analysis and experiment must go hand in hand. This procedure is essential in order that derived formulas for thermal stresses may be used with confidence.

No. 1922

THE GAS ENGINE IN THE STEEL INDUSTRY

By A. C. Danks,[1] Cleveland, Ohio
Non-Member

The paper traces the development of the gas engine operating on blast-furnace gas as a prime mover in the steel industry, from the initial installation in 1900 to the present time. The design problems that have had to be solved in the successful application of the gas engine to this service included methods of governing, valve gear as related to gas analysis, ignition, piston troubles, corrosion due to water cooling, etc. The results of tests are given and precautions to be observed in testing are outlined.

The first installation of gas engines in the steel industry to attract attention was that at the Buffalo works of the Lackawanna Steel Co. This installation was decided on in 1900 as set forth in a paper[2] read before the Society in 1910 by E. P. Coleman. It was of the two-cycle type, was installed for use both in the generation of electric power and to deliver air to the blast furnaces, and was built to operate with blast-furnace gas as the fuel.

2 While this was the first major installation to be made in the steel industry, there were a few other scattered installations of smaller sizes, but tending to show the interest of the steel industry in what was then a comparatively new form of prime mover. When these installations were proposed by the plant engineers only one of the few companies then building gas engines could be persuaded to figure on a gas engine for this service.

The First Four-Cycle Installation

3 At this time the tandem double-acting engine was in the experimental stage, and the type generally used was the single-cylinder or tandem opposed cylinder engine, in nearly all cases

[1] Pres., The Ashmead-Danks Co.
[2] First Large Gas-Engine Installation in American Steel Works, E. P. Coleman. *Trans.* A.S.M.E., vol. 32, p. 1361.

Contributed by the Oil and Gas Power Division and presented at the Spring Meeting, Cleveland, Ohio, May 26 to 29, 1924, of The American Society of Mechanical Engineers. Abridged.

single-acting. Very heavy fly wheels were called for to offset the wide fluctuations in the crank effort.

4 The United States Steel Corporation, a few years later, decided to make the installation of the first four-cycle type gas engine designed to operate on blast-furnace gas in this country. This was a purely experimental installation to determine the characteristics that would enter into the design of larger engines of this type. The engine was of the horizontal tandem double-acting four-cycle type with cylinders $21\frac{3}{4}$ in. in diameter and a stroke of 30 in., having a normal speed of 150 r.p.m. It was direct-connected to drive a 250-kw. 280-volt d.c. generator, and so arranged that it was possible to operate either on part of the plant load or on a water rheostat for testing purposes.

5 As the installation started in November, 1905, and operated until September, 1906, ample opportunity was given to study the characteristics of the blast-furnace-gas-driven engine from all its various angles. A complete set of indicator cards taken at both low load and full load, when compared with cards from the modern engine of today, show that very little improvement along this particular line has been recorded.

Comparison of Methods of Governing

6 The most important problem then confronting the gas-engine designer was to decide upon the most suitable of the two forms of governing then in use: namely, that of constant compression in which the air admission was constant and in which the quantity of gas admitted varied, thus giving a variable heat content per cubic foot of cylinder mixture but a constant compression; or that which maintained the ratios of air and gas constant and as determined by the design of the valve gear, thus insuring a cylinder mixture of the same heat value at all loads, but with amounts of gas and consequently a compression varying with the load conditions. The little experimental engine was of the latter type, and as borne out by experience of that installation this system has proved with its various improvements and modifications to be better suited to the changeable conditions of steel mill service and blast-furnace gas.

7 This opinion is not shared by all of those interested in the gas engine for this service, and while there are several very satisfactory installations using the constant-compression system, the majority of the builders today use the constant-mixture or some combination of the two.

8 The ideal condition to be realized in a gas engine for service on this fuel is to combine both systems of governing as each has its advantage at various loads. This has done much to remove the early feeling of uncertainty that prevailed when by reason of faulty mixtures at times of irregular gas supply from the blast furnaces,

backfiring and preignition would occur to an extent that became serious, whether in an electric drive or a blowing unit, at times serious enough to cause the electrical load to be dropped or the blast to be lost or partly reduced on the furnaces. With the gas engine of today, however, many records are available of continuous runs over periods of 90 days and longer, and with entire freedom from trouble.

Valve Gear in Relation to Gas Analysis

9 Another important point in considering systems of governing is the wide variation of the character of the blast-furnace gas. Table 1 shows what the gas engine may have to handle.

TABLE 1 ANALYSES OF BLAST-FURNACE GAS TAKEN FROM FURNACES ON DIFFERENT KINDS OF IRON

	CO_2	CO	H	N
Bessemer and basic	12.9	25.2	3.7	58.2
Spiegel (20 per cent Mg)	6.5	30.0	3.0	60.5
Ferromanganese (80 per cent Mg)	5.0	34.4	2.7	57.9
Ferrosilicon (10 per cent Si)	3.6	33.0	2.3	61.1

10 It will be seen from these analyses that the amounts of gas required vary widely. This calls for a very flexible valve gear if the entire range is to be handled, without introducing the trouble that often follows such a change.

11 This is not the only reason for flexibility in the valve gear, as the changes occur in the same product, due to the various furnace operations. The following variations in analyses occurred while a single furnace, from which the engines were drawing their supply, was passing through the process of casting. These variations are sufficient to set up backfiring and premature firing if the valve gear is not sufficiently flexible to handle them.

	CO	CO_2	H	N
Sample No. 1	25.90	12.60	11.80	49.70
Sample No. 2	29.60	12.50	12.32	45.58

12 The increased heat value to be handled was considerable and had a decided effect on the valve-gear design. The amount of air, by volume, required for blast-furnace gas is about 75 parts air and 100 parts gas, while with natural gas the ratio may be 10 parts air to 1 part of gas, depending on the composition of the gas. The port ratios of this type of engine thus have less flexibility than with other gases.

13 The author had this very forcibly brought home to him by having to enlarge the air ports on one engine before it could be successfully operated on gas from a furnace making ferrosilicon iron. In order to determine and fix the actual ratio best suited, a series of cards was taken, and as they illustrate very clearly the importance of the port ratios on the different gas compositions, they are given in Table 2. In considering these figures it should

be borne in mind that as designed for this engine the air-gas ratio was 100 parts air to 212 parts gas, and were later changed to 100 parts air to 130 parts gas. To determine under actual conditions the proper port ratios, one valve with adjustable ports was built, the control removed from the governing system, and it was traveled through its entire range and cards taken on both

TABLE 2 SUMMARY OF CARDS TAKEN WHILE USING FERROSILICON GAS

(Figures given are averages of three cards. Gas analysis: CO, 30.93; CO_2, 5.00; H, 2.43; N, 61.74. Heating value of gas, 116.39 B. t. u. per cu. ft.)

Ratio of Mixture Air, parts	Gas, parts	Ports Air	Ports Gas	Card Data Area	Card Data M. e. p.	Remarks
100	130.5		4¼	1.69	72.08	
100	123.0		4	1.63	69.25	
100	115.0		3¾	1.62	68.83	
100	107.5		3½	1.61	68.49	
100	100.0		3¼	1.51	64.10	
100	92.3		3	1.48	62.80	
100	84.6	3¼	2¾	1.62	64.50	
100	76.9		2½	1.43	60.75	
100	69.2		2¼	1.33	56.50	Start backfiring
100	61.5		2	1.27	54.00	Running good
100	53.7		1¾	1.18	50.20	
100	46.2		1½	1.05	44.60	
100	38.5		1¼	0.83	35.20	
100	38.5		1	0.85	36.03	

types of gas, although the original change was made to burn ferrosilicon gas.

14 On trying the same plan after the engine was again running on bessemer-iron gas, very different results were obtained. See Table 3. The theoretical air for the gas composition is given as 88.3 cu. ft. gas per 100 cu. ft. air, but the best results were obtained

TABLE 3 SUMMARY OF CARDS TAKEN WHILE USING BESSEMER-IRON GAS

(Gas composition: CO_2, 10.40; CO, 23.60; H, 2.80; N, 63.20. Heating value, standard, 92.06 B. t. u.; do, as sampled, 79.17 B. t. u.)

Ratio of Mixture Air, parts	Gas, parts	Ports Air	Ports Gas	Card Data Area	Card Data M. e. p.
100	130.5		4¼	1.27	54.56
100	123.0		4	1.35	58.08
100	115.0		3¾	1.39	59.84
100	107.5		3½	1.37	58.88
100	100.0		3¼	1.40	60.16
100	92.3		3	1.42	61.12
100	84.6	3¼	2¾	1.40	60.16
100	76.9		2½	1.35	58.08
100	69.2		2¼	0.98	42.08
100	61.5		2	0.92	39.52
100	53.7		1¾	Stopped burning due to bad mixture	
100	46.2		1½		
100	38.5		1¼		
100	38.5		1		

at 92.3 cu. ft. gas and 100 cu. ft. air or about 11.7 per cent excess air. At no place during the entire range did backfiring occur as with ferrosilicon gas and the maximum m.e.p. is noticeably lower, being reached in a different position in each case.

15 Tables 2 and 3 show clearly the variations that result in port changes with the two gases. One noticeable feature on bessemer-iron gas is that when 100 parts air and 53.7 parts gas

was reached, ignition failed. When comparing the bessemer to the ferrosilicon gas, it will also be noticed that the maximum power is reached on bessemer gas with 100 parts air and 92 parts gas, a very nearly equal proportion, while with ferrosilicon the greatest power was with 100 parts air and 130.5 parts gas. With both gases it will be seen that the range of port ratios is comparatively small.

16 One other factor that has entered into the operation of the modern gas engine that did not exist in the early installations is the adoption of by-product coke for the blast furnaces. This together with the continually lowering coke rates on the furnaces has had a decided effect on the gas engine and will have a still further influence, depending upon future improvement in blast-furnace practice.

17 During the early years of gas-engine operation in the steel plants beehive coke was used exclusively, and the coke rates varied from 2300 to 2500 lb. per ton of iron. This gave a heating value

TABLE 4 TEST SHOWING EFFECT OF CHANGE OF INLET TEMPERATURE ON THE ENGINE MEAN EFFECTIVE PRESSURES

(L. H., No. 1 cylinder, No. 1 engine)

Date, 1908	8/4	9/23	9/28	9/29	10/3	11/4	11/3
Engine-room temp., deg. fahr.	102.0	83.0	71.0	72.0	70.0	65.0	60.0
Air temperature, deg. fahr.	107.0	90.0	82.0	78.0	74.0	68.0	64.0
Gas temperature, deg. fahr.	104.0	84.0	80.0	75.0	70.0	64.0	60.0
Avg. gas and air temperature, deg. fahr.	105.0	87.0	81.0	76.5	72.0	66.0	62.0
Gas Analyses:							
CO (324.27 B.t.u.)	23.4	26.5	26.2	27.5	25.0	26.0	26.5
CO_2	13.6	10.2	4.9	7.2	11.2	10.7	10.8
O	0.0	0.0	0.2	0.0	0.0	0.0	0.0
H (324.73 B.t.u.)	2.4	2.0	0.0	2.0	1.6	1.1	1.8
N	60.6	61.3	68.7	63.3	62.2	62.2	60.9
Effective heating value, B.t.u.	82.7	92.5	85.0	95.7	86.4	87.8	91.84
M.e.p. (at observed heating value)	44.25	50.52	49.5	48.1	51.0	56.8	55.6
M.e.p. reduced to standard 85 B.t.u.	44.9	48.4	49.5	44.8	50.8	55.7	53.2
R.p.m.	48.0	50.0	50.0	50.0	50.0	50.0	50.0

of anything from 95 to 110 B.t.u. per cu. ft. and was ideal fuel with the compressions of from 160 to 190 lb. that prevailed at that time. This gas composition and its corresponding heating value have been gradually declining for the two reasons given. At present the heating value is about 84 B.t.u. per cu. ft. While this loss in the heating value of gas has had a certain effect on the rating of the gas engine for given sizes of cylinders, we are able now to secure cylinders of materially larger diameter than was at first thought feasible, and this in a measure compensates for much of the loss.

18 The first two engines to be installed following the experimental unit just described were for blowing service and had 38¼-in. gas cylinders, 60-in. air cylinders and a stroke of 54 in., and even with these sizes there were certain misgivings as to the possible outcome. The next unit had 40¼-in. cylinders with the same stroke, but built for 75 r.p.m.

19 The successful operation of these three power units made in the year of 1906 and 1907 soon led to larger sizes and higher

speed, and we believe 48 in. is considered the standard gas-cylinder diameter by several builders of today. The compressions carried on the earlier engines have been changed very little and are still held around 200 lb. in some types, while in others there is a tendency to lower this with improved methods of ignition and valve-gear arrangement. It is true, however, that improvements in valve gear, mixing proportions of the gas and air, and improved ignition have made it possible to realize considerably higher m.e.p. than were at first thought possible.

20 In the original engines referred to the rated loads were figured on a basis of 56 lb. per sq. in. m.e.p., while in practice today many tests are available showing as high as 61 to 65 lb., with the highest m.e.p. on gas from basic and bessemer furnaces as will be shown later in test figures.

Number and Location of Igniters

21 It was early recognized that both the proper number and locations of the igniters had much to do with the m.e.p. developed through more complete combustion and many and various arrangements were tried out to secure the desired results. On the experimental engine referred to the "jump spark" was tried and soon found to be unsatisfactory. This was followed by a magnetically operated igniter to avoid the necessity of a separate shaft for driving this set of igniters, which was usually placed on the opposite side of the engine and not accessible to the drives from the other igniters. This, too, was shortly abandoned for the more substantial mechanical make and break for all igniters with an additional lay shaft, and has become the accepted design for this service.

TABLE 5 ANALYSIS OF DELAYS IN VARIOUS PLANTS

Plants	A[1]	B	C	D	E	F[1]	G[1]	H	I	J
Hours operated	6375	1523	3695	1407	4592	7876	7806	5685	3982	8137
Hours Lost:										
Not due to engine	1683	6689	4080	6550	4363	318	381	246	831	476
Due to engine	702	492	627	804	5	566	573	2075	323	147
Per cent of time in operation	72.8	17.3	42.0	16.1	52.5	89.6	89.0	70.2	77.8	92.7
Plants	K	L	M	N[1]	O[1]	P[1]	Q[1]	R[1]	S	T
Hours operated	7788	5324	5128	5625	3091	5997	8008	7813	6647	3317
Hours Lost:										
Not due to engine	444	2825	3563	3135	1253	2763	375	504	592	2437
Due to engine	1528	611	69	....	....	....	377	443	1521	126
Per cent of time in operation	88.5	60.7	58.3	64.2	71.4	68.5	91.4	89.1	76.0	56.5

[1] Blowing units; all others are electric engines.

Effect of Entering Temperatures of Gas and Air

22 One other fact that influences the rating possible, but which is often not appreciated to its fullest extent, is the temperature of the entering gas and air where the installation is located so that these become unusually high in the summer months. Table 4

shows results that were obtained when an end inlet valve was isolated from the governor control, blocked in a predetermined position and allowed to remain so through the entire seasonal changes in temperature. Indicator cards were taken at intervals when the temperature changes were apparent.

Study of Mechanical Reliability

23 During the years of early development careful records were made of the delays encountered in gas-engine operation and the

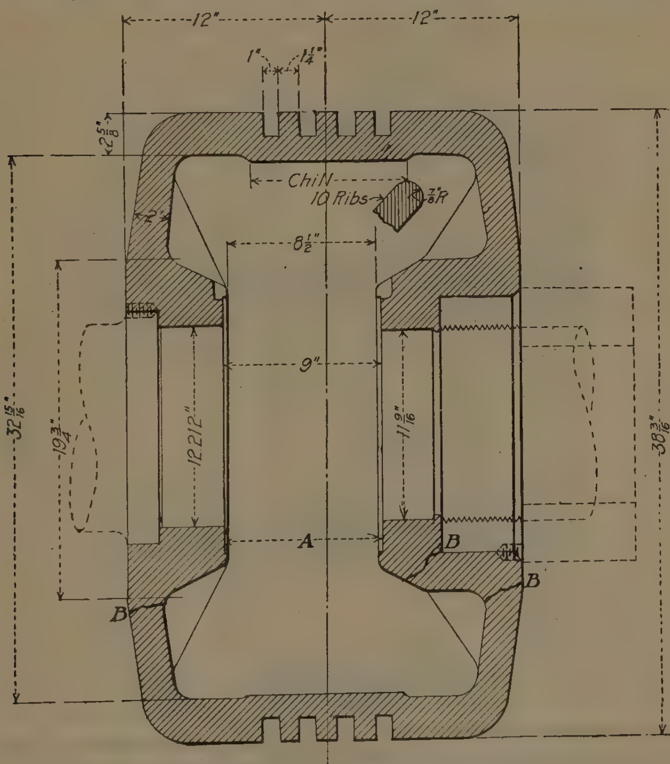

Fig. 1 Original Cast-Iron Piston

(Sectional water-distributing blocks at *A* came loose and piston cracked at points *B*.)

causes responsible for these delays, not so much with the intention of making any record runs but to arrive at some conclusion as to the percentage of operation that could really be expected of the units, and if delays occurred, to what parts of the engine were they chargeable.

24 The results of this study are presented in Table 5 for a number of plants that for obvious reasons have been designated

by letters. These records were all made in the same year, and while there may be some question as to the items "Due" and "Not Due" to engines, the total percentage of time in operation for a year of 8760 hr. is interesting.

Piston Troubles and Design

25 This naturally led to a study of just what the causes of the delays were traceable to, and while local influences were in some cases found responsible, the greater part of the delays were

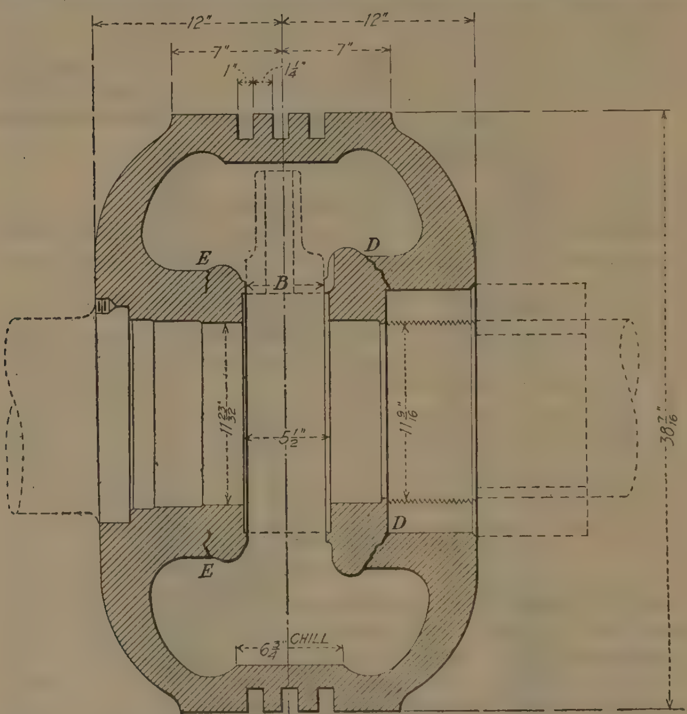

Fig. 2 Second Cast-Iron Piston Designed to Assume Shape of Ball as Much as Possible

(Water-distributing blocks at *B* came loose and cracks developed at *D*. Cracks at *E* form when breaking piston off the rods.)

traceable to piston troubles. To understand this condition it is only necessary to study the development of the large gas-engine piston as shown in Figs. 1 to 5. Fig. 1 was the original cast-iron piston used. It soon developed cracks as indicated. This was followed by the piston of Fig. 2, still of cast iron, with cracks resulting in the same location. Fig. 3 shows the same piston as that of Fig. 2 made in steel. Trouble with the water-distributing

blocks that were clamped by the action of the rod nut against the shoulder on the rod led to the form shown in Fig. 4, and when finally put in shape the piston in Fig. 5 was the result. The pistons of this last design were in operation for many years and none ever cracked, although a few had to be discarded from "old age," due to the width of the ring grooves reaching a prohibitive figure.

Corrosion Due to Water Cooling

26 Many interesting studies could be made of the changes in

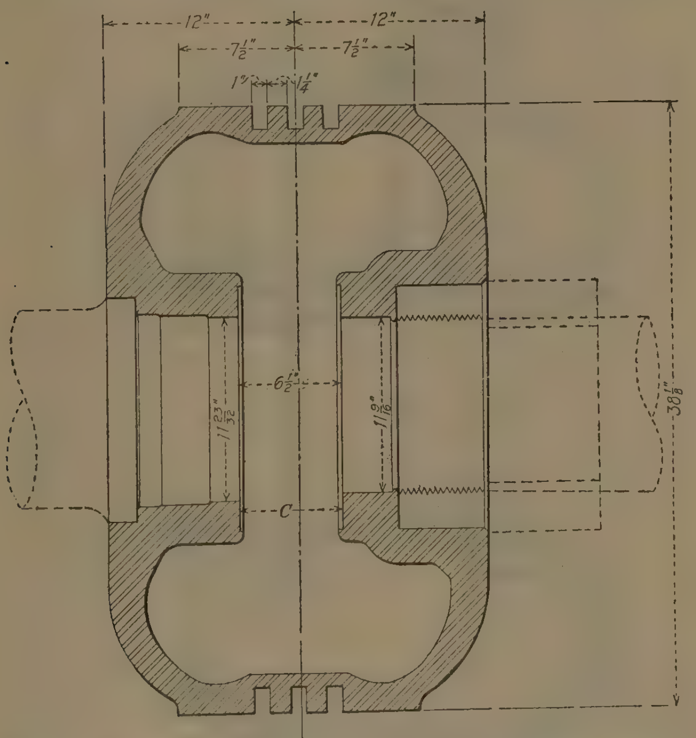

Fig. 3 First Form of Steel Piston, Following Lines of Original Cast-Iron Piston

(No cracks developed but distributing blocks at *C* came loose.)

design that have occurred in the various important parts of the steel-plant gas engine, for design was often influenced by local conditions. This was particularly true of the water-cooling equipment as applied to all parts of the engine. Operators on the Great Lakes with clean water can have very little realization of the troubles resulting both directly and indirectly from the supply that many of the inland plants are obliged to use.

27 This is particularly true of the Pittsburgh district where much of the water supply comes from the Monongahela River and at certain seasons of the year carries a sulphurous acid content as high as 8 grains per gallon. When the gas engine was first being introduced in that district the provisions for treating the water supply were primitive and it was not unusual to have pipework

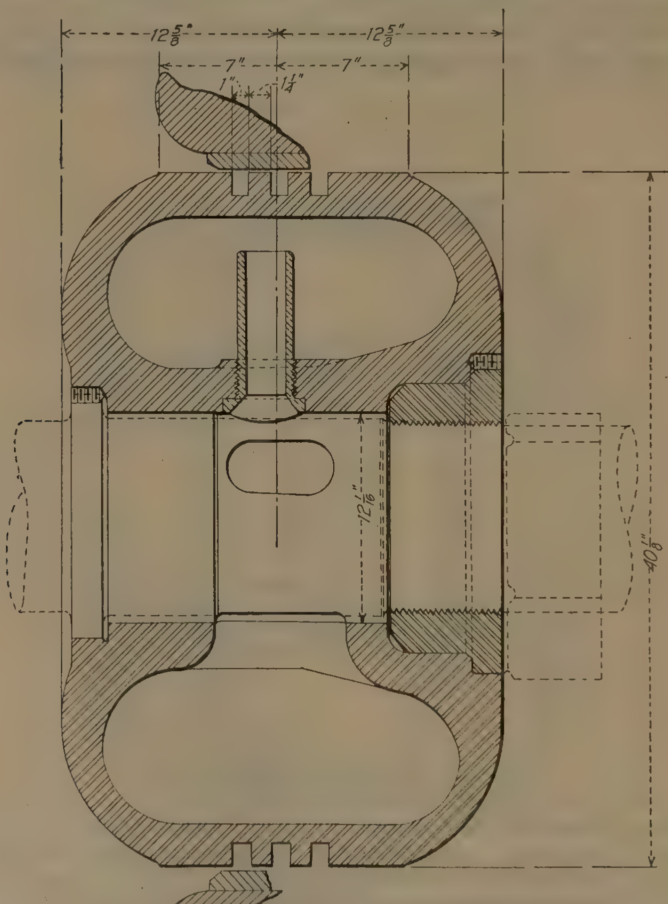

Fig. 4 Final Form of 40-In. Steel Piston
(No cracks and no inside fittings that could come loose.)

last only a few weeks at certain seasons of the year. This eventually led to the use of brass pipe for all water-cooling service.

28 It was noticeable that the attack on cast iron and cast steel, on account of their scale, was much less than on the finished steel on the rods, and as nearly all piston rods were then being drilled for side water connections it became a serious and important

matter to protect them. Many rods were cracked due to weakened sections at these points, and it became necessary to inspect all rods closely for indications of cracks.

29 This led to a construction shown in Fig. 6, which, while it added considerably to the cost of the rod, entirely eliminated the trouble of internal corrosion; and as the water connections were taken from the ends rather than the sides of the rod it also eliminated the danger of cracks that formerly occurred where these

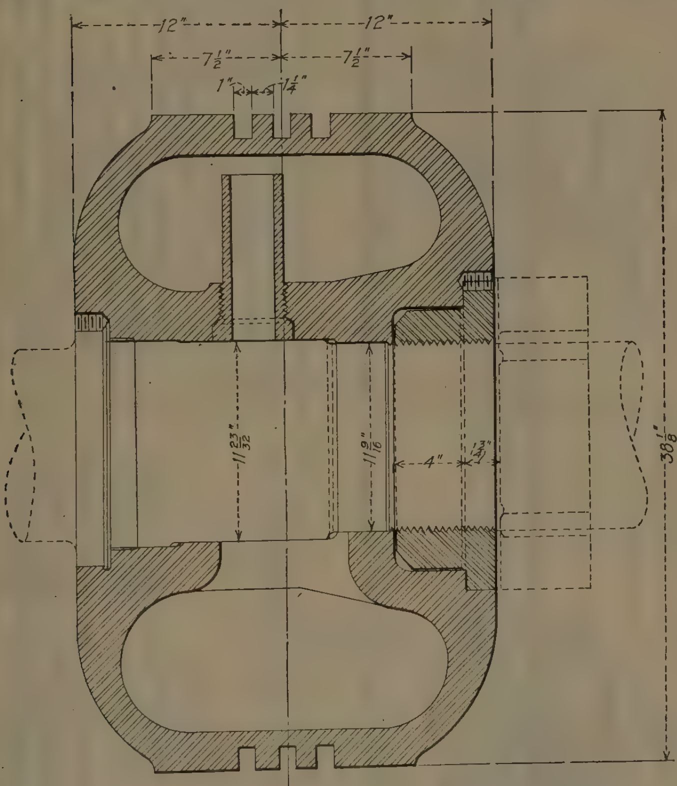

Fig. 5 Final Form of 38-In. Steel Piston
(No cracks and no inside fittings that could come loose.)

holes were drilled through the rods. The construction consisted in lining the rod entirely with bronze or other acid-resisting material. Where the water passed through the rod under the piston bore the holes were lined with tin. Bronze distributing blocks were used at the center of the rod, and standard brass pipe for lining the bore, with suitable bronze water fittings on either end for the entering and leaving water.

30 This particular type of water service to the piston involved the telescoping straight-line sliding plunger. While some types of

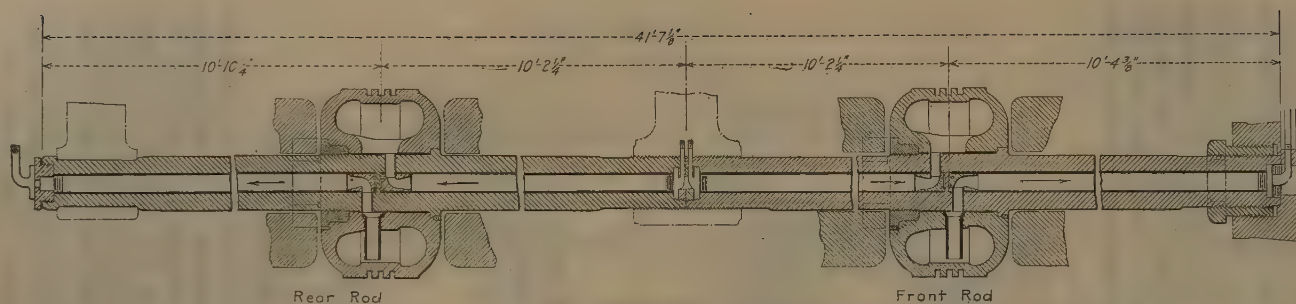

Fig. 6 Modern Arrangement of Piston Rods on a Gas Engine to Eliminate Holes through Rod

engine then used and still do use the swinging joint, at the higher speeds this construction is not as desirable as the straight-line telescope.[1]

Gas-Engine Tests

31 In order to present particulars regarding gas-engine efficiencies that have been obtained both from tests and actual operation over different periods of time, results of a number of tests are given in Tables 6, 7, 8 and 9.

32 When considering the efficiency of any gas-engine installation the first question that is naturally raised is as to the method of measuring the gas, as in handling the large volumes involved in this service the direct-reading meter is out of the question and some form of pitot tube or venturi meter must be resorted to.

33 When pitot tubes are used their location and operation are of extreme importance as the readings are easily affected by eddy currents set up in gas lines containing a number of bends and irregular connections. In short,

[1] At this point in the complete paper the author included charts and tables showing costs of operation and current generation for gas engines, steam engines, and turbines. This information was included in the paper as printed in *Mechanical Engineering*, vol. 46, no. 8, Aug. 1924, pp. 456-459.

TABLE 6 GAS-ENGINE TEST

(44 by 60-in. electric unit, constant-compression type.)

Test No.	1	2	3
Duration of test, hr.	1¼	2	1½
Per cent of rating	100	100	66⅔
B.t.u. per cu. ft. of gas	90.6	90.6	90.6
Avg. max. explosion pressure, lb. per sq. in.	336	374	246
Avg. m.e.p., lb. per sq. in.	29.72	30.72	21.31
Total hp.	4,076	4,087	2,962
Cu. ft. gas per hp-hr	91.8	92	108.9
B.t.u. per hp-hr	8,436	8,451	10,005
Thermal efficiency, per cent	30.17	30.11	25.44

TABLE 7 GAS-ENGINE TEST

(40 by 54-in., 75-r.p.m., direct-connected electric unit operating on blast-furnace gas; constant-compression system of governing.)

Test No.	1	2	3	4	5	6
Duration, hr.	2	8	8	8	8	8
Load, per cent of rating	0	25	50	75	100	125
Avg. compression pressure	60	83	101	140	190	201
Avg. max. explosion pressure	57	137	211	289	353	422
Avg. m.e.p.	11.68	23.84	33.56	46.12	52.44	62.96
Avg. mechanical efficiency	50.16	58.16	73.25	80.18	86.25	94.30
Avg. B.t.u. of gas	90.24	89.15	89.28	87.90	89.98	90.53
Cu. ft. gas per hp-hr	172.2	142.9	120.2	115.4	109.3	....
Gal. cooling water per hp-hr.	42.5	19.5	13.3	11.8	10.8	10.3
B.t.u. per hp-hr	15,452	12,746	10,731	10,143	9,835	....
Thermal efficiency	....	19.97	23.13	23.59	25.89	25.89

NOTE: In another test, No. 7, the duration of which was 8 hr. and the loading variable, the average B.t.u. of the gas was 90.50, and the cooling water required per hp-hr. was 17.7 gal.

TABLE 8 GAS-ENGINE TEST

(38 by 54-in. engine, 60 r.p.m., 4-cycle, blast-furnace gas, constant-mixture type.)

Test No.	1	1	2	3	4
Duration, hr.	2½	2	3	1½	2½
R.p.m.	58	58	58	58	56.5
Inlet gas temperature.	92	92	86	90	90
Total hp.	1,205	1,177	1,155	1,193	1,531
Mechanical efficiency	79.1	81.4	72.5	72.4	77.6
B.t.u. of gas	(Standard)	89.3	92.0	88.1	88.2
Gas per hp-hr., cu. ft.	(Std. 62 & 30)	116.6	118.8	118.7	111.8
B.t.u. pr hp-hr	(Std. gas)	10,423	10,812	10,433	9,793
Thermal efficiency	.....	24.4	23.6	24.4	25.9

TABLE 9 GAS-ENGINE TEST

(21 by 30-in. engine, 160 r.p.m., blast-furnace gas, direct-connected to electric generator, constant-mixture type of governor.)

Test No.	1	2	3	4	5	6
Duration, hr.	5	5	5	5	5	2
Load per cent of rating	25	50	75	100	variable	no load
Avg. r.p.m.	162.5	161.3	158.9	155.4	160.7	169.7
Avg. maximum explosion pressure	160.5	207.4	283.8	309.6	178.7	88.0
Avg. m.e.p.	23.3	29.4	40.9	47.3	25.5	12.43
Avg. hp.	191.1	243.7	324.3	404.1	253.6	88.73
Avg. mech. efficiency	57.9	73.5	63.2	84.3	72.6	....
Avg. B.t.u. of gas	98.2	96.1	98.8	97.8	96.6	99.70
Cu. ft. gas per hp-hr	122.6	118.8	112.5	106.7	120.5	202.2
Gal. cooling water per hp-hr.	35.1	24.8	15.8	18.6	26.0	65.4
B.t.u. per hp-hr	12,057	11,421	11,116	10,439	11,655	20,148
Thermal efficiency	21.3	22.5	23.1	24.6	22.0	12.7

to secure accurate readings the point of average velocity must be found and this in many instances is hard to do.

34 Where pulsations enter into the question they doubly complicate it, and the author recalls his first experience with a proportional meter of rather large size that brought home the many

factors hinging on the final efficiency of the gas engine as reported by test.

35 In this particular case it was both surprising and pleasing to find a gas consumption of 6529 B.t.u. per b.hp., and as this represented about 39 per cent normal efficiency, it was naturally concluded that a new record had been established. But this seemed too good to be true, and in calibrating the meter it was discovered that it was entirely impossible to use one of the proportional type where pulsations were present. Another instance was where a consumption as low as 8000 B.t.u. per b.hp. had been reported and the test published, but upon examination of the installation later it was found that the gas measurements had been made by pitot tube and in locations in the line close to pipe fittings that would render the readings of the tube subject to wide variation. These instances are mentioned in order to show the importance of first knowing how the gas measurements are made before relying too strongly upon a seemingly exceptional result.

36 From these tests and tables it will be evident that the gas engine has a place in the steel industry that will increase in importance as the cost of fuel advances, thus automatically increasing the values of blast-furnace gas; for, unlike the by-product gas of the coke oven, it is limited in its range of application to hot-blast stoves, boilers, and gas engines. There is no reason why in plants where this gas supply is in excess of the needs of the stoves and boilers, that the surplus should not be used for the generation of electric power for sale to the public service companies, and at a figure much below that for which they can make it with any of the existing steam equipment at the prevailing price of coal.

DISCUSSION

Fredrik Ottesen.[1] The piston troubles referred to by the author are past history. We all know how to make good pistons. Cylinder troubles, however, still exist in several steel plants. Tables in the complete paper, omitted from this abridgment, show the various items making up the total cost of producing 1000 kw-hr. It is striking to observe the high maintenance cost of the gas-engine plants. The writer is inclined to believe that the plants referred to must have engines of an older date, because no such high maintenance cost is encountered in modern installations. He further believes that if we could analyze the item "Repairs and Maintenance" over this long period of time, we would find that the repairs and replacements of cylinders would

[1] Chief Engineer, Gas Eng. Dept., Mesta Machine Company, Pittsburgh, Pa. Mem. A.S.M.E.

show the heaviest expenditure. The cylinder is one of the most vital component parts of a gas engine and one of the most expensive parts to replace. Upon it depends to a great extent the continuity of service and the overall cost of repairs.

It is important, therefore, that, first of all, the cylinder be correctly designed; second, that the proper material and workmanship be applied in its manufacture; and third, that it is given the proper attention and care after being put in service.

While it is perhaps not within the scope and purpose of this paper to discuss detail design, yet a few remarks along this line may prove of value to those who are confronted with cylinder troubles. The cylinder must be so designed that it can be cast entirely free from internal stresses (casting stress). The water jackets or water spaces should be large and accessible, in order to afford an easy cleaning from time to time. The inlet and exhaust ports should be designed so as to possess a certain amount of flexibility, and carefully rounded off at the point of intersection with the main cylinder barrel. Finally, the cylinder must be designed with due consideration to expansion and contraction caused by temperature differences, and it must be as nearly symmetrical as possible in order to avoid undue localization of stresses.

In regard to the proper material for gas-engine cylinders, the largest cylinders today of 59 in. bore and 59 in. stroke are made of cast-iron. Personally the writer prefers a high grade of air-furnace iron, specially annealed. Such iron has a tensile strength of from 35,000 to 38,000 lb. per sq. in.

Many gas-engine-cylinder failures are directly chargeable to the operator. He may allow mud and scale-forming substances to build up in the bottom of the water jacket and around the exhaust port, causing "hot-spots" resulting in premature ignition. He may fail to keep the internal surfaces in the cylinder, such as the piston, cylinder heads, valves and igniter points, clean and free from carbon collection, which also causes premature ignition.

Prolonged premature ignition is fatal to all gas-engine cylinders, no matter how carefully the cylinder is designed and made. This phenomenon really warns the operator to start cleaning immediately. During the period of premature ignition, the explosion pressure in the cylinder rises suddenly from 40 to 50 per cent above the normal pressure for which the cylinder is designed, with a corresponding increase in temperature. The combination sets up an excessive stress in the cylinder walls, particularly around the valve ports, causing the surface fibers to yield and form a very small crack. If the premature ignition is allowed to continue, and this small crack is not properly chipped out, the crack will develop deeper and deeper until finally in a short time the entire section of the wall is ruptured, resulting in a replacement of the cylinder and interruption of operation.

Another cause for premature ignition and cylinder failures is the high compression pressure found in older engines. The tuyeres of a blast furnace are liable to leak more or less. Such leakage, particularly during the process of casting, causes the hydrogen content of the gas to rise suddenly above the normal. Hydrogen is a treacherous and snappy gas. When heated in an air mixture, it will not stand much compression without igniting. Therefore, if the high-compression engine happens to carry a heavy load during the period of high hydrogen, we may expect premature ignition, and particularly so if the engine draws gas from one furnace only. On the other hand, if the gas is supplied from several furnaces and a large gas holder arranged between the secondary washer and the engine, conditions are very much better, and the trouble may be entirely eliminated.

However, there is a growing tendency in Europe as well as in this country to reduce the compression pressure for the reason outlined above. The writer therefore wishes to correct the author's statement that the compression pressure of the earlier engines has changed but little, and that it is still held around 200 lb. per sq. in. The modern blast-furnace-gas engine runs with a compression pressure of 150 to 165 lb. per sq. in. The writer recommends that the clearance volume of engines of the old high-compression type be increased, thereby reducing the compression pressure in the cylinder. This will give a better and easier running engine, able to carry a little more load, provided the valve setting is right, and the repair bill will be materially reduced.

Summarizing, we have now reached the point where we know how to prolong the life of a gas-engine cylinder:

1 By the application of simplicity and correctness of design
2 By the use of proper material and skill of production
3 By reducing the compression pressure
4 By the proper care on the part of the operator.

In every gas-engine plant it should be an iron-clad rule to do certain things at regular intervals, such as cleaning cylinders, changing igniters, keying up, etc. The manager of the plant should provide the operator with the necessary help to do this work, which will pay a handsome return in the long run. The average life of a European gas-engine cylinder is considered to be 15 years. It should be the same in America.

W. Trinks.[1] The writer's remarks are to be taken not as a criticism of, but rather a supplement to, the author's paper.

If the "steel industry" means both that of the United States and foreign countries, then we should mention gas engines with enlarged clearance volume and with surcharging, used for the

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purpose of increasing the mean effective pressure without increasing the explosion pressure, and incidentally increasing the horsepower per unit weight or cost. We might also mention the use of waste-heat boilers for the utilization of the heat contained in the exhaust gases.

With regard to the corrosion of pistons and of piston rods, two phases of the problem should be mentioned. One of them is that the steel plants located on the Monongahela River, one after another, turn to liming or neutralizing the service water of the entire plant. While this method removes the acidity of the water, it does not eliminate internal corrosion of piston rods, because that corrosion is caused by entirely different conditions. The writer has found corrosion in piston rods of Westinghouse gas engines in places where there was not even a trace of acid in the water. The underlying cause is cavitation. Exactly the same cause produces corrosion in the runners of centrifugal feed pumps, in the propellers of certain ships, and in the runners of certain water turbines; the reason for the phenomenon is the temporary removal of pressure, with giving off of gases in the nascent state, in which state they are particularly active. Upon the return of the pressure the cavity is closed and a very high, localized pressure results, tending to shatter the metal.

In some of the earlier gas engines, which otherwise followed German practice, the designers omitted a pump for forcing the cooling water through the piston rods. This led to disastrous results. If a pump is supplied or if another source of high-pressure water exists, and if the discharge from the piston rods is throttled sufficiently to prevent the formation of a vacuum in the rod, the latter is always kept full of water, no gases are given off, and no impact occurs to shatter the metal. Even with a considerable amount of acidity, no corrosion results.

The author mentioned the fuel saving of the gas engine versus the high initial cost of that prime mover. The problem of the gas engine is an economic one, the same as all other engineering problems. In this connection the writer desires to point out what he considers a mistake in the accepted method of calculation of return on the investment in internal-combustion engines as compared to steam-power plants. The standard method of comparison figures with the present prices of coal and coke. But a gas engine lasts at least 20 years; in consequence, we should figure with the average price of fuel during that period.

Again, the general tendency of fuel prices is upward. In 1900 coal came down the Monongahela River in barges at 65 cents per ton. Today the price is between \$2.50 and \$3.00 per ton. In 24 years the price of coal has grown 300 per cent. In 10 years it would be almost doubled. For that reason, we should, in our financial calculations, figure with the present price of equipment, but with about twice the present price of coal or coke. If that

were done, there would be many more gas engines at work than are now being installed.

One more thought deserves mention in this connection. That is, the proper time at which to purchase gas-engine equipment. Putting the cost of gas-blowing engines in 1914 at 100 per cent, the cost in 1915 was 110 per cent. In 1918 it had risen to 180 per cent, while in 1919 it was 310 per cent. Today it is approximately 220 per cent. By judicious buying, the fixed charges on gas-engine equipment can be made to be less than those on turbo-generators or turbo-blowers bought at high tide. But the trouble is that the managers of our blast furnaces and steel plants, as a rule, show very little judgment in the buying of gas engines. As an example it may be mentioned that in 1914 it was very difficult to sell gas-electric or gas-blowing engines, because prices of pig iron and of steel were low and because business was slack. On the other hand, a considerable number of gas engines were bought and installed in 1917 and 1918 when prices were high.

If engineers will figure with twice the present coal prices, and if we can get our works managers to buy gas engines when the prices are low, blast furnaces and steel plants will soon be filled with gas engines. For, in spite of modern steam pressures and superheats, the turbo-blower still uses more than twice as much blast-furnace gas per unit of air delivered as the gas-blowing engine requires.

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ANALYSIS OF A MACHINE-SHOP PROBLEM ON A QUANTITY AND FINAL-ECONOMY BASIS

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Member of the Society

This paper discusses the nature of the analysis which should be followed in determining the most economical method of machining work, either in large or small quantities. It leaves out of consideration all items excepting those relating to the actual machining processes. Particular attention is called to the difference between the saving of labor cost and the ultimate economy. As an example some of the operations on an automobile connecting rod are taken.

THE best machine-shop methods are those which give the highest ultimate economy under the existing conditions. This statement of an obvious truth is made here at the beginning of the paper in order to call particular attention to the fact that it is the ultimate economy which counts and not the economy of a single operation, and that some method or methods may be good under one set of conditions and bad, or at least less desirable, under another.

2 To exemplify: What may be the most economical way of turning a piece when it must be ground is not necessarily the best method when the piece is to be finish-turned. Again, what is the best method of machining when a steady stream of work goes through the factory may not be the best method when the work is seasonal.

3 Many factors must be taken into consideration when the method of machining of a piece of work is to be decided on, even if the economy of a single operation only is being investigated. When the ultimate economy must be analyzed the problem becomes even more involved.

4 This paper deals with the problem of analyzing machine-shop operations with an eye on the ultimate economy. However, no

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factors will be considered which do not bear directly on the machining operations. Such items as handling facilities, routing systems, systems of wage payment, etc. will be ignored. They contribute to the ultimate economy of the shop but not to that of the machining methods.

Determining Unit Cost of a Single Operation

5 In order to approach the subject gradually, consideration will be given first to the simplest possible problem, namely, one operation only on only one kind of piece. It will be seen that even under these extremely simple conditions there are still several items that determine which method should be followed.

6 Let n = number of pieces produced per hour
w = wages per hour
N = number of hours per year during which this operation is performed
V = cost of machine, floor space, and permanent equipment
p = percentage charged for interest and depreciation
f = factor (action) denoting that part of the time the machine is usefully employed.

Then fnN is the number of pieces produced per year at a direct labor cost of wN, and pV is the overhead directly chargeable to the machine. The unit cost is then

$$\frac{pV+wN}{fnN}$$

7 This formula does not take all possible factors in consideration but lends itself well for a first approximation. It shows at once that unit cost is inversely proportional to n (the number of pieces per hour) and to f (the factor of employment of the machine). It shows also that there is no proportional relation with the other factors, namely p, V, w, and N.

8 As unit cost is proportionally connected to f and n, these factors may be omitted in further analysis. However, they must be considered when computing actual cost.

9 The formula now becomes: Unit cost is proportional to

$$\frac{pV}{N}+w$$

in which the first term is the machine cost and the second the wage cost. It should be noted here that the machine cost is figured for the number of hours the machine is operating and not for the number of hours it is standing in the shop. This is important when special machinery is introduced.

10 As an example the following figures are chosen:

$V=\$8000$ $N=2400$

$p=15$ per cent$=0.15$ $w=\$0.75$

$$\frac{pV}{N}+w=\$1.25$$

This \$1.25 is the cost of the operation done when the operator can produce n pieces per hour, and when he and the machine can be kept busy the fraction f of the total time. For instance, if the operator works at the rate of 100 pieces per hour but can keep busy only 80 per cent of the time, the unit cost will be $125/(0.80\times 100)=1.5625$ cents.

Additional Factors Entering into the Complete Formula for Unit Cost

11 Certain factors did not appear in the general formula and may be left out for a first approximation. These factors will now be considered.

12 *Tool Expense.* In the first place, there is the tool expense. This consists of two items, first cost and cost of maintenance — sharpening. Where a certain tool is used constantly for one operation only, these two items can be calculated, and should be if the tool or its maintenance is expensive. Where an operation can be performed in various ways and with different tools, the tool cost may well be the deciding element. However, in all cases this item of expense appears as an addition to the value expressed by the formula. The unit cost C_u is then

$$C_u=\frac{pV+wN}{fnN}+t_u$$

in which t_u is the unit tool cost. This unit tool cost is independent of the other factors appearing in the formula.

13 *Cost of Power.* Another item which might be considered is the cost of power. However, power is required whatever machine is used to remove the metal and is in almost all cases a small item. The actual power cost may be fairly great, but if so, it is so because large amounts of metal must be removed — and this is done by heavy machines for which the machine cost per hour is necessarily large.

14 As an illustration a planer may be considered which to remove 20 cu. in of metal per min. requires about 15 hp. This may be assumed to cost 30 cents per hour or \$720 per year of 2400 hours. A planer capable of doing this work may cost, say, \$10,000, so that the machine cost per year would be \$1500. If the nature of the work is such that no other machine but a planer can do it, this power cost will be unavoidable, but if the work is of such a nature that there will be a choice between planer

and milling machine, the difference in power cost may have to be considered. In such a case it may be found that the milling machine consumes 22½ hp. costing 45 cents per hour or \$1080 per year. On the other hand, the machine cost will be less, say, 15 per cent of \$5000=\$750.

15 Furthermore, the full power cost should not be figured in during the entire year. Though there are cases where it may be advisable to analyze cost along these lines, such cases are extremely rare in the actual manufacturing of articles in large numbers. If considered at all, it is to be borne in mind that the power cost is also independent of the factors appearing in the original formula and is merely added to the cost indicated by it.

16 *Set-Up Time.* Still another item is the set-up time. This item modifies the factor f. If this factor is 0.8 during the time the machine operates, then the actual factor to be used is less than 0.8 and becomes so by the fact that machine and operator were non-productively employed during the time required for setting-up. It will be seen, then, that f is variable and depends on the length of the job. This matter is of importance when the job is of short duration; it is of small importance for long jobs and vanishes when a machine is constantly employed on the same work.

17 For the rest, the factor f takes care of a number of items such as tool renewal, temporary absence of the worker, etc. In factories where each machine is assigned to a certain job, the factor f should be determined and should be carried on the record as a constant.

18 The value of the unit cost as expressed by the formula and its additions can be used to compare the economy of various methods of performing a single operation.

Comparison of the Economy of Various Methods of Performing a Single Operation

19 The examples of comparative analysis given in this paper all refer to machining operations on the automobile connecting rod shown in Fig. 1. This piece requires various operations of drilling, reaming or broaching, spot facing, chamfering, tapping, milling, and sawing. Later on combinations of some of these operations will be considered; for the present, however, a comparison will be made of various methods which may be employed for some of these operations singly.

20 As first example the simple operation of sawing the slot at the small end of the connecting rod will be taken. Three cases will be considered: namely, (*a*) when the quantity is 100, (*b*) when it is 10,000, and (*c*) when it is unlimited.

21 In the first case a machine already available must be selected, the quantity being too small to justify consideration of a machine more especially adapted to the work. This is always

true for small quantities, or rather for jobs of short duration. In this case almost any milling machine will do. The piece can be held in a vise with the small end resting against a stop clamped to the table. This brings the head in the proper place and at the proper angle. As the plane of the saw is parallel with the direction of table feed, the opening of the vise must be at right angles to this direction. So placed, it is possible to use either the table feed or the vertical feed.

22 In all milling operations the cutter should be as small in diameter as possible, consistent with the requirements of the work

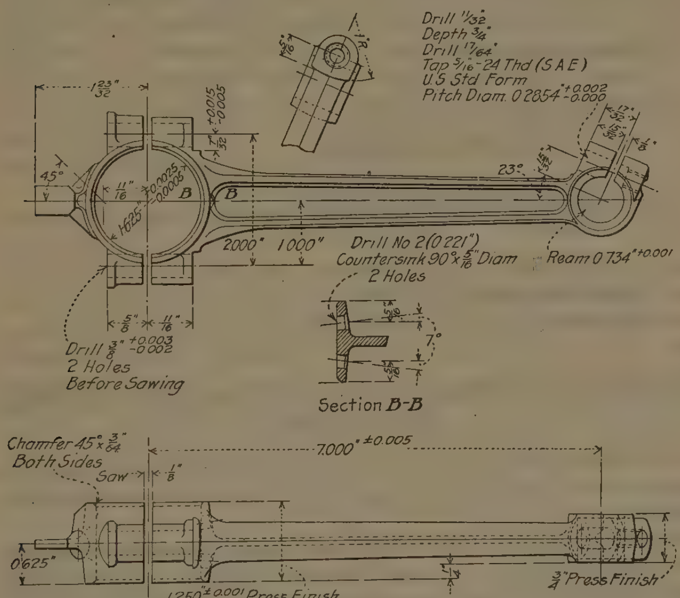


FIG. 1 DROP-FORGED AUTOMOBILE CONNECTING ROD, THE MACHINING OPERATIONS ON WHICH ARE ANALYZED

and with proper size of arbor. Assuming the arbor to be 1 in. in diameter or about $1\frac{1}{2}$ in. over the arbor collars, the size of the cutter must be $1\frac{1}{2}$ in. $+ 2 \times$ depth of cut $= 1\ 11/16$ in. To allow for the proper amount of penetration when the vertical feed is used, and for the fact that a standard-size cutter is desirable, a 2-in. cutter will be selected. The question remains now which feed to select.

23 Using the table feed requires a cutter traverse of $\frac{3}{4}$ in. plus the approach, which in this case is about $\frac{1}{2}$ in. The total length to be milled is therefore $1\frac{1}{4}$ in. If the vertical feed is used the length to be milled is equal to the depth of the slot plus the offset in a 2-in. circle when the chord is $\frac{3}{4}$ in. This is less than 1/16 in.,

so that the distance to be traversed by the cutter is 19/32 in. or, say, $\frac{5}{8}$ in. We therefore select the vertical feed.

24 Ignoring tool cost and power cost, the method of calculating is as follows (certain figures being assumed for obvious reasons):

Speed of cut...............	70 ft. per min. (135 r.p.m.)
Feed per revolution.........	0.030 in.
Feed per minute............	4.05 in.
Time for cut...............	10 sec.
Time for chucking..........	15 sec.
Time for returning table.....	5 sec.
	30 sec.
Time for 100 pieces.........	50 min.
Time for set-up............	25 min.
Total time	75 min.
Value of machines and floor space	$2000

pV = $300 per year = $0.125 per hr.
w = wages = 0.75 per hr.

Total per hr. = $0.875

Total cost of job ... $1.095

Unit cost .... 1.095 cents

Modification of Method Described Where Large Quantities are Involved

25 The question now is, to what extent, if at all, should this method be modified if the quantity is 10,000 (in one lot and no other lots expected).

26 The total time for milling and set-up would be $(100 \times 50) + 25 = 5025$ min. As the cost per hour would be again $87\frac{1}{2}$ cents, the total cost of this job would be $73.28. This amount is not enough to try to reduce it by a special fixture, for, whatever means of chucking may be provided, there will always remain the milling and set-up times which will amount to more than one-third of the total time, so that less than two-thirds of $73.28 can be saved.

27 If, however, the job should be recurrent, say, once a year, a special fixture might be considered provided there were a fair certainty that the job would last a number of years.

28 An entirely different problem is presented when the quantity is unlimited or practically so. In this case savings are calculated not on a given quantity but on a year's production.

29 The productive part of the operation is the sawing. Chucking and manipulation of the machine are necessary but not productive as they interrupt the essential part of the operation, the cutting. As the cutting time is 10 sec. and the total time per

piece 30 sec., it follows that the production could be tripled if the non-productive elements could be eliminated or if they could coincide with the cutting. This means that one piece must be cut while another is being chucked. The most perfect arrangement for obtaining this result is one by means of which the operation becomes continuous, such, for instance, as a rotating fixture, and any such device reaches its highest possible efficiency only when both operator and machine work all of the time. This should not be construed to mean that there shall be no rest for the operator or that he shall be driven, but that the work of man and machine shall harmonize. If the man must wait to allow the machine to finish its task, or if the machine must be slowed down to allow the man to perform his part of the work, then the highest possible efficiency of that combination has not been attained.

30 Testing the case under consideration by this principle and assuming the amount of time for chucking and cutting to be correct as given before, it is found that:

31 In order to cut continuously the feed must be in the direction of the axis of the small hole and not toward the center as heretofore.

32 As it is probably not possible to arrange the fixture so that the small ends of the connecting rods shall touch each other, the length of feed required per piece will be more than $\frac{5}{8}$ in. Assuming that the distance between two adjacent pieces must be $\frac{5}{8}$ in., the length of feed per piece becomes $1\frac{1}{4}$ in., so that now the time for cutting becomes 20 sec. instead of 10. This shows that there has been a loss in both elements of the operation. The cutting time has been increased from 10 sec. to 20 sec. and the chucking time from 15 sec. to 20 sec.

33 The shape of the fixture will be that of a hollow cone, with the pieces of work chucked on the inside, so that that part of the connecting rod on which the work is done shall lie at the small diameter of the cone. The fact that the other end of the connecting rod is wider necessitates this arrangement. If the cut were taken on the large end this condition would have to be reversed, or else the fixture might take the form of a solid cone with the work chucked on the outside.

34 With such a fixture it will be possible to double up, using two hollow cones joined at their small diameter.

35 As a large part of the fixture is relatively far removed from the cutter, it will be possible to chuck two pieces with one manipulation of the clamping bolts, so that it should be possible to chuck two pieces in 20 sec. instead of one piece in 15 sec.

36 The following conditions now obtain: Two pieces are chucked in 20 sec. and two pieces are cut in 20 sec. by two cutters on one arbor — there is no idle time. As a result *one* piece will now be finished in 10 sec. instead of 30 sec. as before.

37 Under the first plan presented, that is, when a lot of 100 pieces had to be milled, the cost per hour was $87\frac{1}{2}$ cents. An entire year's production at that rate would have cost $2400 \times 87\frac{1}{2} = \$2100$.

38 Apart from the cost due to special devices introduced in the last plan, the cost of a year's production will be again \$2100 but the value of the product will be three times as much or \$6300. It should be noted here that the term "value" does not mean the price charged to the public, but what one department of the shop would charge another.

39 The additional value obtained permits the purchase of a certain amount of equipment needed for the new method. The amount itself is limited by a number of items, such as the factor f, the percentage p, and above all by the probable number of years the special equipment will be used. When this number is small the percentage p will be large, yet it cannot be said that p depends upon the number. This is only so when the percentage of depreciation has been increased beyond the normal.

40 The factor f gets new significance when special equipment or devices are used. When nothing but standard equipment is used, delays for repairs, etc. do not necessarily stop production as there is generally a duplicate tool or machine at hand. This is not so with special equipment, and an allowance expressed by this factor f should be made to cover contingencies.

Calculating the Amount That Can Be Spent for Special Equipment for Performing Single Operations

41 Considering these various items a formula can be constructed which will show the amount of money which may be spent for special equipment for the performance of *single* operations. Let

v = value of work in one year by old method
v_1 = value of work in one year by new method if production is uninterrupted
t = estimated number of years
$fv_1 - v$ = gain per year
P = amount to be spent for special equipment
p_1 = percentage of interest charge

Then $tp_1 \times P + P$ is total amount spent at the end of t years, and this amount should be less than the amount of gain in that period.

$$tp_1 \times P + P < t(fv_1 - v)$$
$$P \times (tp_1 + 1) < t(fv_1 - v)$$

or

$$P < \frac{t(fv_1 - v)}{tp_1 + 1}$$

This formula applies when the equipment has no appreciable scrap value and cannot be used for some other standard operations.

42 If the estimated number of years of usefulness is so large that the regular factor of depreciation can be figured, then the formula becomes

$$pP < fv - v$$

or

$$P < \frac{fv - v}{p}$$

43 In the foregoing example the chucking time was assumed to be greater than the cutting time. Conditions are somewhat modified when the reverse is true. As an example, the drilling and spot facing of the two bolt holes will be considered. Various elements will be arbitrarily assumed as in the previous example.

Size of hole.......................................$\frac{3}{8}$ in.
Drill speed..............70 ft. per min. (1130 r.p.m.)
(For the sake of simplicity it is further assumed that a drill press with this speed is available.)
Feed per revolution..............................0.006 in.
Feed per minute.................................6.78 in.
Distance to drill..................................$1\frac{5}{8}$ in.
Time for drilling..................................15 sec.
Time for chucking.................................5 sec.
(The chucking time is assumed for a simple fixture, consisting of an angle plate, a stud for the large hole, a stud slightly flattened top and bottom for the small hole, and a single-jointed lever with cam hook for clamping.)

44 If a single drill were used the time for drilling two holes would be 15+15+5=35 sec.

45 If the cost of machine and floor space is $1000, the wage per hour 75 cents, the time for set-up 25 min., and the quantity 100, then the cost per piece is calculated as follows:

Machine cost per hour........................... $0.0625
Wage cost per hour.............................. 0.75

Total cost per hour............................ $0.8125
Time for drilling 100 pieces..................3500 sec.
Time for set-up...............................1500 sec.
Total time=5000 sec.=$83\frac{1}{3}$ min.
Total cost=$1.13
C_u= 1.13 cents

46 Leaving for the present the spot facing and chamfering out of the problem, it may be well to see whether and when a two-spindle drill head will be economical. Assuming the cost of this

device to be $100, the question is, how many holes must be drillled before this cost is absorbed.

47 On every piece a gain will be made of 15 sec., the money value of which is $\frac{15}{3600} \times 81\frac{1}{4}$ cents. The number of pieces to be operated on to absorb $100 is therefore

$$\frac{100 \times 100}{81\frac{1}{4} \times \frac{15}{3600}} = 30{,}000$$

48 If the two-spindle head is adjustable so that it can be used for other work as well, and if such work is available, it will be sufficient to absorb a proportionate part of the $100, and if such work is of regular recurrence so that the device becomes a part of regular shop equipment, a proportionate part of 15 per cent of $100 only must be absorbed.

49 In addition to drilling there is also spot facing and chamfering. The most common arrangement for performing such operations is a gang drill in which one spindle is used for drilling, one for spot facing, and one for chamfering.

50 Assuming the time required for spot facing to be 8 sec. and for chamfering 2 sec., with 2 sec. for each movement from spindle to spindle, the total time for the combined operation is found thus:

Chuck	5 sec.
Drill	15 "
Drill	15 "
Move to 2nd spindle	2 "
Spot face	8 "
Move to 3d spindle	2 "
Chamfering	2 "
Move to 1st spindle	2 "
Total	51 sec.

51 The time for set-up also is increased, say, to 40 min., making the total time for 100 pieces

$$5100 + 2400 = 7500 \text{ sec.} = 125 \text{ min.}$$

The cost per 100 pieces = $1.69, and

$$C_u = 1.69 \text{ cents}$$

52 It remains to be investigated what can be done if practically unlimited quantities are to be worked up, and it should be noted that single operations are no longer being dealt with but with a combination of operations, though of the simplest kind.

53 The longest cutting operation is 15 sec. and it would be a simple matter to arrange a number of spindles so that all operations were performed simultaneously in that time. This would require a multiple-spindle drill with six active spindles, two for drilling, two for spot facing, and two for chamfering. There would be four fixtures, three of them under the spindles and one at the loading position. This arrangement involves an indexing table, which may be operated by hand. If the operation of indexing requires 2 sec., then a piece will be completed in 17 sec. instead of in 51 sec.

54 However, this arrangement does not give the highest possible economy, for the machine is busy 15 sec. and the operator only 5 (in addition to the indexing). To keep the operator busy it will be necessary to have him load 3 pieces while the machine drills. This would mean that 18 spindles and 12 fixtures are required, while the table indexes to 4 positions. Under these conditions three pieces will be finished in $15+2=17$ sec.

55 Under the conditions assumed for the small lot the cost of the operation was $81\frac{1}{4}$ cents per hour, or \$1950 per year. The output was one piece every 51 sec. while now it is three pieces every 17 sec., so that the output is nine times as large and therefore the value of a year's output has been increased by $8\times$\$1950 =\$15,600. If there is a fair degree of certainty that the article will be manufactured for two years then the total gain is \$31,200, and any amount below this sum would be justified for special equipment. However, here again the factor f should be considered, and furthermore it should be kept in mind that there should be a large margin of profit in sight before such special equipment is purchased. On the other hand, such equipment as has been indicated here, is not necessarily strictly special; it may be standard — though not of general utility — so far as the machine is concerned and special in regard to fixtures. It should further be kept in mind that the output of such a plant would be very large indeed, and that it might not be kept fully occupied all the time.

56 With $f=0.80$ the output would be

$$\frac{2400\times 3600\times 3}{17}\times\frac{80}{100}=1,219,800 \text{ per year}$$

The piece under consideration would lead to many other similar problems and analyses. The two examples given are probably sufficient to indicate the manner in which the various factors which go to make for economy or lack of it should be viewed if one operation only is considered. Before proceeding with the study of the ultimate economy of the entire piece, the author wishes to call attention to the fact that no attempt has been made to exhaust all possible ways of accomplishing the results, and further, that the figures for feed, speed, chucking, time, etc. were assumed. Such items should be determined by actual observation and by

analysis. The author merely attempted to show how factors, once determined, may be *coördinated* and combined.

Analyzing the Cost of Combined Operations

57 The connecting rod shown in Fig. 1 is a drop forging. The rod and cap are forged in one piece, the sawing apart being one of the last operations. There are two holes to be produced with a rather high degree of accuracy, and there are further a number of minor operations such as drilling, tapping, facing, milling, and sawing.

58 There are two possible ways in which the cost of machinery may be reduced: by reducing the cost of the individual operations, and by combining operations. It is a comparatively simple matter to compare various methods and their costs when single operations are being analyzed. Quantity to be made, equipment on hand, and cost of equipment to be bought are controlling elements, taking it for granted that the necessary skill required for any method under consideration is available.

59 To analyze the effect of various combinations is a more complicated problem. At first glance it would appear as if an almost unlimited variety of combinations are possible, but a little study soon shows this not to be the case. Some combinations are not possible, others are clearly not practicable, and still others are so evident that they are hardly considered as combinations at all. If, for instance, a number of holes must be drilled in one surface of the work, it is so natural to do this on a multiple-spindle drill, or with a multiple-spindle drill head, if one is available, that it is hardly considered as a combination of individual operations. Similarly, if two parallel surfaces must be milled, it is but natural to use straddle mills, and this is not considered as a combination.

60 Operations may be combined for various reasons such as:

a To secure greater accuracy
b To avoid handling
c For the sake of economy.

Whatever is necessary to secure the required accuracy must be done, regardless of economy, so that this phase of the matter does not need to be considered. As to the second item, this is really a case of trying to economize. Overhead is saved sometimes even at the cost of direct labor. Where there is a saving in plain sight such combinations are resorted to, but such unmistakable cases are rare. It is generally somewhat difficult to estimate the cost of handling with even a fair degree of accuracy, while labor costs appear in plain figures. The third case, which really includes the second, is the problem with which this paper deals.

61 No general rule or formula can be given for guidance in an analysis of this kind, but it is possible to find the minimum labor cost if conditions are such that the ideal economy can be reached. In other words, it is always easy to establish an ideal to which we can work and from which we will depart only in so far as practical considerations compel us. Such practical considerations may be cost of equipment or perhaps the impossibility of combining certain operations, such, for instance, as planing and boring.

62 If the various operations to be done are denoted by 1, 2, 3, etc., and the times required are A_1, A_2, A_3, respectively, then the greatest possible economy will be reached if all of these operations are performed at the same time, and the time required will be A_m (that of the longest of all the individual operations).

63 It should be kept in mind that this refers to the labor cost only. The overhead incident to such a combination may, and often does, wipe out the saving in labor cost. Besides, the cases where it is practical to combine all of the operations are few and far apart. Nevertheless this ideal should be kept in view, and the proper way is to ascertain to what extent it can be realized.

64 In the case under consideration we have the following individual operations:

Bore large hole
Ream large hole
Drill small hole
Ream small hole
Drill 2 bolt holes (really 2 operations)
Ream 2 bolt holes (really 2 operations)
Face top and bottom 2 bolt holes (really 4 operations)
Mill 2 sides of splasher (really 2 operations)
Drill 3 oil holes (really 3 operations)
Drill for binder bolt
Drill for tapped hole
Tap hole for binder bolt
Face hole for binder bolt
Saw small end
Countersink both sides large end (really 2 operations)
Countersink 3 oil holes (really 3 operations)
Saw cap from body.

There are then altogether 26 operations. It is assumed here that the two main holes are reamed, though they are often broached. This assumption is made so as to avoid unnecessary complications in the problem.

65 It is quite evident that the longest individual operation is either the drilling of the large hole or else the sawing of the cap from the body. A detail analysis of these two operations would show that the drilling requires approximately 100 sec. while the milling can be done in about 30 sec.

The Ideal Scheme, Its Advantages and Disadvantages

66 The duration of the longest operation is now established as 100 sec. The ideal scheme, therefore, is the one which provides for the performance of all of the operations in this length of time.

67 Before making a study of the conditions which may prevent us from reaching this ideal, the advantages and disadvantages of such a scheme should be considered in a general way; and the engineer or executive considering such a scheme should keep in mind that while a single disadvantage may, at times, neutralize all of the advantages, on the other hand, disadvantages, like troubles, are things which either exist in our minds only or else can be overcome by them.

68 The advantages are:

Minimum labor cost
Minimum floor space
Minimum stock and parts in process
Minimum handling
Minimum inspection
Minimum losses on account of faulty operations
Maximum accuracy
Maximum assurance of proper relations of the various elements.

69 The main disadvantages are:

The fact that special machinery is required
The cost of equipment
The low value of equipment in case of disposal
The uncertainty of the life of the piece
The fact that breakage of one tool stops all production
The fact that stand-by machinery is required
The fact that the machinery requires attention of a high order, and that the factor f is necessarily low.

70 The connecting rod forming the subject of this analysis lends itself well to the combining of practically all operations — that is, from a purely technical standpoint. A machine can be readily imagined which would perform all the operations simultaneously. This fact reduces to two the three items which as a rule should be considered: namely,

The technical possibility
The technical practicability
The economical practicability.

71 When it is stated that all operations can be done simultaneously, it is not meant that all the tools shall work at the same time on one piece, but that a station type of machine could be constructed which would permit of performing all, or almost all, of the operations with one chucking. Such machines are usually built with a turret. A certain number of operations are performed at every position of the turret but one — the loading

and unloading position. As a rule, the design of such a machine offers fewer difficulties than that of a standard commercial tool, first, because the machine does not need to take in a wide range of shapes and sizes but needs adjustments only for wear of spindles, slides, and tool, and secondly, because they do not have to compete on price.

72 On the other hand, the building of such a machine requires the highest possible accuracy if the piece of work shall be reasonably accurate, and as this latter point is not always thoroughly understood, an example will be given here to show why this great accuracy is essential in such a turret type of machine.

73 The turret must index with great precision, and the chucks or work-holding devices in general must all be located in the same relation to the center of the turret and to the index slot.

74 As an illustration, it will be assumed that 4 holes must be drilled in a piece of work and that this is done by 4 spindles located opposite the 4 work stations of the turret, the total number of stations being 5. Of course, such an arrangement would not be chosen for the simple task of drilling 4 holes, but this assumption simplifies the problem and makes it easier to visualize. The 5 stations will be called *A, B, C, D,* and *E.* Each one of the index notches varies somewhat from its ideal position, in other words, the notches are not exactly 72 deg. apart, but some intervals are larger and some smaller. The errors will be indicated by a_1, b_1, c_1, d_1, and e_1. The chucks are also more or less wrongly located in relation to the index notches and the centers. These errors will be called

$$\begin{array}{ccccc} a_2 & b_2 & c_2 & d_2 \text{ and } & e_2 \\ a_3 & b_3 & c_3 & d_3 & e_3 \end{array}$$

in which the upper index denotes the amount of error relative to the index notch and the lower one that relative to the center. The spindles also have two errors which are indicated here by

$$\begin{array}{cccc} b_4 & c_4 & d_4 \text{ and } & e_4 \\ b_5 & c_5 & d_5 & e_5 \end{array}$$

The holes to be made are indicated by *P, Q, R,* and *S.*

75 The first piece, chucked at *A,* will have its first hole drilled at *B,* its second at *C,* etc. The second piece, which will be chucked at *E,* will have its first hole drilled at *A,* the second at *B,* etc. The third piece will be chucked at *D* and will have its first hole drilled at *E,* the second at *A,* etc. Each hole will be in error the algebraic sum of the errors of index notch, chuck position, and spindle position, so that the errors for the four holes of the first piece can be expressed thus

$$\begin{array}{cccc} b_1+b_2+b_4 & c_1+c_2+c_4 & d_1+d_2+d_4 \text{ and } & e_1+e_2+e_4 \\ b_3+b_5 & c_3+c_5 & d_3+d_5 & e_3+e_5 \end{array}$$

The second piece will have the following errors:

$$c_1+c_2+c_4 \quad d_1+d_2+d_4 \quad e_1+e_2+e_4 \text{ and } a_1+a_2+a_4$$
$$c_3+c_5 \quad d_3+d_5 \quad e_3+e_5 \quad a_3+a_5 \text{ etc.}$$

76 This shows that not only are the errors of the machines duplicated in the piece, but that the relative position of the holes does not remain constant. The author has found it necessary to employ special methods in the design and construction of such machines to secure a degree of accuracy of a higher order than that required for ordinary machine tools. It is this need for unusual accuracy which causes machines of this kind to be so expensive. However, designers have often made the mistake of ignoring the special requirements of individual cases. In the present instance no great accuracy is required anywhere, and even those dimensions which do not permit of a large variation are between points which can be located in one station, so that there is no fear of undue accumulation of errors.

77 The piece would be clamped on the circumference of the turret. In the first position the two main holes would be drilled, both sides of the large hole would be chamfered, and the tops and bottoms of the bolt-hole lugs would be straddle milled. In the second position the bolt holes and the binder-screw hole would be drilled and the bolt holes chamfered. In the third position all oil holes would be drilled and the binder-screw hole tapped. The fourth position would take care of the reaming of the two main holes, and the fifth position of the sawing for the binder screw and the cap, and straddle milling the splasher.

78 It should be emphasized here that this is not the only possible arrangement but merely one which is feasible and which is given for the sake of focusing on a definite method.

79 The next step is to consider this arrangement from the standpoint of practicability. It is seen at once that to separate the cap from the body in this machine materially reduces the chances for a solid support of the head against the drill. This appears to be of enough importance to warrant dropping this operation.

80 Countersinking the back of the head also seems to be doubtful. However, a closer examination shows that this operation can be included without danger of affecting the success of the other operations.

Economic Soundness of Method Proposed

81 The main question remains yet to be answered: namely, whether such a method would be economically sound. Before going deeply into this matter it should be remarked that a machine such as was imagined would be capable of a certain amount of adjustment so that changes in the dimensions and even in the

distances of the holes would not make it useless. The construction of the connecting rod would have to be changed in its essentials before the machine would become obsolete. This consideration affects the estimate of the useful life of the machine.

82 Assuming that a first machine costs \$25,000, that a second or stand-by machine costs \$15,000, and that the useful life of these machines is five years, the following is found;

Cost of machines	\$40,000
5 years' interest	12,000
Total	\$52,000
Cost per year	\$10,400

Estimating that there are 2400 working hours in a year, the production will be $2400 \times 3600/100 = 86{,}400$ pieces, because it takes 100 sec. per operation. This shows that the machine cost per piece will be a little over 12 cents.

83 If the quantity to be produced is sufficiently large, this machine cost per piece may be reduced in various ways. For instance, double sets of tools and chucks may be provided so that the operator chucks two pieces instead of one; or that part of the plant may be run on the two- or three-shift plan, or else the longest operation, the drilling of the main hole, may be split.

84 The first method is not desirable, partly because it makes the machine too large and complicated, and partly because some of the duplicate tools will be in each other's way. The second method cannot be seriously considered except when other parts of the plant must be run in the same way. The third plan is entirely practical, adds only one station, so that the size of the machine is not much increased, and actually simplifies matters by distributing the tools more evenly. If this third method is followed the machine cost per piece becomes 6 cents.

85 It may be noticed that the machine was supposed to work all the time, no allowance having been made for tool setting and adjustment. This is proper where there is a stand-by machine, as one machine can be in production while the other is being made ready. A small percentage may have to be deducted for absence of the operator, and this may be met by considering the machine cost to be $6\frac{1}{2}$ cents.

86 Single operations performed according to modern methods, using multiple-spindle drill presses and turntables — in short, methods which compare very well with those used in the most productive shops — bring the total time for these operations up to 333 sec. per rod. The single machine therefore saves 233 sec. per piece or not quite 4 min. Estimating the labor cost at 75 cents per hour, the saving in labor cost would be 5 cents per piece, which is less than the machine cost. Against this the machine cost for the single operations must be figured.

87 By splitting the drilling operation and thus reducing the time per piece to 50 sec. the saving per piece becomes practically 6 cents, so that even in this case no profit is shown.

88 If the production requires a piece to be finished every 50 sec. and it takes 333 sec. to finish one by single operations, then there must be more than one set of machines in use; not all the machines, however, would have to be duplicated. This brings the machine cost for the single operations somewhat higher, but not high enough to justify the building of special machinery with its chance of failure and its almost certain period of development.

89 If the quantities required are larger, and especially if the life of the piece is certain to be a great many years, another story would be told.

90 Notwithstanding the fact that the actual machinery cost may not be reduced, there are cases where such special machinery is desirable for the reasons given heretofore and also because present conditions make any method desirable which substitutes machines for men.

91 That there is little or no final economy in a complete combination of all operations does not prove that there can be no economy in the combination of some of them. In the case here considered it would seem, at a first glance, that there should be some advantage in the simultaneous drilling (and also reaming, if reaming is done at all) of the two main holes. In fact, the comparison made above was between the complete machine and single operations, some of which had already been combined.

92 If every operation had been done singly, the final result would have been much more in favor of the turret machine. However, before deciding that such a machine should be built, it would have been necessary to consider various partial combinations and the final result would have been the same.

93 An analysis of this kind may appear to be a slow and tedious bit of work, but when a factory must give up a considerable part of its floor space and a number of machines and tools for the manufacture of a single piece, and in addition provide a number of men, the management is not justified in selecting a mode of procedure unless it has tested out, at least on paper, all possible combinations. A few days are well spent on such a task. That various factories employ radically different methods to obtain the same result is prima facie, if not conclusive, evidence that such an analysis is not always made and that so-called " judgment ", which might more properly be called guesswork, has taken its place.

Interest on Savings Should be Considered in Figuring Costs

94 In estimating the cost of the special machine first cost and interest were both figured, and this is proper, because if the machine had not been built, the fund used for the machine would

have been bearing interest through all the years of the life of the piece. On the other hand if we consider the savings made by the use of such a machine as capital returned to the business, this capital also would be bearing interest.

95 It was found in an earlier paragraph that the cost per year of the special machines was $10,400, and that the machine cost per piece was 6 cents. It was also found that the saving in labor cost per piece was 6 cents, so that the saving per year was also $10,400. This profit is made annually for five years, so that the first year's saving bears interest for 4 years, the second year's saving for 3 years, etc. — altogether the equivalent of 10 years' interest on one year's saving. Assuming, as before, an interest rate of 6 per cent, this would amount to $6240.

96 This item of interest on the savings is often overlooked. In this case the item is not large enough to change the conclusion reached before, but where the life of the piece is long the item becomes of considerable importance.

97 As an example, a case will be assumed where it is estimated that special machinery costing $1000 will produce savings of $2000 per year; and where the life of the piece is estimated to be 10 years, without considering the interest on the savings, we would find the following conditions:

First cost	$10,000
10 years' interest at 6 per cent	6,000
Total cost at end of 10 years	$16,000
Saving per year	$2,000
Total savings at end of 10 years	$20,000
Profit	$4,000

98 This is too small a profit to justify the purchase of the special machine. However, the balance sheet looks different when the interest on savings also is considered, as it should be.

99 The total interest on the savings is the interest for 1 year plus that for 2 years, etc., up to the interest for 9 years. This total equals the interest on $2000 for 45 years and amounts to $5400, so that the total profit is not $4000 but $9400, and this may well be enough to justify the purchase of special equipment.

DISCUSSION

E. J. Bryant.[1] This paper points out the necessity of a very careful analysis of all of the elements entering into the cost of special apparatus before deciding to make an investment, particularly depreciation and interest on investment, which are frequently

[1] Manager, Gage Department, Greenfield Tap & Die Corpn., Greenfield, Mass. Assoc-Mem. A.S.M.E.

overlooked. On the other hand, there are several factors in favor of special equipment, such as the reduction of the amount of the supervision and incidental loss of time due to looking after the several individual operations, and frequently the supervision of a larger number of employees with the necessity of breaking in and training new men to perform the required work. There is also the usual gain in the accuracy of the parts produced.

It would seem that in the case cited by the author one of the fundamental principles in automatic-machine designing had not been followed out, that is, dividing the operation in such a way that the time consumed at each station would be substantially the same. In this instance without doubt it would not be practical to split the milling operation, which is done in 30 seconds, but evidently the other operations could be factored so that the output of the machine could have been figured on a 30-seconds basis instead of on a 100-seconds basis. Undoubtedly this would have added somewhat to the cost of the machine, but on the other hand the increase in production by such a high percentage would probably more than offset the increase in cost necessary.

The writer realizes that the principle involved in cost analysis is what the author is driving at, and of course the number of parts that are ultimately required from the machine is really the controlling factor. It is useless to figure on designing a machine to turn out 100,000 parts in five years if the largest probable number required would be only 10,000. Many jobs that look as though they could be produced at a material reduction by utilizing special equipment will not show a profit on a money investment when a complete analysis of the number of parts required in a given period, which is likely to be the life of the product in question, is made.

John Younger.[1] The most valuable part of the author's paper is that wherein he emphasizes the prime importance of profits, whether the machine employed be standard or special. The vital question is whether or not the machine is profitable and if this can be answered then the nature of the machine can be determined.

One point that should not be lost sight of is that if the machine reduces the time of manufacture it may also reduce the inventory of work of process, and this may be a highly important factor in the final establishment of profits.

Machine troubles may be expected to occur, and like all inevitable matters, they should be provided for.

The interruption of a machine is not profitable but is not a serious matter if care is taken to make the interruption as short as possible.

[1] Editor and Publisher, *Automotive Abstracts*, Cleveland, Ohio. Mem. A.S.M.E.

Ralph E. Flanders.[1] The author has touched incidentally on one point that is important, i.e., changes in design of product. The hypothetical highly specialized machine should be so designed that it is not tied up to a given set of distances between the holes in the particular part under discussion, an automobile connecting rod, but should be built to take care of all pieces of the general design and construction of this part. Otherwise, special machines, designed for a particular part, may become entirely useless and a large investment lost if the design of the part is changed. While the author touched but lightly on this point, it is a highly important one in the design of special machinery.

For the making of parts three kinds of shop equipment are available: (*a*) The standard machine, for comparatively simple operations, and a large subdivision of operations, dividing the cost into a large number of sections; (*b*) the completely special machine, described by the author, doing the maximum possible amount of work at one setting; and (*c*) an intermediate type of standard machine, essentially a framework so designed that highly specialized tooling may be placed on it for more or less complicated operations. Such machines are used in the automobile industry. They provide for machining a range of pieces, and yet lend themselves to specialized tooling that converts them into special machines for any particular piece. This latter type is a good solution of the problem of standard or special machines in many cases. A special machine, if there is any difficulty with it, as usually occurs with new designs, may break down and seriously interrupt production, and is incapable of progressive development or improvement in use. Highly specialized tooling, on the other hand, may be developed and improved in the midst of production without interrupting it. A proper compromise may often be reached by using a standard machine, developed with the idea of highly specialized tooling.

Certain classes of work presuppose the necessity of an accurate jig, say, for boring a number of holes. The jig may be used in one of two ways. It may be fastened to the bed of a high-grade boring machine, in which the spindle is shifted opposite the holes in the jig, boring the holes one after the other. Or, the accuracy of the jig may be recognized, and the necessity for an accurate machine to go with it avoided. All that is necessary in the way of a machine is means for turning the boring bar, of requisite strength and a fair degree of steadiness in the drive. This means a much less expensive mechanism than a high-grade boring machine. Carrying this idea further, if a cheap, simple mechanism is to be built, why stop at driving one boring bar? Why not drive five or six all at the same time? The result will be, that if a special

[1] Manager, Jones & Lamson Machine Co., Springfield, Vt. Mem. A.S.M.E.

boring machine is designed for the jig, and the machine and jig made one mechanism, the total cost will be much less than if a jig and a standard machine were used. Furthermore, the special machine will get out the product in from one-third to one-fifth of the time required by the standard machine, depending on the number of spindles. Such a machine, due to the comparatively low investment in it, can be allowed to stand idle whatever portion of the time may be necessary without serious loss. A large part of the investment will be in the jig which would stand idle in any event when not in use.

One other point that may be mentioned is the possibility of using one of two types of complicated automatic machines. The turret type is one example; the other the writer has seen in but one place, viz., the Waltham Watch Works. For making the various parts of watches — the watch plate, for example — a series of ten to fifteen machines, each to perform a single operation, were arranged along a bench. Transfer arms moved the work from one machine to the next, and reservoirs between the machines permitted stopping any one machine for adjustment, changing tools, etc., without shutting down the whole line.

The turret or station type of machine is exemplified by the mechanism developed at the Elgin Watch Works for doing the same work as described above. All the operations are grouped on one machine, the parts moving around from one station to another, a single operation being performed at each station. The machine is large and expensive and quite complicated. Although the total investment in the Waltham arrangement was much smaller, the writer has been told that this company has gone over to the turret type of machine.

J. Carlton Ward, Jr.[1] In regard to the single-boring jig or the multi-bar jig as mentioned by Mr. Flanders, the question to be decided is again a question of cost and profits. There is a field for both types, and the type to be adopted depends on the quantity of work and on the liability to change in its design. If there is a sufficient quantity of work to permit the additional cost of the multi-bar jig to be divided over a large number of pieces, there is a field for this type. The type of machine required for driving will be the same in either case. If the jig is properly made, the bar can be driven by means of a universal joint in a machine, the accuracy of which is not important. The question of probability of change in design as indicating the type of jig to be used will be decided on the basis of volume of work. A multi-bar jig perhaps will be more economical in operation, but the single-bar jig will involve less loss if it must be scrapped. Hence, the quantity of

[1] Factory Manager, Pratt & Whitney Co., Hartford, Conn. Jun. A.S.M.E.

work over which the initial cost of the jig may be divided becomes an important factor.

A. J. Baker.[1] The writer desires to point out, from the standpoint of the automobile manufacturer, some of the reasons for the analysis of machine operations as suggested by the author. Standard machines are common, using a large percentage of the operator's time in non-productive movements. The size and number of machines and the variation in their relative efficiency often make it impossible for fixed lines of work to pass from one group of machines to another. One operation may be performed by a single efficient machine, while the next may require a battery of ten or more. The difference in the number of lines of work converging at the several machines or groups of machines adversely affects traffic conditions in the plant.

A certain uniformity in products is desirable, to avoid a complete shutdown of a line of machines, as production quantities change. If the machine equipment is not well balanced, the wide variation in the number of machines of various groups that must be utilized for reduced production will introduce serious problems in the labor situation. With reduced production, increased overhead, and a smaller number of parts going through the factory, it is more important to maintain lines of progress than it is when production is at a maximum. It is one of the things, however, that is rather badly planned in many automobile plants. Another is the lack of balance in tool equipment. Tools should be put aside for repairs at regular intervals, just as machines should. The major parts of tool equipment should be duplicated or triplicated. Every precaution is taken to prevent a machine from being under repair at a time when maximum production is demanded. The provision of duplicate parts or duplicate tools will often avoid machine shutdowns at inconvenient times, and would facilitate the use of special machines or make their use more attractive.

The Author. One point that the paper attempted to bring out was that there are cases where it is economical to even design and build special machinery, and that there are other cases where it is not economical to install it. When special machinery is considered, an element that acts as a deterrent to its use is the idea that special machinery gives trouble. The percentage of time that such a machine can be on productive work is necessarily somewhat smaller than the corresponding percentage for a standard tool. The idle time is put down as trouble, but such is not actually the case. There is no trouble when the machine as a normal requirement demands a certain amount of attention from skilled men. If the

[1] Chief Engineer, Willys-Overland Co., Toledo, Ohio. Mem. A.S.M.E.

machine requires, say, ten hours of attention from an operator, and, say, two hours additional from a skilled mechanic, the efficiency of the machine would be computed on the basis of a labor cost of 10 hours of operator's time plus two hours of a skilled mechanic's time.

The paper discusses the possibilities of a complicated type of machine tool for performing a combination of operations. The figures were so chosen as not to show economy in the employment of such a machine. This has been done to avoid any appearance of advocating the use of special machinery in almost any case. The only thing advocated was a clear-cut analysis of conditions to determine whether or not special machinery is economical before it is condemned on the ground of trouble. Trouble, in a machine shop, should mean only breakdowns. And the principal cost of trouble is not the expense of repairs, but the cost of a period of idleness of the machine with its attendant loss of production. Trouble frequently can be avoided by taking down the machine, once or twice a year, or perhaps more often, and making such adjustments and repairs as are necessary. If the machine is viewed from the standpoint that a certain amount of time is necessary for repair and adjustment, it will be considered as a profitable investment. On the other hand, if this period is regarded as all trouble, then the machine may be considered as a poor investment and thrown out. If it is frankly recognized that a certain amount of time must be set aside for overhauling the machine, then the production schedules can be so arranged as not to cause any interruption to the manufacturing activities of the plant as a whole, and there is no trouble.

It is on this basis of purely economical considerations that every machine-shop problem should be analyzed. The old idea that a complexity of tools means trouble, or that the addition of a certain amount of the time of a skilled mechanic to that of the operator means trouble, should disappear. When the conclusion is reached that a machine or equipment is capable of furnishing the product at a lower cost, without any serious interruption to the operation of the plant as a whole, then the installation of such machinery or equipment is justified.

In reference to the two systems of machines used in watch manufacture as cited by Mr. Flanders, an interesting point may be raised. The system of individual machines, even without transfer arms, would be the ideal system so long as the operations have not become absolutely constant. In beginning to build a new product, there is no assurance that small changes will not have to be made from time to time. It would be unwise, therefore, to build entirely special machinery until assurance is had that such changes will not occur within a reasonable time. When such assurance is obtained, probably it would be wise to adopt the station type of machine.

The reason for this is that every transfer from machine to machine, whether by human agency or mechanical means, introduces the possibility of an additional error. The error may not be great, but again, it may be too great for the conditions under which the work is done. While a special machine, performing all the operations, also has its errors, these are always the same, and are not cumulative, as they may be with the transfer type of machine. Therefore when conditions have become stabilized the station type of machine is usually preferable to the transfer type. Prior to that time, however, the transfer type is preferable.

No. 1924

THE EMMET MERCURY-VAPOR PROCESS

By W. L. R. Emmet,[1] Schenectady, N. Y.

Member of the Society

In this paper the author first gives an account of the early experiments made, after which the thermodynamic possibilities of the process are taken up. The 1800-*kw. installation in the plant of the Hartford (Conn.) Electric Light Co. is then described, following which boiler problems, leakage, packing, and related topics are discussed, as well as the question of mercury supply. An appendix to the paper includes a table of the properties of mercury vapor at absolute pressures ranging from* 0.4 *to* 180 *lb. per sq. in. abs., a Mollier chart for mercury, and a chart showing the available energy of mercury vapor in watt-hours per lb. for expansions from absolute pressures of from* 20 *to* 85 *lb. per sq. in. to back pressures of from* 0.4 *to* 1.6 *lb. abs.*

IN THIS PROCESS mercury is vaporized in a boiler at temperatures which can be much higher than those which are practicable with steam. It is then carried through a turbine and does useful work, exhausting into a surface condenser where its latent heat is used to make steam at pressure desirable for use. The condensed liquid is carried back to the boiler preferably by gravity. The characteristics of mercury are such that high temperature can be used without excessive pressure, and the heat of condensation can be delivered at a convenient degree of vacuum and at a temperature suited to making steam at pressures desirable for power uses. The process thus affords means by which the temperature ranges practicable with steam are greatly increased under conditions which afford large gains in efficiency of conversion of the heat energy of fuel into work.

History of the Process

2 The possibilities of this process were first explained in a paper presented by the author at a meeting of the American Institute of Electrical Engineers in 1913. The possible uses of

[1] Consulting Engineer, General Electric Co.

Contributed by the Power Division and presented at the Spring Meeting, Cleveland, Ohio, May 26 to 29, 1924, of The American Society of Mechanical Engineers.

the principle involved were, as there stated, suggested to him by Chas. S. Bradley, who has done much original work in many fields. He did not propose the use of mercury for such a purpose, but thought that some other substance could be found with suitable qualities and a sufficiently high boiling point. Mr. Bradley had taken out a patent involving the general principle of such an application. There had also been one or two others who had previously proposed binary-vapor processes which suggested the use of other substances in temperature ranges higher than those to which steam was well adapted.

3 As a result of these suggestions the author began an effort to find some substance which might have qualities suited to the accomplishment of this very attractive object, and, through various circumstances, finally came to the conclusion that mercury, in spite of its high cost, poisonous quality, and tendency to oxidation, might by sufficient care and study be applied to this purpose.

Experiments

4 A campaign of experimentation was then begun, of which the earlier steps are described in the paper above mentioned. The first steps in this experimentation developed few difficulties, although much time was expended in building operations, and, as a result of these apparent successes, a large equipment designed to deliver 1500 kw. from the mercury turbine was built for installation in the power station of the General Electric Company at Schenectady

5 Work on this equipment was begun in 1915 and, after more than two years of discouraging delays in building operations, it was put in operation. A variety of practical difficulties were encountered and a long time passed before it could be run under load. The long succession of delays, difficulties and disappointments in connection with this installation need not be described now. Their nature for the most part was such as not to discredit the idea and many of the most important features were entirely successful. The equipment was ultimately run on a number of occasions with loads as high as 1000 kw., and, while the efficiency of the one-wheel turbine was only about 60 per cent, the results clearly showed that the economies claimed for the process could be realized and that the knowledge developed by calculation and experiment was substantially correct. While the building, changing, and handling of this apparatus involved much loss of time and much worry and discouragement, it afforded practical experience, without which such a new and complicated development could hardly be expected to advance.

6 The outstanding lesson of this venture was that more must be learned about the principles involved in the construction of a good mercury boiler, and experiments with that end in view

have been going on on a considerable scale ever since. In making boiler experiments which are to be depended upon we cannot work on too small a scale, since we must create conditions of fire temperature and radiant-heat delivery similar to those in actual boilers.

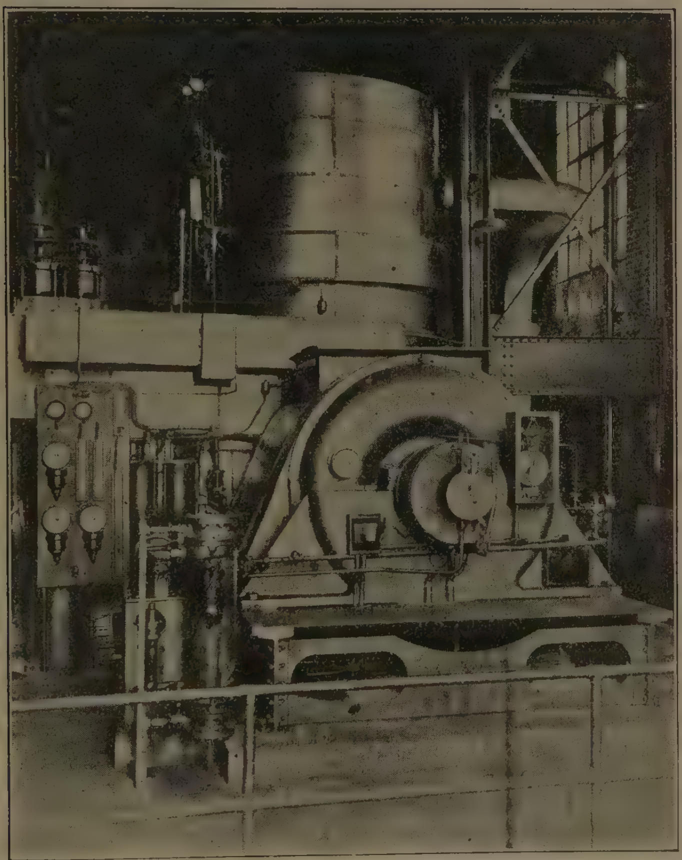


Fig. 1 View of Mercury Condenser, Turbine, and Generator, Hartford Electric Light Co., Hartford, Conn.

7 The boiling conditions of mercury are peculiar and very different from those of water. The metal does not wet the surfaces and there is a wide difference of pressures at different depths. Theories concerning the action have been formed and tested by circulating mercury with compressed air in glass structures, by boiling it in single tubes of various shapes, with measurements of vapor production, liquid circulation, and temperature determina-

tion. Several types of boilers of workable size have been built and tested.

8 When these experiments and studies had given a type of boiler which promised reliability in actual service it was decided to begin work on another large equipment, and it was thought wise to put it in an operating plant, if one could be found to receive it, instead of installing it in Schenectady. About this time the Hartford (Conn.) Electric Light Company made a voluntary offer to the author to make such an installation, and this has been done in their Dutch Point Plant. Their enterprise and foresight have done much toward bringing the process to its present advanced state.

Thermodynamic Possibilities of the Process

9 Before going into the details of the apparatus which has been built and the results which have been accomplished, it may be well to consider the theoretical possibilities as compared with steam processes. An appendix to this paper contains a mercury steam table prepared by Mr. L. A. Sheldon, who has assisted the author since the beginning of work on this process. With this table are given the authorities for the data employed and the calculations. The degree of accuracy of the figures presented is not known, but experience with nozzle flows and turbine results has indicated that they are nearly correct.

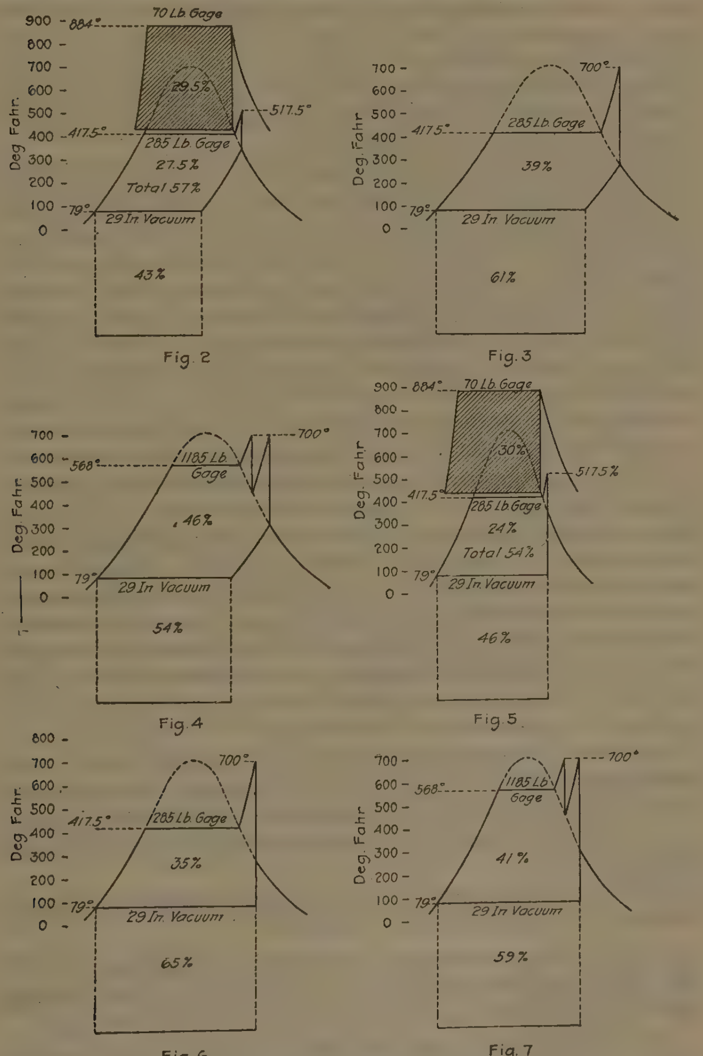
10 In this process the mercury is boiled, carried through a turbine and condensed, giving up its vapor heat to the making of steam. The boiling might theoretically be accomplished at any practicable temperature and pressure, and experience has indicated that temperatures much higher than that here considered may be practicable. The condensation of the mercury and the making of the steam can also occur at any point in the descending temperature scale which gives a practicable mercury vacuum. The final termination of the steam action must be at the temperature of the cooling source: namely, that of the condensing water.

11 For purposes of illustration we will assume a boiling temperature of 884 deg. fahr., corresponding to a pressure of 70 lb. gage in the mercury boiler. As a temperature for condensing mercury we will assume 438 deg. fahr., corresponding to a mercury vacuum of 28.5 in., and being suited with a temperature difference in the condenser of 21 deg. fahr. to the making of steam at 285 lb. gage pressure. The lower limit of steam use is taken at 79 deg. fahr., corresponding to a 29-in. vacuum.

12 The using of a higher mercury vacuum and a lower steam pressure might be expected to give better efficiency, but 28.5 in. is a suitable vacuum for the most convenient design of the mercury turbine.

13 The accompanying diagrams, Figs. 2 to 7, show relative areas of work and loss and are based upon the entropies of the

fluids. They illustrate the theoretical possibilities. The vertical dimensions of these diagrams represent temperature differences. The areas above the line corresponding to 79 deg. fahr., 29 in.



Figs. 2-7 Diagrams Illustrating the Theoretical Possibilities of the Mercury-Vapor Process

vacuum, are proportionate to the work theoretically available. Those below this line are proportionate to the heat energy carried away by the condensing water.

14 Fig. 5 shows the mercury-steam cycle with mercury vaporized at 70 lb. gage, steam made at 285 lb. gage, superheated 100 deg. and expanded by what is known as the Rankine cycle — that is, without heating the feedwater by steam which has been partly expanded and used to give work. In Figs. 5, 6 and 7 this Rankine cycle is shown. In Figs. 2, 3 and 4 a cycle is shown in which the theoretical maximum of feed heating is assumed through extraction of steam from the prime mover at all points in the descending scale of temperature.

15 The 100 deg. of steam superheat in Fig. 5 is that which can be conveniently made after as much heat as easily possible has been delivered from the furnace gases to the making of mercury vapor. The relative expediency of putting added heat from fuel into steam superheat or into more mercury vapor depends upon the relative efficiencies of the mercury and steam turbines and upon the gain by superheating in the steam turbine. With good steam turbines and such mercury turbines as will probably be available there should be some gain in using a higher superheat in the steam, which is easily got by reducing the surface of the mercury heater and increasing that of the superheater.

16 Heating the feedwater by extracted steam will materially improve this process as it does the steam alone, but naturally not so much relatively since the steam alone is affected. As in steam cycles the benefit of full feed heating by extracted steam will be greater with higher steam pressures, but, while under such conditions there will be a gain in efficiency by using higher steam pressures with the mercury, this gain will not be large and moderate steam pressures will generally be considered desirable. The gap in the work diagrams, Figs. 2 and 5, corresponds to the temperature difference in the condenser.

17 In all the diagrams the total areas are the same, representing equal heat energy delivered from the fuel, and the proportions of areas representing work available and heat energy rejected to condenser are given by percentages which show the theoretical efficiency.

18 Fig. 6 shows steam at 285 lb. gage, superheated to 700 deg. fahr. and expanded to a 29-in. vacuum.

19 Fig. 7 shows steam at 1185 lb. gage and superheated to 700 deg. fahr., expanded until it loses its superheat, then reheated again to 700 deg. fahr., and then expanded to a 29-in. vacuum.

20 Figs. 5, 6 and 7 show the same as Figs. 2, 3 and 4, except that the theoretical maximum of feed heating is assumed by steam extraction after the steam finally reaches saturation.

21 Since perfect efficiency of apparatus is impossible — and for many other reasons — none of these conditions can in practice be accomplished, and many variations of the conditions might be

made. The diagrams, however, serve to illustrate the principle and are useful for purposes of comparison.

The Hartford Installation

22 The equipment at Hartford was designed to operate with 35 lb. gage pressure on the mercury boiler, to deliver about 1800 kw. from the mercury turbine, and to make steam at 200 lb. pressure with about 100 deg. of superheat. The character of the apparatus can best be observed from the drawings and photographs reproduced in Figs. 8-13. The furnace gases pass in order through the mercury boiler, a mercury-liquid heater, a steam superheater, and a feedwater heater. It would be more economical to substitute for the last-named device an air heater, since it would put this residuum of heat into the mercury instead of into the steam. The feed heat would then be accomplished by bleeding the steam turbine. The approximate temperatures in the gases at different points, and the temperatures and pressures of fluids, are given in Figs. 8 and 9.

23 Referring to Fig. 9, the mercury vapor formed in the boiler passes through a governing and emergency valve to the turbine, the single wheel of which is overhung on the end of the generator shaft and operates in the condenser space. Two safety valves are arranged to bypass the vapor into the condenser if the governor causes its valve to close, so that the mercury vapor continues to be condensed and to make steam whether it is passing through the mercury turbine or not. The shaft passes through a packing between the generator and the turbine wheel and this is sealed by mercury vapor above atmospheric pressure. The outward leakage from this packing is sucked into a cooler where it is all condensed and returned to the system.

24 The discharge from the turbine wheel is delivered directly against the condensing surface which consists of dead-ended tubes hanging vertically from a cylindrical steam and water drum, to which they are attached by rolling and also arc welding.

25 The condenser shell and all the pipes and containers which carry mercury are put together by welding. From the condenser the liquefied mercury runs through a sump to the mercury heater and boiler by gravity.

26 On the side of the condenser shell a large blowout diaphragm is provided which communicates to the stack and will give way if, through escape of steam or water or other cause, the pressure in the condenser should begin to rise unduly.

27 The mercury boiler at Hartford is made up of fire tubes, the lower two-thirds of which are hexagonal with slightly convex faces. These are nested together like a honeycomb (Fig. 10), and for each 7 tubes a space is left for a duct which returns the circulating liquid. This liquid rises and the vapor forms in clear-

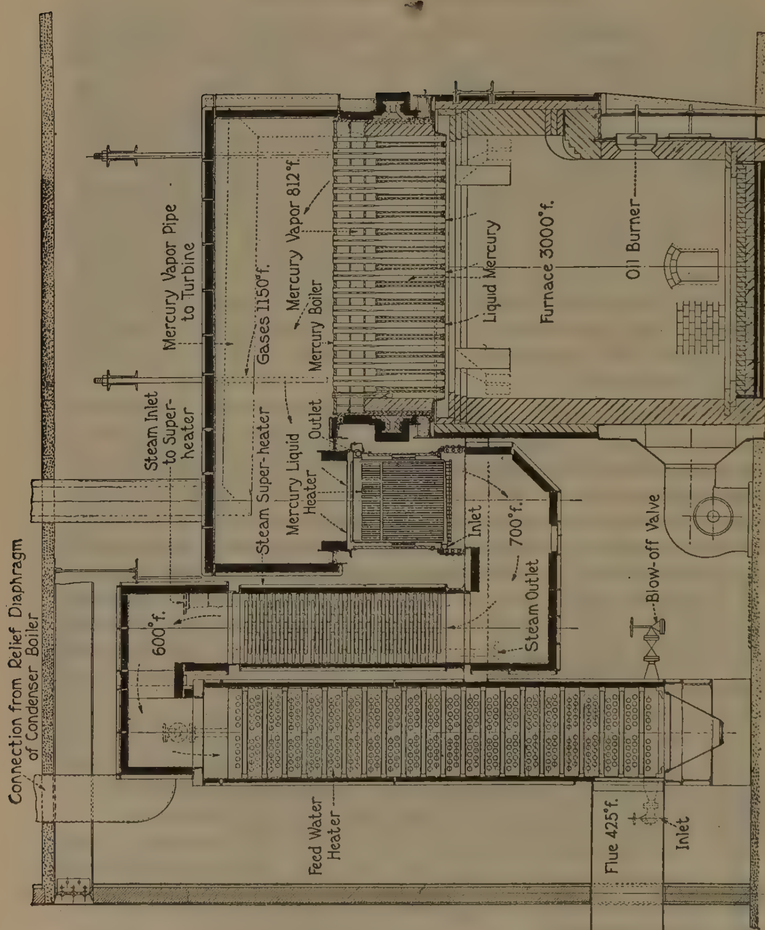


Fig. 8 Section Through Mercury Boiler, Heater, Superheater, and Feedwater Heater

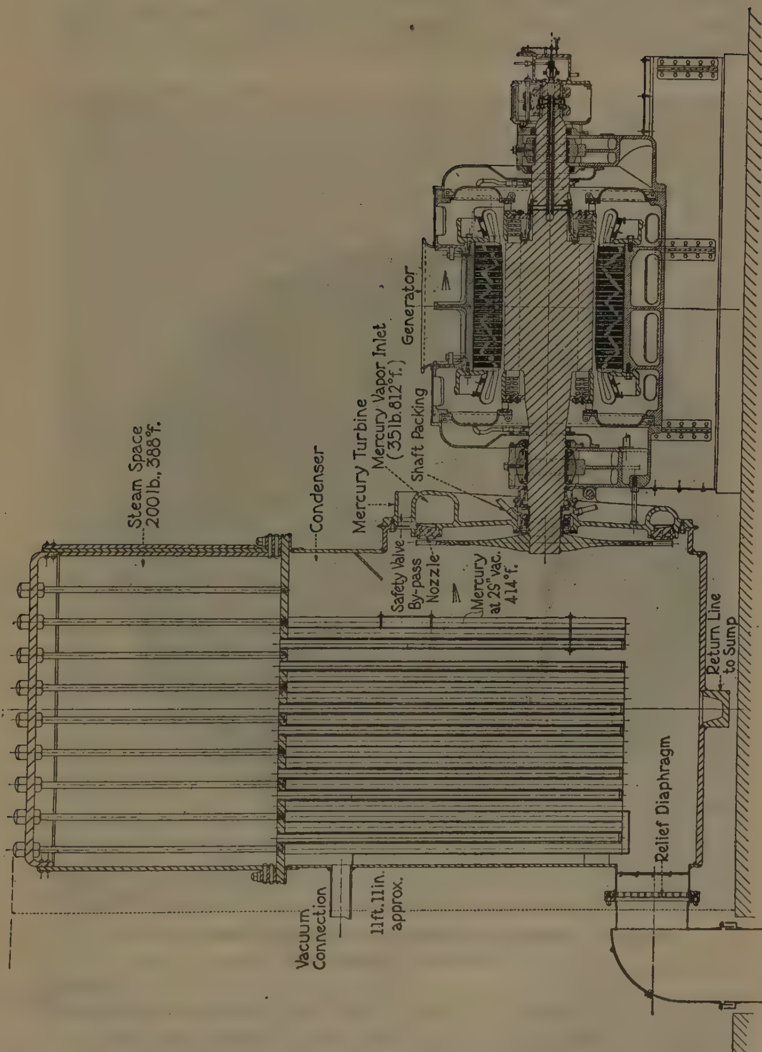


FIG. 9 SECTION THROUGH MERCURY CONDENSER, TURBINE, AND GENERATOR

ances between the tubes at the corners. The use of such tubes in a mercury boiler was proposed by Mr. B. P. Coulson, the author's assistant. The upper round part of the tubes acts as superheating surface and the vapor is delivered with a little superheat, which is desirable for the reason that it probably tends to prevent cutting

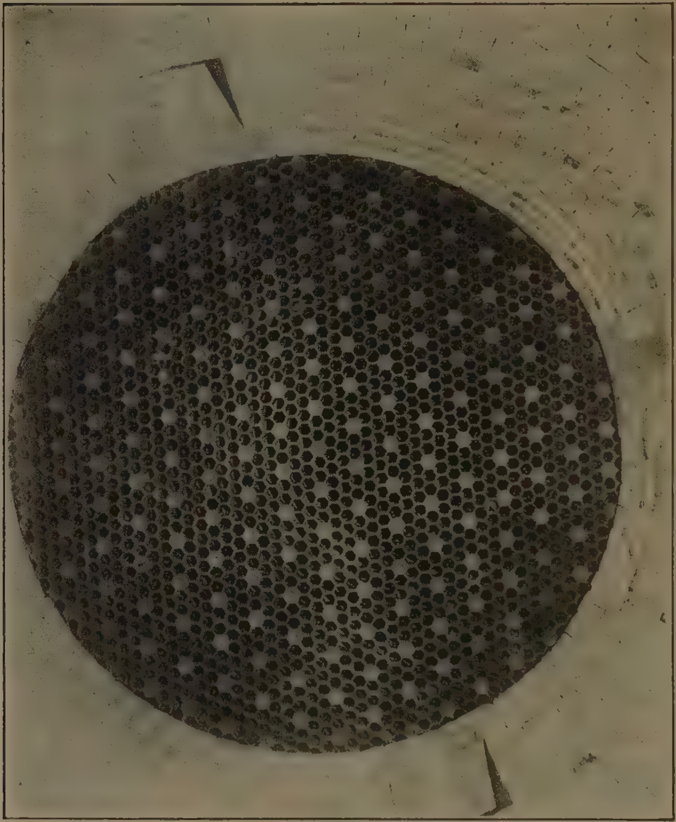


Fig. 10 Bottom of Mercury Boiler Set in Furnace, Hartford Installation

of the turbine blades — of which there has been none at Hartford, although a slight amount was observed in former turbines.

28 This equipment has been run experimentally for several months. Few troubles have been experienced and nothing of a serious nature. It has delivered power to the circuits for about 800 hours. The highest load carried has been 1500 kw. and the load carried most of the time has been 1200 kw. There has been some uncertainty as to the safe capacity of the boiler and, since experi-

ence with continued operation has been the most important need, it has been thought best to avoid risk of any possible serious trouble. The conditions of running cannot be considered commercial, but the results secured give a very good indication that commercial results are not difficult of attainment.

Boiler Problems

29 The most difficult and novel part of this process is the boiler. While mercury is a good heat conductor, it does not wet steel and considerable temperature differences are required to deliver heat to it at a high rate. When it begins to produce vapor on the heating surfaces this condition becomes rapidly worse. Successful mercury boilers are dependent upon a rapid circulation of the liquid and a large difference of pressure between the top and the bottom which prevents boiling of the circulating liquid until it is well started on its upward course. That is, the rapid heat delivery must be confined to the part of the surface which is not much disturbed by boiling. Rapid circulation is essential and the spaces for discharging vapor must be so proportioned that the circulation will not be checked by the escape of the vapor. If these spaces are too small the liquid will be given motion like a charge of shot, and prohibitive back pressure will result which will check circulation. The vapor must slip by the liquid, impelling it but not giving it too much velocity.

30 In the type of boiler which is now considered most promising these functions are accomplished in dead-ended tubes with interior structures which provide the circulation conditions for each tube independently. These tubes are arranged in such a manner that they receive radiant heat from below, the gases passing upward in rather small and converging spaces between them. The arrangement is such that all tubes receive nearly equal quantities of heat both from radiation and from contact with the gases. A section of such a boiler is shown in Fig. 11. Twenty of these sections would be used with a 7500-kw. mercury-turbine unit, boilers in a number of sections operating in parallel and any section being removable for cleaning or repair if necessary.

31 This type of boiler has the advantage that its heating surface is free from expansion strains. It also makes possible the calorizing of the tubes. This process, which was developed in the General Electric Research Laboratory, aluminizes the surface of the tubes so that they are immune from burning at high temperatures. It is used successfully in petroleum stills at temperatures much higher than those which need concern us in the mercury process. Such treatment will render possible the use of higher pressures in the process, since the pressures already employed approach the point at which iron begins to scale destructively. Higher pressures will afford better economy, and there seems to

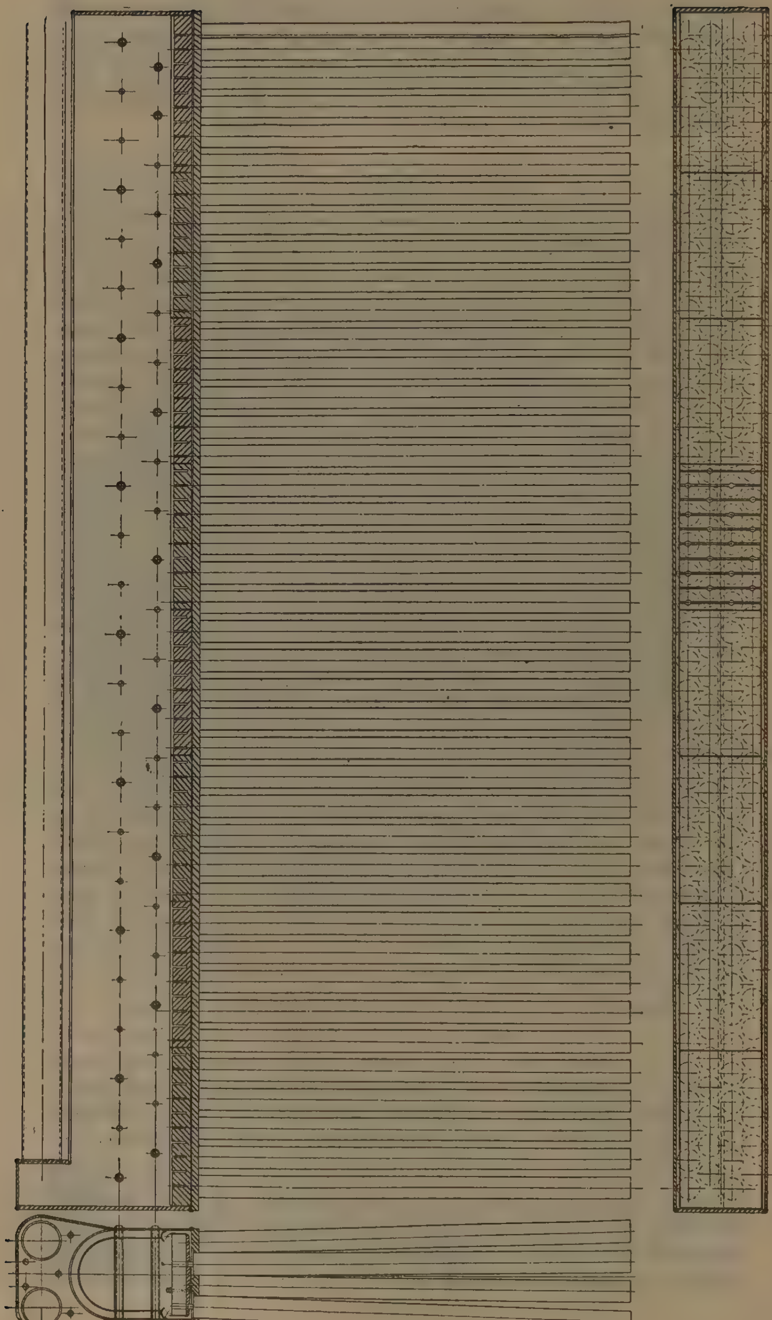


Fig. 11 New Type of Boiler Unit with Closed-End Tubes

be nothing against their use outside of the boiler. The turbine is of low speed and there are no heavy centrifugal strains as in steam turbines.

Oxide and Impurities

32 Mercury does not oxidize very rapidly even when exposed to air at high temperatures, but a small degree of oxidation seems

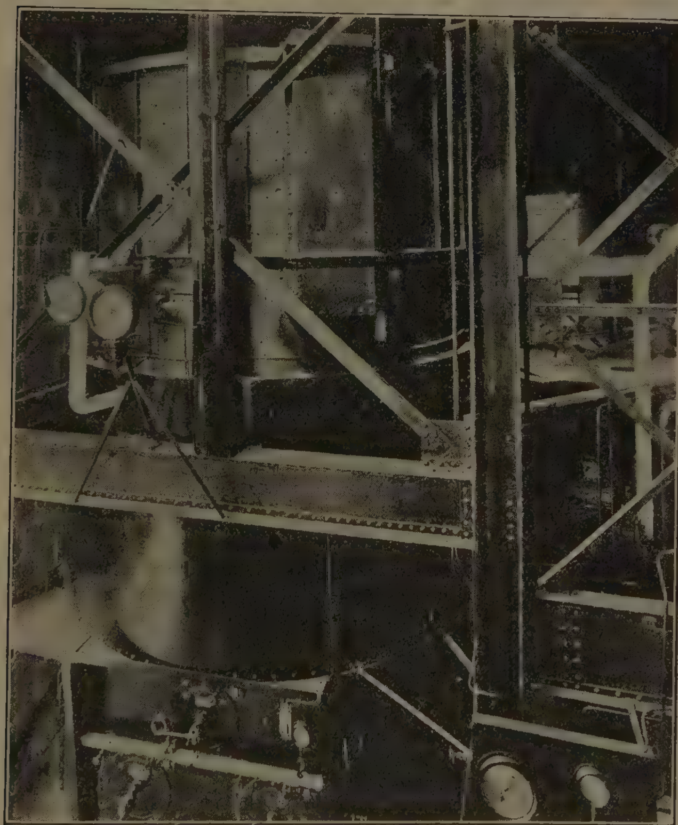


Fig. 12 Mercury Boiler Installed on Lower Floor at Hartford

to render the metal adhesive and makes it very difficult to separate finely divided foreign matter. Experience has shown that if the scum formed by oxidation, accompanied by scale, dirt, cuttings from the steel, etc., is allowed to freely form and circulate in a mercury-boiler system, accumulations are likely to occur in interior passages, and that in these accumulations large crystals of red oxide are likely to form, increasing the bulk of solids and causing

danger of stoppages. For these reasons it has been found desirable to carefully protect the mercury from frequent or continued exposure to oxygen, and this has been done by filling the boiler and condenser with illuminating gas when they are shut down. Arrangements have also been made by which gas is kept in the cleaning sump and can be kept in contact with the mercury which leaks from the packing, although this small quantity could be cleaned continuously as it returns to the system if desirable. Hydrogen and carbonic oxide are reducing agents for mercury

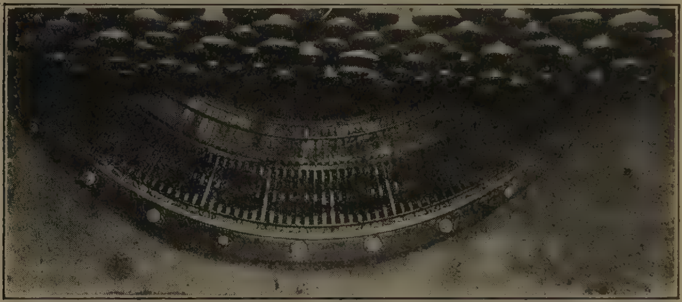


Fig. 13 View of Turbine Wheel and Condenser-Boiler Tubes

and, working with them, the liquid is not adhesive and cleaning is relatively easy.

Expansion Strains

33 In such apparatus the temperature differences are very large and expansion strains must be scrupulously guarded against. The problem is similar to that in properly designed steam apparatus, but much more important because leaks are prohibitive and any repeated excess strain will make a crack. In all welded joints the weld should be made much stronger than the metal adjacent to it; otherwise small movements may concentrate there and develop cracks.

Mercury Turbines

34 In the first ventures which have been made in mercury turbines, single wheels of the impulse type, like the original DeLaval turbines, have been used. This was done for the sake of simplicity, the speed conditions being suited to such a wheel. It was not known how much the mercury might wear the blades. Mercury discharged at high velocity and high temperature does cut ordinary steel rapidly.

35 It has been found, however, that tempered high-speed tool steel is hardly at all affected under the severest conditions The

Hartford turbine was fitted with buckets of this metal and some of soft steel. Neither has been even slightly affected and the inference is that a turbine with a plurality of stages and buckets of ordinary steel can be used. It may be that it will be found desirable to use special buckets in the higher stages where the temperature is higher. The Hartford turbine has a hooked-on bucket construction by which the wheel can be quickly rebucketed by a man working inside of the condenser.

36 In many respects the problems of mercury turbines are similar to those of steam turbines. The velocities involved, however, are of the order of one-third and the mechanical problems are therefore much easier notwithstanding the high temperature. The single-wheel turbine at Hartford will in a short time be replaced by a three-wheel machine, also overhung inside the same condenser. It is hoped that this will give a fair efficiency and good durability.

Packings

37 Two types of shaft packings have been successfully used in mercury turbines, in one case the seal being centrifugal with liquid mercury, and in the other being made with mercury vapor. The latter type has been found most convenient. Since the conditions are peculiar and it is not expedient to use split or jointed structures, it has been found desirable to develop a special type of self-centering packing ring. In this a light cast-iron ring is riveted to, and held circular by, a deep ring flange of chrome steel, the whole being held in place frictionally by springs of a material suited to the high temperature.

Condensation and Steam Making

38 The condensing conditions with mercury are peculiarly good. The liquid does not wet the surface and the falling drops tend to move the vapor and also to give added condensing surface. In Hartford we have run with 770 B.t.u. transfer per degree fahrenheit temperature difference per square foot per hour.

39 The limitation of heat delivery is probably governed largely by heat conduction through the heavy steel tubes, and delivery as indicated by the falling mercury, which has been observed through a lighted window, seems to be quite uniformly distributed over the condensing surface. This makes a very good condition for the production of steam. There can be no overheat or strain, and scaling can only result ultimately in impairment of mercury vacuum. The steam space can be made very large without inconvenience or appreciable added expense.

40 The tubes are accessible from above through hand holes at the top of the drum, and can be cleaned by inserting into the top of the tube a nozzle connected to a piece of flexible pipe or hose.

Through this another small hose can be run down to the bottom of the tube and scale or dirt can be flowed out with water after it has been loosened by the ordinary means.

Method of Operation

41 The arrangement of valves, the governing devices, etc., devised for this system are quite new and have proved to be very practical and successful, but their details cannot be described within the limits of the present paper. Since it is economical to use the mercury turbine as much as possible, the normal governing is done on the steam end, and the governor of the mercury turbine operates only as a speed-limiting device, being set to operate above the running speed fixed by the steam-turbine governors.

42 The furnace is operated, and pressure and level in the steam part are held, just as in the case of an ordinary steam boiler. If the mercury turbine cannot use the vapor made by the fire, it is by-passed through the safety valves. If the steam made cannot be used, the steam safety valves must blow as in any boiler. The mercury level in the boiler is indicated to the fireman by float gages, the motion of which is transmitted magnetically through disks of non-magnetic steel welded into the ends of the boiler sections when, as in proposed design, the boiler is built in sections. If a loss of mercury should be observed, the temperatures should be checked as quickly as possible so that the loss may be reduced as much as possible.

Leakage

43 The whole apparatus for this purpose has been developed with a view to avoiding the possibility of leaks, and experience with much experimental apparatus over a period of years has indicated that the desired results are not very difficult of attainment. With proper methods of welding and of proportioning of parts, leaks which may occur through imperfect work, imperfection of material, or expansion strains improperly provided for, will generally be of the nature of minute cracks — and mercury leaks slowly on account of its low pressure and high density. Means have been developed by which the presence of a very small quantity of mercury in the flue gases can be shown by the discoloration of a strip of treated paper so that inspection will show conditions. All parts of the apparatus will be readily accessible for repair if required. Most of the parts which carry mercury under pressure are in the flue-gas space which, being subject to stack suction, cannot deliver fumes to the air which is breathed. The parts which are not so situated will be completely surrounded by sheet-metal enclosures outside the lagging, which spaces are also subject to stack suction.

Poisoning

44 In ten years of experimentation with many kinds of mercury apparatus we have experienced no case of poisoning, but nevertheless realize that the utmost care is necessary. On several occasions men have been in contact with much mercury vapor in hot places, and in one or two cases men have been temporarily quite sick as an apparent result. There has been no case of salivation or continued symptoms of mercury poisoning, although most of these men have been on the job for long periods. The experience of mercury production and its use in other arts would indicate that, with reasonable precautions, danger of poisoning should not be appreciable.

Economies

45 The most economical method of operating this process will generally be to get as much heat as possible into the feedwater by bleeding the steam turbine as in best modern steam practice. Since such units will not be run at heavy overloads, as is common with steam boilers during peaks, it will be practicable to put a large quantity of heat into the incoming air without danger of burning the brickwork, and it is thought that such a device as the Ljungström air heater can be used to great advantage in bringing the flue gases to a low temperature and delivering their heat to the furnace. The heat from the furnace will be used to heat and vaporize mercury and to give such superheat to the steam as may be expedient with the steam apparatus used.

46 With such arrangement, if we assume 70 per cent efficiency, 70 lb. pressure for the mercury cycle, and the most desirable steam conditions, we should be able to operate on base load for about 10,000 B.t.u. from fuel per kilowatt-hour.

47 In Hartford, where oil is burned and measured, and where steam flow and feed are both measured, it has been estimated that if the steam produced were used effectively, the fuel rate would be about 12,000 B.t.u. per kw-hr. This is with only 1200 kw. load, a single-wheel turbine of only about 60 per cent efficiency, and with only 22 lb. mercury pressure.

48 For the purpose of giving an idea of possibilities in existing stations, the cases of three large plants — among the best in the country, operating for the month of January, 1924, have been considered. The following table shows the conditions in these plants and the gain in net output which would have resulted if the same fuel had been burned under mercury boilers, with the same auxiliary and flue-gas conditions; it being assumed that the mercury turbines with generators are 70 per cent efficient and that a mercury pressure of 70 lb. gage is used.

	Plant 1	Plant 2	Plant 3
Capacity in kilowatts	180,000	50,000	100,000
Economizers	No	No	Yes
Steam pressure, lb. per sq. in.	219	286	235
Superheat, deg. fahr	217	245	200
Load factor, per cent	57	50	50
B. t. u. in fuel per kw-hr	19,850	19,700	18,250
Gain in output if fuel had been burned under mercury boilers, per cent	65	58	51

49 These estimates do not admit of much error. The combustion conditions would be the same in both cases and no additional auxiliary load would be occasioned by the change. There would be some difference in banking and starting since the mercury equipment would not generally be designed for heavy overloading at peaks as is common with steam boilers.

50 If in such cases the conditions described above as desirable were used instead of the flue-gas and feed-heating arrangements employed in these stations, the fuel efficiency would be considerably better.

51 With the same steam turbines and condensers in each of these cases plant capacity could be more than doubled by providing full mercury-boiler equipment. From this fact it may be inferred that the process affords great advantages in the matter of investment as well as of operation.

Mercury Supply

52 The demand for mercury has always been strictly limited and it is probably not safe to predict positively the consequences of a greatly increased demand. The cost is governed largely by the richness of ore. Ore of various grades exists in many places and it only now pays to work the best of it. A well-informed mercury mine operator has estimated that a maintained price of $2 per lb. would call forth from known sources in the United States enough mercury, if used as we expect, to correspond in plant capacity to the largest yearly output of General Electric turbines. Several other experienced persons have expressed opinions generally agreeing with such a view. Unworked deposits are known in Alaska, South America, New Zealand, and elsewhere, and a rise of price will undoubtedly bring much more to light. It is thought, therefore, that we need not slacken our efforts for the present through fear of a shortage of mercury.

Other Substances

53 Mr. Parkman Coffin of the General Electric Research Laboratory has made for the author a search for substances which might be used in this general way instead of mercury. Some substances of considerable stability and desirable thermodynamic properties have been found, but none of the possibilities, with the exception of sulphur, have stood continued boiling under pres-

sure without gradual change and deterioration. Among the materials tried were diphenyl, diphenyl ether and benzophenone. Means might be devised by which sulphur could be used as a thermodynamic fluid. The principal objections to it are that it attacks steel at the temperatures needed, and that it is viscous and a very poor heat conductor even at the temperature of the highest-pressure steam which might be used to take heat from it in a condenser. Certain aluminum-iron alloys might make practicable containers, and the rates of condensation to the temperature of 1200-lb. steam have been measured and are not entirely prohibitive. It has peculiar thermodynamic characteristics and is obviously much less desirable for such a purpose than mercury. These studies have inclined the author to the idea that nothing other than mercury is ever likely to be used for such a purpose.

Plans for the Future

54 As soon as certain boiler experiments now in progress are satisfactorily completed, it is proposed to build a new boiler of a different type for the existing Hartford installation. This boiler we propose to adapt for a pressure of 70 lb. gage, the design pressure of the present boiler being 35 lb. gage. We also intend to build a new three-stage turbine instead of the one-stage turbine now used. When these changes are made it is hoped that this installation will be representative of types which can be repeated indefinitely on a large scale and with such resultant economies as have been outlined in this paper.

APPENDIX NO. 1

PROPERTIES OF MERCURY VAPOR

By L. A. Sheldon,[1] Schenectady, N. Y.

Member of the Society

55 *Mercury Vapor Pressure.* According to Alexander Smith and Alan W. C. Menzies,[2] the Kirchoff-Rankine-Dupé formula for the vapor pressure of mercury is—

$$\log\ p=A+\frac{B}{T}+C\ \log\ T$$

or with constants inserted,

$$\log_{10} p=9.9073436-\frac{3276.628}{T}-0.6519904\ \log_{10} T \quad . \quad . \quad [1]$$

where $\log B=3.5154272$

$\log C=\bar{1}.8142412$

p = pressure in millimeters of mercury

T = temperature in degrees absolute $=(t+273)$

t = temperature degrees centigrade.

56 In English units Equation [1] reduces to

$$\log_{10} p=8.3601791-\frac{5897.9304}{T}-0.6519904\ \log_{10} T$$

where p = pressure in pounds per square inch absolute

T = temperature in degrees absolute $=(t+459.64)$

t = temperature in degrees fahrenheit.

57 The experimental values covered by these experiments ranged from 482 deg. fahr. to 815 deg. fahr. Values in Table 1 above and below these temperatures are extrapolated.

58 *Heat of Vaporization* (r). According to Kurbatoff the latent heat of vaporization (r) is 122.1 B t.u. per lb. at 677.1 deg. fahr., and according to Loomis, 128.1 B.t.u. at 415 deg. fahr. and 125.7 at 504 deg. fahr. Plotting these values there is obtained the equation

$$r=128.2-0.022\ (t-400)$$

59 *Specific Volume.* Having r, substitute in the equation $r=AUt\ \frac{dp}{dt}$ and solve for the specific volume U, giving

$$U=\frac{r}{\frac{dp}{dt}AT}$$

where $A=\frac{1}{778}$

U = specific volume of vapor minus specific volume of liquid $=V_S-V_L$

Since V_L is very small compared to V_S it may be neglected, or $U=V_S$.

$\frac{dp}{dt}$ may be found from the vapor-pressure curve

[1] M. E., General Electric Co.

[2] Jour. Am. Chem. Soc., vol. 32, p. 1441.

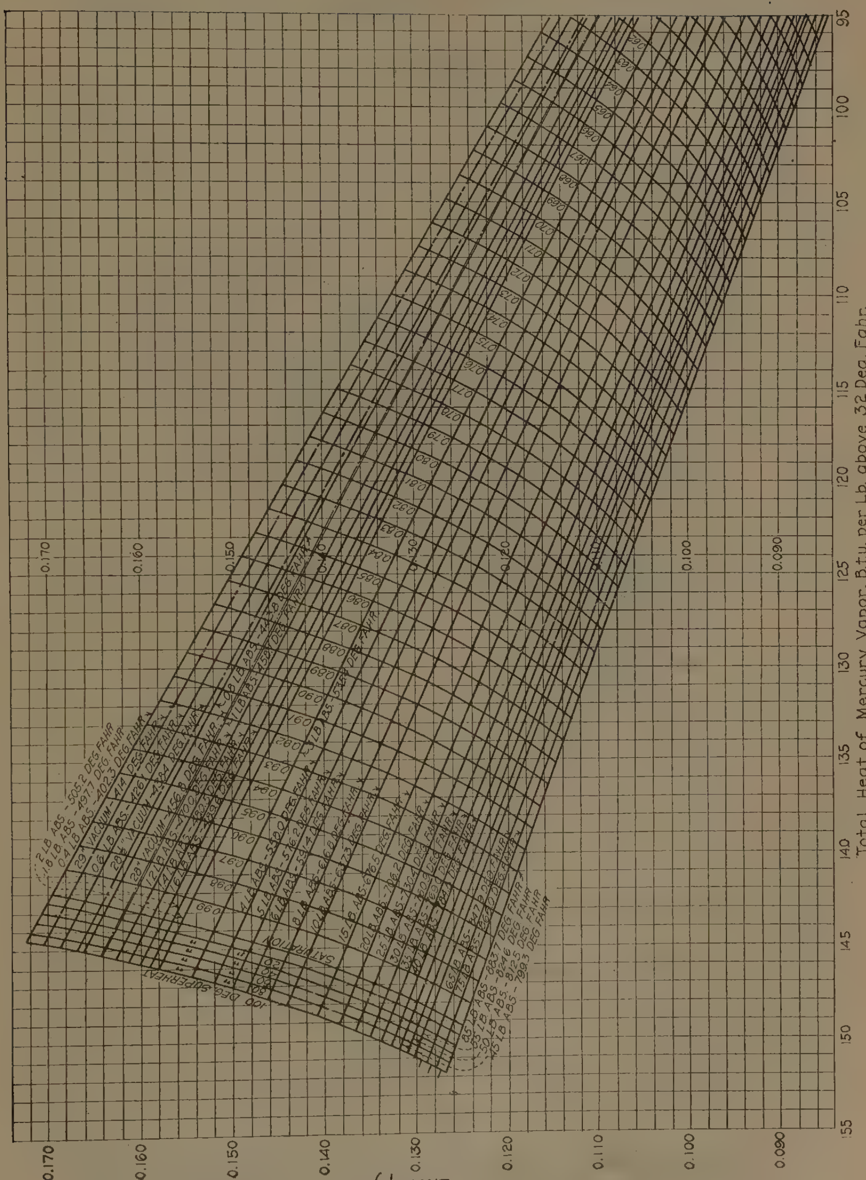


FIG. 14 MOLLIER CHART FOR MERCURY

$$\log p = A + \frac{B}{T} + C \log T$$

which, differentiated gives

$$\frac{dp}{pdt} = \frac{13580.4866186}{T^2} - \frac{0.6519904}{T}$$

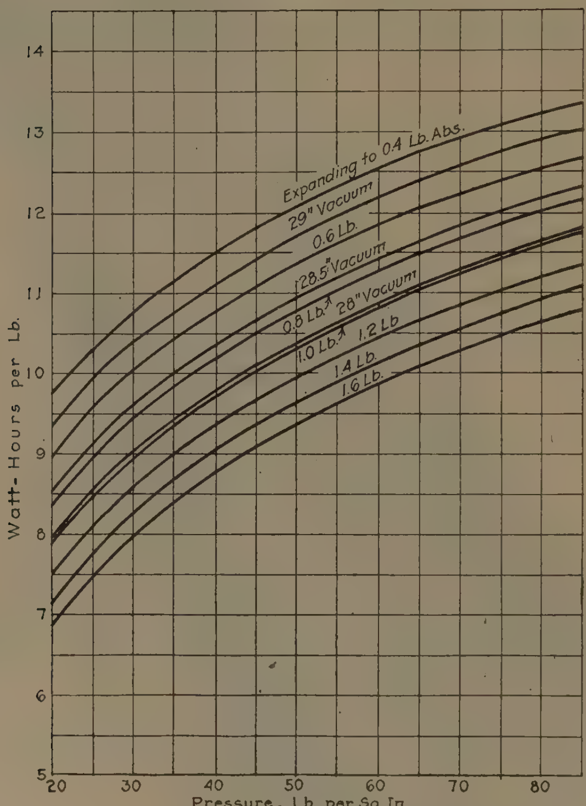


FIG. 15 AVAILABLE ENERGY OF MERCURY VAPOR

60 *Density.*

$$\text{Density} = \frac{1}{\text{Specific Vol.}}$$

$$= \frac{1}{U}$$

61 *Specific Heat.* According to Kurbatoff[1] the average of the specific heat of liquid mercury between 19 deg. cent. and 335 deg. cent. is

$$S = 0.0373$$

[1] Zeit. Phys. Chem., vol. 43 (1903), p. 104.

62 The ratio of the specific heat at constant pressure (C_p) to the specific heat at constant volume (C_v) is 1.666+ for mercury vapor. From this fact and the equation

$$C_p - C_v = \frac{1.985}{200}$$

the specific heat at constant pressure is found to be

$$C_p = 0.02481$$

63 *Available Energy.* A convenient equation for available energy is

$$E = \left[Q_1 - Q_2 \left(1 + \log_e \frac{Q_1}{Q_2} \right) + \frac{x_1 r_1}{Q_1} (Q_1 - Q_2) \right] 778$$

where E = available energy
$Q_1 = T_1 \times 0.0373$
$Q_2 = T_2 \times 0.0373$
r_1 = heat of vaporization at temperature T_1
x_1 = quality of vapor at temperature T_1

64 *Entropy of the Liquid.* The change of entropy of the liquid is

$$Q_1 - Q_2 = C \log_e \frac{T_1}{T_2}$$

65 *Entropy Due to Vaporization.* The change of entropy is

$$Q_1 - Q_2 = \frac{x_1 r_1}{T_1}$$

The values for entropy given in Table 1 are those for entropy above 32 deg. fahr.

TABLE 1 PROPERTIES OF MERCURY VAPOR

Pressure, lb. per sq. in. abs.	Temp., deg. fahr.	Abs. temp., deg. fahr.	Heat of liquid above 32 deg., $q=0.0373\ (t-32)$	Heat of vaporization $r=128.2-0.022\ (t-400)$	Total heat $H=q+r$	Entropy of liquid above 32 deg.	Entropy of vaporization	Total entropy	Specific volume, cu. ft. per lb. V_s	Weight, lb. per cu. ft. $1/V_s$
0.4	402.3	861.94	13.81	128.15	141.96	0.020940	0.14867	0.169610	114.50	0.008733
0.49 (29 in. vac.)	414.0	873.64	14.25	127.89	142.14	0.021443	0.14638	0.167823	94.627	0.010567
0.5	415.2	874.84	14.29	127.87	142.16	0.021491	0.14619	0.167681	92.850	0.010770
0.6	426.1	885.74	14.70	127.63	142.32	0.021957	0.14410	0.166057	78.233	0.012782
0.7	435.4	895.04	15.05	127.42	142.47	0.022350	0.14238	0.164730	67.695	0.014772
0.735 (28.5 in. vac.)	438.4	898.04	15.16	127.36	142.52	0.022472	0.14182	0.164292	64.644	0.015469
0.8	443.8	903.44	15.36	127.24	142.60	0.022693	0.14085	0.163543	59.718	0.016745
0.9	451.4	911.04	15.64	127.07	142.71	0.023005	0.13947	0.162475	53.474	0.01870
0.98 (28 in. vac.)	456.8	916.44	15.85	126.95	142.80	0.023228	0.13851	0.161738	49.372	0.020254
1.0	458.1	917.74	15.89	126.92	142.81	0.023280	0.13828	0.161560	48.450	0.020639
1.1	464.3	923.94	16.12	126.79	142.91	0.023530	0.13722	0.160750	44.290	0.022578
1.2	470.0	929.64	16.34	126.66	143.00	0.023760	0.13624	0.160000	40.830	0.024491
1.3	475.4	935.04	16.54	126.54	143.08	0.023978	0.13533	0.159308	37.876	0.026400
1.4	480.5	940.14	16.73	126.43	143.16	0.024180	0.13450	0.158680	35.370	0.028268
1.5	485.1	944.74	16.90	126.33	143.23	0.024361	0.13373	0.158091	33.141	0.030174
1.6	489.6	949.24	17.07	126.23	143.30	0.024540	0.13300	0.157540	31.206	0.03204
1.7	493.7	953.34	17.22	126.14	143.36	0.024700	0.13232	0.157020	29.470	0.03393
1.8	497.7	957.34	17.37	126.05	143.42	0.024858	0.13168	0.156538	27.940	0.03579
1.9	501.5	961.14	17.51	125.97	143.48	0.025007	0.13108	0.156087	26.562	0.03765
2.0	505.2	964.84	17.65	125.89	143.54	0.025143	0.13048	0.155623	25.318	0.03949
3.0	535.4	995.04	18.78	125.22	144.00	0.026299	0.12584	0.152139	17.340	0.05767
4.0	558.0	1017.64	19.62	124.72	144.34	0.027140	0.12259	0.149730	13.262	0.07540
5.0	576.2	1035.84	20.30	124.32	144.62	0.027800	0.12003	0.147830	10.774	0.09281
6.0	591.4	1051.04	20.87	123.99	144.86	0.028346	0.11793	0.146276	9.0962	0.10993
7.0	605.0	1064.64	21.37	123.69	145.06	0.028821	0.11618	0.145001	7.8825	0.12686
8.0	616.8	1076.44	21.81	123.43	145.24	0.029230	0.11469	0.143920	6.9630	0.14361
9.0	627.5	1087.14	22.21	123.20	145.41	0.029601	0.11332	0.142921	6.2448	0.16013
10.0	637.3	1096.94	22.58	122.98	145.56	0.029938	0.11211	0.142048	5.6610	0.17664
11.0	646.3	1105.94	22.91	122.78	145.69	0.030242	0.11102	0.141262	5.1808	0.19301
12.0	654.6	1114.24	23.22	122.60	145.82	0.030520	0.11001	0.140530	4.7812	0.20915
13.0	662.3	1121.94	23.51	122.43	145.94	0.030776	0.10912	0.139896	4.4393	0.22526
14.0	669.7	1129.34	23.79	122.27	146.06	0.031019	0.10826	0.139279	4.1443	0.24129
14.7	674.5	1134.14	23.97	122.16	146.13	0.031172	0.10771	0.138882	3.9630	0.25233
15.0	676.5	1136.14	24.04	122.12	146.16	0.031245	0.10749	0.138735	3.8923	0.25691
16.0	683.0	1142.64	24.28	121.97	146.25	0.031458	0.10674	0.138198	3.6665	0.27273
17.0	689.2	1148.84	24.51	121.84	146.35	0.031660	0.10602	0.137680	3.466	0.28851
18.0	695.0	1154.64	24.73	121.71	146.44	0.031849	0.10538	0.137229	3.2877	0.30416
19.0	700.8	1160.44	24.95	121.58	146.53	0.032035	0.10477	0.136805	3.1268	0.31981
20.0	706.2	1165.84	25.15	121.46	146.61	0.032208	0.10418	0.136388	2.983	0.33523
21.0	711.3	1170.94	25.34	121.35	146.69	0.032371	0.10363	0.136001	2.8520	0.35061
22.0	716.4	1176.04	25.53	121.24	146.77	0.032531	0.10308	0.135611	2.733	0.36589
23.0	721.2	1180.84	25.71	121.13	146.84	0.032683	0.10258	0.135263	2.6228	0.38127
24.0	725.8	1185.44	25.88	121.03	146.91	0.032830	0.10210	0.134930	2.5215	0.39658
25.0	730.4	1190.04	26.05	120.93	146.98	0.032975	0.10162	0.134595	2.429	0.41169
26.0	734.6	1194.24	26.21	120.84	147.05	0.033108	0.10118	0.134288	2.342	0.42698
27.0	738.8	1198.44	26.36	120.75	147.11	0.033238	0.10076	0.133998	2.2623	0.44202
28.0	742.9	1202.54	26.52	120.66	147.18	0.033362	0.10033	0.133692	2.1872	0.45720
29.0	747.0	1206.64	26.67	120.57	147.24	0.033492	0.099922	0.133414	2.1177	0.47221
30.0	750.9	1210.54	26.81	120.48	147.29	0.033609	0.099530	0.133139	2.053	0.48709
31.0	754.7	1214.34	26.96	120.40	147.36	0.033727	0.099151	0.132878	1.9921	0.50198
32.0	758.3	1217.94	27.09	120.32	147.41	0.033840	0.098789	0.132629	1.9343	0.51698
33.0	761.8	1221.44	27.22	120.24	147.46	0.033947	0.098443	0.132380	1.8811	0.53160
34.0	765.5	1225.14	27.36	120.16	147.52	0.034057	0.098080	0.132137	1.8298	0.54650
35.0	769.0	1228.64	27.49	120.08	147.57	0.034162	0.097740	0.131902	1.7815	0.56132
36.0	772.3	1231.94	27.61	120.01	147.62	0.034263	0.097419	0.131682	1.7367	0.57580
37.0	775.5	1235.14	27.73	119.94	147.67	0.034358	0.097100	0.131458	1.6936	0.59051
38.0	778.6	1238.24	27.85	119.87	147.72	0.034452	0.096798	0.131250	1.6523	0.60519
39.0	781.8	1241.44	27.97	119.80	147.77	0.034549	0.096500	0.131049	1.6134	0.61976
40.0	784.8	1244.44	28.08	119.73	147.81	0.034640	0.096218	0.130858	1.5762	0.63440
41.0	787.8	1247.44	28.19	119.67	147.86	0.034730	0.095934	0.130664	1.5405	0.64913
42.0	790.7	1250.34	28.30	119.61	147.91	0.034815	0.095667	0.130482	1.5071	0.66352
43.0	793.6	1253.24	28.41	119.54	147.95	0.034902	0.095386	0.130288	1.4747	0.67810
44.0	796.5	1256.14	28.52	119.48	148.00	0.034989	0.095110	0.130099	1.4441	0.69247

(Continued on following page)

TABLE 1 PROPERTIES OF MERCURY VAPOR—CONTINUED

Pressure, lb. per sq. in. abs.	Temp., deg. fahr.	Abs. temp., deg. fahr.	Heat of liquid above 32 deg.; $q=0.0373\ (t-32)$	Heat of vaporization $r=128.2-0.022\ (t-400)$	Total heat $H=q+r$	Entropy of liquid above 32 deg.	Entropy of vaporization	Total entropy	Specific volume, cu. ft. per lb. V	Weight, lb. per cu. ft. $1/V_s$
45	799.3	1258.94	28.62	119.42	148.04	0.035070	0.094868	0.129938	1.4147	0.70686
46	802.0	1261.64	28.72	119.36	148.08	0.035152	0.094600	0.129752	1.3861	0.72144
47	804.7	1264.34	28.82	119.30	148.12	0.035232	0.094358	0.129590	1.3593	0.73567
48	807.4	1267.04	28.92	119.24	148.16	0.035310	0.094095	0.129405	1.3327	0.75035
49	810.0	1269.64	29.02	119.18	148.20	0.035388	0.093865	0.129253	1.3080	0.76452
50	812.5	1272.14	29.11	119.13	148.24	0.035462	0.093646	0.129108	1.284	0.77881
51	814.9	1274.54	29.20	119.07	148.27	0.035532	0.093411	0.128942	1.2611	0.79295
52	817.4	1277.04	29.30	119.02	148.32	0.035608	0.093200	0.128808	1.2389	0.80716
53	819.9	1279.54	29.39	118.96	148.35	0.035682	0.092965	0.128647	1.2174	0.82142
54	822.2	1281.84	29.48	118.91	148.39	0.035748	0.092775	0.128518	1.1970	0.83542
55	824.6	1284.24	29.56	118.86	148.42	0.035817	0.092552	0.128369	1.1767	0.84983
56	827.0	1286.64	29.65	118.81	148.46	0.035887	0.092339	0.128226	1.1573	0.86408
57	829.3	1288.94	29.74	118.76	148.50	0.035951	0.092135	0.128086	1.1388	0.87811
58	831.6	1291.24	29.83	118.71	148.54	0.036020	0.091940	0.127960	1.1210	0.89206
59	833.8	1293.44	29.91	118.66	148.57	0.036082	0.091738	0.127820	1.1032	0.90645
60	836.1	1295.74	29.99	118.61	148.60	0.036148	0.091540	0.127688	1.0865	0.92038
61	838.3	1297.94	30.08	118.56	148.64	0.036213	0.091350	0.127563	1.0700	0.93457
62	840.4	1300.04	30.16	118.51	148.67	0.036275	0.091161	0.127435	1.0545	0.94831
63	842.6	1302.24	30.24	118.46	148.70	0.036339	0.090970	0.127309	1.0391	0.96237
64	844.8	1304.44	30.32	118.41	148.73	0.036400	0.090780	0.127180	1.0243	0.97627
65	847.0	1306.64	30.40	118.37	148.77	0.036460	0.090600	0.127060	1.0100	0.99009
66	848.8	1308.44	30.466	118.326	148.792	0.036509	0.090433	0.126942	0.9958	1.0042
67	850.8	1310.44	30.541	118.283	148.824	0.036565	0.090262	0.126827	0.9822	1.0181
68	852.8	1312.44	30.616	118.238	148.854	0.036621	0.090090	0 126711	0.9689	1.0321
69	854.7	1314.34	30.687	118.197	148.884	0.036677	0.089928	0.126605	0.9560	1.0460
70	856.6	1316.24	30.758	118.155	148.913	0.036729	0.089767	0.126496	0.9436	1.0597
71	858.5	1318.14	30.828	118.113	148.941	0.036785	0.089605	0.126390	0.9313	1.0737
72	860.4	1320.04	30.899	118.071	148.960	0.036837	0.089445	0.126282	0.9195	1.0875
73	862.3	1321.94	30.970	118.030	149.000	0.03689	0.08928	0.12617	0.9080	1.1013
74	864.2	1323.84	31.041	117.988	149.029	0.03695	0.08912	0.12607	0.8968	1.1150
75	866.0	1325.64	31.108	117.948	149.056	0.03700	0.08897	0.12597	0.8859	1.1287
76	867.8	1327.44	31.175	117.909	149.084	0.03705	0.08882	0.12587	0.8750	1.1428
77	869.6	1329.24	31.242	117.869	149.111	0.03710	0.08867	0.12577	0.8617	1.1564
78	871.3	1330.94	31.306	117.831	149.137	0.03715	0.08853	0.12568	0.8545	1.1702
79	873.1	1332.74	31.373	117.792	149.165	0.03720	0.08838	0.12558	0.8447	1.1838
80	874.8	1334.44	31.436	117.754	149.190	0.03725	0.08824	0.12549	0.8349	1.1977
81	876.6	1336.24	31.504	117.715	149.219	0.03730	0.08809	0.12539	0.8256	1.2112
82	878.4	1338.04	31.571	117.675	149.246	0.03735	0.08794	0.12529	0.8164	1.2248
83	880.1	1339.74	31.634	117.638	149.272	0.03739	0.08780	0.12519	0.8073	1.2386
84	881.9	1341.54	31.701	117.598	149.299	0.03744	0.08766	0.12510	0.7987	1.2520
85	883.7	1343.34	31.768	117.559	149.327	0.03749	0.08751	0.12500	0.7901	1.2656
86	885.2	1344.84	31.824	117.526	149.350	0.03754	0.08739	0.12493	0.7816	1.2794
87	886.8	1346.44	31.884	117.490	149.374	0.03758	0.08726	0.12484	0.7734	1.2929
88	888.4	1348.04	31.944	117.455	149.399	0.03763	0.08713	0.12476	0.7655	1.3063
89	890.0	1349.64	32.003	117.420	149.423	0.03767	0.08700	0.12467	0.7575	1.3201
90	891.6	1351.24	32.063	117.385	149.448	0.03771	0.08687	0.12458	0.7497	1.3338
92	894.8	1354.44	32.182	117.314	149.496	0.03779	0.08661	0.12440	0.7343	1.3618
94	897.9	1357.54	32.298	117.246	149.544	0.03788	0.08637	0.12425	0.7201	1.3886
96	900.9	1360.54	32.410	117.180	149.590	0.03797	0.08613	0.12410	0.7068	1.4148
98	903.9	1363.54	32.522	117.114	149.636	0.03805	0.08589	0.12394	0.6937	1.4415
100	906.9	1366.54	32.634	117.048	149.682	0.03813	0.08565	0.12378	0.6811	1.4682
102	909.9	1369.54	32.746	116.982	149.728	0.03821	0.08542	0.12363	0.6687	1.4954
104	912.8	1372.44	32.854	116.918	149.772	0.03829	0.08519	0.12348	0.6570	1.5220
106	915.6	1375.24	32.958	116.857	149.815	0.03837	0.08497	0.12334	0.6457	1.5487
108	918.4	1378.04	33.063	116.795	149.858	0.03844	0.08475	0.12319	0.6349	1.5750
110	921.1	1380.74	33.163	116.736	149.899	0.03852	0.08454	0.12306	0.6242	1.6020
112	923.8	1383.44	33.264	116.676	149.940	0.03859	0.08434	0.12293	0.6141	1.6283
114	926.5	1386.14	33.365	116.617	149.982	0.03866	0.08413	0.12279	0.6044	1.6545
116	929.2	1388.84	33.466	116.558	150.024	0.03874	0.08392	0.12266	0.5949	1.6809
118	931.8	1391.44	33.563	116.500	150.063	0.03880	0.08373	0.12253	0.5856	1.7076

(Continued on following page)

TABLE 1 PROPERTIES OF MERCURY VAPOR—CONTINUED

Pressure, lb. per sq. in. abs.	Temp., deg. fahr.	Abs. temp., deg. fahr.	Heat of liquid above 32 deg., $q=0.0373\ (t-32)$	Heat of vaporization $r=128.2-0.022\ (t-400)$	Total heat $H=q+r$	Entropy of liquid above 32 deg.	Entropy of vaporization	Total entropy	Specific volume, cu. ft. per lb. V_s	Weight, lb. per cu. ft. $1/V$
120	934.4	1394.04	33.660	116.443	150.103	0.03887	0.08353	0.12240	0.5767	1.7340
122	936.9	1396.54	33.753	116.388	150.141	0.03894	0.08334	0.12228	0.5681	1.7602
124	939.4	1399.04	33.846	116.333	150.179	0.03901	0.08315	0.12216	0.5597	1.7866
126	941.9	1401.54	33.939	116.278	150.217	0.03908	0.08296	0.12204	0.5515	1.8132
128	944.3	1403.94	34.029	116.225	150.254	0.03914	0.08278	0.12192	0.5437	1.8392
130	946.7	1406.34	34.118	116.173	150.291	0.03920	0.08261	0.12181	0.5360	1.8656
132	949.1	1408.74	34.208	116.120	150.328	0.03927	0.08243	0.12170	0.5287	1.8914
134	951.4	1411.04	34.294	116.069	150.363	0.03933	0.08226	0.12159	0.5216	1.9171
136	953.7	1413.34	34.379	116.019	150.398	0.03939	0.08209	0.12148	0.5145	1.9436
138	956.0	1415.64	34.465	115.968	150.433	0.03945	0.08192	0.12137	0.5077	1.9696
140	958.3	1417.94	34.551	115.917	150.468	0.03951	0.08175	0.12126	0.5012	1.9952
142	960.6	1420.24	34.637	115.867	150.504	0.03957	0.08158	0.12115	0.4948	2.0210
144	962.8	1422.44	34.719	115.818	150.537	0.03963	0.08142	0.12105	0.4886	2.0466
146	965.0	1424.64	34.801	115.770	150.571	0.03969	0.08126	0.12095	0.4824	2.0729
148	967.2	1426.84	34.883	115.722	150.605	0.03974	0.08110	0.12084	0.4765	2.0986
150	969.4	1429.04	34.965	115.673	150.638	0.03980	0.08094	0.12074	0.4706	2.1249
155	974.7	1434.34	35.163	115.557	150.720	0.03994	0.08056	0.12050	0.4566	2.1900
160	979.9	1439.54	35.357	115.442	150.802	0.04007	0.08019	0.12026	0.4438	2.2531
165	985.0	1444.64	35.547	115.330	150.877	0.04020	0.07983	0.12003	0.4316	2.3191
170	989.9	1449.54	35.730	115.222	150.952	0.04033	0.07949	0.11982	0.4200	2.3807
175	994.8	1454.44	35.912	115.114	151.026	0.04046	0.07915	0.11961	0.4091	2.4440
180	999.6	1459.24	36.091	115.009	151.100	0.04058	0.07881	0.11939	0.3990	2.5062

DISCUSSION

LIONEL S. MARKS.[1] The writer's discussion of the paper will be devoted to the efficiencies possibly attainable with the mercury-vapor process. Unfortunately, the physical properties of mercury are not well established. There is a considerable error in the values of heat of the liquid tabulated in the paper. They are based on a specific heat of 0.0373 quoted from Kurbatoff (*Zeit. Phys. Chem.*, 1903), which value was later found to be in error. The same investigator, repeating his work, found a value of between 0.0325 and 0.0331 (Russian Physical-Chemical Society, 1908), which agrees well with the values of Bronsted (1912), Barnes, and all the other investigators. A value of about 0.033 is probably correct and this is 13 per cent less than the value used in this paper. The tables prepared by Kearton (Proc. Inst. M. E., 1923) seem to be preferable and have been used in the writer's calculations.

Mercury has an advantage as a fluid for extending the temperature range, in that its Rankine-cycle efficiencies are higher than those for steam operating through the same range. This is due to the fact that the heat of the liquid has a smaller ratio to the latent heat. The specific heat of liquid mercury is about one-

[1] Professor of Mechanical Engineering, Harvard University, Cambridge, Mass. Mem. A.S.M.E.

thirtieth that of water; the latent heat is about one-eighth. The ratio of specific heat to latent heat for mercury is consequently about one-fourth that for steam. Fig. 16 shows the ratio of Rankine-cycle efficiency to Carnot-cycle efficiency for these two fluids. It will be seen that, whereas the Rankine-cycle efficiency with high-pressure steam is about 85 per cent of the corresponding Carnot efficiency, with mercury vapor it is over 95 per cent. This gives an initial advantage to mercury over steam of about 11 per cent. By regenerative feed heating, the cycle efficiency of steam can be increased; with mercury, regenerative heating would have little value.

In calculating theoretical efficiencies for this binary-vapor system, maximum efficiencies will apparently be obtained with heat

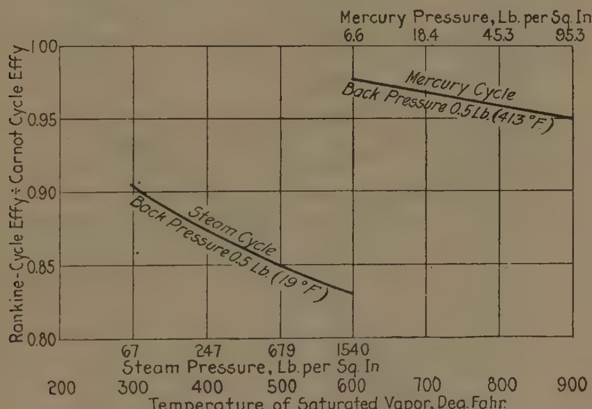


Fig. 16 Ratio of Rankine-Cycle to Carnot-Cycle Efficiencies for Steam and Mercury

taken in only at the mercury boiler, and with regenerative heating of the liquid mercury and the feedwater. But under these conditions the flue gases will escape at about 1150 deg. fahr., and the boiler efficiency will be very low. If the flue gases are also used for heating the liquid mercury, for superheating the steam and for feedwater heating, the cycle efficiency will be considerably lower, but the flue gases will be well cooled and the boiler efficiency correspondingly raised. It is important, therefore, to scrutinize the proposed cycles carefully to insure that they permit reasonable boiler efficiencies.

The curves of Figs. 17 and 18 show cycle efficiencies for two conditions which permit of usual boiler efficiencies. In Fig. 17 the mercury vapor is assumed dry and saturated, and is considered generated at four different pressures from 20 to 70 lb. per sq. in. abs. The condenser temperature for the mercury is assumed the same as the temperature of steam generation, and calculations

have been made for mercury-condenser temperatures corresponding to steam pressures from 200 to 800 lb. per sq. in. abs. The four curves given show the efficiencies when the steam is superheated to 700 deg. fahr. and expands to 1 in. mercury pressure. The flue gases heat the liquid mercury and evaporate it, superheat the steam, and heat the feedwater as in Fig. 8. For comparison, the efficiency of a steam regenerative cycle up to 1200 lb. pressure (taken from the paper by Hirshfeld and Ellenwood [1]), is included in Fig. 17. It will be seen that cycle efficiency falls with decrease in the temperature range in the mercury turbine and consequent increase in the temperature range of the steam turbine.

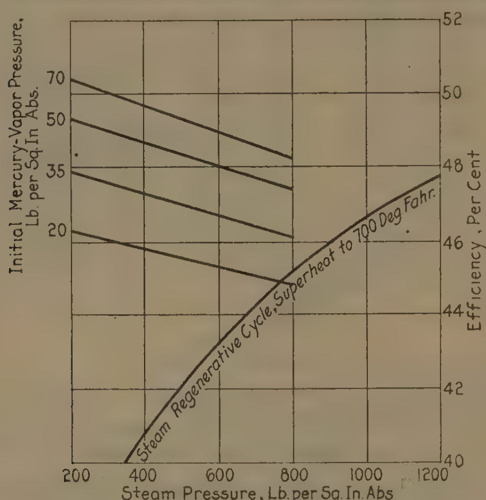


Fig. 17 Efficiency Curves of Mercury-Vapor Cycle and of Regenerative Steam Cycle

(Mercury vapor dry and saturated; steam superheated to 700 deg. fahr.; complete feedwater heating by use of flue gases. Steam-condenser pressure 1 in. for all cycles.)

The curves of Fig. 18 when compared with those of Fig. 17 show the effect of regenerative feedwater heating. In Fig. 18 this is supposed to heat the feedwater from the condenser temperature to that temperature at which the steam becomes dry and saturated as a result of adiabatic expansion. In all other respects the conditions are as for Fig. 17. The feedwater heating is completed to the boiler temperature by use of the flue gases. This condition will permit of high boiler efficiency, especially if air preheating is used. The efficiencies are notably improved, as compared with Fig. 17, and they increase with increased steam boiler pressure.

[1] Trans. A.S.M.E., vol. 45, 1923, p. 663.

The maximum of 54 per cent is shown for 70 lb. mercury pressure and 800 lb. steam pressure.

No attempt has been made to estimate probable actual efficiencies, but a comparison with the theoretical steam efficiencies will indicate the order of magnitude of the possible economy by the use of the mercury-vapor system.

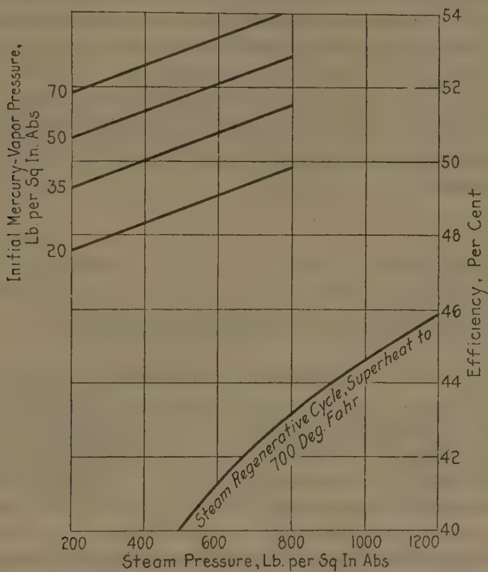


Fig. 18 Efficiency Curves of Mercury-Vapor Cycle

(Mercury vapor dry and saturated; steam superheated to 700 deg. fahr. by chimney gas; feedwater heated by regeneration to adiabatic dewpoint temperature and thence by chimney gases to boiler temperature. Steam-condenser pressure 1 in. for all cycles.)

Joseph Pope.[1] There have recently been made some careful tests of the equipment as installed in the Dutch Point Station of the Hartford Electric Light Co., the results being briefly,

Fuel oil burned per hour	2,046 lb.
Net electricity generated by mercury turbo-generator	1,258 kw-hr.
Weight of steam made per hour	24,750 lb.
Heat value of oil as fired	18,534 B. t. u.
Temperature of water fed to economizer	197 deg. fahr.
Gage pressure of steam delivered	188 lb. per sq. in.
Superheat of steam delivered	49 deg. fahr.
Weight of oil fired per net kw-hr, generated by mercury turbo-generator only	1.63 lb.
Weight of steam delivered per lb. of oil fired	12.1 lb.

[1] Steam Power Research Engineer, Stone & Webster, Inc., Boston, Mass. Mem. A.S.M.E.

Attention is called particularly to the last two figures. If we ignore entirely the steam made by the process, we find that on this test a kilowatt-hour of electric energy was obtained about as cheaply as any straight steam plant of corresponding size can deliver it, i.e., for slightly less than $1\frac{3}{8}$ lb. of oil per kw-hr. If, on the other hand, we choose to ignore the electric power produced and look only at the steam output, we find that again the performance is favorably comparable with straight steam-plant practice, for the actual evaporation is 12.1 lb. water per pound of fuel, or an efficiency of about 70 per cent.

In order to show what the effect would be of combining mercury equipment identical with that in Hartford with an existing steam plant, some calculations, of which the following is a brief abstract, have been made:

Existing Steam Plant

Peak load	9,600 kw.
Annual output	57,850,000 kw-hr.
Annual load factor................	70 per cent
B.t.u. per kw-hr....................	26,451

(These figures are matters of actual record.)

Four Mercury Units Combined with Existing Steam Plant

Peak load	9,600 kw.
Estimated output by mercury........	30,050,000 kw-hr.
Estimated output by steam..........	27,800,000 kw-hr.
Total output	57,850,000 kw-hr.
Estimated B.t.u. per kw-hr..........	19,020
Saving in fuel over steam plant......	28 per cent

It will be observed that these figures are for the same steam-plant generating equipment and for the same total output.

Another way of making the comparison is to indicate the additional capacity to be obtained from the replacement of fuel-fired steam boilers with mercury-vapor units. Using the same existing plant as a basis, it appears that six mercury units of the Hartford size would be necessary to furnish the steam required to develop the present electrical output, and that 98 per cent additional power would be generated by the mercury turbo-generator at an expenditure of 22 per cent more fuel. The net heat consumption under these circumstances would be 16,300 B.t.u. per kw-hr.

T. H. Soren.[1] We have been operating the mercury-vapor turbine in Hartford since last September. The author, so far as we can see, has overcome all the operating difficulties. The plant has now operated for somewhere over 800 hours, without causing any trouble. We had to shut down once for a short time because the oil had carbonized in the heater, and are now about to shut down to clean the boiler flues. We have had a little trouble with

[1] Vice-President, Hartford Electric Light Co., Hartford, Conn.

the air pump, but the mercury-vapor process itself has caused no trouble whatever. The General Electric Company, through two of its engineers, established the method of operating and instructed two of our best men — a turbine man and a fireman — and these men have trained others of our force. This crew has operated the plant week in and week out 24 hours per day on three shifts. One man handles the turbine and another the fire. During the time the plant has operated we have had to shut down, start up, etc. This operation has been carried on regularly with our own forces without the presence of a representative of the General Electric Company. The troubles that the author has described as part of the development are non-existent in operation. We are quite ready to agree with the author that it will be possible to obtain one kilowatt-hour for 11,000 B.t.u., although it may be several years before we attain this economy.

R. D. De Wolf.[1] A point of interest to the operating engineer is the question as to what would happen in a straight mercury plant, that is, a mercury-vapor plant with no steam plant in connection with it, in the event of a sudden increase in the load on the plant. In the steam plant there is a large amount of heat energy stored in the water of the boilers. Energy can be transferred from the coal into the water in the boilers very quickly, and in the case of a sudden load a draft can be made on the heat stored in the boilers and coal in the stokers to carry the load. In the case of the mercury turbine, the mercury boiler and turbine appear to form a buffer between the steam turbine and the fuel-burning apparatus, and in order to obtain additional power from the steam turbine there must first be put into the mercury boiler the additional power that the mercury turbine will use plus the additional heat energy that must go to the steam turbine. A question then arises as to whether this involved process can be carried out quickly enough to hold or pick up the load in as satisfactory a manner as in the case of a straight steam system.

Henry M. Lane.[2] It will be of interest also to know the possibilities of the mercury-vapor plant, when the reverse of the conditions outlined by Mr. De Wolf obtain. That is, how will the mercury plant act when the entire load suddenly goes off the unit?

W. L. R. Emmet. In reply to the questions by Messrs. De Wolf and Lane, the mercury boiler under the circumstances cited is exactly like a steam boiler making steam with a certain fixed

[1] Chief Operating Engineer, Rochester Gas & Elec. Corpn., Rochester, N. Y. Mem. A.S.M.E.
[2] President, H. M. Lane Co. Detroit, Mich. Mem. A.S.M.E.

condition of fire. There is heat stored in the steam, and a certain amount of heat also will be stored in the mercury boiler. The net output cannot be greater than the energy the fire is giving out, either with steam or with mercury. In a steam plant, with increased load, the firing is forced so that the load may be held. It would be undesirable to do that with mercury. The mercury equipment is designed to run normally at one pressure, but not at a much higher pressure. After it has reached its normal load the mercury turbine could be bypassed, the excess mercury vapor being used to make steam, but it is not advisable to provide enough mercury and mercury boiler capacity to thus provide for an overload condition. However, there is a possibility that with calorized tubes, which will stand overheat, the boiler might be forced and the safety valves allowed to blow and make more steam without injuring the boiler. In such an event there would be nothing to prevent forcing the fire within certain limits. We have not intended to do this, for the reason that it is best to consider the mercury-vapor equipment as a base-load proposition, using it up to a certain limit of load, which is high, as compared with a steam boiler.

In regard to the load going off suddenly, the action would be the same as with steam. There being no hot fires under the steam-boiler end of the plant, the safety valves might blow and empty the boilers of water without damage. The mercury boiler would continue to operate and pump mercury vapor through the same nozzles. If a load came on or off the turbine the valve would open or close, but the vapor would pass on through the steam boiler just the same.

Replying to certain other questions that have been asked: When we first began to consider a binary-vapor turbine we investigated many substances that might work as does the mercury, with the result that the author has decided that there is no other substance than mercury that is available. We thought that certain hydrocarbons derived from benzol might be used. We experimented with diphenyl and diphenyl ether, and benzophenone. We boiled them all at high pressure and condensed them, but they decomposed and degenerated, being unable to stand the high rate of heat delivery. Trichloride of antimony, which has a high boiling point and which seemed to have certain characteristics of a rather simple combination, also decomposed. Sulphur was found to be stable in boiling. It can exist in various forms, ranging from S_2 to S_8, which would involve very different vapor volumes. However, it seems to remain as S_8 throughout the range in which we would use it. Its use, however, would involve various difficulties. It is viscous at any available condensing temperature; it is one of the worst heat conductors in the world; and it almost solidifies at temperatures in the neighborhood of

400 deg. fahr. We conducted certain tests to ascertain the size of condensing apparatus necessary for using sulphur and found that it would have to be very large although not entirely prohibitive. The relative cheapness of sulphur as compared to mercury is of little advantage when the size of the apparatus is considered. Furthermore, the sulphur would have to be pumped through the apparatus in a viscous condition, and the pumps would be very large.

While sulphurous anhydride has been proposed as a fluid for binary-vapor machines, it was to overcome the low efficiency at the vacuum end which is unavoidable with reciprocating steam engines. The introduction of the turbine has removed the reason for the SO_2 engine, because we now can work down to the temperature of the condensing water with good efficiency.

Replying to an inquiry as to the rate of heat transfer in the condenser, the plant at Hartford, owing to the low steam pressure, is operating at an abnormally high vacuum, much higher than is desirable in a mercury condenser. We do not know to what extent the vacuum is limited by the air pump, nor do we know the exact rate of condensation. We do know, however, that we are delivering heat to the steam from the mercury vapor at a rate of about twice the highest rate that we know of for steam practice. It is quite possible that a still higher rate could be obtained. Condensing conditions with mercury are peculiarly favorable. Instead of wetting the surface, the condensate is constantly falling off and leaving it exposed. The falling drops of condensate themselves are cooled and keep stirring up the vapor in the presence of the condensing surface. Steam can be made with a remarkably small condensing surface.

As regards pressures that can be attained in the mercury boiler, while we have designed for 70 lb. per sq. in. gage, a pressure of 180 lb. should be practicable. This corresponds to a temperature of 1000 deg. fahr. The temperature drop in the turbine would then be about 565 deg., so that steam would be made at 435 deg. fahr., 350 lb. pressure.

It has been asked whether or not it will be practicable to use waste gases, as from a gas engine, to generate mercury vapor, to be utilized in a turbine, and then to condense the mercury and generate steam for a steam turbine. Any form of waste heat whatever can be used in the mercury-vapor process, as well as in any steam process.

No. 1925

PERFORMANCE OF CENTRIFUGAL FANS FOR ELECTRICAL MACHINERY

By Carl J. Fechheimer,[1] East Pittsburgh, Pa.

Non-Member

The ordinary commercial centrifugal fan is provided with a stationary collecting device, generally a volute, which usually improves the performance materially. In the fans used for cooling electrical machinery, however, such collecting devices cannot be readily applied and the revolving impeller therefore forms the sole means for developing the pressure needed to drive the air through the vent ducts. The present paper gives results of an extended series of tests recently made to determine the performance of centrifugal fans without collecting devices, and to obtain sufficient data to enable designers to select types of fans intelligently and to predict their performance.

In THE construction of most of the modern electrical machinery, air is the cooling medium, and fans are frequently attached to the rotating element to drive the air through the vent passages. While in some machines, such as in certain direct-current machines, the coils or other machine parts act as impellers for the air and additions thereto are unnecessary for cooling, in many others, particularly in enclosed machines, such as steam-turbine-driven generators, there are definite fans added near the ends of the rotor body. Even in open machines it has been found in many cases that the addition of fans and the gain in cooling afforded thereby are amply justified, although the back pressure against which the fan then works is nearly negligible.

2 The fan problem in electrical machinery differs from that in ordinary applications. Practically every commercial fan is provided with a stationary collecting device, generally a volute, and that adjunct usually improves the fan performance materially. In the electrical machine it is usually found that a collecting device cannot readily be applied, and that the revolving impeller as

[1] Research Engineer, Power Engineering Dept., Westinghouse Elec. & Mfg. Co.

Presented at the Spring Meeting, Cleveland, Ohio, May 26 to 29, 1924, of The American Society of Mechanical Engineers.

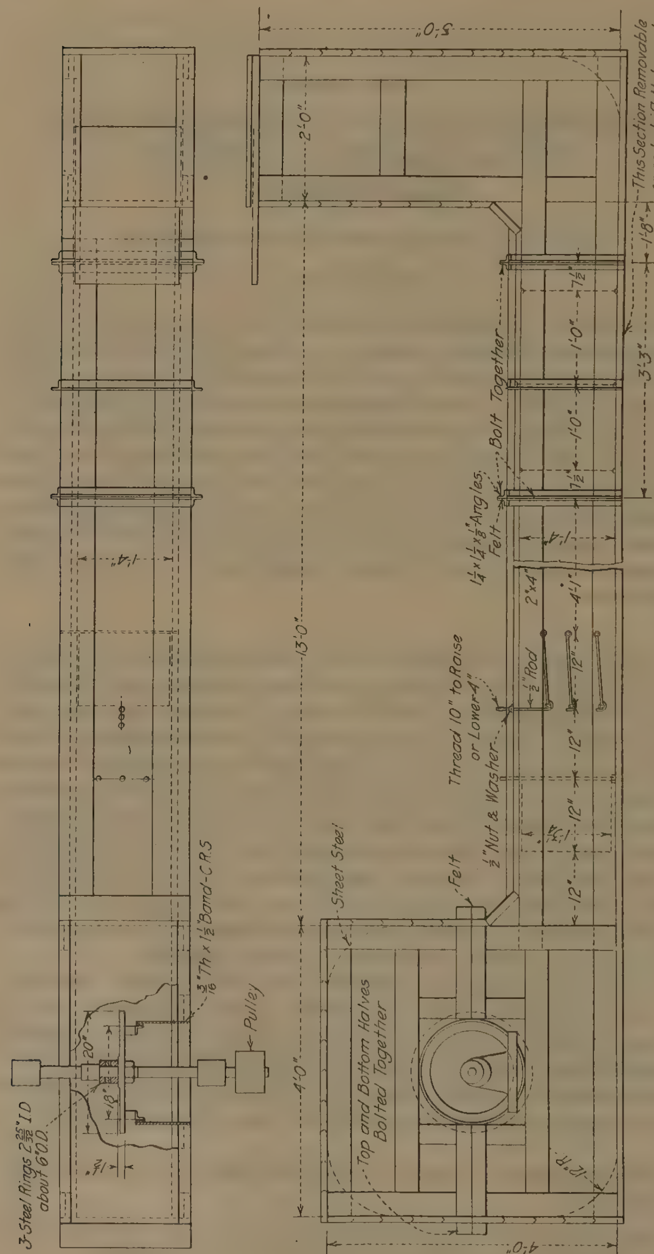


Fig. 1 Plan and Elevation of Equipment Used in Testing Centrifugal Fans for Cooling Electrical Machinery

such must be the sole means for developing the pressure needed to drive the air through the vent ducts.

3 Our principal problem was to determine the performance of centrifugal fans without a collecting device, and to obtain sufficient data to enable them to be applied by the designer of electrical machinery, when specifying the proportions of the particular electrical machine. So far as we are aware, no data on this subject have hitherto been published. Furthermore, the published methods for testing fans were not suitable for our requirements, particularly in regard to the measurement of pressure, and it became necessary to devise our own ways. The methods adopted were described in the May and July, 1923, issues of the *Electric Journal,* but they are in part repeated here.

4 The earliest systematic tests on centrifugal fans intended for use in electrical machinery which were conducted at the works of the Westinghouse Electric & Manufacturing Company were made in 1916. These tests were carried out on quite a large scale, and in consequence did not cover the range that would be feasible if they had been made on a small scale. The measurements of pressure and volume were subsequently considered to be somewhat inaccurate, and therefore the tests were later repeated on a smaller scale, covering a wider range, and the errors of measurements reduced. The latter tests, which are described in this paper, were made in the early part of 1921 at the East Pittsburgh Works of the Westinghouse Company, and the results obtained were incorporated in a book which has not yet been published. Parts of that work appeared in the *Electric Journal* between August, 1922, and August, 1923, under the title Some Elements of Air Flow in Electrical Machinery.

5 It was our aim to cover, in part, the performance of centrifugal fans as influenced by the following independent variables: (1) Shapes of blades; (2) number of blades; (3) width of blades; (4) depth of blades; (5) influences of leakage paths with different kinds of blades; (6) influence of intake restrictions; and (7) effects of certain external influences.

The Test Rig

6 The construction of the equipment used in making these tests is shown in Figs. 1, 2, and 3. The rig includes a wheel mounted on a suitable shaft supported in bearings at either side and driven by belt from an electric motor. This wheel rotates in a wooden box, at one end of which a chute is joined. After flowing the length of the chute the air turns through a right angle and is discharged upward, the outlet from the vertical duct being controlled by means of a horizontal door which is used as a valve.

7 Inasmuch as a number of different widths of fans were to be tested, collars were placed on the shaft, which could be moved from one side of the wheel to the other so as to permit axial

movement of the wheel as desired. In order to minimize leakage of air for most tests, a $1\frac{1}{2}$-in. by $1\frac{1}{2}$-in. angle iron was bent in a circle to form a flange ring, its external diameter (18 in.) corresponding with that of the fan. The horizontal part of this flange ring cleared a stationary ring by about $\frac{1}{16}$ in. In consequence the shaft could have the usual axial play in its bearings

FIG. 2 VIEW OF TESTING RIG SHOWN IN FIG. 1

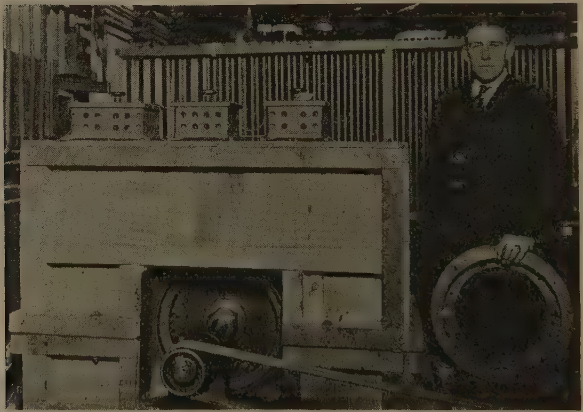


FIG. 3 ANOTHER VIEW OF TESTING RIG SHOWN IN FIG. 1

without much leakage through the $\frac{1}{16}$-in. clearance; and, what is more important, the conditions under which all the fans were tested were identical in this respect when the rotating flange ring was used.

8 The box, chute, and outlet were made of suitable tongue-and-groove lumber, supported by heavier members. To minimize the leakage at the joints the wooden structure was lined with

muslin pasted on, the muslin being painted and then sandpapered. The top cover of the chute was fastened by means of screws, soft felt gaskets being used to reduce leakage to a minimum. The outlet valve (at the top of the discharge), which could be moved horizontally as required, thereby regulating the flow, was also protected against leakage by means of felt fastened to the non-movable part. The box was split horizontally along the center line of the shaft to facilitate changing fans, the two halves being bolted together, with felt gaskets between the halves. In each upper and in one lower corner of the box pieces of sheet steel curved at a suitable radius were placed in order that the air would not pocket in the corners.

9 There are six air guides in the chute, three of which are vertical and the other three nearly horizontal. These air guides consist of flat sheet steel, one dimension of which is just a little less than the corresponding dimension of the chute, the other dimension being 12 in. One end of these steel sheets is curved and bent around a suitable rod so that it can turn about this rod freely. At the other end of the sheets, steel straps are connected, the ends being brought out through the wall of the chute, thereby affording a means of adjusting the guides without opening the chute. When the tests were started, these guides were so set that the air velocities were nearly uniform when a large volume was being discharged from the fan.

10 Toward the end of the chute near the vertical outlet is a removable section 39 in. long, and well braced with suitable angle irons. In this section was the volume meter, which is described subsequently.

11 The air was discharged upward because horizontally discharged air would have been objectionable to other persons working in the same room. The cross-section of the duct (1.78 sq. ft.) was sufficiently large to give low velocities with the maximum volumes handled, so that the pressure drop was never very great.

The Volume Meter

12 The meter, as used, was a modified form of the Thomas meter,[1] and consisted of a heater, which was placed at the central part of the section of the duct intended for the meter, and two

[1] The principle of operation of the meter is that the heat developed electrically is picked up by the moving air streams, the temperature of the air being dependent upon (1) the mass per minute, (2) the number of heat units developed in the heater per minute, and (3) upon the specific heat of the air. In the meter developed by Prof. Carl Thomas, the temperature rise of the air (or other gas) is measured by means of resistance exploring coils connected with other fixed external resistances in a Wheatstone-bridge network. For our particular work, thermocouples were found to be preferable for measuring the air-temperature rise. The power input to the heater, measured electrically, gives the heat developed per unit of time.

sets of thermocouple junctions, one set about 12 in. in front of, and the other set the same distance behind, the heater. The heater was made of No. 17 B. & S. gage Advance[1] wire, the resistivity of which is about 28 times that of copper at 25 deg. cent. Sixty-two wires were strung vertically, being fastened to the upper and lower walls by means of screw eyes. The wires were arranged in two equal rows staggered with respect to each other. The resistance of the entire heater was then approximately 12 ohms.

13 The heater wires should be vertical rather than horizontal, in order that sagging and short-circuiting be avoided. In fact, it was found advisable, even with the comparatively low temperatures used, to brace the heater wires by means of a glass rod about half-way up.

14 The thermocouples consisted of No. 20 copper and Advance wire, the junctions having been welded in a percussion welder. There were 25 junctions in each side, so arranged as to have them uniformly distributed throughout the cross-section. Thus the square section (16 in. by 16 in.) was divided into 25 equal squares and a thermocouple junction was put into each of the small squares. The thermocouples in the two sets were connected in series; thus a cold junction in front of the heater was joined in series with the corresponding hot junction located behind the heater. In joining these couples in series it is of course essential to connect the Advance wire in one set to the corresponding wire in the other set and similarly for the copper. The terminals were then brought out from the copper wires at either end.

15 The scheme of connections is shown in Fig. 4. A potentiometer reading in millivolts was used for determining the difference in temperature between the entering and outgoing air of the volume meter as indicated by the thermocouples. For simplicity, only two hot and two cold couples are shown, and the potentiometer is indicated as a simple slide-wire bridge. As accuracy of the measurements is largely dependent upon the potentiometer, a reliable instrument, which has been carefully calibrated, should be used.

16 The equation[2] for volume, taking into account atmospheric conditions, namely, pressure and temperature, is as follows:

$$Q=\frac{0.178nEW(273+t)}{1000eP}\text{ cu. ft. per min.}$$

where n = number of hot or cold thermocouples
e = millivolts read on potentiometer
W = watts lost in heater
t = temperature of air entering heater, degrees centigrade
E = microvolts per degree from thermocouple calibration
P = barometric pressure in inches of mercury.

[1] Advance is a trade name for constantan, a copper-nickel alloy.
[2] The derivation of this equation is given in the Appendix, p. 337.

This equation is based upon a specific heat of 0.2418 and a density of 0.074 lb. per cu. ft. at 25 deg. cent. and 29.92 in. of mercury.

17 In determining the value of E it is well to have a calibration curve (microvolts and temperature) of a number of the thermocouples that are used; since this calibration curve is not quite a straight line, the microvolts per degree should be read at approximately the average temperature of the air by drawing a tangent to the curve at that point.[1] The slope of this tangent is the value desired.

18 It was thought that the heat which escapes from the outside of the volume meter might be sufficient to influence the readings. But it was found that the temperature of the outside of the meter with or without heat-insulating material was substantially the same as that of the surrounding air, so that the amount of heat escaping was of no consequence.

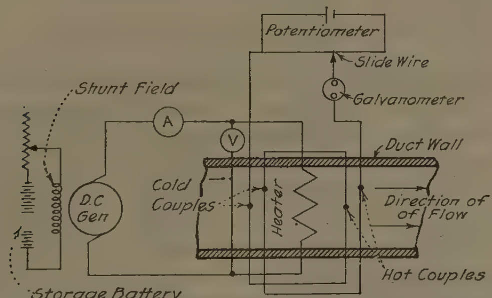


Fig. 4 Arrangement of Heater Coils and Thermocouples in the Volume Meter

19 A considerable error may be introduced with this type of volume meter if the velocities are materially different for different sections, but departures from uniformity should not cause the errors in measurement to be more than about one per cent. If the sectional area of the duct be broken into a number of small, equal areas, then the ratio of the volume that would be measured were the velocities uniform to the volume that is measured, is given in the equation[2]

$$R = \frac{(\Sigma v)\left(\Sigma \frac{1}{v}\right)}{n^2}$$

[1] Another way is to apply the derivative of the equation $E=at+bt^2$, where E is microvolts and t, temperature. a and b are constants, evaluated by selecting two points on the curve.

[2] The derivation of this equation is given in the Appendix, p. 339.

where v is the velocity considered uniform in any small area and n the number of small areas.

20 In order to determine the distribution of velocities in the duct, a pitot tube was used, which was inserted to admit of exploration of velocities — along the vertical and subsequently along the horizontal center lines — beyond the volume meter. The air guides previously described were adjusted until the error due to non-uniformity of velocities was less than one per cent. The readings with the pitot tube were taken with a large volume of air, as otherwise the manometer readings would have been too small; and in using the pitot tube it was not of importance to know the actual velocities, as only the relative velocities from one position to the next were desired.

Static-Pressure Measurement

21 Various difficulties were encountered in the measurement of static pressures in our fan tests. The air inside the box in which the fan rotated was undoubtedly in motion, whirling around with the fan, the velocity of whirl decreasing as the radius increased. Consequently higher pressures were developed at the outer periphery of the box than near the fan; perhaps, too, a little of the velocity head was converted into pressure. The conditions that obtain in an electrical machine, as far as the developed pressure of the fans is concerned, are believed to be about the same as those secured near the periphery of the box in which the fan rotated. If air near the inside of the wall of the box were trapped so as to be stationary and if the pressure of this trapped air were then measured, an accurate equivalent of the useful pressure in the end bell of an electrical machine would be obtained. Three curved pieces of sheet steel were put in the corners of the box shown in Fig. 1 principally for this purpose, there being enough leakage at the ends of these sheets axially to permit air to enter the "trap" back of the curved sheet and assume the same pressure as in the box near them. A hole was bored from the outside into one of these traps and a brass tube was inserted which was joined by a hose to the manometer. This brass tube was pushed through a hole drilled through a soft rubber stopper, and this stopper was pushed tight into the hole bored into the wall of the box. This stopper then prevented leakage of air around the outside of the tube, which was important, because any leakage would have meant that air flowed from the box into the trap behind the sheet steel, producing a pressure drop.

22 Nevertheless it was felt that the method of measuring pressure by this means might have been subject to error; thus,

pulsations and turbulence around the fan might produce alternate flow into and out of the trap. Accordingly a special pressure gage was devised which is shown in Fig. 5. The principle used in this instrument is that, with a fairly steady flow through the duct, the static pressure is that which is exerted against the side walls and is independent of the velocity head. This gage has a diaphragm about 0.01 in. in thickness which is flush with the inside surface of the upper wall of the duct. A "test indicator" such as is used for measuring small deflections and reading to 0.001 in. per division, has its plunger in contact with the upper side

Fig. 5 Special Pressure Gage Devised for the Tests

of the diaphragm. When the pressure in the duct is above that of the atmosphere, the diaphragm moves upward and the needle of the gage deflects. A small hand pump is then used to increase the pressure in the small chamber above the diaphragm until the indicator reads the same as when no air flows. A manometer reads the pressure in the small chamber above the diaphragm, which, by means of the pump, is kept the same as the pressure in the duct. This results in a zero method of measurement, like a potentiometer.

23 The pressure in the chute is necessarily lower than the pressure in the box whenever air flows, and there is a loss of head at the exit of the box. It would be expected, therefore, that the

pressure readings taken with this special mechanical gage would be the same at zero volume as those in the trapped chamber in the box, but would read less with appreciable flow. Points taken on one test, shown in Fig. 6, indicate that the pressure drop after the air leaves the box is about as would be expected; thus, at the maximum volume, in this case 1900 cu. ft. per min., the velocity was 1060 ft. per min., corresponding to a velocity head of 0.07 in. of water. There was probably about that loss in getting out of the box and a little more drop from the box to the pressure gage. The actual difference between the trapped-air and mechani-

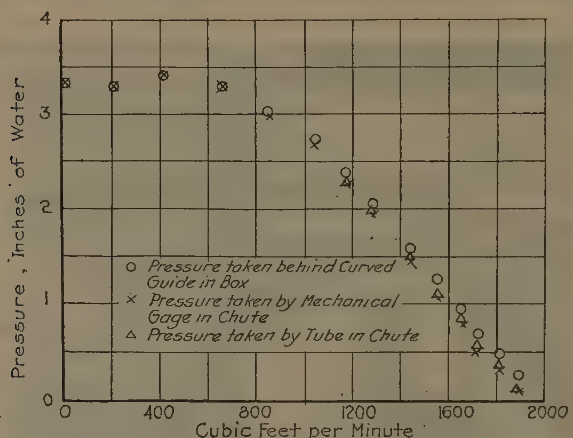


FIG. 6 PRESSURE-VOLUME CURVES OF FAN NO. 11

cal-gage readings was 0.17 in. In any case the reading seemed to indicate that the observations taken of the air trapped in the box were sufficiently reliable for any work with which we have to deal.

24 The observed pressure readings were corrected to 25 deg. cent. and 29.92 in. of mercury, the following equation being used:

$$\text{Standard pressure} = 0.1003 \left(\frac{273+t}{P}\right) \times (\text{Test pressure})$$

where t is the temperature in deg. cent. and P the barometric pressure in inches of mercury.

INPUT READINGS

25 A small motor of about 5 hp. capacity was belted to the fan and the input to this motor was measured in the usual way. The idle losses were obtained by measuring the input to the driving motor when driving the wheel without the fan, the bolts being in place during that test and increasing the loss by about 20

watts. It is believed that service conditions for electrical machinery were substantially duplicated by not charging to the fan the losses due to the wheel, bearings, etc., because in electrical machinery losses arising from the equivalent parts are independent of the input to the fan.

26 The watts input as measured should be corrected back to standard conditions by means of the following equation:

$$\text{Watts standard} = 0.1003\left(\frac{273+t}{P}\right) \times (\text{Watts test})$$

where t is the temperature in deg. cent. and P the barometric pressure in inches of mercury.

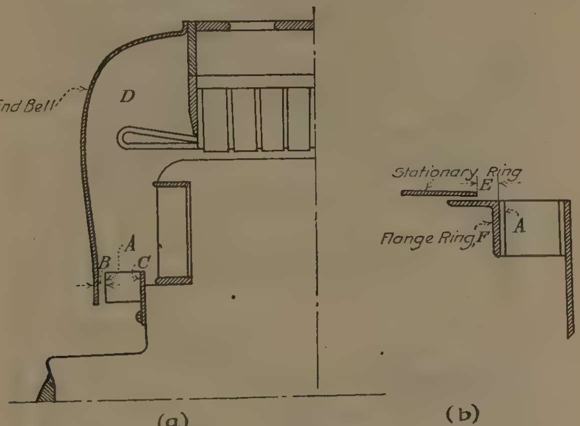


FIG. 7 METHODS OF APPLYING CENTRIFUGAL FANS TO ELECTRICAL MACHINERY

POWER OUTPUT AND EFFICIENCY

27 The power output, neglecting the power represented by the velocity head multiplied by the volume, is:

$$\text{Horsepower} = \frac{\text{Pressure} \times \text{Volume}}{6350}$$

$$\text{Watts} = \frac{\text{Pressure} \times \text{Volume}}{8.51}$$

The pressure is the static pressure in inches of water column, and the volume is in cubic feet per minute.

28 The velocity head of the air stream after leaving the fan has been neglected because (1) it is impossible to evaluate the velocity head when a collecting device is not used, and (2) nearly all of the velocity head is destroyed, and only the static pressure must be relied upon to drive the air through the vent ducts in the electrical machine.

Use of Test Data for Other Sizes of Fans

29 It has been shown that when two fans are similar, the performance of one fan may be determined from the performance of the other on which tests were conducted by means of the following proportionalities:

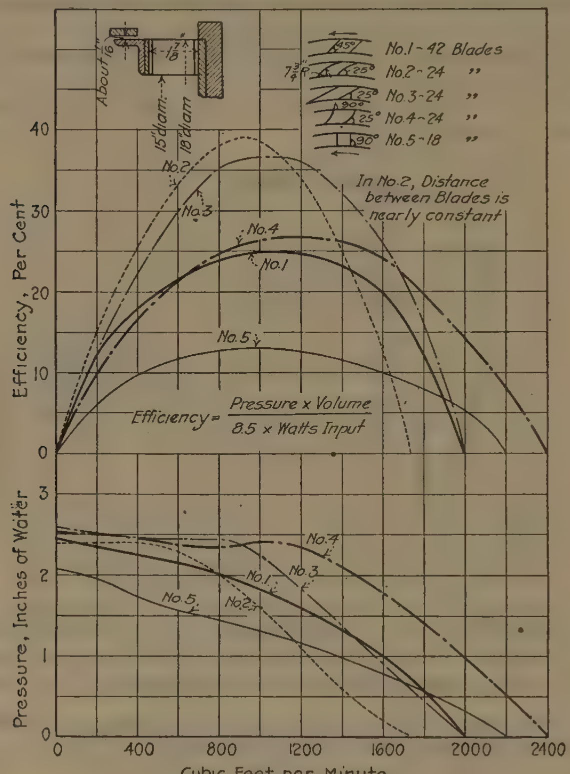


Fig. 8 Performance Curves of Fans of Various Types

(All fans have the same general dimensions and operated at 1800 r. p. m. with rotating shroud and flange rings; blade slopes were different, as shown. For performance curves with other numbers of blades, see Fig. 9.)

1 Pressure is proportional to the square of the peripheral velocity

2 Volume (cu. ft. per min.) is proportional to the peripheral velocity × diameter × width

3 Power input is proportional to the cube of the peripheral velocity × diameter × width

4 Efficiency is independent of peripheral velocity or dimensions.

30 These proportionalities were checked in so far as the influence of angular velocity was concerned. The published data on the largest and smallest sizes of lines of "similar" fans built by a number of leading manufacturers were compared, using the above proportionality relationships, and it was found that the pressure-volume, efficiency, and horsepower-input curves for the largest and smallest fans were coincident, or nearly so. It is therefore believed that for fan work in electrical machinery the

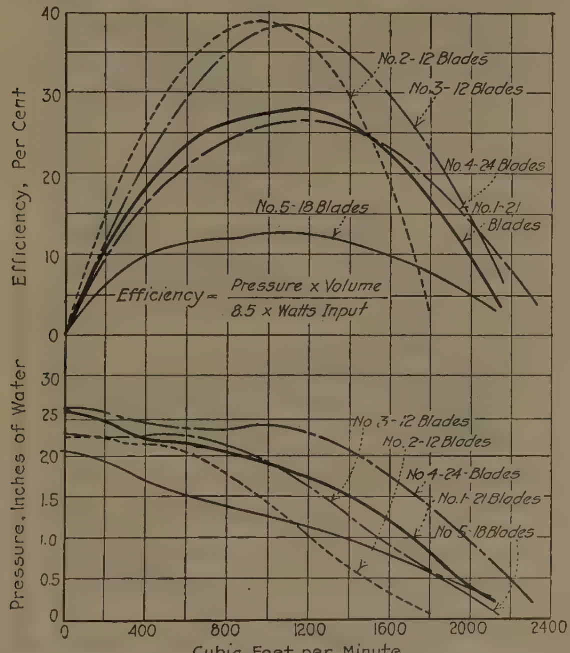


FIG. 9 PERFORMANCE CURVES OF FANS OF VARIOUS TYPES

(See note under caption of Fig. 8 and sketches at top of that figure. For watts input, see Fig. 10. For performance with different number of blades, see Fig. 8.)

proportionality multiplying factors are sufficiently accurate, and that the errors which they introduce will be of the order of only a few per cent.

RESULTS OF TESTS ON FANS

31 All fans tested had the same external diameter of 18 in. and were tested at 1800 r.p.m. In laying out the schedule of tests, they were classified in regard to the performance as affected, first, by internal influences, that is, the impeller only, and second, by influences external to the impeller. These two classes were then subdivided as follows:

A — Internal Influences (the impeller only):
1 The shapes of the blades (Figs. 8, 9, 10, 17)
2 The number of blades (Figs. 11, 13, 14, 15, 18)
3 The width of the blades (Figs. 12, 24)
4 The depth of the blades (Figs. 11, 22, 23)
5 The use of a partition (Fig. 25).

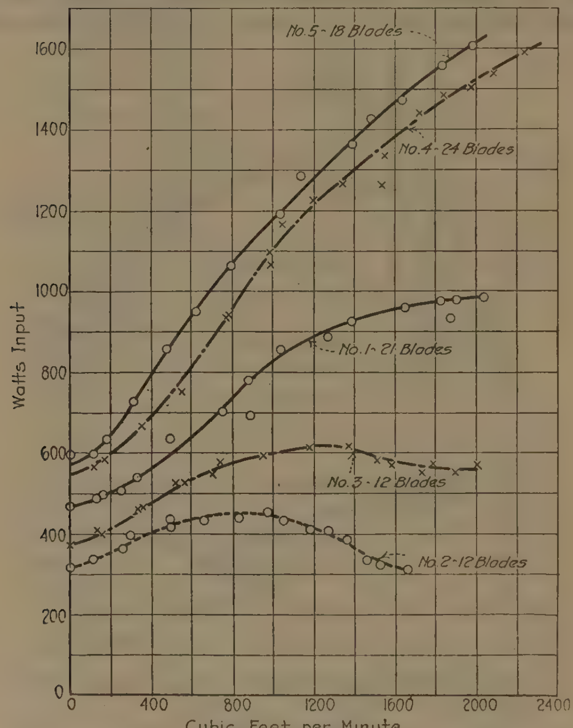


FIG. 10 WATTS INPUT OF VARIOUS TYPES OF FANS

(See note under caption of Fig. 8. For pressure-volume and efficiency curves, see Fig. 9.)

B — External Influences:
1 Leakage paths (including omission of shrouds and rotating flange rings (Figs. 15, 16, 17, 19, 21, 23)
2 Stationary end windings (Fig. 20)
3 Diffuser and stationary guide vanes (Figs. 26, 27)
4 Intake restrictions (Figs. 28, 29, 30).

32 *A1 — Shapes of Blades.* Regarding the shapes of blades, it was believed from theoretical considerations, that about 25 deg. blade angle at entrance would be best for ratios of internal to external diameters between 0.8 and 0.85. Accordingly three fans

were built with that angle (Figs. 8, 9, 10), one with curved blades terminating radially (No. 4), one with curved-back blades with substantially uniform section between blades (No. 2), and one with straight blades which for these shallow fans should not differ very much from the curved-back-blade fan (No. 3). Fans with a 45-deg. entrance angle have frequently been built for use

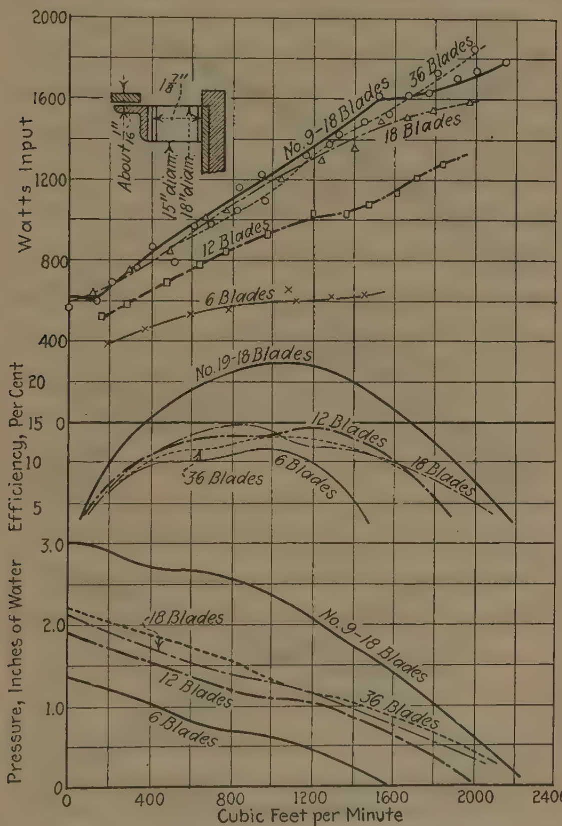


Fig. 11 Performance Curves of Various Fans with Radial Blades

(Fans operated at 1800 r. p. m. with rotating shroud and flange ring. All fans like sketch, except No. 9, for which internal diameter was 12 in.; otherwise like sketch.)

in electrical machinery, and therefore that type was also tested (No. 1). Inasmuch as radial blades must be used for machines that operate in either direction of rotation, that type was also made (No. 5). The performance of another curved-blade fan which was built to check the performance on a fan previously built for a turbo-alternator, will be found in Fig. 17. With the exception of the curves on Fig. 17, all data on the other curve

sheets (Figs. 8, 9, 10) pertain to fans of the same dimensions, (18 in. external diameter, 15 in. internal diameter, 1.875 in. wide) and operated under identical conditions. The only difference between Figs. 8 and 9 is in the number of blades, and in Fig. 10 the watts input are plotted for the smaller number of blades as in Fig. 9.

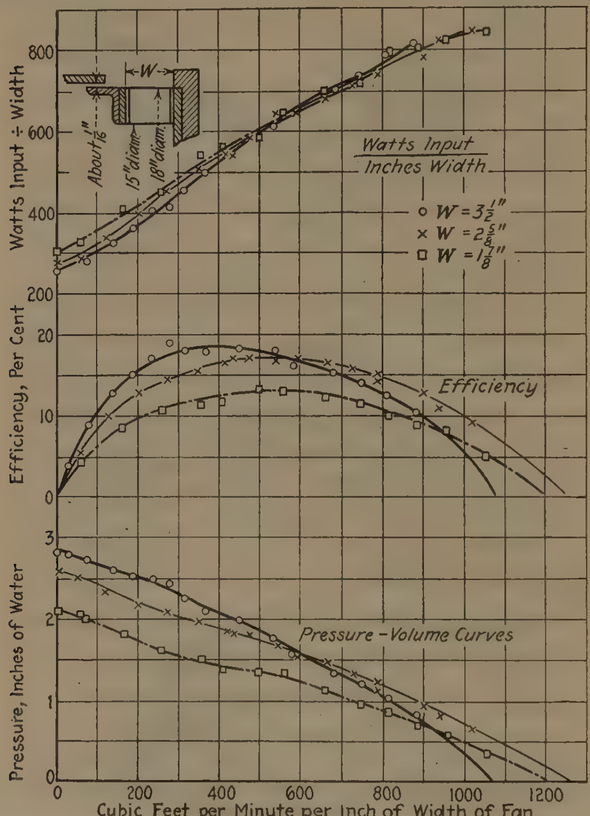


Fig. 12 Curves Showing Comparative Performance of Radial-Blade Fans of Various Widths

(All fans with 18 blades; speed in all cases, 1800 r. p. m.)

33 *A2 — Number of Blades.* The influence of the number of blades can perhaps be best comprehended from an inspection of Fig. 11, which pertains to shallow radial blades. However, Figs. 13, 14, 15 and 18, for curved and inclined shallow blades and for deep radial blades, are also helpful. The surprisingly small gain by the use of a large number is particularly evident; in fact, in many of the fans the performance is better with the smaller number. The thickness of blades was $\frac{1}{16}$ in. in all fans tested.

34 *A3 — Widths of Blades.* A comparison of fans of different widths will be found in Figs. 12 and 24, the former for shallow radial and the latter for shallow 25-deg.-inclined blades. In both figures the abscissas are the volumes per inch of width, so that the influence of the outer whirl on the gain in pressure and that of non-uniform axial distribution upon the reduction in volume

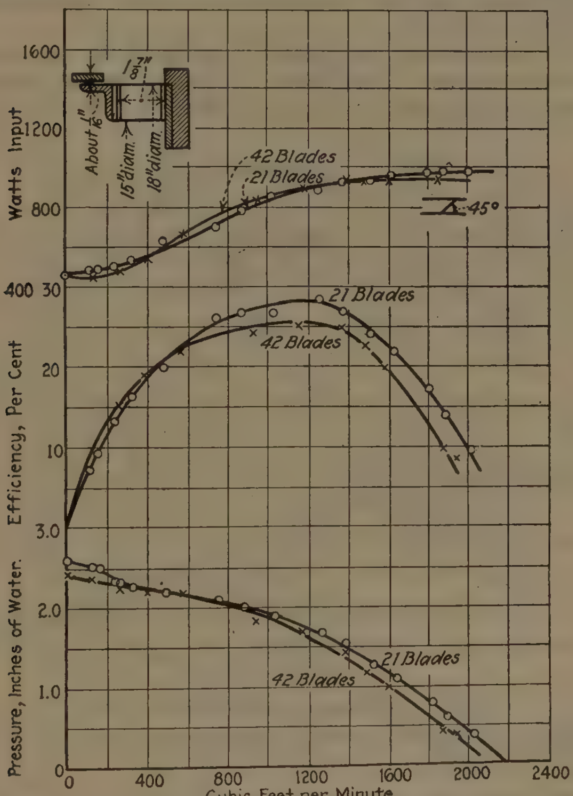


FIG. 13 PERFORMANCE CURVES OF FAN NO. 1

(Tested at 1800 r. p. m. with rotating shroud and flange ring.)

can be detected at a glance. The width is the only independent variable on either of the curve sheets.

35 *A4 — Depth of Blades.* The influence of the depth of blades in radial fans will be seen from an inspection of Fig. 11, the number of blades being 18 for the fan with the smaller internal diameter. The improvement in efficiency with the deep blades is particularly marked. For a comparison of performances with deep and shallow blades for 45-deg.-inclined blades see Fig. 22,

the conditions of test being the same for the curves indicated by the small circles and triangles. The change in performance of the 25-deg. straight-blade fan due to change in depth will readily be seen from an inspection of Fig. 23. There is, with the deeper blades, an increase in pressure at small volumes, a slight decrease in volume at low pressures, and a decrease in maximum efficiency. It would seem that the shock loss at entrance is increased by deepening the blades.

36 *A5 — Partition.* In general a partition is used only to reduce the stresses in the blades. Only one test was made to determine its influence upon the performance (Fig. 25). In this

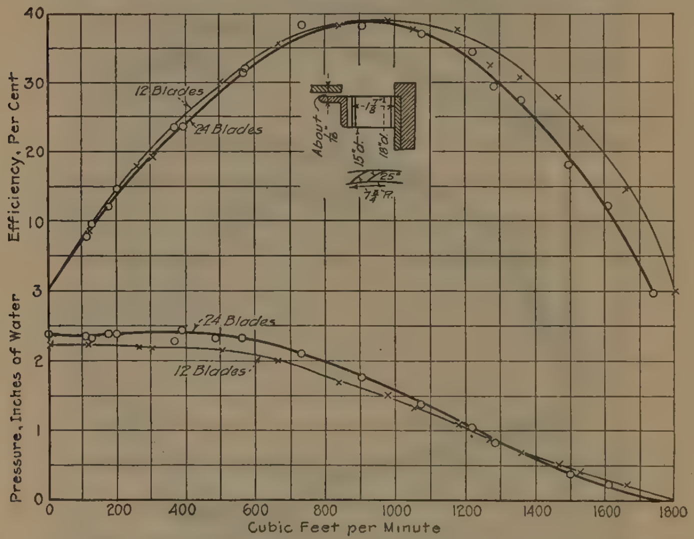


Fig. 14 Performance Curves of Fan No. 2

(Tested at 1800 r. p. m. with rotating shroud and flange ring. Distance between blades nearly uniform.)

case the 25-deg. straight-blade fans were the ones investigated. At low volumes the pressure is reduced due to the reduction in effective depth of whirl. Furthermore the partition reduces the sectional area through which the air flows, and that combined with the reduced pressure (at least at small volumes) reduces the volume. However, the curves cross at the higher volumes, showing that the partition reduces the eddying between blades. In general, it would seem that the partition is detrimental to the performance. It will be noted that the fan is the widest of those tested.

37 *B1 — Influence of Leakage Paths.* In applying centrifugal fans to electrical machinery, the construction is often considerably

simplified by omitting a shroud ring. For example, Fig. 7(*a*) shows a section of part of a salient-pole alternator with solid end bells. A portion of the end bell, *B*, acts as a stationary shroud; the rotating shroud ring is present on one side, *C*, but is omitted on the other side, *A*. In such cases, the air passing through the fan is not confined within its boundaries, *A* and *C*. What, then,

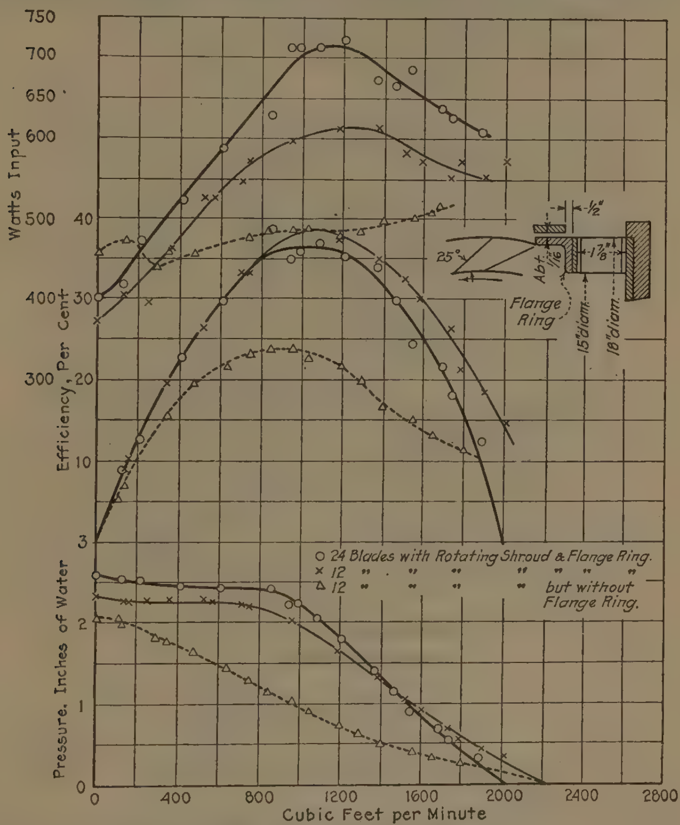


FIG. 15 PERFORMANCE CURVES OF FAN NO. 3

(Tested at 1800 r. p. m.)

is to be gained by the use of the rotating shrouds which might be located at *C* and *A*?

38 Or, with or without a rotating shroud at *A*, if there is a certain clearance between the fan blade and the stationary shroud (end-bell wall) as *B*, what volume of air is lost through this clearance space when there is a definite pressure in the end-bell chamber *D* above atmospheric pressure? Or, if with a construction like Fig. 7(*b*) the rotating flange ring *F* is removed, air can leak out

through the opening E, and consequently the fan characteristics are changed.

39 In the curves various test results showing the influence of leakage are given. These are classified as follows:

B1a With and without rotating flange ring F in Fig. 7(b); the rotating shroud A being in place in all tests in this group:

B1a1 Shallow Radial Blades — Narrow
18 blades Fig. 16

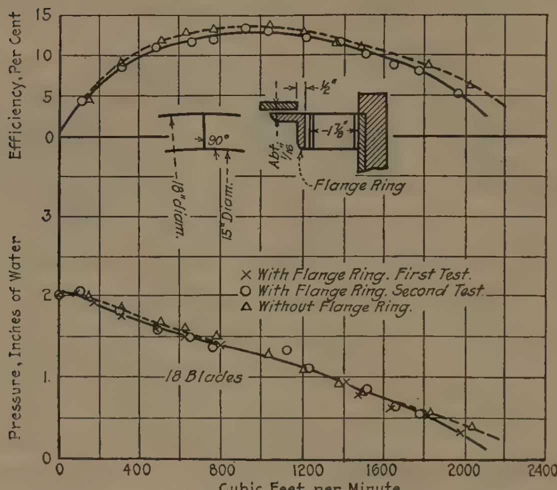


Fig. 16 Performance Curves of Fan No. 5

(Reduced to 1800 r. p. m.)

B1a2 Deep Radial Blades — Narrow
9 blades Fig. 19

B1a3 Inclined Straight Blades — 25 Deg., Shallow, Narrow
12 blades Fig. 15

B1a4 Curved Blades — 27-Deg. Entrance, 63-Deg. Exit
15 blades Fig. 17

(Other tests to determine influence of leakage with radial blades were made, but are not published. Results were similar to those in curves in Fig. 16 in this paper.)

B1b With and Without Rotating Shroud:

B1b1 Shallow Radial Blades — Narrow
18 blades Several Combinations in Fig. 21

B1b2 Deep Radial Blades — Narrow
9 blades Fig. 19

B1b3 Inclined Straight Blades — 45 Deg., Deep, Narrow
12 blades Fig. 22

B1b4 Inclined Straight Blades — 25 Deg., Deep, Narrow
12 blades Fig. 23

B1c With and Without Stationary or Rotating Shrouds, Shallow Radial Blades — Fig. 21.

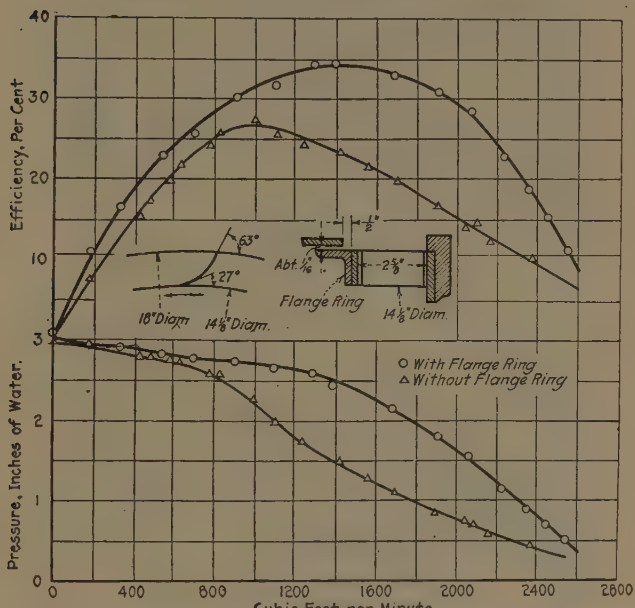


FIG. 17 PERFORMANCE CURVES OF FAN NO. 6A

(15 blades. Tested at 1800 r. p. m.)

DISCUSSION OF TEST RESULTS

40 The performances of the fans tested are plotted in families of curves. The same curve is often repeated on other sheets in order that the influence of the particular independent variable may be seen at a glance and readily studied. In the following paragraphs the influences of these independent variables, as shown in the group classifications in families of curves, will be discussed briefly.

A1 — Shapes of Blades

41 The families of curves given in Figs. 8, 9, and 10 show the performance of fans that are alike except for shapes of blades and number of blades. The five shapes are:

No. 1 Straight blades, inclined at 45 deg. with the tangent at the inner diameter

No. 2 Curved blades, making an angle of 25 deg. with the inner tangent, and so curved that the area between adjacent blades is approximately constant. Radius of curvature is constant

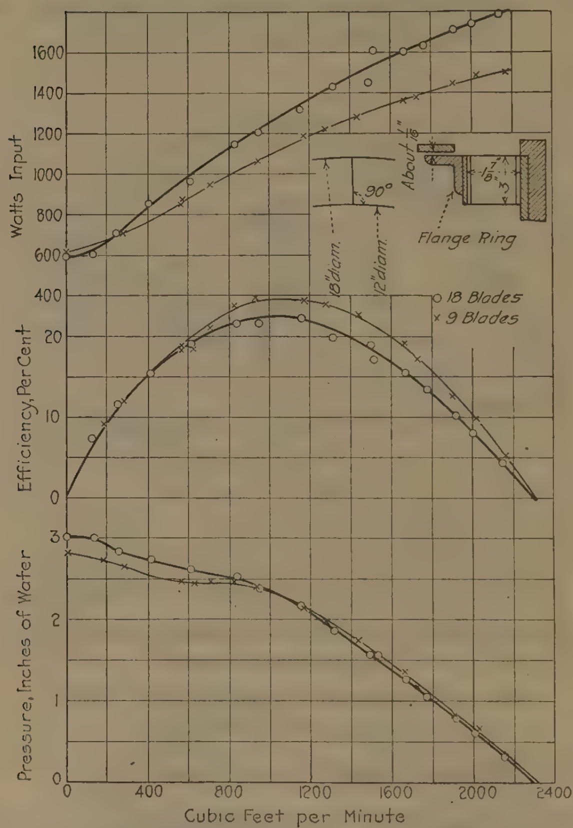


Fig. 18 Performance Curves of Fan No. 9

(Tested with rotating shroud and flange ring at 1800 r. p. m.)

No. 3 Straight blades, inclined at 25 deg. with the inner tangent

No. 4 Curved blades, inclined at 25 deg. with the inner tangent, and terminating radially. Radius of curvature is constant

No. 5 Simple radial blades.

42 Forty-five-degree straight blades like No. 1 had frequently been used in electrical machinery, and it was desired to determine

their performance. There are three fans in which the entrance angle is 25 deg. It was believed that the curved-back blades like No. 2 would prove to be most efficient, as (*a*) the shock loss at entrance is small, (*b*) there is no appreciable change in velocity between blades from entrance to exit, and (*c*) the velocity head at discharge is small. As straight 25-deg. blades like No. 3 are not much different from No. 2 they were also tried, as their manu-

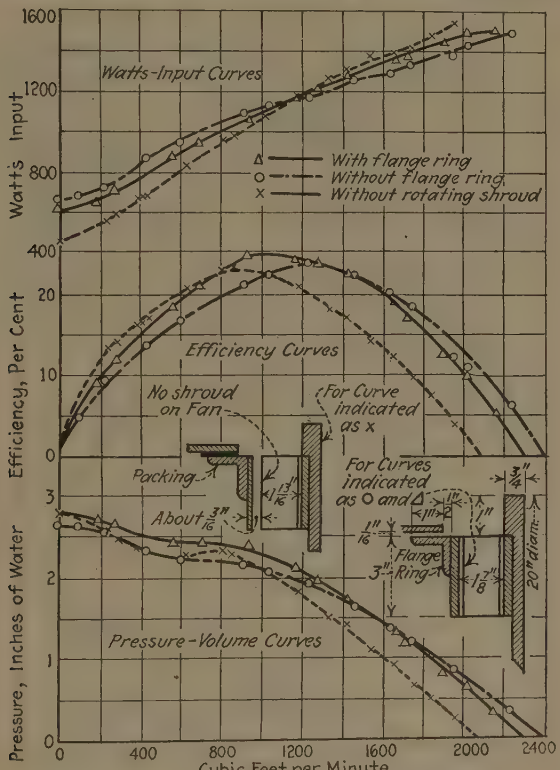


FIG. 19 PERFORMANCE CURVES OF FAN NO. 9

(9 blades, radial; tested at 1800 r. p. m.)

facture is much simpler. No. 4 has the advantage over the radial blades of No. 5 of low shock loss at entrance, but the exit angle is the same. The differences in performance for the five fans are indeed remarkable.

43 As has been stated, an entrance angle of about 25 deg. was believed to be best from theoretical considerations for fans in which the ratio of inner to outer diameter is approximately 0.8 to 0.85. This opinion was reached when laying out the program of tests to be made, and was based upon the direction of flow

relative to the blades at entrance, obtained by drawing a parallelogram of velocities. The usual entrance parallelogram is obtained by assuming the absolute direction at entrance to be radial. [See Fig. 31(*a*).] It was, however, recognized that the motion of the fan causes the air to follow it circumferentially at its external and internal peripheries for appreciable depths. The internal whirl causes the absolute direction of flow to depart from the

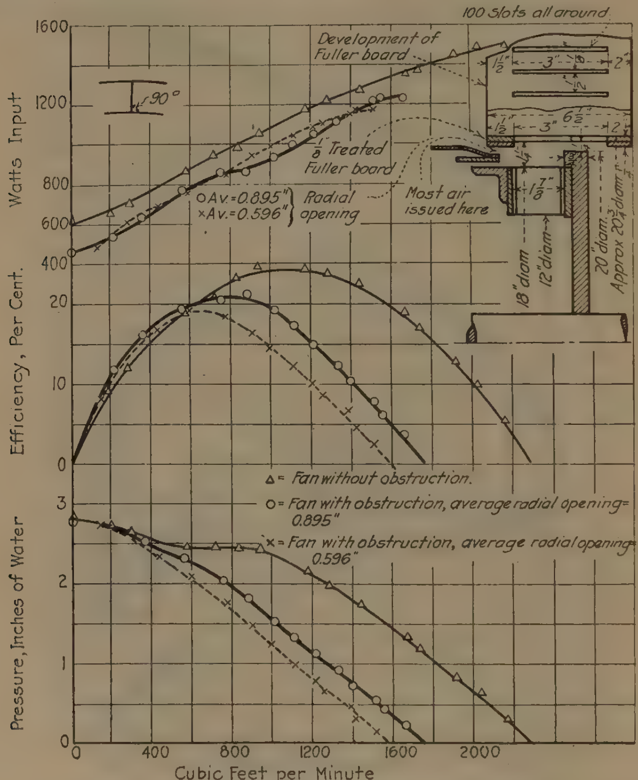


Fig. 20 Performance Curves of Fan No. 9

(9 blades, 1800 r. p. m. To imitate obstruction offered by end windings.)

usually assumed radial line. With the air entering radially, and with the magnitude of the entrance velocity not far from that obtained by dividing the volume for maximum efficiency (approximated from earlier tests) by the inner circumferential area, the relative angle to the tangent is about 15 deg. The effect of the inner whirl is to increase this angle. Furthermore, since the internal peripheral area is decreased by the departure from radial entrance, the effect is to increase the absolute velocity. [See Fig. 31(*b*).] To be sure, it is extremely difficult to determine just

what the angle at entrance should be to eliminate shock loss there; but it is well known from tests on stationary elbows that for angles of a few degrees' departure from the initial direction of flow, the loss is not great. With an angle as small as 15 deg., the area normal to the blades is restricted and the velocity relative to them is increased, thereby raising the loss. Consequently the increase from 15 deg. to 25 deg. reduces the velocity relative to the

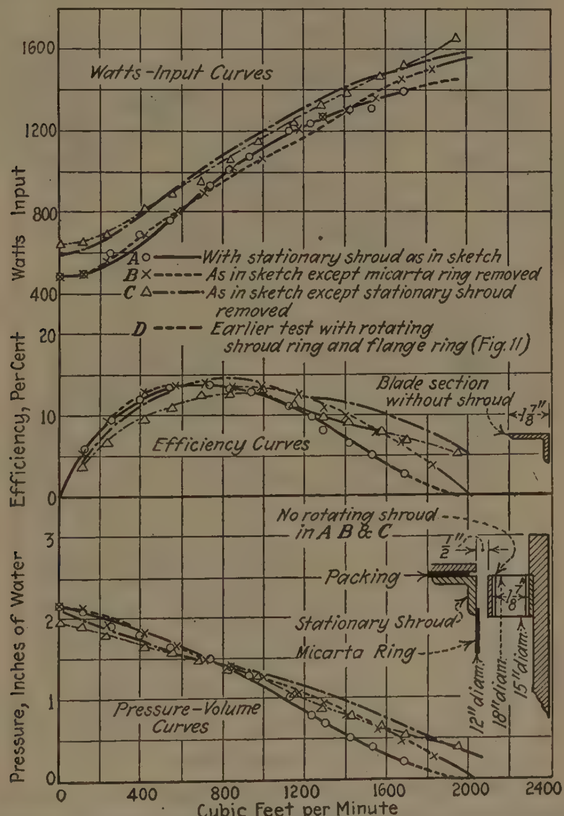


Fig. 21 Tests to Determine Effect of Shroud Rings

[All tests on Fan No. 5 (18 radial blades) at 1800 r. p. m.]

blades. The angle would then be a little larger than would be needed to compensate for the departure of direction of flow in space from radial; at the same time, the shock loss at entrance would be small.

44 It will be seen from Figs. 8 and 9 that No. 2 fan with blades curved backward gives maximum efficiency in accordance with our reasoning. However, the restricted area between blades decreases the maximum volume. It should be especially noted

that No. 3 fan with straight blades, but with the same entrance angle as No. 2, has nearly as high maximum efficiency, and delivers a larger volume of air for a given pressure. There is a material improvement in the efficiency of 25-deg. straight-blade fans over 45-deg., as might be expected from our line of reasoning.

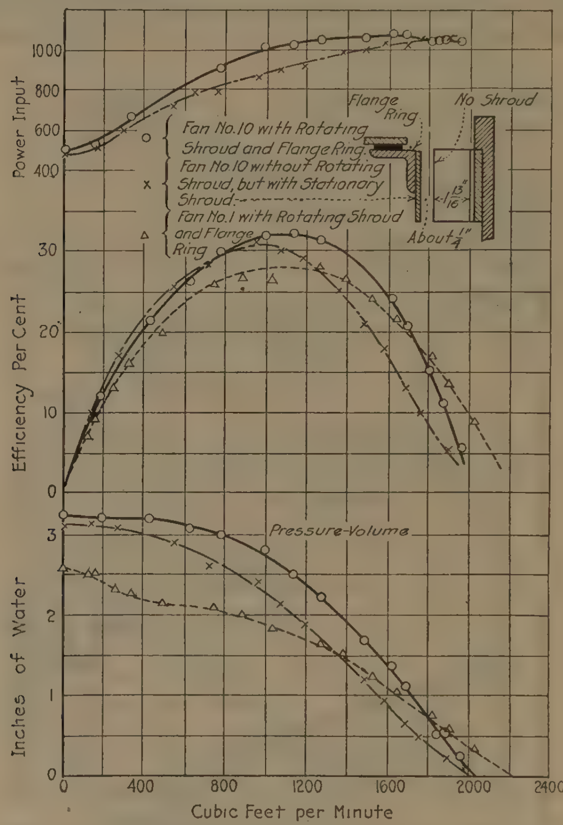


FIG. 22 PERFORMANCE CURVES OF FANS NOS. 1 AND 10

(Both fans 18 in. in external diameter with straight blades inclined at 45 deg. to tangent at inner periphery. Fan No. 1 has 21 blades; fan No. 10, 12 blades Internal diameters: Fan No. 1, 15 in.; fan No. 10, 12 in.)

45 Many electrical machines must operate in either direction of rotation, and then radial blades must be used. An inspection of a parallelogram of velocities for entrance for shallow radial blades will indicate a high shock loss there. With radial blades the absolute velocity at discharge is high, and consequently the loss of head, without a collecting device, is large. Therefore, with radial blades the efficiency is low, and is lower the shallower the blades due to the higher entrance shock loss. This expectation

was substantiated by the tests, as may be seen from Fig. 8 or 11. By decreasing the inner diameter the shock loss at entrance is lowered and the efficiency increased, as will also be seen from fan No. 9, Fig. 11.

46 An idea of the increase in efficiency obtained by changing the entrance angle from 90 deg. to 25 deg. and curving the blade

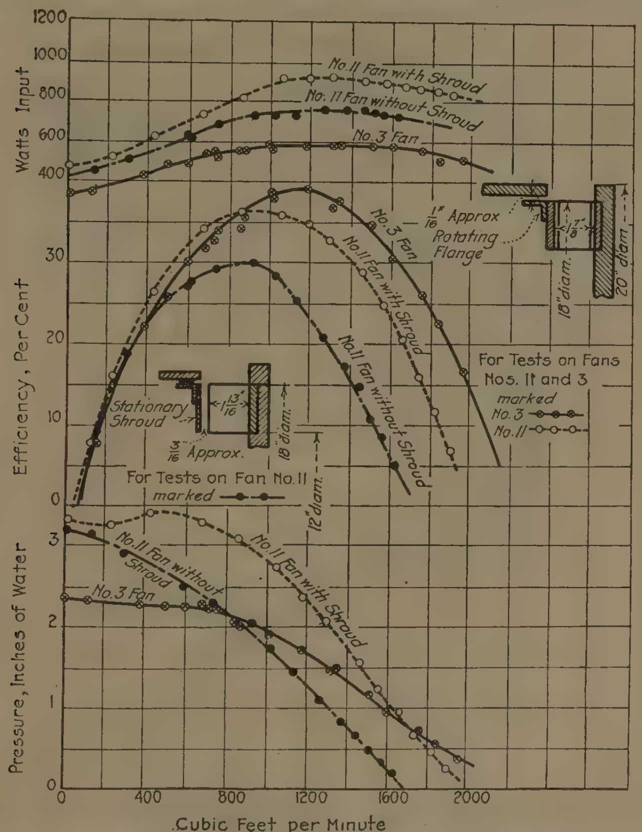


FIG. 23 PERFORMANCE CURVES OF FANS NOS. 3 AND 11

(Both fans have 12 straight blades inclined at 25 deg. to tangent at inner periphery. Internal diameter of fan No. 3, 15 in.; of fan No. 11, 12 in. All tests at 1800 r. p. m. Note that curves of fan No. 3 differ slightly from earlier tests in other figures. "Without shroud" means without rotating but with stationary shroud, as in sketch.)

will be seen from an inspection of curves for fan No. 4, Fig. 8. Thus, the maximum efficiency with radial blades (No. 5, Fig. 8) is 13 per cent, but with the curved blades it is 26.5 per cent. With deep radial blades (No. 9, Fig. 11) the maximum efficiency is 22.2 per cent.

47 The influence of blade shape upon the pressure-volume characteristic is also of considerable interest. Thus, in Fig. 8 the pressure at zero volume is nearly the same for four of the types tested, but with radial blades it is less. If v_1 is the external and v_2 the internal peripheral velocity, both in feet per second, then the generated pressure at zero volume, neglecting disturbing

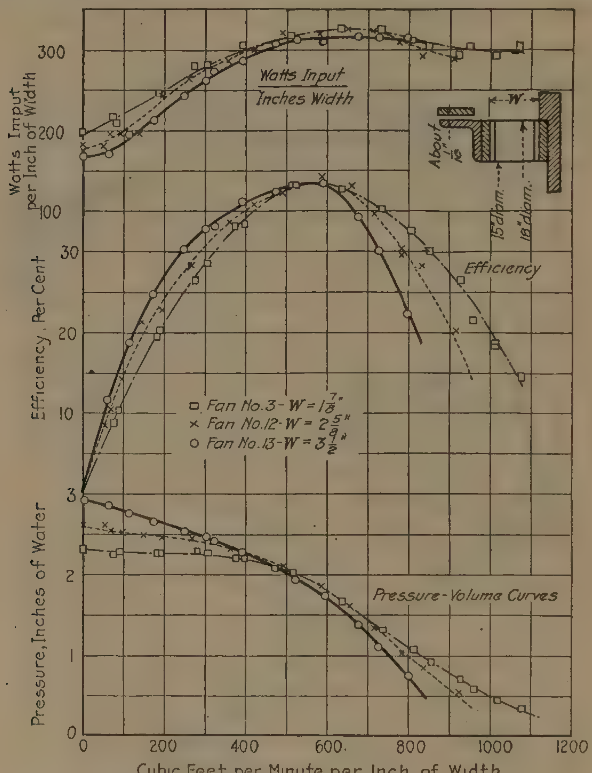


FIG. 24 CURVES SHOWING COMPARATIVE PERFORMANCE OF INCLINED-BLADE FANS OF VARIOUS WIDTHS

(All fans have 12 straight blades inclined at 25 deg. to tangent at inner periphery. All tests at 1800 r. p. m.)

influences, has been shown to be $(v_1^2 - v_2^2)/2g$. This gives 1.35 in. of water for the conditions of test plotted in Fig. 8, whereas in every case it is somewhat higher. The explanation lies in the influence of the external and internal whirls which increase the difference in effective diameters, and effective velocities. The higher pressure at zero volume with blades that are inclined at entrance over that with radial blades, may be explained by considering that the inclined blades "scoop up" the air.

A1 — Blade Shape as Affecting the Power Input

48 The family of curves in Fig. 10 is especially interesting. The rapid increase in power consumption with volume of delivery is well known for radial-blade fans and many other shapes. The power curve for a fan with small entrance shock loss but large

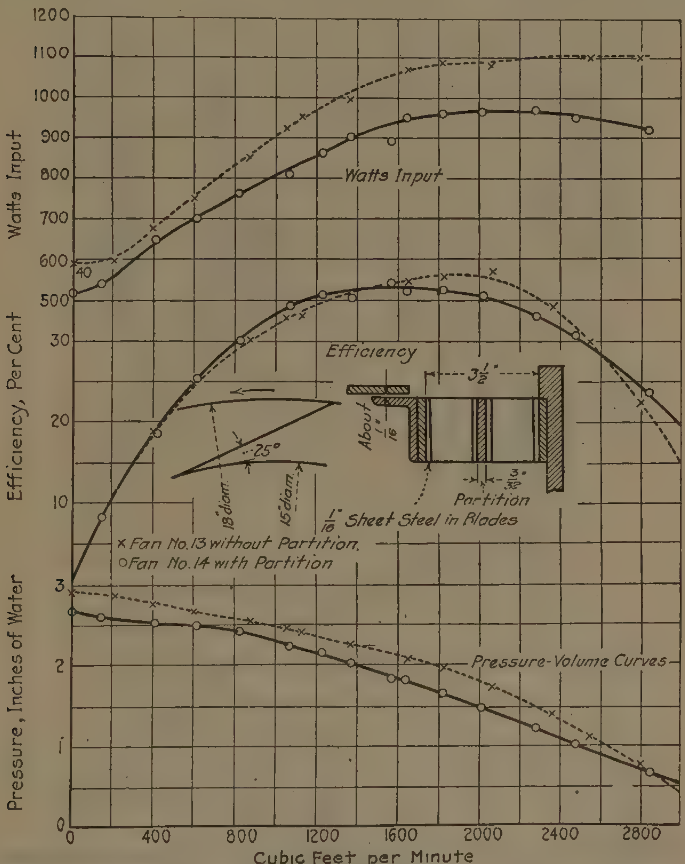


Fig. 25 Performance Curves of Fans Nos. 13 and 14, Showing Influence of Partition

(Blades spaced 1/12 of circle in both fans. All tests at 1800 r. p. m.)

loss at exit (No. 4) is of much the same shape. But it may surprise many to see that with backwardly curved blades (No. 2) the power consumption rises to a maximum and then falls, there being about the same input for maximum volume as for zero volume. A somewhat similar condition of reaching a maximum and then falling slightly holds for straight blades with the same

entrance angle (No. 3). This curve is especially interesting because the power input is nearly constant over what might be considered to be working range (Fig. 10).

49 The loss at entrance to the fan impeller is, for a given angle, of minimum value for some volume of delivery, as will be

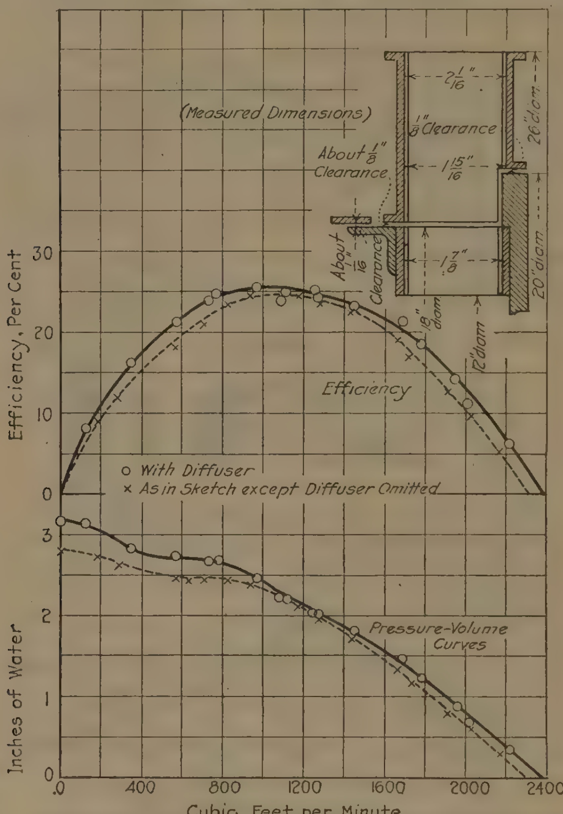


FIG. 26 PERFORMANCE CURVES OF FAN NO. 9, SHOWING INFLUENCE OF DIFFUSER

(9 radial blades; r. p. m., 1800.)

seen from an inspection of parallelograms of velocity at entrance. Yet there is undoubtedly quite a range of volumes, especially with blades tilted backward at the inner periphery, over which the loss at entrance does not change much. It is conceivable that over that range, with an angle that is such as to minimize the shock loss, the energy loss at entrance is not of great magnitude. On the other hand, with blades that are radial at entrance the

shock loss[1] is considerable: compare the efficiency curves of fan No. 4 with fan No. 5 on Fig. 9, the former being with small, and the latter with large, entrance loss, the maximum efficiencies being 26.5 and 13 per cent, respectively. In both fans the blades are

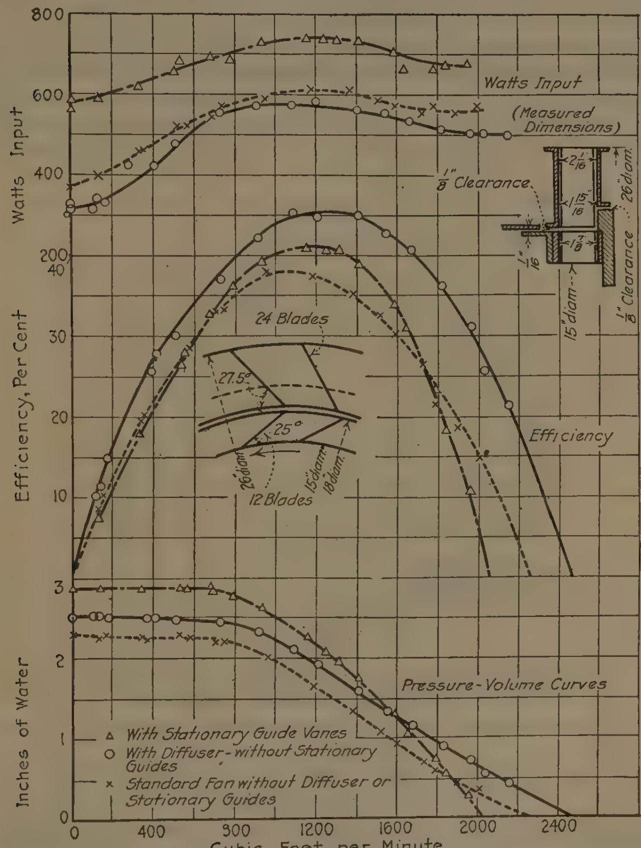


FIG. 27 PERFORMANCE CURVES OF FAN NO. 3, SHOWING INFLUENCE OF DIFFUSER AND STATIONARY GUIDE VANES

(Diffuser tests taken first, after which stationary guides were added.)

radial at the outer periphery; the fans are of same dimensions, the principal difference being in the blade shape.

[1] Perhaps instead of considering the loss at entrance to be due directly to shock, it would be better to consider that loss as being due to eddies formed between blades, following the shock at entrance. These eddies persist, not only between blades after entrance but to some extent after leaving the fan tips at the outer periphery.

50 In general, the losses due to eddies between blades and to friction on the blades are probably not of much influence except at large volumes, and except in so far as the eddies are produced by shock at entrance. Thus, on Fig. 9, No. 3 fan with divergent passages between blades has substantially the same maximum efficiency as No. 2 fan with constant area between blades.

51 As was previously pointed out, the loss at exit, with an inefficient collecting device, is the largest loss, and attention will

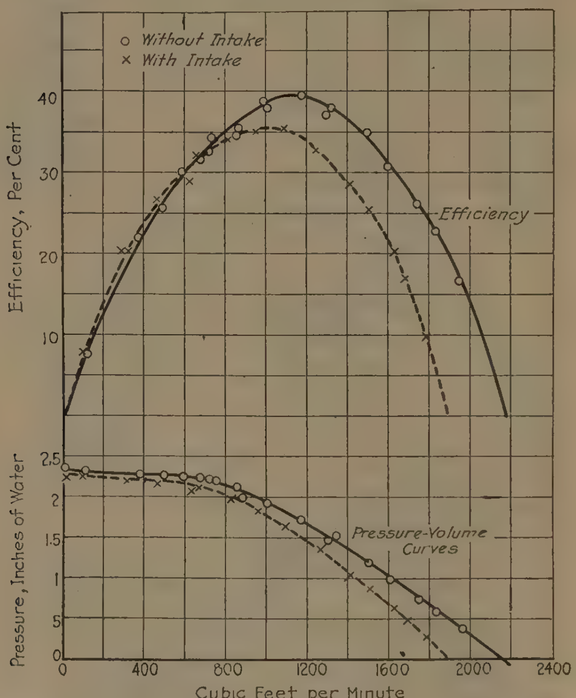


Fig. 28 Performance Curves of Fan No. 3, Showing Influence of Intake

(Fan has 12 straight blades inclined at 25 deg. to tangent. External and internal diameters 18 in. and 15 in., respectively; r. p. m., 1800. Intake shown in Fig. 30.)

be given to that loss as affecting the shape of the power-input curve. It has frequently been thought that the power input continually increases with the volume, and this is the case with the types of commercial fans usually built: viz., those with straight radial blades and those with blades tipped forward. In both types the velocity of air from the external diameter increases with the volume, the equivalent pressure drop corresponding to this resultant velocity is proportional to the square of the velocity, and

the power proportional to that equivalent pressure drop multiplied by the volume. With radial blades the power has a definite value at zero volume, and approximates a straight line from there up to the maximum volume (see No. 5, Fig. 10).

52 With backwardly curved or inclined blades the shape of the power-input curve is quite different, due principally to the lowering of the velocity head at discharge with increasing volume. As the curved-back blade with nearly uniform section between blades and with small shock at entrance over a fairly large range of volume approaches nearest to streamline flow, it affords a means of using the parallelogram of velocities at the outer periphery

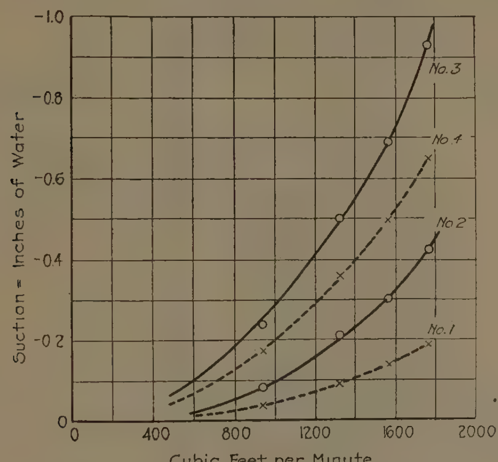


FIG. 29 CURVES SHOWING PRESSURE (SUCTION) AT VARIOUS POSITIONS OF INTAKE TO FAN FOR VARIOUS VOLUMES

(See Figs. 28 and 30. Numbers on curves refer to positions in intake given in Fig. 30.)

with the possibility of attaining calculated results comparable with the test figures.

53 In order to try this, velocity parallelograms were drawn for fan No. 2, with 12 blades (Figs. 9 and 10) at discharge from the wheel, for various volumes (Fig. 32). From these parallelograms the resultant velocities and velocity heads were determined. In Fig. 33, curve 1, these velocity heads are plotted as functions of the volume discharged from the fan. It will be of interest to note that the pressures corresponding to these velocity heads drop rapidly with increasing volumes. The pressure corresponding to these velocity heads multiplied by the volume (cubic feet per minute) and divided by 8.51 gives the equivalent power in watts, for which see curve 2, Fig. 33. These watts must necessarily start at zero (for zero volume) and rise with increasing volume; but

owing to the rapidly dropping velocity-head curve they do not continually increase but, after reaching a maximum, drop again. The shape of this curve accounts in part for the peculiar shape of the power-input curve.

54 In the upper part of Fig. 33 the test pressure-volume curve 6 is plotted. The volumes multiplied by the pressures from that curve and divided by 8.51 give the useful output in watts (curve 3). The sum of this and the velocity-head power curve is also plotted (curve 4). If all the other losses in the fan were constant, the ordinates on the power-input curve should equal those on the

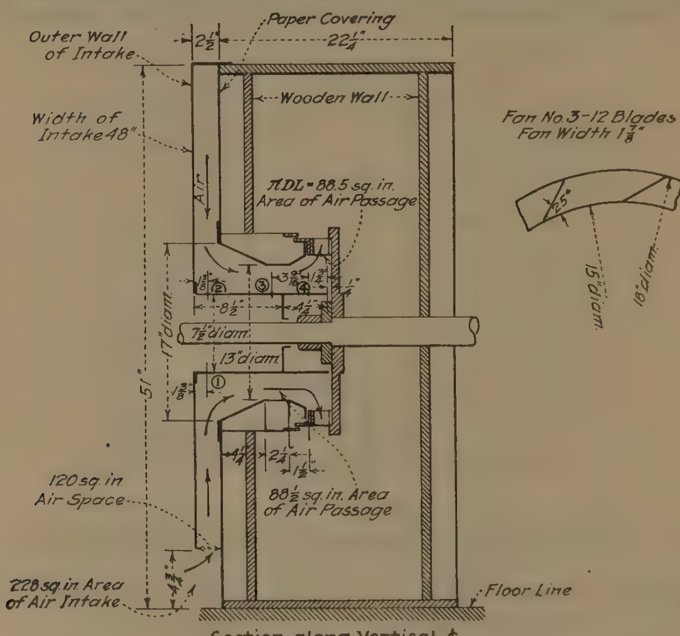


FIG. 30 VERTICAL SECTION OF FAN, CASING, AND INTAKE

(Positions for pressure readings indicated by numbers in circles. See Fig. 29.)

summation curve plus a constant. Within a certain range when the shock loss at entrance is small and nearly constant, and the character of flow between blades is smooth, the extra losses may be expected to be approximately constant. An inspection of Fig. 33 will reveal this to be the case between abscissas of about 900 and 1550 cu. ft. per min., and also the fact that over this range the pressure-volume curve is practically a straight line. The relatively large extra losses at small volumes may in part be accounted for by the eddies incident to the inner and outer whirls which gradually give way to the comparatively smooth flow with increasing volumes.

55 In the upper part of Fig. 33 are plotted the efficiency curves, curve 7 computed from the ratio of watts output to watts input (the same as generally given for "static" efficiency curve); and curve 8, the efficiency calculated from the ratio of the sums of the watts output and watts represented in the velocity head

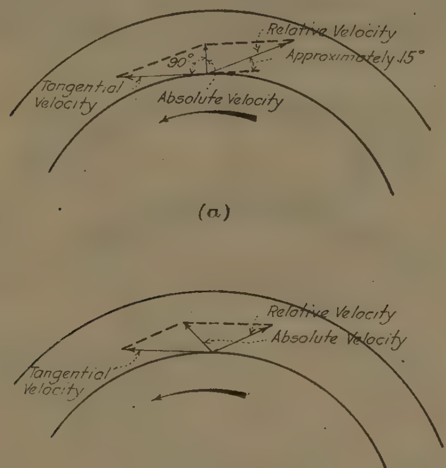


FIG. 31 VELOCITY PARALLELOGRAMS

(curve 4) to the watts input (curve 5). The latter is called the "maximum possible efficiency." Thus, if all the velocity head could become useful, a maximum efficiency for the particular fan of 84 per cent would be obtained. The curve was drawn to illus-

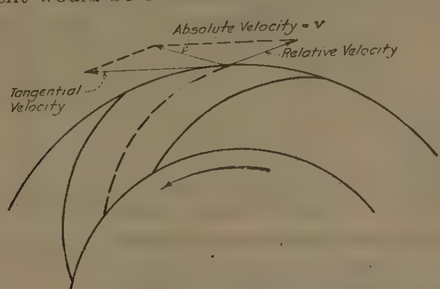


FIG. 32 VELOCITY PARALLELOGRAM FOR FAN NO. 2
(Loss of head at discharge $= v^2/2g$.)

trate how much might be gained by restoring the velocity head, and to emphasize the magnitude of its loss.

56 In Fig. 17 are shown results of tests on a special curved-blade fan which is not directly comparable with those previously discussed: the depth and width were greater. That fan was built

and tested in order to compare with results obtained with a similar fan used in a turbo-alternator.

A2 — The Number of Blades

57 At zero volume the radial depths of the inner and outer whirls are undoubtedly affected by the number of blades. Thus,

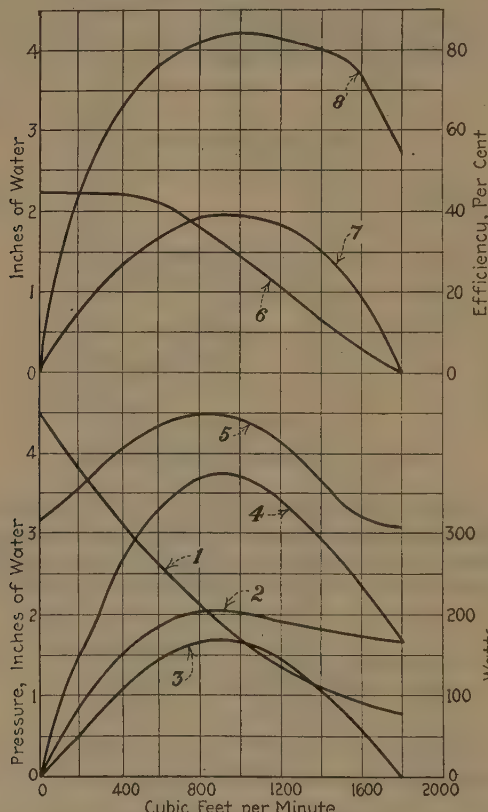


Fig. 33 Curves Obtained from Tests of Fan No. 2

1 — Velocity head, inches of water.
2 — Watts equivalent to velocity head.
3 — Watts output from curve 6.
4 — Sum of curves 2 and 3.
5 — Watts input—test curve.
6 — Test pressure-volume curve.
7 — Efficiency from curves 3 and 5.
8 — Maximum possible efficiency from curves 4 and 5.

in Fig. 34 are indicated the probable tendencies of the effective external diameter to be increased and the internal diameter to be decreased by the presence of the blades. That is, the effective

depths of the outer and inner whirls are increased and the developed pressures are increased thereby.

58 For a given thickness of the individual blade, the available section through which the air passes is reduced by an increase in the number of blades. The velocity with respect to the blades is thereby increased for a given volume; or, other things being equal, for a given velocity the maximum volume should be reduced

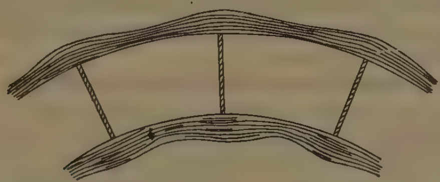


Fig. 34 Radial Depths of Inner and Outer Whirls as Affected by Presence of Blades

by increasing the number of blades. This tendency to increase the maximum volume by decreasing the number of blades is augmented by the decrease in friction with the smaller surfaces then obtaining. These statements do not always apply, but only between limits. If the number of blades is small there are insufficient guides for the air, so that conditions of instability, non-

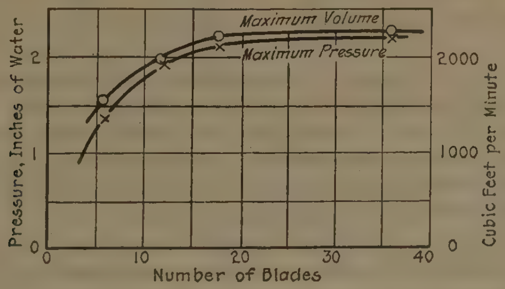


Fig. 35 Curves Showing Influence of Number of Blades on Volume at Zero Pressure and on Pressure at Zero Volume

(Radial-blade fans, 18 in. external and 15 in. internal diameter, 1¾ in. wide; 1800 r. p. m.)

uniform flow, and eddies may be produced between blades, which would tend to reduce the maximum volume.

59 The results of tests on inefficient and efficient fans are submitted. The first results pertain to shallow radial blades, which are quite inefficient (Fig. 11), and therefore an increase in the number of blades should increase the maximum volume up to the point where the blades choke up the fan. Plotting the pressures at zero volume and the volumes at zero pressure against the

number of blades, as was done in Fig. 35, shows at a glance the gains obtained by increasing the number. It will be seen that for the particular range used in the tests (6 to 36 blades) the pressures as well as the volumes increase continually with increasing number of blades, but that there is but little difference between 12, 18, and 36 blades. However, there is quite a large dropping off in pressure, in volume, and in efficiency from 12 to 6 blades. The second results pertain to fairly efficient fans, and the influence of the number of blades upon performance may be seen from an inspection of Figs. 14 and 15.

60 Table 1 records the terminal points from the curves of Figs. 14 and 15. For blades as shallow as these (ratio of internal to external diameter=0.833), 12 blades is quite a small number; yet it would seem that the number is sufficient to permit of smooth flow. The pressures at low volumes are in both types lower with the smaller number of blades.

61 The number of blades needed, when they are curved or inclined, is sometimes thought to be such that they should nearly

TABLE 1 INFLUENCE OF NUMBER OF BLADES ON PERFORMANCE OF FANS

Type of fan	Maximum pressure, in. water	Maximum volume, cu. ft. per min.	Maximum efficiency, per cent	Number of blades
No. 2, Fig. 14......	2.40	1730	38.5	24
	2.24	1800	39.0	12
No. 3, Fig. 15......	2.59	2020	36.5	24
	2.31	2170	38.7	12

overlap. That so great a number may be more than necessary or desirable will be seen from an inspection of Fig. 15. The condition of "nearly overlapping" is fulfilled when the number of blades is slightly less than 24; yet the performance, from the standpoint of maximum volume and efficiency, was found to be slightly better with 12 blades than with 24.

62 In the tests on 45-deg. straight blades (Fig. 13), with the smaller number of blades the volumes are undoubtedly larger at low pressures and the maximum efficiency seems to be higher. The pressures at low volumes were unsteady and unreliable. With deep radial blades (Fig. 18) the pressures at small volumes are lower and the volumes at low pressures about equal, but the efficiencies are higher with the small number of blades.

63 It is important to note that the smaller the ratio of internal to external diameter, the smaller may be the number of blades. This will be evident for radial blades from an inspection of Fig. 11 for shallow, and Fig. 18 for deep, blades. In the latter there was no decrease in volume at low pressures when the number of blades was decreased from 18 to 9, but in the former there was an appreciable lowering with a change from 18 to 12 blades. It is further of value to note that if the same blade shape and ratio

of internal to external diameters are preserved, the best number of blades is substantially independent of the external diameter.

64 A leading authority on water-wheel design claims that the number of blades in a vertical water-wheel runner should be from 13 to 24,[1] depending on the size (up to about 35,000 kw. had been built), the head employed, the diameter, speed, and other conditions. Our own experience would indicate that from 9 to 24 blades for the types of fans tested are best from the standpoint of performance.

A3 — The Width of Blades

65 In Fig. 36 a narrow fan is shown at (*a*) and a wide one at (*b*), the corresponding radial depth of whirl above each fan being indicated. Thus it will readily be seen that the pressure at zero volume, which is somewhat dependent upon this effective radial depth, should increase with the fan width. This was found to be the case, as will be seen from an inspection of Fig. 37.[2] The points in this figure shown as circles apply to radial-blade fans which were duplicates in every way except width, while those indicated by crosses are for inclined-blade fans (α=25 deg.) which were also duplicates except for width. The pressure calculated from the physical diameters (difference between the squares of the peripheral velocities) is 1.35 in. of water, and it is interesting to note that that pressure is nearly the same as obtained with radial blades projected back to zero width. The pressure at zero width with inclined blades is somewhat higher than for radial blades, indicating the influence of "scooping action."

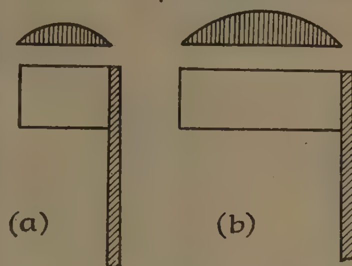


Fig. 36 Radial Depth of Whirl for Narrow (*a*) and Wide (*b*) Fans

66 With a large discharge area the volume may not be proportional to the width, as the air distribution axially becomes less uniform as the width is increased, thereby giving rise to non-uniform velocities between blades. The greater pressure developed with the wide fan tends to offset this somewhat, but it has been found that the maximum volume per inch of width may decrease as the fan is widened; there being one width of fan for which the volume (at zero pressure) per inch of width is a maximum.

[1] W. M. White: Water-Wheel Designs and Settings, Jour. A. I. E. E., Aug., 1921, p. 668.

[2] Fig. 37 is plotted from data obtained from Figs. 12 and 24.

67 Between the points of zero volume and zero pressure the volume and pressure must necessarily be affected by the eddies and the greater developed pressure with increased width, one tending to offset the other, the result being that if the volumes per inch of width be plotted as abscissas, the pressures are highest for the widest fans (for the low volumes); and there are definite widths which give the highest pressures at the large volumes, neither the very narrow nor the very wide fans giving the large pressures at the large volumes. A better understanding of this will be had by inspecting Figs. 12 and 24.

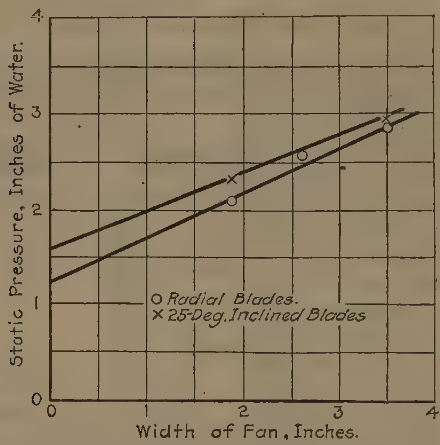


FIG. 37 CURVES SHOWING INFLUENCE OF WIDTH OF FAN ON STATIC PRESSURE AT ZERO VOLUME

(Pressure calculated from $(v_2^2 - v_1^2)/4520 = 1.35$ in.)

68 As the width is increased the volume is increased nearly in proportion, and the area of inlet (between, say, the inner diameter of the impeller and the shaft) remains constant. Consequently the velocity through this section increases with the fan width, and as this is accompanied by a loss of head proportional to the square of the velocity, there is a limit imposed beyond which the fan width should not be increased.

A3 — THE DEPTH OF BLADES

69 The equation for the pressure developed by a rotating impeller at zero volume, as previously written, is $(v_1^2 - v_2^2)/2g$, where v_1 and v_2 are respectively the external and internal tangential velocities in feet per second and g the acceleration of gravity. The result is in feet of air column. If the result is to be in inches of water and the density of air is taken as 0.074 lb. per cu. ft., the pressure is $(v_1^2 - v_2^2)/4520$. This equation ignores the influence of increase in external and decrease in internal

effective diameters due to the air following the fan peripheries for finite depths, also the effect of the "scoop" at the inner periphery. It would appear, however, that as the internal diameter is decreased the pressure at zero volume should increase, other things remaining the same. The value of this increase should be given approximately by the above equation, where now v_1 is the inner tangential velocity with the shallow blades and v_2 the inner tangential velocity with the deep blades.

70 Referring to Fig. 11, it will be seen that with deep radial blades (18 in. external and 12 in. internal diameter) the pressure developed at zero volume is 3 in. of water; with shallow blades (18 in. external and 15 in. internal diameter, and same number of blades) the pressure is 2.1 in. The tangential velocities corresponding to diameters of 15 and 12 in. are 117.9 and 94.2 ft. per sec., respectively, at 30 rev. per sec. Then, according to the formula, the difference in pressure for the two fans would be:

$$\frac{117.9^2-94.2^2}{4520}=1.1 \text{ in. of water}$$

The difference from test is 0.9 in., which is slightly less than calculated. The discrepancy is probably due in part to error in observation, and in part to the fact that the depth of whirl at the inner periphery was slightly less with the smaller diameter, thereby lowering the developed pressure slightly.

71 An attempt was made to estimate the effective depth of the whirl from the above data. If the assumption be made that the depth is proportional to the diameter, then if the depth of the external whirl be 0.4 in. radially (0.8 in. on diameter) and the internal whirls in proportion, the pressure calculated for the deep blades is 3.05 in. and for the shallow blades, 2.06 in., not very different from the test figures.

72 A glance at Table 2 will show the gain in pressure by decrease in internal diameter for radial blades, for straight blades inclined at 45 deg., and for those inclined at 25 deg. with a tangent at the inner periphery. The increase with 25-deg. blades is 1.02 in., nearly the same as computed for radial blades.

73 A number of writers on centrifugal pumps give the pressure developed at zero volume as a function only of the external peripheral velocity and not of the internal. Such a conclusion would apply only with very deep blades, in which a change in blade depth could have no appreciable influence upon the developed pressure; or possibly in which the fluid enters, at least in part, axially along the blade depth. The conclusions reached by these writers are liable to be misleading, and, if applied to fans, may conduce to erroneous results. The blades tend to crowd together with decrease in internal diameter, so that, as indicated in Table 2, the maximum volumes are slightly greater (with inclined blades) for the shallow blades.

74 In Table 2 the maximum efficiencies are recorded, and it will be seen that, whereas the decrease in internal diameter results in an increase in efficiency for radial and 45-deg. blades, it actually causes a slight reduction in efficiency with 25-deg. blades. The reason for the increase in efficiency for the two former is that the shock loss at entrance is reduced. On the other hand, with 25-deg. blades the angle at entrance is probably too small with the small internal diameter; and an improvement, both in efficiency and in maximum volume, could be effected by increasing the angle at entrance slightly. However, if that were done the velocities at exit would be increased; in fact, they are already slightly higher than desirable for high efficiency with these deep blades. Therefore, since nearly all of the velocity head at exit is lost in practically all electrical machinery, the solution, if deep

TABLE 2 INCREASE OF PRESSURE WITH REDUCTION OF INTERNAL DIAMETER FOR RADIAL AND INCLINED BLADES

Type of blade	No. of blades	Internal diam., in.	Pressure at zero volume, in. of water	Volume at zero pressure, cu. ft. per min.	Max. efficiency per cent.	Fig. No.
Radial	18	15	2.1	2200	13.5	16
	18	12	3.0	2300	22.5	18
	9	12	2.8	2300	24.5	18
45 deg. at inner radius	21	15	2.6[1]	2200	28	13
	12	12	3.25	2050	32	22
25 deg. at inner radius	24	15	2.58	2030	36.5	15
	12	15	2.33	2230	38.5	15
	12	12	3.35	1990	36.5	23

[1] This particular value may not be reliable, as the observed pressure with double the number of blades was 2.4 in.

blades are to be used and high efficiencies are to be obtained, is to curve the blades backward, with the entrance angle slightly more than 25 deg.

75 From Table 2 it will be seen that for the inclined blades tested the volumes at zero pressure are slightly greater for the shallow than for the deep blades. This is as would be expected, since the entrance area is more restricted the deeper the blade, even though the developed pressure is greater with the deeper blades. On the other hand, with deep radial blades the entrance loss is less and the maximum volume slightly greater than with shallow blades.

B1 — Influence of Leakage Paths

76 Although tests were made on radial-blade fans of several numbers and widths, with and without the flange ring, to determine the influence of leakage through the space which had been occupied by it, only the tests on the $1\frac{3}{4}$-in.-wide 18-blade fan are given here. In all cases the results showed but little change due to the removal of the flange ring. See Fig. 16.

77 Generally, the volume was increased when the flange ring was removed for the low-pressure parts of the curves. It is believed that the air was thrown from the fan tips at a fairly high velocity, and that the air did not leave in the theoretically logarithmic-spiral paths but had a velocity component in an axial direction. The moving stream from the fan tips was accompanied by whirls, and the whole effect was to place the equivalent of a curtain above the opening E in Fig. 38, thereby reducing the tendency for air to leak out. That the maximum volume was actually increased by the removal of the flange ring at low pressures may be accounted for by the tendency for the air to be drawn in by the outside surface of the rotating shroud A in Fig. 38; as then the pressure in the chamber D is small, the pressure due to centrifugal force of the air drawn by friction against the rotating shroud is sufficient to overcome it and cause air to flow in parallel with the principal paths between the blades.

78 When the flow is fairly smooth, such as obtains with fans having blades inclined backward, the conditions are altered because the air leaving the fan tips probably has too small an axial velocity component to prevent the escapement of air due to the pressure built up in the end-bell chamber or equivalent thereof, at any but the low-pressure parts of the curve. Then, as will be seen from Fig. 15, the performance of the fan, both as to the pressure-volume and efficiency curves, is very greatly influenced. The volume at low pressure is not far from that which obtains with the flange ring in use, and this shows that the leakage is of little consequence in those electrical machines in which the air is discharged from the fan tips into the atmosphere.

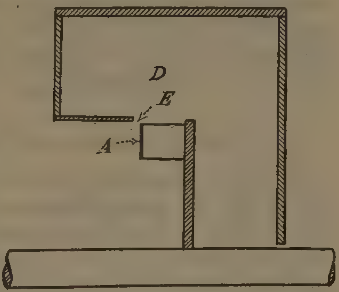


Fig. 38 Section of Radial-Blade Fan without Flange Ring

79 The other curves showing the influence of leakage, Figs. 17, 19, 21, and 23, will not be further commented upon here. Similar arguments apply to them. Each set should be carefully studied in order that the results may be applied intelligently.

B2 — Influence of End Windings

80 In many electrical machines with stationary end windings the coil ends are far enough away from the fan to offer negligible obstruction to the flow of air thrown from the fan tips. In induction motors, however, in which the clearance between stator bore and rotor is small and the coil ends may not be flared back very

much, the obstruction offered by the coil ends may not be negligible.

81 The end-winding obstruction was imitated by cutting $\frac{1}{8}$-in. slots in a sheet of fullerboard and this sheet was placed around the test fan. Between slots, solid spaces $\frac{1}{2}$ in. wide imitated the coil ends. Obstructions were further placed in the path of the air axially to imitate the rings used for supporting the coil ends (see Fig. 20).

82 As induction motors are usually to operate in either direction of rotation, tests were made with radial blades of proportions which would probably be not far from those in induction motors. Two sets of tests were made with two different radial clearances (at one end only) to imitate two different backward flares of the coil ends. All of the tests are plotted in Fig. 20.

83 It was noted, while making the tests, that for the maximum volume the velocity of the air issuing from the slots was quite low, and that most air escaped through the radial clearances between the stationary fullerboard and the rotating member. This is as one might expect, because the air passes from the fan tips nearly tangentially and does not have a large radial velocity component; the tangential component is not effective in causing air to issue along a radial direction.

B3 — Diffuser and Stationary Guide Vanes

84 Tests were made on the deep-radial-blade fan with diffuser; and on the fan having shallow blades inclined 25 deg. both with diffuser and with stationary guides. (See Figs. 26 and 27.) With both fans there is a gain in pressure and efficiency due to employing the diffuser, but it is considerably greater with the inclined-blade fan than with the one having radial blades. The reason therefor is believed in part to be that the flow is smoother with the inclined blades.

85 With the inclined blades the pressure at zero volume was increased 10 per cent, the volume at zero pressure 12 per cent, and the maximum efficiency from 38 to 45.5 per cent. It is of interest to note that the curves with and without diffuser have the same general shape. That the pressure at zero volume was increased can be accounted for by the protection offered by the diffuser to the outer whirl of air surrounding the fan periphery.

86 In order to obtain much of a gain by means of a diffuser the latter should be fairly deep radially. Thus, in the tests made the external diameter of the diffuser was 44 per cent greater than the external diameter of the fan, and the radial depth of the diffuser was 2.65 times the radial depth of the fan with inclined blades; yet at the larger volumes there was still quite a large velocity head in the stream of air issuing from the external periphery of the diffuser.

87 With the idea of simplifying construction, the only type of stationary guide tried in the tests had straight blades. These vanes were fitted into the diffuser which had previously been used. (See Fig. 27.) It is of interest to note that the pressure at low volumes was increased by means of the vanes to values appreciably higher than those with the diffuser alone. It is believed that this is due to the "scooping" of the air in the external whirl surrounding the fan, just as the pressure at low volumes is increased by the "scooping" of the air at entrance to the fan by suitably inclined blades.

88 With stationary guides the volumes for the lower pressures are less than without them; the characteristic curve with the vanes crosses the curve with the diffuser and even the curve for the case when no collecting device is employed. The poor performance at the larger volumes is undoubtedly due to the losses incident to the incorrect angles of the blades.

B4 — Influence of Intake Restrictions

89 When centrifugal fans are used for cooling turbo-alternators, it is customary to provide a suitable intake to which external ducts may be connected if desired. When such an intake is used, the entrance for the air is usually near the floor line, the air then passing vertically upward and some of it being turned through 180 deg. before it enters the fan. There is usually a restricted section shortly before entrance to the fan, but in most cases the area of the section is not less than the area represented by the internal periphery of the fan multiplied by the width.

90 The test rig which was used for obtaining the curves was modified so as to simulate the intake conditions for a turbo-generator. A vertical section of the fan, the casing about it, and the intake is shown in Fig. 30. The cylinder near the center of the outside cover was used to restrict the section before the air entered in the fan.

91 The curves which were taken with the intake are shown in Figs. 28 and 29. An inspection of the pressure-volume curve, Fig. 28, shows that the pressure drop due to the intake is approximately proportional to the square of the volume, as would be expected. There is, of course, a corresponding reduction in efficiency due to the loss of pressure.

92 It was believed to be desirable to account for the various pressure drops. Referring to Fig. 30, the positions for pressure readings are indicated by the numbers 1, 2, 3, and 4 in circles. These pressures were taken by soldering small brass tubes to the inside of the cylinder. Very small holes were drilled through the cylinder at the centers of the brass tubes. It will be seen from Fig. 29 that the pressure near the entrance to the fan (position No. 4) is less than the pressure in the restricted section (position

No. 3). Thus, some of the velocity head is restored to static pressure, although there is quite a loss, as is the case whenever a fluid is slowed down fairly rapidly.

NOTES ON THE VOLUME DELIVERED BY A ROTATING IMPELLER

93 In all probability the effective depths of the whirls at the inner and outer peripheries change with the volume of delivery, thereby altering the total developed pressure. This variation is dependent upon the blade shape; thus, with blades in which the flow approaches nearest to streamline, the depth of the external whirl seems to increase slightly with increase in flow for the small

TABLE 3 VELOCITIES AND VELOCITY RATIOS FOR THE VARIOUS TYPES OF FANS TESTED

(For kinds of blades see Fig. 39)

Fan No.	Number of blades, N	Internal diameter, in.	Max. volume, Q, cu. ft. per min.	Distance between adjacent blades, d, in.	$\text{Area}_1 = \frac{d \times N \times \text{width}}{144}$, sq. ft.	$\text{Vel.}_1 = \frac{Q}{\text{Area}_1}$	$\text{Vel.}_2 = \frac{Q}{\text{Area}_2} = \frac{Q}{0.735}$	$\frac{\text{Vel.}_1}{\text{Peripheral velocity}}$	$\frac{\text{Vel.}_2}{\text{Peripheral velocity}}$	Maximum efficiency, per cent
1	42	15	2100	0.82	0.445	2860	4720	0.338	0.556	25.0
1	21	15	2200	1.70	0.464	3000	4740	0.254	0.560	28.3
2	24	15	1730	0.82	0.255	2355	6790	0.278	0.800	38.5
2	12	15	1800	1.70	0.265	2450	6800	0.289	0.802	39.0
3	24	15	2020	1.00	0.310	2750	6500	0.324	0.770	36.5
3	12	15	2170	2.06	0.322	2950	6740	0.348	0.795	38.7
4	24	15	2400	1.63	0.510	3270	4710	0.386	0.556	27.0
5	18	15	2200	2.56	0.600	3000	3660	0.354	0.432	13.0
9	18	12	2280	2.03	0.476	3100	4790	0.366	0.565	22.5
9	9	12	2300	4.04	0.474	3140	4850	0.370	0.573	24.5

volumes, and then gradually diminishes, which would explain the shape of the pressure-volume curves in Fig. 14. Following that curve for 12 blades, as the volume increases the pressure remains nearly constant up to about 550 cu. ft. per min. (there being a slight rise in pressure around 100 cu. ft.), indicating a probable increase in depth of whirl to compensate for the pressure drop with increasing volume. Around 550 cu. ft. there is a change in character of flow and the air passes between blades quite smoothly but with a gradual reduction in depths of whirls,[1] thereby reducing the developed pressure, which causes the curve to drop nearly as a straight line instead of as a parabola from about 800 to 1600 cu. ft. With shallow radial blades, in which

[1] A thread held near the inner periphery of this fan showed a change in direction of the air stream around 500 cu. ft., which bears out this statement.

there is very turbulent flow, the characteristic is nearly a straight line from zero volume to maximum volume, which perhaps indicates a gradual reduction in whirl depth throughout the range.

94 It was previously pointed out that when a given type of fan is worked at a given percentage of its maximum delivery, the volume is proportional to the peripheral velocity. If the velocity of the air leaving the impeller be determined from the ratio of the maximum volume to the area of section, it would be expected that that velocity should bear some relation to the peripheral velocity. The area may be calculated in two ways:

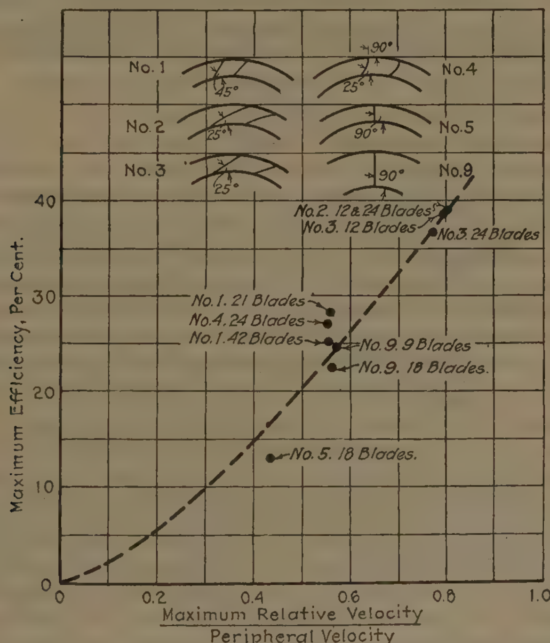


FIG. 39 VELOCITY RATIOS PLOTTED AGAINST MAXIMUM EFFICIENCIES

(Values taken from the last two columns of Table 3. All fans 18 in. in external diameter, 1¾ in. wide, and 15 in. in internal diameter except No. 9, which is 12 in.)

1 It may be taken as the circumference times the width, or
2 It may be taken as the summation of areas between blades at the minimum section, this being the distance between adjacent blades normal to a median line multiplied by the width multiplied by the number of blades.

95 In Table 3 these velocities and velocity ratios are given for the various types of fans, the test curves of which appear in an earlier portion of this paper.[1] All fans were 18 in. in external

[1] In calculating the results in Table 3, the blade thickness ($\frac{1}{16}$ in. each) were allowed for.

diameter and were run at 1800 r.p.m., so that the peripheral velocity was 8480 ft. per min. The width for all was 1.875 in. In the fifth column are given the distances between adjacent blades normal to a median line. In the sixth column the areas computed from these distances in square feet are recorded. In the seventh column the velocities as obtained by the maximum volume divided by the peripheral·area (0.735 sq. ft.) are given. In the eighth column the relative velocities as determined from the maximum volumes and areas between adjacent blades will be found. The ratios of these velocities to the peripheral velocities are given in the two succeeding columns. These are subsequently referred to as No. 1 and No. 2 velocity ratios. In the last column the maximum energy efficiencies of these fans are recorded.

96 From the standpoint of maximum volume of air delivered for given external dimensions, it would appear that fan No. 4, with blades curved at entrance for small shock loss and terminating radially, is the most economical, although the radial-blade fans (No. 9) are nearly as good. There is less variation in the No. 1 velocity ratio between the various types of fans than there is in the relative velocity ratio No. 2. The latter, however, means more from the standpoint of economical operation. It would be expected that the fan which has the least loss due to eddies, shock, etc. should have the maximum velocity between blades when delivering maximum volume, provided that a negligible amount of the velocity head of the stream leaving the fan tips is converted into pressure (as was the case in these tests). In other words, the higher the maximum efficiency, the higher should be the relative velocity ratio. It will be seen that this is so, the maximum velocity ratio being about 80 per cent for No. 2 fans, with curved-back blades. The straight-blade fans, No. 3, which differ comparatively little from No. 2, have nearly as high velocity ratios and maximum efficiencies. The shallow radial-blade fans are the poorest for velocity ratio and efficiency. The various values in the last two columns are plotted in Fig. 39.

97 If the fan is provided with an efficient collecting device, such as a well-proportioned volute, in which a considerable portion of the velocity head is converted into static pressure, the maximum volume is greater than for fans not so equipped, because there is then more pressure which is useful in overcoming the internal losses of head in the fan. Thus, from catalog data on forward-tipped-blade fans built by one of the leading blower manufacturers, the velocity ratio No. 1 (based upon peripheral area) is 0.655, or considerably higher than for any of the fans listed in Table 3 which were not provided with a collecting device.

Methods of Using Fan Curves for Electrical Machinery

98 To apply the test data in this paper to any specific case, the proportionalities given under Use of Test Data are used. If

P = pressure
Q = volume (cu. ft. per min. or equivalent)
W = power input
V = peripheral velocity
D = external diameter
L = length axially (width) and
η = efficiency,

the proportionalities may be written for "similar" fans as follows:

$$P \propto V^2 \quad \text{[a]}$$
$$Q \propto VDL \quad \text{[b]}$$
$$W \propto V^3DL \quad \text{[c]}$$
$$N = \eta \quad \text{[d]}$$

99 There are two general cases: (A) The fan proportions are unknown, and are to be decided upon; (B) The fan proportions are given, the fan having been, say, previously applied, and the volume, pressure, and power are to be determined.

100 *A — Fan Proportions are Unknown.* (1) Decide upon the number of cubic feet per minute (for one fan) needed to cool the apparatus. This is given for standard conditions (25 deg. cent. and 29.92 in. of mercury) by the equation:

$$\text{Cu. ft. per min.} = \frac{1.77 \times \text{watts taken up by air from one fan}}{\text{Degrees centigrade rise of air}}$$

(2) Determine the approximate pressure (in inches of water) needed to drive the required air volume through the machine. This may be done by comparing with similar apparatus, in which the approximate velocities and pressures have been measured.

(3) Decide upon the blade shape, and possible external influences.

(4) Decide upon the external and internal diameters. The external diameter is usually fixed by conditions in the electrical machine. The internal should preferably bear the same ratio to the external diameter as in some fan as tested. If this is not found desirable, the test curves may be interpolated.

(5) Select the fan curve which is representative of the same blade shape, ratio of internal to external diameters, same number of blades, and approximate ratio of width to external diameter, as will be used.

(6) Calculate the peripheral velocity, in feet per minute, of the fan to be built. Then, since the test-fan peripheral velocity is 8480, the pressure on test-fan curve [from proportionality (a)] is equal to

$$\left(\frac{8480}{\text{Per. vel.}}\right)^2 \times \text{pressure needed}$$

(7) Read the volume (cu. ft. per min.) from the test pressure-volume curve corresponding to pressure as computed under (6). Figure width, using proportionality (*b*); allow for peripheral velocity of 8480 and diameter of 18 in. of test fan:

$$\text{Width} = \frac{(\text{Per. vel.} \times \text{ext. diam.} \times \text{width}) \text{ of test fan}}{(\text{Per. vel.} \times \text{ext. diam.}) \text{ of fan needed}} \times \frac{\text{Vol. needed}}{\text{Vol. test fan}}$$

(8) If the ratio of width to diameter for the selected curve of the test fan was much different from the ratio as found after computing the width, then refigure, using the curve corresponding to about the same ratio. (It may be found advisable to interpolate in some cases.)

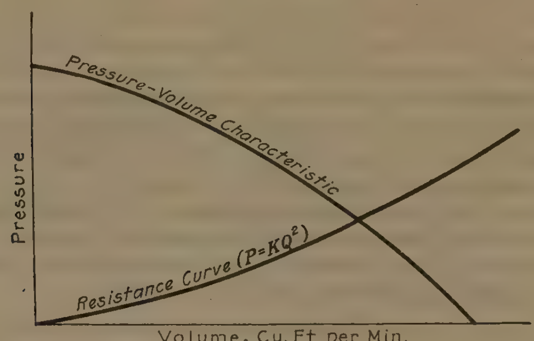


FIG. 40

(9) The efficiency may be read from the same curve sheet. Then the watts input to one fan $= \dfrac{\text{Volume} \times \text{pressure}}{8.51 \times \text{efficiency}}$. If the pressure is so low that the nominal efficiency is nearly zero, the watts-input curve may be used. The watts are then [see proportionality (*c*)] equal to

$$\text{Watts test curve} \times \frac{(\text{Per. vel.})^3 \times \text{width} \times \text{ext. diam. of new fan}}{(\text{Per. vel.})^3 \times \text{width} \times \text{ext. diam. of test fan}}$$

(10) The stress due to bending of the blade should be estimated, and the thickness should be made sufficient to warrant safety from this point of view.

101 *B — Fan Proportions are Given.* (1) Select the test curve of the fan having the nearest proportions to the one which is to be investigated. (Same ratio of diameter, of width to diameter, of number and shape of blades, and corresponding external influences.)

(2) Figure a new pressure-volume curve from the test curve by selecting a number of points on the test curve, and changing as follows:

$$\text{Pressure} = \left(\frac{\text{Per. vel.}}{8480}\right)^2 \times \text{test pressure}$$

$$\text{Volume} = \left(\frac{\text{Per. vel.} \times \text{ext. diam.} \times \text{width of fan to be calculated}}{\text{Per. vel.} \times \text{ext. diam.} \times \text{width of test fan}}\right) \times \text{test volume}$$

(3) Plot a new pressure-volume curve. From data on similar apparatus the resistance of the air circuit may be approximated by, say, taking the pressure drop to be the same for the same velocity. The intersection of the resistance and pressure-volume curves (Fig. 40) gives the volume and pressure desired.

(4) The volume equation under (2) should be solved for the volume from test, and the volume as found from the intersection of the two curves (Fig. 40) used in the equation. With that volume, read the efficiency on the test curve sheet. Then figure the watts input from:

$$\text{Watts input} = \frac{\text{Volume} \times \text{pressure}}{8.51 \times \text{efficiency}}$$

Acknowledgments

102 The author wishes to express his thanks to the Westinghouse Company for the use of apparatus, and to a number of co-workers, especially Messrs. G. E. Luke and T. S. Taylor. He also acknowledges the helpful suggestions of men not connected with that company, particularly Messrs. F. W. Caldwell, N. E. Fales, and George de Bothezat, of McCook Field, Dayton, Ohio.

APPENDIX

DERIVATION OF EQUATIONS FOR VOLUME METER AND CORRECTIONS FOR PRESSURE AND POWER

103 If there are n cold and n hot thermocouple junctions, all connected in series, then if the potentiometer reads e millivolts when the galvanometer reads zero, the millivolts per couple are e/n. The calibration curve of the thermocouple shows that there are E microvolts per degree at the approximate working temperature. The temperature rise of the air in degrees centigrade is evidently the potentiometer reading reduced to microvolts per couple divided by the calibration microvolts per degree. Or if θ is the air temperature rise,

$$\theta = \frac{e \times 10^3}{nE}$$

104 The specific heat of air at constant pressure is 0.2418, and the density of air under conditions assumed as standard (29.92 in. of mercury and 25 deg. cent.) is 0.074 lb. per cu. ft. Calling Q the volume

in cu. ft. per min., and W the watts in the heater, the former may be calculated from the equation[1]

$$Q = 1.77 \frac{W}{\theta} = 1.77 \frac{nEW}{1000e}$$

105 We do not usually have standard atmospheric conditions and there may be an appreciable error introduced by neglecting to correct for the departure. Especially is this the case if comparisons of the performance of different fans are to be made, the fans being tested on different days. The thermal volume meter is necessarily an instrument that indicates the weight, or mass, of air per minute, and what is desired is the volume per minute.

106 Other things being equal, the generated pressure is proportional to the density of the fluid ($P = K_1 \times$density); and the resistance in the air circuit is also proportional to the density (Resistance $= K_2 \times$ density). The volume is proportional to the square root of the generated pressure divided by the resistance. Thus,

$$\text{Volume} = \sqrt{\frac{\text{Generated pressure}}{\text{Resistance}}} = \sqrt{\frac{\text{Density} \times K_1}{\text{Density} \times K_2}} = \text{constant}$$

That is, the *velocity* and *volume* of the fluid leaving the impeller are independent of the density of the fluid.

107 Under test conditions the density is D, and under standard conditions it is 0.074 lb. per cu. ft.; or, the air temperature rise is inversely as the density. The equation for volume (under standard conditions) is $Q = 1.77W/\theta$; to change this to test conditions, multiply the temperature rise by $D/0.074$; or,

$$Q = \frac{1.77W}{\theta_{\text{test}}} \times \frac{0.074}{D} = \frac{1.77nEW}{1000e} \times \frac{0.074}{D}$$

108 Now D, the weight per cubic foot, is the reciprocal of the specific volume. The well-known equation for a perfect gas connecting specific volume, V, absolute temperature, T, and pressure, P, is $\frac{PV}{T} = \frac{P_1V_1}{T_1}$ = constant. Taking D as $\frac{1}{V}$, $\frac{D}{D_1} = \frac{PT_1}{P_1T}$. The absolute temperature in centigrade degrees is $273+t$, and for standard conditions $t =$ 25 deg. cent.; so that $T_1 = 298°$. $T = 273+t$, $P_1 = 29.92$ in. of mercury, $P =$ barometric pressure under test conditions, and $D = 0.074$. Therefore

$$D = \frac{0.074P \times 298}{29.92(273+t)} = \frac{9.96 \times 0.074P}{273+t}$$

109 Substituting this value of D in the equation given in Par. 107, the volume is finally

$$Q = \frac{0.178nEW}{1000e}\left(\frac{273+t}{P}\right)$$

110 The pressure developed by the fan (above atmosphere) should also be corrected for the departure of atmospheric conditions from the reference standard. The pressure is evidently proportional to the mass density of the fluid. To correct back to standard, multiply by $\frac{0.074}{D}$. By using the equation $\frac{PV}{T}$ = constant, it is then found that:

$$\text{Pressure standard} = 0.1003\left(\frac{273+t}{P}\right) \times \text{pressure in test}$$

[1] The derivation of this equation is given by the author in an article entitled Some Elements of Air Flow in Electrical Machinery, in the *Electric Journal*, Aug., 1922.

where t and P are respectively the temperature in degrees centigrade and the barometric pressure in inches of mercury.

111 The question may arise whether the volume as computed for test condition should not be again corrected to correspond to the pressure as corrected back to standard conditions. It was previously shown that the average velocity, and therefore the volume of flow, is independent of the mass density, as the generated and consumed pressures are both proportional to the first power of the density. Therefore there is no further correction for volume.

112 The same factor $\left(\frac{273+t}{P}\right)$ enters into the pressure and volume equations. For pressure it means a correction from test to standard conditions. For the volume, however, it enables the calculation of volume for test conditions, or for standard conditions. Since power is proportional to the product of volume and pressure, the power input from test should be multiplied by the same correction factor, that is,

$$\text{Watts standard} = 0.1003 \left(\frac{273+t}{P}\right) \times \text{watts test}$$

Meter for Measuring Air Volumes by Heating Air — Error Due to Non-Uniformity of Velocities

113 Assume that the cross-section of the duct is broken into a number of equal areas a through which the velocity is v and the volume per minute is q. Both v and q are variables, but v is constant through a given small section a. The temperature rise of the air through the total section is the variable θ, but θ is constant in any one section. (That is, if the variation in velocities be great, each section a may be taken as infinitely small and equal to da.) The temperature rise in any section is inversely as the volume q. Call q_1 that volume per section which would obtain if there were uniform velocities v_1 throughout, and the resulting uniform temperature rise would then be θ_1. Then

$$\frac{\theta}{\theta_1} = \frac{q_1}{q} = \frac{v_1}{v}$$

whence

$$\theta = \frac{\theta_1 v_1}{v}$$

114 The average temperature rise of the air under conditions of test is

$$\theta_{av.} = \frac{\Sigma(\theta a)}{\Sigma a} = \frac{\Sigma(\theta a)}{A}$$

where $A = \Sigma a =$ total cross-sectional area. Substituting in this equation the value just obtained for θ,

$$\theta_{av.} = \frac{\Sigma\left(\frac{v_1\theta_1 a}{v}\right)}{A} = \frac{v_1\theta_1\Sigma\left(\frac{a}{v}\right)}{A}$$

115 The ratio of the average temperature rise of the air ($\theta_{av.}$) to the assumed rise for uniform flow (θ) is

$$R = \frac{\theta_{av.}}{\theta_1} = \frac{v_1\theta_1\Sigma\left(\frac{a}{v}\right)}{\theta_1 A} = \frac{v_1\Sigma\left(\frac{a}{v}\right)}{A}$$

But v_1 = average velocity across the section $= \Sigma(va)/A$; therefore

$$R = \frac{\Sigma(va)\Sigma\left(\frac{a}{v}\right)}{A^2}$$

116 If there are n small areas a equal throughout, so that $A = na$, the preceding equation becomes

$$R = \frac{(\Sigma v)\left(\Sigma\frac{1}{v}\right)}{n^2}$$

117 The potentiometer must necessarily read the average temperature rise of the air, θ_{av}. The temperature rise that would be read for uniform flow is θ_1. The volumes, being inversely as these temperatures, are

$$\frac{Q_1}{Q_{av.}} = \frac{\theta_{av.}}{\theta_1} = R$$

Thus, the volumes as measured are

$$Q_{av.} = \frac{Q_1}{R} \text{ or } Q_1 = RQ_{av.}$$

That is, the volume that would obtain with uniform flow is the measured volume multiplied by R.

118 Suppose that there were velocities of 1000, 1050, 1100, and 1200 ft. per min. in four equal sections. Then R would be equal to 1.005, or only $\frac{1}{2}$ of one per cent in error.

DISCUSSION

SANFORD A. MOSS[1] and I. H. SUMMERS.[2] In work, such as that described by the author, there is always difficulty with measurement of static pressure due to jets and impacts. The close agreement of the author's pressures, as measured in the chute, and behind the curved guides, shown by Fig. 11, seems to indicate that the problem has been solved satisfactorily. It must be noted, however, that the mechanical pressure gage used is liable to the same errors caused by jets and impacts as is a static hole in the wall of a chamber. This is because the diaphragm of the gage would be deflected by such jets in the same way as by a difference in static pressure. This is indicated by the agreement of the pressures by the mechanical gage and by the tube, shown in Fig. 11. The mechanical gage is therefore merely a substitute for a U-tube, or other pressure-measuring instrument used with a static hole in the wall, and does not overcome the effects of jets and eddies on a static hole.

The designer of a fan for electrical machinery must secure the required volume and pressure rise, and satisfy all the limitations

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imposed by the electrical design, and yet strive for good fan efficiency. The author's tests show that attention to curvature and angles of the blades will give important gains in efficiency, and that merely using straight blades at some assumed angle causes much greater fan losses than would occur if attention were paid to blade design. The well-known radial-blade fan is shown to have efficiencies of from 10 to 12 per cent, while fans with inclined blades for machines with known direction of rotation give the same amount of air with from one-half to one-third the power consumption. At the same time, even the best of the author's fans is below the efficiency of a good fan blower. In electrical machines with high rotative speeds it is possible to get the requisite pressure rise by having the blades turned backward at exit to a greater extent than the author has done. It is also possible to go further than he has in improving the various details investigated. With attention to detail in all of these respects, fans for electrical machinery can be made with efficiencies equal to those of fan blowers. There is always some conversion of exit velocity into pressure, and a properly small exit velocity secured by careful design, with the inevitable conversion, gives a very small loss due to the lack of a volute or diffuser.

In a number of places in the paper the author speaks of "whirling motion" of the air preceding and succeeding the fan for appreciable depths. What is meant by "appreciable depths," and how can it be known that there is such whirling? The investigation by the writers of fans with carefully designed inlets and exits, by means of threads, smoke and incandescent particles, indicates that the air at inlet flows perpendicularly to the surface generated by the blade tips, at least to within 1/16 in. of this surface. This is theoretically as it should be. If a fan inlet is properly designed to suit the relative angle and relative velocity of the air, the blade tips will make comparatively small disturbance, and the air will flow past the surface generated by the blade tips with little disturbance from them. The air on the fan just within the surface generated by the inner tips of the blades is not whirling, but is threading its way between the blades which, when properly designed, are at the angle that permits this. It is, of course, proper to draw a velocity triangle or parallelogram at inlet which assumes that the air has the velocity of the wheel together with a velocity relative to the wheel. The latter velocity exists just as much as the former one, however, and the vector sum of the two must be considered. This obviously is the absolute velocity of the air, which is, as stated, perpendicular to the surface. Similarly the air at exit has the velocity of the wheel plus a relative velocity. The vector sum of these two is turned somewhat toward the direction of motion, but does not even approximately have the whirling velocity of the fan periphery. If the air on the fan were whirling with it, the pressure rise produced would be proportionate

to the square of the velocity of the fan periphery. As the author points out, the pressure produced by the centrifugal effect is the difference of the squares of fan velocities at periphery and inlet, which shows that the actual amount of whirl which the air acquires is very small with normal flow and with blades designed for proper streamline action.

G. E. LUKE.[1] Tests of fan performance involve the determination of three important factors, namely, power input to the fan, and the volume and pressure delivered by the fan. The writer would recommend an electric torque dynamometer as a better means of measuring the fan input, as this would eliminate the possibility of errors due to variable motor and belt losses. The measurement of air volumes by the electric air meter is an ideal method and permits of high accuracy with comparatively short length of air ducts.

The measurement of correct static pressures is difficult, especially where the air flow is irregular. The special static air-pressure gage devised by the author is ingenious. However, the writer is of the opinion that this gage will also give a correct reading only when the air flows parallel to the face of the disk and will read incorrectly if the air impinges on the disk.

With self-ventilating electric machines the air is forced through the passages by fans on the motor. The air is turbulent due to the absence of a fan volute and due to the constantly changing cross-sectional areas through the machine. The air in passing through a typical electrical generator changes its velocity as much as ten times through a range varying from zero to the maximum velocity, and this change in velocity is usually accompanied by changes in direction of air flow. The average length of air path may not be over 20 ft., which again tends to accentuate the irregularities of the air flow. These conditions make an accurate exploration of the static air-pressure drop along the air path impossible, but considerable information can be obtained from a knowledge of the total volume of air passing and the maximum air pressure in the end bell or chamber surrounding the fan.

Radial-blade fans are used almost universally on the general-utility electric motors and generators. This is due to simplicity of construction and to the fact that such fans are independent of direction of rotation. Fans of this type are usually limited to shallow blades, and the performance, especially the efficiency, is very poor. The efficiency of fans for use on slow-speed machines of the above class usually is not of great importance. Of two types of fans available, the designer will usually select the one giving the greater volume and pressure, regardless of fan effi-

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ciencies. In fact, on the above class of machines the fans are often made with fan widths greater than advisable for maximum efficiency; in other words, the fan may be worked high up on the pressure-volume curve in order to get the maximum volume through the machine.

The noise produced by fans for such machines is seldom objectionable. However, for large electric machines with high peripheral velocities, fan noise may be a factor in determining the type of fan. Also, as the speed and size of the fan increase, the

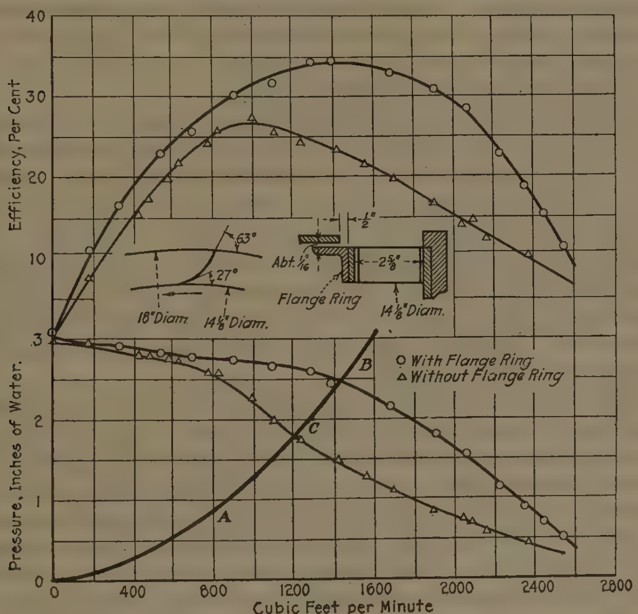

FIG. 41 PERFORMANCE CURVES OF FAN NO. 6A
(15 blades tested at 1800 r.p.m.)

efficiency of the fan becomes of greater importance. Thus, on turbo-generators with peripheral speeds of 25,000 ft. per min. and volumes of 50,000 cu. ft. per min., fan efficiency and noise are of great importance. The fan windage may be in such a case as much as 50 per cent of the total generator losses, so that an improvement in fan efficiency means a material increase in overall generator efficiency. Thus the performance of fans such as No. 2 and No. 3 shown in Fig. 8 explains why this type of fan is used almost exclusively for turbo-generators.

The maximum possible efficiency curve shown on Fig. 33 of the "25 degree inclined-back" fan indicates the goal for such a fan with a properly designed scroll or volute. Such a fan will

not deliver as high a static pressure as an "inclined-forward" blade for a given peripheral speed. However, this may not necessarily be a disadvantage in many applications of fans for a direct motor drive. High rotative speed is desirable in many cases for direct electric drive in order to lessen the cost and size of the driving motor.

EDGAR KNOWLTON.[1] In discussing this paper it may be well to consider briefly what takes place when a fan is combined with an electrical machine. Take as an example the fan whose characteristics are shown in Fig. 41, which is Fig. 17 of the paper with one curve added. This is curve *A*, the assumed pressure-volume curve of a machine which the fan ventilates.

To simplify, we will neglect the fan effect of the rotor to which the fan is attached. The combination of fan and machine will work at the intersection of the pressure-volume curves. When the fan having the flange ring is used, this point is at *B*; and with the fan having no flange ring, the point is at *C*. In the first case the quantity of air will be 1400 cu. ft. per min., and in the second case, 1200 cu. ft. per min. The pressures are 2.5 in. and 1.8 in., respectively, and the power input about 11.7 and 108 watts, respectively. Therefore, with 10 per cent more power about 17 per cent more air is obtained.

Those unfamiliar with the ventilating of electrical machinery sometimes ask the impossible. Just at the point where air leaves a rotating member its velocity may exceed that of the member by a small amount. This velocity is soon decreased and cannot be restored to anywhere near its original value. Experience seems to show that the highest velocity later obtained cannot exceed about one-half that of the highest peripheral speed of the rotor. The author's opinion on this point would be of value. Even a rough approximation may be of considerable value when considering novel methods of ventilating electrical machinery.

WALTER C. DURFEE.[2] To make the valuable and accurate information in the paper wholly available to the readers of the TRANSACTIONS, the writer believes that the exact contour of the blade cross-sections should be published, including the thickness and shape of both edges. For a rounded front edge moving through air which is in process of varied acceleration because of blade forces, it is not possible to determine by simple vector analysis what part of the round is the real front edge, or to determine how to trace from that point onward the true center line.

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[2] Boston, Mass. Mem. A.S.M.E.

Also it is impossible to name the true blade angle at entrance, although something may be guessed if the shape is known.

Can the authors explain why the best results seem to be limited to a so-called " efficiency " a little under 50 per cent? For the type of apparatus tested how much more could be obtained? Is there a choice among the good blades in relation to noise, and where does the noise originate?

THE AUTHOR. Replying to Dr. Moss in regard to the accuracy of the mechanical gage for measuring static pressures in the duct, his criticisms are, in general, well taken. If jets impinge upon the diaphragm, the reading of the gage is liable to be incorrect. However, the degree of error is believed to be considerably less than with a static hole, because the area of the diaphragm (in this case 16 sq. in.) was sufficiently large to annul the influence of a jet here and there. The average pressure over the diaphragm cannot be far from the true static pressure. This argument does not apply to the static hole, but it does to a more limited extent to a " piezometer ring," or equivalent. Furthermore, the duct area was made so large that the velocities therein were usually below 1200 ft. per min.[1] The pressure corresponding to this low velocity is less than 0.1 in. of water. Since the real object of the gage was only to serve as a check on other measurements, whatever error existed was of relatively slight importance.

It may be true that slightly higher efficiencies could be obtained by curving the blades back further, but it is doubtful whether any considerable gain can be effected in that way. From Fig. 9 it will be seen that the efficiencies for fan No. 2 with curved-back blades are nearly the same as for fan No. 3 with straight inclined blades. Yet the velocity heads of the air streams leaving the impeller of fan No. 2 were considerably lower than for those leaving fan No. 3. For the curved-back blades the fan must be made wider than for straight inclined blades, and must be wider the more the blades are thrown back. This increases the space occupied, and the cost as well. Furthermore, curved blades are more expensive to manufacture than are straight inclined blades.

In regard to the criticism of the statement that the air does not move perpendicularly to the inner periphery of the impeller, tests were made with threads at low volumes, when the air moved substantially tangentially, as would be the case with a solid wheel. In general, there is too much disturbance to explore carefully, particularly when there is much shock at entrance. It is well known that there is only one volume (for a given r.p.m.) which gives zero shock loss at entrance; that is, for which it is possible to draw a parallelogram of velocities assuming the inflow to be

[1] The maximum duct velocity for any test (with the widest fans) was 1570 ft. per min.

radial and the relative velocity to be determined by the volume divided by the area between blades. For other volumes either the relative flow is not along the blades at entrance, or the space flow is not radial. Both departures probably apply. The fact that the space flow is nearly tangential at low volumes (that is, that the action is like that of a solid wheel) would mean that the same action, but to a more limited extent, applied for larger volumes. If we were able to draw parallelograms of velocities for many volumes and be sure of the magnitudes and directions of the sides, a large part of the fan problem would be solved.

If the air does not whirl at the external and internal peripheries for appreciable depths, how does Dr. Moss explain the various phenomena observed in the tests? Or how does he account for the fact that in the usual multi-vane fan with forward-tipped blades the generated pressure at zero volume is approximately twice $V^2/2g$, where V is the external peripheral velocity? Even at the maximum-efficiency point the pressure is about 70 per cent higher than $V^2/2g$.

The method of applying the data given by Mr. Knowlton is one that the author has used also. It is described in the section of the paper beginning with Par. 98, which was omitted from the preprint copies available at the meeting.

Mr. Knowlton states that the maximum velocity in vent ducts in electrical machinery (presumably salient-pole alternators—see Fig. 7) is about 50 per cent of the peripheral velocity. That, of course, is an empirical relation, and can only be obtained experimentally. The velocity head of the air as it leaves the rotor is largely lost before the air enters the stator vent ducts, and most of the flow through the stationary vents is due to the static pressure generated. The path through them is tortuous, and consequently the flow is reduced by the obstructions. The pressure generated by the poles, which act as fans, plus that of the fans is undoubtedly more than would give a velocity of half its peripheral velocity. There are no data available at present on these relations, but it is hoped that they will soon be forthcoming.

Replying to Mr. Durfee, in regard to the shapes of the blades, these were made as simple as possible. Sheet steel 1/16 in. thick was cut square without any taper at the entrance or exit edges. This undoubtedly affected the performance adversely to a slight extent. The fans must be manufactured quickly, and they are only accessories to the electrical machine. Usually less than one per cent of the power output of the electrical machine is required to drive the fans, so that a small gain in efficiency which might be obtained by sharpening the exit or rounding the entrance blade edges is not warranted. The blades were simply bent and held in place by two rivets at each side, as is usual in practically all commercial makes of fans.

As was pointed out in the text, nearly all the velocity head of the air discharged from the external fan periphery was lost, as no collecting device was provided. That constitutes a very considerable portion of the loss. In commercial fans provided with a volute, much of that velocity head is recovered, and the efficiencies are consequently much higher.

The fans were not noisy in the commonly accepted sense of that term. Noise depends not only upon the amplitude of the sound wave, but upon the pitch, or combination of pitches. Whereas the radial-blade fans were more noisy than the others, the sound was of a fairly low pitch, and therefore was not objectionable.

No. 1926

SYMPOSIUM ON EFFECT OF TEMPERATURE UPON THE PROPERTIES OF METALS

INTRODUCTION

GREATER economy and security in central-station development and in oil-refinery operations require the designing engineer to have an accurate knowledge of the properties of metals under stress at high temperatures. Lack of this knowledge limits the use of higher pressures and temperatures, and the presentation of the following papers at a joint session held by The American Society of Mechanical Engineers and the American Society for Testing Materials at Cleveland on May 29, 1924, during the A.S.M.E. Spring Meeting, was most timely. These papers set forth the importance of securing full and authentic information, discuss the difficulty of comparing tests that have already been made on metals at high temperatures, and review the knowledge now existing in regard to the properties of metals at high and low temperatures.

This session was the direct outcome of the activity of Sub-Committee No. 3 on Steel Flanges, Sectional Committee on the Standardization of Pipe Flanges and Fittings, organized under the procedure of the American Engineering Standards Committee. Interest in the subject was stimulated by the discussion on the paper presented at the 1922 Annual Meeting of the A.S.M.E. by G. A. Orrok and W. S. Morrison on the Commercial Economy of High Pressure and Superheat in the Central Station, and by the papers by Frederick N. Bushnell and Charles H. Merz before the meeting of the National Electric Light Association in June, 1923.

The symposium at Cleveland was in charge of the following Committee on Arrangements: V. T. Malcolm, *Chairman,* H. J. French, W. F. Graham, R. S. MacPherran, L. W. Spring, and A. E. White.

Out of the presentation of these papers and the discussion thereon is being evolved a definite program of cooperative standardization and research upon which the commercial uses of metals at high and low temperatures may be based. Under the joint auspices of the two societies, a Research Committee on the Effect

of Temperature upon the Properties of Metals has been appointed, the personnel of which is George W. Saathoff, *Chairman,* R. L. Duff, George K. Elliott, H. J. French, John B. Johnson, V. T. Malcolm, John A. Mathews, and L. W. Spring. The three immediate purposes of the committee are:

1 Accumulation of existing unpublished data covering satisfactory and unsatisfactory service of various metals in different fields.

2 Standardizing procedure for testing materials at high and low temperatures. This should preferably include new comparative tests of metals by the principal methods now in use in various laboratories and likewise, a critical examination of data already published.

3 Outlining new research work to be done. The first and most important materials to be investigated are considered to be carbon and alloy steels, so-called "heat-resisting" alloys consisting of various combinations of nickel, chromium, iron, tungsten, molybdenum, etc., and trimming materials (chiefly alloys of nickel and copper) for valves and equipment intended for high-temperature service in power stations, oil refineries, etc.

No. 1926a

INDUSTRIAL APPLICATIONS OF METALS AT VARIOUS TEMPERATURES

By L. W. Spring,[1] Chicago, Ill.

Non-Member

From the time that James Watt utilized a pressure of 10 lb. per sq. in. in his steam-expansion engine, pressures have gone up very gradually to the 350 lb. per sq. in. now in common use. Increase in superheating temperature has been more rapid, but steady, during the twenty years since its general introduction. The author points out, however, that at the present moment a drastic and perhaps unwise change is taking place, since working pressures of 600 lb. per sq. in. will shortly be used in two or three plants, experiments with 1200 lb. pressure are to be tried out in small units in a commercial way and in England steam at its critical pressure of 3200 lb., when it is as heavy as water, is to be essayed. This increase in pressure holds out fascinating possibilities of high efficiency, and in the author's opinion can be taken care of by increase in metal thickness. But since metals lose considerably in strength above 600 deg. fahr., he advises caution in going beyond the present limit of 315 deg. superheat or a total temperature of about 750 deg. fahr.

There is another direction in which the properties of metals at high temperatures will have to be studied, and that is in the numerous gasoline cracking processes, where pressure has never exceeded 350 lb., but temperatures have risen to the high value of 1000 deg. fahr. in some instances. While little or no trouble with regard to materials in cracking processes has been experienced as yet, anxiety exists as to the consequences should any material containing flaws get into service. Cast steel has served well under these conditions of moderate pressure and high temperature. For steam-turbine blades and valve seats certain non-ferrous materials (alloys containing a considerable percentage of nickel and copper) are being used and must be thoroughly studied at high temperatures. It may also be necessary to consider ferrous alloys in lieu of the plain carbon steels which have given satisfaction in steam work thus far.

[1] Chief Chemist and Metallurgist, Crane Co.

Presented at a joint session of The American Society of Mechanical Engineers and the American Society for Testing Materials, at Cleveland, Ohio, May 29, 1924, during the A.S.M.E. Spring Meeting.

ABOUT one hundred and fifty years ago James Watt improved Newcomen's "atmospheric" steam engine and went to the then "enormous" steam pressure of ten pounds per square inch, utilizing in addition the expansive force of steam as well as its direct pressure. From that time till now, though quite slowly, steam pressures have gone up and up, through 50, 100, 150, 200 and 250 lb. per sq. in., each with its increasing temperature, until of late years pressures of 350 lb. per sq. in. have become more or less common. Though the superheating of steam before using was suggested by Watt it was not actually tried out by him and its use did not become at all common until within the last twenty years. In superheat practice the steam is given an additional 150 to 300 deg. fahr. (85 to 170 deg. cent.) of temperature, obtained, of course, by passing the steam from contact with the water in the boiler into another chamber, or through superheater pipes. Here it is converted by the further heating into "dry" steam with the additional heat mentioned, having then a total temperature of from 550 to 750 deg. fahr. (288 to 400 deg. cent.).

2 While the increases in pressures and temperatures during the approximately one-hundred-and-fifty-year period mentioned have been quite gradual, at the present time a seemingly rather drastic change is taking place which at first thought may appear to some to be very sudden, and possibly unwise, though it must be admitted that technical and mechanical talent have made wonderful strides within the last two decades. This change is the very decided increase in working steam pressures, which almost everywhere is occupying the thought of power-plant designing engineers and which, in some cases, is already being translated into actual plant installation. Regular working pressures of 600 lb. per sq. in. are being or shortly will be used in two or three plants which are looking toward the greatest possible efficiency to be obtained with safety. Undoubtedly, increases in working pressures are not to stop here. Three or four power-plant designers in this country already have arranged to try out, in a commercial way, in small units, steam at 1200 lb. working pressure. These have as antecedents a German experimental plant which operated at 900 lb. pressure for a period of nine months, and a plant in Sweden, which, though built for 1500 lb., is now operating at 900 lb. per sq. in. In one installation in England the full critical steam pressure of 3200 lb. per sq. in. is to be tried shortly.

3 When generated freely from water (that is, without pressure) steam occupies approximately seventeen hundred times the volume of the water it came from and has a specific gravity or density of about 0.0006. Under the critical pressure of 3200 lb. per sq. in., however, and at a temperature of at least 706 deg. fahr. (375 deg. cent.), it occupies only the equivalent of the volume of water from which it was generated and has, therefore, a specific

gravity of approximately one. In other words, under these conditions it has the same specific gravity as the water in the boiler and it is possible that the steam may be in and through the water or even below it in the boiler, since the two have the same density. The speculative possibilities and the promises of largely increased efficiencies by operating at 3200 lb. pressure and 750 deg. fahr. (400 deg. cent.) are very interesting and have been quite thoroughly discussed elsewhere.

4 Without dwelling longer on the subject of pressures, which have been referred to mainly to indicate the rather extreme tendency toward higher efficiencies and therefore toward new designs and probably new materials in steam power practice, we will pass to the matter of temperatures.

5 As told above, present-day superheated-steam practice has developed rapidly during the past twenty years with the result that a large number of power plants now use total operating temperatures of 550 to 750 deg. fahr. (288 to 400 deg. cent.). A superheat of 315 deg. fahr., that is, 750 deg. fahr. (400 deg. cent.) total temperature, has been and is yet considered the tentative top limit of temperature for high-pressure power-plant work. The reason for this is that metals which at present are available for steam-generating purposes undergo considerable losses in strength as their temperatures rise above 600 deg. fahr. (315 deg. cent.). Just where the point of recession begins and exactly where the curves of tensile strength, elastic limit, elongation, and reduction of area lie with temperature increases, are matters upon which metallurgists are not yet quite fully agreed. Safety requires that we go slow in increasing operating temperatures above the 750 deg. fahr. (400 deg. cent.) total temperature. How much this tentative maximum can be raised with safety eventually will depend in great measure upon agreement of results of metallurgists who are determining the strengths of materials to be used for power-plant purposes and also upon more satisfactory materials which probably will be devised. While working pressures are being increased now by leaps and bounds, because these can be taken care of by increase in metal thickness, it is probable that increases of working temperatures will be made much more slowly and cautiously.

6 There is a second extensive use of this class of materials at high temperatures, and here working temperatures have considerably exceeded the 750 deg. fahr. (400 deg. cent.) mark. This is the application of these metals — cast steel, malleable iron, and other boiler, piping, and valve materials—to the oil-refining industry. In the early days of petroleum refining it was chiefly the "coal oil" or kerosene which was utilized. There were at that time no internal-combustion engines with their demand for the more volatile gasoline. Likewise, there was little demand for petroleum lubricants, animal oils being considered the proper

lubricants for machinery. With the expanding use of the automobile and the tractor, the demand for gasoline and the volatile portion of petroleum has outrun their production through straight distillation methods. Hence there have been made extensive efforts through high temperature or combined temperature and pressure "cracking" to increase the production of gasoline or a serviceable equivalent by such artificial breaking up of the less valuable, heavier petroleum oils. Without going into the history of their development it can be said that there are now somewhere around fifteen or twenty recognized "cracking" processes. These vary considerably in type of apparatus used and in the application of heat, the taking off, separation, and condensation of the vapors formed, as well as in pressures and temperatures used. In general, it may be said that the pressures are comparatively low, though there seems to be a trend toward the use of higher ones. The majority of these processes use pressures of around 100 lb. per sq. in. with temperatures of from 750 to 850 deg. fahr. (400 to 450 deg. cent.). Three or four use 350 lb. pressure and 900 deg. fahr. (480 deg. cent.), while one uses as high as 600 lb. pressure and 900 deg. fahr. (480 deg. cent.). Pressures in the latter process probably surge as high as 750 lb. per sq. in., and temperatures may go as high as 950 deg. fahr. (510 deg. cent.). Two such processes are utilizing as high as 1000 deg. fahr. (540 deg. cent.) of temperature, with, however, only 350 lb. of pressure. While, apparently, there has been little or no trouble as to materials standing up so far as actual strength is concerned, even at these temperatures, anxiety naturally exists in the minds of both user and producer over consequences should any material containing flaws get into service. Furthermore, failures might occur through corrosion, breaking of parts under pipe strains, etc. Any failure is serious since the uncondensed vapors from the cracking stills are extremely inflammable and explosive.

7 As will be shown by the contributions of others to this program, losses in strength of cast steel at such high cracking temperatures as 900 and 1000 deg. fahr. (480 to 540 deg. cent.) are rather great, considerably more so than at 750 deg. fahr. (400 deg. cent.), the tentative top point for superheated-steam service. However, under existing pressures and other service conditions, cast steel is serving well. Whether much will be gained by further increase in cracking temperatures and what further pressure increases may be made remains for the future to tell.

8 There are certain non-ferrous metals which are made use of in equipment for power plants and oil refineries. These are the metals or alloys of which steam-turbine blades, valve seats, and sometimes other parts of valves are made. In general, valve-seat alloys contain as a base considerable percentages of nickel and copper. Under increased temperatures some of these alloys do not suffer more than cast steel, so far as strength is concerned.

In addition to good strength under working conditions, such alloys must have sufficient hardness to resist the cutting action of steam, scale or grit, for, to be satisfactory, valve seats must remain tight. Seating metals must have approximately the same coefficient of expansion under heat as that of the valve body and disk metal, and they should be as non-corrosive as possible, for corrosion also is an enemy to valve tightness. While considerable research is constantly going on, the ideal valve-seating material for high-pressure, high-temperature steam probably has not been found, and, certainly, the proper seating metal for oil-cracking vapor service is yet a matter of doubt.

9 While in the ferrous alloys the plain carbon steels have given satisfactory service so far and undoubtedly can be used safely at somewhat higher temperatures than are considered wise at present, it may be that alloy steels eventually will be found desirable.

10 Such alloys as chrome-nickel, chrome-iron-nickel, and others which are often used for carbonizing boxes, annealing pots, supports for articles in enameling ovens, floors and arches of furnaces, where temperatures of from 1600 to 2000 deg. fahr. (870 to 1100 deg. cent.) prevail, are hardly within the scope of this paper. It is possible, however, that these indicate the trend toward other materials which may be necessary to keep pace with the demand for higher operating pressures and temperatures in various industrial applications.

No. 1926 *b*

METHODS OF TESTING AT VARIOUS TEMPERATURES AND THEIR LIMITATIONS[1]

By V. T. Malcolm,[2] Indian Orchard, Mass.

Non-Member

The paper points out that in the use of temperatures above normal, characteristics of materials must be carefully studied; failure in such cases generally has been due not to the quality of the metal or alloy but to improper application. The testing of metals at high temperatures therefore becomes a very fruitful field for research. Experiments of this character date back to 1820, *and extensive literature on the subject is already available, although the modern line of investigation dates back only to* 1912. *Early methods consisted in heating a specimen in a furnace and then transferring it quickly to a testing machine. Such methods were necessarily crude. Also temperatures in early experiments were taken at points distant from the test specimen, thus introducing further error.*

In the past ten years, methods of furnace construction, location of the thermocouple, calibration of the pyrometer, and general checking up of the work, have led to much greater accuracy. In work by French, the electric furnace was incorporated in the testing machine itself, thermal equilibrium was very carefully obtained, and temperature determinations were made under actual test conditions, by placing thermocouples in holes located at various points in the specimen, carrying the entire auxiliary apparatus. Bregowsky and Spring extended their studies to determine variation of temperature of test specimens at several internal and external points, using specimens drilled axially. Priester and Harder gradually heated specimens to the desired temperature and held the temperature steady for thirty minutes to obtain thermal equilibrium. Spooner noted that with addition of nickel, chromium, and tungsten the color of fractured bars changed considerably. Other investigators, both in America and abroad, proceeded on similar, constantly more refined lines. In experiments conducted by the author himself, interesting results were obtained showing variation both from point to point along

[1] For discussion and closure see pp. 489-534.

[2] Metallurgist, The Chapman Valve Manufacturing Co., Indian Orchard, Mass.

Presented at a joint session of The American Society of Mechanical Engineers and the American Society for Testing Materials, at Cleveland, Ohio, May 29, 1924, during the A.S.M.E. Spring Meeting.

the surface and from surface to center. Jeffries and Sykes extended the range of the work to copper, tungsten, Armco iron, nickel, and molybdenum wires, with appropriate variations in methods of test, sometimes using boiling water, and sometimes crisco, which can be heated to 480 deg. fahr. without volatilization. In impact tests, Guillet and Revillon used representative methods, heating specimens slightly above the temperature required in an electric furnace, placing them on the anvil, and measuring temperatures a short distance from the cross-section to be fractured. Methods similar to those in tensile tests have been applied to torsion experiments, but very little has been done in alternating stress tests. X-ray tests and "long-time" tests are in process of development.

In conclusion, the author points out that the effect upon the physical properties of metals of raising the temperature cannot yet be stated in terms of a definite law, although it may be generally assumed that the tensile strength and elastic limit of steel decrease, and elongation and reduction of area increase, as the temperature is raised. Disagreement between tests on steel and results with alloys in practice are explained on the ground that alterations in physical properties of metals and alloys due to variations in temperature are not always of the same nature. In particular, the author stresses the importance of directing research toward such tests as approximate working conditions rather than allowing it to retain an academic character.

Introduction

IN THE design of apparatus for use at temperatures either above or below normal, it is of importance that the designer be familiar with the physical characteristics of the metals or alloys specified at the temperature to which they will be subjected in service. As many processes require the use of metals at temperatures other than normal, the degree of success of a metal or alloy must be measured by its stability under such working conditions. As a matter of fact, most failures at temperatures other than normal are due not to the quality of the metal or alloy but to the application of certain metals in connection with work for which they are entirely unsuited. For use at other than normal temperatures a careful study must be made of an alloy's characteristics, for with the advent of the central power station and oil refinery operating at great pressure and high temperature, failure of material in service is likely to be disastrous both to life and property.

2 At the Power Session (**190**)[3] of the Annual Meeting of The American Society of Mechanical Engineers held December 7, 1922, in New York, a statement was made that power-plant

[3] The bold-face numbers in parentheses refer to the references in the bibliography on pages 477-487.

equipment had kept pace with the demand, except information regarding the properties of metals at elevated temperatures. Again at the Annual Convention(195) of the National Electric Light Association held June 4, 1923, at New York, it was claimed that while higher pressures were assured, higher temperatures must be provided for in metallurgy. In *Mechanical Engineering*(211), March, 1924, a résumé of power-plant progress is given, and the author calls attention to the fact that the use of temperatures above 750 deg. fahr. (400 deg. cent.) has not been contemplated owing to the lack of reliable information.

3 The importance of these statements to the mechanical engineer is profound, as he is directly responsible for the safety of the structures he designs, and he is probably unwilling to take for granted that a metal is suitable for other than normal temperatures without being convinced that tests have been carried out in a correct manner.

4 The testing of metals at temperatures other than normal is probably one of the most vital fields of engineering research to-day and will in the near future be one of the most actively exploited fields of metallurgy in the search of alloys, both ferrous and non-ferrous, to withstand temperatures other than normal. A most cursory examination of scientific and technical literature will convince any one of the extent of scientific interest in this subject, and we believe it is one of the most serious problems confronting industries today.

5 The ultimate aim of all temperature tests should be to devise a comprehensive series of tests to which standard specimens of metals or alloys may be subjected and by which the relative physical properties of these metals may be predicted for certain service conditions. At this time any one who undertakes a survey of the literature on the temperature problem is certain to be overwhelmed by the different types of furnaces, extensometers, strain gages, thermocouple location, etc., that have been used by the various investigators, and, on the other hand, numerous curves and data are shown which are comparatively worthless to the designing engineer unless the method of making tests and reaching thermal equilibrium are shown also. Therefore, the problems at hand are of such importance that it becomes necessary to give statements on the apparatus used, method of testing, and manner of reaching thermal equilibrium.

6 It is with this idea in mind that an attempt has been made to review part of the literature with reference to tests of metals at various temperatures by investigators who are thoroughly familiar with their work, and it is hoped in this way to reach some definite plan by which the work of these men may be correlated, and standardized apparatus, method of testing, etc., be developed for use in all future tests.

Historical

7 In reviewing the literature on the testing of metals at various temperatures, it is well to go back to the beginning and carry the work forward to the present time in order to show the remarkable progress that has been made in methods of test in this branch of engineering research.

8 As early as 1828 Tremery and Proirier Saint-Brice (1) carried out a series of experiments on the tensile strength of wrought iron. In 1837 Sir William Fairbairn(2) made a number of experiments on cast iron at various temperatures. In 1837 research was carried on by a Committee of The Franklin Institute(3) on the Effect of Temperature on Boiler Plate. In 1856 Fairbairn(4) carried out a series of experiments on rolled iron at various temperatures. In 1860 David Kirkaldy(5) of Glasgow, Scotland, carried out a number of interesting experiments on the value of iron and steel at various temperatures, especially investigating the action of frost upon the metals. William Brockbank(7) describes some very interesting experiments in determining the effect of cold upon cast iron. In 1863 a series of tests was conducted by the Royal Technological Institute of Stockholm(6), on the properties of irons and steels at various temperatures. In 1871 Peter Spence(8) carried out a number of investigations with cast iron at low temperatures. Jouroffsky(12) of St. Petersburg in 1879 conducted a number of tests on rails at low temperatures.

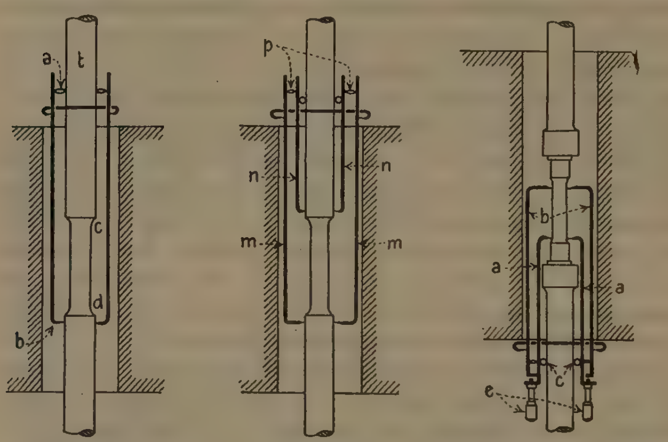
9 Between 1885 and 1905 a great deal of attention was given the subject of testing metals at various temperatures by such well-known investigators as Rosenhain and Humfrey(74, 93), Le Chatelier(39), Martens(18), Bach(44, 50), Unwin(16), Hopkinson and Rogers(55), Rudeloff(20, 36, 64), Stribeck(48, 52), Howard(17), Charpy(21, 25), Carpenter(24), and Hadfield(54), and a considerable number of articles appeared in the German and British engineering literature. Very little work was done in the United States during this period with the exception of the work of Howard.

10 The modern line of investigation was begun about 1912 and is still being carried on by such investigators as Huntington(85), Bengough(82), Hansen(101), Dewrance(98), Aitchison(123), Lea(187), Dickenson(173), and Dupuy(150), in England and France, and in this country by such men as Meyers(128), Spring(83, 216), Jeffries(127), Sykes(163), Merica(168), McNiff(144), French(153, 179), Spooner(162), MacPherran(158), Langenberg(200, 201), Epps and Jones(116), Perrine and Spencer(104), Priester and Harder(206), White and Upthegrove(147), Spieler(215), d'Arcambal(146), and the author(188, 189, 203, 212).

Early Methods of Testing

11 The old methods used for testing metals at various temperatures consisted of heating a specimen in a furnace or forge or freezing it in some liquid medium, transferring it to a testing machine, and quickly conducting the test at normal temperature, allowing for loss or gain of heat in transferring test bars and in conducting the test. In some cases the temperature was merely judged by the oxide film on the specimen and in other cases by the use of thermometers. These methods were of course very crude and the results unreliable, but as this was before the day of our modern equipment the tests served their purposes.

12 In 1888 a most extensive series of tests was made by Howard(17) at the Watertown Arsenal, and as this was also before



(a) Martens' Arrangement (b) Rudeloff's Arrangement (c) National Physical Laboratory Arrangement

Fig. 1 Diagrams of High-Temperature Extensometers

the day of the electric furnace and the pyrometer the specimens were heated by a series of Bunsen gas burners and the temperature estimated from the coefficient of expansion of the heated specimen. No temperature correction for the coefficient of expansion was used. These tests received and still receive considerable favorable comment.

13 In 1890 Martens(18) published his investigations on the tensile properties of iron, steel, and copper at elevated temperatures. He used a bath of paraffin for temperatures up to 400 deg. fahr. (200 deg. cent.) and a bath of lead or lead-tin alloy for temperatures from 400 to 1100 deg. fahr. (200 to 600 deg. cent.). The gas jets were located on the sides in conjunction with a vertical testing machine. For measuring temperatures he used

in the former case a mercury thermometer and in the latter case an air thermometer. For determination of the elastic limit he adapted his mirror extensometer shown in Fig. 1. The test piece was turned down in the center at *c d* and the extensometer clips were attached at *b* on the lower enlarged end and were carried out of the furnace, for the attachment of the measuring rhombs *a*.

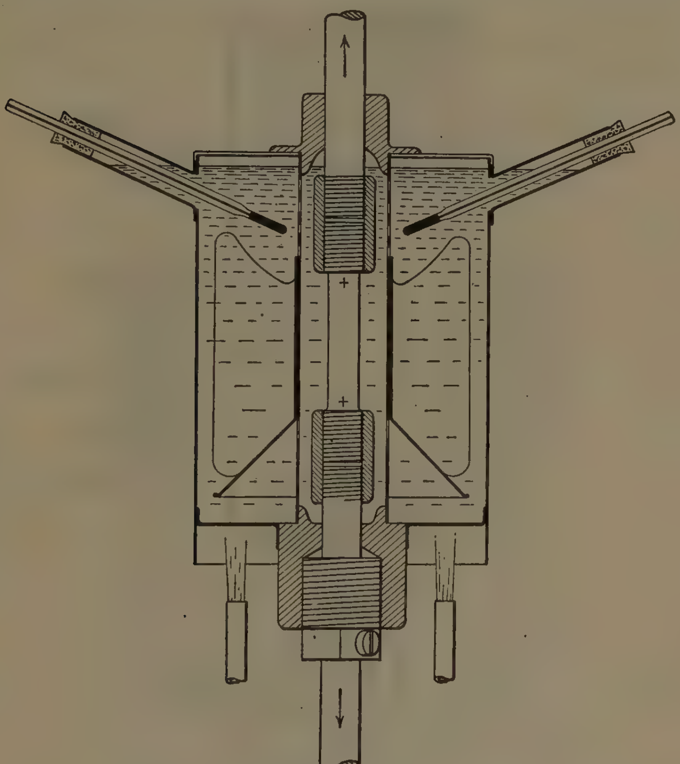


FIG. 2 DIAGRAM OF TESTING APPARATUS USED BY UNWIN

The extension, therefore, was measured on the length *a b* and correction was made for the extension of the enlarged ends, in order to obtain the extension of the gage length, *c d*.

14 Unwin(16) in 1889 published results of tests of metals at elevated temperatures. These tests were carried out by heating the specimens in an oil bath from below by gas burners, the whole apparatus being placed between the jaws of a testing machine as shown in Fig. 2. Thermometers were used to measure the temperature. Charpy(25) in 1896 also used a bath with burners located below. Le Chatelier(39) used a horizontal testing machine

with gas jets located above. Batson(80) used steam coils in the testing of 50-ft. lengths of copper wire up to temperatures of 140 deg. fahr. (60 deg. cent.). Stribeck(48, 52) used a cylindrical oven, electrically heated, through which the bar passed. Other investigators used methods more or less similar, the main features being air, gas, or liquid baths heated by blast lamps, Bunsen

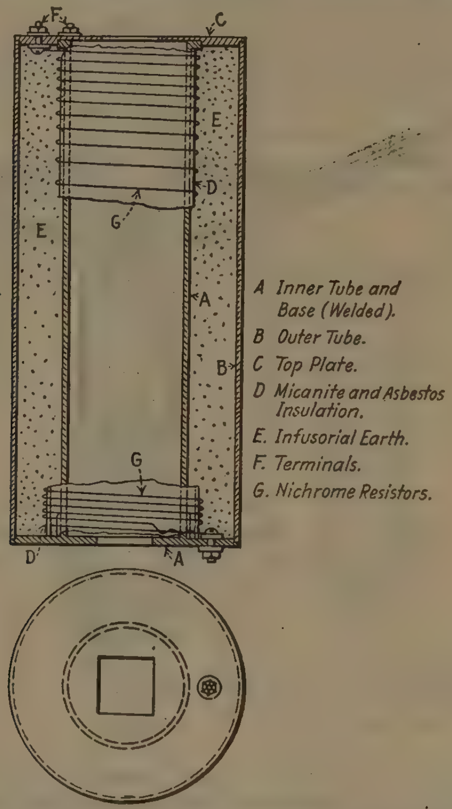


Fig. 3 Electric Furnace Used by French in High-Temperature Tests

burners, or electric current, and in the case of low temperature a freezing solution.

15 In nearly all the early investigations we find that temperatures were taken at points distant from the test specimen itself, the assumption being that temperature of bath and test bar must be alike.

16 It is therefore our conclusion that most of the early investigations can be only approximate determinations at best.

Modern Methods of Tension Testing

17 During the past ten years methods of furnace construction, location of the thermocouple, calibration of pyrometer, and the general checking up of the work in hand has received close attention and great accuracy has been attained, so that at this time conclusive results have been obtained and industrial applications certainly may be based on these findings.

18 The apparatus and methods of test of some of the more prominent investigators during the past ten years will be described in detail. Attention will first be devoted to tension tests, after which reference will be made to other tests of metals at various temperatures.

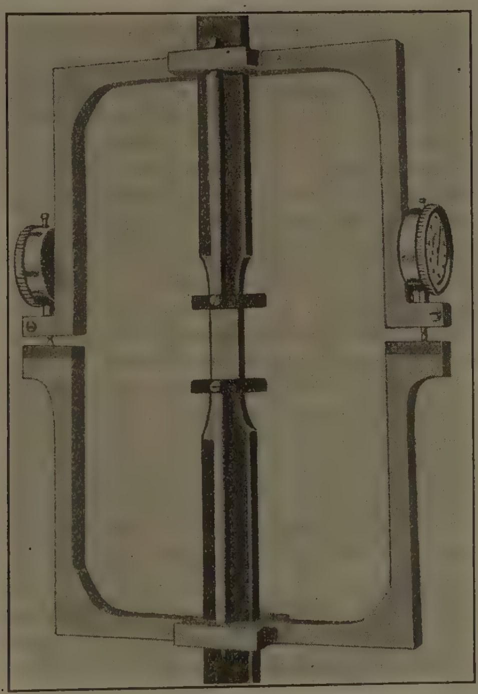


Fig. 4 Apparatus Used by French for Determining Proportional Limit

H. J. FRENCH(153, 179)

19 *Furnace.* The test specimens are heated by means of an electric furnace shown in Fig. 3. Two spiral resisters in series are used: One covers the entire length of the inner tube (11 in.) and the other is concentrated at the ends, the two requiring about

80 ft. of No. 22 nichrome wire. Yokes and the greater part of the test bar and rods are contained in the heating chamber, which is 11 in. long. The furnace is operated on either 110 or 220-volt direct current, close regulation being obtained by a variable resistance in series in the circuit.

20 *Proportional-Limit Apparatus.* The apparatus used in the determination of the limit of proportionality at various temperatures, illustrated in Fig. 4, consists primarily of two aluminum-alloy frames each rigidly fastened to a quenched-and-tempered steel yoke by two annealed low-carbon steel rods. The specimen passes freely through holes in the base of each of the frames. Yokes are clamped to the specimen by three quenched-and-tempered high-speed steel screws, while the spreading of the former is overcome by the long screw. The flanges on the upper frame are so arranged that dial micrometers for indicating deformation may readily be securely fastened to them, while those of the lower frame are capped with polished steel plates in order to give a smooth bearing surface to the plungers of the dials.

21 The smallest division on the instruments used is equal to 0.001 in., but estimated readings to the nearest 0.0001 in. are readily obtained. When stress is applied to the specimen, one-half the algebraic sum of the deformation recorded by the two dials represents the deformation of the specimen, which is centrally located with respect to the entire apparatus.

22 *Test Procedure.* The method of setting up the apparatus together with the procedure followed in actually carrying out the tests is substantially as follows: A specimen is marked on the surface with a double-pointed center punch leaving marks 2 in. apart. The yokes are attached to the specimen by setting the single screw into the impressions. Then by lightly tapping the opposite side of the yoke containing the two screws, a light impression of their exact location on the test bar is obtained. These points are then enlarged by use of the double-point center punch, and the yokes carrying rods and frames are firmly attached to the test piece. Bolts holding the upper frame to the two rods are next taken off and the upper frame removed. The specimen is then passed up through the furnace until the rods appear above the top, when the upper frame is again fastened to the rods. After the furnace is placed on the stand and the specimen is in the jaws of the testing machine, the dials are attached to the frame and adjusted to zero. The completely assembled apparatus is shown in Fig. 5.

23 When thermal equilibrium at the desired temperature is reached, an initial load of about 1500 lb. is applied and the dials read or, as a matter of convenience, set at 17.2. Readings are then made at increments of 500 or 1000 lb. actual load until the proportional limit is passed. The dials are then removed and the

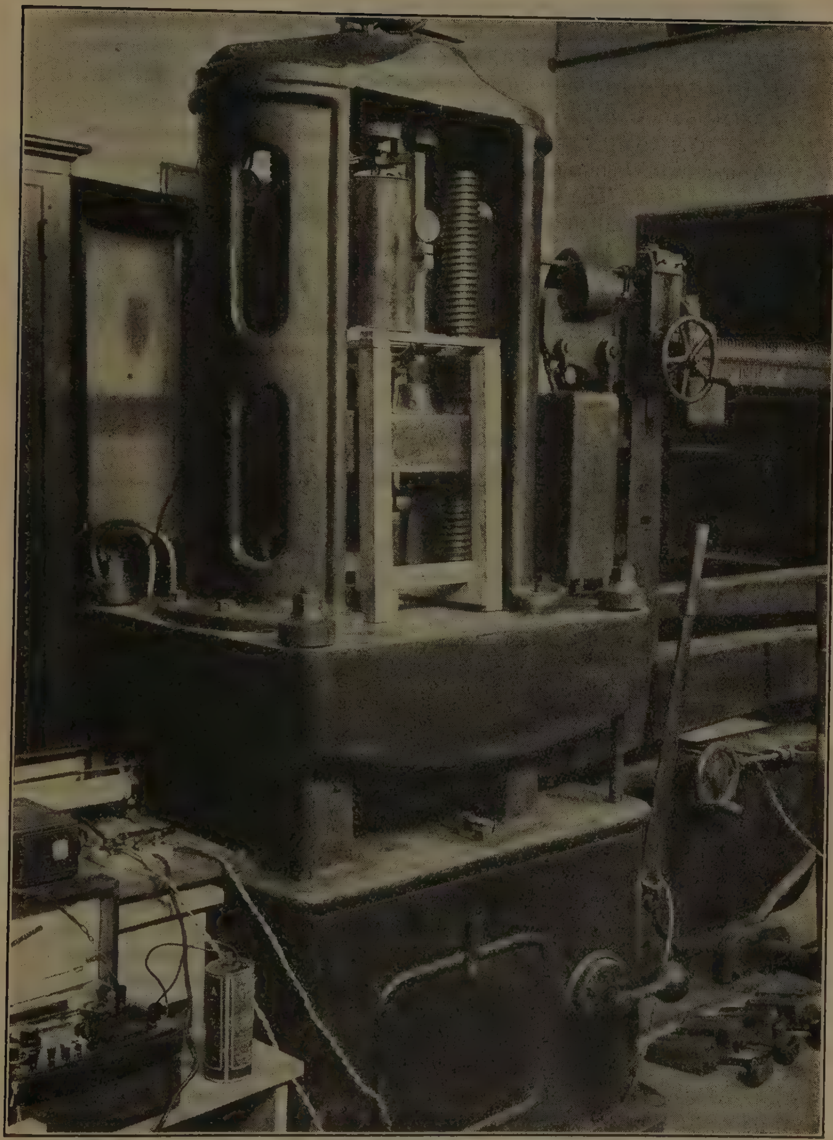


FIG. 5 ASSEMBLED APPARATUS USED BY FRENCH FOR DETERMINING THE TENSILE PROPERTIES OF METALS AT HIGH TEMPERATURES

specimen is broken in the usual manner with a low rate of extension which approximates the intermittent increases of stress applied during determination of the limit of proportionality. Tests at each temperature are made in duplicate or triplicate and the proportional limit is obtained from the stress-strain diagram.

24 The temperature is measured by a No. 22 standardized chromel-alumel couple connected to a Leeds and Northrup portable potentiometer. The end of the couple is inserted directly into a small hole drilled in the specimen at the fillet, its exact location being shown in Fig. 6.

25 *Thermal Equilibrium.* In order to obtain reliable and satisfactory results with the method described in the preceding paragraphs, thermal equilibrium must be reached prior to the start

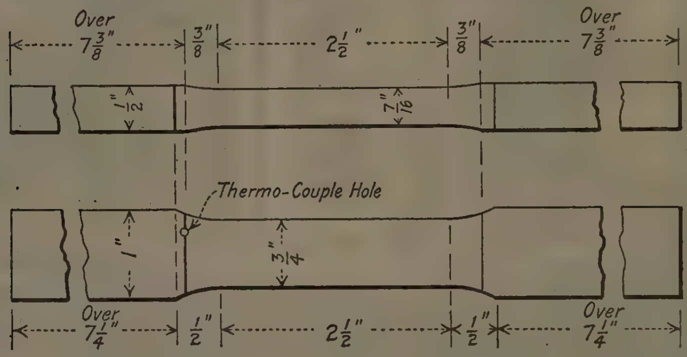


FIG. 6 FORM AND DIMENSIONS OF TEST SPECIMEN USED BY FRENCH

of the loading and maintained during the actual 8 to 15 minutes during which the test is being carried out. The adjustable resistance in series in the electrical circuit makes current adjustment possible, so that the loss of heat from the heating unit, ends of test specimen and auxiliary apparatus by radiation, convection, and conduction balances the energy added to this entire system. The effect of temperature variations may be large unless care is taken to allow sufficient time for the specimen to become uniformly heated throughout after the potentiometer has once indicated the desired temperature. The dial readings will assist in determining when equilibrium has been reached and is being maintained.

26 Temperature determinations under actual test conditions, made by placing thermocouples in holes located at various points in the specimen carrying the entire auxiliary apparatus, show that the position chosen for the single thermocouple (in the fillet) is representative of about the mean gradient throughout the gage length, where the temperature gradually decreased from top to bottom. Fig. 7 shows a partial reproduction of these variations,

which are within 45 deg. fahr. (30 deg. cent.). It is the greatest in the upper temperature ranges under consideration, and does not exceed 36 deg. fahr. (20 deg. cent.) at the lower temperature used. However, as the thermocouple, specimen with auxiliary apparatus, and furnace are in the same relative position in each test, the results obtained at various temperatures throughout the range, 70 to 870 deg. fahr. (20 to 465 deg. cent.), are comparable.

I. M. BREGOWSKY AND L. W. SPRING (83, 216)

27 *Furnace and Testing Apparatus.* The heating apparatus consists of a $7\frac{1}{2}$ by $1\frac{9}{16}$-in. sleeve of $\frac{1}{32}$-in. sheet iron wrapped with one layer of $\frac{1}{16}$-in. sheet asbestos for insulation, and wound

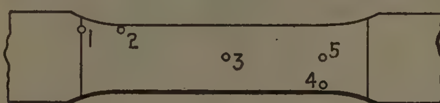


FIG. 7 TEMPERATURES AT VARIOUS PARTS OF TEST SPECIMEN

Desired Temperature, deg. fahr.	Desired Temperature, deg. cent.	Point No. 1, deg. fahr.	Point No. 1, deg. cent.	Point No. 2, deg. fahr.	Point No. 2, deg. cent.	Point No. 3, deg. fahr.	Point No. 3, deg. cent.	Point No. 4, deg. fahr.	Point No. 4, deg. cent.	Point No. 5, deg. fahr.	Point No. 5, deg. cent.	Time after couple no. 1 first reached desired temperature, minutes	Average Temperature of Couples Nos. 2, 3, 4, and 5, deg. fahr.	Average Temperature of Couples Nos. 2, 3, 4, and 5, deg. cent.	Maximum Temperature Variation, deg. fahr.	Maximum Temperature Variation, deg. cent.
329	165	329	165	343	173	333	167	316	158	329	165	15	299	166	27	15
608	320	608	320	621	327	621	327	590	310	612	322	0	580	322	31	17
		617	325	633	334	633	334	608	320	644	329	5	592	329	25	14
		617	325	637	336	633	334	604	318	630	332	20	594	330	29	16
752	400	756	402	779	415	774	412	739	393	756	402	10	729	405	40	22
		756	402	779	415	774	412	739	393	756	402	20	729	405	40	22

on each end with twenty turns of No. 18 or No. 20 chromel wire, leaving a gap of 3 in. in the middle, except for the approximate one-half-turn connection between the end windings. The wire is plastered thinly with alundum cement to prevent excessive oxidation at high temperatures. Outside layers of sheet asbestos, shrunk on, insulate the wires and retain the heat. The coil, test bar, and hole drilled in it for the thermocouple are illustrated in Fig. 8. Test bars are turned to 0.505 in. in diameter over the breaking section and the ends are threaded to fit the self-centering holders of the testing machine. In the top end of the bar a $\frac{3}{16}$-in. hole is drilled axially to within about $\frac{3}{8}$ in. of the turned-down breaking section. In testing, the coil is suspended from the upper holder. The test bar, heating coil, and self-

centering holders into which the threaded ends of test bars screw are heat-insulated from the testing machine by ½-in. pads of asbestos, bolted between steel flanges which form parts of the self-centering holders. The general assembly of the apparatus showing insulating flanges, heating coil, extensometer, and pyrometer is given in Fig. 9.

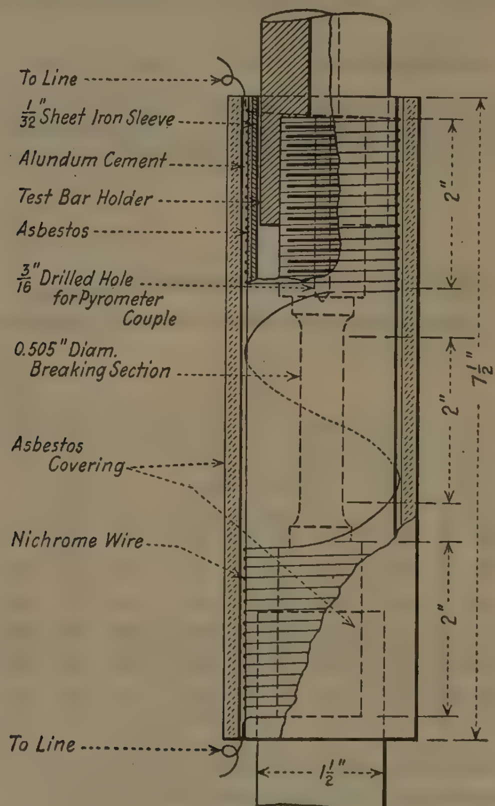


Fig. 8 Heating Coil and Test Specimen Used by Bregowsky and Spring

28 In the hole in the top end of the bar a thermocouple is inserted, which gives the temperature inside the bar very near the turned-down breaking section. By careful regulation, any desired temperature between 70 and 1200 deg. fahr. (21 and 650 deg. cent.) can be obtained in from 20 minutes to 1½ hours and held without more than a few degrees' fluctuation. Readings for proportional limit and yield point with the aid of an extensometer attached to the bar holders and breaking the bar (all of which

requires four or five minutes) are accomplished within a few degrees of the desired temperature. Temperatures are taken with chromel-alumel couples, carefully standardized against the freezing points of chemically pure tin, zinc, and aluminum. Measurements are read on a millivoltmeter.

29 On various occasions tests were made to determine variation of temperature of test specimens at several internal and external points, using for this purpose specimens drilled axially nearly to the lower end. The thermocouple inserted in the drilled hole

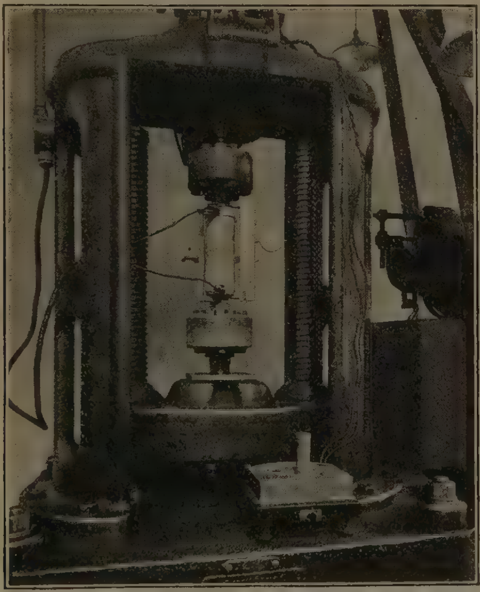


Fig. 9 Testing Machine Used by Bregowsky and Spring with Test Specimen, Heating Coil, and Temperature-Measuring Apparatus in Place

was raised or lowered from time to time and readings were taken at various locations inside the bar. When using coils wound over their full length it was found practically impossible to avoid a higher temperature in the center or breaking section of the test bar than at the ends. Part of this variation is attributed to conduction of heat away from the ends of the test bar through the bar holders. Therefore, coils were developed wound on the ends only, and by throwing all the heat upon the ends of the test bar and none along its middle or heating section a quite uniform temperature in the breaking section and adjacent parts was obtained, the variations not exceeding 35 deg. fahr. at 1200 deg. fahr. (20 deg. cent. at 650 deg. cent.).

30 Tests were made to determine the accuracy of taking temperatures on the surface of the bar, by strapping the tip of another thermocouple to the outside of the drilled test bar at the middle of the breaking section and taking comparative readings after thermal equilibrium had been attained. Of course, these tests showed that temperature readings taken at the surface are somewhat higher than readings taken inside, and it is believed that an

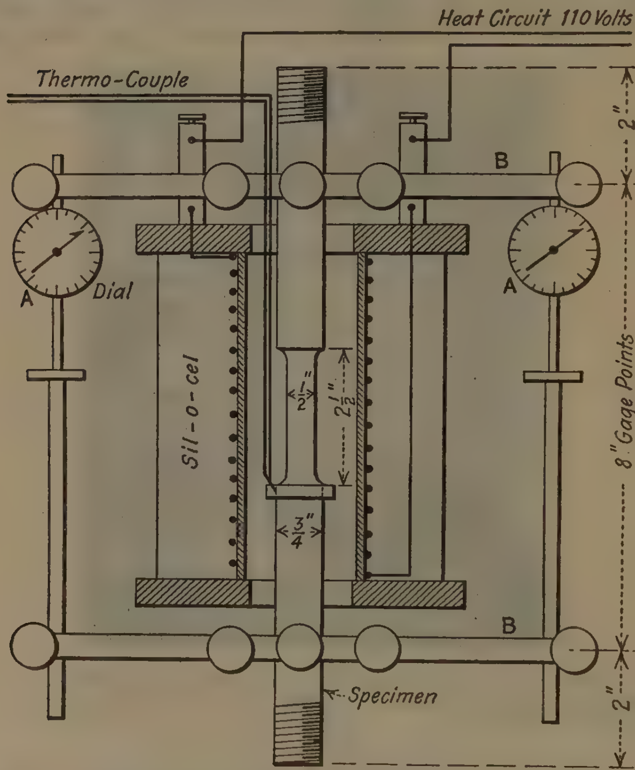


FIG. 10 SKETCH OF APPARATUS OF PRIESTER AND HARDER FOR MECHANICAL TESTS AT ELEVATED TEMPERATURES

"outside" thermocouple gets direct radiant heat from the heating coil and therefore registers higher. Moreover, even without direct radiant heat and assuming that the thermocouple registers the temperature of the outer fibers of the bar perfectly, the outer fibers naturally have a higher temperature than the center of the bar, because the heat goes from the surface toward the center.

G. C. PRIESTER AND O. E. HARDER(206)

31 In order to make tests at elevated temperatures a special apparatus as shown in Fig. 10 was developed. The temperature

of the specimen is measured by a carefully calibrated thermocouple, the hot junction of which is clamped in contact with the lower shoulder of the specimen. The specimen is gradually heated to the desired temperature by means of an electric furnace and is held at this temperature for 30 minutes to establish thermal

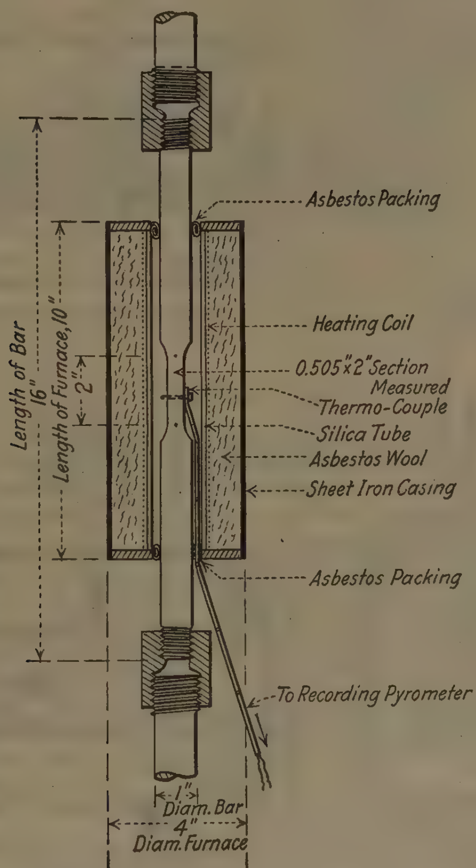


Fig. 11 Priester and Harder Arrangement of Test Specimen in Furnace

equilibrium. Under these conditions it is believed that any heat changes that take place during the heating of the specimen are negligible. It is believed that there can be no question about the accuracy of the temperature measurements made in this way.

32 Ames dial gages attached to yokes are used. The test pieces are heated to the required temperature in the furnace for one-half hour before testing.

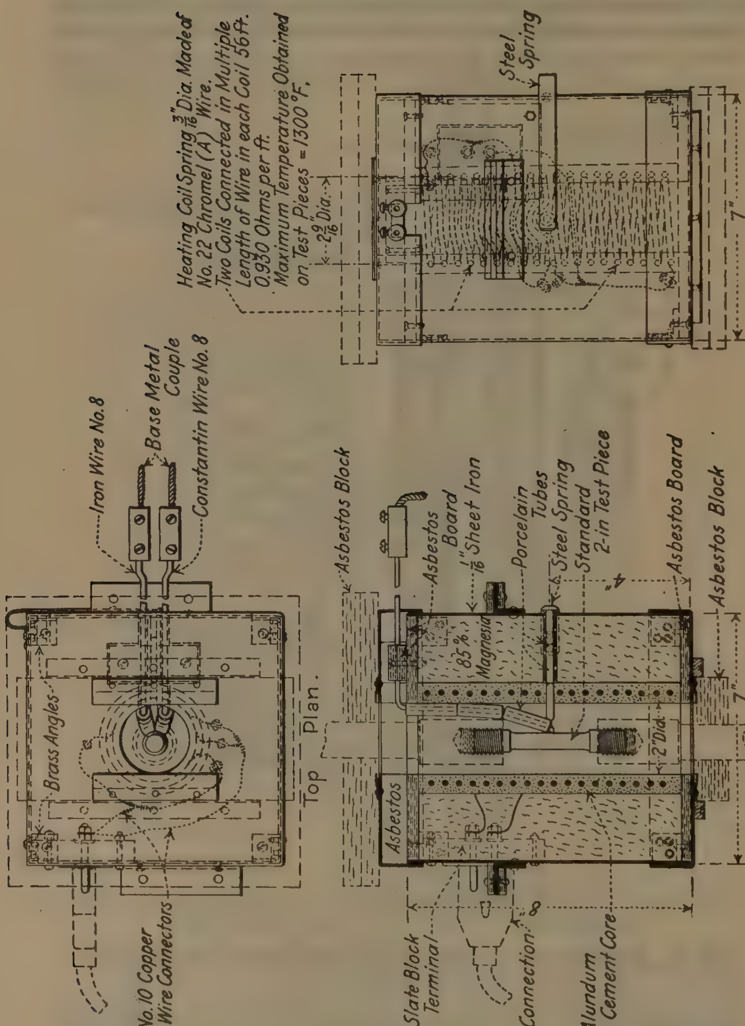


FIG. 12 DIAGRAM OF ELECTRIC HEATER USED BY MACPHERRAN, SHOWING SPECIMEN IN POSITION

A. P. SPOONER (162)

33 Specimens are heated in a circular electric furnace, with a thermocouple inserted against the middle of the pull section and connected with a Leeds and Northrup recording pyrometer. The thermocouple is wired to the necked-down section of the specimen and asbestos packing used to close the ends of the furnace tube, which is 1½ in. in diameter. All specimens were held at the testing temperature for 30 minutes before the load was applied. The apparatus is illustrated in Fig. 11.

34 The specimens are pulled in a 100,000-lb. Emery testing machine. The yield point is determined by the drop of the beam and occasionally checked up with the dividers.

35 An interesting feature to be noted in the tests reported by Spooner is the fracture of the broken test bars at the different temperatures. The usual commercial, structural and carbon steel fractures look very much alike, but with the addition of certain per centages of nickel, chromium, and tungsten the colors of the fractured bars change considerably.

R. S. MacPHERRAN (158)

36 The furnace used is a heating box 7 in. square by 9 in. high. The hole in which the test specimen is inserted is 2 in. in diameter and is lined with a core of alundum cement wound with No. 22 chromel wire. This wire is wound in two coils, one over the top and one over the lower half of the alundum core. They are connected in multiple. The coils are first covered with alundum cement and then with magnesia. The thermocouple enters at the top of the box and runs down in such a position that the point is opposite the center of the test specimen. A spring on the outside of the box forces a porcelain rod through a hole against the base metal couple, holding the point against the test specimen. The temperature is measured with a Leeds and Northrup pyrometer using a base metal thermocouple. The apparatus is illustrated in Fig. 12.

37 The specimens used are the standard 0.505-in. test bars with threaded ends. All tests are held at constant temperature from 15 to 30 minutes before pulling. The proportional limit was not measured but the yield point was determined by the drop of the beam, and above 600 deg. fahr. (315 deg. cent.) it was uncertain.

38 MacPherran states there are two ways to determine the temperature in testing metals at elevated temperatures: One with the thermocouple in contact with the outside of the specimen at the center of its gage length and the other by inserting it in the specimen outside the gage length. If the thermocouple is in contact with the surface of the specimen at the center, the test specimen cannot be hotter than the couple. If it is inserted in

the specimen outside the gage length the couple cannot be hotter than the specimen. The possible error of the first method is to get temperature readings that are too high and of the second method to get readings that are too low. After carefully con-

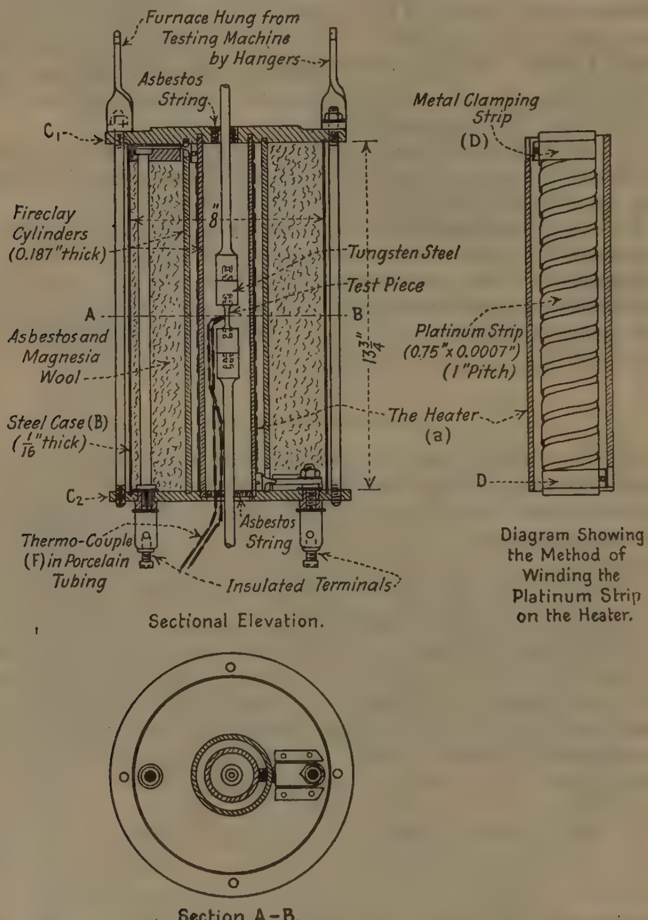


FIG. 13 NATIONAL PHYSICAL LABORATORY PLATINUM HIGH-TEMPERATURE FURNACE FOR TENSION TESTS

sidering both methods the former was selected. It is possible that the higher temperatures found by several investigators are due to the placing of the thermocouple. It would seem, however, that there is more to be said in favor of the system adopted than for the one requiring the insertion of the thermocouple junction in the test specimen. It is very difficult to maintain a constant

temperature for a considerable distance in the furnace tube, and it is believed by MacPherran that the temperature should be taken as near as possible to the point of rupture.

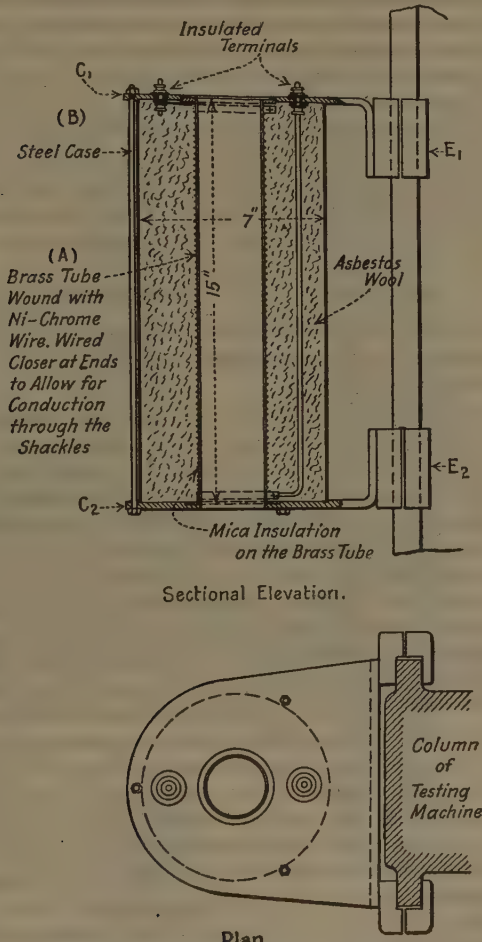


Fig. 14 National Physical Laboratory Furnace for Tension Tests up to 1100 deg. fahr. (600 deg. cent.)

39 A series of tests were made with one thermocouple touching the outside of the center of the test specimen as in the regular tests, and another couple adjusted so that the point was in the exact center of a hole drilled through the specimen in the same horizontal plane as the point of the first couple. To protect the point of this second couple, the hole was then plugged with

asbestos. The two couples therefore indicated the temperatures at the surface and center of the test specimen. To emphasize the difference between center and outside, a test specimen 0.75 in. in diameter was used in place of the regular 0.505-in. diameter bar. Three sets of readings were taken at temperatures in excess of 500 deg. fahr. (260 deg. cent.) with the result that the center of test specimen was found to be 22 to 36 deg. fahr. (12 to 20 deg. cent.) below the temperature of the outside of the bar. With the 0.505-in. bar this difference would of course be less. As the area represented by the outer layers is much greater than that represented by the center, MacPherran believes that the central outside thermocouple location as used in these tests gives a better indication of average temperature of the specimen under test.

NATIONAL PHYSICAL LABORATORY(169)

40 In the work of the National Physical Laboratory in England, two types of furnace were used in connection with a vertical testing machine.

41 A platinum furnace, Fig. 13, was used for temperatures up to 2200 deg. fahr. (1200 deg. cent.), the heating element consisting of a platinum strip 0.75 in. by 0.0007 in. wound on a fireclay cylinder with an outside diameter of $\frac{2}{25}$ in., a thickness of 0.187 in. and a length of $13\frac{7}{8}$ in. The ends of the strips are clamped in position on the cylinder by metal clips. A steel case surrounds the heater, and the space between the two is packed with asbestos and magnesia wool. The whole apparatus is clamped between two end plates on one of which two insulated terminals are fixed, these being connected to the two ends of the platinum heating coil. The furnace itself is suspended from the top shackle of the testing machine and uses a current of 15 amperes and 105 volts.

42 The second furnace used is for temperatures up to 1100 deg. fahr. (600 deg. cent.) and is shown in Fig. 14. The heating element consists of nichrome wire wound on a brass tube. The tube is bound with mica, before winding the wire, in order to insulate it, and asbestos string is wrapped over the wire so as to keep the wire in position when it expands with the temperature. The heating chamber is surrounded by a steel shell 7 in. in diameter and the space between the shell and heating element is filled with asbestos wool. Two steel plates are bolted together clamping the heater and outer shell between them, and are arranged to connect the furnace to the frame of the testing machine. The ends of the heating coil are connected to two insulated terminals on the top plate. No. 18 nichrome wire is used for the heating element. The wire is coiled closer to the ends to allow for conduction of heat through the grips of the testing machine and to give a more uniform heating over the central three to five inches of the furnace. The temperature is measured with a thermocouple placed

at the outside middle section of the gage length and protected by a porcelain tube.

43 The extensometer used is a combination of the best features of both Rudeloff's and Lea's extensometers and had two micrometers for measuring extension beyond the elastic limit. The clips are attached to the reduced part of the test piece by springs and protrude from the furnace. The inner clips are guided on flats

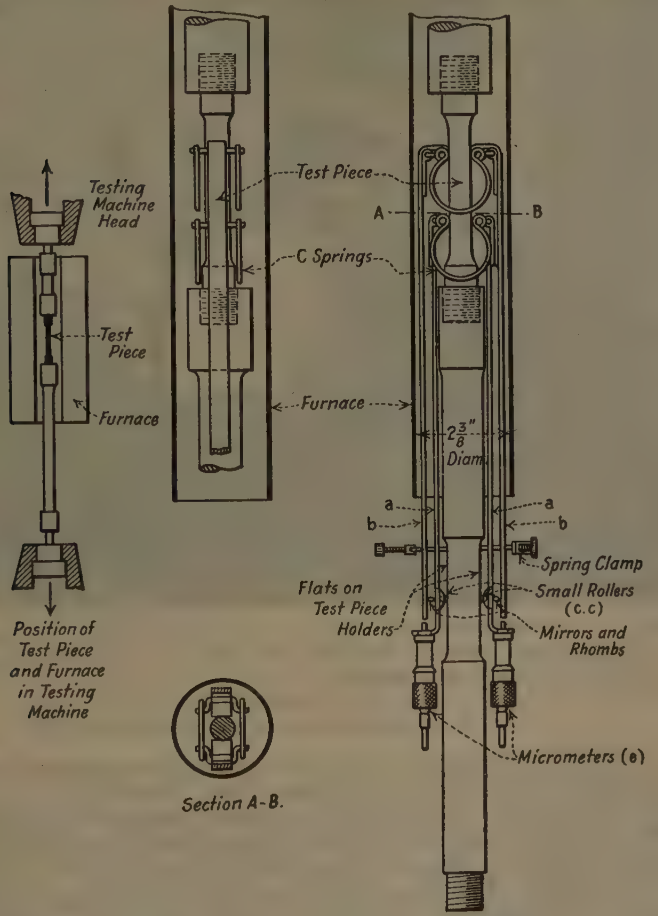


Fig. 15 National Physical Laboratory High-Temperature Extensometer

on the test-piece holders by small rollers. Mirrors and rhombs are placed between the clips and the whole is clamped together by a special spring attached to notches on the outer clip. The

extension of test pieces causes relative movement of the clips and therefore rotation of the mirror rhombs which is measured in the usual way by a telescope and scale. The relative movement of the clips is also measured by two micrometers attached to the inner clips and working against the outer ones. Sketches of these extensometers are shown in Figs. 1 and 15.

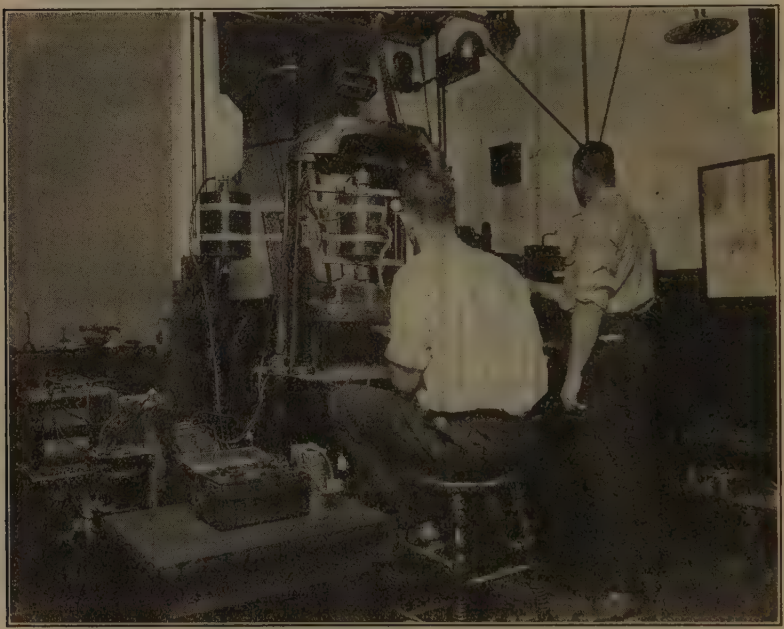


Fig. 16 Apparatus Used by Malcolm in Tension Tests at High Temperatures, Showing Furnaces, Transformers, Potentiometer, and Elastic-Limit Apparatus

V. T. MALCOLM (188, 189, 203, 112)

44 The tension tests of metals at elevated temperatures are made on a 100,000-lb. standard Olsen testing machine.

45 *Pyrometers.* The pyrometer equipment consists of a portable Leeds and Northrup indicating potentiometer and thermocouples of 0.025-in. (No. 22) alumel and chromel wire.

46 *Furnace.* The furnace equipment was made in duplicate in order to facilitate the heating and testing of specimens. The furnaces are attached to the outer leg of the testing machine by hinges so that they swing alternately into place between the heads for testing purposes. The general arrangement of apparatus is shown in Fig. 16, and Fig. 17 shows a section of the furnace with specimen in place and with extensometer gages attached and pyrometer inserted.

47 The test bars used are machined in the form of the standard 2-in. screw-end tension specimen 0.505 in. in diameter as shown in Fig. 18. In addition, special bars were used for calibration tests for each of the various metals tested, and were also used for certain temperature explorations and calibrations of the furnace. (See Fig. 19.)

48 In Fig. 17, A is an alundum tube upon which is wound the heating element B. This heating element is connected to the

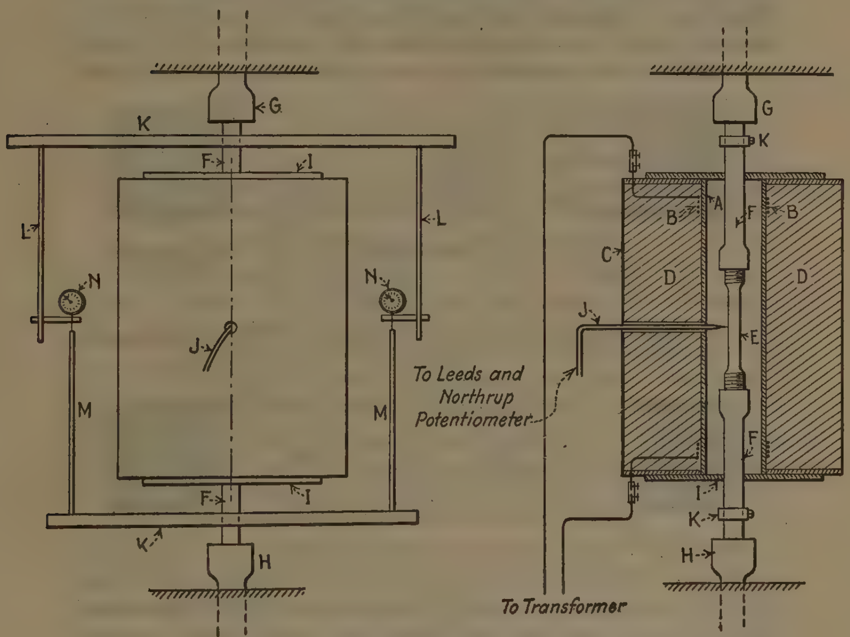


Fig. 17 Diagram of Apparatus Used by Malcolm in Tension Tests at High Temperatures, Showing Specimen in Place and Location of Thermocouple

secondary of a transformer. The tube A is held in the container C and the space between D is filled with kieselguhr. The specimen under test, E, screws into the specimen holders F, which in turn screw into the testing-machine adapters G and H. G is the fixed member and H the moving member. When the specimen is in position, the doors I are closed and the thermocouple J placed against the test specimen at its mid-point. The doors I are made of asbestos board and fit snugly around the holders F so that no air currents can get into the heating chamber. As a further precaution against air currents, asbestos rope is wound around the holders F just inside the doors I. Arms K are rigidly fastened to specimen holders F. The rods L are fastened to the upper arm

K and rods M to the lower arm K. Ames dial gages are connected to the fixed rods L and rest on the moving rods M.

49 *Method of Test.* The test specimen is inserted in the heating unit and fixtures described. After the specimen has been at the desired temperature for at least one-half hour, an initial load of 100 lb. is applied in order to eliminate all lost motion. The Ames gages are then set at zero after which equal increments of load are applied and simultaneous readings of gages recorded. The size of the increment applied depends upon the kind of material under test and varies from 100 to 500 lb. Observations of elongation per increment of load are recorded until the yield

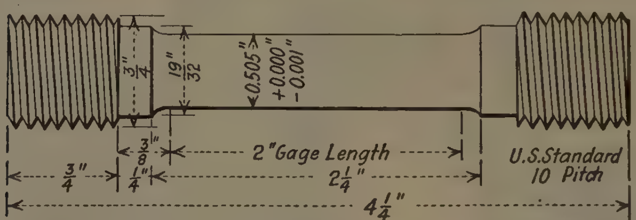


FIG. 18 TEST SPECIMEN USED BY MALCOLM IN HIGH-TEMPERATURE TESTS

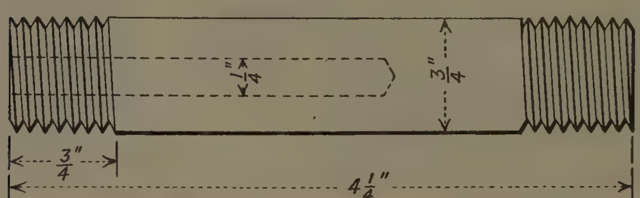


FIG. 19 CALIBRATION BAR USED BY MALCOLM IN HIGH-TEMPERATURE TESTS

point is passed. The Johnson elastic limit is then determined from a curve drawn through points obtained by plotting the elongation per increment of load.

50 *General Discussion of Accuracy of Test Results.* The accuracy of the results obtained in testing metals at elevated temperatures depends upon (a) the rate at which the load is applied, (b) the length of time the specimen is held under load at a given temperature, (c) the accuracy of the thermocouple, and (d) the true temperature of the material under test. These points will be considered in order.

(a) In general it may be said that the faster the rate of load application, the higher the tensile properties. In the author's tests a slow rate was used in order to observe the deformations which were plotted to determine the

elastic limit. In this respect, the values reported are on the low side as compared with values at ordinary testing speeds.

(*b*) The tensile properties decrease with the length of time the specimen is held under load at a given temperature. This may be dismissed from consideration inasmuch as all specimens were tested in approximately the same length of time — which was short.

(*c*) In order to determine the accuracy of the thermocouples, the following procedure was adopted: One of the thermocouples used was checked against a standard and was found to be correct within experimental error. All other thermocouples used were then checked against this one and found to be within limits of experimental error.

(*d*) The fourth point, regarding the true temperature of the bars, appears simple on its face, but after some of its phases are considered it is found to be very complex. The temperature of a specimen itself may vary in two ways: (1) From point to point along the surface and (2) from the surface to the center. The variation in temperature along the surface is easily determined by fastening the thermocouples to the points whose temperature is desired. The temperatures of three points along the surface of the pull section of the specimen have been determined, namely, at the top fillet, at the mid-point, and at the bottom fillet. The results are shown in Fig. 20.

51 In order to determine the difference in temperature between surface and center of this specimen, a set of test bars known as calibration specimens were used. These bars were $\frac{3}{4}$ in. in outside diameter, with a $\frac{1}{4}$-in. hole bored from one end to $\frac{1}{2}$ in. beyond the center. This gave a wall thickness of $\frac{1}{4}$ in., equivalent to one-half the diameter of the standard $\frac{1}{2}$-in. bar. A thermocouple was placed against the outside, as was done in the regular tests, and another couple inserted in the longitudinal hole, both registering the temperature at the same section with $\frac{1}{4}$ in. of metal between. The whole apparatus was placed in the testing machine under the same conditions that obtained during the regular tests and comparative observations made from room temperature to 1000 deg. fahr. (540 deg. cent.). The temperature differences observed on several materials tested were plotted as shown in Fig. 21.

52 One of the specimens was then suspended in the furnace by means of a wire, with grips removed, the thermocouples being in the same relative positions, and the temperature observations duplicated. The thermocouple readings with this set-up coincided within the limits of experimental error, which indicates that the

difference in temperature between surface and center is due to loss of heat by conduction to adapters, holders, extensometer arms, etc.

53 Although the apparent difference in temperature is indicated to be 146 deg. fahr. (80 deg. cent.) as the maximum at 600 deg. fahr. (315 deg. cent.) with the calibration bar used, this apparent difference exceeds appreciably the true difference existing between the surface and the center of the standard ½-in. specimen. The reasons for this are as follows:

54 When these differences are considered in connection with the temperature differences noted along the surface, the existence

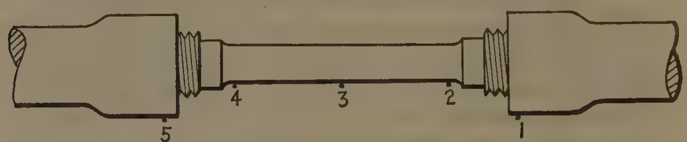


Fig. 20 Sketch Showing Temperatures at Various Points of Test Specimen in Tests by Malcolm.

Temperatures of Specimens at Points Indicated

Point No. 1		Point No. 2		Point No. 3		Point No. 4		Point No. 5	
deg. fahr.	deg. cent.	deg. fahr.	deg. cent.	deg. fahr.	deg. cent.	deg. fahr.	deg. cent.	deg. fahr.	deg. cent.
295	146	330	166	365	185	308	153	273	134
508	264	547	286	600	315	521	272	477	247
677	358	738	392	791	422	717	381	636	336
877	469	938	503	991	532	921	494	832	444

Differences in Temperature at Points Indicated from that at Point 3

Point No. 1		Point No. 2		Point No. 3		Point No. 4		Point No. 5	
deg. fahr.	deg. cent.	deg. fahr.	deg. cent.	deg. fahr.	deg. cent.	deg. fahr.	deg. cent.	deg. fahr.	deg. cent.
—70	—39	—35	—19	0	0	—57	—32	—92	—51
—92	—50	—53	—29	0	0	—79	—44	—123	—68
—114	—63	—53	—29	0	0	—74	—41	—155	—86
—114	—63	—53	—29	0	0	—70	—39	—159	—88

of isothermals along the specimen must be assumed, which does not seem warranted.

55 The rate of heat transfer along a bar which is not at uniform temperature may be represented by the equation:

$$\frac{dH}{dt} = \pi r^2 K \frac{dT}{dl}$$

where $\frac{dH}{dt}$ = rate of heat flow

πr^2 = area

K = coefficient of conductivity

and $\frac{dT}{dl}$ = rate of change of temperature with change in distance from the hotter portion.

56 From this equation it will be seen that the rate of flow of heat and therefore the temperature difference varies directly with the area. Reference to the calibration bar will show that the area is twice the area of the standard specimen. Inasmuch as the area is the only varying quantity between the two equations for standard and calibration bars, it is apparent that from this correction alone the temperature difference between surface and center would be reduced very materially below the apparent differences shown. Assuming that there is a temperature gradient between points on the surface of the standard test bar and the center of the section through any of these points, the temperature

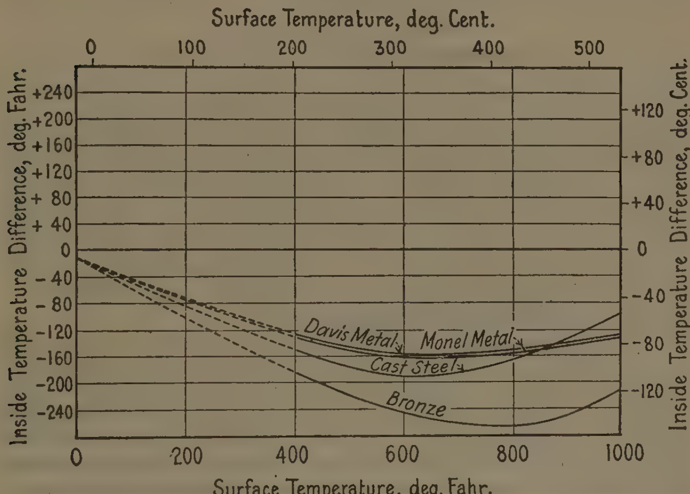


FIG. 21 MALCOLM'S TEMPERATURE CALIBRATION CHART

measured at the surface would approximate more closely the mean effective temperature, which would lie at the point of mean effective area, namely, one-third the distance in from the surface of the bar. For this reason, the temperature as registered by a thermocouple placed against the mid-point of the specimen has been used by the author.

57 Referring particularly to variations in temperature along the length of the pull section of the standard test bar, observations of the manner in which the bars broke, under load, cast considerable doubt on the possibility of there being any appreciable difference in temperature along the bar.

58 Considering, therefore, all the experimental and theoretical observations made, we are of the opinion that the effective temperatures, whether they be in the center of the bar or at

the ends, do not vary more than 40 deg. fahr. (22 deg. cent.), and from the temperature indicated by the pyrometer, in some cases may be considerably less.

59 The work of Speller(215), Germer and Woods(209), d'Arcambal(148) and several other investigators, shows that the tendency of present investigation is to use the cylindrical electric furnace, with the thermocouple wire placed against the outside surface of the test specimen.

60 Before leaving the subject of tension tests of metals at elevated temperatures to discuss special tests of metals at various temperatures, tests of Jeffries and Sykes on wires at both low and high temperatures will be described.

ZAY JEFFRIES(127) AND W. P. SYKES(163)

61 The investigations of Jeffries and Sykes were mainly in connection with copper, tungsten, Armco iron, nickel, and molybdenum wires and the methods of test will be described in detail. In these tests wires were subjected to tension at temperatures from −310 deg. fahr. (−190 deg. cent.) to a maximum of from 1650 to 1830 deg. fahr. (900 to 1000 deg. cent.). No attempt was made to determine the elastic limit but the other tensile properties, namely, tensile strength, elongation, and reduction of area, were determined at all temperatures.

62 The apparatus used for these tests is composed of two main parts, the base and the loading mechanism. The base consists of two cast-iron disks, a bottom and a top, that are fastened together by three pieces of steel pipe. The base weighs a little over 100 lb. and is placed on a platform scale. The top of the base portion is provided with a steel tube 0.875 in. in outside and 0.5 in. in inside diameter to which the test pieces are clamped by special clamps. The upper clamp is connected with the loading mechanism. When the load is applied by the hand-wheel, the test wire pulls on the base and the amount of its pull is measured on the weighing mechanism of the scale. The zero point on the scale is equal to the weight of the base; the scale is kept balanced continually during the test until the wire breaks. The scale reading at the breaking load of the test specimen is then subtracted from the zero reading and the difference gives the breaking load of the specimen.

63 *Liquid-Air Tests.* Punch marks 2 in. apart were made on all wires and the clamps were set about 2.5 in. apart, leaving about 0.25 in. between each clamp and the closest punch mark. The test wires were locked in clamps and inserted in the machine and a 1-qt. wide-mouth thermos bottle of the food-jar type, more than three-quarters filled with liquid air, was raised in such a manner that the steel tube containing the test wire was immersed

in liquid air. The test wire was completely immersed and hence its temperature was that of boiling liquid air. As soon as violent boiling ceased, the load was applied until the wire broke. The vacuum jar containing the liquid air was then lowered from the steel tube and another vessel containing warm water was substituted for it. When the temperature had been raised by water the clamps were removed.

64 *Tests at* 212 *deg. fahr.* (100 *deg. cent.*). These tests were made in boiling water. An electric percolator heater was placed on the platform instead of the vacuum jar and a coffee pot was used for holding the water. The water was kept boiling vigorously till the end of the test.

65 *Tests at* 400 *deg. fahr.* (204 *deg. cent.*). These tests were made in hot crisco. Crisco was used because it could be heated to 480 deg. fahr. (250 deg. cent.) with very little volatilization; one of the chief advantages of crisco is that it is very fluid at higher temperatures. The crisco was heated in the same electric heater that was used in the 100 deg. cent. tests. The temperature during the test may have varied 3 or 4 deg. cent. Several pounds of crisco, however, were used so that the temperature changes were slow. Immediately after each test piece broke, the temperature of the crisco was measured with a mercury thermometer.

66 *Tests above* 400 *deg. fahr.* (204 *deg. cent.*). A different scale and different loading mechanism were used and the base of the apparatus modified so that an electric furnace could be used to obtain the proper temperature. A Kron scale with a 30-in. dial graduated in quarter-pounds but sensitive to less was used. The electric furnace consisted of an alundum tube 1 in. in inside diameter, and 12 in. long, wound with nichrome ribbon enclosed in a gas-tight steel cylinder. Powdered silica was used as a heat-insulating medium between the alundum tube and the cylinder. The cylinder was provided with a connection to a tank of compressed argon, so that a neutral atmosphere could be maintained in the furnace at higher temperatures. The electric furnace was so fastened that the load on the wires was transmitted to the base portion by the lower end of the furnace. The bottom clamp was flanged to fit a seat at the bottom of the furnace housing; the flange was so large that it could not be drawn through the furnace tube. This clamp was fastened to the loading mechanism by means of a clevice.

67 A platinum-platinum-rhodium thermocouple was used to measure the temperature. The hot junction was placed in the furnace tube about half way between the two clamps, that is, at about the central point of the test wire. The thermocouple was connected with a Wilson-Maeulen galvanometer with both millivolt and temperature scales. The temperature of the electric furnace could be maintained constant with a wire-wound rheostat.

68 Details of the furnace and tube with the specimen in position are shown in Fig. 22.

Impact Tests

69 Guillet and Revillon(60) carried out tests on a Guillery 60-kg-m. impact machine. The test pieces were heated in an electric furnace to slightly above the temperature required for the

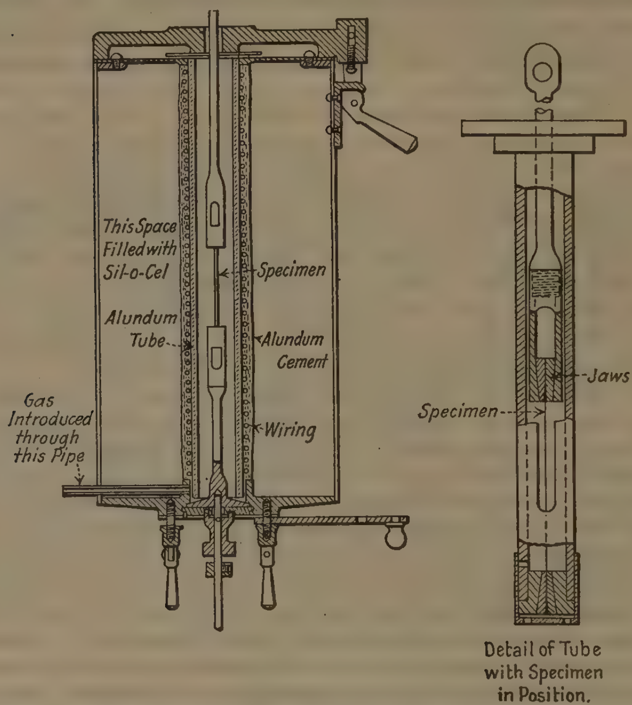


Fig. 22 Diagram of Furnace Used by Jeffries and Sykes for the Testing of Metals at High Temperatures

tests; they were then placed on the anvil and temperature noted at time of fracture. The temperature was determined by the use of a thermocouple inserted in a small hole drilled in the specimen and penetrating up to about 3 mm. from the cross-section to be fractured. The ends of the test pieces were covered with asbestos to prevent cooling of the extremities when in contact with the anvil. Tests at as near 212 deg. fahr. (100 deg. cent.) as possible were obtained by using boiling water.

70 A. C. Langenberg(200, 201) has made impact tests at Watertown Arsenal at the following temperatures: −80, −40, −20,

0, 15, 32, 50, 70, 90, 110, 130, 150, 175, 200, 225, 250, 350, 500, 750 and 1000 deg. fahr. (−60 to 540 deg. cent.), two test specimens being tested at each temperature. Temperatures below the

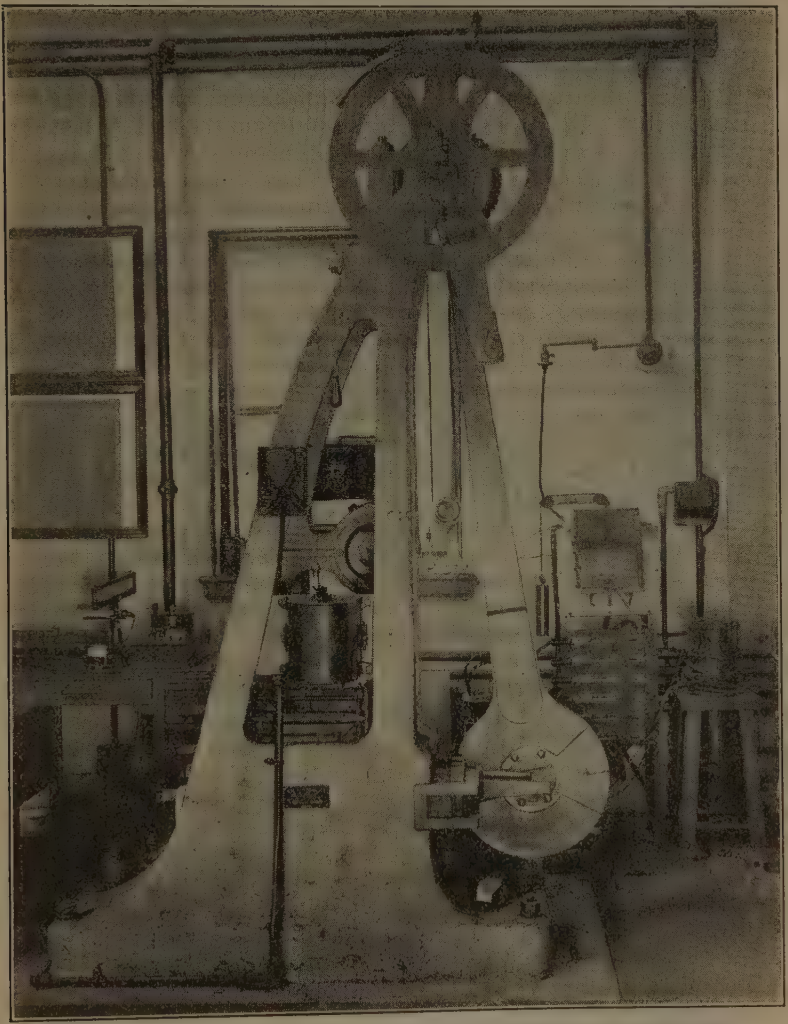


Fig. 23 Charpy Impact Testing Machine Used by Langenberg

atmospheric were obtained by immersing the test bars in a bath of acetone. The acetone bath was cooled to the desired temperatures down to 0 deg. fahr (−18 deg. cent.) by means of a calcium-chloride solution cooled by an ordinary ammonia refriger-

ating apparatus. The lower temperatures were obtained by direct addition to the acetone of carbon dioxide snow. Temperatures from 90 to 350 deg. fahr. (32 to 175 deg. cent.) were obtained by heating the test bars in a Freas constant temperature oven, and from 500 to 1000 deg. fahr. (260 to 540 deg. cent.) the test bars were heated in a Hoskins electric muffle furnace.

71 The lower temperatures were ascertained by alcohol thermometers, the medium by mercury thermomoters, and the higher temperatures by means of platinum-platinum-rhodium thermocouples. In tests from −80 to 350 deg. fahr. (−60 to 175 deg. cent.) a thermometer was inserted in a hole drilled in a dummy test specimen and packed with magnesium powder, and the dummy and thermometer were placed in the cooling bath or heating furnace in the center of the group of test bars. An additional thermometer indicated the temperature of the heating or cooling medium. When two thermometers gave readings as near alike as it was found possible to obtain, the temperature of each test bar was considered to be very close to the temperature of the dummy test bar as shown by the inserted thermometer. At temperatures from 500 to 1000 deg. fahr. (260 to 540 deg. cent.) a thermocouple was inserted in the center of the notch of each test bar and the bar was tested when the thermocouple showed that the desired temperature was reached.

72 In testing, the heating or cooling apparatus was placed as conveniently near the impact testing machine as possible, and each specimen transferred quickly to the machine, the time in transferring being observed by means of a stop watch. It was found that the temperature of the test bars at the moment of impact closely approximated the desired temperature and it is believed that each specimen was tested at a temperature within 2 deg. fahr. above or below the temperature recorded.

73 The large Charpy impact machine used in the tests (Fig. 23) has the following constants:

Weight of pendulum..................	212.3 lb.
Velocity of impact..................	25.7 ft. per. sec.
Capacity	2199.5 ft-lb.

74 The specimen used in these tests was the large notched Charpy impact specimen, 6.102 in. long and 1.181 in. square in cross-section, with the notch 0.591 in. deep by 0.158 in. wide.

Alternating-Stress Tests

75 Not a great amount of research work has been carried out on the effect of various temperatures on the strength of materials under alternating stress. Tests were made by Batson and Hyde(89) at the National Physical Laboratory in England in a machine of the Wohler type, the test piece running at a speed of 2000 alternations per minute in an oil bath heated electrically.

76 During the past year a considerable number of repeated-stress tests of steel at temperatures up to 875 deg. fahr. (465 deg. cent.) have been made in the laboratory of the Investigation of the Fatigue of Metals at the University of Illinois (213) under the direction of Prof. H. F. Moore with the coöperation of the National Research Council, General Electric Co., Illinois Engineering Experiment Station, Engineering Foundation, Western Electric Co., Allis-Chalmers Manufacturing Co., and the Copper and Brass Research Association. The testing machine used in these experiments is shown in Fig. 24 and is a reversed-bending testing

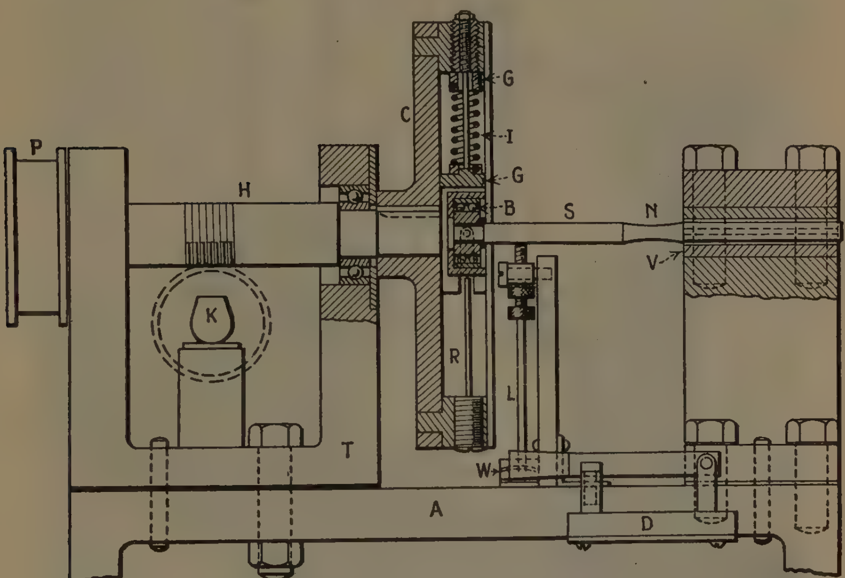


Fig. 24 Diagram of Rotating-Spring Reversed-Bending Testing Machine Used by Moore

machine. In this machine one end of the specimen S is held rigid in the vise of V and the other end which runs in the bearing B, is rotated in a small circle. Sidewise pressure, which can be adjusted by means of a screw, is brought on the bearing B by a calibrated indicator spring I. The compression of the spring, and hence the load on the specimen, is measured by means of a strain-gage spanning the gage holes GG shown near the ends of the spring. The rotating spring is carried in the crosshead C. Sidewise motion of the bearing B is prevented by placing the bearing in a slot, and excessive displacement of the bearing, when the specimen breaks, is prevented by the rod R. The crosshead is driven by a shaft H, a pulley P, and a motor not shown in the

figure. The number of revolutions of the crosshead is measured by the revolution counter K which is driven by a worm on the drive shaft H. When a test to destruction is carried out, the fracture of the specimen occurs at the necked-down part N, and the broken end of the specimen hits a screw and kicks out a lever L. This releases the spring W, which then opens the motor switch D, thus stopping the motor.

77 In the elevated temperature investigation a small electric furnace not shown in the figure was added in order to heat the

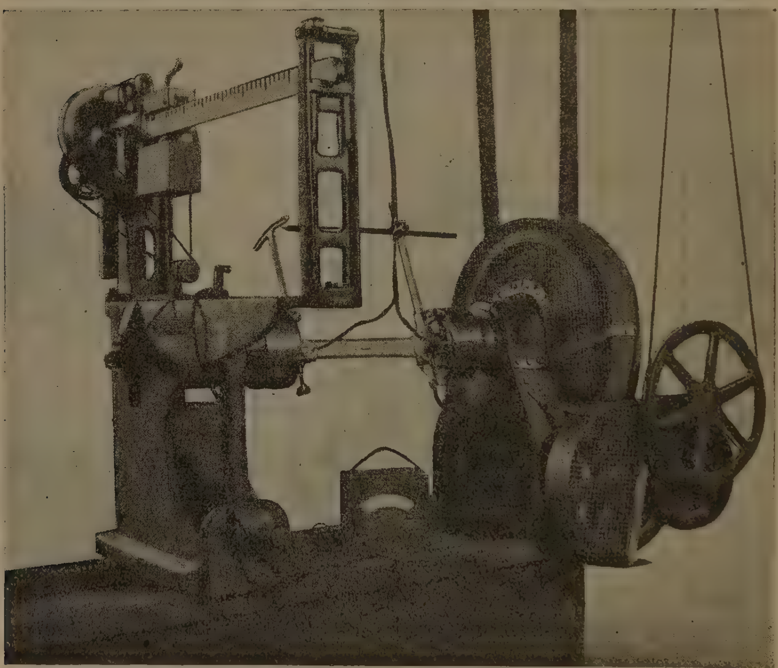


FIG. 25 TORSIONAL MACHINE USED BY BREGOWSKY AND SPRING, WITH TEST SPECIMEN AND APPARATUS IN PLACE

specimen to the required temperature for the test. Temperatures were measured by means of a thermocouple in contact with the specimen at its region of maximum stress and temperature was recorded and controlled by a Leeds and Northrup temperature recorder and automatic controller.

78 Endurance limits (fatigue limits) for completely reversed stress were determined from S-N diagrams described in Bulletins Nos. 124 and 136 of the University of Illinois.

79 The results obtained by Moore are regarded for the present merely as preliminary. However, they do give certain interesting

indications, and we may expect to find some startling results in tests of this nature as the work progresses.

Torsion Tests

80 Bregowsky and Spring made a number of torsional tests on rolled rods at elevated temperatures. In order to insure as nearly comparable results as possible, rods of the same size ($1\frac{1}{8}$ in. in diameter) were purchased in the market and test bars cut off and turned down to a uniform diameter of 0.855 in. The length of the turned-down section was 8 in. in nearly all cases.

81 The same sort of heating apparatus was used as noted under tension tests by Bregowsky and Spring(83). The coil was 12 in. long. Fig. 25 shows the bar and apparatus in position. For high temperature, mica plates and asbestos sheets were inserted between the jaws and head of the testing machine to retard the loss of heat.

82 Elastic limits were determined by plotting readings taken by means of the troptometer, with increments of load applied very slowly and the corresponding distortion read off the scale. Ultimate strength and elastic limit were given in pounds per square inch and total twist in turns (revolutions of 360 deg. each) with number of degrees in excess.

Hardness Tests

83 Aitchison(123) gives some results of hardness tests at elevated temperatures on high-speed steels of various compositions. Prof. C. A. Edwards(120) recently carried out a series of investigations into the hardness of steels at elevated temperatures, and in order readily to compare his work with the standard Brinell machine he had to develop an entirely new apparatus.

84 This hardness testing at elevated temperatures is of great advantage in order to determine the cutting efficiency of tools at various temperatures, and a simple accurate hardness test at elevated temperatures is certainly needed today. As this method of test is really in process of development, little can be stated at this time regarding the testing or apparatus.

X-Ray Spectograph

85 Arne Westgren(165-6) of Gothenburg has carried out in the Physical Institute of the University of Lund, Sweden, considerable interesting and instructive work with the X-ray spectrograph on iron and steel at elevated temperatures. Lately the X-ray has been used to study the defects in iron and steel castings and it has opened up a field that is both instructive and fascinating. While the X-ray analysis is in its infancy, the author believes that

it will be one of the future methods of testing metals and alloys at various temperatures.

86 Westgren has shown the importance of the X-ray method of investigation in metallographic research of iron and steel, both at ordinary and elevated temperatures. According to the spectograms obtained at elevated temperatures it has been shown that a fundamental difference exists between the apparent transformation point A_2 and the critical point A_3. Westgren reported he was unable to find any structural change of iron either above or below the A_2 point. At A_3, however, he finds the atoms of iron are completely rearranged and the iron passes from one crystal

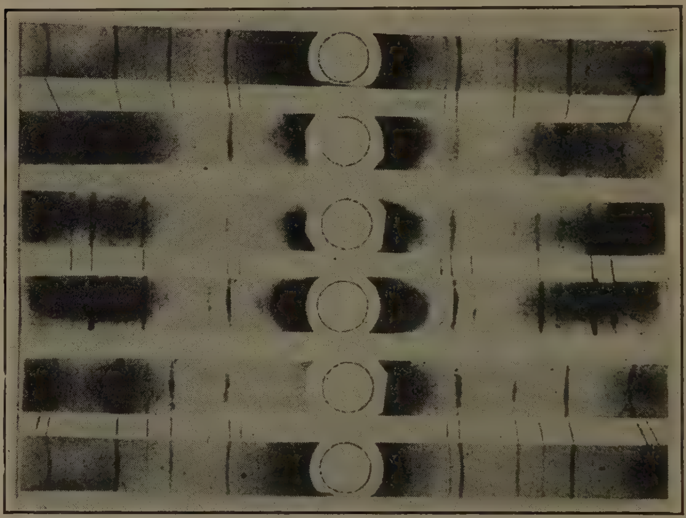


FIG. 26 PHOTOGRAMS OF PURE IRON AT DIFFERENT TEMPERATURES

class into another. Consequently, the difference between iron of the alpha and beta states is not of the same kind as between the alpha and gamma states. Fig. 26 gives photograms of pure iron at different temperatures.

87 This investigation has shown that the iron atoms of martensite are oriented in exactly the same way as ferrite, and the difference in hardness between these two structural constituents is not due to the iron but to the carbon present in the martensite. This has an important bearing on the testing of metals at elevated temperatures, for by the use of the X-ray spectrograph, problems now solved only by empirical methods may be open to theoretical treatment and become based on rational scientific knowledge.

88 It is the belief of such investigators as Rosenhain, Jeffries, and Lester that the real future advancement of a knowledge of the

internal structural changes in metals at various temperatures lies in the direction of the X-ray spectrum. It is to be hoped that investigators will actively take up this very interesting method of research, as it gives great promise of revealing, in a simple manner, the more complicated problems of structural changes in metals at various temperatures.

Long-Time Tests

89 In the foregoing series of tests the time consumed in most cases has been so short as to give little idea as to the probable behavior of the metals or alloys when maintained for considerable periods under the applied conditions of stress and temperature. Recent experimental work by J. H. S. Dickenson(173) calls attention to the influence of the time factor in determining the temperature up to which certain steels can support a given load and, by inference, the load which can be borne at any given temperature.

90 Two series of tests were run by Dickenson to determine duration, one at constant load and constant temperature, and the other at constant load and uniform rate of temperature increase. In both series of tests a load of 19,000 lb. per sq. in. on the specimen was used and the temperature varied from 932 to 1769 deg. fahr. (500 to 965 deg. cent.). The tests were carried out on six samples of steel made into bars of 8-in. gage length and 0.40 in. in diameter.

91 The tests were carried out with one exception on heat-treated steel, the exception being a cast chrome-nickel steel. The apparatus used was arranged for six tension tests simultaneously, with electric furnaces wired in parallel, the temperature being recorded by a Cambridge thread recorder, the couples being checked before and after use. The testing machines were of special construction and designed for this investigation.

92 *Duration Tests at Constant Load and Constant Temperature.* In conducting these tests, care was taken in applying the load so that no live load was momentarily produced and the temperature was steadily increased to the desired point where it was maintained day and night until fracture occurred or until it was deemed unnecessary to continue. A daily measurement was made of the change in distance between the gage points on the specimen, this measurement being readily obtained because the gage marks were outside the furnace. More frequent observations were made when rapid extension of the specimen required it or when rupture became imminent at the end of long runs.

93 A series of test pieces for each class of steel was run at 90 deg. fahr. (50 deg. cent.) ranges, such as 1022 to 1112 deg. fahr. (550 to 600 deg. cent.), 1112 to 1202 deg. fahr. (600 to 650 deg. cent.), and so on.

94 *Tests at Constant Load and Uniform Rate of Temperature Change.* In these tests, after the test piece had been placed in position in the furnace and the load applied, the gage length was measured. The temperature was slowly raised by means of a specially constructed rheostat at the uniform rate of 180 deg. fahr. (100 deg. cent.) per hour, an overall measurement being made at 212 deg. fahr. (100 deg. cent.) and thereafter on reaching each additional 90 deg. fahr. (50 deg. cent.), care being taken to make this coincide with half-hourly periods. Later as the rate of extension increased, more frequent measurements were made, usually in six-minute periods, and finally the temperature at which the test specimen fractured was carefully noted.

95 Tables were worked out and graphs made showing estimates of the probable lives of test specimens subjected to constant load and temperature for long periods, and eventually withdrawn but not broken. An estimate of the probable time before rupture would have occurred, had the test proceeded, was obtained in each case by comparing the time required to produce the same extension in unbroken and broken specimens of the same steel. The estimates given, however, are only of general interest.

96 In 1922 the author(188, 189) carried out a series of tests for 400 hours on cast steel at constant load and constant temperature. The standard A.S.T.M. test bars were used, of 2-in. gage length and 0.505 in. in diameter, and the apparatus used was the same as described in the tension tests. The temperature of the specimen was raised at a uniform rate until a maximum of 1100 deg. fahr. (593 deg. cent.) was reached, and a load of 21,000 lb. per sq. in. placed on the specimen. This temperature and load were maintained night and day for 400 hours, after which the load was removed and the temperature allowed to drop to normal. The original gage length was remeasured for any permanent extension or deformation.

97 The U. S. Bureau of Standards is now working on apparatus for testing metals at constant temperature over long-time periods, but as yet little work has been accomplished.

98 A knowledge of the behavior of metals at elevated temperature over long periods would undoubtedly be of great technical importance for the reason that these tests tend to cast grave doubts on our ideas as to the yield point or elastic limit of certain metals in common use, and it would appear that we may be compelled to revise our ideas along these lines. Since it is now generally agreed that design should be based on the yield point or elastic limit, the matter becomes important.

Metallic Oxidation at Elevated Temperatures

99 One neglected type of study in the testing of metals at elevated temperatures is the metallic oxidation of the metals or

alloys when exposed to temperatures considerably higher than the atmospheric range, and more especially the attack of atmospheric oxygen upon the exposed metallic surfaces. That the problem is a difficult one to contend with is connoted by the practical dominance of a certain alloy in electric heating. Proprietary interest may account in part for the paucity of information available in technical literature upon even the most general facts on the behavior of metals and alloys when exposed to elevated temperatures. A very interesting and instructive paper on this subject was published by Pilling and Bedworth(191) in March, 1922.

100 Dickenson(173) published results of scaling tests on metals at elevated temperatures. In his experimental work eight typical steels were selected for examination. From each sample nine cylinders each 0.50 in. in diameter by 2 in. in length were machined, polished with emery, and weighed. Each of these cylinders was heated for a total time of 100 hours. In order to maintain throughout the 100 hours a practically uniform rate of oxidation, which slows down as the adhering scale increases in thickness, the heating was carrried out in 18 periods of approximately 5½ hours each, the specimens being scraped free from scale and weighed after each cooling. Two types of furnace were used for heating the specimens. An electric furnace surrounded by air was used for the lower temperatures, while for the higher ranges the less pure atmosphere of a gas furnace was employed.

101 In a table by Dickenson the scaling rate in ounces per square inch per hour at mean temperatures indicated is given, including the results from both electric and gas furnaces. It appears that the rate of scaling at 1600 deg. fahr. (870 deg. cent.) is much the same in the two types of furnace, at any rate when the gas-muffle front is slightly open and burners are receiving full air as in the present case, so that the lower series and the upper series may be considered satisfactorily linked.

102 A series of tests was carried out by the author (189) in 1922 on the rate of scaling of cast steel. In the experimental work 1-in. cubes were used, each cube being carefully ground on an emery wheel to remove any foreign substance adhering to it, after which it was carefully calipered and weighed. These cubes were then subjected to 100 hours' treatment in an electric furnace in which were maintained ordinary atmospheric conditions. In order to maintain a uniform rate of oxidation, the specimens were removed approximately every six hours, scraped free of adhering scale, and weighed. A graphical chart was plotted showing the amount of scale in ounces at the temperatures noted. It was found from these tests that little or no scaling existed when steel was exposed to temperatures below 1100 deg. fahr. (593 deg. cent.) and that the formation of scale started at about that temperature and increased rapidly, while at 1700 deg. fahr. (925 deg. cent.) the amount of scaling may be considered excessive.

Conclusion

103 The author has endeavored to describe in outline the latest methods of testing metals at various temperatures. It will be noted that the furnaces and other apparatus, procedure, etc., used by the various investigators differ to some extent and are by no means standardized, each investigator believing his method to be the correct one. Very little definite information on the rational mechanical testing of metals at elevated temperatures has been obtained.

104 The effect upon the physical properties of metals of raising the temperature cannot as yet be stated in terms of a definite law. It may be generally stated that the tensile strength and elastic limit of steel decrease and the elongation and reduction of area increase as the temperature is raised. Of course there are exceptions to this rule. It has been noted in nearly all the investigations that at certain temperatures between 500 and 800 deg. fahr. (260 and 425 deg. cent.) the tensile strength rises and the elongation and reduction of area decrease, but the elastic limit continues to decline steadily. Therefore, to use the tensile strength as a basis of design would be far from correct. This condition of marked rise in the tensile strength and fall in the elongation and reduction of area is known as "blue brittleness" from the original German term "Krupp-Krankheit" and is to some extent the limit of our general knowledge. Rosenhain and Archbutt(130) found a formation of intercrystalline cracks which eventually produced failure in boiler plate. These plates were used at elevated temperatures and it is thought that the long-sustained load at these temperatures caused the amorphous cement at the grain boundaries to flow and eventually break without deformation of the grains. Jeffries(184) has made quite a study of the physical changes in iron and steel at various temperatures and the conclusions he reached are both interesting and instructive. Langenberg(186) believes that "blue brittleness" is a distinctive property of free ferrite, and furthermore that "blue brittleness" is not the property of free ferrite at blue heat, but rather is a property resulting from a mechanical deformation of free ferrite at blue heat or lower temperatures. We believe that this is a subject that should be given careful consideration in the testing of metals at elevated temperatures.

105 Again, we find that results obtained in actual practice have not always been in accordance with those obtained experimentally. One instance with which the author is acquainted is a set of valves operating at 950 deg. fahr. (510 deg. cent.) for several years which are still giving excellent service, yet most of our experimental work in testing metals at elevated temperature shows that steel is very weak at this temperature.

106 The explanation of such inconsistencies is probably to be found in the fact that the alterations in the physical properties of metals and alloys due to variations of temperature are not always of the same nature. With any increase in temperature and consequent molecular activity we may expect a gradual falling off in tensile strength until at the melting point of the metal the tenacity becomes nothing. Allotropic changes in metals are accompanied by changes in physical properties. For instance, iron undergoes certain changes at certain temperatures. Zinc is brittle at ordinary temperatures but when heated to certain temperatures it becomes malleable and again loses this property at higher temperatures. Tin at low temperature undergoes a molecular change and falls to powder. Such changes are abnormal, and, except in the case of iron, very little is known as to what takes place when metals are alloyed and subjected to various temperatures.

107 Some metals and alloys undergo a gradual change in their crystalline character, which is greater at elevated or low temperatures. This change may be simply an increase in size of crystals or may be a change in crystalline structure. For instance, tests show that brass or bronze when heated to temperatures beyond 400 deg. fahr. (205 deg. cent.) becomes very treacherous, the tensile strength and elongation both decreasing as the temperature is raised, and the crystal size becoming very coarse. Alloys containing two or more constituents are more likely to suffer failure at elevated temperatures than those containing only one constituent, especially if one of the constituents is a eutectic. The eutectic often has a melting point lower than the constituent metals and therefore its strength is affected at a lower temperature; and if the eutectic forms a network or cement around the grains or crystals, its strength represents the strength of the alloy.

108 In these several causes of failure, the gradual change of structure occurs only after a lapse of time, and this is one reason for failure of metals or alloys that have shown good results when tested in a short time at elevated temperatures. Tests carried out on an alloy at short duration are not always sufficient to indicate the behavior of a metal in service.

109 Another example which may be cited is the large columnar structure often found at certain temperatures in nickel-copper alloys.

110 Very little appears to be known about the changes that take place in metals or alloys when subjected to high-temperature service, and it seems advisable that a complete structural study with the aid of a microscope should be made of metals when tested at various temperatures. The microscope together with the X-ray, we believe, will be found valuable in future tests.

111 In summing up, we would say that in carrying out research into methods of testing metals at various temperatures, it

should be our aim to carry out such tests as will approximate the trying conditions of service. This fact is often lost sight of in making an investigation. To be of any value the investigation should have a definite aim and be carefully planned. A general survey of the entire field should be made, as one method of test may be of value to a single consumer or producer, but of little or no value to others.

112 The author hopes that we shall soon be able to standardize our methods of test in this field of great importance, in which, it may not be amiss to mention, continental Europe has made rapid progress.

No. 1926 c

AVAILABLE DATA ON THE PROPERTIES OF IRONS AND STEELS AT VARIOUS TEMPERATURES[1]

By H. J. French[2] and W. A. Tucker,[3] Washington, D. C.

Non-Members

The authors point out that while a vast amount of test work has been carried out on the properties of ferrous metals at high and low temperatures, the information available is incomplete and unsatisfactory. They summarize effectively the information available for temperature effects on such mechanical properties as tensile strength, elongation, hardness, strength in torsion and crushing for cast irons, steels, cast steels, and malleable iron. There is invariably a loss in mechanical properties with increase in temperature, but this varies for each metal and alloy. The importance of the time factor in all tests is emphasized and short-time tests are considered as worthless by some investigators in indicating true behaviors under service conditions.

A brief note is given regarding the thermal expansion of some metals, some alloys, such as invar and the related nickel steels, departing completely from the usual laws of thermal expansion. "Growth" in cast iron with repeated heating is considered, and "semi-steel" is recommended for use in annealing ovens, rolls, fire bars and other cases where repeated heating occurs.

Chemical inertness is shown to be an important factor in the selection of metals for use at high temperatures, some alloys being more resistant to chemical change than others. Oxidation of steels is briefly studied. While it is evident that all steels weaken with considerable rise in temperature, the composition and treatment of steels may be varied to meet specific requirements, and emphasis is laid on the fact that heat-resisting alloys which are now being produced commercially have very desirable properties at very high temperatures. Many of these alloys cannot be called steels or even ferrous alloys, and in some cases are practically free from iron, but they are extremely valuable, nevertheless. Nickel-chrome-iron base

[1] Published by permission of the Director of the U. S. Bureau of Standards. For discussion and closure see pp. 489-534.

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alloys and copper-nickel and other non-ferrous alloys are among those shown to be promising for heat-resisting purposes. There is also a possibility of the development of coated metals for high-temperature service. Aluminum and chromium coatings are particularly promising in this regard.

Introduction

DESPITE the fact that attention has been called repeatedly during the past few years to the incomplete and unsatisfactory nature of available information on the properties of ferrous metals at high and low temperatures, a vast amount of test work has been carried out by many investigators. Not all of the results have been published but much of the information is now in print, and there can be developed many interesting and important comparisons in addition to the special features emphasized in each report. However, it is not intended to present a complete résumé of all this material but rather to give a brief sketch of the character of published information. Detailed study of any of the phases covered in the literature may be made by consulting the references in the bibliography.[1]

2 No attempt will be made to reproduce or discuss results for all the ferrous alloys which have been tested, nor can the tests made by all investigators of a given alloy be mentioned. However, there will be given representative graphs or other forms of data to show the general effects of temperature variations upon the properties of irons and steels, particularly with respect to those features of interest to engineers. Reference will also be made to some of the "heat-resisting" alloys now in use or proposed for high-temperature service.

Mechanical Tests at Various Temperatures

IMPORTANCE OF THE TIME FACTOR AT HIGH TEMPERATURES

3 Before summarizing some of the principal features shown by the available data, attention should be called to the importance of the time factor in mechanical tests of metals at high temperatures. Howard**(17)**[1] reported that the "rate of speed of testing which might modify the results somewhat with ductile material at atmospheric temperature had a very decided influence upon the apparent tenacity at high temperature." Steel containing 0.81 per cent of carbon was tested at a slow speed which produced rupture in from 5 to 10 minutes and also under rapidly applied stresses (in which case the time was from 2 to 8 seconds). Nearly the same strength was displayed whether slowly or rapidly frac-

[1] The bold-face numbers in parentheses refer to references in the bibliography on pages 477-487.

tured at temperatures below about 600 deg. fahr. (315 deg. cent.), this being a comparatively brittle metal at moderate temperatures. Above this temperature the apparent strength of the rapidly fractured specimens largely exceeded the strength of the others. The higher the temperature the wider apart were the results. An extreme illustration of this kind was furnished by a specimen tested at 1410 deg. fahr. (765 deg. cent.) which, when ruptured

TABLE 1 EFFECT OF SLOW LOADING ON THE TENSILE PROPERTIES OF FIREBOX BOILER PLATE AT DIFFERENT TEMPERATURES [FRENCH(179)]

(C, 0.19; Mn, 0.43; P, 0.020; S, 0.031)

Temperature of test, deg. fahr.	Temperature of test, deg. cent.	Rate of loading	Proportional limit, lb. per sq. in.	Tensile strength, lb. per sq. in.	Elongation in 2 in., per cent	Reduction of area, per cent	Remarks
315	155	Adopted standard [2]	26,600	58,100	24.9	49.3	Average of 3 tests.
315	155	6⅔ hours from 22,000 to 47,00 lb. per sq. in.[1]	......	64,300	22 8	45.9	Average of 2 tests.
565	295	Adopted standard [2]	14,300	66,400	25.9	53.1	Average of 3 tests.
565	295	3½ hours from 9,000 to 20,000 lb. per sq. in	......	60,000	36.0	59.2	
865	465	Adopted standard [2]	13,200	47,500	33.6	68.5	Average of 3 tests.
865	465	6 hours from 9,000 to 30,000 lb. per sq. in.	......	33,600	42.0	78 4	

[1] Note the apparently anomalous behavior with respect to rate of stress application at 315 deg. fahr. (155 deg. cent.) as compared with higher temperatures.
[2] Adopted standard averages about 0.05 in. per minute extension.

in 2 seconds, showed a tensile strength of about 62,000 lb. per sq. in., whereas at ordinary speed of testing a corresponding bar fractured at 33,240 lb. per sq. in.

4 Similar effects are observed in comparison of extremely slow and ordinary rates of loading as shown in Table 1, which is taken from tests by one of the authors(**179**).

5 Hopkinson and Rogers(**55**) reported that as the temperature rose the stress-strain relations in steel underwent remarkable changes which might best be expressed by saying that the variously called "creeping," or "*elastische nachwirkung*," or "time effect," increased greatly with temperature. While such effects might be detected at ordinary temperatures they attained a different order of magnitude at red heat, 1100 deg. fahr. (600 deg. cent.). The effect of "creeping" was found to make the determination of Young's modulus a matter of some uncertainty for the extension of a bar stressed at 1110 deg. fahr. (600 deg. cent.) varied 15 per cent or more depending on the time of application of load. For very short applications of the order of one or two seconds, the strain produced approached a definite limiting value which, if

used in determination of the modulus, made it independent of the manner of loading and a physical constant.

6 Many other tests including those of Robin(71) and, more recently, Chevenard(124) and Dickenson(172) throw light upon the

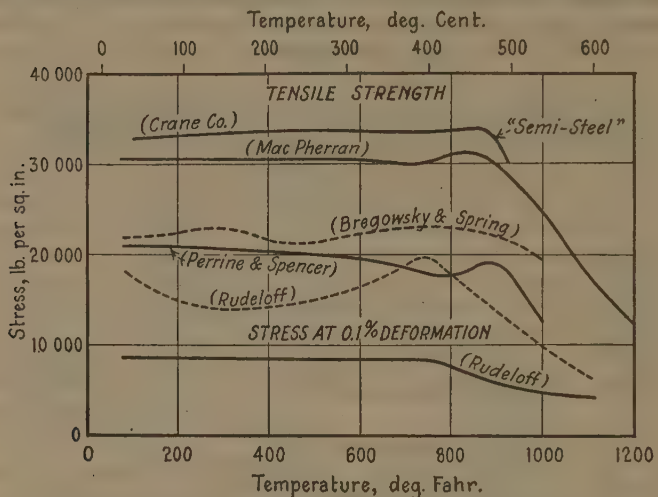

FIG. 1 HIGH-TEMPERATURE TENSILE PROPERTIES OF CAST IRONS AND SEMI-STEEL AS REPORTED BY VARIOUS INVESTIGATORS

		Chemical composition, per cent							
Investigator	Bibliography reference no.	Combined carbon	Graphitic carbon	Total carbon	Mn	Si	P	S	Remarks
MacPherran	(183)	....	....	....	0.64	1.84	0.52	0.11	Annealed at 1100 deg. fahr. (595 deg. cent.) before test.
Bregowsky and Spring	(83)	0.17	3.31	....	0.60	2.57	0.73	0.10	
Perrine and Spencer	(104)	....	....	2.69	....	....	....	....	Curve based on very few tests.
Rudeloff [a]	(36)	....	....	3.56	0.93	2.64	0.52	0.05	Curves are averages from both wet- and dry-sand castings.
Crane Co.	(216)	"Semi-steel" or "ferro-steel"				....	....	....	In reality a low-carbon cast iron with high Mn and low Si.

[a] 1000 lb. per sq. in. = 0.7031 kg. per sq. mm.

time effect and its importance in any discussion of the high-temperature properties of metals.

7 Under the conditions outlined it should therefore be recognized that terms such as "proportional limit," "yield point," "tensile strength," etc., which are used in this report, represent

values obtained in each case under given conditions of test and do not necessarily have the same significance throughout a large part of the temperature range considered as in tests at room temperatures.

TENSION TESTS

8 *Cast Irons, Semi-Steel and Malleable Iron.* The results of tension tests reported by a number of investigators for cast irons, semi-steel and malleable iron are shown graphically in Figs. 1 and

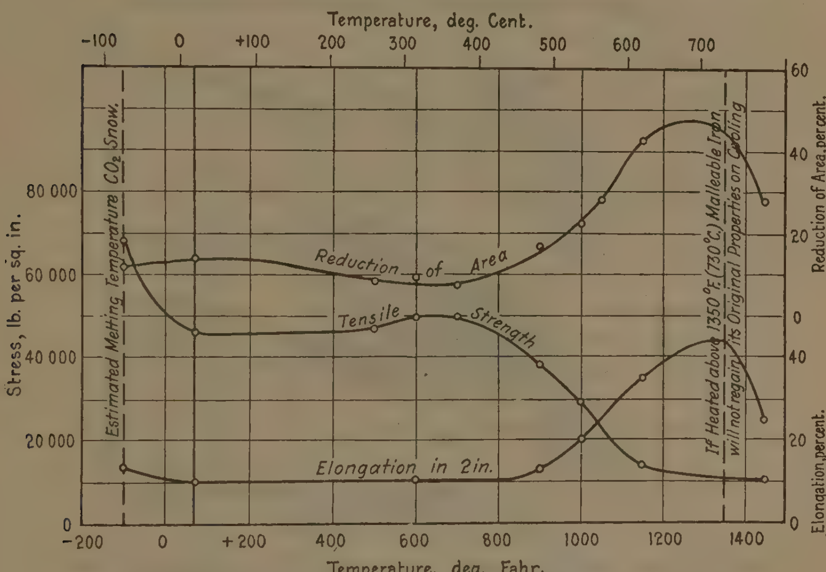

FIG. 2 TENSILE PROPERTIES OF MALLEABLE CAST IRON AT VARIOUS TEMPERATURES [SCHWARTZ (131)]

Tests made on specimens 0.634 in diameter, ground to size before annealing. The results are known, according to Schwartz, to be unaffected by shrinkage or other defects.

2. The discrepancies in numerical values for the various irons, which are not in all cases completely identified, are relatively unimportant for the moment. The principal feature to be observed for the three types of product is the small change in tensile properties with temperature rise from 70 to about 800 deg. fahr. (20 to 425 deg. cent.). There are some indications of a maximum in the tensile strength-temperature curves at about 600 to 800 deg. fahr. (315 to 425 deg. cent.), but this is smaller than in the case of carbon steels and for most practical purposes the tensile values may be considered nearly constant throughout the specified range. With further increase in temperature there begins a "softening"

which becomes quite rapid above about 900 deg. fahr. (480 deg. cent.).

9 Decrease below 70 deg. fahr. (20 deg. cent.) results in a "stiffening" of the metal as shown by increased tensile strength and some decrease in elongation and reduction of area in the case of malleable iron. In general the effect is accompanied by increased brittleness.

10 *Wrought Iron and Mechanically Worked Steels.* By far the largest number of tests in tension have been made on mechan-

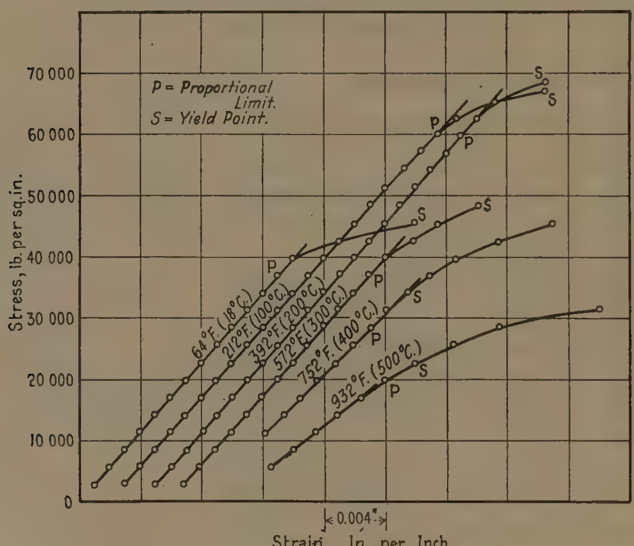

FIG. 3 EFFECT OF TEMPERATURE ON THE STRESS-STRAIN RELATIONS IN TENSION OF 0.37 PER CENT CARBON STEEL [WELTER(164)]

1000 lb. per sq. in. = 0.7031 kg. per sq. mm.

ically worked steels with or without subsequent thermal treatments. Many interesting comparisons are possible, but it is impracticable to include in this report more than a brief summary of features having very general interest and to reproduce data for more than a few steel types.

11 Figs. 3 to 8, inclusive, show the effect of temperature variations upon the proportional limit, tensile strength, elongation, reduction of area, and the stress-strain relations, including the modulus of elasticity, of carbon and some alloy steels.[1] While an attempt has been made to choose representative results for these graphs it should be kept in mind that they are based on tests

[1] Graphs are not given for wrought iron as the changes in properties are quite similar to those shown for low-carbon steels.

of individual heats under specific test conditions. Variations in numerical values may be expected when comparing tests of additional heats of the same type in one laboratory or of the same heat in different laboratories.

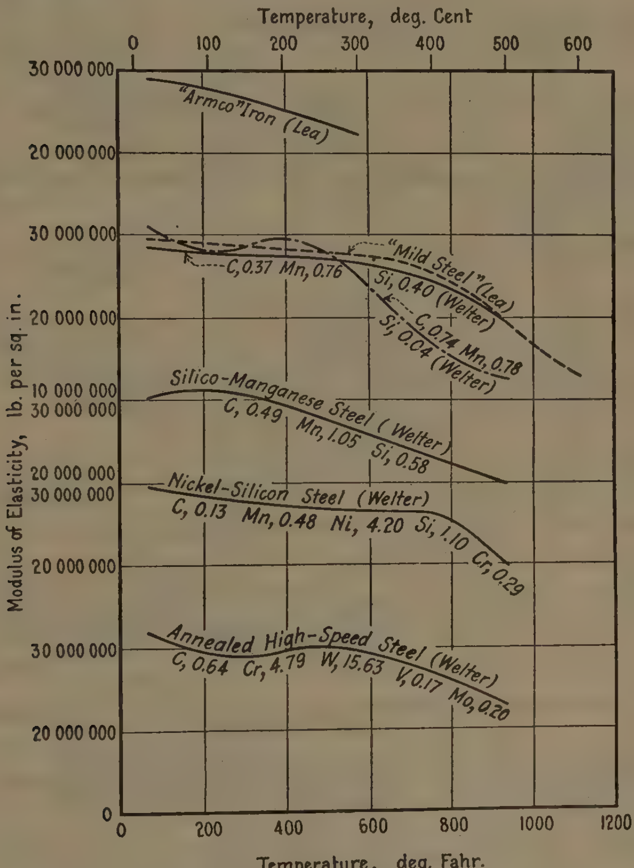

Fig. 4 Effect of Temperature on the Elastic Modulus of Various Steels in Tension

The work of Welter is given in item (**164**) of the bibliography; that of Lea in items (**103**) and (**187**).

12 Some of the important facts, which may be deduced from available results of a large number of similar tests, may be summarized as follows:

(*a*) The effect of temperature rise to about 1100 deg. fahr. (600 deg. cent.) is to reduce tensile strength, proportional limit, and the elastic modulus, and greatly increase

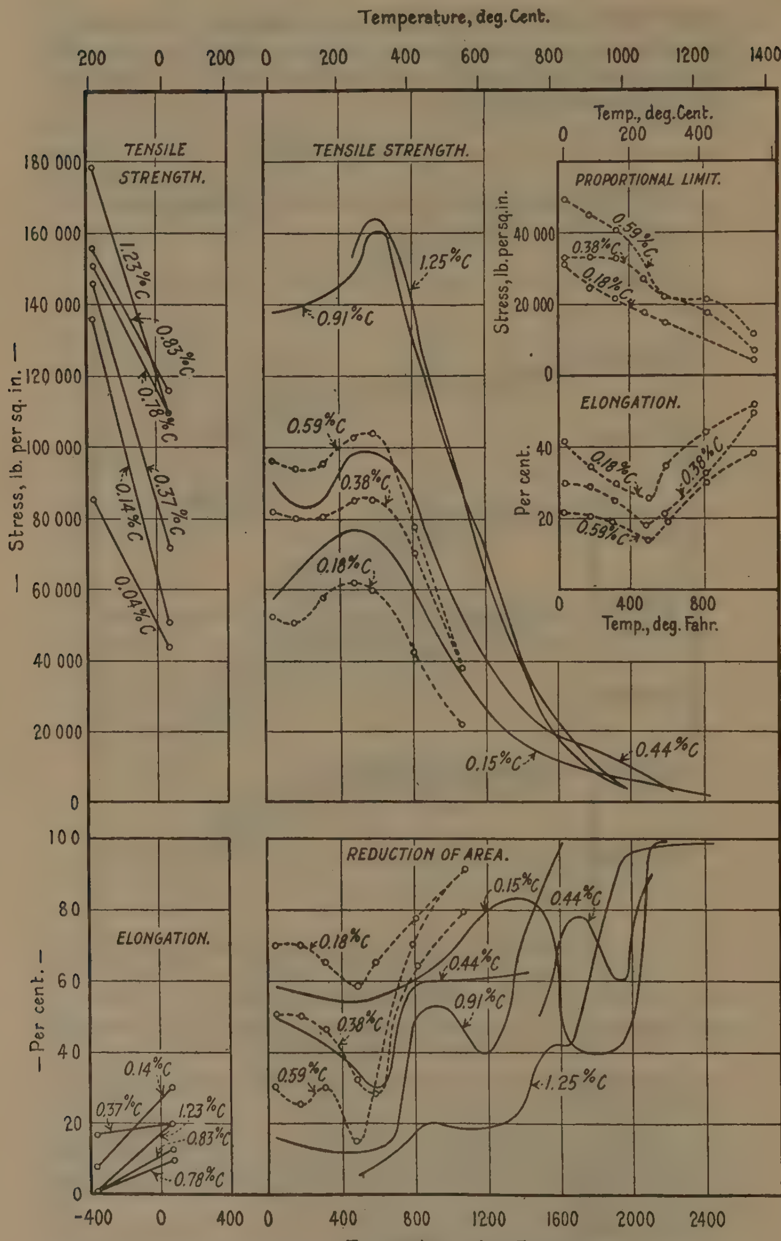

FIG. 5 EFFECT OF TEMPERATURE ON THE TENSILE PROPERTIES OF CARBON STEELS AS DETERMINED BY VARIOUS INVESTIGATORS

(See note on following page)

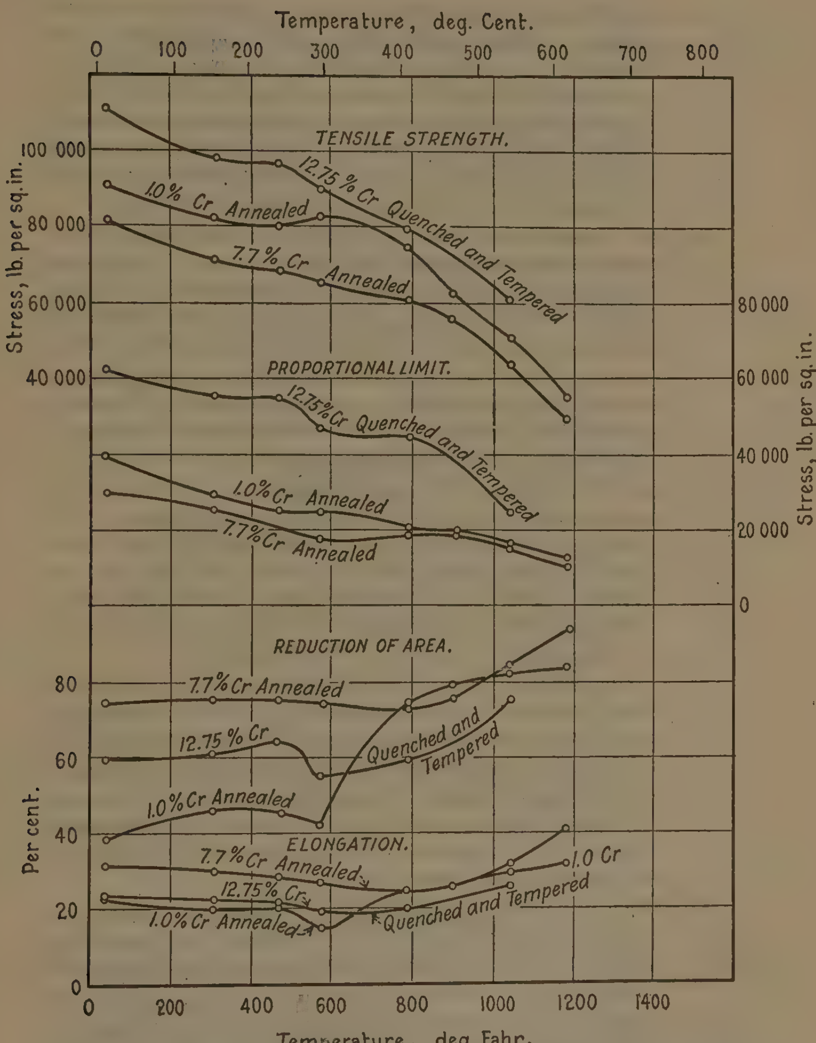

FIG. 6. EFFECT OF TEMPERATURE ON THE TENSILE PROPERTIES OF VARIOUS CHROMIUM STEELS CONTAINING 0.3 TO 0.4 PER CENT OF CARBON. (TESTS BY THE AUTHORS.)

Annealed = Annealed by heating for 45 min. at 1650 deg. fahr. (900 deg. cent.) and furnace cooling.

Quenched-and-Tempered = Oil quenched from 1750 deg. fahr. (955 deg. cent.); then tempered 45 min. at 1250 deg. fahr. (675 deg. cent.) and air cooled.

NOTE TO FIG. 5: Solid lines at elevated temperatures are results reported by Dupuy(150) for normalized steels; dotted lines are results on normalized steels obtained by the authors(196). Results below room temperature are those reported by Hadfield(54) for annealed steels containing about 0.1 to 0.3 per cent of manganese.

ductility and the tendency to creep in wrought iron and steels. Certain combinations of composition and treatment, notably normalized chromium-vanadium, quenched-and-tempered stainless, and air-cooled 28 per cent nickel steels are strongest at ordinary temperatures, but carbon and the majority of alloy steels show maximum tensile-strength values and minimum ductility in the range of 400 to 650 deg. fahr. (205 to 350 deg. cent.).

(*b*) The proportional limit of medium or low-carbon steel which has been largely relieved of stress by suitable treatment decreases with rise in temperature. In highly stressed metal, resulting from cold or blue work or quenching, and that having residual stress, such as often exists in thin sections of hot-finished steel, the proportional limit either remains at approximately its room-temperature value over a well-defined interval or shows an increase with first rise in temperature.

(*c*) From the standpoint of high strength and limit of proportionality of steels at elevated temperatures, the temperature scale may be divided roughly into three parts: (1) the range 70 to about 850 deg. fahr. (20 to 450 deg. cent.); (2) the range 850 to 1100 deg. fahr. (450 to 600 deg. cent.); (3) above about 1100 deg. fahr. (600 deg. cent.).

(*d*) In the lowest range, high strength and proportional limit are functions of composition and heat treatment, and in general, combinations giving highest strength at ordinary temperatures show similar superiority throughout the entire range. It is advisable, however, to keep the carbon low since decreased ductility becomes more marked with increase in carbon content, particularly in the blue-heat range.

(*e*) The upper limit in the second or "transition" range requires nearly full tempering following hardening for stability, so that in most cases the benefits to be derived from heat treatment are limited (except in the lower portion of the range) and high strength and limit of proportionality are more largely functions of composition. While short-time tests reported do not give quantitative comparisons for steels subjected to sustained loads, on account of the importance of the time factor, it would be reasonably expected that steels having highest limits of proportionality would be able to sustain higher loads than those with low proportional limits, though not necessarily in direct proportion to observed values. On this basis of comparison it appears possible

to improve the properties of steel by adding such elements as chromium, cobalt, uranium, molybdenum, and vanadium.

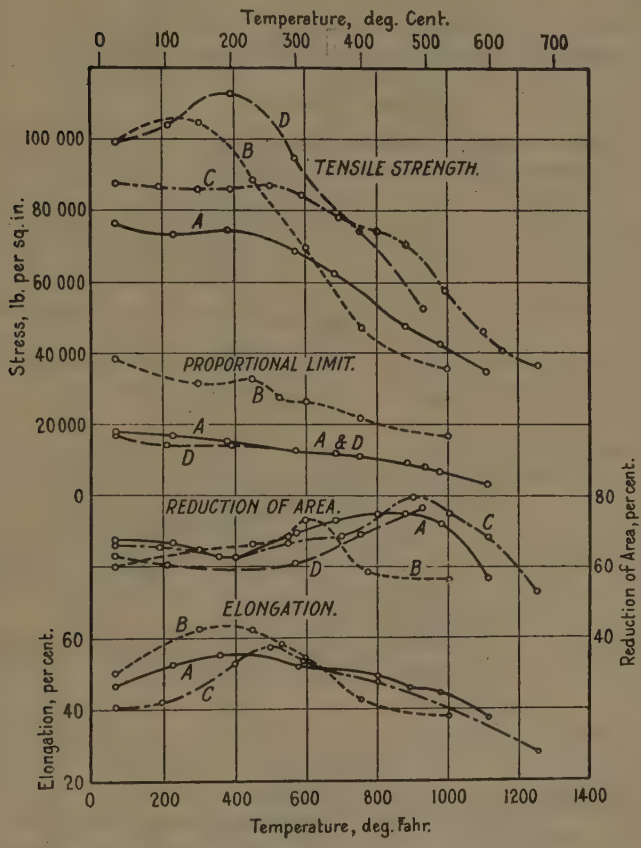

Fig. 7 Effect of Temperature on the Tensile Properties of Steels Containing from 25 to 35 per cent Nickel as Determined by Various Investigators

A = 28 per cent nickel steel reported by French(196); first air cooled from 1475 deg. fahr. (800 deg. cent.).
B = 31 per cent nickel steel reported by Bregowsky and Spring(83); tested in condition "as received."
C = 34 per cent nickel steel reported by MacPherran(158); tested as forged.
D = 25 per cent nickel steel reported by Welter(164).

(*f*) The drop in strength and proportional limit of steels at temperatures around 1025 deg. fahr. (550 deg. cent.) is permanent for most practical purposes, so that it would appear improbable that commercial steels can be produced to withstand continuously fairly large loads at

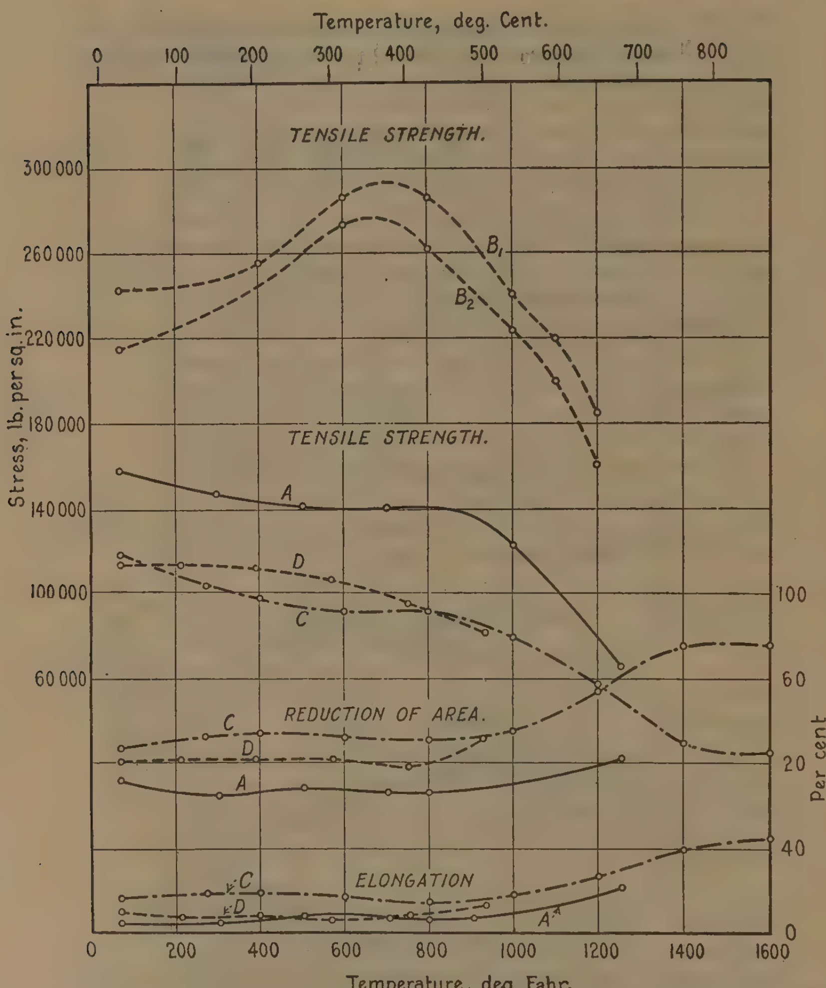

FIG. 8 HIGH-TEMPERATURE TENSILE PROPERTIES OF ANNEALED OR QUENCHED-AND-TEMPERED HIGH-SPEED STEELS REPORTED BY VARIOUS INVESTIGATORS

	Investigator and reference	Type composition, per cent C	Cr	W	V	Preliminary treatment
A	MacPherran(158)	0.68	3.36	19.3	0.88	2300 deg. fahr. (1260 deg. cent.) oil; tempered 1400 deg. fahr. (760 deg. cent.).
B_1	d'Arcambal(148)	0.65	3.62	17.8	0.95	2350 deg. fahr. (1290 deg. cent.) oil; tempered 1100 deg. fahr. (595 deg. cent.).
B_2	d'Arcambal(148)	0.69	3.13	13.9	1.68	2350 deg. fahr. (1290 deg. cent.) oil; tempered 1100 deg. fahr. (595 deg. cent.).
C	Spooner(162)	0.66	3.15	15.9	0.70	Annealed at 1660 deg. fahr. (905 deg. cent.).
D	Welter(164)	0.64	4.79	15.6	0.17	"Annealed."

temperatures above about 1200 deg. fahr. (650 deg. cent.), except when large proportions of one or more alloying elements are added to reduce the iron content to such a low value that the resulting product cannot correctly be called steel, or in special cases where extremely large proportions of special compounds are present.

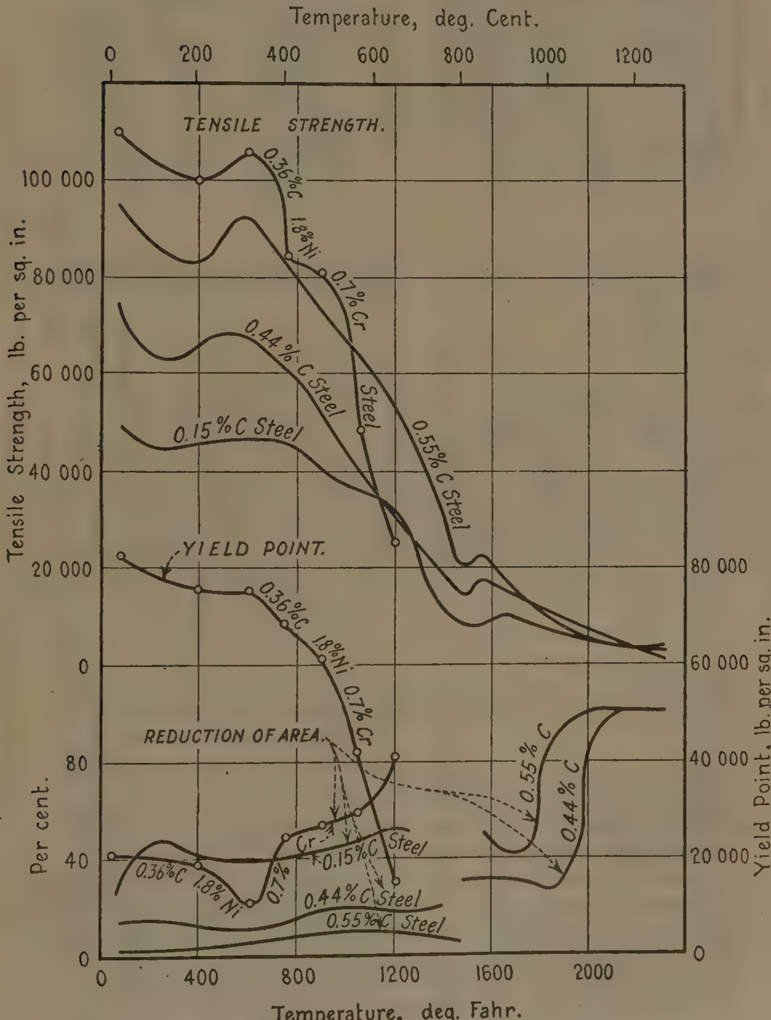

FIG. 9 EFFECT OF TEMPERATURE ON THE TENSILE PROPERTIES OF CAST STEELS (VARIOUS INVESTIGATORS)

Results on carbon steels are those reported by Dupuy(150); those for the nickel-chromium steel were obtained from V. T. Malcolm, Chapman Valve Manufacturing Co.

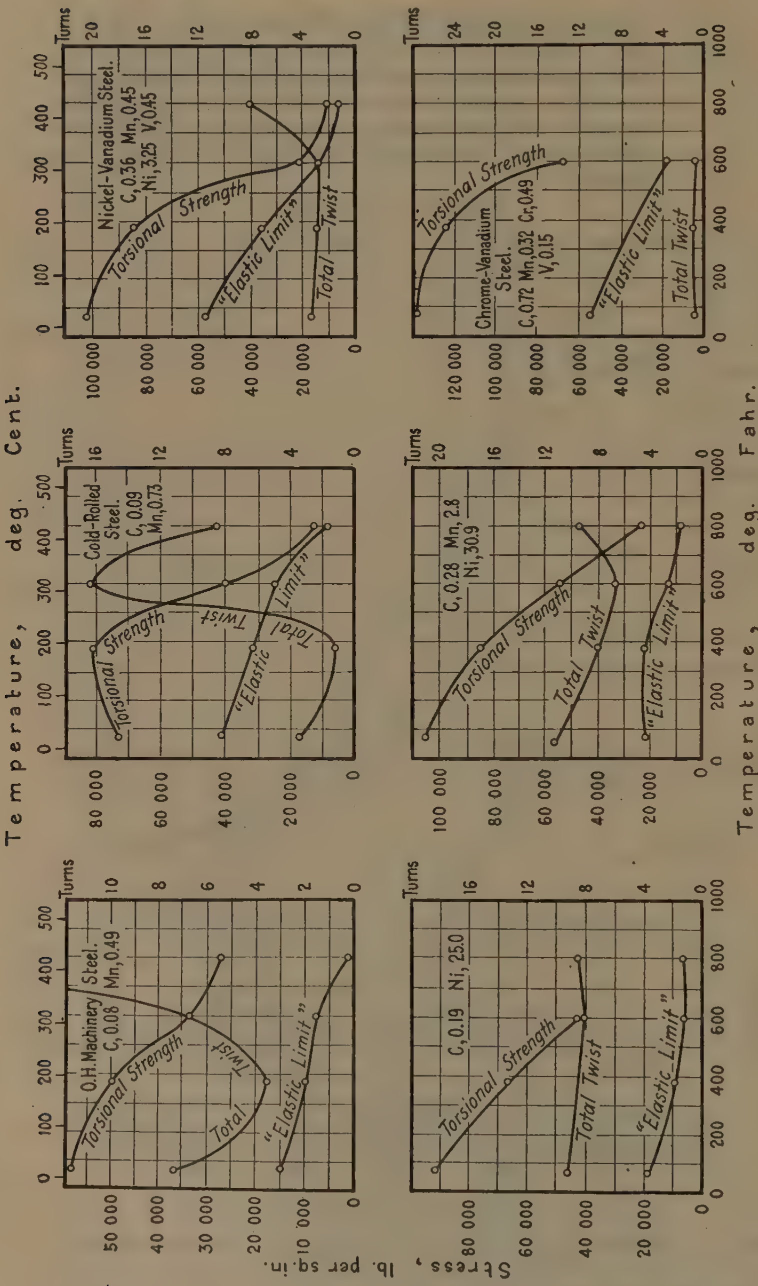

FIG. 10 EFFECT OF TEMPERATURE ON THE TORSIONAL PROPERTIES OF VARIOUS STEELS [BREGOWSKY AND SPRING(83)]

(See note on following page)

13 *Cast Steels.* In general, the effect of temperature increase on the mechanical properties of cast steels is similar to that on mechanically worked steels. However, the actual values obtained in tests will differ materially up to from about 800 to 1000 deg. fahr. (425 to 540 deg. cent.) and to a somewhat less extent at higher temperatures from those values observed in similar steels after mechanical work. As both chemical and physical characteristics play a predominant part at slightly elevated temperatures in the latter class of steels, so will these same factors, as reflected in the details of casting practice and heat treatment, be of prime importance in determining the properties observed in cast metals. They are, of course, to be considered of importance at all temperatures, but the weakening effect on all alloys containing large proportions of iron becomes so marked when the temperature becomes high that it obscures, at least in large part, the differences referred to. Representative results obtained on cast carbon and nickel-chromium steels are shown graphically in Fig. 9, and in view of the previous discussions no further comments will be added.

TORSION TESTS

14 Very little has been published concerning the torsional properties of ferrous metals at various temperatures. The report of Bregowsky and Spring(**83**) is the only one that has so far come to the authors' attention giving results in the range 70 to 800 deg. fahr. (20 to 425 deg. cent.) and some of the results are reproduced in Fig. 10. While insufficient data are given from which to draw general conclusions, a marked "softening" is observed in all steels tested when the temperature is raised from that of the room to 800 deg. fahr. (425 deg. cent.). However, there appears to be a range of minimum ductility in the neighborhood of 400 to 600 deg. fahr. (205 to 315 deg. cent.) as shown by the small number of turns (twists) before failure. This coincides quite closely with minimum values of elongation and reduction of area observed in tension tests of similar materials.

15 While considering the torsional properties of ferrous metals attention should be drawn to the qualitative experiments described by Brearley[1] to show the effects upon steels of temperatures in the neighborhood of those used in hot working. A bar of steel, either rectangular or appropriately marked so that the twisting could readily be followed, was heated to about 1800 to 2000 deg. fahr. (982 to 1093 deg. cent.) at one end and then removed from the furnace to allow the heat to taper down until, within 3 or 4 in.

[1] Discussion of report by Dickenson(**172**).

Note to Fig. 10: "Elastic limits" determined from stress-strain relations obtained with a troptometer. Specimens had 8-in. gage length and were 0.855 in. in diameter, except the 30 per cent nickel steel which was 1 in. in diameter. All tests made on material as received.

from the colder end, it was at perhaps 1100 deg. fahr. (600 deg. cent.). The hot end was then placed in a vise and the bar twisted from the colder end. In nearly all cases there was a twist of short pitch at the hotter end; then somewhere down the metal at intermediate temperatures came a twist of longer pitch and finally there was a twist of shortest pitch where the metal was coldest.

16 Very recently similar experiments were carried out more carefully by Sauveur(214) who found that such discontinuities in the twist were associated with an independent A_3 transformation;

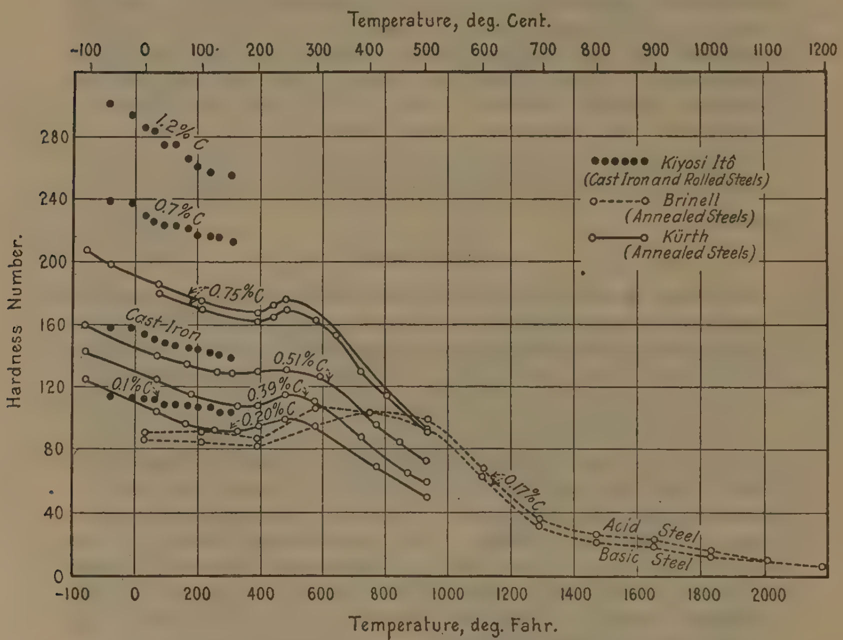

FIG. 11 BRINELL HARDNESS OF CAST IRON AND CARBON STEELS AT VARIOUS TEMPERATURES (VARIOUS INVESTIGATORS)

Refer to bibliography, items(53) (Brinell),(62) (Kürth), and(198) (Ito.)

hence they were observed in iron or steels containing less than about 0.40 per cent carbon. This temperature range within which a "critical twist" was observed may be called a zone of "reduced malleability" and coincides with the so-called "hot-short" range long recognized by mill men for the very pure iron known as Armco or ingot iron. Discontinuities in mechanical properties-temperature curves in this range have also been shown in tension tests by Rosenhain(93) and others.

HARDNESS TESTS

17 Hardness tests, using the Brinell method, have been reported by Brinell(53), Kürth(62), Robin(63), Ito(198), etc. As shown in Fig. 11, the hardness decreases progressively with temperature rise from −80 to 2200 deg. fahr. (−60 to 1200 deg. cent.) with the exception of a fairly narrow temperature range around "blue heat" where a rise in temperature results in an increase in hardness. This effect is observed between 400 and 600 deg. fahr. (205 and 315 deg. cent.) in the hardness-temperature curves of Kürth

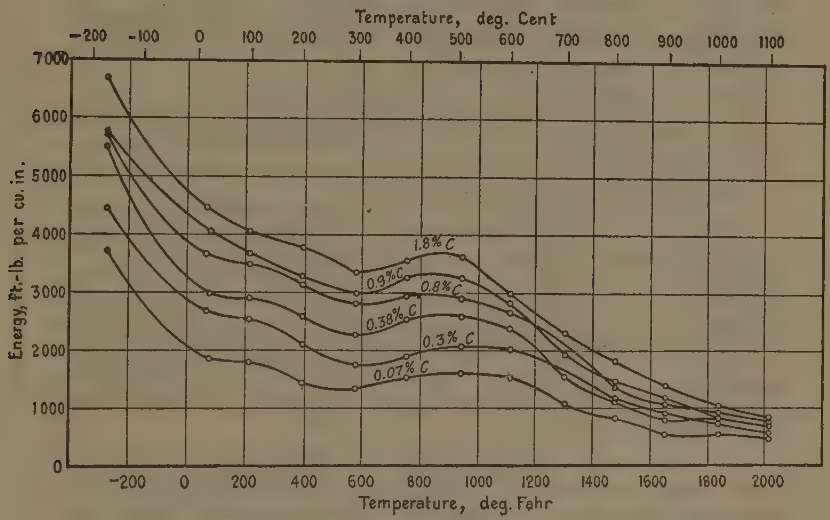

Fig. 12 Loads Required at Various Temperatures to Reduce the Height of Rolled Carbon-Steel Cylinders (With Ratio of Length to Diameter of 1) by 20 per cent When the Force is Applied at 6.56 ft. (2 meters) per second [Robin(71)]

for carbon steels and coincides with the zone of minimum ductility or maximum strength shown in most tension and torsion tests of similar alloys; in the case of Brinell's curves it extends over a wider range and occurs at somewhat higher temperatures.

CRUSHING TESTS

18 What may be called the "deformational characteristics" of irons and steels have been very carefully studied throughout a wide temperature range by Robin(71). The extent of his investigations prevents a complete summary, but there are a number of features relating particularly to crushing tests and comparisons of crushing resistance with other mechanical properties which

should be referred to in some detail. Among these are the following:

The work necessary to effect a given crushing varies according to the number of blows of a given intensity which produced this crushing. The curves of crushing or of the resistance to crushing, in terms of the number of blows, are hyperbolic and depend on the hardness and on the elasticity of the metal. The direction of the curves changes when the heat diminishes the resistance of the metal, whereupon the latter behaves like a soft metal.

The resistance to crushing of a straight steel with circular base diminishes when the ratio of its depth to the diameter of its base increases. The law which correlates this resistance with the relative dimensions of cylinders is represented graphically by the hyperbolic paraboloid. In cylinders with constant dimensions of the base and with increasing depths the resistance to crushing diminishes hyperbolically; in cylinders of constant depth but increasing diameter, the resistance to crushing increases in proportion.

The rate of testing influences the resistance of metals to crushing. As in the case of a number of different blows it acts in opposite ways according as it is a question of hard and elastic metals or soft metals. At each temperature of crush in any metal the rate of speed produces specific variations in the numerical results.

The resistance to crushing (of carbon steels), which is relatively considerable at liquid air temperatures, −310 deg. fahr. (−190 deg. cent.), diminishes very rapidly up to 30 deg. fahr.(0 deg. cent.) and then slowly up to 570 deg. fahr. (300 deg. cent.) where the minimum resistance is found. The resistance increases, reaching a maximum at about 930 deg. fahr. (500 deg. cent.), followed by a rapid fall at 1560 deg. fahr. (850 deg. cent.) and a very slow fall at higher temperatures. [Refer to Fig. 12.] Lack of cohesion in steels containing high percentages of carbon and the intervention of fusion in soft steels restrict the experiments, reducing the resistance to crushing to an exceedingly low value.

19 Robin further pointed out that "interstrained" steels give more marked variations in crushing resistance than do the same steels after annealing; on the other hand, phosphorus diminishes the variations but "increases the value in common, generally speaking, with other elements dissolved in iron."

Pearlitic steels undergo the same variations as carbon steels; variations in resistance to crushing may be greatly reduced or even obliterated by the presence of a sufficient amount of an element in solution, such, for example, as chromium.

Martensitic steels yield a decreasing curve which possesses neither maximum nor minimum; the greatest fall in resistance commences at 930 deg. fahr. (500 deg. cent.).

Austenitic steels vary little in their resistance to crushing up to about 1000 or 1100 deg. fahr. (550 or 600 deg. cent.). The resistance to crushing increases considerably at liquid-air temperatures. Starting from 30 deg. fahr. (0 deg. cent.), the curve is generally rectilinear up to about 1110 deg. fahr. (600 deg. cent.) where the most important fall in resistance occurs. Special steels containing the free carbide and the high-speed steels investigated behave similarly. Their resistance at ordinary temperatures and particularly at about −310 deg. fahr. (−190 deg. cent.) is, generally speaking, high. Some steels preserve a high degree of resistance to crushing at high temperatures, a resistance much greater than that of carbon steels. The presence of nickel favors this resistance at high temperatures.

20 In comparing the static and dynamic tests of steels at various temperatures, Robin pointed out that:

The static tensile and hardness tests correspond with one another. Compression appears to indicate corresponding variations: The rate of testing affects the observed results at higher temperatures up to a limit which apparently cannot in practice be exceeded and relates to shocks of any rate or intensity whatever. Brittleness as the result of static effects appears to occur at 570 deg. fahr. (300 deg. cent.) but brittleness under shock is practically in the neighborhood of 930 deg. fahr. (500. deg. cent.).

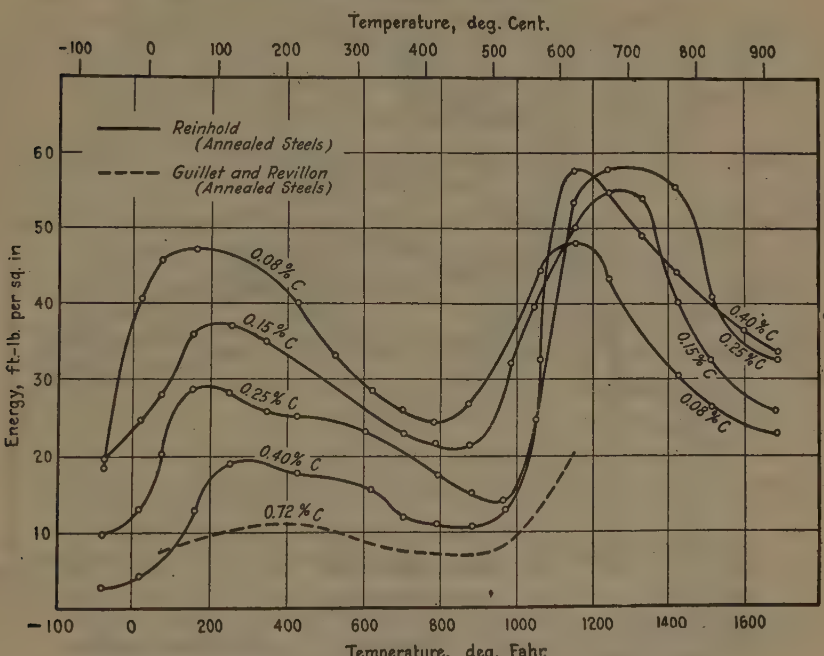

FIG. 13 NOTCHED-BAR IMPACT RESISTANCE OF CARBON STEELS AT VARIOUS TEMPERATURES

Refer to bibliography, items (59) [Guillet and Revillon (Guillery test)] and (113) [Reinhold (Charpy test)].

The properties of steel, so far as the dynamic and static effects are concerned, vary in totally different ways according to the temperature and according to the nature of the steels. The correlation of these effects at the normal temperature in the case of certain steels appears, therefore, to be due purely to coincidence.

IMPACT TESTS

21 The results obtained by Reinhold(113), Charpy(97), Guillet(59), and more recently Langenberg(200-201) will serve to show the effects of temperature variations upon the impact resistance

of steels as determined on notched bars. Representative results are shown graphically in Figs. 13 and 14.

22 The general form of the impact energy-temperature curves is quite similar for the majority of steels tested. As the temperature is progressively raised from about −100 deg. fahr. (−75 deg. cent.) the absorbed energy increases and reaches maximum

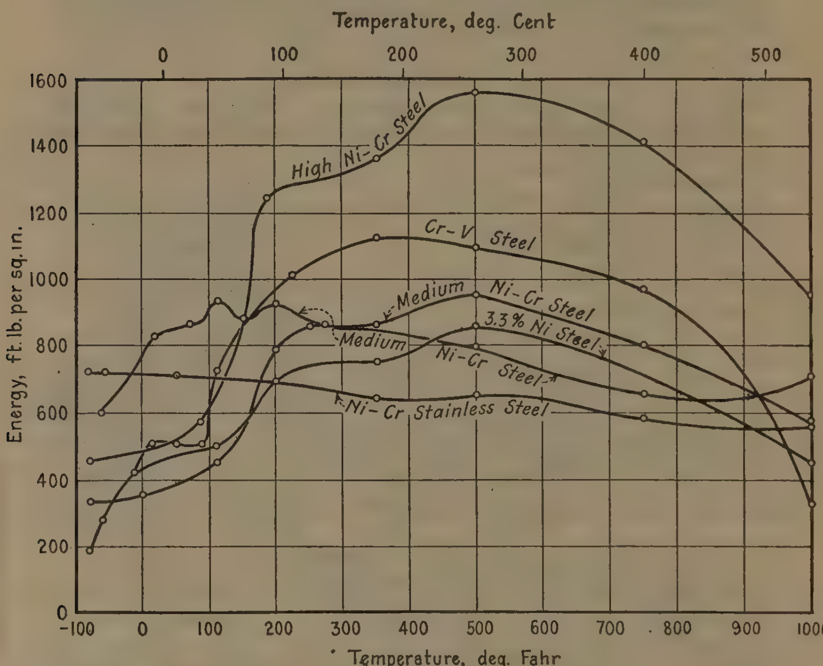

Fig. 14 Notched-Bar Impact Resistance (Charpy Test) of Some Alloy Steels at Various Temperatures [Langenberg (201)]

High-nickel-chromium steel: 0.39 per cent C; 3.44 per cent Ni; 1.58 per cent Cr. 1450 deg. fahr. (790 deg. cent.) oil; tempered 1150 deg. fahr. (620 deg. cent.).
Chrome-vanadium steel: 0.38 per cent C; 0.79 per cent Cr; 0.15 per cent V. 1600 deg. fahr. (870 deg. cent.) oil; tempered 1150 deg. fahr. (620 deg. cent.).
Medium-nickel-chromium steel: 0.21 to 0.36 per cent C; 1.93 per cent Ni; 0.99 per cent Cr. Annealed at 1650 deg. fahr. (900 deg. cent.) (upper curve) 1580 deg. fahr. (860 deg. cent.) oil; 1400 deg. fahr. (760 deg. cent.) oil; tempered 500 deg. fahr. (260 deg. cent.) (lower curve).
Nickel-chromium stainless steel: 0.38 per cent C; 23.75 per cent Ni; 7.08 per cent Cr. Annealed at 1290 deg. fahr. (700 deg. cent.).
3.30 per cent nickel steel: 0.31 per cent C; 3.30 per cent Ni. 1515 deg. fahr. (825 deg. cent.) oil; tempered 1110 deg. fahr. (600 deg. cent.).

values in the range 150 to 400 deg. fahr. (65 to 205 deg. cent.); it then decreases. According to the results obtained by Reinhold, Guillet, and Charpy, a second rise in impact resistance begins in the neighborhood of 800 to 1000 deg. fahr. (425 to 540 deg. cent.) and is followed by maximum values which in general are greater

than the first maximum between 150 and 400 deg. fahr. (65 and 205 deg. cent.). As the temperature is raised above about 1200 to 1400 deg. fahr. (650 to 760 deg. cent.) the absorbed energy decreases rapidly.

23 These changes appear to be quite generally characteristic of the majority of steels for which data are available, but the magnitude of the observed effects is dependent to a large degree upon composition, previous mechanical and thermal treatments

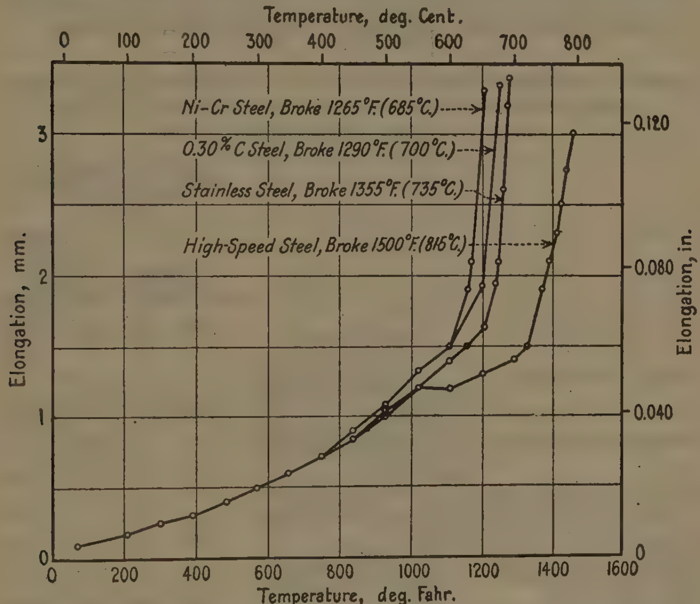

FIG. 15 "FLOW" OF VARIOUS STEELS UNDER A STATIC LOAD OF 19,000 LB. PER SQ. IN. (8.5 TONS PER SQ. IN.) WITH RISING TEMPERATURE [DICKENSON (172)]

Compositions and treatments of the various steels are given in Table 2.

and upon the methods used in the test. The most notable exception is the high-nickel-chromium steel containing also about 1.5 per cent silicon, tested by Langenberg. In this case the absorbed energy shows a very gradual and comparatively small decrease as the temperature is raised from −80 to 1000 deg. fahr. (−60 to 540 deg. cent.).

24 Attention has already been called to the low ductility in steels in the neighborhood of "blue heat" (400 to 600 deg. fahr.) (205 to 315 deg. cent.) as shown in both tension and torsion tests. This effect is apparently distinct from and not accompanied by brittleness, as the energy absorbed in the notched-bar impact tests does not show low values in the specified temperature range.

25 Langenberg has pointed out and special consideration should be given to the marked influence on impact resistance of ordinary atmospheric temperature variations. Also steels and treatments giving highest impact resistance at 70 deg. fahr. (20 deg. cent.) do not necessarily show the same degree of superiority at higher or lower temperatures and in quite a number of cases the order is reversed. A similar condition is observed in the short-time tension tests and shows the need for readjustment in methods of design of equipment for high or low temperature service.

TABLE 2 LIMITING TEMPERATURES SET BY DICKENSON(172) AT WHICH A LOAD OF 19,000 LB. PER SQ. IN. CAN BE SUSTAINED IN BOTH SHORT- AND LONG-TIME TENSION TESTS

Type of steel	Heat treatment [a]	Maximum temperature to which steel will withstand stress of 19,000 lb. per sq. in. — For short duration of loading [b], deg. fahr.	For short duration of loading [b], deg. cent.	For considerable time without sensible deformation, deg. fahr.	For considerable time without sensible deformation, deg. cent.
0.30 per cent carbon steel.	1560 deg. fahr. (850 deg. cent.) oil; tempered at 1065 deg. fahr. (575 deg. cent.	1425	775	930	500
0.45 per cent carbon steel.	1600 deg. fahr. (870 deg. cent.) water; tempered at 1110 deg. fahr. (600 deg. cent.).	1480	805	840	450
Nickel - chromium steel, 0.25 per cent C; 3.6 per cent Ni; 0.6 per cent Cr.	1525 deg. fahr. (830 deg. cent.) oil; tempered at 1110 deg. fahr. (600 deg. cent.).	1470	800	970	520
Stainless steel, 0.26 per cent C; 14.7 per cent Cr; 0.6 per cent Si; 0.4 per cent Ni.	1700 deg. fahr. (925 deg. cent.) oil; tempered at 1200 deg. fahr. (650 deg. cent.	1650	900	1065	575
High-speed steel, 0.6 per cent C; 17.4 per cent W; 4.0 per cent Cr; 0.7 per cent V.	Annealed 1470 deg. fahr. (800 deg. cent.).	1770	965	1110	600

[a] Fahrenheit temperatures given to nearest 5 deg. in conversion from centigrade scale.
[b] This refers to the tensile-strength values.

26 Another feature of special interest is the superior resistance to impact of the commercial nickel-chromium steels compared to $3\frac{1}{2}$ per cent nickel steel and likewise the superiority of chromium or chromium-vanadium steels compared to nickel-chromium steel.

27 The available impact tests show, among other things, that chromium is highly beneficial in the quantities ordinarily used and in comparison with the other alloying elements at present employed in commercial steels appears to "toughen" as well as "stiffen" the resulting metal in those temperature ranges in which composition is of prime importance as compared to heat treatment.

LONG-TIME OR "FLOW" TESTS

28 Mention has already been made in this report of the importance of the time factor in testing metals at high temperatures. In 1919, Chevenard(124) published results obtained under sustained loading on an air-hardening nickel-chromium steel and among other things gave definite values for the rate of flow for various loads at different temperatures. More recently Dickenson(172),

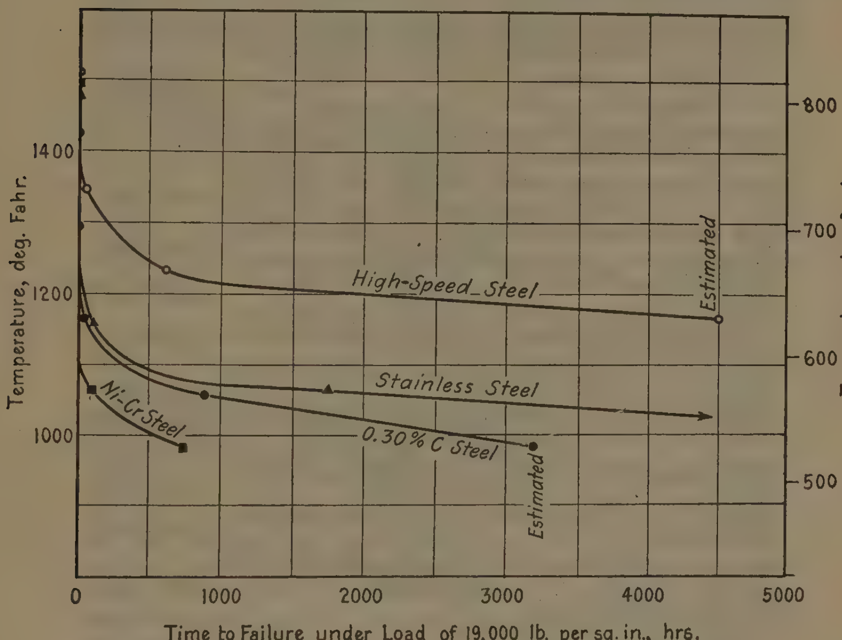

FIG. 16 TIME REQUIRED TO PRODUCE FAILURE IN VARIOUS STEELS AT DIFFERENT TEMPERATURES UNDER A LOAD OF 19,000 LB. PER SQ. IN. (8.5 TONS PER SQ. IN.) [DICKENSON(172)]

Composition and treatment of the different steels are given in Table 2.

following the same principles, reported results obtained in both short-time and prolonged tension tests of various steels. Two carbon steels, a nickel-chromium steel, stainless (13 per cent chromium) and high-speed steels were investigated and results are reproduced in part in Figs. 15 and 16.

29 Table 2 is also taken from Dickenson's report and shows quite clearly that the ordinary tensile strength values at high temperatures give no direct indication either of the limiting temperatures up to which fixed loads can be maintained without sensible flow or the limiting loads which can be sustained at various temperatures.

30 While freely condemning the short-time tension test as being worthless for use by designing engineers, Dickenson unfortunately neglected the only factors which might show a direct relation to the results obtained so laboriously in the long-time tests. No attempt was made to determine proportional or elastic limits and the stress-strain relations in the ordinary tests or to correlate existing data of this type with the limiting values determined under prolonged loading.

TABLE 3 COMPARISON OF SOME DATA IN BOTH LONG- AND SHORT-TIME TENSION TESTS AT HIGH TEMPERATURES

Steel	Heat treatment [a]	Temperature deg. fahr.	Temperature deg. cent.
MAXIMUM TEMPERATURE AT WHICH STEEL WILL SUSTAIN A STRESS OF 19,000 LB. PER SQ. IN. FOR LONG PERIODS WITHOUT SENSIBLE FLOW [a]			
0.30 per cent carbon steel.	1560 deg. fahr. (850 deg. cent.) oil; tempered 1065 deg. fahr. (575 deg. cent.).	930[b]	500[b]
0.45 per cent carbon steel.	1600 deg. fahr. (870 deg. cent.) water; tempered 1110 deg. fahr. (600 deg. cent.).	840[b]	450[b]
Nickel-chromium steel, 0.25 per cent C; 3.6 per cent Ni; 0.6 per cent Cr.	1525 deg. fahr. (830 deg. cent.) oil; tempered 1110 deg fahr. (600 deg. cent.).	970[b]	520[b]
Stainless steel, 0.26 per cent C; 14.7 per cent Cr; 0.6 per cent Si; 0.4 per cent Ni.	1700 deg. fahr. (925 deg. cent.) oil; tempered 1200 deg. fahr. (650 deg. cent.).	1065[b]	575[b]
High-speed steel, 0.6 per cent C; 17.4 per cent W; 4.0 per cent Cr; 0.7 per cent V.	Annealed 1470 deg. fahr. (800 deg. cent.).	1110[b]	600[b]
TEMPERATURE AT WHICH A PROPORTIONAL LIMIT OF 19,000 LB. PER SQ. IN. IS SHOWN IN THE SHORT-TIME TENSILE TEST [a]			
0.33 per cent carbon steel.	1555 deg. fahr. (845 deg. cent.) water; tempered 1005 deg. fahr. (540 deg. cent.).	790[c]	420[c]
Nickel-chromium steel, 0.39 per cent C; 3.1 per cent Ni; 0.9 per cent Cr.	Air cooled from 1560 deg. fahr. (850 deg. cent.).	930[d]	500[d]
Stainless steel, 0.26 per cent C; 12.8 per cent Cr.	1750 deg. fahr. (955 deg. cent.) oil; tempered 1210 deg. fahr. (655 deg. cent.).	1065[e]	575[e]
High-speed steel, 0.6 per cent C; 17.4 per cent W; 4.0 per cent Cr; 0.2 per cent V; 0.2 per cent Mo.	Annealed.	860[f]	460[f]

[a] Fahrenheit temperatures given to nearest deg. in conversion from centigrade scale.
[b] From data reported by Dickenson (172).
[c] From data obtained by one of the authors.
[d] From data reported by French (153).
[e] From data reported by French (180).
[f] From data reported by Welter (164).

31 In Table 3 are given the limiting temperatures determined by Dickenson at which a load of 19,000 lb. per sq. in. (8.5 tons per sq. in.) can be sustained for long periods by each of the five steels without sensible deformation, and also the temperature at which a proportional elastic limit of 19,000 lb. per sq. in. is shown in the short-time tension test. These latter values were, of necessity, collected from various sources so that many variables are introduced when comparisons are attempted, such as, for example, individual heat characteristics of the steels, variations in chemical composition and heat treatment, methods of test, etc. Despite

these variations the limiting temperatures determined from the stress-strain relations in the short-time tests are comparable to those deduced from the prolonged loading. In fact, in all but one case the former values are somewhat lower than the latter. For the two stainless steels which are quite similar in composition and heat treatment identical values are obtained.

32 It is not intended to give the impression that proportional limits determined by methods so far employed can give as accurately as sustained-loading tests the limiting loads or temperatures for various steels, but at least the comparisons cited point to the possibility of obtaining from the stress-strain relations in a short-time test with slow rates of loading quite satisfactory information for most practical purposes. Dickenson's values may be accepted as quite accurate, but it should be kept in mind that they are based on individual heats tested under specific conditions. Similar tests carried out on additional heats of each type of steel would undoubtedly show variations which might possibly be as great as the differences shown by the two methods in Table 3.

Special High-Temperature Properties and Tests

thermal expansion

33 Thermal expansion of ferrous metals is of particular interest in the design and installation of equipment for high-temperature service and there are included in the bibliography[1] references relating to such data for irons and steels. It may be well to point out that the average coefficient for annealed structural carbon and the majority of current commercial structural alloy steels recently tested by Souder(193, 194) is between about 6.5×10^{-6} and 7.5×10^{-6} parts per unit length per 1 deg. fahr. over the temperature range 75 to 570 deg. fahr. (25 to 300 deg. cent.); in the range from 570 to 1110 deg. fahr. (300 to 600 deg. cent.) it is between 8.0×10^{-6} and 9.3×10^{-6}. Among the principal exceptions are invar and some of the related high-nickel steels which have very low expansion coefficients at slightly elevated temperatures but show exceptionally high values in the neighborhood of 750 to 1110 deg. fahr. (400 to 600 deg. cent.). Likewise stainless steel (13 per cent chromium) shows a somewhat lower expansion than the representative values given above. The sample tested by Souder had an average of 6.1×10^{-6} from 75 to 570 deg. fahr. (25 to 300 deg. cent.) and 7.4×10^{-6} in the range from 570 to 1110 deg. fahr. (300 to 600 deg. cent.).

"growth" in cast irons

34 Cast irons, as is well known, are subject to permanent changes in volume upon repeated heating and as a result their field of usefulness has been restricted. This effect, which is gener-

[1] See pp. 477-487.

ally an increase in volume for commercial materials and commonly called "growth," is not only dependent upon temperature and time but also upon composition. According to Rugan and Carpenter(65)[1] white irons shrink and gray irons grow, but in white irons containing appreciably more than 3 per cent carbon there is a tendency to deposit temper carbon upon prolonged heating and the metal will then tend to grow. An alloy of practically constant volume under repeated heating at 1650 deg. fahr. (900 deg. cent.) was found to be a white iron containing about 3 per cent of carbon and only small quantities of other constituents, and in particular less than 0.2 or 0.3 per cent of silicon.

35 Carpenter(65) stated that phosphorus tends to diminish "growth" and that if 0.3 per cent is present, growth is lessened by about 3 per cent. The amount of sulphur present in commercial cast irons is not usually sufficient to have more than a minor influence. Manganese is one of the most important elements to be considered and not only retards the rate of growth but in the majority of cases diminishes the absolute amount. The effect of dissolved gases is negligible in the presence of more than 3 per cent of silicon, but may cause a growth of 1 to 2 per cent in irons containing 1.75 to 3.0 per cent of this element and at least 10 per cent when silicon does not exceed 1 per cent.

36 As a result of his investigations, Carpenter recommended the use of a "semi-steel" containing about 2.6 per cent of carbon, 0.6 per cent of silicon and 1.6 per cent of manganese for annealing ovens, rolls, fire bars, high-pressure steam valves and turbine casings whose growth, when made from gray irons, is so objectionable a feature. Such metal showed no growth after 150 heatings to high temperatures, but on the contrary a slight contraction of about 0.13 per cent.

CHEMICAL STABILITY

37 In addition to suitable mechanical properties, chemical stability or, as described by Chevenard(170), "chemical inertness" is an important factor in the selection of metals for use at any temperature. It is not considered within the scope of this paper to attempt even a partial review of the large mass of literature relating to corrosion at ordinary or low temperatures, but mention should be made of the extensive bibliography relating to this subject recently prepared by Van Patten(207).

38 While it is well recognized that certain alloys are generally more resistant to chemical changes than others, the greater part of published data on the high temperature stability of ferrous metals is concerned with failures or observations made under

[1] Attention should be called to the very early studies of the "growth" in cast irons by A. E. Outerbridge, Jr.(51), though the more recent experiments cited better serve the purposes in view for this paper.

specific laboratory or service conditions. Few comparisons of alloy steels at relatively high temperatures have been recorded. Because of this and the fact that many variables are encountered in different types of service only brief reference will be made to typical effects.

39 *Intercrystalline Deterioration.* Failures have not been uncommon in iron or steel parts due to intercrystalline attack resulting in what has been called intercrystalline brittleness. Stress, which may be either internal or external but probably restricted in intensity, nature, and distribution, is a necessary condition for such selective attack. However, the active agents producing intercrystalline brittleness may vary widely. Thus in power-plant installations the embrittlement of boiler steels may be the result of reactions with dissolved alkaline substances in the feed-water(118). In a case recently brought to the attention of the authors, embrittlement was observed in wrought-iron stirring rods immersed in molten copper and was accompanied by intercrystalline penetration of the copper.

40 The intercrystalline cracking of mild steel in caustic soda solutions has been attributed to the weakening of grain boundaries due to hydrogen absorption by the intercrystalline material alleged by one school of metallurgists to be amorphous. Jones(167) has shown that solutions of nitrates also yield a product having a selective action on the intercrystalline material and suggests that this may be nitrogen or an oxide of nitrogen. However, intercrystalline fracture of metals may possibly occur, according to Hanson(167), "as a result solely of the stresses but corrosive agents might act in accelerating them" and likewise cases are cited where material, relatively free from stresses, has developed intercrystalline fracture due to cementite envelopes, etc.

41 It is not intended to discuss all possible causes of intercrystalline brittleness and fracture, but merely to point out (1) that such effects are observed in steels under a variety of conditions in which stress and corrosive agents have been present, (2) that few data are available for comparison of various ferrous alloys and (3) that with increasing demands upon materials for high-temperature service, both with respect to stresses and temperatures, further attention must be given to this important subject.

42 *Oxidation of Steels.* The relative resistance to oxidation of alloy steels in air has recently been studied by Aitchison(123) and Dickenson(172). As the result of extensive tests of carbon, chromium, nickel, nickel-chromium and tungsten steels, the former concluded:

That the high-chromium steels present the greatest resistance to scaling at high temperatures of any of the steels. Those of the "stainless" type (13 per cent chromium) give a very high resistance whilst those containing about 7 per cent of chromium give a very

fair resistance though not quite so good as that of the stainless steels. In the latter case, however, the scale is more adherent than in the case of the stainless steel type.

That the nickel-chromium steels (ordinary structural types) scale to a greater extent than do the steels of any of the other types.

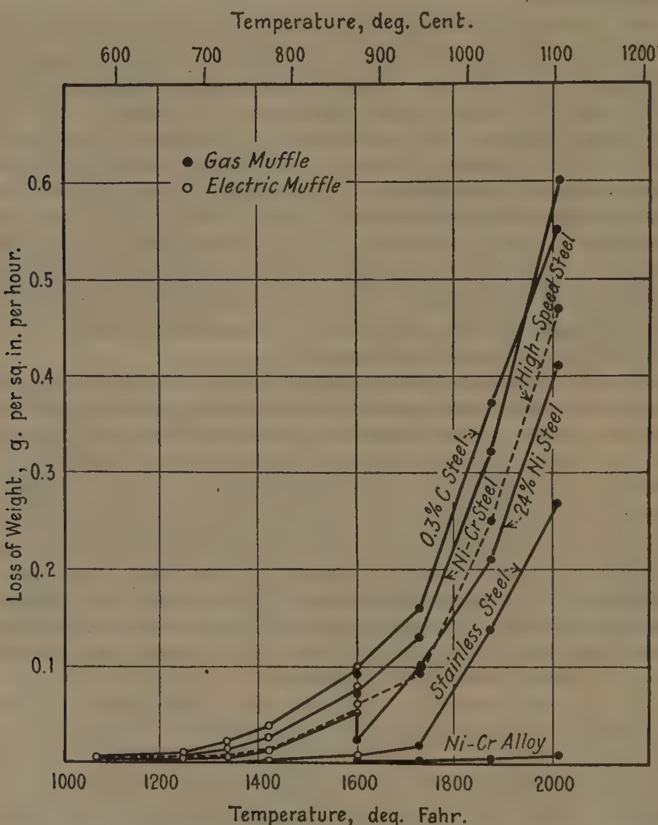

Fig. 17 Comparison of the Rate of Scaling of Various Steels and a Cast Nickel-Chromium Alloy at Elevated Temperatures [Dickenson (172)]

Composition and treatment of the steels are given in Table 2. The nickel-chromium alloy contains 0.54 per cent C; 0.73 per cent Si; 0.10 per cent Mn; 69.9 per cent Ni; 15.5 per cent Cr. Tests carried out in either gas-fired or electric furnaces as indicated.

That the tungsten steels scale comparatively little up to temperatures of about 1560 deg. fahr. (850 deg. cent.), but beyond that they are liable to scale very considerably.

43 Results obtained by Dickenson(172) are partially summarized in Fig. 17 and likewise show the superiority of the stainless steel in comparison with carbon, ordinary nickel-chromium, high-

nickel and high-speed steels. However, an iron alloy containing high proportions of nickel and chromium is much superior to stainless steel. Dickenson also pointed out a marked difference in the formation of the scale produced in steels containing nickel and those without appreciable quantities of this element. The former all showed the characteristic "double scale" described by Stead(115) in which the lower layer contains a large proportion of metal.

TREND OF DEVELOPMENT OF "HEAT-RESISTING" ALLOYS

44 The foregoing sketch of available data, while incomplete, shows clearly that irons and steels have serious limitations for high-temperature service. In the case of cast and malleable irons and semi-steels this is further substantiated by the recommendations of one manufacturer of power-plant equipment: [1]

> Cast iron is recommended for a total temperature of 500 deg. fahr. (260 deg. cent.) when the pressure does not exceed 25 lb. gage and for 100 deg. fahr. (40 deg. cent.) when the gage pressure is not greater than 75 lb. Malleable iron and semi-steels are recommended for temperatures up to 500 deg. fahr. (260 deg. cent.) when the pressure does not exceed 200 lb. gage.; at higher temperatures and pressures cast or forged steels are recommended.

45 As already indicated, the composition and treatment of steels may be varied to meet specific requirements at temperatures above 500 deg. fahr. (260 deg. cent.). However, it is also evident from the described data that all steels "weaken" with considerable rise in temperature. While this weakening may be delayed by additions of relatively large proportions of such elements as chromium, the resulting product retains, in this respect at least, the characteristic properties of the iron which forms the largest part of the alloy, and steels would not generally be expected to stand up under fairly high stress at temperatures exceeding about 1200 deg. fahr. (650 deg. cent.). Thus for service at higher temperatures it would appear necessary to seek alloys in which the iron plays a secondary rôle instead of forming the largest part of the product.

46 In this connection it will be of interest to cite briefly the result of experiences encountered with the direct synthetic ammonia process as summarized by Vanick.[2]

> The fixation of nitrogen by the synthetic ammonia processes requires metal tubes and containers capable of conveying or holding corrosive gas at high temperatures and high pressures. A gas-proof, forgeable, machinable, corrosive-resisting, and heat-resisting material is required. The Fixed Nitrogen Research Laboratory of the Depart-

[1] Data obtained from correspondence.

[2] This summary was prepared at request of the authors by Mr. J. S. Vanick, formerly of the Fixed Nitrogen Research Laboratory, U. S. Department of Agriculture.

ment of Agriculture has completed an extensive investigation of plain carbon and alloy steels that might be applied to this service.

Of the commercially obtainable alloy steels in the 0.30 to 0.40 per cent carbon range, those containing chromium or tungsten or both elements showed a superior resistance to deterioration over other alloy steels. A series of chromium-vanadium steels containing up to 21 per cent chromium showed an improvement in resistance to deterioration which increased with the percentage of chromium in the alloy. For the purposes which the laboratory had in view, a 2½ per cent chromium steel was selected as suitable for the type of service demanded of a metal in the direct synthetic ammonia process. These results apply to the special conditions of test representing 100 atmospheres pressure of synthetic mixtures at 930 deg. fahr. (500 deg. cent.). No significant improvement in tensile properties at elevated temperatures was obtained with these steels.

Long before this position in the development of materials had insured temporary security, new achievements in the same field of chemistry were requiring better materials to hold the same gases at super-pressures and perhaps super-temperatures, with respect to earlier processes. Pressures which can be held at ordinary temperatures exert stresses at the operating, elevated temperatures that would correspond to or exceed the limit of elasticity for practically all steels and ferrous alloys. So important has high-temperature strength become for this service that the property of resistance to corrosion which had thus far been associated with it, may be subordinated; partly because most of the elements which possess the property of strength or coherence at elevated temperatures also possess, in this case, a superior resistance to corrosion or deterioration.

New investigations are leading into alloys of the inelastic type; alloys which possess a yield point at ordinary temperatures that closely approaches the ultimate strength. At elevated temperatures some plasticity would be expected which would not necessarily imply elasticity. At present these alloys are expensive, unmachinable, must be cast to shape with the difficulty that attends viscous fusions, and in the present state of development improve the high-temperature strength very slightly. For service as tubes or containers for gas under pressure, such defects as porosity and segregation delay their acceptance.

Work on these important alloys will clear many of the obstructions now encountered in their preparation and lead to new developments in the strength-at-high-temperature field.

47 Some of the features pointed out by Vanick are in substantial agreement with experiences encountered in France in the production of ammonia by the Claude process. The development of materials in this case is reported by Le Chatelier[1] as follows:

Mr. Claude had begun his experiments with a mild-steel tube under a pressure of a thousand atmospheres, water-cooled and heated internally by a helix through which was passed electric current. He [Professor Le Chatelier] had made the suggestion that as iron possessed considerable tensile strength at 750 deg. fahr. (400 deg. cent.) it would be sufficient for the purpose to plunge the tube into a bath of lead and thus reduce considerably the consumption of electric energy. From the first the experiment succeeded, and Mr. Claude was able to obtain from the outlet of his tube liquid ammonia, the problem being thus apparently solved. Unfortunately, after a run of six hours the plant exploded. . . . Experiments made under the same pressure of

[1] Discussion of report by Dickenson (172).

a thousand atmospheres and at a temperature of 1110 deg. fahr. (600 deg. cent.) on iron wires showed that the nitrogen had no action but that the hydrogen caused rapid alteration in the metal similar to that observed in the case of the tube referred to above. It was necessary, therefore, to find a metal which would resist the action of hydrogen. . . . Mr. Chevenard suggested in the first instance a steel having a composition similar to that of high-speed steel. . . . That steel underwent without difficulty a pressure of a thousand atmospheres at 1110 deg. fahr. (600 deg. cent.). It was no longer necessary to resort to internal heating: the heat evolved by the reaction was sufficient to a great extent to maintain a temperature of 1110 deg. fahr. (600 deg. cent.) at the point where the external cooling was not too great. Once again it was assumed that a final solution of the problem had been found but after a hundred hours of working the tube again exploded.

Mr. Chevenard then suggested the use of another steel having the following composition: Nickel, 60 per cent; iron, 25 per cent; chromium, 12 per cent; manganese, 2 per cent; carbon, 0.5 per cent. The tubes from that metal yielded excellent service and tubes of larger dimensions were therefore ordered capable of being employed in normal manufacture. It was found impossible to forge the large ingots and necessary to employ the metal as cast. The tubes behaved, nevertheless, very well in service and some of them had already been in use for five thousand hours.

48 Quite a number of "heat-resisting" alloys are now produced commercially in this country, and others have been shown to have desirable properties at very high temperatures. Many of these cannot be called steels or even ferrous alloys and in some cases are practically free from iron, but they will be briefly considered to show the present trend in development of metals well adapted for various types of high-temperature service under which ordinary steels fail to meet at least some requirements.

49 In Table 4, in which is given a partial but representative list of such metals, an arbitrary division has been made depending upon the iron content. The first group comprises steels with relatively large proportions of special elements but containing over 50 per cent of iron; the third group consists of alloys practically free from this element or with proportions up to 4 or 8 per cent in the nature of an impurity; the second is an intermediate group with from about 10 to 50 per cent of iron. Commercial non-ferrous alloys for service at steam temperatures are not considered.

50 Except in two cases the steels of Group I, Table 4, are low in carbon and are based upon chromium additions of from 11 to 30 per cent. However, they may also contain varying amounts of one or more of the elements, nickel, tungsten, silicon, and copper, and such additions may be expected to modify the properties obtained. Steels similar to No. I(h) have exceptionally good resistance to oxidation at high temperatures and have been discussed by Johnson(155).

TABLE 4 SOME "HEAT-RESISTANT" ALLOYS NOW IN USE OR PROPOSED FOR HIGH-TEMPERATURE SERVICE OF VARIOUS TYPES[1]

(Approximate type compositions unless otherwise specified)

Manufacturer or name		C	Mn	Ni	Cr	W	Co	Si	Fe	Cu	Remarks
		Chemical Composition, Per Cent									
GROUP I STEELS											
I(a) Stainless iron.		Under 0.12	....	....	11 to 14	....	..	....	....	..	
(b) Stainless steel.		0.2 to 0.4	....	....	11 to 14	....	..	....	....	..	
(c) Manufacturer	A	Under 0.8	....	....	18 to 20	....	..	....	....	..	
(d) "	B	0.5	....	....	25	....	..	0.4	....	..	
(e) "	C	0.3	....	....	28 to 30	....	..	....	....	..	
(f) "	D	0.30	0.30	0.30	19.0	3.0	..	2.75	....	..	P, 0.02 S, 0.02 Actual analysis
(g) "	E	0.3	....	....	20	....	..	....	....	1	
(h) "	F	0.4	0.23	15.3	16.0	....	..	3.22	....	..	Actual analysis
(j) "	G	1.75 to 2.00	....	5	20	....	..	....	....	..	
(k) "	H	0.4	....	21	7.5	....	..	1.5	....	..	
GROUP II NICKEL-CHROMIUM-IRON-BASE ALLOYS											
II(a) French patent 469,929.		0.3 to 1.0	1 to 5	50 to 80	8 to 25	0.5 to 8.0	..	....	....	..	See Footnote 2
(b) French alloy "ATG."		....	....	60	10	4	..	....	Remainder	..	See text and Fig. 18
(c) French alloy.		0.5	2	60	12	....	..	....	25	..	See text
(d) Manufacturer	J	....	....	62	11	....	..	....	25	..	
(e) Manufacturer J, K, L, M, N.		0.5	2	68	17	....	..	1	10.75	..	Mo, 0.75
(f) Manufacturer	G	0.2 to 1.25	....	35	15	....	..	....	47	..	
GROUP III ALLOYS FREE FROM OR LOW IN IRON											
III(a) Manufacturer	J	....	....	80	20	....	..	....	....	..	
(b) "	J	....	3	94	....	....	..	1	....	..	Al, 2
(c) "	O	....	....	80	11	....	..	....	6	..	
(d) "	J	0.2	1.5	83.5	13.5	....	..	0.3	0.5	..	
(e) German alloy (Tammann).		....	....	....	25	....	75	....	....	..	See text
(f) German alloy (Tammann).		....	....	....	30	....	70	....	....	..	See text
(g) Aluminum-nickel		Under 0.15	0.3	97	....	....	..	0.3	0.5	..	Al, 1.8
(h) Manganese-nickel		0.2	3	95+	....	....	..	....	1	..	
(j) "A" nickel.		0.15	0.2	99	....	....	..	....	Remainder	..	

[1] Copper-nickel and other non-ferrous alloys especially adapted for service at steam temperatures are not considered.

[2] Tungsten may be partially replaced by molybdenum.

51 The alloys in Group II are primarily nickel-chromium-iron alloys with or without additions of tungsten, manganese, silicon, and molybdenum, and in the majority of cases are quite close to or within the limits given under No. II(a), Table 4. They have been used to advantage for a variety of purposes including furnace parts, containers employed in the heat treatment of metals, etc. One of these, as has already been indicated, has been used in France for tubes employed in production of ammonia by the Claude process. The mechanical properties of two of the alloys in this group are shown in Fig. 18 in comparison

with heat-treated stainless steel and a chromium-molybdenum structural steel.

52 In addition to alloys of nickel with chromium, manganese, aluminum and silicon, Group No. III contains two cobalt-chromium combinations, mentioned by Tammann,[1] which are reported

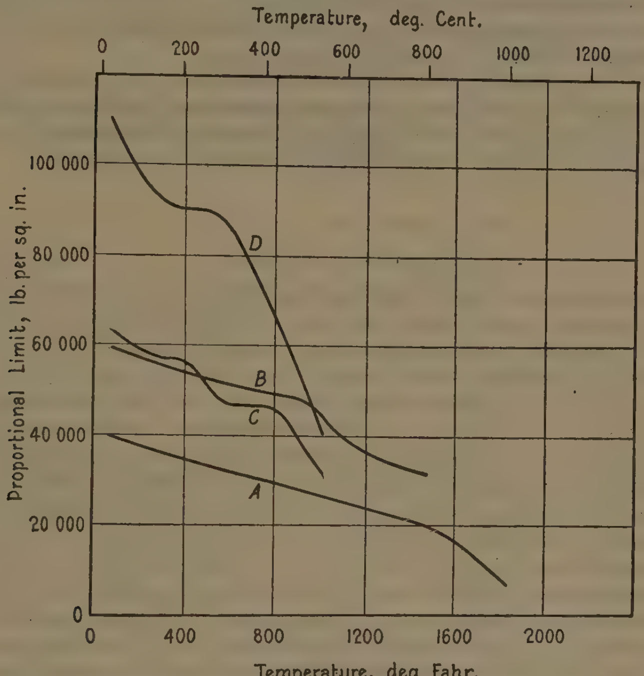

FIG. 18 EFFECT OF TEMPERATURE ON THE PROPORTIONAL LIMIT OF CHROMIUM-MOLYBDENUM AND STAINLESS STEELS AND TWO NICKEL-CHROMIUM-IRON ALLOYS (COLLECTED FROM VARIOUS SOURCES)

A—Nickel-chromium-iron alloy containing approximately 0.8 per cent C; 35 per cent Ni; 15 per cent Cr. Tested as cast. Data obtained by the authors.
B—Nickel-chromium-iron-tungsten alloy containing about 60 per cent Ni; 10 per cent Cr; 4 per cent W; 25 per cent Fe [Guillet(100)].
C—Steel containing 0.27 per cent C; 0.99 per cent Cr; 0.41 per cent Mo. Oil quenched from 1550 deg. fahr. (845 deg. cent.); tempered at 1110 deg. fahr. (600 deg. cent.). [French and Tucker(196)].
D—Stainless steel containing 0.31 per cent C; 12.75 per cent Cr. Oil quenched from 1750 deg. fahr. (955 deg. cent.); tempered at 1250 deg. fahr. (675 deg. cent.) [French(180)].
Note the rapid decrease in the proportional limit of the steels above 800 deg. fahr. (425 deg. cent.) as compared with the two nickel-chromium-iron alloys.

to have very desirable mechanical properties in the neighborhood of 1300 to 1500 deg. fahr. (705 to 815 deg. cent.). Alloy No. III(*e*), Table 4, showed an elastic limit of about 40,000 lb. per sq. in. at 1470 deg. fahr. (800 deg. cent.) and No. III(*f*) about

[1] Refer to discussion of Dickenson's report (172).

65,000 lb. per sq. in. at 1330 deg. fahr. (720 deg. cent.). Nickel, or alloys containing very large proportions of this element, as shown in Group III, have excellent resistance to oxidation at temperatures up to 1830 deg. fahr. (1000 deg. cent.) in atmospheres relatively free from sulphur but not high strength when considering sustained loads. Some, as in the case of Nos. III(*a*), (*b*) and (*c*) have special electrical properties which make them particularly useful.

53 In conclusion, mention should be made of the possible developments in the use of coated metals for high-temperature service. Aluminum (calorizing) and chromium (chromizing) coatings appear to offer promising developments in this field.

No. 1926 *d*

AVAILABLE DATA ON THE PROPERTIES OF NON-FERROUS METALS AND ALLOYS AT VARIOUS TEMPERATURES[1]

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Non-Member

and

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Member of the Society

The properties of non-ferrous materials at high temperatures have received far less attention than has been given ferrous materials, and even present interest is less than their importance warrants. The purpose of the paper is not to present all the work that has been done in this field but to set forth typical properties of various non-ferrous materials. Annealed electrolytic copper, for example, falls off rapidly in strength with increasing temperatures, while the elongation decreases somewhat slowly up to 400 *or* 500 *deg. fahr., and then very rapidly to a minimum. In the rolled condition, electrolytic copper shows a slightly greater decrease in strength at lower temperatures. But this applies to tests in air. When copper is tested in an atmosphere of carbon dioxide, both strength and elongation remain practically unchanged until a temperature of about* 950 *deg. fahr. is reached. Similar phenomena indicate how readily the properties of copper at elevated temperatures are influenced by other factors than temperature, such as the presence of impurities, oxidizing or non-oxidizing conditions, degree of annealing, and so forth. Similar decreases in value with high temperature are observed for hardness of copper. As a rule similar effects are also observable for copper-tin, phosphor-bronze, gun metal, and other alloys, but the properties vary largely with the composition of any given type of alloy. Copper-nickel alloys in particular seem to possess very desirable properties for use at elevated temperatures.*

[1] For discussion and closure see pp. 489-534.

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Presented at a joint session of The American Society of Mechanical Engineers and the American Society for Testing Materials, at Cleveland, Ohio, May 29, 1924, during the A.S.M.E. Spring Meeting.

The paper is a thorough and authoritative digest of existing information embracing almost the entire range of non-ferrous metals and alloys used on a large scale industrially.

Introduction

THE physical properties of the non-ferrous metals at elevated temperatures have attracted the attention of relatively few investigators. While considerable study has been given to the properties of iron and steel at elevated temperatures, non-ferrous materials have received but little attention until very recently. Moreover, the present attention given is not in keeping with their importance.

2 One of the earliest investigations of non-ferrous alloys at elevated temperatures was carried out in 1877 by the British Admiralty at the Portsmouth Dockyard. The test bars were first heated in an oil bath and then transferred as quickly as possible to the testing machine and broken. Even this very crude arrangement gave remarkably accurate results. In 1890, Martens reported, in connection with a series of tests on iron and steel at elevated temperatures, the results of some tests on copper. This seemingly represents the beginning of a series of tests by Rudeloff, Stribeck, Unwin, Charpy, Bach, and Le Chatelier. Their investigations of non-ferrous metals were confined mainly to copper, although Rudeloff made a rather extensive investigation of high-manganese bronzes and Delta metal. The results of these were given quite largely in Rudeloff's report on The Influence of Increased Temperatures on the Mechanical Qualities of Metals.

3 In more recent years the properties of non-ferrous metals at elevated temperatures have been contributed to very largely by Huntington, Bengough, Hanson, Edwards, Rosenhain, Lea, Doernickel and Trockels in Europe, and by Bregowsky and Spring, Jeffries and Sykes, and very recently by Malcolm in this country. Of this work, both in this country and abroad, a considerable proportion has had as its primary purpose the determination of certain scientific facts rather than physical properties, although the latter have not been ignored. As a result many of the tests have been carried out under very widely varying conditions as to the nature of the material, that is, composition, degree of working or of annealing, size of specimens, method of heating, temperature measurement, nature of atmosphere in which the tests were made, and method and rate of loading. The work of Lea and of the National Physical Laboratory of England on aluminum alloys stands forth as that of well organized and systematic tests, the primary purpose of which is the determination of the physical properties. Similarly, in this country, the work of Bregowsky and

Spring, and the more recent investigations of Malcolm, represent work which has had as its primary purpose the determination of the physical properties.

4 The purpose of this paper is not to present all of the work which has been done on the physical properties of non-ferrous metals at elevated temperatures, but to present as far as possible

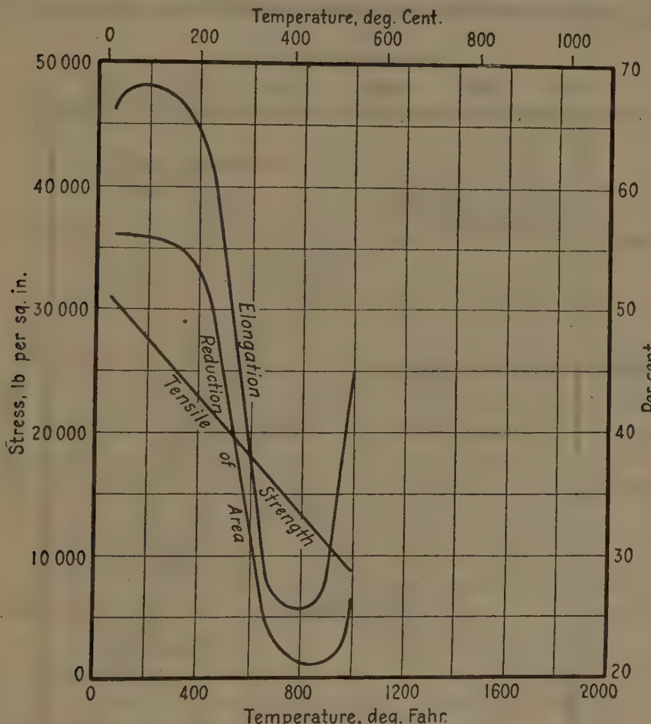


Fig. 1 Effect of Temperature on Tensile Properties of Electrolytic Copper, According to Huntington (85)

Annealed at 1112 deg. fahr. (600 deg. cent.)

typical properties for the various metals which have been investigated. In many cases alloys have been investigated by only one individual, or investigators have used materials so widely different in composition as to represent in reality two entirely different alloys. In view of this condition and the limited development of the art of the testing of non-ferrous alloys at elevated temperatures, it has seemed unwise to include values from any other than the original sources. In one case, where the original author has drawn his curves to emphasize certain inflections or critical

points, the values have been replotted and the curves drawn as average curves. (Figs. 2, 13, and 14.)

5 No attempt has been made to consider the methods of testing, that being considered beyond the scope of this paper.

Copper

6 Copper, though somewhat limited in its field of application at high temperatures, has apparently invited the attention of a

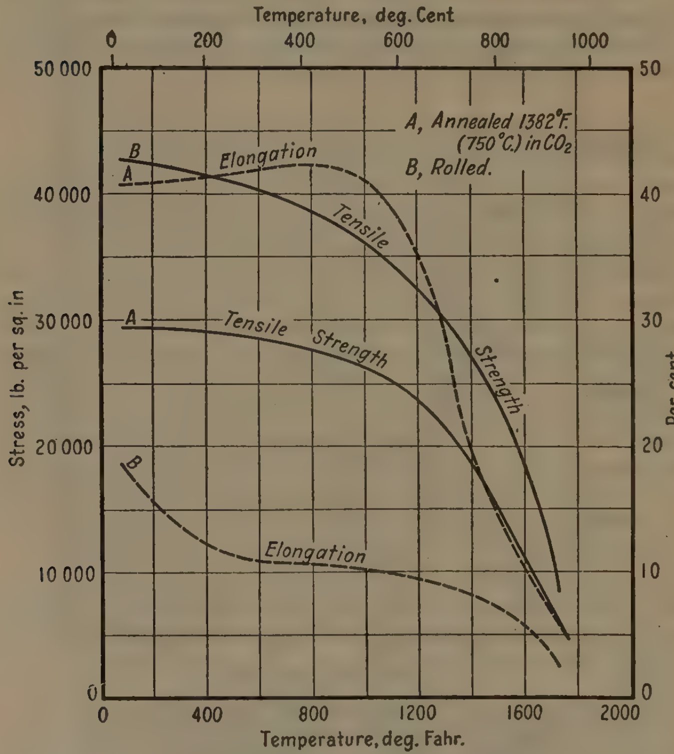


Fig. 2 Effect of Temperature on Tensile Properties of Electrolytic Copper, According to Bengough and Hanson (**96**)

Broken in carbon dioxide

number of investigators. Rudeloff (1893-1898), Unwin (1899), Le Chatelier (1901), and Stribeck (1903) made tests on copper at elevated temperatures. The results of these tests were presented by Rudeloff (**27, 64**)[1] in his official report, Influence of Increased Temperatures on the Mechanical Qualities of Metals, made to

[1] The bold-face numbers in parentheses refer to references in the bibliography on pages 477-487.

the International Association for Testing Materials in 1909. The results presented, while extending over narrower ranges of temperature than used by later investigators, showed in addition to the actual temperature effects, the influence of composition, of cold working, and of the rate of loading. Rudeloff found the tensile strength of cold-worked copper superior to that of annealed copper at the lower temperatures. At higher temperatures this difference disappeared. Tin in copper was found not only to aid

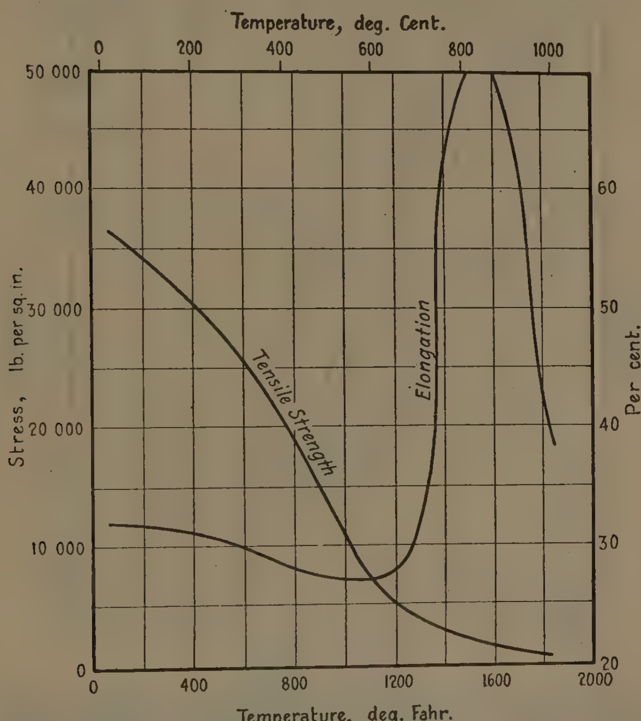


FIG. 3 EFFECT OF TEMPERATURE ON TENSILE PROPERTIES OF COPPER, ACCORDING TO BENGOUGH (82)

Chemical composition, per cent: Cu, 99.84; As, 0.05; Mn, 0.08; Fe, trace; S, 0.005

in retention of the tensile strength but to increase in a very marked manner the tensile strength at the higher temperatures. Stribeck (48) found lower tensile values but used a slower rate of loading. These tendencies have all been confirmed by the later investigators, though the degree to which these factors influence properties or the temperatures at which the effects are most marked are not always in agreement with the earlier results.

7 In the period following the presentation of Rudeloff's report and up to the present time, investigations of copper at elevated temperatures have been made by Huntington, Bengough and Hanson, Jeffries, Hughes, Doernickel and Trockels, and others. While the methods of testing used by both Huntington and Bengough have been subjected to considerable criticism, particularly

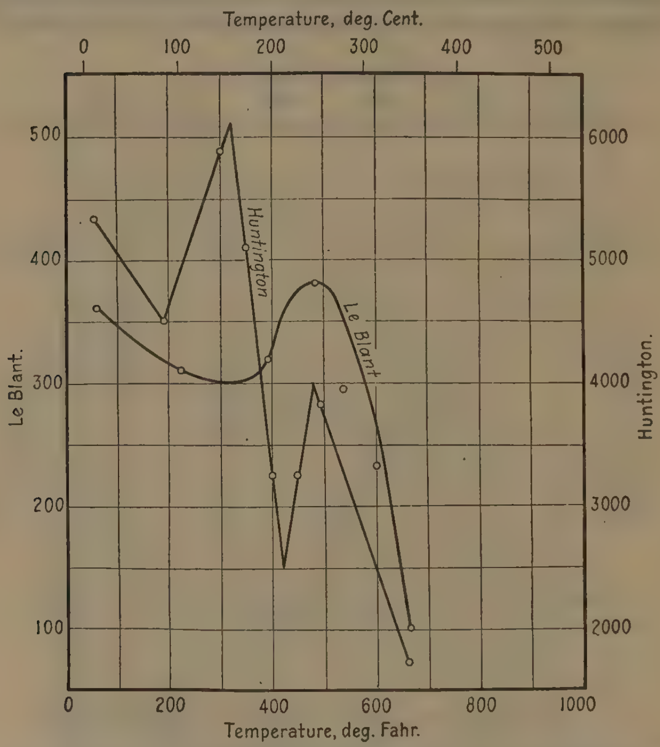


Fig. 4 Effect of Temperature on Resistance to Alternate Bending of Copper, According to Huntington (107) and Le Blant (34)

the method of heating used by Huntington and the method of loading employed by Bengough, their results are of decided interest.

8 The tensile properties of electrolytic copper as determined by Huntington (85) and by Bengough and Hanson (96) are shown in Figs. 1 and 2. In Fig. 3 Bengough's (82) tensile properties of a cold-rolled copper carrying 0.05 per cent arsenic are shown. Annealed electrolytic copper, according to Bengough and Hanson, shows when tested in an atmosphere of carbon dioxide quite different properties than when tested in air. In air, tensile strength

falls off rapidly with increasing temperatures, while the elongation decreases somewhat slowly up to 400 to 500 deg. fahr. (205 to 260 deg. cent.) and then very rapidly to a minimum. Tested in an atmosphere of carbon dioxide, both tensile strength and elongation are retained practically undiminished up to temperatures of 950 to 1050 deg. fahr. (510 to 565 deg. cent.). Above these

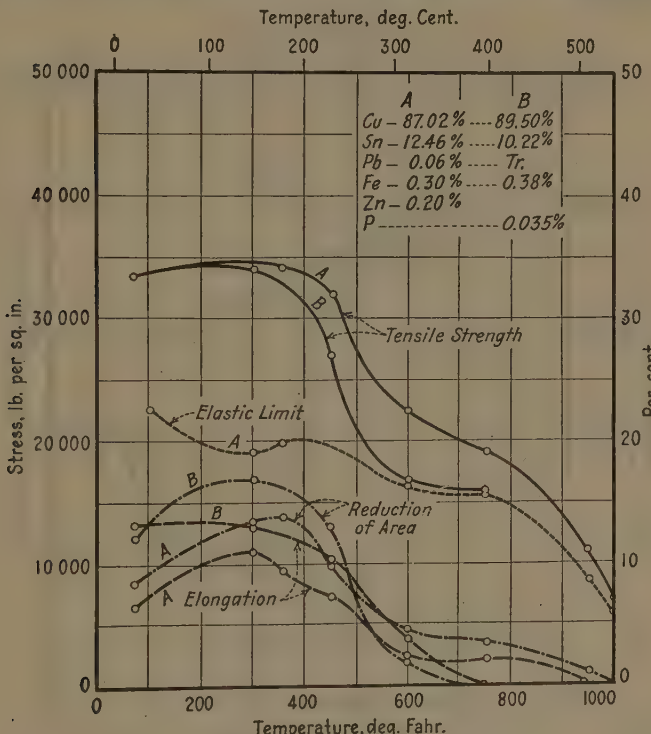


FIG. 5 EFFECT OF TEMPERATURE ON TENSILE PROPERTIES OF COPPER-TIN BRONZE, ACCORDING TO BREGOWSKY AND SPRING (83)

Chemical composition as indicated

temperatures both tensile strength and elongation decrease rapidly. In the rolled condition, the electrolytic copper shows a slightly greater decrease in strength at low temperatures. The elongation decreases rapidly up to temperatures of 400 to 500 deg. fahr. (205 to 260 deg. cent.), remains unchanged up to 1100 to 1200 deg. fahr. (600 to 650 deg. cent.), followed by a rapid decrease. It will be noted that neither the rolled nor the annealed electrolytic copper shows a reversal in elongation if tested in carbon dioxide. Bengough, however, did obtain a reversal in elongation for cold-rolled copper tested in air (Fig. 3). In his

opinion, this marked difference in behavior of the tensile properties of copper at the higher temperatures was due to the influence of arsenic in the copper and the presence of the oxidizing atmosphere. The decrease in elongation between 400 and 500 deg. fahr. (205 to 260 deg. cent.) observed by Huntington and by earlier investigators, it was suggested, might be due to the influence of the annealing.

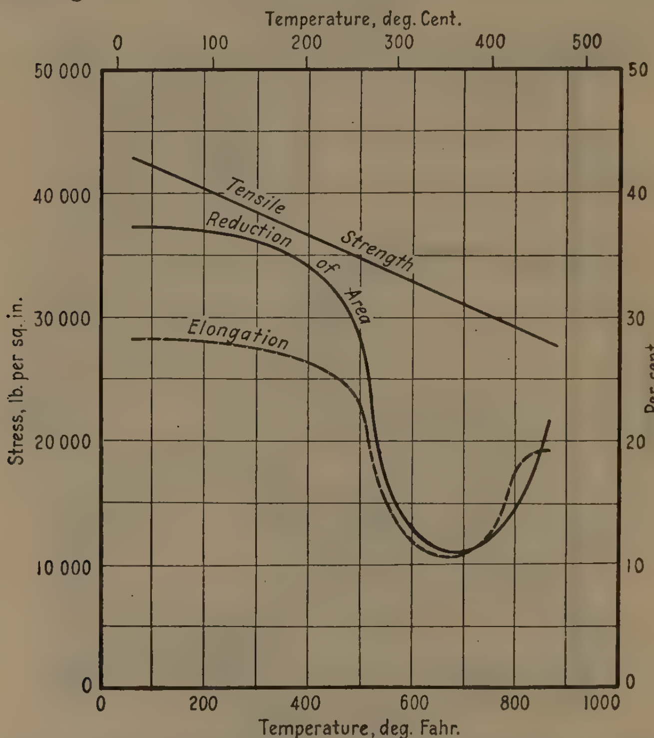


Fig. 6 Effect of Temperature on Tensile Properties of Copper-Tin Bronze, According to Huntington (85)

Chemical composition, per cent: Cu, 97.673; Sn, 2.408; Pb, 0.024

9 Jeffries' work (127) on the tensile properties of copper wire at temperatures above and below atmospheric temperature is not entirely in agreement with that of Bengough and Hanson. Jeffries used electrolytic copper and an atmosphere of argon for all tests above 400 deg. fahr. (205 deg. cent.). With annealed copper wire a decrease in elongation was observed at 400 to 500 deg. fahr. (205 to 260 deg. cent.) by Jeffries but no indication of a reversal at 800 deg. fahr. (425 deg. cent.), the elongation decreasing continuously. With cold-drawn electrolytic copper, Jeffries shows a

decrease in elongation followed by a very rapid increase at a temperature lying between 800 and 900 deg. fahr. (425 and 480 deg. cent.), a change similar to that observed by Huntington for annealed copper.

10 Although Jeffries' results were obtained on small wires and a faster rate of loading was used by him than by Bengough and

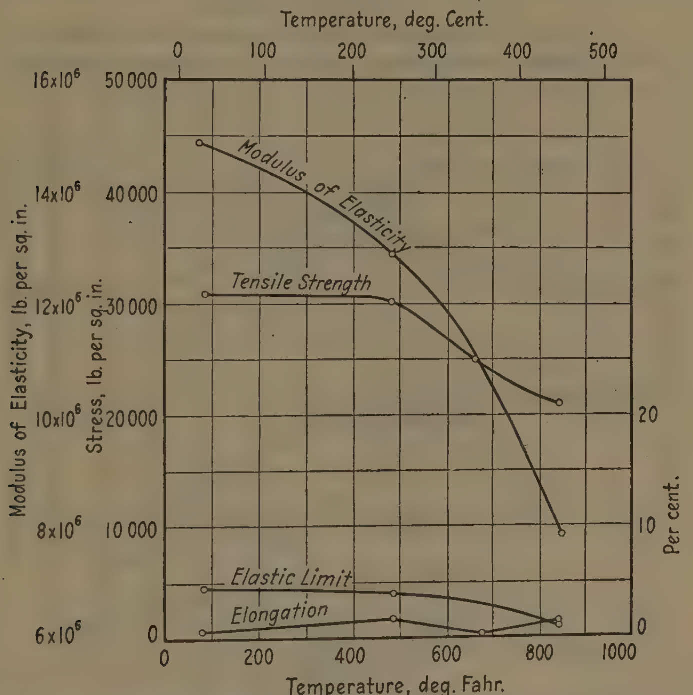


FIG. 7 EFFECT OF TEMPERATURE ON TENSILE PROPERTIES OF PHOSPHOR BRONZE, ACCORDING TO LEA (142)

Chemical composition, per cent: Cu, 85.38; Sn, 12.55; Zn, 1.01; P, 0.24; Fe, 0.02; Pb, 0.61; Ni, 0.11

Hanson, the regularity with which the reversal in elongation in the neighborhood of 800 deg. fahr. (425 deg. cent.) occurred on the cold-rolled copper, leaves the question of the behavior of cold-rolled electrolytic copper when tested as above somewhat in doubt. Either the results obtained by Bengough and Hanson and by Jeffries are open to question or factors other than the presence of oxygen or arsenic contribute to this difference.

11 The above variations in the tensile properties of copper at elevated temperatures have been pointed out primarily to show how very readily the tensile properties, particularly the elongation,

are influenced by the presence of factors other than the temperature alone. The presence of impurities, oxidizing or non-oxidizing conditions, cold working, the degree of annealing, the rate of loading, method of testing—all may have a very decided influence on the tensile properties, though usually to a greater degree on the elongation.

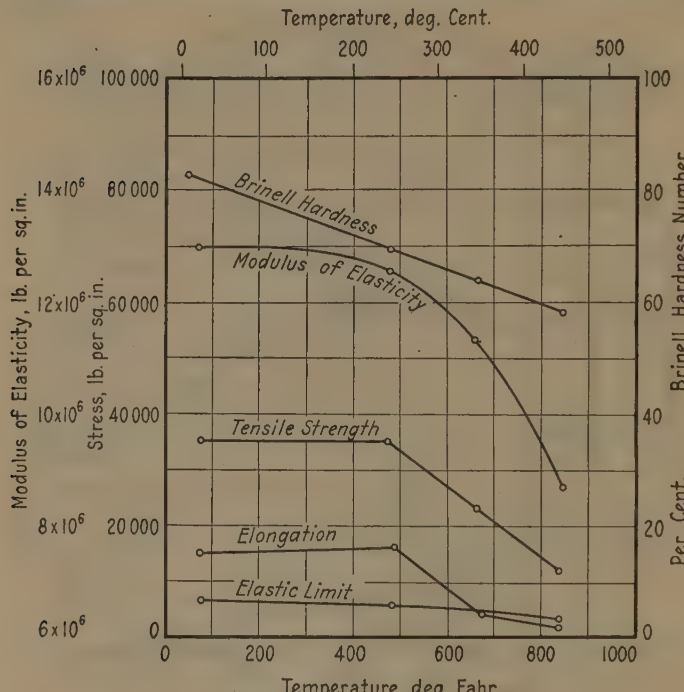


Fig. 8 Effect of Temperature on Tensile Properties and Brinell Hardness of Gun Metal, According to Lea (141, 142)

Chemical composition, per cent: Cu, 86.52; Sn, 10.2; Zn, 3.29; Pb, 0.12

12 For average conditions the changes in tensile properties of copper at elevated temperatures will conform in general to the changes noted by Huntington (Fig. 1). Rolled copper will show a tensile strength superior to that of annealed copper at the lower temperatures. Both tensile strength and elongation differences for rolled and annealed copper will disappear at higher temperatures, or even at lower temperatures if held sufficiently long for an annealing effect to occur.

13 Additional tests on copper at elevated temperatures which may properly be mentioned are hardness tests by Kürth (62) and Ludwik (112), low-temperature tests by Jeffries (127), alternate-stress

or bending tests by Huntington(107), modulus determinations by Iokibe and Sakai(154) and crushing tests by Doernickel and Trockels(149).

14 Kürth(62) has shown the hardness of copper to decrease gradually with increasing temperatures. Confirmatory results

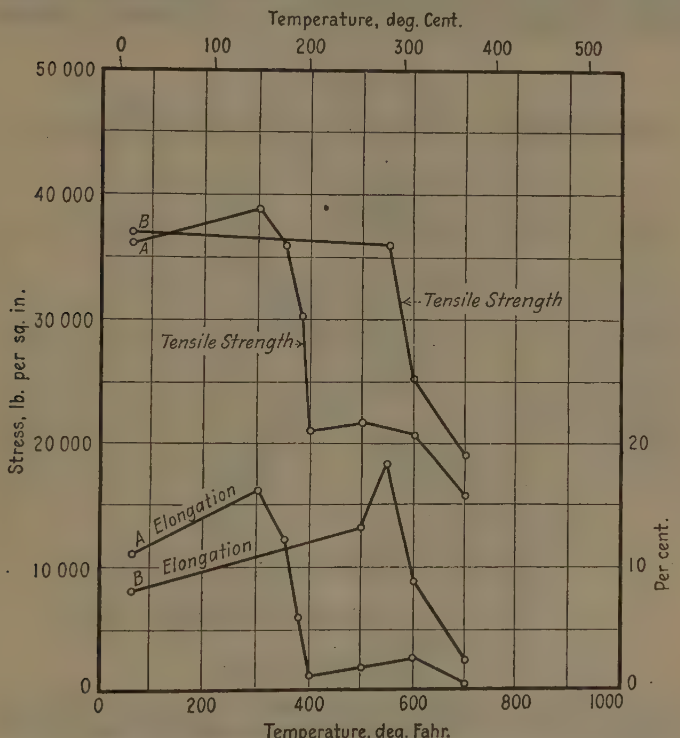


FIG. 9 EFFECT OF TEMPERATURE ON TENSILE PROPERTIES OF GUN METAL, ACCORDING TO DEWRANCE (98)

Chemical composition, per cent.: A. Cu, 88; Sn, 10; Zn, 2. B. Cu, 87.5, Sn, 10; Zn, 2; Pb, 0.5

have been obtained by Ludwik(112), who also investigated hardness of lead, zinc, aluminum, tin, antimony, cadmium, and bismuth. According to Ludwik the influence of the time of loading becomes very important at higher temperatures, a 15-second and a 300-second loading resulting in a 40 to 50 per cent difference in hardness values at 900 to 1000 deg. fahr. (480 to 540 deg. cent.). Huntington (Fig. 4) found the resistance to bending stresses decreases very little at temperatures up to 350 to 400 deg. fahr. (175 to 205 deg. cent.). Le Blant(34) in earlier investigations found

little decrease up to 500 deg. fahr. (260 deg. cent.). Jeffries in his tests of metals below atmospheric temperatures obtained a tensile strength of 80,000 lb. per sq. in. and 6 per cent elongation for cold-drawn copper at −301 deg. fahr. (−185 deg. cent.). Tensile strength and elongation of both cold-drawn and annealed copper are increased at temperatures below atmospheric. Doernickel and Trockels (Figs. 19 and 20) show the crushing strength

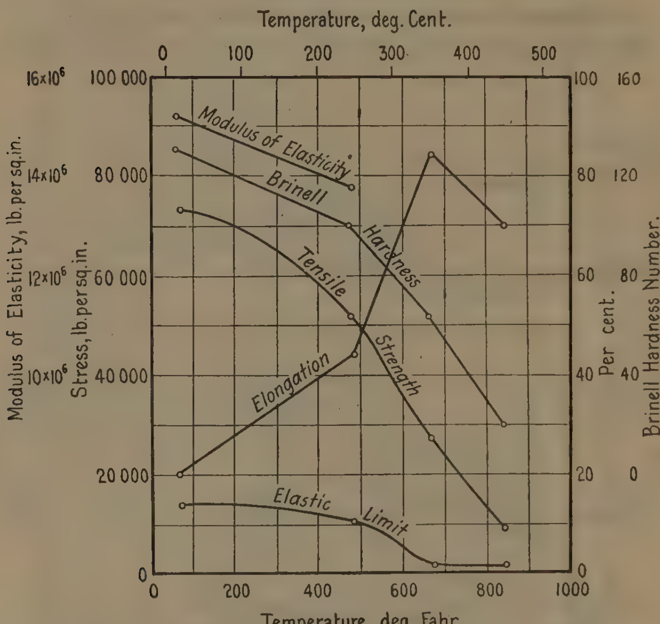


FIG. 10 EFFECT OF TEMPERATURE ON TENSILE PROPERTIES AND BRINELL HARDNESS OF CAST MANGANESE BRONZE, ACCORDING TO LEA (141, 142)

Chemical composition, per cent: Cu, 58.61; Zn, 36.9; Sn, 0.08; Mn, 1.88; Al, 1.01; Fe, 1.46; Pb, 0.06

to decrease gradually with temperature. Modulus of elasticity, according to Iokibe and Sakai, decreases according to a parabolic law.

COPPER-TIN BRONZES

15 The influence of temperature upon the properties of copper-tin bronzes (Fig. 5), according to Bregowsky and Spring (83), becomes most marked between 400 and 600 deg. fahr. (205 and 315 deg. cent.), all of the properties except the elastic limit decreasing very rapidly within that range of temperature. At temperatures below 300 to 350 deg. fahr. (150 to 175 deg. cent.) no appreciable difference in tensile strength is found for the bronzes

carrying 12 per cent of tin. At all higher temperatures the 12 per cent bronze is superior in strength to the bronze of a lower tin content. The influence of the additional tin is also shown in the effect on elongation and reduction of area at temperatures above 750 deg. fahr (400 deg. cent.). Variations in properties at elevated temperatures seemingly are dependent upon the presence of the delta constituent, the final decrease in elongation and tensile

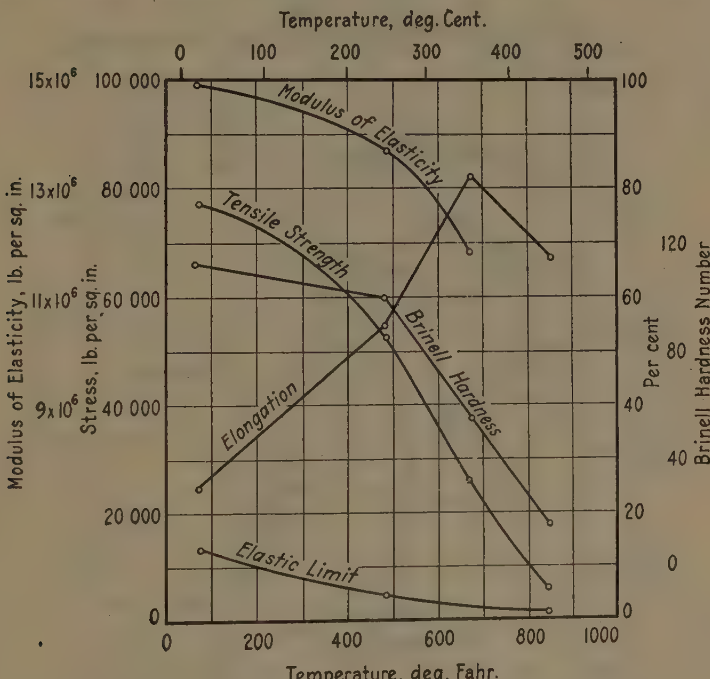


Fig. 11 Effect of Temperature on Tensile Properties and Brinell Hardness of Drawn Manganese Bronze, According to Lea(141, 142)

Chemical composition, per cent: Cu, 56.91; Zn, 40.28; Sn, 0.75; Mn, 0.19; Fe, 0.82; Pb, 0.66; Al, 0.18; Ni, 0.21

strength coming at the temperatures corresponding to the absorption of this constituent. In view of the much higher elastic-limit values obtained by Bregowsky and Spring on gun-metal bronzes than have been obtained by other investigators, the elastic limit values given in Fig. 5 may be higher than further investigation will prove to be the case.

16 Low-tin bronze (Fig. 6), according to Huntington(85), shows a gradual decrease in strength with increasing temperature. Elongation and reduction of area decrease rapidly at 500 deg. fahr.

(260 deg. cent.), reaching a minimum at 700 deg. fahr. 370 deg. cent.). The results obtained by Huntington are very similar to the results obtained in 1893 by Rudeloff(20, 64) with copper carrying 1.86 per cent tin. This copper-tin alloy shows a tensile strength superior to that of copper at all temperatures up to 900 deg. fahr. (480 deg. cent.).

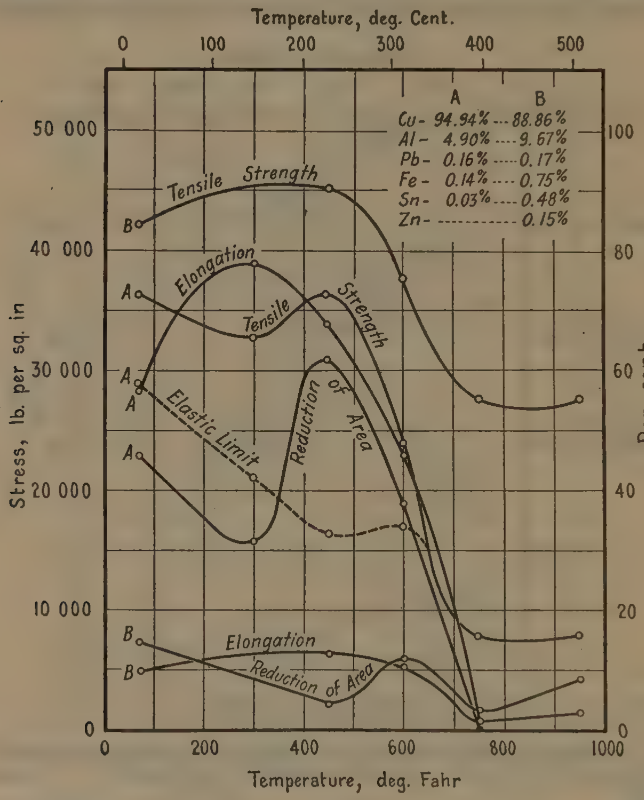


Fig. 12 Effect of Temperature on Tensile Properties of Aluminum Bronze, According to Bregowsky and Spring(83)

Chemical composition as indicated

Phosphor Bronze

17 Phosphor bronze (Fig. 7) retains its tensile strength undiminished up to temperatures of 450 to 500 deg. fahr. (230 to 260 deg. cent.). Above 500 deg. fahr. (260 deg. cent.) the strength and elastic properties decrease rapidly. According to Lea(142), the elastic limit becomes zero and the tensile strength decreases to less than 5000 lb. per sq. in. above 900 deg. fahr. (480 deg. cent.).

Gun Metal

18 Tensile properties of gun metal, according to Lea(141, 142) (Fig. 8), undergo no appreciable change up to 450 to 500 deg. fahr. (230 to 260 deg. cent.). Above 500 deg. fahr. (260 deg. cent.) the tensile strength and elongation decrease very readily. The elastic limit decreases more slowly but reaches a value of less than

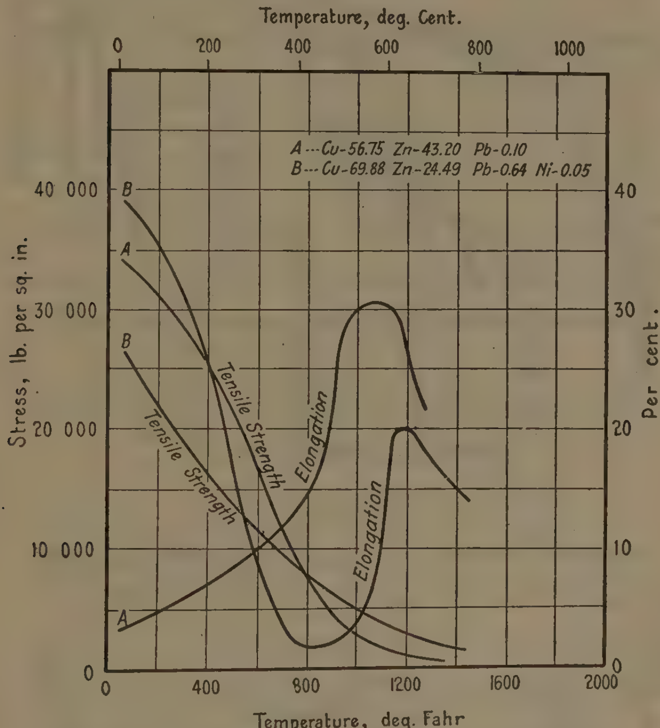
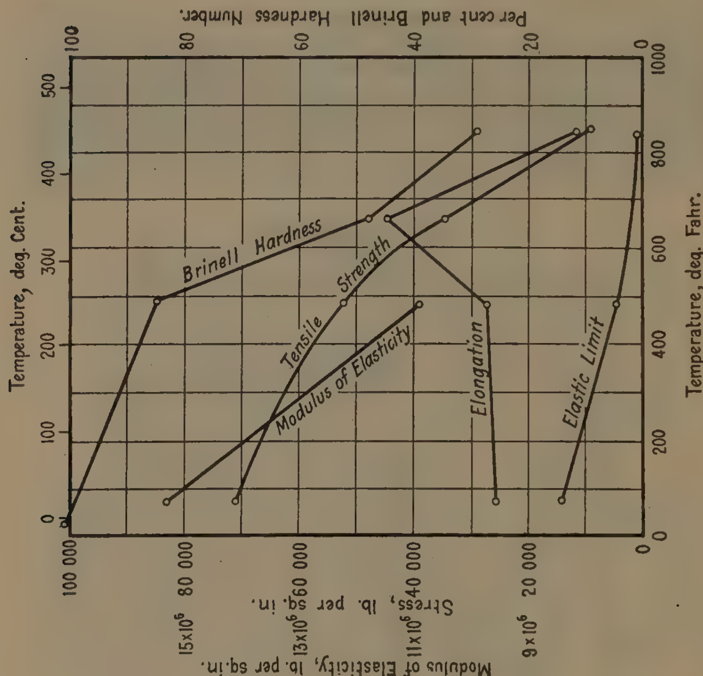


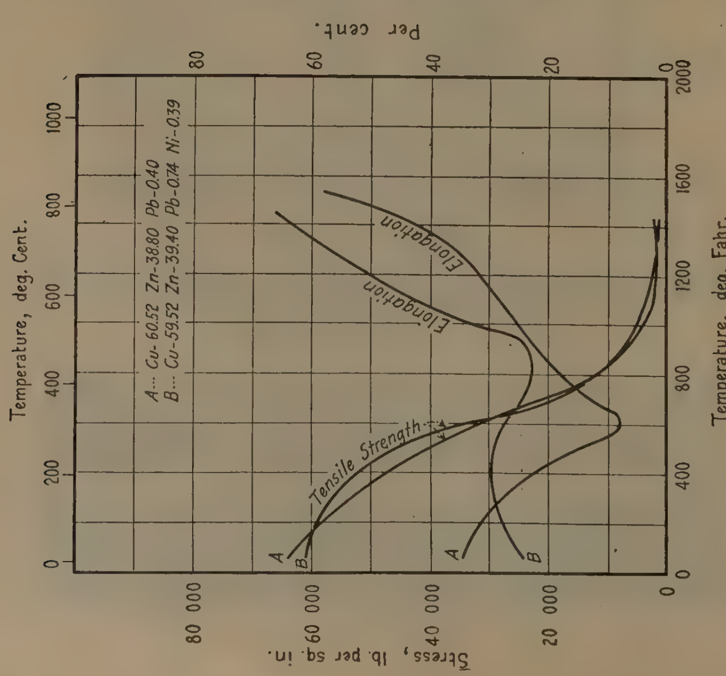
Fig. 13 Effect of Temperature on Tensile Properties of Cast Brass, According to Bengough(82)

Chemical composition as indicated

4000 lb. per sq. in. at 850 deg. fahr. (450 deg. cent.). Hardness decreases slowly with increase in temperature. The tensile properties of gun metal show a slight superiority over phosphor bronze for temperatures up to 450 to 500 deg. fahr. (230 to 260 deg. cent.).

19 Bregowsky and Spring(83), in an earlier investigation, obtained very similar results for U. S. Navy Gun Bronze, though slightly lower values are given for tensile strength and for elongation than are obtained by Lea. Values for permanent set differ





FIGS. 14 AND 15.
(See captions at foot of following page.)

very widely from those obtained by Lea. The initial value at normal temperature is given as 25,100 lb. per sq. in. as opposed to 7150 lb. per sq. in. by Lea. At 750 deg. fahr. (400 deg. cent.)

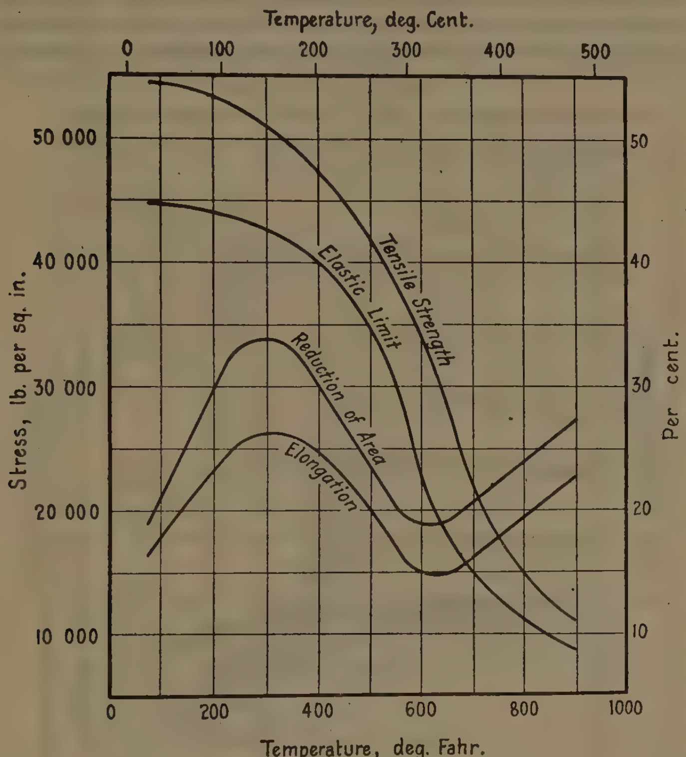


FIG. 16 EFFECT OF TEMPERATURE ON TENSILE PROPERTIES OF LEADED BRASS, ACCORDING TO CRANE CO. (216)

Chemical composition, per cent: Cu, 62.5; Zn, 35; Pb, 2.5

a value of 17,500 lb. per sq. in. is given as opposed to less than 5000 lb. per sq. in. by Lea. This difference indicates the necessity for a careful check.

FIG. 14 EFFECT OF TEMPERATURE ON TENSILE PROPERTIES OF ROLLED MUNTZ METAL, ACCORDING TO BENGOUGH (82)

Chemical composition as indicated

FIG. 15 EFFECT OF TEMPERATURE ON TENSILE PROPERTIES AND BRINELL HARDNESS OF MODIFIED MUNTZ METAL, ACCORDING TO LEA (141, 142)

Chemical composition, per cent: Cu, 58.96; Zn, 39.77; Sn, 0.56; Pb, 0.67

20 Influence of 0.5 per cent of lead has been considered by Dewrance(98). Gun metal without lead (Fig. 9) undergoes no decrease in its tensile properties up to 350 deg. fahr. (175 deg. cent.). Above that temperature tensile strength and elongation decrease rapidly up to 400 deg. fahr. (205 deg. cent.), remaining constant up to 600 deg. fahr. (315 deg. cent.), and again decreasing.

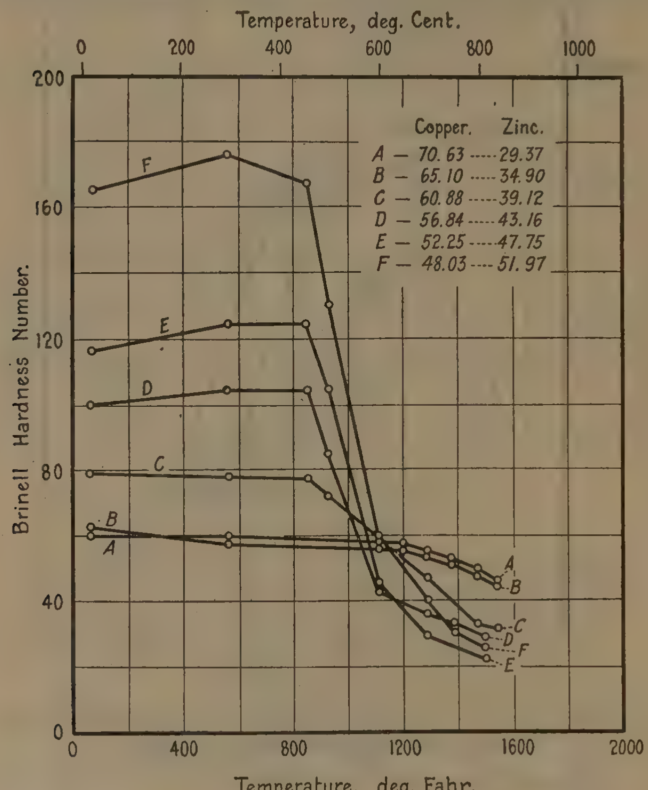


FIG. 17 EFFECT OF TEMPERATURE ON PLASTICITY OF COPPER-ZINC ALLOYS, ACCORDING TO EDWARDS AND HERBERT(151)

Chemical composition as indicated

With 0.5 per cent of lead present, the tensile strength and elongation remain undiminished up to a temperature of 550 deg. fahr. (290 deg. cent.). Above that temperature tensile strength and elongation decrease rapidly. The retention of the tensile strength at higher temperatures with the second alloy is attributed by Dewrance to the presence of the lead. As neither Bregowsky and Spring nor Lea reported tests at 550 deg. fahr. (290 deg. cent.), their results cannot be used directly to confirm or deny the results obtained by

Dewrance, although Lea used an alloy with 0.12 per cent of lead and Bregowsky and Spring an alloy with 0.39 per cent of lead. However, the fact that Lea, using an alloy with comparatively low lead content (0.12 per cent), shows the tensile strength unchanged up to a temperature of 482 deg. fahr. (250 deg. cent.) leaves the suggested superiority of the 0.5 per cent lead bronze somewhat in doubt. Dewrance gives no analysis of the metal

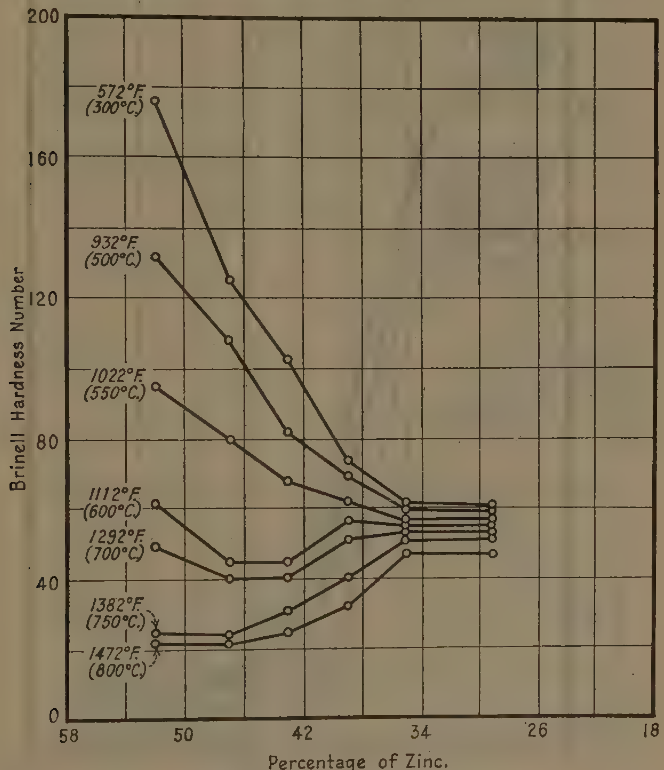


Fig. 18 Effect of Chemical Composition on Plasticity of Copper-Zinc Alloys at Various Temperatures, According to Edwards and Herbert (151)

actually tested, but it should be pointed out that his gun metal alloy without lead was made from copper with a purity of 99.55 per cent and carrying 0.08 per cent lead. The sharp increases in elongation noted by Dewrance were not observed by either Lea or Bregowsky and Spring.

Manganese Bronze

21 The tensile properties of cast and drawn manganese bronze (Figs. 10 and 11) change rapidly with increasing temperatures.

Tensile strength decreases to 60 to 70 per cent of its original value at 500 deg. fahr. (260 deg. cent.). Above that temperature the rate of decrease is more rapid. Elongation of both cast and drawn manganese bronze increases with increasing temperatures, reach-

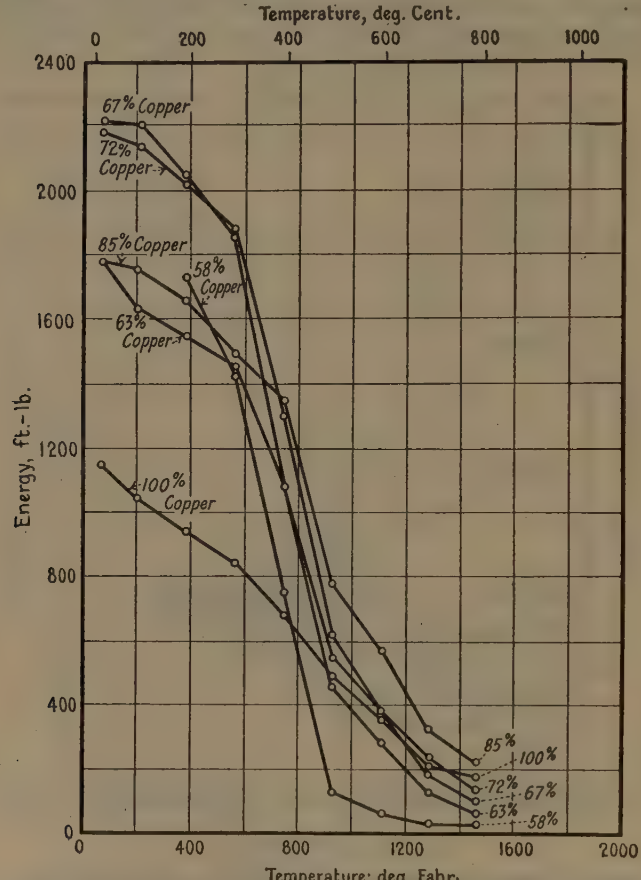


Fig. 19 Effect of Temperature on Resistance to Compression of Copper-Zinc Alloys, According to Doernickel and Trockels (149)

Chemical composition as indicated

ing a maximum at 650 to 700 deg. fahr. (345 to 370 deg. cent.), according to Lea(141, 142). Brinell hardness changes with tensile strength. Elastic limit decreases slightly up to 500 deg. fahr. (260 deg. cent.), then falls rapidly for cast metal. The elastic limit of drawn metal shows a continuous decrease.

22 Earlier results of Bregowsky and Spring(83) are in agreement with those of Lea as to tensile-strength changes and elonga-

tion except for the maximum noted by Bregowsky and Spring at 500 deg. fahr. (260 deg. cent.) and by Lea at 650 to 700 deg. fahr. (345 to 370 deg. cent.). As regards elastic limit the shape of the curve is the same for both, no appreciable drop occurring below 450 deg. fahr. (230 deg. cent.). Original values, however, are

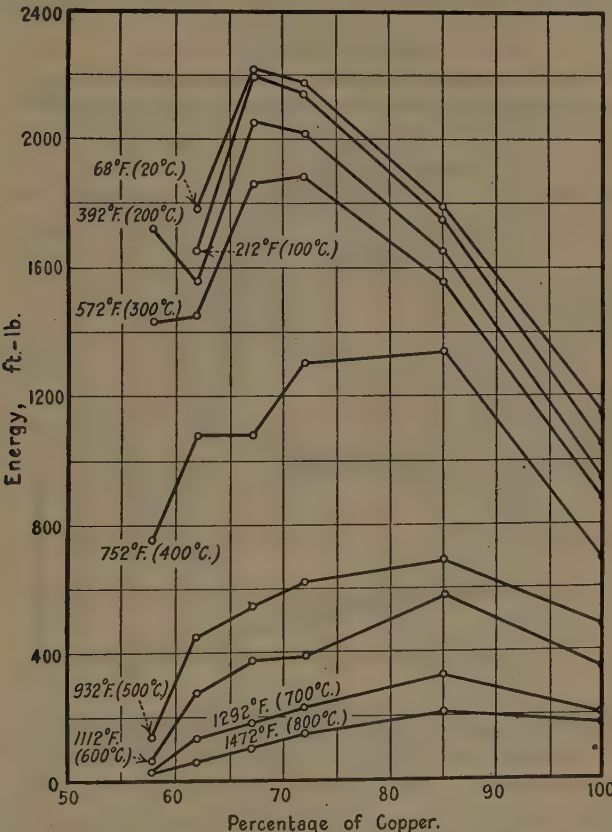


FIG. 20 EFFECT OF CHEMICAL COMPOSITION ON RESISTANCE TO COMPRESSION OF COPPER-ZINC ALLOYS AT VARIOUS TEMPERATURES, ACCORDING TO DOERNICKEL AND TROCKELS (149)

decidedly different, a difference which undoubtedly reverts to the method of determining elastic limit. In either case the elastic properties of manganese bronze practically disappear between 700 and 800 deg. fahr. (370 to 425 deg. cent.).

ALUMINUM BRONZE

23 Early tests by Rosenhain(57) on rolled aluminum bronze showed the superiority at all temperatures up to 800 to 900 deg.

fahr. (425 to 480 deg. cent.) of the 10 per cent aluminum bronze over the 5 per cent bronze. Elongation of the 10 per cent or two-constituent bronze increases up to 900 deg. fahr. (480 deg. cent.), while the 5 per cent bronze shows a continuous decrease in elongation. The influence of increasing the percentages of manganese

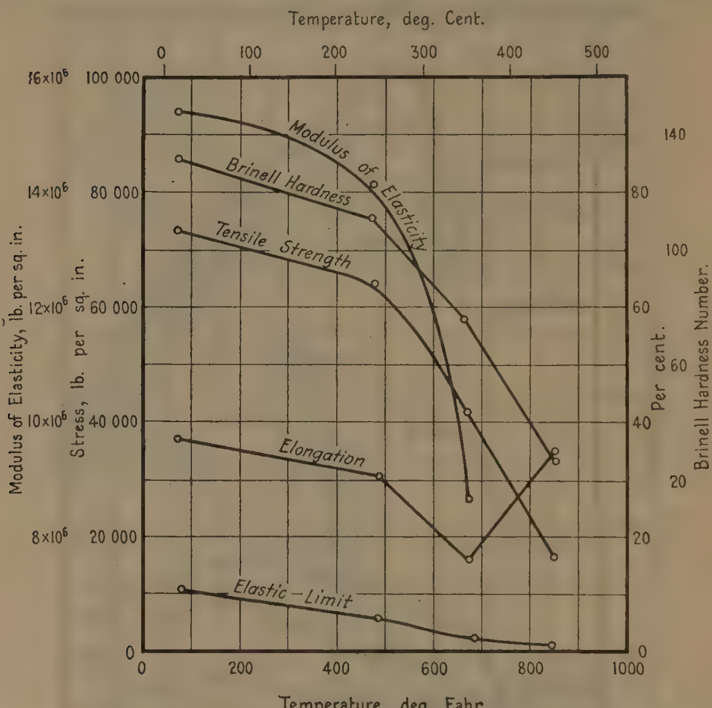


Fig. 21 Effect of Temperature on Tensile Properties and Brinell Hardness of Delta Metal, According to Lea(141, 142)

Chemical composition, per cent: Cu, 58.27; Zn, 39.05; Fe, 0.13; Sn, 0.06; Ni, 2.2; Mn, 0.14 Pb, 0.15

was also tried by Rosenhain but this did not result in material improvement in the properties of the bronze.

24 Bregowsky and Spring(83) (Fig. 12) found the cast 10 per cent aluminum bronze superior in tensile strength at all temperatures to the 5 per cent bronze. No appreciable decrease in strength occurs in the 10 per cent bronze below 600 deg. fahr. (315 deg. cent.). Above that temperature and up to 900 deg. fahr. (480 deg. cent.), the strength remains very nearly equal to the initial strength of the 5 per cent bronze.

25 Edwards and Herbert(151) have also carried out dynamic tests, referred to under brasses, on copper-aluminum alloys.

Brasses

26 Tensile properties of brasses at elevated temperatures have been investigated by Bengough and Hanson(96), Huntington(85) and Lea(141, 142). Edwards and Herbert(151) have investigated the plasticity of brasses by means of dynamic rather than tension

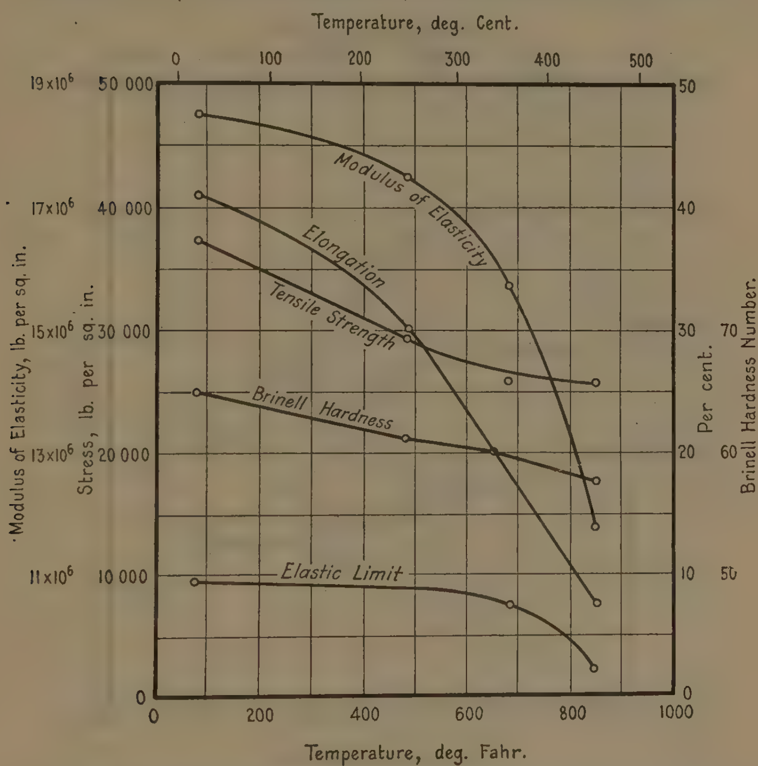


Fig. 22 Effect of Temperature on Tensile Properties and Brinell Hardness of Copper-Nickel Alloy, According to Lea(141, 142)

Chemical composition, per cent: Cu, 97.80; Ni, 2.0; Al, 0.20

tests, while Doernickel and Trockels(149) have investigated the compressibility.

27 The tensile strength of cast brasses (Fig. 13) decreases rapidly with increased temperatures, falling to values of less than 5000 lb. per sq. in. at 1000 deg. fahr. (540 deg. cent.), according to Bengough(82). The elongation of the 70-30 brass, single-constituent type, falls rapidly to a minimum value at 800 deg. fahr. (425 deg. cent.) and again increases to a maximum at 1200 deg. fahr. (650 deg. cent.). The elongation of the 60-40, or

two-constituent brass, increases somewhat slowly at first, but above 800 deg. fahr. (425 deg. cent.) it increases rapidly to a maximum with absorption of the alpha constituent.

28 The tensile properties of extruded brass vary with increasing temperatures in practically the same way as for cast brass, according to both Huntington and Bengough.

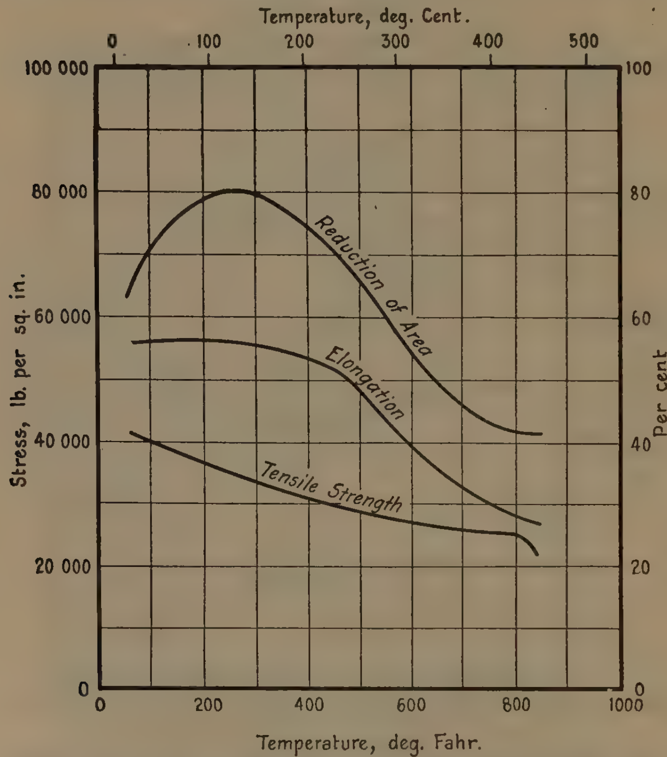


FIG. 23 EFFECT OF TEMPERATURE ON TENSILE PROPERTIES OF COPPER-NICKEL ALLOY, ACCORDING TO HUNTINGTON (85)

Chemical composition, per cent: Cu, 88; Ni, 12

29 Rolled brass of the Muntz-metal type (Fig. 14) undergoes a somewhat slower change in tensile strength up to temperatures of 400 to 500 deg. fahr. (205 to 260 deg. cent.). Above 500 deg. fahr. (260 deg. cent.) the tensile strength of Muntz metal decreases to less than 10,000 lb. per sq. in. between 800 and 900 deg. fahr. (425 to 480 deg. cent.), or slightly below, according to Lea (Fig. 15). Slight changes in composition, while apparently having little influence on tensile strength, may have a very marked effect on elongation as is shown in Fig. 14.

30 Rolled rod brass, 62.5 per cent copper and 2.5 per cent lead, according to the Crane Co. (Fig. 16), retains its tensile strength and elastic properties with very small changes up to a temperature of from 400 to 500 deg. fahr. (205 to 260 deg. cent.). At temperatures above 500 deg. fahr. (260 deg. cent.) tensile strength and elastic limit drop very rapidly. Compared to the

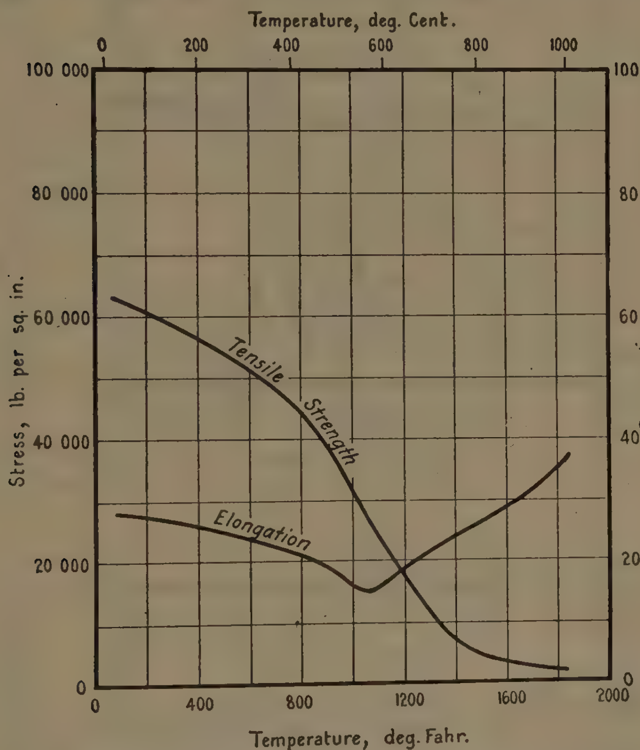


FIG. 24 EFFECT OF TEMPERATURE ON TENSILE PROPERTIES OF COPPER-NICKEL ALLOY, ACCORDING TO BENGOUGH (82)

Chemical composition, per cent: Cu, 79.99; Ni, 19.6; Fe and Mn, 0.41

B composition Muntz metal (Fig. 14) there appears to be very little difference in the properties of the two alloys.

31 Edwards and Herbert have investigated the plasticity of brasses at elevated temperatures, employing a dynamic test rather than the more commonly used static test. Plasticity is measured in terms of the indent made by the application of a 63-in-lb. blow. This is converted by the formula $\frac{7455}{d^3} = H$ to Brinell numbers, d being the diameter of the indent. In making the tests the

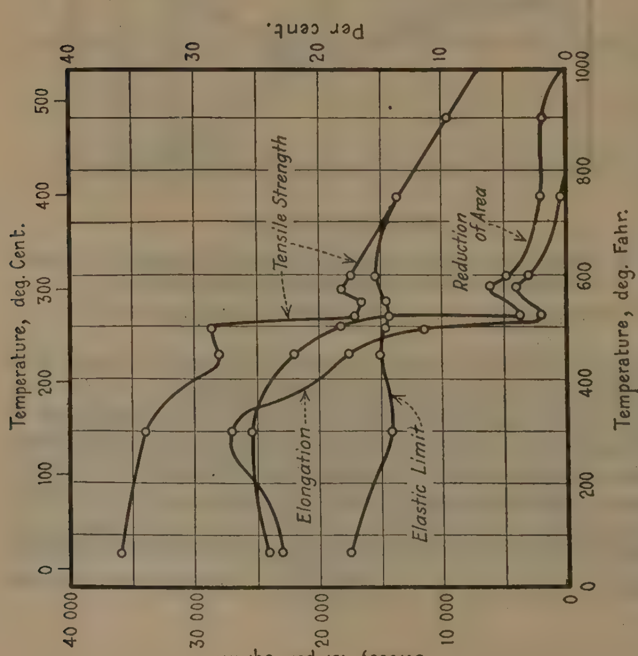


FIG. 25 EFFECT OF TEMPERATURE ON TENSILE PROPERTIES OF U. S. NAVY BRONZE M, ACCORDING TO BREGOWSKY AND SPRING (83)

Chemical composition, per cent: Cu, 86.92; Sn, 7.72; Zn, 3.62; Pb, 1.22; Fe, 0.23.

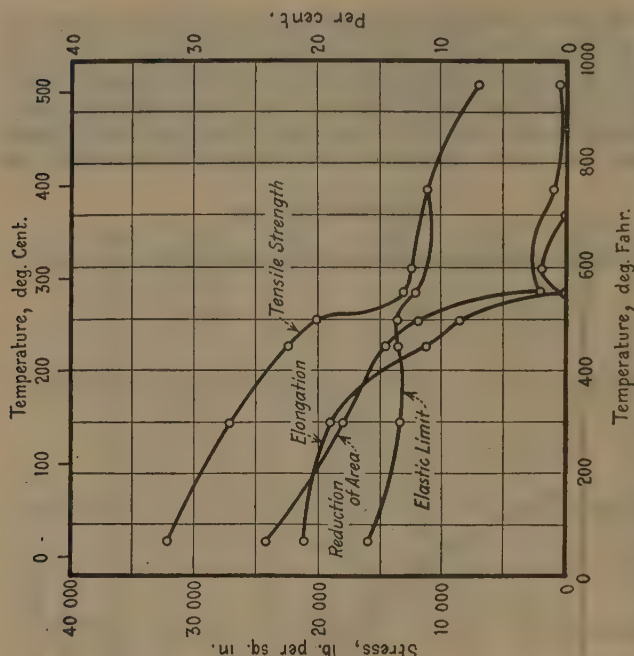


FIG. 26 EFFECT OF TEMPERATURE ON TENSILE PROPERTIES OF BRASS, ACCORDING TO BREGOWSKY AND SPRING (83)

Chemical composition, per cent: Cu, 86.19; Sn, 5.69; Zn, 5.03; Pb, 3.02; Fe, 0.20

samples were supported on a steel dummy and held at temperature for 15 minutes previous to the test. The time required for the blow was estimated as not exceeding 2 to 3 seconds. With this method of testing, alpha brasses show no change in plasticity up to 1100 deg. fahr. (600 deg. cent.). See Figs. 17 and 18. Above 1100 deg. fahr. (600 deg. cent.) they become very slightly more plastic. With an alpha-beta brass — 31.12 per cent zinc, 61.9

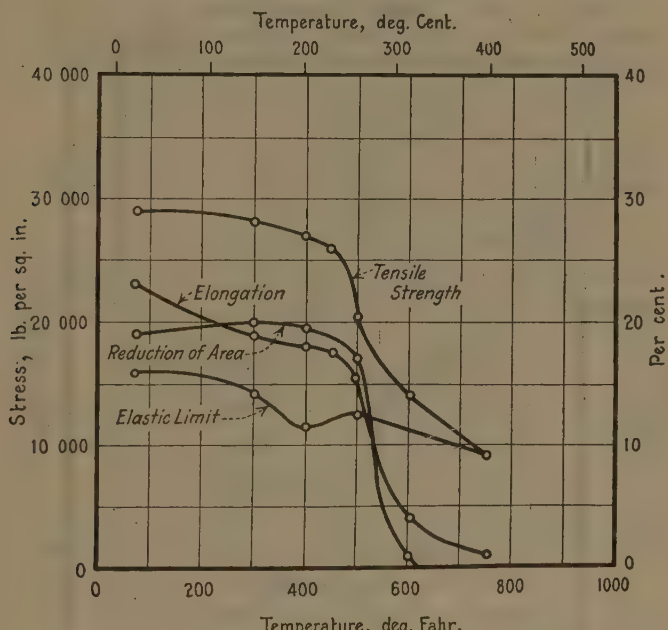


FIG. 27 EFFECT OF TEMPERATURE ON TENSILE PROPERTIES OF U. S. NAVY BRASS S-C, ACCORDING TO BREGOWSKY AND SPRING (83)

Chemical composition, per cent: Cu, 80.32; Zn, 12.80; Sn, 3.98; Pb, 2.78; Fe, 0.24

per cent copper — a slight softening occurs below 850 deg. fahr. (450 deg. cent.). Above that temperature softening is rapid. Alloys *C*, *D* and *E* (Fig. 17), with less than 60 per cent copper, show a slight decrease in plasticity up to 850 deg. fahr. (450 deg. cent.), followed by a very rapid increase up to 1100 deg. fahr. (600 deg. cent.). Alloys of copper and zinc, in which the beta constituent is present, become much more plastic above 850 deg. fahr. (450 deg. cent.). Edwards and Herbert also observed that the degree of plasticity was much greater at 1600 deg. fahr. (870 deg. cent.) if the brass was cooled down to the temperature rather than heated up to the temperature. In Fig. 18 the curves are plotted to show

the effect of composition for the temperatures at which the tests were made.

32 Compression or crushing tests have been made on brasses at elevated temperatures by Doernickel and Trockels (Figs. 19 and 20). In these tests the work required to compress cylinders 18 mm. in diameter and 36 mm. long to 50 per cent of their length was determined. With copper the decrease in work required to

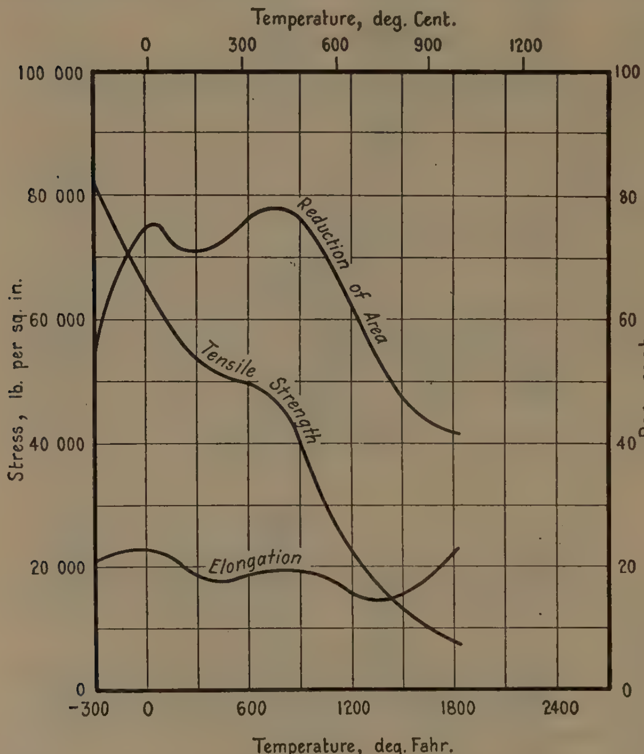


FIG. 28 EFFECT OF TEMPERATURE ON TENSILE PROPERTIES OF ANNEALED NICKEL, ACCORDING TO SYKES (146)

produce 50 per cent compression is very nearly continuous, dropping from slightly less than 1200 ft-lb. to less than 200 ft-lb. at 1400 deg. fahr. (760 deg. cent.). The curve is not unlike the tensile strength curve. The brasses show a critical point or inflection above which the work required to produce a given compression falls off rapidly. Above 932 deg. fahr. (500 deg. cent.) low-zinc brasses offer the greatest resistance to crushing. Below 572 deg. fahr. (300 deg. cent.) the 67 to 72 per cent copper brasses offer the greatest resistance to crushing. Here, as with Edwards'

dynamic tests, the changes due to composition are most marked at the lower temperatures.

Delta Metal

33 The tensile strength and elastic properties of Delta metal (Fig. 21) decrease slowly up to 500 deg. fahr. (260 deg. cent.).

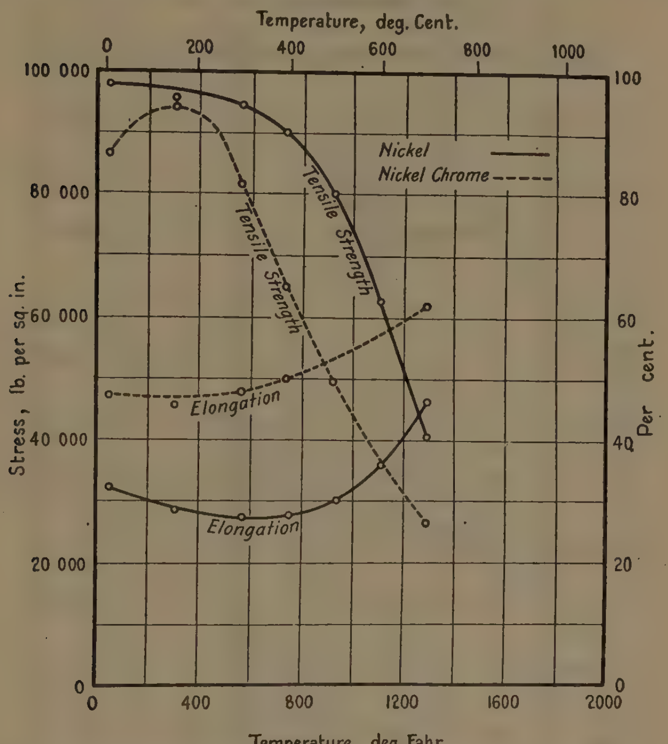


Fig. 29 Effect of Temperature on Tensile Properties of Nickel and Nickel-Chrome, According to Lea (142)

Chemical composition, per cent: *Nickel* Ni, 95.53; Impurities, 4.47
Nickel-Chrome Ni, 59.8; Cr, 12,1; C, 0.4; Mn, 2.9; Fe, 23.7

Above that temperature the tensile strength decreases rapidly and the elastic limit approaches zero between 700 and 800 deg. fahr. (370 to 425 deg. cent.). Delta metal is shown to retain its tensile strength somewhat better than Muntz metal (Fig. 15) up to 400 deg. fahr. (205 deg. cent.) and is slightly superior at all temperatures up to 800 deg. fahr. (425 deg. cent.). There appears to be little difference in the elastic properties of the two metals.

Copper-Nickel Alloys

34 Copper-nickel alloys, according to Lea(141, 142), Huntington(85) and Bengough(82) (Figs. 22, 23 and 24) retain their strength

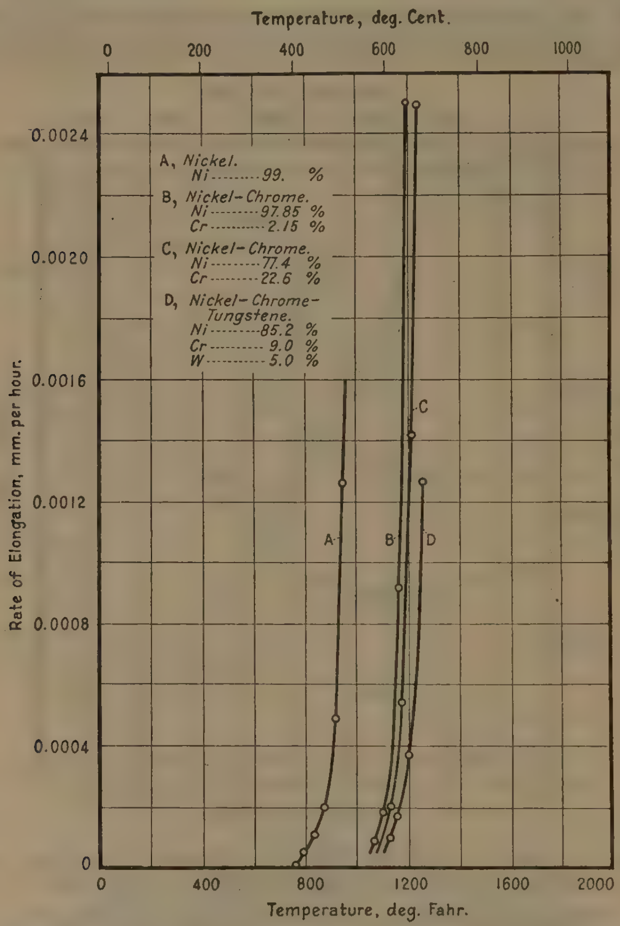


Fig. 30 Effect of Temperature on Elongation of Nickel Alloys Under a Constant Load of 14,223 lb. per sq. in., According to Chevenard (170) and Le Chatelier (35)

Chemical composition as indicated

very well up to temperatures of 600 to 800 deg. fahr. (315 to 425 deg. cent.), depending upon the nickel content. The influence of the nickel content up to 12 per cent appears to be most marked at temperatures below 400 deg. fahr. (205 deg. cent.). The elastic

limit in the 2 per cent alloys remains practically unchanged up to 600 deg. fahr. (315 deg. cent.), and suggests to Lea that the copper-nickel series may offer very desirable alloys for elevated temperatures. The elastic limit has not been determined for the higher nickel contents. However, neither the 2 per cent nor the

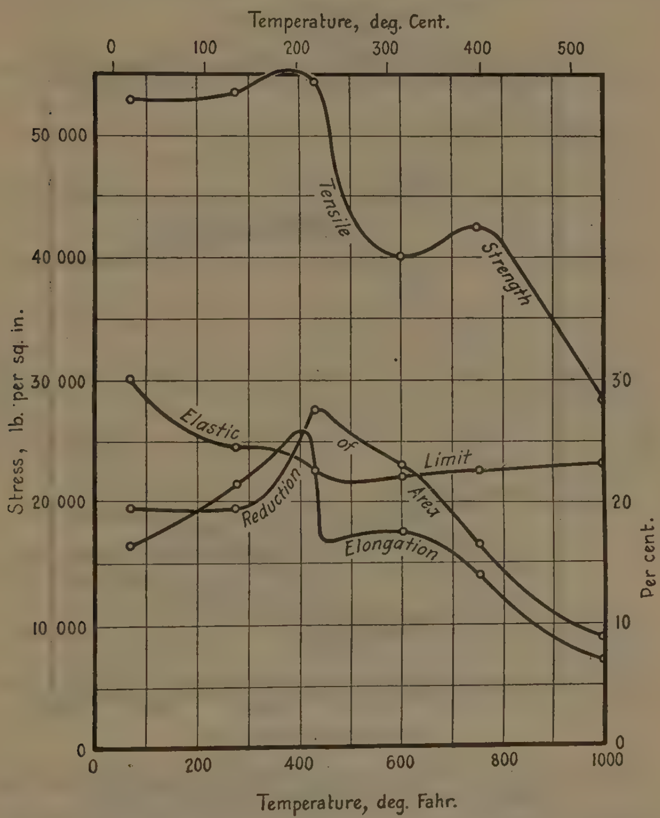


Fig. 31 Effect of Temperature on Tensile Properties of Cast Monel Metal, According to Bregowsky and Spring (83)

Chemical composition, per cent: Cu, 27.11; Ni, 64.79; Sn, 0.08; Pb, 0.13; Fe, 5.46; Mn, 2.33; C, 0.32

12 per cent nickel alloy retains its strength to any greater extent than the 2.5 per cent tin bronze (Fig. 6).

35 The 20 per cent nickel alloy when cold-rolled behaves very similarly to Muntz metal at the same temperatures as regards the tensile strength and elongation.

COPPER-TIN-ZINC-LEAD ALLOYS (STEAM BRONZE TYPE)

36 Alloys of this type usually decrease in both tensile strength and elongation between 500 and 600 deg. fahr. (260 and 315 deg. cent.). The properties of the three alloys investigated by Bregowsky and Spring(83) are shown in Figs. 25, 26 and 27. The U. S.

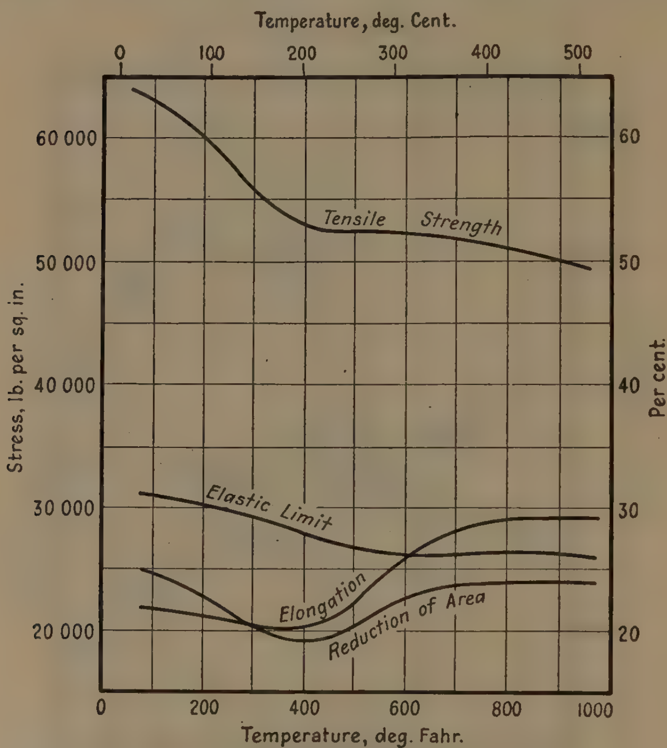


FIG. 32 EFFECT OF TEMPERATURE OF TENSILE PROPERTIES OF CAST MONEL METAL, ACCORDING TO MALCOLM (212)

Chemical composition, per cent: Cu, 29.48; Mn, 0.98; Ni, 66.63; C, 0.15; Fe, 2.36; Si, 0.40

Navy Bronze shows slightly better tensile and elastic properties up to 500 deg. fahr. (260 deg. cent.) than the other two. Between 500 and 600 deg. fahr (260 and 315 deg. cent.) the tensile strength of each of the alloys has decreased to about one-half of the initial value. Above 600 deg. fahr. (315 deg. cent.) the tensile strength decreases more slowly. The elongation approaches zero.

37 Malcolm(212) recently presented curves for steam metal approaching the composition of the brass shown in Fig. 26. Decrease in tensile strength is slight up to 400 deg. fahr. (205

deg. cent.). From 400 to 600 deg. fahr. (205 to 315 deg. cent.) the decrease is rapid. From 600 deg. fahr. (315 deg. cent.) up the decrease is again more gradual. The elastic limit decreases slowly from 19,800 to 8700 lb. per sq. in. at 1000 deg. fahr. (540 deg. cent.). The ductility decreases somewhat more slowly.

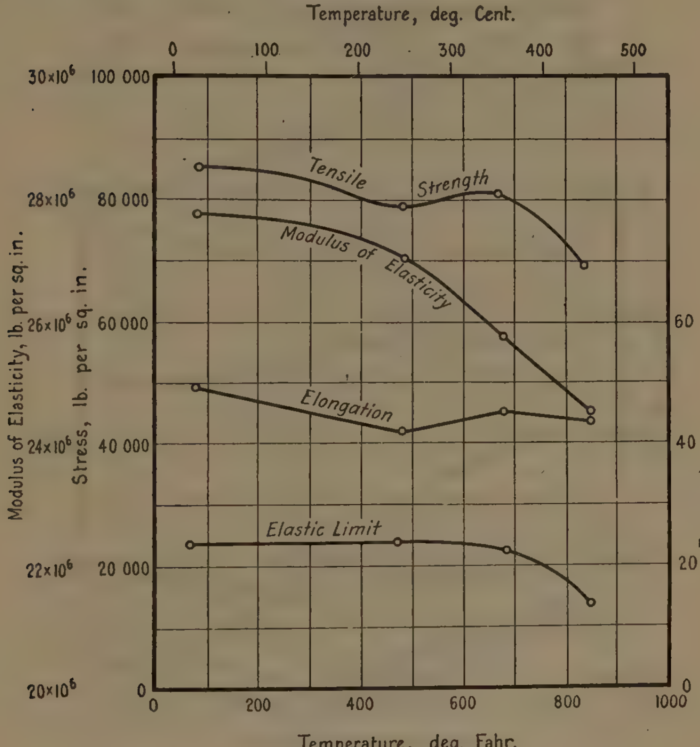


Fig. 33 Effect of Temperature on Tensile Properties of Monel Metal, According to Lea(142)

Chemical composition, per cent: Cu, 29.96; Ni, 66.31; Mn, 1.13; Fe, 2.24

38 Alloys of the leaded type invariably show loss in tensile strength and decrease in ductility when the melting point of the lead is approached, as indicated by the decrease in strength and ductility slightly below 600 deg. fahr. (315 deg. cent.). The high-tin alloy with low lead content shows a similar drop at 500 deg. fahr. (260 deg. cent.).

Nickel and Nickel Chrome

39 Results obtained by Sykes(146) (Fig. 28) on tests of nickel wire, purity 99.8 per cent, indicate that the tensile strength of

nickel decreases slowly from atmospheric temperature — or in fact from below atmospheric temperatures — to temperatures approaching 750 deg. fahr. (400 deg. cent.). Above this temperature

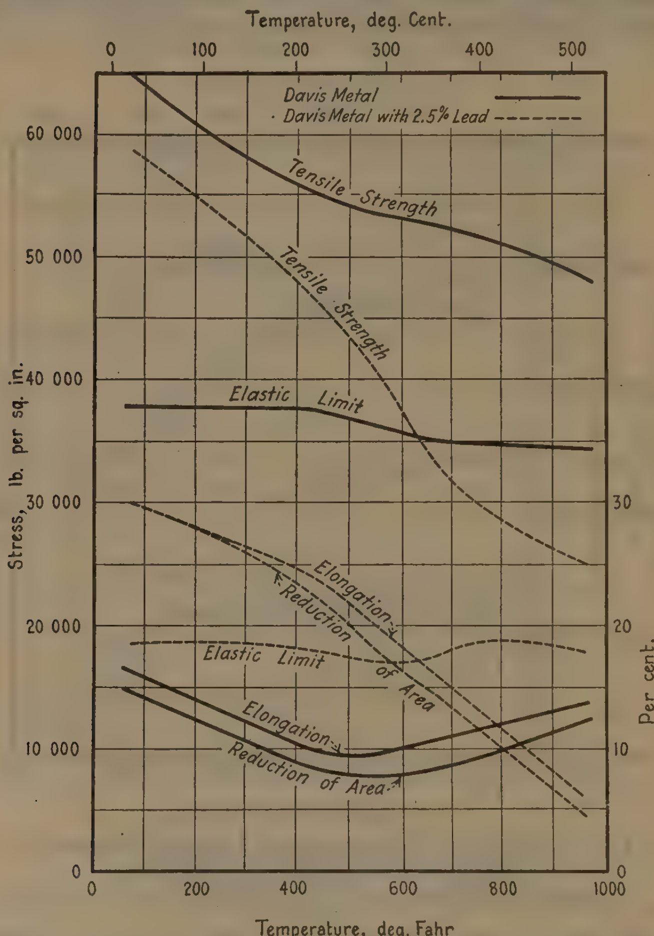


Fig. 34 Effect of Temperature on Tensile Properties of Davis Metal, According to Malcolm(212)

Chemical composition, per cent: Ni, 30; Cu, 65; Mn, 1; Fe, 3; remainder, C + Si.

the decrease is very rapid. Elongation also decreases from atmospheric temperatures to 575 deg. fahr. (300 deg. cent.), but shows an increase at 750 deg. fahr. (400 deg. cent.). Very similar results were obtained by del Regno(192), a rapid decrease in strength and increase in ductility being observed at approximately 750 deg.

fahr. (400 deg. cent.). Lea(142) (Fig. 29), using an impure nickel, shows a rapid decrease in tensile strength beginning between 750 and 850 deg. fahr. (400 and 450 deg. cent.) with an increase in elongation at the same temperature. This constitutes an excellent example of the influence of impurities upon tensile properties.

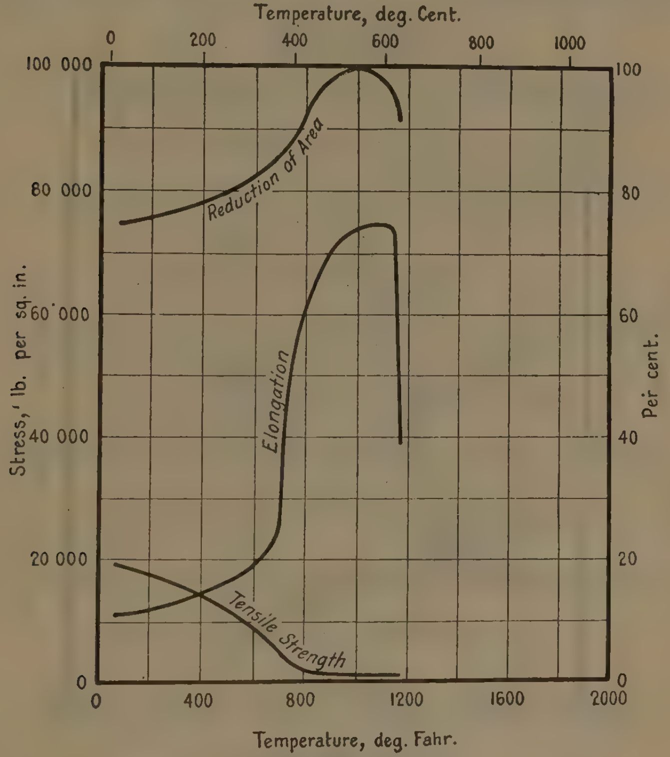


FIG. 35 EFFECT OF TEMPERATURE ON TENSILE PROPERTIES OF ALUMINUM, ACCORDING TO BENGOUGH (82)

Chemical composition, per cent: Al, 99.56; Fe, 0.22; Si, 0.22

40 The nickel-chrome alloy behaves similarly to the impure nickel, but decreases in its strength at lower temperatures (Fig. 29).

41 Some recent tests by Chevenard(170) and Le Chatelier(35) (Fig. 30) on nickel and nickel-chrome alloys indicate the necessity of greater consideration for the time element when determining properties of metals at elevated temperatures. The rate of elongation in millimeters per hour is shown for different alloys under constant load.

Monel Metal

42 The tensile properties of cast monel metal presented by Bregowsky and Spring(83) (Fig. 31) are not in agreement with those recently presented by Malcolm(212) (Fig. 32). Bregowsky and Spring indicate little or no decrease in tensile strength up to

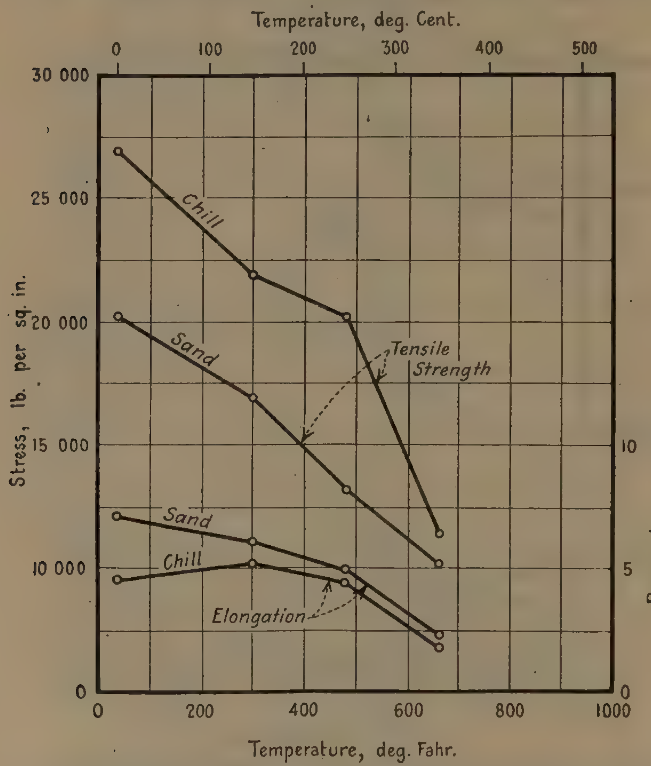


Fig. 36 Effect of Temperature on Tensile Properties of Copper Aluminum Alloy, Chill and Sand Castings, According to Lea(187)

Chemical composition, per cent: Al, 90; Cu, 10

450 deg. fahr. (230 deg. cent.); above that temperature the tensile strength decreases rapidly to 600 deg. fahr (315 deg. cent.), and then increases slightly to 800 deg. fahr. (425 deg. cent.), above which temperature the fall is rapid. Malcolm indicates the tensile strength as decreasing from atmospheric temperatures up to 400 deg. fahr. (205 deg. cent.), above which the tensile strength decreases very slowly. Both show the elastic limit to decrease but slightly up to 900 deg. fahr. (480 deg. cent.), although the de-

crease is more marked according to Bregowsky and Spring. Malcolm also disagrees with Bregowsky and Spring in regard to the form of the ductility curves. Bregowsky and Spring show a maximum at 400 deg. fahr. (205 deg. cent.) while Malcolm shows a minimum. Bregowsky and Spring show a reduction in ductility above 400 deg. fahr. (205 deg. cent.), while Malcolm shows an

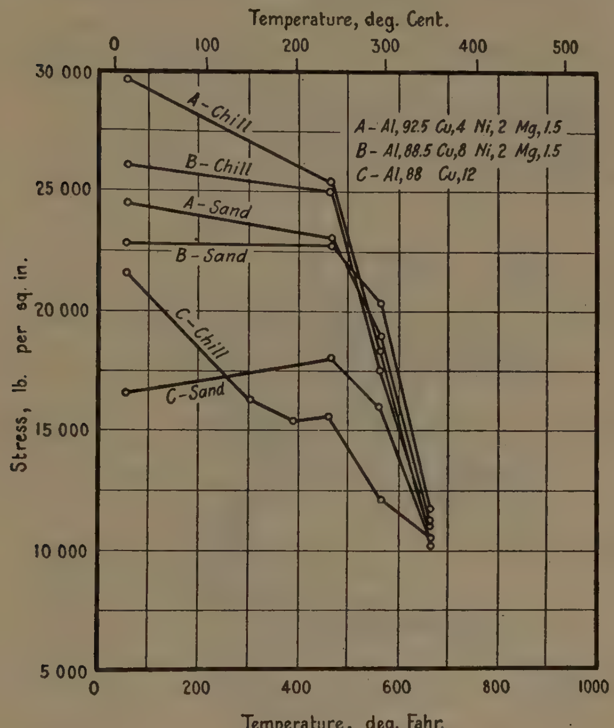


FIG. 37 EFFECT OF TEMPERATURE ON TENSILE PROPERTIES OF ALUMINUM ALLOYS, CHILL AND SAND CASTINGS. ELEVENTH REPORT TO BRITISH ALLOYS RESEARCH COMMITTEE

Chemical composition as indicated

increase. Malcolm's curves follow more closely the changes observed by Lea and the Allis-Chalmers Co. for worked monel metal.

43 The form of curve obtained by Bregowsky and Spring, and by Lea(142) (Fig. 33) for worked monel metal is very much the same, though there exists some doubt as to where the final drop in strength begins. All show the strength as decreasing from atmospheric temperature to 300 or 400 deg. fahr. (150 or 205 deg. cent.). At temperatures above 400 deg. fahr. (205 deg. cent.) the tensile

strength remains unchanged up to 500 to 600 deg. fahr. (260 to 315 deg. cent.), up to 650 to 700 deg. fahr. (345 to 370 deg. cent.) or up to 800 to 850 deg. fahr. (425 to 450 deg. cent.), depending

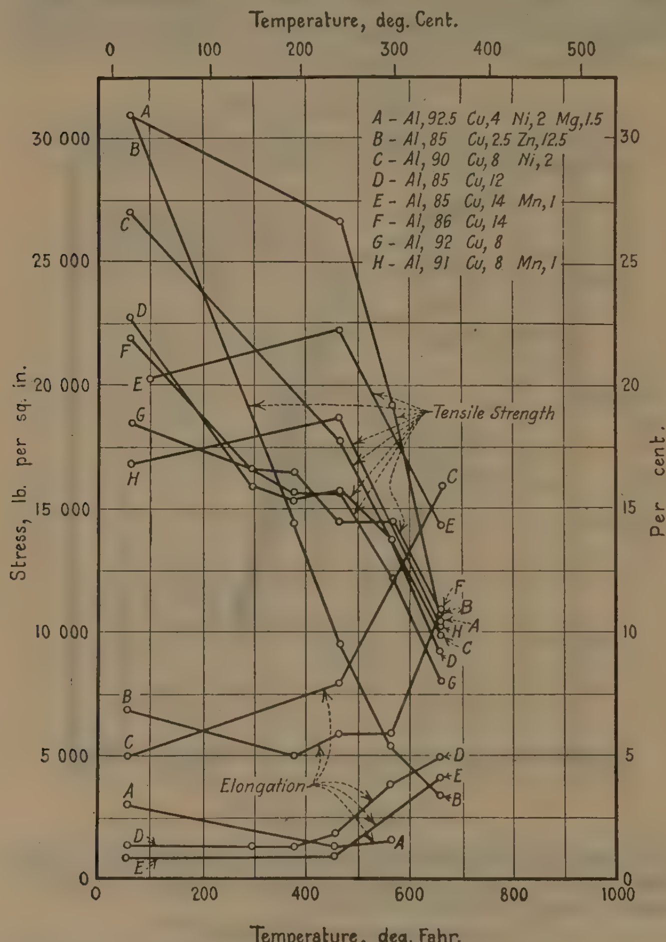


FIG. 38 EFFECT OF TEMPERATURE ON TENSILE PROPERTIES OF ALUMINUM ALLOYS, CHILL CASTINGS. ELEVENTH REPORT TO BRITISH ALLOYS RESEARCH COMMITTEE.

Chemical composition as indicated

upon the investigator. According to Lea, the final and more rapid decrease in tensile strength begins between 650 and 700 deg. fahr. (345 and 370 deg. cent.). The elastic limit undergoes a very marked decrease from 70 to 300 deg. fahr. (21 to 150 deg. cent.)

and from 600 to 800 deg. fahr. (315 to 425 deg. cent.), according to Bregowsky and Spring, while Lea shows practically no decrease up to 700 deg. fahr. (370 deg. cent.). Initial values obtained by Lea are relatively much lower than are those obtained by Bregowsky and Spring, and in fact are appreciably lower than those usually reported for monel metal.

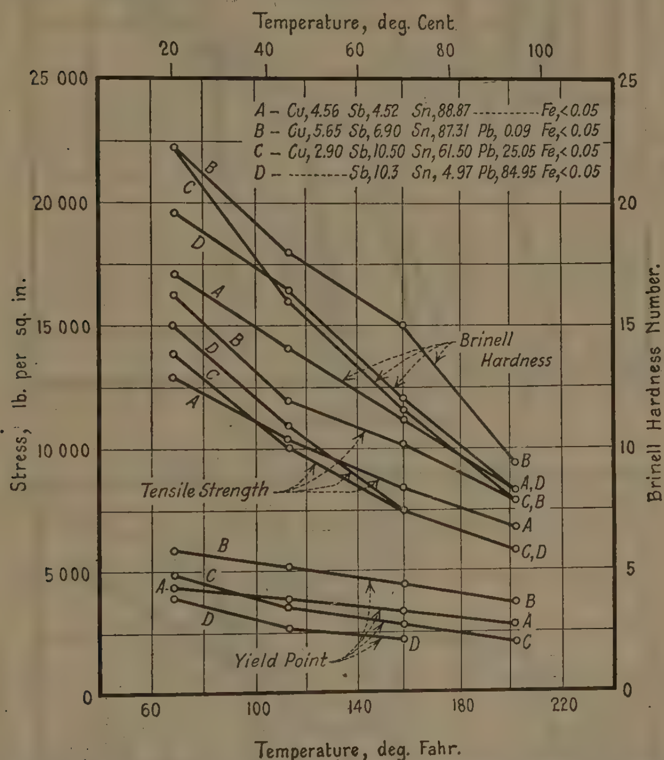


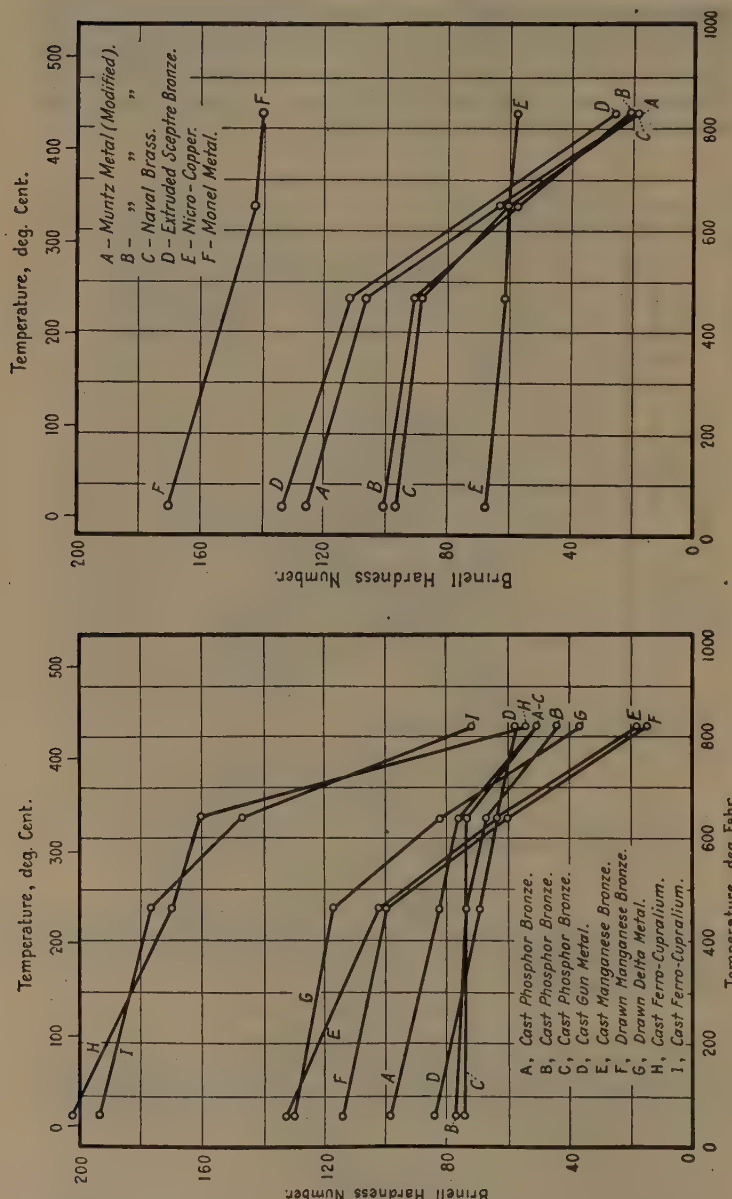
Fig. 39 Effect of Temperature on Tensile Properties and Brinell Hardness of White Metal Bearing Alloys, According to Freeman and Woodward (152)

Chemical composition as indicated

44 The work of both Malcolm and Lea indicates that monel metal retains its elasticity but with relatively small decreases up to 600 to 700 deg. fahr. (315 to 370 deg. cent.).

Davis Metal

45 Davis Metal (Fig. 34), a copper-nickel alloy made up approximately of 30 per cent nickel, 65 per cent copper, 1 per cent



FIGS. 40 AND 41 EFFECT OF TEMPERATURE ON BRINELL HARDNESS OF COPPER ALLOYS, ACCORDING TO LEA (141)

manganese, 3 per cent iron, and the balance silicon and carbon, the properties of which have been very recently presented by Mal-

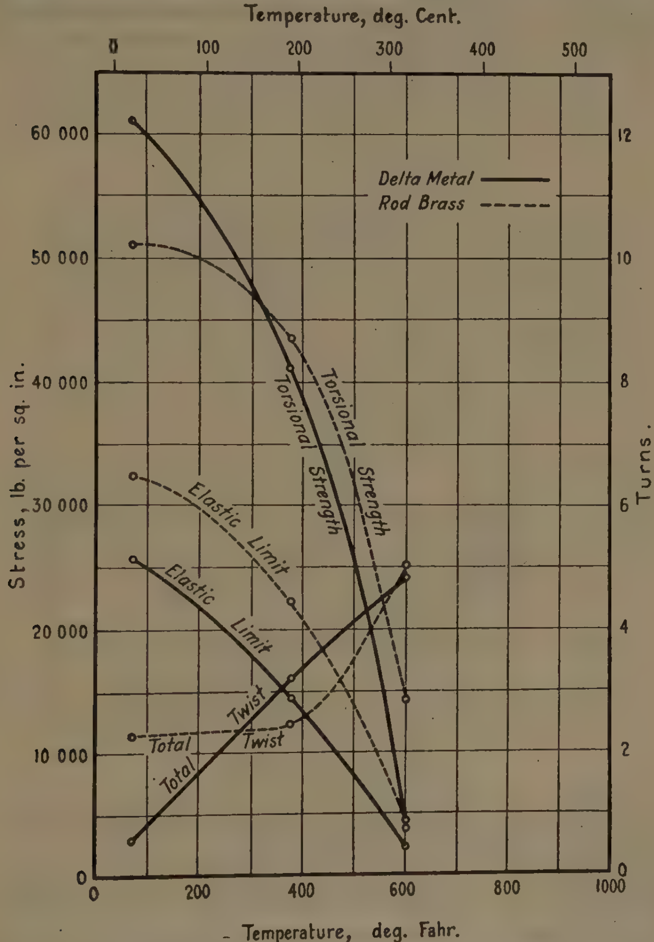


FIG. 42 EFFECT OF TEMPERATURE ON TORSIONAL PROPERTIES OF ROD BRASS AND DELTA METAL, ACCORDING TO BREGOWSKY AND SPRING (83)

Chemical composition, per cent: *Rod Brass*—Cu, 61.08; Zn, 35.72; Pb. 2.34; Fe, 0.42; Sn, 0.18; *Delta Metal*—Cu, 56.56; Zn, 39.36; Fe, 2.40; Sn, 0.76; Pb, 0.56; P, 0.004

colm (212), not only retains its tensile strength at high temperatures but shows an elastic limit superior at all temperatures to monel metal. The substitution of 2.5 per cent lead (Fig. 34) decreases the strength more rapidly and cuts the elastic limit in half. The elongation and the reduction of area decrease continually with increasing temperatures.

Aluminum and Aluminum Alloys

46 The tensile strength of aluminum (Fig. 35) decreases continuously to less than 2000 lb. per sq. in. at approximately 800 deg. fahr. (427 deg. cent.), while the elongation increases slowly up to 700 deg. fahr. (370 deg. cent.) and then suddenly goes to a maximum between 1000 and 1100 deg. fahr. (540 and 600 deg.

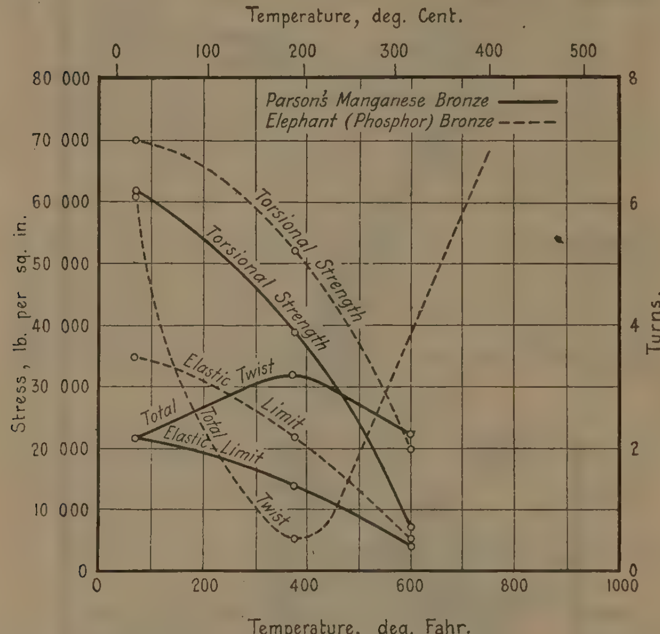


Fig. 43 Effect of Temperature on Torsional Properties of Parsons' Manganese Bronze and Elephant (Phosphor) Bronze, According to Bregowsky and Spring (83)

Chemical composition, per cent: *Parsons' Manganese Bronze*—Cu, 59.58; Zn, 38.08; Fe, 1.22; Sn, 0.64; Al, 0.34; *Elephant (Phosphor) Bronze*—Cu, 95.52; Sn, 3.87; P, 0.307; Fe, 0.16

cent.) according to Bengough(82) A very striking similarity is to be noted between the behavior of aluminum and cold-rolled copper (Fig. 3).

47 Many data relative to the behavior of aluminum alloys at elevated temperatures have been presented in the Eleventh Report of the Alloys Research Committee(161), also in the various reports of the British Light Alloys Sub-Committee, Advisory Committee for Aeronautics. Hardness tests showed that at all temperatures from 70 to 750 deg. fahr. (21 to 400 deg. cent.) the alloys containing copper were harder than those free from copper.

Zinc-aluminum alloys lose their hardness very rapidly, and zinc in copper-aluminum alloys causes them to become softer at higher temperatures. Manganese and iron have the opposite effect. Tension tests on three typical aluminum alloys at low temper-

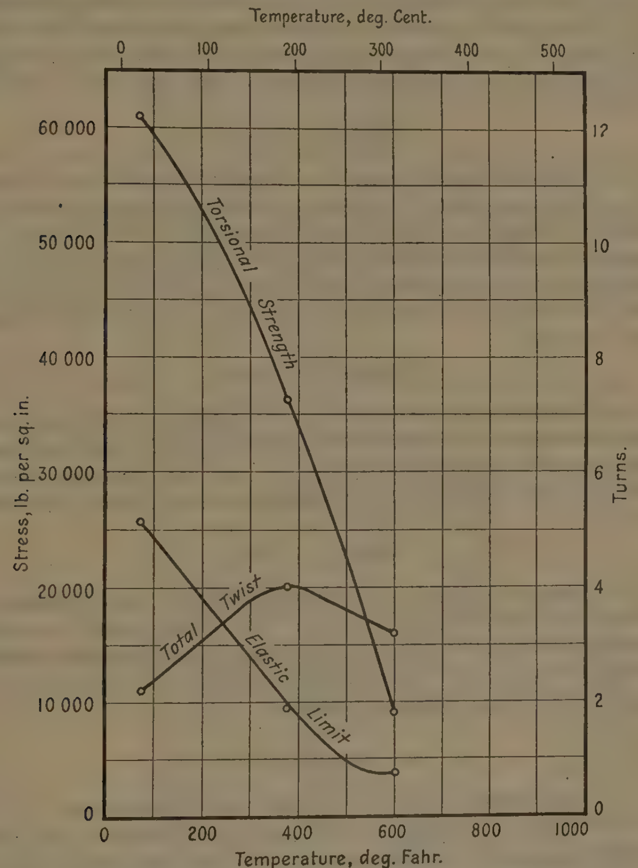


FIG. 44 EFFECT OF TEMPERATURE ON TORSIONAL PROPERTIES OF TOBIN BRONZE, ACCORDING TO BREGOWSKY AND SPRING (83).

Chemical composition, per cent: Cu, 59.86; Zn, 38.94; Sn, 0.80; Fe, 0.46; P, 0.0015

atures, −112 deg. fahr. (−80 deg. cent.), showed no decrease in tensile properties. Impact properties of the copper-manganese-aluminum type show no change up to 480 deg. fahr. (250 deg. cent.). Zinc-copper or zinc-copper-tin alloys showed a marked reduction. Duralumin decreases in resistance to impact above 150 deg. fahr. (65 deg. cent.). In Figs. 36, 37, and 38 are given curves for aluminum alloys at elevated temperatures. Because of

the large number of alloys which have been investigated, no attempt is made to present more than a few which may be considered typical.

48 Lea(187) in summing up the influence of temperature upon the properties of aluminum alloys points out that while the properties of most of the aluminum alloys change at temperatures above 480 deg. fahr. (250 deg. cent.), there is little danger of aluminum pistons failing at temperatures below 650 deg. fahr. (345 deg. cent.).

Bearing Metals

49 Very little work relative to bearing alloys has been done at elevated temperatures. Freeman and Woodward(153), of the U. S. Bureau of Standards, have made compression and hardness tests on white metal bearing alloys (Fig. 39). The tensile strength, yield point, and Brinell hardness decrease with increasing temperatures. Alloy *B* is superior to the other alloys in the retention of strength and elasticity.

Hardness of Alloys

50 Variations in hardness with increasing temperatures have been determined by Lea (141) and others. Typical hardness curves for a number of alloys according to Lea are shown in Figs. 40 and 41. Lea states that the hardness curve follows the tensile curve.

Torsion Tests

51 Torsion tests on five non-ferrous alloys were reported by Bregowsky and Spring(83) in 1912. The values obtained are shown in Figs. 42, 43 and 44. Torsional strength, elastic limit and number of turns are given. No additional tests of this type have been reported in the intervening period.

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Index to References in Paper No. 1926*c*, Covering Ferrous Metals

(Includes the majority of references subsequent to 1914 and a few earlier ones)

Type of Test	Cast Iron	Malleable Iron and Semi-steel	Cast Steels	Mechanically Worked Metals: Wrought Iron and Carbon Steels	Mechanically Worked Metals: Ordinary Structural Alloy Steels	High-Alloy-Content Steels and Heat-Resisting Alloys
Tension	119 183 147 197	131 216 147	147 212 150 216	109 147 164 196 113 150 172 204 116 153 177 206 119 155 178 208 125 156 179 214 127 158 181 132 160 187 144 162 190	123 162 196 133 164 204 153 170 208 157 172 210 158 174 212 160 179	109 158 172 123 160 180 127 162 196 133 163 204 148 164 216 157 170
Torsion				214 216		216
Hardness	198			53 63 156 62 71 198	63 71 133	71 123 133
Crushing	71		71	71	71	71
Notched-Bar Impact				113 156 200 125 160 201	160 201 200 210	123 160 201
"Flow" Tests				172	124 172	170 172
Expansion	193	131		117 193	117 143 193	117 193 126 194
Intercrystalline Deterioration	111 185 145			118 137 167 127 138	137 172	172
Oxidation (includes mainly scaling tests)	182			123 182 172 205	123 133 172	115 133 205 123 172
Special References or Descriptive Reports	145			105 129 154 167 110 134 155 199 114 137 159	110 140 199 137 167	126 136 135 154

Index to References in Paper No. 1926*d*, Covering Non-Ferrous Metals

(Includes the majority of references subsequent to 1910 and a few earlier ones)

Type of Test	Copper	Brasses	Bronzes	Cupro-nickel	Nickel and Nickel Alloys	Aluminum and Aluminum Alloys	Miscellaneous
Tension	64 127 79 141 82 142 85 154 96 171 102 187 107	64 141 82 142 83 171 85 187 102 216 107	57 141 64 142 73 171 79 187 83 190 85 212 98 216 122	79 141 82 142 85 171	106 190 142 192 146 212 154 216 170	82 154 85 161 146 187	100 146 127 154
Torsion	108	108	108	108	108		108 154
Hardness [1]	112 151 120 175	120 175 141	120 175 141	120 151 141 175	112 141	112 151 120 161 141 175	112 151 120 175
Crushing	149	149					
Impact	120 175	120 175	57 73	120 175		120 175 161	
Flow Test	151			151		151	151
Miscellaneous			176				

[1] Hardness tests on bearing metals(**152**).

DISCUSSION[1]

R. B. Wilhelm.[2] This discussion deals with the tensile properties of medium-carbon steel at temperatures between 20 and 500 deg. cent. A description of the apparatus used for these tests is given, together with certain numerical data obtained. The discussion describes only the beginning of an investigation planned by the Westinghouse Electric and Manufacturing Company, the main points of which will be to investigate the properties of certain materials under prescribed conditions. The effect of time at high temperatures is also included in the program. From the results of the investigations so far published on this subject, the behavior of the material either under normal test conditions or under a long time effect are separately considered. It is felt that some valuable results from both methods of testing can be obtained if these tests are made on the same materials. Such is the object in view of which the results given are preliminary.

In the normal tension test, special care was taken to determine the modulus of elasticity and proportional limit. From the modulus of elasticity given in the paper, it is possible to obtain some data on the value of the modulus of rigidity at different temperatures by considering the values for Poisson's ratio for different temperatures as found and published by H. Carrington.[3]

Since it is our opinion that a difference in one of the constituent elements of the steel may materially affect the properties at high temperatures, the complete chemical analysis is given of the material tested. This may possibly help to clear up the seemingly contradicting results found by different investigators. We note, for instance, that for carbon steels of similar carbon content the values of proportional limit and yield point show an increase in some cases and a decrease in others, with increase of testing temperature. Further tests may decide to what extent either the composition or the state of the material is responsible for this fact.

For the first series of tests a carbon steel was chosen in order to form a basis on which the results found for alloy steels may in the future be compared.

Apparatus Used for High-Temperature Tests

Testing Machine. All the tests were carried out on the latest type of hydraulic 100,000-lb. tension testing machine supplied by

[1] Complete discussion of four papers preceding is followed, on pp. 527-534, by authors' closures.

[2] Research Dept., Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

[3] *Engineering,* 1924, No. 3029.

the Alfred J. Amsler Co., Schaffhousen, Switzerland. The machine is equipped with an attachment for taking autographic diagrams. Shortly before executing these tests the machine was checked up by means of a standardizing box, supplied by the builder of the machine. The errors in the readings of the load applied by the machine proved to be within ±0.5 per cent.

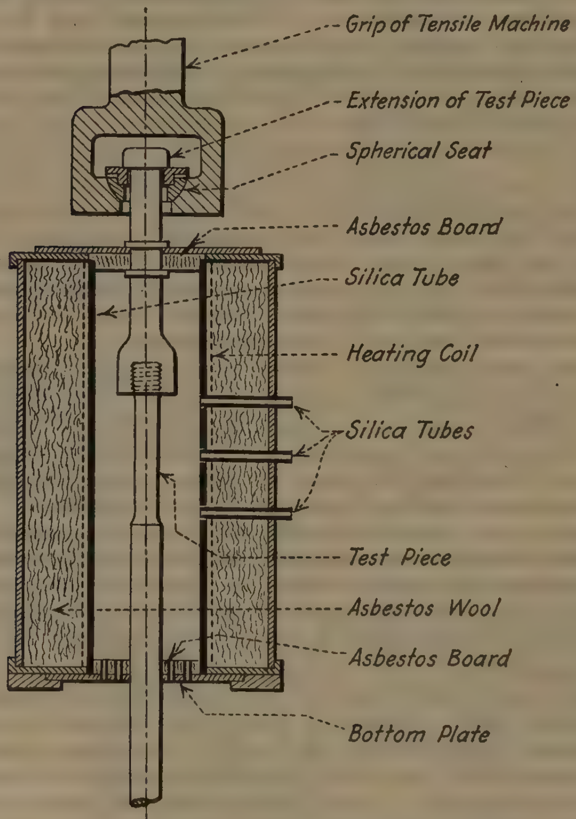

Fig. 1 Electric Furnace for Tensile Tests

(Coil wound from ⅛-in. × 0.032-in. nichrome wire; resistance, 125 ohms per 1000 ft.)

Furnace and Temperature Measurements. According to tests carried through by Welter,[1] for temperatures up to 500 deg. cent. and for the duration of the tests no scaling occurs on medium-carbon steel which might otherwise affect the results. As a rule alloy steels show a greater resistance against scaling than plain

[1] Forschungsarbeiten auf dem Gebiete des Ingenieurwesens, Heft 230, p. 11.

carbon steels. Considering these facts, it was decided to use an air-bath furnace instead of a liquid. In many respects an air-bath furnace offers advantages over the liquid-bath furnace. In the first case the extensometer with the mirrors can be inserted from the bottom, and the mirror carriers and the mirrors are thus subjected to much less heating. Besides, an air-bath furnace is easier to handle before and after the test.

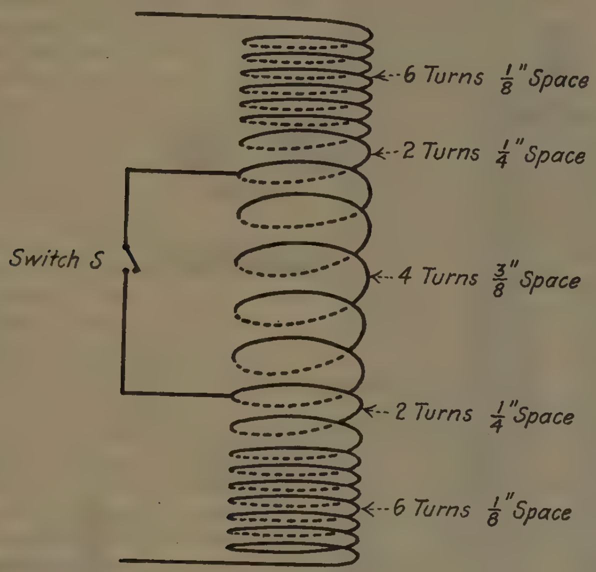

Fig. 2 Coil Unequally Spaced to Compensate for Cooling

A section through the electrically heated furnace is given in Fig. 1. The furnace consists essentially of a silica tube (2½ in. I. D., 3 in. O. D. and 9 in. long) the heating coil, an outer brass tube (6 in. O. D.), top and bottom plates, and the necessary asbestos board and wool for heat insulation. The length of the furnace was limited by the dimensions of the machine. By means of the top plate the furnace is suspended from the test piece. Sliding bottom plates are provided to remove the high-temperature extensometer during the test after having passed the proportional limit. The small silica tubes are inserted horizontally to take care of the thermocouples. Fig. 2 shows the coil unequally spaced to compensate for cooling at the ends. In addition to this, several turns in the center can be put in parallel with a line of low resistance by closing switch *S*. In further development, switch *S* may be replaced by a resistance which would permit regulation of the amount of current going through each of these branches.

The thermocouple readings in the center of the furnace and at 2 in. distance toward top and bottom are given in the following table for three different ranges of temperatures:

Location of thermocouple	Temperature, deg. cent.		
	200	300	500
Top	200	296	499
Center	197	300	502
Bottom	206	308	505

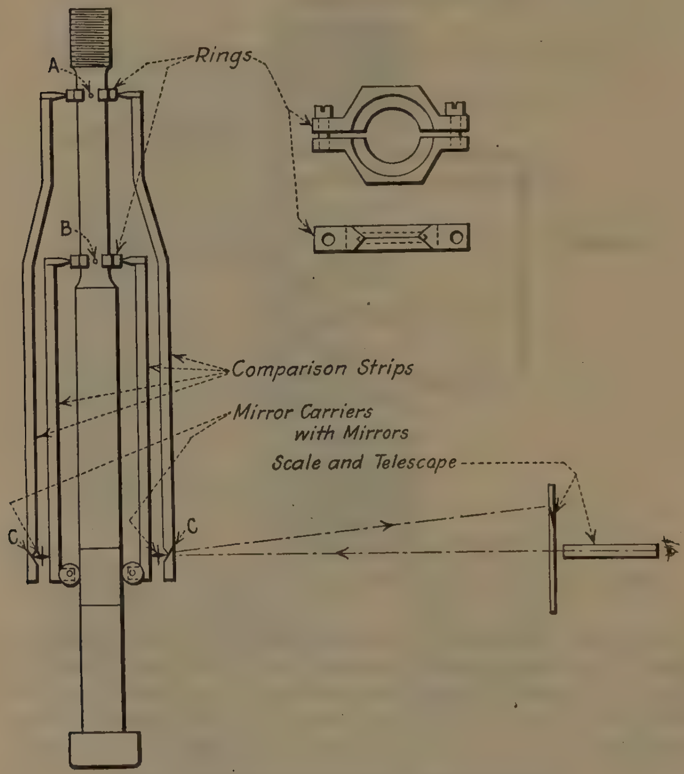

Fig. 3 High-Temperature Extensometer

Comparison tests of thermocouples welded on, and others pushed against, the test specimen gave at the desired temperatures a difference of 12 to 25 deg. cent., the couple welded on the test piece showing the lower temperature.

The test piece being exactly in the middle of the furnace and the gage length with the fillets being 4 in. long, the thermocouples of the top and the bottom at 4 in. distance will coincide with the increased section of the test piece, where due to conductivity

a decrease in temperature was noticed. Therefore the top and bottom thermocouples proved to be more valuable in indicating the fluctuations of temperature rather than giving any absolute value, which exclusively was determined by the pyrometer in the middle of the gage length.

The materials used for the thermocouples were copper and Advance. For taking the readings they were pushed slightly against the test piece. A potentiometer in conjunction with a calibration curve for the above mentioned metals was used for determining the temperature.

High-Temperature Extensometer. The telescopes and the mirrors of the Martens mirror apparatus were used in the ordinary way for measuring strains. The comparison strips had to be subjected to a change in order to transmit the extension of the gage length outside of the furnace. This has been done on the same principle as in tests executed by other investigators and is shown schematically in Fig. 3.

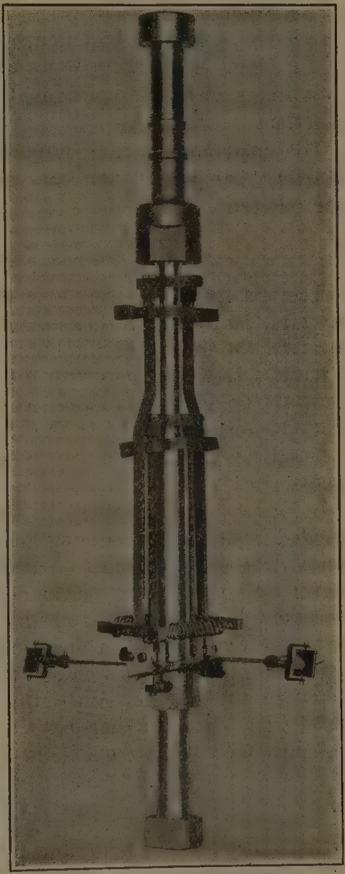

Fig. 4 Test Piece with Extension Grip and High-Temperature Extensometer

Two rings with knife edges inside and V-notches outside secure the position of the comparison strips. Spring attachments hold the strips in their position. The mirror carriers rest in a V-notch on one strip and on a cylindrical surface on the other. In the design special care was taken that no part of the instrument was supported in more than three points to secure its position. Any extension between the points A and B will produce a displacement of the points C and therefore affect the position of the mirrors, and, in consequence, the reading on the scale through the telescope. With regard to the accuracy of the readings, the distance AB on the test piece ought to be as long as possible; on the other hand, the difficulty of having a uniform temperature over a great

length had to be taken into consideration. A length of 2.75 or 3 in. seems to comply best with both requirements.

The material used for the instrument had to show the following characteristics:

a Machinability
b No scaling at high temperatures
c Good tensile properties at high temperatures.

Stainless steel was considered suitable for the temperatures in question.

The instrument was compared with the normal-temperature Martens equipment and the difference found to be less than one per cent.

Tension Test Specimens

The test specimens were machined from 2-in.-diameter steel bars in pairs. In further tests a smaller diameter may be used in order to avoid the milling of the bars in the longitudinal direction. Fig. 4 shows a test specimen with the extension grip and the high-temperature extensometer mounted on it. The cylindrical part of the test specimen subjected to the test has a diameter of 0.505 in. and a length of 3 in., which are used for the extensometer measurements.

In order to determine the elongation the whole length was divided in parts 0.25 in. long and a fine mark made with a center point. The fact that most of the test specimens broke between two marks and not in the marks itself may prove that there is no considerable influence on ultimate strength due to this dividing method. The value of elongation was determined for a standard gage length of 2 in. as well as for 3 in. and fracture was supposed to be in the middle. With the intention to save material and machining costs, further tests with grips on both ends of the test specimen will be made.

Execution of Tests

After checking the main dimensions and dividing the gage length, the extensometer was assembled without the mirror carriers. The test specimen and instrument were inserted in the furnace and all together put into the tension testing machine. After putting the mirror carriers in place and adjusting the mirrors a test was made at normal temperature and at a low stress (within the proportional limit) to insure that the extensometer and mirrors were set correctly. Heating of the furnace was then commenced, and after it reached the desired temperature, it was kept for at least two hours before the test specimen was put under load. Most of this time was necessary for an accurate adjustment of the electric current to maintain a state of equilibrium in temperature, the latter

being indicated by thermocouple as well as by extensometer readings, provided that there was no, or at least no variable, stress on the test specimen. Then a gradually increasing load was applied and extensometer readings were taken at equal intervals. After passing the yield point, or after an equivalent increase in strain at temperatures without an accentuated yield point, the mirrors were removed. In order to avoid any disturbance of the constant temperature the comparison strips were removed only after passing the ultimate strength and previous to fracture.

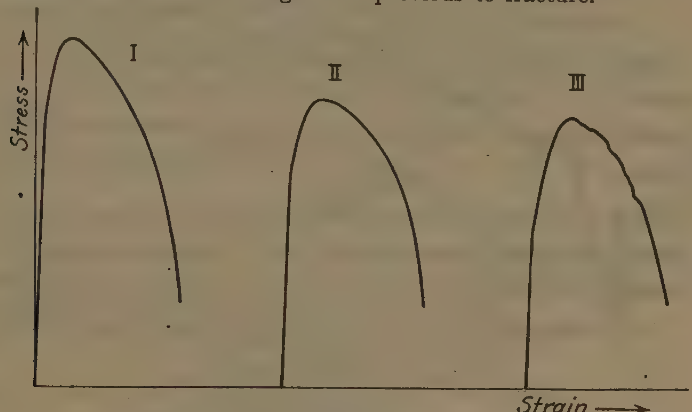

Fig. 5 Speed Effect at 500 Deg. Cent., Medium-Carbon Steel

Test number	I	II	III
Duration of test, min	6	70	240
Ultimate strength, lb. per sq. in	53,500	44,500	41,600

Preliminary Tests on Medium-Carbon Steel

A series of tests was first carried out at various temperatures without the high-temperature extensometer. This showed an increasing effect of testing speed especially upon the ultimate strength. Fig. 5 shows the diagrams taken at 500 deg. cent. with different speeds, the durations of the whole test being 6, 70 and over 240 min., respectively.

A second series was carried out with the high-temperature instrument. Two speeds were chosen, one which just allowed readings being made and the other which was a very low one, i.e., 1000 and 100 lb. increase of load on the machine per minute, respectively. This test, however, did not show a very distinct speed effect. This fact might have influenced Mr. French of the Bureau of Standards to use a photographic method for recording the stress and strain values at high testing speed.

For our test it was decided to use a normal testing speed and to study the time effect, especially of long time, on a separate apparatus. Another phenomenon worthy of note is the fact that

fracture of the test piece occurred at the point of minimum temperature, provided that the ultimate strength increased with increase of temperature, and at the point of highest temperature if the ultimate strength decreased with increase of temperature.

Properties Determined in Tension Tests

The elastic properties such as proportional limit and modulus of elasticity were considered the most important features of the

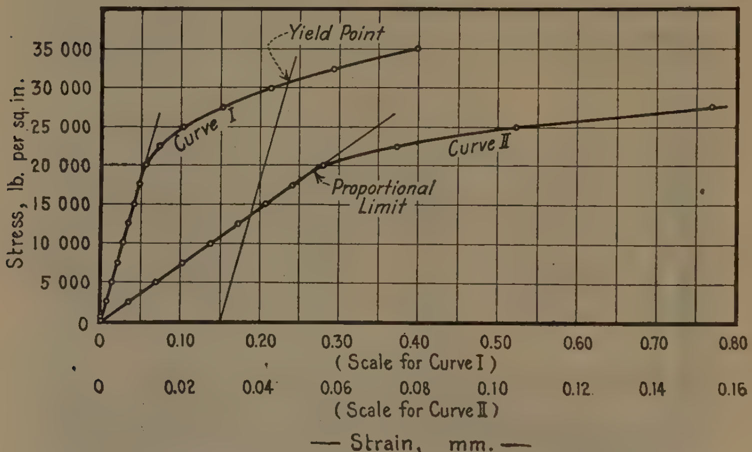

Fig. 6 Stress-Strain Curve Illustrating the Definitions for Proportional Limit and Yield Point

(Test piece No. 721-7, medium-carbon steel. Testing temperature, 300 deg. cent.; proportional limit, 19,500 lb. per sq. in.; yield point, 30,600 lb. per sq. in.; modulus of elasticity, 27.6×10^6 lb. per sq. in.)

tests. In addition to these the values of yield point, ultimate strength, elongation and reduction of area were determined.

Proportional limit was defined as the stress value at which the stress-strain curve showed a distinct deviation from a straight line. It is necessary to add that the same scale for stress and strain had to be used in these determinations.

Since the yield point was not accentuated at higher temperatures, a certain method of expressing this value was adopted. In these tests, therefore, the yield point is taken as the intersection of the stress-strain curve and a straight line parallel to the line of proportionality and passing through the abscissa at a strain of 0.2 per cent of the gage length. That means that by unloading at this stress value the permanent set would amount to 0.2 per cent of the gage length.

This definition is widely adopted in European laboratories for materials without accentuated yield point. A point determined

in such a manner gives valuable information with regard to the shape of the curve beyond the proportional limit. Fig. 6 shows the application of these definitions on a stress-strain curve, taken at a testing temperature of 300 deg. cent.

The modulus of elasticity was calculated from the formula:

$$E = \frac{\Delta P \cdot l}{s \cdot \Delta\lambda}$$

where ΔP = increase in load in lb. from reading to reading
l = distance A-B in inches (Fig. 3)
s = section of test piece in square inches
$\Delta\lambda$ = average increase in extension, from reading to reading up to proportional limit in inches.

The other values are determined in the usual manner and need no further explanation.

Characteristics of Material Tested and Results Obtained

a Chemical Analysis. The material used for these tests was taken from the same heat and all the bars analyzed. The values obtained varied within the limits given below:

C = 0.37 to 0.40	P $\sim$ 0.012
Mn = 0.63 to 0.69	S $\sim$ 0.037
Si = 0.11 to 0.14	

Phosphorus and sulphur were only determined from some of the bars.

b Treatment of Material. Before machining the material (2 in. in diameter) was normalized at 875 deg. cent., soaked three hours at temperature, and air-cooled.

Discussion of the Results

From Fig. 7 the decrease in slope of the stress-strain curve with increasing temperature will be seen. As this slope determines the modulus of elasticity, it may be noted that there is only a slight decrease at the beginning and a more accentuated one at higher temperatures. Fig. 8 gives the reproduced autographic diagrams. The suppression of the yield point at temperatures higher than 260 deg. cent. is clearly shown. Furthermore, it is interesting to note that the diagram taken at 200 deg. cent. shows accentuated vibrations before reaching the ultimate stress. The same phenomenon was observed by Portevin and Le Chatelier.[1] These vibrations may partly be due to the pendulum of the testing machine, but it appears logical to state that a primary cause has to be sought in a change of the material itself.

[1] *Comptes Rendus,* vol. 176, p. 507.

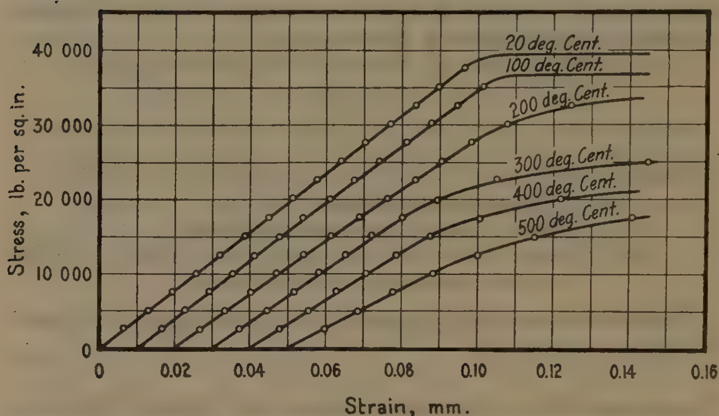

FIG. 7 STRESS-STRAIN CURVES AT VARIOUS TEMPERATURES, MEDIUM-CARBON STEEL

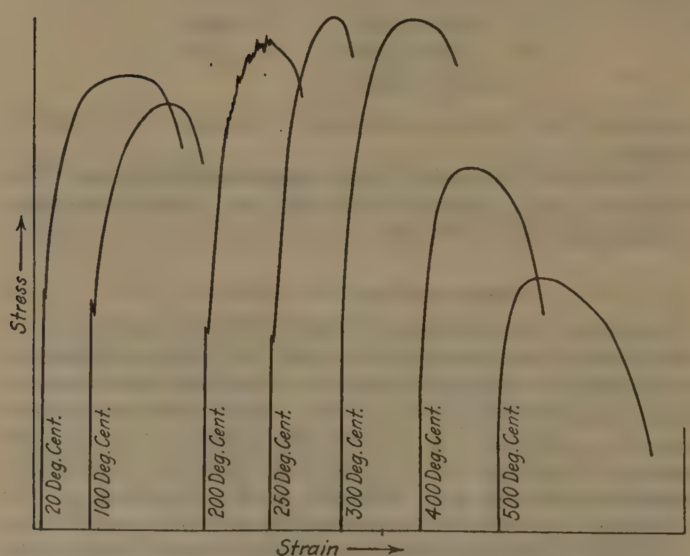

FIG. 8 AUTOGRAPHIC DIAGRAMS FOR VARIOUS TEMPERATURES, MEDIUM-CARBON STEEL

Fig. 9 gives the complete results of the tension tests. It will be observed that the curves of the proportional limit and yield point show from the beginning a decrease in stress with increase

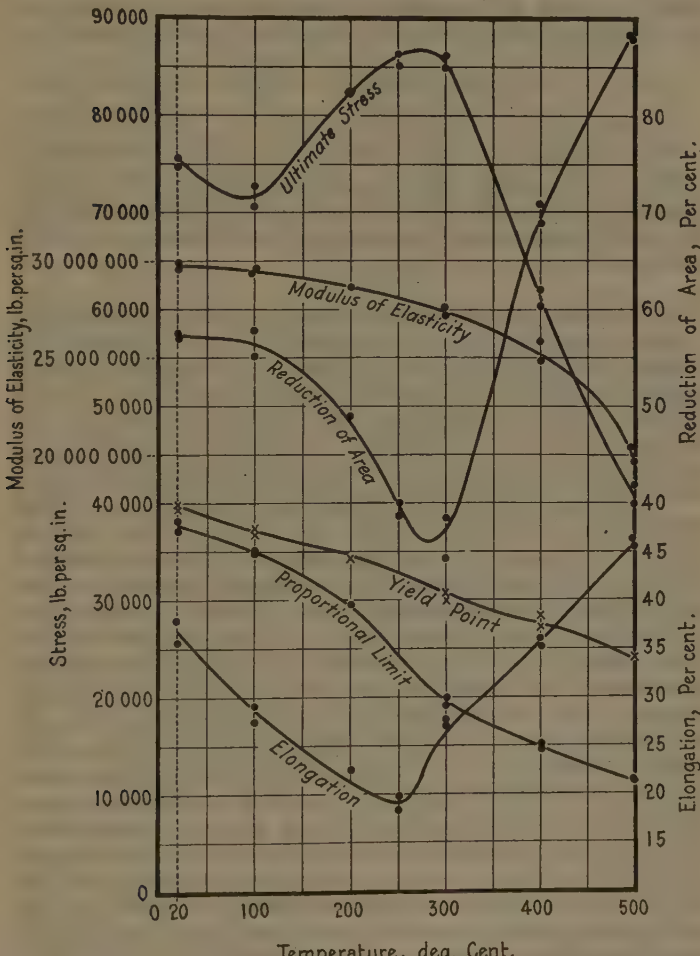

FIG. 9 TENSILE-TEST RESULTS AT HIGH TEMPERATURES, MEDIUM-CARBON STEEL

of temperature. The ultimate strength shows a minimum at about 100 deg. cent., but attains a maximum at about 260 deg. cent., from where it falls sharply, up to the highest testing temperature. The curves for elongation (on 2 in. gage length) and reduction

of area are of similar character, but it may be noted that the minimum elongation is reached at a somewhat lower temperature than that of reduction of area.

The change in the modulus of elasticity expressed by the ratio E_t/E_{20} in which E_t = modulus at temperature t deg. cent., and E_{20} = modulus at temperature 20 deg. cent., is given below:

	Temperature, deg. cent.					
	20	100	200	300	400	500
E_t/E_{20}	1	0.987	0.961	0.920	0.853	0.657

On this occasion the writer wishes to express his indebtedness to Mr. J. M. Lessells and Dr. Timoshenko for their valuable suggestions concerning these tests and to the Westinghouse Electric and Manufacturing Company for the permission to publish these results.

H. H. Lester.[1] Mr. Malcolm's reference to X-ray testing touches a field so new that no one can predict now the influence this method will have on production methods. Dr. Westgren in Sweden pioneered in the field of applying X-ray analysis to the structure of steel at high temperatures. Mr. Malcolm referred to the original article by Dr. Westgren. Since this was published he has continued this work and there was presented at the May, 1924, meeting of the British Iron and Steel Institute a further contribution along the same line. In this paper the previous results with regard to δ iron are confirmed. That is, there is a fourth critical point in the steel constitution diagram. Apparently molten steel in freezing changes to cubic crystals of the body centered type. At around 1450 deg. cent. the body-centered cubes change to the face-centered cubic crystals characteristic of austenite. At around 760 deg. cent. the face-centered type changes to the body-centered type characteristic of α iron, the iron usually found at room temperature in ordinary steels. With regard to carbon in steel at high temperature, Westgren shows that the face-centered crystals of austenite are enlarged due to the presence of carbon. This indicates that the carbon atoms are forced into the interstices between the iron atoms at temperatures where γ iron is formed. Work by Bain, McKeehan, and others has shown that metallic solid solutions are formed by atoms of the solute replacing atoms of the solvent in the crystal structure of the solvent. Westgren's work points out the possibility of another type of solid solution in which the solute atoms are forced between the solvent atoms without breaking up its structure. According to this, alloy steels may be made up of three types of constituents; that is, mechanical mixtures of different metals and two kinds of solid solutions. In addition we may have chemical compounds. Chemical analysis may be used to distinguish definite

[1] Watertown Arsenal, Watertown, Mass.

compounds, but nothing we know of except X-ray tests will distinguish between mixtures and the two types of solid solutions.

That a knowledge of solid solutions and mixtures is highly important in steel practice is indicated by the fact that in Watertown Arsenal four different solid solutions were found in a single specimen of high-speed tungsten tool steel. Control of these solutions probably will be effected through heat treatments. It is necessary to correlate the X-ray data with physical tests to determine the value of these solutions. This information is being gradually accumulated in the Watertown laboratories and elsewhere.

When we consider the various changes in structure that iron undergoes in cooling from the liquid state to room temperature, the fact that alloy constituents often tend to delay or prevent these changes, and the fact that different rates of cooling also affect these changes, we would expect to find in castings, particularly in chilled castings, metal that is by no means in a state of equilibrium. It may be full of partially completed physical reactions. These arrested developments are no doubt responsible for many physical peculiarities. Martensite is an example of arrested developments and its hardness is due to arrested physical reactions. X-ray investigation probably will slowly unravel the tangle of cast-metal structures and give us the knowledge to control to our profit these partially completed reactions.

Herman A. Holz.[1] Referring to the paper by Mr. Malcolm on the methods of testing metals at abnormal temperatures in which the periods of stress application must often be extended to hours, days and weeks, it will be of interest to mention here the special apparatus (Fig. 10) recently developed by Dr. Alfred Amsler for the automatic maintenance of a constant load, independent of the deformation of the specimen, in his well-known tension and compression testing machine. This load-maintaining device has been developed especially for research work on the physical properties of metals at elevated temperatures extending over long periods and operates automatically for any desired period, for days or weeks. It thus produces the same effect as the direct hanging of known weights onto a test piece, as used by Dickenson in his high-temperature researches.

The Amsler pendulum dynamometer with its self-contained "primary standard" of pressure measurement against which the load in the testing machine is automatically balanced during the entire testing operation, is too well known to require a detailed description. The automatic load-maintaining apparatus consists of a very simple and effective electric-contact attachment to the load dial of this dynamometer. The contact is made and broken through the load-indicating pointer. The contact arrangement is

[1] Testing Engineer, New York, N. Y.

connected to a mercury tilting switch and closes or opens the electric circuit. The apparatus actuates, through a solenoid, a device which stops the oil-pressure pump of the testing machine immediately the limit of load allowable is reached, and starts it again as soon as the oil pressure in the testing machine sinks below

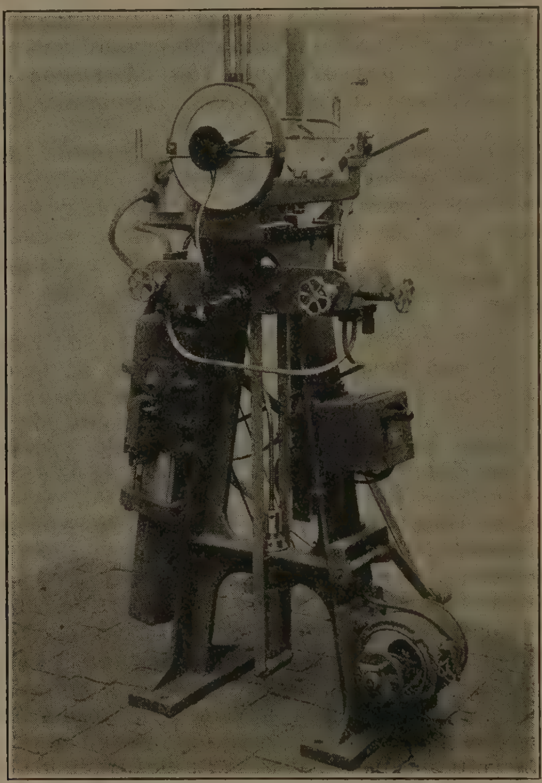

FIG. 10. AMSLER PENDULUM DYNAMOMETER WITH APPARATUS FOR AUTOMATIC LOAD MAINTENANCE, INDEPENDENT OF THE DEFORMATION OF THE SPECIMEN

the lower limit which had been set. A movement of two millimeters covered by the pointer suffices for reversal. To prevent any abrupt action of the mechanism, a regulating valve is fitted in the pressure regulator.

Mr. Malcolm has described some of the difficulties usually encountered in attempting to obtain a uniform temperature over the entire length of a heated test bar. In the design of electric resistance furnaces of this type it may be useful to make the heating element not of one continuous wire spiral, but to use a

number of separate wire spirals, arranged along the heating tube, with no electrical connection between the various sections making up the entire heating element. If each of these spirals is controlled separately by means of rheostats, it will be possible to obtain a uniform temperature over the entire length of the test bar. The influence of the cold-specimen heads which conduct a considerable quantity of heat away can thus be offset and eliminated.

There is one method of heating the specimens which Mr. Malcolm has not mentioned. The writer refers to the method of using the tension-test specimen itself as resistor in an electric circuit of low voltage and high amperage, by the utilization of alternating current and a suitable transformer. If this method of heating could be successfully applied, it would possess the important advantage that the bars would be heated progressively from the inside toward the outer portions, and not vice versa. An arrangement of this kind may also simplify the application of optical extensometers.

Welter, who developed a special gas furnace for his researches, calls attention to the possible influence of magnetic induction, as produced by electric furnaces made from a coil of resistance wire, on the strength and elastic properties of steel specimens. He claims that his gas furnace, constructed in two parts connected by hinges for convenient opening and closing, permits a temperature regulation and maintenance to 0.5 deg. cent. at low and high temperatures. This accurate control, due mainly to the design of the furnace, is facilitated by the insertion of a pressure regulator in the gas duct. Although the tendency during recent years has been to apply electricity to the production of high temperatures in the laboratory, Welter points out the disadvantages of the electric furnace for this particular work in testing practice and describes means developed by him for producing and maintaining accurate temperature control by the use of gas which can hardly be improved upon, if at all equaled, by using electric current. If the writer is not mistaken, Welter's furnaces for use in testing machines are now being produced abroad on a commercial scale.

Regarding the methods to be applied in high-temperature researches on metals, there are two methods which we will have to develop for routine tests at high temperatures, while a third one will yield much information of value in research. The writer is referring, first of all, to hardness tests at elevated temperatures, because resistance to indentation under static load and strength of the material run more or less parallel. Unfortunately, the standard static Brinell test is not suitable under these circumstances, mainly for the reason that it does not produce impressions of geometric similarity, so that the results are not independent of the load applied and of the impressions produced. At normal temperatures

this is not very serious; we can standardize the load applied and most of the other testing conditions. In working at high temperatures, however, we have to figure with considerable variations in the plastic properties of the materials under investigation, and we therefore cannot use a method which compares the various materials, in entirely different states of deformability, to entirely different degrees of deformation.

The writer believes that the Ludwik cone test, which does not possess these disadvantages, will be quite suitable for high-temperature tests and that Ludwik's researches on "the variation of internal friction of metals with temperature" can be extended from the non-ferrous to the ferrous field. The conical indenting tool as designed by Ludwik could be constructed from a suitable cobalt-chrome alloy. The application of the Ludwik test, in this instance, would certainly be preferable to the use of kinetic hardness tests as recently developed by Edwards in England and by Wuest and Bardenheuer in Germany. He does not believe that the various formulas at which these investigators arrived during their comparative static and kinetic ball-hardness tests at room temperatures will hold good at elevated temperatures. Furthermore, static tests such as Ludwik's will always yield data of greater accuracy and reliability than kinetic tests, because in static tests all forces applied and energy absorbed are under perfect control and measurable with accuracy.

The second routine test which is urgently needed, not only in high- but also in normal-temperature tests, is one permitting the determination of the elastic limit and of the limit of proportionality of metals exposed to impact forces. Mr. Malcolm calls attention to the important fact that it would be quite incorrect to use the tensile strength of metals at high temperatures as a basis of design. The writer believes that it would be still more dangerous, in many cases, to base the design of metal structures exposed to impact on the data of their resistance to fracture by impact. Almost nothing has been done so far in elastic-limit determinations under impact, although it is undoubtedly of the greatest practical importance to study the impact-elastic range of materials, at normal and elevated temperatures.

The third method, which the writer previously called a research method and which must be extended to the field of high temperatures, is the so-called "looping" method developed by Dalby. It is one of the most sensitive methods ever devised on the microstructure of metals and is particularly suitable for investigation of the metals in their plastic state. Dalby's methods and researches are now so well known that it will not be necessary here to go into further details. Looping tests at high temperatures, by the use of the Martens optical extensometer, have been carried out on iron and copper by Mauksch in Germany, and valuable data

have been obtained. It would be very desirable to extend these high-temperature looping tests to the alloy steels.

Kirtland Marsh.[1] In order to make the data obtained by several observers comparable, the temperature of the test specimens should be accurately determined, and in some of the apparatus described in Mr. Malcolm's articles it seems improbable that the temperature of the specimens could have been accurately measured with the equipment arranged as shown. The results, given in the symposium, of tests made by some of the observers show the temperature differences which may have existed in the test specimen and the differences in temperatures as measured by thermocouples mounted in different manners. Data on physical properties at other than room temperatures would be of much greater value if it were definitely known that the reduced section of the specimen were at a uniform temperature throughout, and that the temperatures given were the actual temperatures of the specimen. Therefore, since there may be some doubt as to the uniformity of temperatures throughout a test specimen and also as to the accuracy of the specimen temperatures as measured, every observer, not only for his own benefit but also to accredit the results of his work to others, should carefully determine the temperature gradient throughout the specimen and the accuracy with which the actual temperature of the specimen is measured.

A thermocouple measures the temperature of its hot junction, but if a temperature gradient exists close to the junction it can not be safely assumed that the hot junction is at the same temperature as another object, even closely adjacent to it, whose temperature it is desired to measure. In the case of a furnace where the holders or the specimen itself extend outside of the heating chamber, as is necessary for this work, a large amount of heat is conducted out of the furnace through the holders or specimen with the result that the temperature of the specimen easily might be 150 deg. fahr. or more below the temperature of the medium surrounding it. Under such conditions, a couple with its hot junction held in contact with the specimen would probably indicate a temperature more nearly equal to the temperature of the medium surrounding the specimen than the temperature of the specimen itself, due to conduction of heat along the wires of the thermocouple to the hot junction. This is particularly liable to obtain in the case of a couple, the hot junction of which consists of a weld at the end of a twisted section of the wires, for in such a couple the hot junction would be at the first point of electrical contact between the two thermoelements, which in some cases might be at the beginning of the twist rather than at the welded

[1] Pyrometric Engineer, Aluminum Co. of America, New Kensington, Pa.

portion. Even in the case of a couple welded without any twisting of the wires and placed in contact with the specimen there would be such poor thermal contact between the hot junction and the specimen that it is highly probable that more heat would be conducted along the wires to the hot junction than would be con-

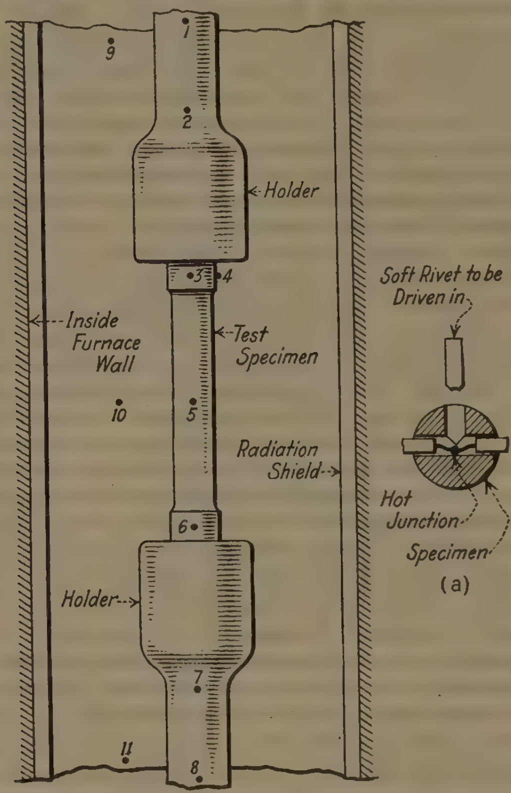

FIG. 11 LOCATION OF THERMOCOUPLES IN TEMPERATURE SURVEY OF TEST SPECIMEN

ducted from the junction to the specimen so that the junction would be at a higher temperature than the specimen.

The conduction of heat by the specimen or the holders extending outside the furnace cannot be eliminated, but it is perfectly possible to practically eliminate the conduction of heat away from that section of the specimen which it is desired to maintain at a uniform and constant temperature, namely, the reduced section. This can be accomplished if the heat, which ordinarily would be drawn from the reduced section of the specimen and conducted away by the holders or specimen extending beyond the furnace, is

otherwise furnished by providing enough heat absorbing surface between the ends of the reduced section of the specimen and the ends of the furnace or by using auxiliary heaters around that part of the holders or specimen which projects beyond the end of the furnace.

If the conduction of heat away from the specimen is eliminated, a practically uniform temperature throughout the specimen can be secured and the temperature of the specimen, after temperature equilibrium in the furnace is reached, will agree more closely with the temperature of the medium adjacent to it. A radiation shield between the heaters and specimen may be found necessary in some cases. Under these conditions the temperature of the specimen can be easily and accurately determined.

To determine if the above conditions exist, a very careful temperature survey should be made and the following general method will serve this purpose. Locate thermocouples as shown in Fig. 11 the couples at positions 1, 2, 3, 5, 6, 7, and 8 being located with the hot junction on the longitudinal axis of the specimen and inserted as illustrated at (*a*) in the same figure This latter shows a small-gage thermocouple with laid asbestos insulation and with a butt-welded hot junction. The couple is inserted in a hole, drilled diametrically through the specimen, with the hot junction on the center line of the specimen; a soft rivet driven into a second radial hole perpendicular to the first forces the hot junction firmly against the specimen thereby making good thermal contact. The asbestos insulation on the wires extends inside the hole and prevents the wires from coming in contact with the specimen and forming another junction and also helps to retard heat flow between the wires and surrounding atmosphere. This arrangement, it is believed, will measure the temperature at the center of the specimen very accurately.

Couple No. 4 is held against the specimen by as small a band as possible; the hot junction of this couple should be in the form of a bead flattened out a little with a hammer to provide greater contact surface. The bead should be in direct contact with the specimen but should be insulated from the band with a little asbestos and the two wires should touch neither the specimen nor the band. A couple attached in this way would satisfactorily measure the temperature of the specimen after the proper conditions have been attained and a comparison of the readings from couple No. 4 with readings from No. 3 during the survey will show how closely No. 4 can be relied upon to do so.

Couples Nos. 9, 10, and 11 are suspended in the medium surrounding the specimen and are for the purpose of indicating the temperature gradient from top to bottom of the furnace and to show how closely the specimen temperature agrees with the furnace temperature when equilibrium has been reached.

After the proper design of furnace has been attained there should be no temperature gradient within the specimen and only a very slight temperature difference between the specimen and the surrounding medium.

If it is deemed inadvisable to drill thermocouple holes in the holders intended for the actual physical tests, duplicate holders could be made up for the temperature survey.

In subsequent routine physical tests two thermocouples should be used, one at position 10 for furnace control and another strapped to the specimen as at position 4, to measure the temperature of the specimen. The difference between the readings from both will show when equilibrium has been reached. If auxiliary heaters are used around the ends of the holders or specimen projecting beyond the furnace two more thermocouples should be used to control the temperature in these heaters.

H. E. Moore.[1] In the paper by V. T. Malcolm, reference was made to the methods used by the Investigation of the Fatigue of Metals (University of Illinois, National Research Council, Engineering Foundation, and various coöperating firms) for making fatigue tests of metals at elevated temperatures. The results obtained to date are regarded as tentative, but as a matter of interest they are summarized in Table 1:

TABLE 1 ENDURANCE (FATIGUE) LIMITS OF STEEL AT VARIOUS TEMPERATURES

Temperature, deg. fahr.[1]	Endurance limit, lb. per sq. in.[1]	Temperature, deg. fahr.[2]	Endurance limit, lb. per sq. in.[2]
70	36,000	70	105,000
555	39,000	330	96,000
715	42,000	580	85,000
875	44,000	845	78,000

[1] 0.49 per cent carbon steel, normalized; tensile strength, 88,700 lb. per sq. in.; Brinell hardness number, 164.

[2] 1.02 per cent carbon steel, spring temper; tensile strength, 200,400 lb. per sq. in.; Brinell hardness number, 415.

The slight increase in fatigue strength up to 875 deg. fahr. for normalized 0.49 per cent carbon steel checks results obtained by Professor Lea of Birmingham, England. It is suggested as a hypothesis that within the range of temperature studied, increase of temperature has two contradictory effects: (1) Increased temperature tends to soften the steel and hence to reduce the endurance limit; (2) increased temperature tends to increase the ductility of steel and to diminish internal strain, and tends to retard or even to inhibit the formation and spread of fatigue fractures, and hence tends to increase the endurance limit. For the 0.49 per cent carbon steel the latter tendency predominates, and for the 1.02 per cent carbon steel the destructive tendency predominates. Of course,

[1] Research Professor of Engineering Materials, University of Illinois, Urbana, Ill. Mem.A.S.M.E.

further tests are necessary to give a satisfactory basis for any theory. Such tests are now in progress.

F. N. Speller.[1] Lap-welded steel pipe for steam pipe in boiler plants is now made (under the A.S.M.E. Boiler Code Specifications) of low-carbon open-hearth steel with average analysis and physical

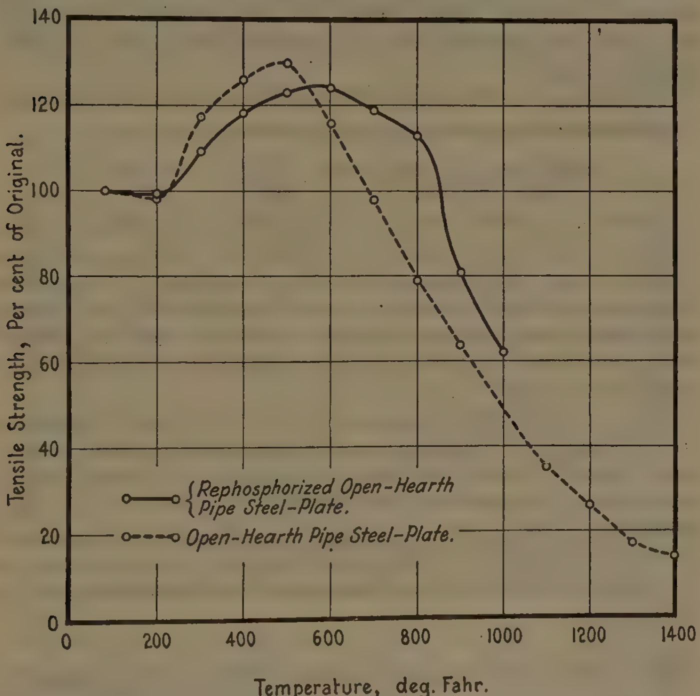

Fig. 12 Effect of Temperature on the Longitudinal Tensile Strength

properties (at normal temperature) as in Table 1, which also includes data regarding the same grade of steel to which ferro-phosphorus was added in the ladle.

TABLE I

	Chemical composition				Physical properties			
	C	Mn	S	P	Ultimate strength	Yield point	Elongation in 2 in., per cent	Reduction in area, per cent
Regular O. H.	0.09	0.44	0.034	0.013	50,690	29,730	42.0	67.7
Rephos. O. H. ...	0.09	0.43	0.050	0.103	61,120	38,785	37.0	63.8

[1] Metallurgical Engineer, National Tube Co., Pittsburgh, Pa. Mem. A.S.M.E.

The tensile strength of this steel at normal temperature compared with the strength at higher temperatures is shown in Fig. 12, expressed in percentage of the original strength at normal temperature. On the same chart is shown the strength of open-hearth steel of the same carbon contents to which ferrophosphorus has been added in the ladle. These tests indicate that the latter retains a somewhat larger proportion of its original strength at the higher temperatures without much loss of ductility or resistance to impact.

The average tensile strength of these steels at 1000 deg. fahr., as determined from this test, run as follows:

Regular basic open-hearth pipe.	26,500 lb per sq. in.
Rephosphorized pipe	36,000 " " " "

Attention is called to this, as phosphorus is one of the very few elements which can be added to welding steel without interfering with welding. Molybdenum seems to be another. In fact, the rephosphorized steel is easier and safer to forge weld, and apparently gives a sound steel equal to the regular open-hearth product with a much higher factor of safety. Endurance tests should be made on this steel.

The investigation now being carried out by the Joint Committee on Investigation of Phosphorus and Sulphur in Steel should determine whether in fact these elements have any detrimental effect when added to low-carbon steel which is originally low in these elements. If not, in the interest of all concerned the question of revising American standard specifications with reference to the sulphur and phosphorus limits should be considered without further delay.

H. A. Schwartz.[1] The writer offers in Fig. 13 the results of tests made under his supervision by Messrs. W. W. Flagle and C. S. Fuller, which show the effect of low temperature upon the impact resistance of commercial malleable cast iron. These tests were made in connection with a problem which focused our attention upon the lower temperatures. They serve as a comparison, however, throughout at least part of the temperature range with the data of French and Tucker's Figs. 13 and 14.

It may be added that the results have been confirmed as typical of normal malleable by much other work in this laboratory. It is possible to produce malleable cast iron of still higher impact resistance by special methods, and equally possible to produce inferior material which suffers much more rapidly from brittleness as the temperature is lowered.

[1] Research Department, National Malleable Castings Company, Cleveland, Ohio. Mem.A.S.M.E.

S. R. Puffer.[1] In the various tests reported of the physical properties of steel at high temperatures, were the test specimens machined before or after heat treating? In other words, was any difficulty encountered in machining steel having high physical properties?

In modern superheated-steam and gas-turbine practice, it is very essential that the turbine buckets retain high physical properties at high temperatures. Dovetail shapes on such buckets are often very complicated. If heat treating is done after machining, hardening cracks are likely to be introduced. If machining is

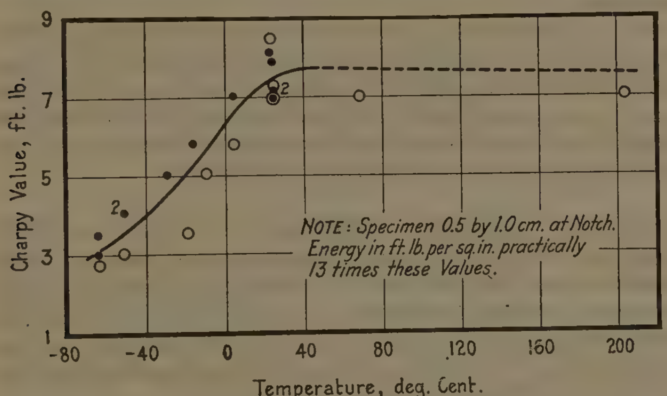

Fig. 13 Notched-Bar Impact Resistance of Normal Malleable Cast Iron

Open and solid circles represent metal from different sources.

done after heat treating, will it not be necessary, in order to do the machining, to sacrifice a large percentage of the strength which might be available?

In Fig. 14 of the paper by French and Tucker, it is evident that high-nickel-chromium steel has excellent resistance to impact. Are any data available as to the tensile properties of this material at temperatures above 900 deg. fahr.?

In Fig. 8 of the same paper, are shown two samples of tungsten high-speed steel, classified as B_1 and B_2, which show excellent properties at high temperatures. We understand that they have no elongation or reduction, even at the high temperatures, and are so hard that they could not be machined except by grinding. Is this correct?

Are any data available as to the high-temperature tensile properties of the high-nickel-chromium alloys, such as nichrome, chromel, etc.?

[1] Thomson Laboratory, General Electric Co., Lynn, Mass.

JEROME STRAUSS.[1] The authors in their brief discussion of the chemical stability of steels and associated metals have touched upon a subject of extreme importance not only to the general engineering profession as we normally visualize it but of particular importance to the chemical engineer and his associates. Increased application, for production purposes, of high temperature processes involving the reaction or production of chemically active materials, and the extension of the temperature and pressure ranges of these processes, have forced upon metallurgists the development of materials for progressively increased utility in these fields.

Even the service that has heretofore been obtained from ordinary metallic containers at atmospheric temperature no longer satisfies the requirements of continuous economical production. And in many cases metals have been required to withstand the action of corrosives through cyclic variations of temperature, pressure, and concentration over rather wide ranges.

FRANK A. FAHRENWALD.[2] Referring to the paper by Messrs. French and Tucker, the writer believes greater emphasis should be placed upon the time factor as affecting the working strength of material at high temperature. The time factor as developed under tests running for only a short time does not give the effective strength of the material when subjected to stresses at temperatures above the recrystallization point.

Much of the information that has been hoped for and suggested by the various authors of these papers has already been worked out and has been available for some years to the trade. The physical strength of heat resisting alloys at elevated temperatures has been determined in terms of permissible safe-load stresses for use in design.

This property apparently has nothing to do with the elastic limit, nor is the modulus of elasticity in any way involved, and it seems that the fundamentals which govern the flow of viscous materials—such as ordinary road paving pitch—have more to do with the behavior of these alloys at high temperatures than do the factors which we ordinarily associate with metals and alloys. A piece of road-paving pitch, placed between thumb and finger, can be slowly flattened with steadily applied pressure but if hit by a sharp blow with a hard object it will immediately fly to pieces like glass.

This same type of behavior seems to be common to metals and alloys as well. The rate of application of the load and the time during which it is applied are more important in determining the

[1] Material Engineer, Naval Gun Factory, Washington, D. C.
[2] Consulting Engineer, Cleveland, Ohio.

ability of an alloy to resist stress at high temperatures than all the other factors involved.

The writer has determined the high-temperature characteristics of various steels, and particularly of alloys for resisting chemical corrosion and mechanical stress at high temperatures, and while some of this information is bound up in proprietary interests and professional obligations, most of it is available to any one who cares to ask for it.

At high temperatures the relationship between the apparent mechanical strength of a metal or alloy as revealed in ordinary tensile tests, compared with the ability of the same material to resist continuously applied stress, is truly surprising.

At 1750 deg., for instance, the strength of the nickel-chromium alloy, under a quick-pull test, will be more than fifty times that under a stress extending over a period of a year.

Most of the data that the writer has developed in this line are not taken from laboratory tests, but have been interpreted from practical commercial operations and with this information it is possible to design beams or structural members for operation at any given temperature up to, say, 2200 deg. with the same assurance of success that obtains in the design of ordinary mechanical structures.

Thermal expansion is perhaps one of the most powerful and destructive agencies encountered in mechanical operations at high temperatures, due to dimensional changes that accompany changes in temperature.

In even the most simple mechanisms it is almost impossible to prevent temperature differentials of from ten to several hundred degrees between one point and another on the same alloy unit and as a result the cold part is under tension and the hot area under pressure with resulting plastic flow under either tension or compression, followed by a reversal of stresses perhaps with further temperature changes and final failure. Here is a problem of fatigue from alternating compression and tension beyond the plastic deformation limits of the material, and whether this corresponds to fatigue as we ordinarily understand it the writer does not know.

The problem of the application of metals and alloys at high temperatures is far more complicated and difficult than is ordinarily supposed. If physical strength or chemical resistance or thermal expansion or elastic limit or any one single factor is considered without correlating it to all of the other factors of the problem, failure will result.

These remarks may confuse the issue rather than clear it up, but this phase of engineering is indeed very complicated and anything that will serve to call attention to the need for considering and correlating the numerous essential factors will be of help.

A. G. Christie.[1] The summary of the principal published work on the strength of non-ferrous alloys at various temperatures as presented by the authors indicates the increasing demand for alloys which will exhibit properties required by designers of apparatus which is stressed at high temperatures. The ideal condition as regards the strength of the material used in apparatus, such as valves and fittings, operating under superheated steam conditions would be that the physical properties remain constant from room temperature up to some point above the range of the temperature of operation. It is obvious that with such material the designers could be assured that no failure due to weakness would develop when the temperature is increased from normal to 800 or 900 deg. fahr., which is above present operating temperatures.

In connection with securing data on such properties, it should be noted that where the curve of the elastic limit is falling off rapidly with increasing temperature, a slight experimental error in the measurement of the true temperature of the specimen affects its value for a given temperature to a very great degree. While allowance could be made by the designer for the decrease in strength of a material in which this property is affected by an increase in temperature as is the case in many steels and ferrous alloys, it is apparent from a study of the result of methods of testing and of values presented for various materials of this type that it is difficult to arrive at the exact amount of decrease of strength for a given temperature. Hence, it would be very much safer to use a material for which the curve of the elastic limit is nearly flat throughout the range of working temperatures.

Other considerations governing the choice of material by the designer or operator for valves and related parts subject to high temperatures are freedom from oxidation or corrosion and ability to grind the seat with facility.

N. L. Mochel.[2] Referring to the paper by Professors Upthegrove and White, it may be of interest for us to record a peculiar type of failure which is apt to take place in the use of copper-tin alloys at elevated temperatures. A number of curves are given for copper-tin and copper-tin-zinc alloys, and in general there is a marked change, a sudden drop or a more rapid falling off in strength in the neighborhood of 450 to 500 deg. fahr.

Fig. 14 is a micrograph ($\times 75$) of a specimen of drawn phosphor bronze, containing 2 per cent tin and low phosphorus, after service for one year at 650 deg. fahr. The stresses were quite low. There is a peculiar intercrystalline action which has taken

[1] Consulting Engineer, Curtis Bay Copper and Iron Works; Professor, Mechanical Engineering, Johns Hopkins University, Baltimore, Md. Mem. A.S.M.E.

[2] Metallurgical Engineer, Westinghouse Elec. & Mfg. Co., Philadelphia, Pa.

place at the surface and is rapidly growing inward. The same condition has been observed on similar material after service at 500 deg. fahr. The action has not been limited entirely to the drawn material, but has been observed as well on cast bronze of the 88-10-2 type, resulting in a falling away or deterioration of the metal and in the carrying away of " chunks " of the material, or its absolute failure. The action seems to be peculiar to those alloys of copper with low-melting-point materials such as tin, although quite similar deterioration has been reported with copper-aluminum alloys at approximately 500 deg. fahr. The action is marked by an embrittling of the affected material.

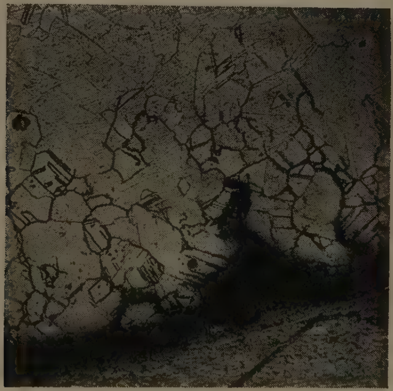

FIG. 14 MICROGRAPH OF SPECIMEN OF DRAWN PHOSPHOR BRONZE

The short-time tests may show fair strength for certain materials of the type mentioned above, at temperatures above 500 deg. fahr., but deterioration is an item and must be considered.

It is also felt that a valuable and interesting addition to the bibliography of the paper would be a paper presented before the Institute of Metals, in March, 1924, by Bunting, on The Brittle Ranges in Brass. It has been summarized as follows:

The brittle ranges exist in brasses of composition varying from 90 to 52 per cent copper. The brittle range of the 52 per cent alloy extends from 220 to 540 deg. cent., and as the percentage of copper increases the range reaches a minimum at 57.5 per cent copper, extending from 320 to 450 deg. cent. With further increase of copper the range extends upward indefinitely, and at 65 per cent exists from 325 deg. cent. until the solidus is entered. At 75 per cent an upper limit to the range is once more observed, the range now extending from 350 to 725 deg. cent. Beyond this point the lower limit (hitherto practically constant at 325 deg. cent.) rises until at 80 per cent the range extends from 430 to 630 deg. cent. The range now narrows, and finally terminates in the neighborhood of 90 per cent.

George K. Elliott.[1] There is need of close coöperation between chemist and metallurgist in our present problem of metals for high-pressure and high-temperature steam. At present the metallurgist is given little other information than the temperature and the pressure at which the central station is to be operated, and possibly this information is sufficient, but the writer for one would like to have fuller data concerning the chemical composition of this new steam which our metals are to handle.

Endless literature has been written about the chemistry of the boiler and boiler waters and the most of it is of great value, especially in studying corrosion, but if we search for information concerning the chemical reactions which do or are likely to take place in boilers operating at the new pressures of 400, 600, 900 lb. per sq. in. or even higher, we are doomed to disappointment; little is available. The reason for laying stress on this is that the writer feels insufficiently assured that the chemical reactions taking place in water containing certain dissolved salts and gases, at 100 lb. pressure and 337 deg. fahr., are going to take place when the pressure is raised to 400 or 600 lb. and the temperature to 750 deg. fahr. Will the steam generated under the new conditions contain compounds which were not present in the vapor from the old-fashioned boilers?

Only one possible but admittedly speculative condition will be given as an example. Some investigating chemist in England, I believe, has made somewhat of a study of this new boiler chemistry and is on record as having evidence that alkaline boiler water containing sodium carbonate, for example, under certain conditions of concentration, pressure, and temperature, will react to form a series of organic acids such as formic, glycollic, and others of a series made by successive deductions of an atom of oxygen from the compound. This is extremely interesting if true, and even more so when we are told that there is a possibility that volatile organic compounds of a corroding nature such as formaldehyde may be formed in the boiler. If, therefore, it is discovered that corrosive compounds of a kind hitherto unknown to boiler chemistry are likely to be formed in high-pressure boilers, the metallurgist should have definite information concerning the exact nature of these new ingredients in the steam to be handled by the metallic piping, valves, fittings, and turbines.

It may be that there is no real cause for alarm, but the writer would like to see the question of boiler-water chemistry thoroughly studied by competent chemists, preferably organic chemists, since the reactions foreseen are of a decidedly organic chemical nature. There is a probability that the new high-pressure boilers are chemical manufacturing units, operating on the dissolved sub-

[1] Chief Metallurgist, The Lunkenheimer Co., Cincinnati, Ohio.

stances of the water, in which a great number of complex reactions probably take place with the production of many compounds, some of which may well be viewed with suspicion by the metallurgist who is prescribing metals to handle these chemical products when mixed with steam.

Once the program of purely chemical research is completed, and the chemical nature of the impurities in steam generated at high pressures and then superheated is determined — if such impurities be found — obviously the next step would be to conduct high-temperature physical tests with the test pieces immersed in atmospheres which are similar to the steam we are describing. This would add immeasurably to the labor and time necessary for making these tests, especially the time, for time would be important directly in proportion to its duration; but the results might well prove to be worth all the trouble multiplied many times. The writer is not ready to predict how important the matter of surrounding atmosphere may be in making these high-temperature tests upon all the metals and alloys now used in handling steam, but it has been demonstrated by Bengough and Hanson that in the matter of copper it is of the greatest importance. So-called season cracking, so frequently met with in wrought non-ferrous metal, also is somewhat related to the point of this discussion. It was shown some years ago that this kind of metal failure has its beginning with surface corrosion of the piece under stress, the corrosion being caused often by gaseous impurities of a corrosive nature in the surrounding atmosphere.

To sum up, the writer suggests the following research:

1 That chemical reactions in high-pressure boiler water and in superheated steam, up to 800 deg. fahr. be investigated

2 That, if corrosive compounds are found to be a possibility in such steam, high-temperature physical tests be made with test pieces surrounded by an atmosphere similar to this steam.

R. S. MacPherran.[1] As many plants are operating with steam at from 800 to 850 deg. fahr., it is necessary for us to know the properties of materials at these temperatures. Until recently, most high-temperature testing was done by heating the specimen until the desired temperature was reached, and then running the usual tension test. As referred to in these papers, however, Mr. Dickenson has made a series of most interesting tests by maintaining a constant load and a constant temperature until failure of specimen. We are preparing to make tests along these lines and hope to work under various temperatures and loads.

We are now making some tests along a little different line by holding the specimen at constant load and slowly increasing the

[1] Chief Chemist, Allis-Chalmers Mfg. Co., Milwaukee, Wis.

temperature until failure occurs. In each test, the specimen was held at a definite temperature until the beam remained in balance for 15 minutes. This would allow for any expansion due to increased temperature, and for any extension which might take place in this short period. The final period or period of maximum temperature before failure at this increasing temperature and constant load lasted several hours or more. For example, one specimen was held at 1000 deg. fahr. under a load of two-thirds of its elastic limit for over 15 hours before it finally failed. The load was maintained, of course, during the entire test by keeping the beam in balance as the specimen elongated. While our results on these tests have been very interesting we are not yet in a position to report.

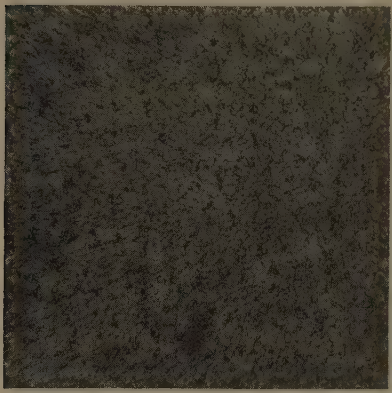

Fig. 15

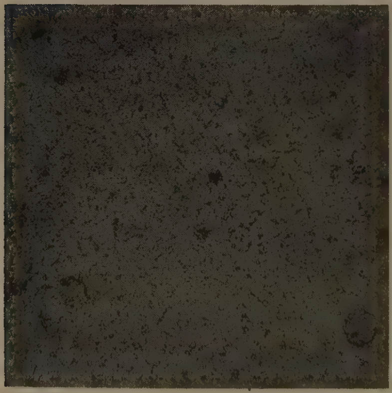

Fig. 16

We would be much interested in learning how the various bronze test specimens were prepared. Were they cast or rolled? And was any trouble found in obtaining a uniform material? We have cast several sets of bronze bars for high temperature tests but find great difficulty in obtaining specimen of the necessary uniformity for a series of these tests.

One of the most interesting ideas in this discussion is that advanced by Mr. Elliott.[1] We have all seen examples of steam corrosion for which there seemed to be no explanation or definite cause. It is possible that investigation of the chemical combinations formed in the boiler under high temperatures and pressures may lead to the solution of some of these problems.

[1] See preceding discussion.

J. C. LINCOLN.[1] This contribution to the discussion is at the request of the American Welding Society. It has to do with the action of metal deposited by the metallic arc and the effect of repeated heating on such metal. The metal deposited by the metallic arc process has a tensile strength of about 50,000 lb. per sq. in., if properly deposited. The ductility of this metal is low, about 5 per cent.

Repeated heating of such weld metal in an oxidizing atmosphere at a temperature of about 1550 deg. fahr. changes the structure of the weld metal and decreases both its strength and ductility. So far the writer has been unable to assign the cause for the change in structure and for the change in physical properties. If anyone has done any work along this line and can throw some light on this problem, he will be doing the art of electric welding a service.

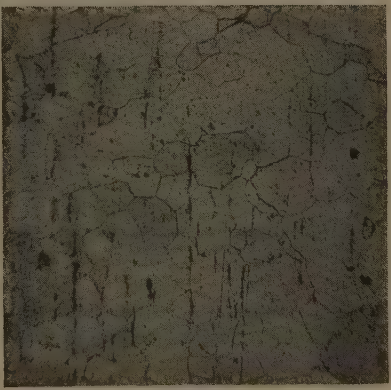

FIG. 17

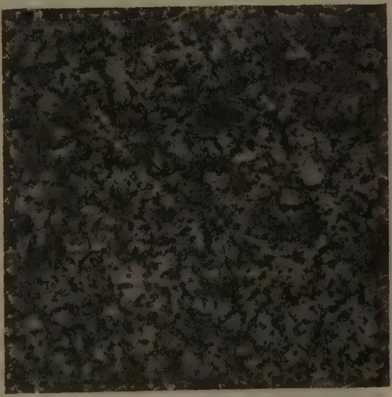

FIG. 18

Fig. 15 is a microphotograph of metal deposited by the metallic arc after polishing and etching.

Fig. 16 is a microphotograph of the same metal after repeated heating at about 1500 deg. fahr.

Fig. 17 is a microphotograph of ordinary open-hearth low-carbon steel after exposure to the same heating and cooling that produced the change in structure shown in Fig. 16.

Fig. 18 is a microphotograph of the same sample shown in Fig. 16, except that the magnification is 200 diameters instead of 100 diameters.

[1] Lincoln Electric Company, Cleveland, Ohio.

Sanford A. Moss.[1] The temperatures of 700 to 800 deg. mentioned in the papers are really quite moderate. Mr. Emmet[2] mentioned temperatures of 1000 deg. fahr. in connection with the mercury turbine. There now exist a number of gas turbines in series with internal-combustion engines, with the wheel rotating at high speeds, and operating at temperatures of about 1000 deg. fahr. Hence the authors of the various papers must not stop at 700 or 800 deg. fahr. but must go on.

One important point is proof that the specimen has the temperature alleged. Many of the tests in the bibliography cited are worthless because there is nothing to show that they do not fall in the large group of tests with erroneous temperature measurements. One way to be certain that the specimen has the supposed temperature is to make two separate sets of tests with two separate furnaces, one with a certain length of furnace and specimen, and the other with the same length of specimen and a longer furnace. If the same results are obtained reliance can be placed on them. No high-temperature tests can be given credence unless there is given actual proof in some such way of the validity of the temperatures.

It is quite possible that the time element which has been mentioned is largely a matter of thermal equilibrium. In other words, it may be that some of the experimenters who have thought that their results showed the effect of the time element really did not have conditions of thermal equilibrium originally, so that the effect was one of temperature measurement error only. The certainty of the time element can be established only when it is first proved that the temperature of the specimens is the temperature alleged.

Very little attention has been paid to the hardening and drawing temperatures for the ferrous materials tested. It is probable that, in order to secure the best performance at given high temperature, there must be quite a different combination of hardening and drawing temperatures than for some other temperatures. This, of course, opens up a vast field for research. In other words, it is possible that for every temperature of use of ferrous materials, there must be a special hardening and drawing temperature.

The matter of the effect of the atmosphere on the testing has been mentioned. It is sufficiently difficult to make tests at high temperature without the added complication of having to maintain a certain atmosphere. The only materials in which we are interested for high temperatures, are materials which will not be affected by the atmosphere. An independent set of tests could be

[1] Engineer, Mechanical Research Department, General Electric Company, West Lynn, Mass. Mem.A.S.M.E.

[2] Paper No. 1924, p. 285.

made at high temperature in a given atmosphere, without any tension tests, simply to establish as to whether or not the material was affected. If it is found that the material is affected at the temperature involved, there is really little use of making further tests. If it is found that the material is not affected, then the high-temperature tensile tests may be made.

NEVIN E. FUNK.[1] This symposium adds to our knowledge of the performance of materials at high temperatures, but the writer questions the advisability of using the exact values given on account of the fact brought up by other discussors, that it may not be possible to duplicate the results with different apparatus.

Since we are following practice with knowledge rather than knowledge with practice, and since the possibilities of considerably higher temperatures are of great interest in obtaining better efficiencies, material manufacturers should not consider the problem solved, but should endeavor to produce materials that will withstand these high temperatures better than the ones that are now available.

This symposium apparently does not contain information as to the effect of temperature variations over a continuous period on the performance of these materials.

It is generally known that cast iron, unfortunately, grows at temperatures as low as 550 so that in, say, eight years' time, parts made of that material must be replaced. With these higher temperatures, the same thing may happen to steel or steel alloys. We have not yet had enough practical experience to know, but the field should be investigated.

The effect of oxidation on these metals at high temperature has been touched upon. If the higher temperatures produce more rapid corrosion, the factor of safety may soon become inadequate.

While all the information that has been presented is valuable as an indication of the trend of the subject, so far as the writer's personal feeling is concerned, only a beginning has been made toward the developments necessary to the intelligent design of higher temperature stations and the certainty that the life and safety of these stations will be as accurately anticipated.

GEO. A. ORROK.[2] From the standpoint of the power engineer this subject presents three distinct problems dealing with (1) the turbine blade and the interior of the turbine, which remain at practically one temperature as long as the machine runs; (2) the boiler, whose external temperature may be quite high while its internal temperature is limited to the steam temperature used;

[1] Operating Engineer, Philadelphia Electric Company, Philadelphia, Pa. Mem.A.S.M.E.

[2] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

and (3) the steam pipe, in which the internal temperature is fixed while that outside varies.

In regard to the first of these problems, materials which we have been using apparently stand any temperatures that have been attained so far and probably materials can be obtained which will permit turbine blades to be operated up to 800, possibly 900 deg. Whether we shall be able to find materials for the gas turbine with its higher temperatures is something to be worked out.

As for the boiler, the outside of the metal may be heated to almost any degree. The writer recently had the opportunity of looking into a boiler where the internal surfaces were at a bright red heat. Probably the temperature of those internal surfaces was 900 or 1000 deg. since the temperature inside the boiler was about 850 deg. What happens in such cases is not known and much good work must be done before reliable information can be obtained.

Regarding the steam piping, it appears that materials are fairly well understood and perhaps pipe manufacturers will be able to provide almost anything that may be needed in the next twenty-five or thirty years.

One of the particular things in this kind of research is getting specimens of material which are alike and in duplicating them in the actual material that we buy to put into our plants. The writer is very certain that most of these specimens which have been tested, while ostensibly of a reasonably close chemical composition, are not alike, and probably two pieces cut from the same bar will show rather wide variations, both in chemical composition and in crystalline structure.

Ernest L. Robinson.[1] The writer calls attention to a point which has not been emphasized elsewhere in the symposium, namely, the importance of the effect of temperature on the modulus of elasticity. The other discussions deal with questions of strength, proportional limit, and extension, but in tuning a turbine wheel so as to control its natural frequencies of vibration, it is important to have information as to the modulus of elasticity. We have thought heretofore that, throughout the temperature range to which turbine wheels are subjected, the change in modulus of elasticity is so gradual as to be of little importance. But some of the curves presented in these papers show rather sudden changes in the modulus. If these indications are substantiated, it will be important to give attention to the modulus in future investigations of the effect of temperature on steels.

[1] Turbine Engineering Department, General Electric Company, Schenectady, N. Y. Mem.A.S.M.E.

C. C. Trump.[1] To Mr. Orrok's three and Mr. Robinson's fourth problems the writer would like to add a fifth. That is the problem of the oil refinery which has been referred to already in the papers. We not only have boilers, turbines and piping for steam but we also have stills which carry oil vapors at increasingly high temperatures and pressures. Within those oil vapors are associated not only high temperature and pressure, but also corrosive media such as sulphureted hydrogen and other gases which at those temperatures do attack the metals. In fact, they attack them to such an extent that we have to make our pipes thick enough to withstand a considerable reduction in thickness. Our problem of strength is one of years. We want to know about strengths of materials at high pressures, and we hope that there will be further information coming along these lines.

Another point in which we are interested is the growth of materials other than cast iron, especially of the non-ferrous metals, because we sometimes want to use non-ferrous alloys in our valves and fittings. There is nothing in any of these papers concerning the growth of materials other than cast iron.

Samuel L. Hoyt.[2] The writer would like to point out a certain relationship which he believes to be a general one connecting the load on a specimen and the life of the specimen at that load.

He first experimented on tin, a metal which is fairly soft and plastic at room temperature, with loads varying over a considerable range, so that the time element varied from less than one second to over a million seconds. In order to interpret the data he plotted them two or three different ways and finally found that if the logarithm of the time were plotted against the square root of the load the result was a straight line. That such a straight-line relationship is desirable can be shown very briefly.

According to one method of plotting the data, all of the points came on a smooth curve. When the data were plotted as a straight line it was noticed that the time for some of the loads was twice what it should be, according to the straight-line relationship. By examining the samples it was seen that they had fractured according to a different method than the samples which had longer lives. This indicates that tin extends and fractures according to two different mechanisms, depending on the rate of deformation.

The writer has also noticed a difference between cold-worked metals and annealed metals in the way they follow this relationship, and, further, that the relationship holds over a wide range of temperature. He has examined tungsten at high temperatures

[1] Engineer of Tests, Atlantic Refining Co., Philadelphia, Pa. Assoc-Mem.A.S.M.E.

[2] Research Laboratory, General Electric Co., Schenectady, N. Y.

and is beginning to examine iron and other metals at a somewhat lower temperature. From these observations the relationship seems to be a general one.

The writer suggests that those who are interested in this particular feature plot their data as the square root of the load versus the log of the time. In that way, if there are any exceptional conditions present they will probably be brought out at once by the fact that the points do not fall on the curve or do not come as close to the curve as they should according to the experimental error involved.

A long-time test at a high temperature is not a simple thing, but if we knew that this relationship held, it would be possible to extrapolate from high loads, in order to get the probable life of low loads.

R. L. Templin.[1] We have been taking temperature tension tests on aluminum and some of the alloys of aluminum from time to time during the past five years. In general our results on pure aluminum check fairly well those given in the symposium.

In connection with such tests it should be noted that in the material which was used by Professors Upthegrove and White there existed an appreciable amount of cold work. If the material had been annealed to start with, the tensile strength curves would not be parallel to the curve given in their paper. If the plotting of the data, however, is done so as to give the ratio of the tensile strength at any high temperature to tensile strength at room temperature, there would be fairly close agreement of results. That is, differences due to different amounts of cold working and perhaps even variations in composition of the material will be practically eliminated by such a method of treatment, usually in a very satisfactory manner.

It has been observed that tests on the cold-worked metal tend to give higher values for the range of temperature in which we are normally interested for design purposes. It is thought, however, that if the specimens were maintained at these higher temperatures for a considerable period of time, varying with the temperature, the values obtained would approach those which are normally obtained, starting with annealed material.

In addition to the work that has been done on pure aluminum we have done some work comparatively recently on pure magnesium and one of its alloys. In connection with these tests, it has been interesting to note that while we obtain one set of data with pure magnesium and another set of data with a magnesium alloy consisting approximately of 4 per cent aluminum, yet when treated in the manner just indicated, the tensile strength results are identical.

[1] Chief Engr. of Tests, Aluminum Co. of America, New Kensington, Pa.

In applying these data in a practical way our inclination is first to evaluate the effects of temperature upon the material, then for design purposes to use in our formula the annealed tensile strength of the material at room temperature and show the effect on it rather than to take the tensile strength of some harder or cold worked metal and depend upon that as the basic value.

We have run into serious difficulties in our methods of testing because the materials with which we have been concerned have a rather high thermal conductivity. This, of course, tends to decrease the difference in temperature that exists between the center of the specimen, say, and the outside surface, but at the same time it is harder to keep the temperature uniform throughout the length of the specimen. Again, the values for ductility or elongation are rather high in these materials, running sometimes over 200 per cent. That means that furnaces used in testing them must be quite long in order to maintain the specimens at the desired temperatures throughout.

We ordinarily consider these data as being roughly divided into two phases. The first is the one which extends usually to about 400 deg. cent. Temperatures from below room temperature to this point are the ones which concern the designer. Those beyond that point are usually of prime interest only to the manufacturer of such materials.

Zay Jeffries.[1] Probably in all the fields for the use of materials at high temperature there is none in which so much effort has been spent as in the electric-lamp field. Artificial illumination with incandescent lamps depends upon the maintenance of a body at a very high temperature and for a considerable time. The tungsten filaments used in incandescent electric lamps sometimes reach a temperature of 2900 deg. cent. and are maintained at that temperature over a period of at least 100 hr., and some lamps with a temperature of 2700 deg. cent. may maintain their temperature for 1000 hr.

The material used is nearly a pure metal, about 99.9 per cent tungsten. The writer calls attention to the effect of grain size on the maintenance of the characteristics of the metal at a high temperature. If the grains are maintained small at the high temperature the filament sags during the course of its use and the coils get out of shape. If a special treatment is made so that a larger grain size is produced, the elastic limit of the material is relatively high at a temperature just under the melting point and the coils maintain their positions. That has been found generally true, not only in tungsten but in other metals, and would indicate that at very high temperatures we should strive to obtain large grains in pure metals in order to maintain permanency of shape of the material.

[1] Metallurgical Engr., Research Bureau, Aluminum Company of America, Cleveland, Ohio.

There are so many complications in the temperature effects of pure metals that one hesitates to bring in the added complications of alloys, but they must be considered because they are important commercially. In alloys, the inter-metallic compounds are the hard constituents at high temperatures. They correspond in high-temperature properties more to the non-metallic substances like fire clay or silica. They are the hard bricks which strengthen the soft metallic matrix in materials at high temperatures.

The writer compliments Mr. Wilhelm upon the determination of the proportional limit and the modulus of elasticity of steel at various temperatures. The subject of the modulus is one which has been worrying people for a long time, and the true proportional limit is one which is masked by the blue heat effect in iron. His results are certainly the best that have been seen in that field, and the writer looks forward to seeing further results from the same apparatus.

In conclusion the writer suggests that the general research be divided into two groups, as follows: First, the fundamental properties of materials at high temperatures, which may be carried on by the metallurgist and chemist; and second, the use of materials at high temperatures. The latter study may involve tests simulating use tests, and should be made by the engineer who is in close contact with the actual utilization of the metals at high temperature.

John A. Mathews.[1] As manufacturers of these materials, we have found some which proved to be tough at the lowest temperatures and malleable at liquid air temperature, while others showed only an oxidation film (similar to the temper colors of a tool) at 2200 deg. fahr. We have worked out a series of them for various conditions of corrosion, a point that has been only slightly touched upon in this symposium.

There is no one material that will answer all purposes. For each of the milder acids—acetic, formic, lactic and butyric—we must meet the condition of maximum serviceability through special compositions. We have studied our products one at a time to attain maximum resistance to corrosion for scores of reagents under various concentrations and temperatures. In some cases a single material answers for many of them but not for all. For use at temperatures from 700 to 1600 deg. fahr. these nickel-chromium-silicon alloys, known under the trade name of "Rezistal," seem to be stronger, tougher and more resistant to fatigue than any other type of ferrous or non-ferrous alloy.

[1] Vice-President, Crucible Steel Company of America, New York, N. Y.

CLOSURES TO DISCUSSION[1]

L. W. Spring. Several discussions have referred to possible inaccuracy of results due to uneven heating of the specimens, loss of heat through conduction, difficulties of temperature measurement, etc. The author believes that no one realizes more than the investigator himself, who has tried to do some of these things and to do them as accurately as he could, the difficulties that are in the way of obtaining accurate physical properties of materials at high temperatures. One would be unwise to claim that any of the work done is perfect. Since all who have worked in the high-temperature field have proceeded along more or less different lines, and since, therefore, there has never been anything like a standard method of making such tests, it is highly desirable that some properly formed committee very carefully work upon and determine the most satisfactory method or methods of high-temperature testing, so that, hereafter, such routine testing may be done according to something like standard methods.

A point was made by Mr. Holz regarding the possible effect of magnetic induction upon the results. Years ago in our laboratory we made tests along that line, using coils that were exact duplicates, except that one was our usual heating coil of nickel-chromium or nickel wire and the other coil was wound with pure copper wire, which gave only 12 deg. fahr. rise in the bar, when using much greater amperage than we ever applied in actual work. These results were reported in 1913[2] but are repeated in Table 1. As is shown by the figures, on neither brass, bronze, ferro steel, nor cast iron do results differ from results obtained without any coil at all, showing that the effect of induction is negligible.

Mr. Morrison referred to insulation of the holders. The author believes that such has not been done commonly, so the holders and heads of the testing machines are constantly conducting away considerable amounts of heat, which, of course, means less equal temperature throughout the length of the bar or at least difficulty in maintaining equal temperatures. Of recent years we have interposed horizontal disks of asbestos ½ in. thick between the ends of the bar and the machine heads as shown in Fig. 9 of the paper by Mr. Malcolm (see p. 369). These disks are held between 6-in. cast steel flanges, the bolts of which are also insulated from contact with the flanges by asbestos winding.

[1] Authors' closures to Papers Nos. 1926*a*, p. 351; 1926*b*, p. 356; 1926*c*, p. 399; and 1926*d*, p. 433.

[2] *Valve World,* vol. 10 (1913), no. 1, p. 3; also references (83) and (216) of the bibliography (see pp. 481 and 487).

TABLE I EFFECT OF ELECTRICAL CURRENT WITHOUT HEAT ON STRENGTH OF TEST BARS

Material	Test		Tensile strength, lb. per sq. in.	Elastic limit, lb. per sq. in.	Elongation in 2 in. per cent	Reduction of area, per cent
Crane Valve Brass	Test I[a]		30,800	19,900	12.5	21.8
			30,800	18,900	18.8	19.2
		Average	30,800	19,400	15.7	20.5
	Test II[b]		31,600	18,775	15.6	19.6
			30,800	19,150	15.6	19.3
		Average	31,200	18,960	15.6	19.5
	Test III[c]		30,120	18,180	12.5	....
		Average	30,120	18,180	12.5	....
	Test IV[d]		32,300	21,000	15.6	18.9
			32,000	19,600	14.1	16.5
		Average	32,150	20,300	14.9	17.7
Crane Hard Metal	Test I[a]		38,500	23,430	7.8	14.6
			36,700	26,010	9.4	16.3
			34,180	27,000	9.4	15.1
		Average	36,480	25,480	8.9	15.3
	Test II[e]		35,350	....	9.4	....
			36,950	23,130	9.4	13.9
			37,100	26,745	12.5	16.6
			34,200	27,600	6.3	9.9
		Average	35,910	25,825	9.4	13.5
Crane Ferro Steel	Test I[a]		39,135			
			35,130			
			32,710			
			36,920			
			37,210			
		Average	36,220			
	Test II[e]		38,640			
			40,270			
			35,900			
			38,000			
			34,650			
		Average	37,490			
Crane Cast Iron	Test I[a]		23,700			
			21,960			
		Average	22,830			
	Test II[e]		21,750			
			23,800			
		Average	22,775			

[a] Without coil.
[b] With copper wire coil; ½ ampere, 690 ampere-turns. Broke at once.
[c] With copper wire coil; ½ ampere, 690 ampere-turns. Current on 5 hours.
[d] With copper wire coil; 2½ amperes, 3450 ampere-turns. Broke at once.
[e] With copper wire coil; 1500 ampere-turns.

In many of our tests the bottom of the hole which is drilled axially into the bar, as shown in Fig. 8 of Mr. Malcolm's paper (see p. 368), contains a few drops of metallic mercury to insure perfect contact of the pyrometer tip with the bar itself. This

eliminates any possibility of getting incorrect readings because of poor contact.

We have been using the gap-wound coil, also shown in Fig. 8. With coils wound over their full length we found it almost impossible to avoid a higher temperature in the center of the breaking section of the test bar than at the ends. Part of this variation may be attributed to conduction or radiation of heat from the ends of the test bar. By using a coil with a 3-in. gap over the breaking section of a calibration bar drilled axially all the way down to the lower shoulder, the pyrometer tip showed very close temperature readings at the lower shoulder, the center of the breaking section, and the upper shoulder where temperatures usually were taken.

V. T. Malcolm. The author has been very much interested in the various comments regarding the details of the test methods and in the several suggestions that have been made. Mr. Wilhelm is to be congratulated on the thoroughness of his work in this field of research. However, we must keep in mind that our tests must be as simple as possible because they are destined to become routine tests in the laboratories of the producers of high-temperature materials. In fact, this is true to some extent today; certain specifications now require that several test bars from each lot of steel of 200 lb. or over be tested at elevated temperatures. There is considerable difference between research and routine testing. For routine work apparatus must be developed that is accurate within certain limits and with which tests at elevated temperatures can be readily made.

In the entire discussion very little attention was given to the condition of the material before test. The author believes this is one of the most important points to be taken into consideration if reliable results are to be obtained, for the reason that inclusions, gas pockets, segregation, etc., tend to give false results. The structural composition, both before and after testing at various temperatures, should be investigated, because we know that structural changes take place at elevated temperatures, especially in the non-ferrous materials. Elevated temperatures, combined with corrosion or a cycle of normal temperatures, then elevated temperatures and back to normal, will give results quite different in service than with the use of elevated temperatures alone. Laboratory tests should be compared with actual service conditions and the results carefully studied and tabulated for use.

Mr. Lester's remarks regarding the X-ray method of test are quite pertinent. The author is personally familiar with Mr. Lester's work, and is in a position to appreciate the value of the X-ray as applied to testing of steel.

Mr. Holz's references to certain apparatus, and especially his remarks regarding routine testing, are to the point, and the

author hopes that he may have the opportunity of studying the methods described by Mr. Holz.

Mr. Marsh's discussion regarding temperature difference is one that is of vital interest to investigators carrying out this type of work, as the proper location of the thermocouple is of great importance in the reporting of reliable results; this, as well as the means of reaching thermal equilibrium so that there will be no doubt as to the correctness of the temperature of the material under test, are matters for further study.

A point which has not been touched upon is the use of steel and a non-ferrous alloy together at elevated temperatures. The author believes that this should be given careful consideration on account of the difference in the coefficients of expansion. For example, in a cast-steel valve with a bronze seat ring, the difference in coefficients of expansion between steel and bronze at a temperature of 750 deg. fahr. is so great as to cause the bronze ring to be stressed beyond its elastic limit and when the temperature returns to normal the ring would be loose and probably fall out. The improper application of some materials to services for which they are totally unsuited is the cause of a number of failures.

H. J. French. A suggestion which the author finds especially interesting is that of Mr. Hoyt regarding the straight-line relations between load and time of failure in metals. It may at some time clear up very readily for us, in tests made in the laboratory, some of the questions relating to practical service.

Referring to points raised in Mr. Puffer's discussion, a question was asked regarding the machinability of the various heat-treated steels. Following the hardening operation, which we shall assume momentarily fully hardens the steel, it is necessary to temper at a temperature somewhat in excess of the service temperature for stability. If the steel is merely hardened and we then attempt to make use of it at high temperatures, say, in the neighborhood of 1000 deg. fahr. or above, the tempering will automatically take place in service and in many cases undesirable effects may be observed. The ordinary structural steels are of such a nature that a temperature of about 1000 deg. fahr. or above will materially soften the alloy and leave it in a machinable condition. In the case of hardened high-speed steel somewhat higher tempering temperatures are necessary to effect this softening. Steels B1 and B2 of Fig. 8 of the paper by the authors (see p. 410), referred to by Mr. Puffer, are not in an initially machinable condition. They have fair ductility, at least consistent with such high tensile-strength values in ferrous alloys. These results were included only for comparison with the more complete data for annealed steels given in the same figure, and as already pointed out tensile-strength values in short-time tests are not proper criteria for design purposes.

Reply to the question regarding the tensile properties at temperatures above 900 deg. fahr. of a high-nickel-chromium steel similar to that in Fig. 14 may be found by referring to the bibliography (p. 477) and the index to the bibliography (p. 488).

The same applies to the last question having to do with the tensile properties of high-nickel-chromium alloys. However, it may be stated that we are now making a series of high-temperature tests on a wide range of compositions in the nickel-chromium-iron series, including both commercial and special alloys.

In reference to the remarks of Mr. Moss, it has already been mentioned that tempering subsequent to hardening must be carried out at temperatures at least equal to and generally above the proposed service temperature to produce stability. Attention should also be called to the fact that the weakening effects of high temperatures tend to diminish differences observed between different materials or treatments at ordinary temperatures. That, therefore, has a direct bearing on what we can do by varying hardening heats to produce exceptional properties in a given alloy. In other words, by varying the hardening temperature and subsequent tempering we may produce very different results at ordinary temperatures, but the necessity of tempering at or above the service temperature for stability and the weakening effects of temperatures in the neighborhood of 1000 to 1600 deg. fahr., for example, limits materially what can be done by varying preliminary heat treatments. These features are discussed at some length on page 408 et seq.

Another point raised by Mr. Moss had to do with the time factor versus thermal equilibrium. Both the papers presented and the discussion have thoroughly emphasized the importance of the time factor, which is distinct from what the author understands Mr. Moss to mean by thermal equilibrium. The following paragraph, appearing on page 417, and quoted from a report by Robin, may clear up this question of thermal equilibrium and the time element:

> The properties of steel, so far as the dynamic and static effects are concerned, vary in totally different ways according to the temperature and according to the nature of the steels. The correlation of these effects at the normal temperature in the case of certain steels appears, therefore, to be due purely to coincidence.

Everyone must agree with Mr. Fahrenwald that the time factor is of great importance in any discussion of the high-temperature properties of metals and this has been emphasized in the author's paper. However, Mr. Fahrenwald is too optimistic and not in agreement with any of the other speakers in stating that "Much of the information that has been hoped for and suggested by the various authors of these papers has already been worked out and has been available for some years to the trade." It is not quite

consistent with his later statement that "The problem of the application of metals and alloys at high temperatures is far more complicated and difficult than is ordinarily supposed."

Another subject which has been touched upon in the discussion is cast metals, including so-called heat-resisting alloys, but no one has mentioned, or at least emphasized, the importance of foundry practice in the production of these alloys.

We have recently tested some of the nickel-chromium-iron alloys and the uniformity of the results obtained from ostensibly the same lot of material has not been at all satisfactory. In one case at moderate temperatures, values of about 75,000 lb. per sq. in. tensile strength were obtained, and the duplicate determination gave about 50,000 lb. There were no visible flaws in the specimen to account for such a difference and the cause was not apparent offhand. This question of uniformity is of prime importance from several angles, including the interpretation of test data. Numerical values given should not be used unless there is sufficient evidence that the stated properties can be uniformly produced in any product. While this is primarily a metallurgical problem, it affects materially the engineering application of these materials.

A. E. White. In this discussion three things seem to stand out. The first relates to the need for information regarding the modulus of elasticity. That seems an outstanding need upon which very little work has been done.

The second is the need for the development of a short-time test which will enable one to duplicate the changes resulting from long-time exposures to the given condition. The author thinks the contribution of Mr. Hoyt, in which he mentions the possibility of plotting the logarithm of the time against the square root of the load, is a very valuable suggestion and one which should be given very careful consideration.

The third matter is the need for a suitable classification of metals and alloys. What are some of the outstanding metals which seem to enable an alloy to maintain its properties at elevated temperatures and what are the particular characteristics of these metals? On looking over the charts which have been prepared from the non-ferrous data, it is noted that in the main when nickel is added to copper one gets decidedly beneficial effects. We might therefore make a statement that nickel seems to be beneficial to the extent of enabling metals to maintain their properties at elevated temperatures. If we look over the data in the field of the ferrous metals, we shall find that chromium is of decided benefit. We may therefore say that chromium is a metal of decided value from this standpoint.

We can then go a step further and think of an alloy of nickel and chromium. This alloy, of course, constitutes the base for most of our present-day heat-resisting alloys. Then we can go just a step further and ask what the particular properties are in chromium and nickel which enable these metals, when alloyed or by themselves, to undergo less change at elevated temperatures than most other metals and alloys. That gets us into the field of basic fundamentals. It is a field on which much important work is being done to-day. Every one will appreciate that when the splendid fundamental work which is being done with regard to atomic structure is better understood and appreciated and when it is carried a bit further, we shall be able to convert that information into our engineering needs so as to perfect a metal or an alloy which will approximate at elevated temperatures the properties it has at atmospheric temperatures.

Closure by Committee on Arrangements. Measurable progress should be made from this symposium, which has directed attention to many phases of the application and testing of metals at high and low temperatures. Already, definite results have been obtained, for it has (1) brought together widely scattered and important published information not in all cases readily accessible and likewise served to give a picture of the present state of our knowledge of the subject; (2) developed, through the general discussion, new data of value; and (3) focused attention upon the needs of industry and some of the most important problems to be solved separately or jointly by the metallurgist, chemist, and engineer. It has, of course, been valuable in these respects and in promoting a widespread exchange of ideas, but of greater importance will be the future developments or what may be called the superstructure built upon the foundation of the symposium.

As has been brought out in the symposium, it is common knowledge that rather widely dissimilar results have been obtained by investigators in their tests of practically the same materials. This is not surprising, since they used different types of furnaces for heating the test pieces, different means of reading temperatures, various periods of time under heat, unlike speeds of loading, etc. It has become increasingly important that we know as accurately as possible the strengths of various engineering materials at elevated temperatures under approximate operating conditions. For this reason it is highly desirable that definite means be taken to determine the sources of error in the methods being used today and to determine upon a certain method or methods, which in the hands of investigators may be depended upon to give concordant results. Such methods when properly worked out and

proved accurate, should be accepted as standard methods until superseded by other methods proved to be more accurate or advantageous.

In the belief that the time is now ripe for an organized and systematic effort along these lines, the Committee on Arrangements for the symposium suggests the formation of a special committee under the joint auspices of The American Society of Mechanical Engineers and the American Society for Testing Materials to foster and coördinate and possibly also to carry out service and laboratory investigations relating to the application and testing of metals at various temperatures.[1]

[1] A Joint Research Committee was formed following the Cleveland Meeting. See the introduction to the symposium, p. 350.

No. 1927

POWER RESOURCES, PRESENT AND PROSPECTIVE

By Fred R. Low, New York, N. Y.

President of the Society

AT A RECENT gathering of engineers and scientists one of the speakers quoted Bagehot to the effect that during the early ages of civilization slavery was essential to progress because only through the enforced labor of the many could the few find time to think.

It is within only comparatively recent times that the burden of supporting the race, the drudgery of the struggle for existence, has been transferred to power-operated machinery.

Up to the nineteenth century man was largely dependent upon the work of his own muscles and those of the animals that he had domesticated for his sustenance, shelter, and transportation. His only help from natural resources was in crude adaptations of water and wind power.

Today power-operated machinery is doing in these United States alone more work than could be performed by all the able-bodied men in the world working like slaves from sunrise to sunset.

In a century we have come to be dependent upon our supply of artificial power to such an extent that any interruption of or serious diminution in it would mean industrial and social chaos, a relinquishment of much of the comfort and convenience and culture of our present civilization, and a serious retrogression in the progress of the race.

It may not be uninteresting, therefore, or irrelevant to the occasion to take account of the power resources at our command and the increasing rate at which we are drawing upon them, to pass in review the various processes by which these resources are converted, to see how nearly we have come to ultimate efficiencies, to glance in the direction of possible improvements, and to speculate on other sources of power which research may disclose before our unrenewable supplies of fuel are depleted.

The least important factor in the direct production of power is the wind, although in its propulsion of sailing vessels and the operation of a multitude of windmills, large and small — principally

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Presidential Address at the Annual Meeting, New York, December 1 to 4, 1924, of The American Society of Mechanical Engineers.

small — the power that it furnishes must be, in the aggregate, considerable. The wind is, however, an important factor in carrying the water that has been evaporated from the oceans in the form of clouds and moisture-laden air across the continents and depositing it upon the mountains and highlands, endowed with the power to generate in its descent energy equivalent to that consumed in its elevation. As the evaporation is effected by heat and the movement of the winds is due to differences of temperature, both of these forms of power are, in the last analysis, manifestations of heat energy.

The natural application of running water to the turning of a wheel by weight or impact led to simple reaction types and eventually to the turbine.

The possibility of generating considerable amounts of power by the use of this compact prime mover made possible the development of sizable industries, and the establishment of these industries upon rivers where hydraulic power in the required amounts was producible was the determining factor in the location of many cities and manufacturing districts.

Power has thus played an important rôle in determining the geographical distribution of industry, the occupation and manner of livelihood of whole sections, the attraction to certain localities of different classes of population, and the parts that different divisions of the country play in our social economy.

It will be interesting when some historian, sufficiently removed in point of time to have the right perspective, evaluates the effect of invention and engineering, rather than of conquest and politics, upon the development of this nation. What differences would there have been in our local characteristics and industrial set-up if, in the beginning electrical distribution had made it unnecessary to locate the factory at the dam and had made profitable the development of water powers then too inaccessible or remote from established demand?

At the time of the organization of this Society the most powerful turbine in existence had a capacity of only a few hundred horsepower, and usual efficiencies were below 70 per cent.

There are now in operation at Niagara three units of 70,000 horsepower each, capable of converting over 90 per cent of the energy of the water passing through them into useful work.

Various authorities estimate the work that a man turning a winch is capable of doing as from 1,250,000 to 2,500,000 foot-pounds per day — and these estimates were made when a day's work was more than eight hours. Taking it at 2,000,000 foot-pounds per day, each 70,000-horsepower turbine, which can work 24 hours a day at full load if it has to, can do the work of more than 1,633,000 men.

Working 24 hours a day at full load for a year, it would develop as much power as would over 600,000 tons of coal at only two pounds per horsepower-hour.

This is more than one-thousandth of all the coal that we mine, and more than one six-hundredth of all the coal that is burned for power production.

This estimate may be modified by a very moderate use factor and still leave a significant figure.

Power Resources of the Country and the Increasing Rate at Which They are Being Drawn Upon

The potential water-power resources of the United States are estimated in round numbers by the Geological Survey at 34,000,000 horsepower available 90 per cent of the time or 55,000,000 available 50 per cent of the time.

The total amount of power used, or even of the prime-mover capacity installed, in the country is impossible of close estimate. A survey made jointly by the *Electrical World* and *Power* shows that there are installed in the mills and factories of the United States some 34,000,000 horsepower, and in the central electric stations, 24,600,000 horsepower. There is some duplication on account of the fact that the industrial-plant figure includes motors, current to drive which is purchased from the central station. The combined installed capacity of prime movers may be guessed at as about 45,000,000 horsepower. The total of all prime-mover capacity in the United States is given in the following table:

	Installed capacity, Hp.
Central stations and industrial	45,000,000
Electric Railways (Jan. 1, 1923)	4,119,000
Mining (Jan., 1920)	5,147,000
Stationary, non-industrial	4,000,000
Steam railroads	130,000,000
Navigation	16,000,000
Agricultural and traction	200,000,000
Automotive	300,000,000
Total	704,266,000

If these figures are correct, there is installed for each unit of our population prime-mover capacity capable of generating about seven horsepower, and at our previous estimate of 2,000,000 foot-pounds as a day's work for a man, this would be equivalent to the ability to produce for each man, woman, and child of our population if they demanded it, physical service equivalent to that which could be rendered by nearly 150 slaves.

That is what we could have if we had to with the power-producing machinery already at our service; but it is not running all the time or at full load when it does run. Mills run only forty-odd hours a week; the load factor of most central stations is less

than 40 per cent. The average generating capacity of automobiles has been taken at 20 horsepower, but they utilize that amount of power only at brief intervals even when running, and are parked or in the garage the greater part of the time. Few factories, public buildings, hotels, etc., can tell anywhere nearly how many horsepower-hours they use per year.

It is impossible, therefore, to tell how hard our mechanical slaves are working or how many horsepower-hours of actual service are actually produced by them per year. Only in the case of the public utilities that make power as a commodity, the establishments that buy it by the meter, and the comparatively few of the industrial concerns that keep any intelligent record of the amount of power that they produce, can reliable statistics be had. It is evident, however, that assuming a very low use factor for the seven hundred and odd million horsepower of installed prime movers, the 55,000,000 potential water horsepower of the United States would be vastly inadequate for our present demands even for the 50 per cent of the time that it would be available.

The returns show that the electric public utilities alone will produce this year 80,000,000,000 horsepower-hours. Data available indicate that the rate of electrical production by central stations is increasing at the rate of about 10 per cent yearly. If this rate of increase continues that long, they will have doubled their production in a little over seven years.

We are, then, mainly dependent for our power, and shall be unless and until some other source is discovered, upon our fuel supply.

Conversion of Energy of Fuel into Power

Fuel is capable of producing heat by reason of the attraction between its atoms, mainly those of carbon and hydrogen and those of oxygen.

A candle burns and apparently disappears; but for every pound of paraffin so burned there are discharged into the room 1.32 pounds or 35 cubic feet of water vapor and 3.13 pounds or 27 cubic feet of carbonic acid gas or carbon dioxide.

A substance is hotter because its molecules move more briskly. As the atoms approach each other under the influence of their mutual attraction, their velocity increases as does the velocity of a body falling toward the earth or the velocity of a planet as it approaches the sun. It is not the clash of these atoms that produces the elevation of temperature, but the velocity and momentum acquired as they fall together and take up their positions, whirling about one another like minute planetary systems, forming the molecules of the resulting substance, the temperature of which depends upon their mass and average velocity; that is to say, upon their average momentum.

In a boiler furnace we have the molecules of the incandescent fuel and the gaseous products of its combustion and the heated furnace walls vibrating at a rate corresponding to something between two and three thousand degrees fahrenheit. The steel walls of the boiler are composed of molecules, too, vibrating and circulating among one another in regular orbits separated by distances vast as compared with their own diameters. And yet so strong is the attraction between these widely separated molecules that it takes a force of thirty tons to separate as many of them as are exposed when a square inch of section is torn apart.

To these molecules of the steel the momentum of the furnace molecules is communicated and by them passed along to those of the water inside.

When that water was ice, the positions and orbits of its molecules were fixed. Their mutual attraction far exceeded the centrifugal force due to their rotation. But when their velocity had reached that corresponding to 32 degrees, their centrifugal force became so nearly that of their mutual attraction that they could only feebly resist displacement.

In this condition the mass can no longer hold its shape, but takes that of the containing vessel. One can push the molecules aside without effort, as when dipping his finger into the bowl, but some cohesion persists and draws the water into the drop that remains suspended upon the finger when it is withdrawn or into the crystal globe of the dewdrop. As the temperature of the water, that is, the velocity of its molecules, is raised in the boiler, their centrifugal force increases until a point is reached when they overcome the combined effect of their attraction for one another and the pressure about them and fly off, like a stone from a sling-shot, into space, producing by their bombardment upon the containing surfaces the effect that we know as pressure.

How many of these molecules do you suppose there are in a cubic foot of steam at 250 pounds pressure? Write down 82,126 and then add 20 ciphers to it and you will have the number pretty nearly.

Such numbers convey no impression. Suppose that each of the molecules in that cubic foot of 250-pound steam were magnified until its diameter was 1/200th of an inch, about as large as the dot that one would make with the point of a well-sharpened pencil; how much space do you think they would occupy? Do you suppose they would fill this room? They would fill a cube the side of which is as long as it is from here to Yonkers, or they would cover the whole surface of the earth — land and sea — to a depth of over an inch.

A pellet of lead going 100 times as fast as the bullet from a rifle or revolver would have to weigh only 1/10,000th as much to have the same energy and hit as hard, because the energy varies as the square of the velocity. If the bullet weighs 200 grains the pellet

would have to weigh only 0.02 of a grain, and would require, if spherical and of lead, to be only 0.05 of an inch in diameter. The molecules of steam are small in mass but, as we have seen, great in number, and their velocity is measured in miles instead of in feet per second. There are enough of them in that cubic foot of steam, and they are traveling fast enough and hitting hard enough and often enough to maintain upon each square inch of their container a pressure of 250 pounds.

The carbon and hydrogen of the fuel have been converted into carbon dioxide or carbonic acid, the gas that makes soda water and champagne bubble, and into water vapor. A power station burning 100 tons of carbon per hour is pouring enough carbonic acid gas into the atmosphere in that time to cover a plot 100 feet square with a column 630 odd feet in height. Every fire that is burning, every animal that is breathing, is pouring out carbon dioxide, and yet there is no measurable increase in the CO_2 content of the atmosphere.

The boundless ocean and all the water that is seeking to return to it are the result of the combustion of hydrogen.

All that we need do to get hydrogen or carbon is to decompose the water or the carbon dioxide; but it takes as much energy to pull those atoms of hydrogen or carbon away from those of oxygen with which they are combined as they generated when they fell together, just as it takes as much energy to raise a weight against the attraction that exists between it and the earth as the weight can generate by falling the same distance.

A cubic inch of carbon in the form of anthracite would, in burning, make enough carbon dioxide to make a bubble about 15 inches in diameter.

It would take as much energy to pull the atoms of carbon and oxygen united in the molecules of that 15-inch bubble of gas apart as it would take to lift a ton weight almost 200 feet or to run a one-horsepower engine almost 12 minutes.

To separate these elements in the laboratory, we are obliged to resort to the most powerful chemical agents and to conduct the process in vessels composed of the most refractory materials under all the violent manifestations of light and heat; but in the economy of nature this process is constantly going on, not with the noisy demonstration of prodigious effort, but quietly, in the delicate structure of a green leaf waving in the sunlight.

In some mysterious manner in the frail and microscopic vegetable cell the energy received from the sun is made to separate these atoms against their mutual attraction — to wind up the clock that has run down. The carbon is built into the structure of the growing plant and the oxygen returned to the atmosphere.

And it has been by this process that the energy of the sunlight of forgotten ages has been absorbed, built into vegetation and

stored in strata of coal and pools of oil, rendering possible this age of power.

The earth's surface absorbs from the sun heat energy equivalent to some 3900 foot-pounds per square foot per minute. Referred to its cross-section, this means that the earth is absorbing energy from the sun at the rate of over 162 trillion horsepower.

Most of this energy is immediately radiated back to space. A small portion of it is absorbed by vegetation, some in evaporating water and inducing air currents, some in warming surfaces by day to cool off at night, and some in other ways. But it cannot be retained as heat without a rise in the earth's temperature, and there appears to be no large-scale storage of energy going on in other forms as when the coal measures were in process of formation. The energy temporarily stored in the growing tree or grain is reconverted into heat when the combustion of the vegetation takes place, either by the slow process of decay or as fuel in the furnace or food in the animal organism, and is radiated in various forms, as is that from wind and water, back into the universe.

When we shall have found the secret of the vegetable cell, there may be a possibility of accelerating and intensifying this slow process of nature and of utilizing more directly and immediately than by our round-about process of accelerating and retarding molecules, a larger proportion of this vast stream of energy that comes to us from the sun.

In the meantime practically all the use that we are making for power purposes of our current supply or solar energy, is what we get from wind and falling water and growing vegetation. That which we get from the wind is negligible, and of our present installation of power-producing apparatus in the United States about 9,000,000 horsepower is hydraulic. For the rest, as well as for most of our heating and industrial processes, we are drawing upon the energy stored up years ago, when the crust of the earth was in its making and the luxurious vegetation of the Carboniferous age was compacted into its forming strata in the form of coal.

How Long Present Fuel Resources Will Last at Present Rates of Consumption

The end of the known supply of anthracite is approaching. There are estimated to be, of recoverable fuel of this type in the United States, some eleven billion tons, which at our present rate of consumption will last only about 100 years. It is used mostly for domestic purposes, although the smaller sizes, formerly wasted, are now used for steam making.

Of bituminous coal and lignite there are estimated to be still in the United States some 3½ million million tons, of which about 60 per cent would be recoverable by present methods. We have

already used 12 billion tons. At our present rate of consumption the rest would last some 4000 years. But our rate of consumption has been increasing for the past 25 years at a fairly uniform rate of about 18 million tons yearly. If this rate of increase continued so long, we should use up our visible supply in less than 500 years.

How long and at what rate we shall continue to increase our yearly draft upon these resources depends upon our capacity to absorb light and power now made conveniently ready to our hand, what new power-absorbing processes and inventions may be discovered and developed, improvements in our processes of mining, and upon the increasing efficiency with which we may be able to use our fuel supply.

Mr. Julian D. Sears, of the United States Geological Survey, in a paper to be presented at one of the sessions of the meeting, says that the American petroleum industry began in 1859 and it took over 41 years to produce the first billion barrels. The seventh billion was produced in a little over a year and a half. If what now remains could be continuously extracted and consumed at the 1923 rate, it would last less than 11 years. But Dr. Sears asks us not to accept this as a prediction.

As the true coal becomes scarcer and more costly, we shall doubtless learn to use peat, of which we have large supplies, which will serve to ward off for a time that doleful finish of humanity so vividly pictured by one of my predecessors in this chair when the surviving inhabitants of this cooling planet will be engaged in exterminating one another in a fight for the few remaining heat units.

Nevertheless it is not too early to have that possible plight in mind and plan to delay it as much as possible if it cannot be averted. Two thousand years is not such a long time in the history of a race that has been on earth a million years or more, and if old Tut Ankh Amen had run his kingdom by steam instead of with slaves, and the world generally had followed the practice, we should be in that predicament now.

Power is of such vital and increasing importance that its control would give its possessor a mastery over his fellows and opportunities for tyranny and extortion possessed by no autocrat of any previous empire, visible or invisible, feudal or industrial. The people may well be concerned at any gesture in that direction. Happily, their interest in the water powers has been guarded by the Federal Water Power Act of 1920, which, maintained in its integrity and faithfully administered, will retain the title of the nation in these resources, under conditions that offer opportunity to initiative, security to capital, and freedom from extortion to the consumer. Control over the distribution and sale of power by public-utility corporations is in the hands of Public Service Commissions in most of the states. But an uninterrupted and abundant supply of power cannot be assured to the nation at

reasonable rates so long as the fuel from which most of it is made is subjected to the uncontrolled manipulation of private interest and the organized will — or won't — of labor.

It is to be regretted that a resource so vital to industry, so essential to their continued existence upon the present and coming plane, should have been permitted to pass out of the control of the people and be subjected to the possibilities of manipulation for private gain. The one crumb of comfort in the report of the recent Coal Commission is its declaration that the mining and distribution of coal is charged with public interest, but there appears to be little probability of the government, as at present constituted, taking any active steps toward their public regulation and control.

Efficiencies Attained in Conversion of Fuel Energy into Power

How nearly have we come to possible perfection in the process of converting the potential energy of this fuel into power? Mr. George A. Orrok in a paper presented recently to the Society places the consumption of Savery's engine, built around the year 1600, at 100 pounds of coal per horsepower-hour, and Newcomen's engine of 1750 at about 22. A common figure for the Watt engine of the last quarter of the eighteenth century was 10 pounds, although some of the Cornish pumping engines got down to remarkable efficiencies, several records being reported in the first half of the last century of less than 2 pounds per horsepower-hour.

Trials made at Woolwich Dockyard in 1847 and 1848 gave evaporations of eight or ten pounds, averaging about 9.5 for 8 tests.

Professor Unwin reports a boiler test at about the time of the organization of this Society, showing an efficiency of 80 per cent.

With all our increased knowledge and refinements we have been able to get this up to around 90 per cent, and this only in exceptional cases in our best-designed and most skillfully operated plants. The average for the smaller and less expertly handled plants is below 60 per cent.

There is evidently not much opportunity for improvement in the maximum efficiency, but a great opportunity for improvement in the general efficiency. This can be effected by greater attention to the design and operation of plants that are able to make their power cheaper than they can buy it, and by abandoning those that cannot justify their continued operation in favor of power from an efficient central supply.

This step should not be taken, however, without giving full weight to the value of the exhaust or extracted steam from one's own engines or turbines for heating and manufacturing processes. It takes only about 50 more heat units to make a pound of steam

at 250 pounds than at atmospheric pressure, and after it has done its work in the engine it will have over one thousand heat units left in it, the difference between which and the temperature at which it is desired to use it will be available for heating and process work.

A station developing 100,000 kilowatts at 15 pounds of steam per kilowatt-hour will discharge into the river over a billion and a half B.t.u. per hour, to make the equivalent of which in steam for heating or manufacturing processes would take over 85 tons of coal.

In steam-operated prime movers, too, we are approaching the limit of attainment for existing conditions, turbine efficiencies exceeding 90 per cent having been claimed and performances in the 80's substantiated.

A turbine of 85 per cent efficiency would need only 6.2 pounds of steam at 350 pounds absolute, 750 degrees, per horsepower-hour, and a boiler of 85 per cent efficiency could make this with about 0.7 of a pound of 12,000-B.t.u. coal. These are possibilities but not practice. A larger proportion of the coal burned for power purposes will be used at this efficiency as more of it is used in large and skillfully designed apparatus by experts in its operation.

Our progress in the boiler art has been not so much in being able to evaporate more water per pound of coal as in being able to evaporate more water per pound of boiler and to evaporate it at higher pressures.

We have single boilers today that evaporate over 150 tons of water an hour, and one boiler is being built to carry a pressure of 1200 pounds.

Our present effort toward reduction in fuel consumption is in the direction of increased initial pressures. We have already gone about as far in initial temperature as the materials at present available will stand.

The lowest steam consumption attainable by any combination of suggested processes such as bleeding, reheating, etc., would be with 1500 pounds pressure, 750 degrees initial temperature, 29 inches vacuum, 85 per cent efficiency, about 4 pounds of steam per horsepower-hour. This steam, including the heating during isothermal expansion, requires about 1400 B.t.u. per pound, and a boiler of 85 per cent efficiency would require a little over half a pound of 12,000-B.t.u. coal to evaporate and reheat the 4 pounds of it required to produce a horsepower under these conditions.

Just as the amount of power that can be gotten out of a given quantity of water depends upon the height through which it can be made to fall, so the amount of power that can be gotten out of a given quantity of heat depends upon the range of temperature through which it can be made to drop. Just as by shooting steam through a nozzle we imbue the rapidly moving jet with kinetic energy, so in raising the temperature of a body we add to the kinetic energy of its molecules. As in the turbine we recover and

convert that kinetic energy by bringing the jet as nearly as possible to rest, so in the heat engine we recover as much as possible of the kinetic energy added to the molecules by slowing them down as much as possible or, in other words, by reducing the temperature to the lowest possible limit.

This lower limit is set for the steam engine by the temperature of the condensing water, the upper limit by the temperature that the available materials will stand and the pressures that increase of temperature generates in the medium.

As the temperature of water increases, the addition of a given amount of heat produces rapidly increasing incrcments of pressure. At 600 degrees its pressure has become over 1500 pounds. There is a limit to what we can do with steam in this direction.

By using a substance that has a higher boiling point and less pressure at these higher temperatures, the higher level may be raised without involving the limitations of uncontrollable pressures, and Mr. W. L. R. Emmet has developed this idea in his mercury turbine, with which he expects to be able, using it in series with a steam turbine, to develop a horsepower-hour on about 7500 B.t.u. equal to about three-quarters of a pound of 12,000-B.t.u. coal used in a boiler of 85 per cent efficiency.

In the internal-combustion engine there is in the cylinder itself the high initial temperature of the burning fuel, but the heat drop is limited by our inability to expand the heated gases to a low temperature level, the exhaust leaving the cylinder of a Diesel engine at a temperature higher than the initial temperature in a steam engine. Diesel engines produce a horsepower, however, on 0.4 of a pound of fuel oil, equivalent in heat value to 0.6 of a pound of good coal.

Possible Sources of Energy Other Than Fuel

Although the efficiency of our heat engines depends upon the range of temperature through which they work, the animal organism converts the latent energy of food or fuel into work without any perceptible difference of temperature. I do not know what the thermal efficiency of a mule may be, but it is inconceivable that one could get the number of foot-pounds of work that he develops, by burning the hay and grain that he eats in the furnace of a steam boilcr. Perhaps in this direction lies the development of a process that will enable us to apply usefully that large proportion of the heat derived from fuel that we are obliged by our present process to reject in the low-temperature exhaust. In the primary battery we have another instance of the conversion of the energy of fuel into electricity with no perceptible temperature range and a high degree of efficiency.

We have seen that it is fruitless to think of making fuels by decomposing the products of previous combustion unless the energy

to do so can be obtained, as it is by the vegetable cell, from the sun. It may, however, prove to be possible to obtain energy by the transmutation of one substance into a substance the atoms of which possess less energy of relative position or motion. It is believed that this process still going on in the sun is the source of its apparently constant supply of heat.

Dr. F. W. Aston, in a lecture before the Engineers' Club some time ago, said that if the hydrogen in 9 cubic centimeters of water could be reduced to helium, 200,000 kilowatt-hours of energy would be released, and that some time somebody may discover how this may be done. But in doing it he may set off a train of decomposition that will cause the record of his achievement to be written on the firmament in a new star.

Recent researches in electrochemistry and molecular physics have been productive of marvelous results and are full of suggestion of future possibilities. The time is not inconceivable when the tools of our present wasteful processes of power generation will be as archaic as the turbine of Hero and the engine of Papin. For the present we must use our best endeavors along established and developing lines to make the best and most efficient use of our diminishing visible resources in the face of the rapidly growing demand.

No. 1928

ANNUAL REPORT OF THE COUNCIL 1924

IN THIS annual report of the Council no attempt will be made to cover in detail the many activities of the year, either of the Society as a whole, of the Council and its many standing and special committees, or of the joint activities of the Society. This report does aim, however, to give an outline of the varied and progressive work of the Society during the presidential term of F. R. Low.

The detailed reports of the standing committees of the Council are made a part of the official records of the Council.

The membership of the Council and its officers appears with the list of committees who have served during the same period in the Society Affairs section. (See page 7.) The Council wishes to record its deep sense of obligation and appreciation for the splendid work and unselfish devotion of committeemen, who without remuneration, and often at great personal sacrifice, have carried on and are carrying on the work of the Society, their only objective being to best serve their profession and the public welfare.

FINANCES

The certified accountant's report of the audit of the books for the fiscal year from October 1, 1923 to September 30, 1924 is made an appendix to this report. Quoting from the annual report of the Finance Committee to the Council, it is pointed out that heretofore "the so-called 'reserves' of the Society were largely in inventory on which the purchase price could not be realized. For the past five years, due to the great expansion of the Society's business and lack of working capital, the Society had to borrow money to carry it into the next year when these notes were paid from the dues for the new year. Under the policy laid down for the year 1923-1924, ably administered and wholeheartedly supported by the various agencies of the Society, the Society has ceased to make these annual borrowings."

In other words, we have placed our balance sheet, income and expenditures, on a business basis. We maintain the cost of service to the member approximately equal to the dues. Then we use the net income from the business activities to support the professional service and service to the public, and to build up the "rainy-day fund" and "working capital" within the limits of our income.

TRUST FUNDS

The Trust Funds of the Society, not inclusive of joint trust funds, such as The Engineering Foundation and part ownership in the Engineering Societies Building, are listed in the official audit appended to this report.

MEMBERSHIP

Table 1 shows the changes in membership during the fiscal year. The considerable shrinkage has no relation to, nor is in consequence

of, the increased dues which went into effect after the period of this record, but is the result of the earlier action of the Council to clear the Society's books of delinquent members.

There have passed through the hands of the Membership Committee during the fiscal year 2697 applications, of which 2209 have been recommended for membership. The classification in the various grades is as follows:

Members	298
Promotions to Member	152
Associates	57
Transfer to Associate	1
Associate-Members	375
Promotions to Associate-Member	137
Juniors	456
Juniors (R 5 Rule 1)	733
Total	2209

Elections declared void during fiscal year 1923-1924......... 103

Memorial notes of members who have died during the year have appeared currently in the *A.S.M.E. News* and *Mechanical Engineering*, and are recorded in this volume. We number in these our Past-President and Honorary Member, Henry R. Towne, one of the early leaders in the development of our Society. By his will $50,000 was left to the Engineering Foundation. The roll of honorary members has also lost by death Gustav Eiffel.

MEETINGS

The Spring Meeting was held in Cleveland, May 26-29, 1924. This meeting invites comparison with a meeting of the Society held in Cleveland in 1883, with a registration of 77; in 1924 the registration was nearly 1000. The Forty-Fifth Annual Meeting of the Society was held in New York, December 1-4, and included technical sessions, committee meetings, and public hearings for standardization codes which are now gaining steadily in both national and international recognition. Further details of these meetings are given in the Society Affairs section of this volume.

PROFESSIONAL DIVISIONS

Satisfactory progress in developing the concept of the Professional Divisions of the Society has been made this year. These divisions are working through the activities of the Local Sections, the general meetings, the publications, and the professional and technical committees. The new grouping of the Professional Divisions as approved by recent Council action is shown in Fig. 1.

The referendum sent out with the letter-ballot for officers this year brought replies from over 2000 members of the Society volunteering their services for active work in Professional Divisions. A portion of those responding offered papers, while others have suggested the formation of new divisions.

The Professional Divisions of the Society present a number of interesting problems which will probably take several years to develop and in which progress will necessarily be slow on account of the number and new character of these problems, for which there is no precedent. Perhaps the largest of these is the classification of the entire membership.

TABLE 1 STATUS OF MEMBERSHIP—OCT. 1, 1923 TO SEPT. 30, 1924

	Oct. 1, 1923	Oct. 1, 1924	Losses: Transferred from	Losses: Resigned	Losses: Dropped	Losses: Died	Additions: Transferred to	Additions: Elected	Totals: Loss	Totals: Gain	Totals: Total
Honorary Members	20	19	....	....	....	1	....	....	1	....	— 1
Life Members	75	77	....	....	....	1	....	3	1	3	2
Members	8139	8019	....	119	467	76	154	388	662	542	—120
Associates	873	803	10	32	95	8	....	75	145	75	— 70
Associate-Members	4414	4147	112	85	506	8	132	312	711	444	—267
Juniors (20)	932	1010	57	43	171	1	350	....	272	350	78
Juniors (10)	3485	3519	457	73	491	8	....	1063	1029	1063	34
TOTALS	17938	17594	636	352	1730	103	636	1841	2821	2477	— 344

Management Week has become an annual event, occurring this year October 20-25; the topic was on Budgeting for Better Business, and the work of the A.S.M.E. and the five coöperating national engineering societies was supplemented by business men's clubs, chambers of commerce, and similar local organizations. During this week hundreds of meetings devoted to this subject were held all over the country.

LOCAL SECTIONS

The activities of the Local Sections Committee had to do with the work of 64 geographical groups of the membership of the Society. These local groups are coming more and more into the work of assisting the Membership Committee in keeping the membership of the Society up to a high standard and checking applicants locally.

During the year 328 local meetings were held. For reasons of economy, regional meetings were restricted to only one, at Chattanooga. The attendance at this meeting was comparatively small but the high standard of the papers, the local interest, and the necessity to serve the district fully justified the meeting. Two new sections have been established, one at Savannah, Georgia, and one in Vermont, the latter called the Green Mountain Section.

The Local Sections Committee held meetings at New York, Columbus, Cleveland, and Chicago. In addition, at least one member of the committee participated in the local meetings throughout the country. The President and Secretary of the Society have also visited other local groups.

Where Student Branches of the Society are established, members of the Local Sections Committee have made a point of meeting the student members.

A notable feature of the Annual Meetings is the Local Sections Delegates' Conference at which is appointed the Nominating Committee for the selection of the officers of the Society.

A plan of coöperation in the several centers of the United States without loss of individuality has been worked out for the Local Sections activities of the Founder Societies. It is hoped that this will do much toward the elimination of waste and duplication of effort of these societies.

Under the joint auspices of the New Haven Section and Yale University the fourth Annual Machine Tool Exhibit was held at New Haven September 15-18, the Machine Shop Practice Professional Division and 16 Local Sections coöperating in the combined exhibit and technical meetings. Approximately 15,000 persons visited the Exhibit in the Mason Laboratory during these four days; they came from 20

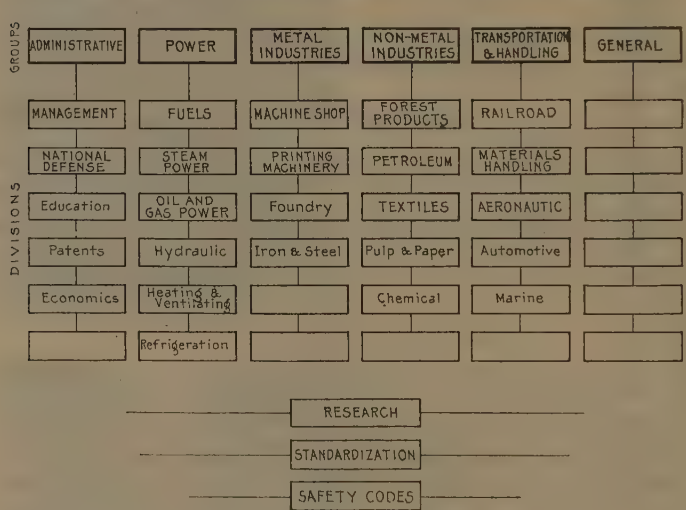

FIG. 1 NEW GROUPING OF PROFESSIONAL DIVISIONS

(Names of divisions in capital letters indicate those already in existence; others, those which can be formed as members require.)

states in the Union and from France, Germany, Australia, and Czechoslovakia; the exhibitors increased 25 per cent over last year and during the exhibition 130 machines were in operation.

PUBLICATIONS

It has been an active year in Society publications. Volume 45 of Transactions, now on the press, contains 948 pages.

Mechanical Engineering has published 1107 pages of text and over 1100 illustrations, not including advertising pages. Twelve regular issues have been supplemented with the Mid-November number to take care of the papers for the Annual Meeting.

The issuance of the Year Book under the new policy of the Council only to such members of the Society as requested it resulted in a saving of approximately $7000. The listing showed the total membership, checked to February, 1924, to be as follows: In the United States, 16,618; foreign countries, 770; addresses unknown, 64; total, 17,452.

The fifth General Index to the entire TRANSACTIONS of the Society was published in October. The last General Index to the TRANSACTIONS was that published in Volume 27 of the TRANSACTIONS. This General Index is the first to be issued for 20 years, and covers 45 years of engineering literature (1880-1924), two generations of engineering achievement.

The publication of the Autobiography of John A. Brashear is another marked achievement for the year and is to be followed, it is planned, by the biographies of John E. Sweet and Robert H. Thurston, organizers and Past-Presidents of the Society. These books are self-supporting, the Society underwriting their publication. History contains few records of the lives of great engineers, and much favorable comment has been received on the policy of securing and publishing such records.

Condensed Catalogues has reached its fourteenth annual edition. The Catalogue Section contains 430 listings; 4400 firms are classified under 3900 different subject headings, making a total of more than 31,000 separate listings. A distinctive feature of the volume is the Professional Engineering Service Directory in which 575 professional engineers and engineering organizations are listed. The policy of confining this Directory to unbiased professional service is strictly enforced and as in previous years A.S.M.E. members are given one listing free.

CONSTITUTION, BY-LAWS AND RULES

The Constitutional amendment passed by referendum of the membership raising the dues became effective at the Spring Meeting in Cleveland. The Constitution now provides as follows:

ARTICLE C5—FEES AND DUES. *Section* 2. The annual dues for membership in each grade shall be: Member, $20; Associate, $20; Associate-Member, $20; Junior, for the first six years of his membership, $10, and after six years, $20.

Under request of the Council, the work of the Committee on Constitution and By-Laws involved modifications of procedure, the revision of By-Laws and Rules in connection with members in arrears for dues, billing, etc., and Professional Divisions appointments. The statement of duties of the Finance Committee and method of handling funds also have been revised.

AWARDS AND PRIZES

Holley Gold Medal. This year the Society was the recipient of a gift from George I. Rockwood, Vice-President of the Society, an endowment for a Gold Medal of Great Achievement. The medal has been called the Holley Medal after Alexander L. Holley, presiding officer at the organization meeting of the Society. It is to be awarded "in those rare cases where an individual has succeeded by the exercise of his genius and character in powerfully assisting the fortunes of our country, or the general engineering progress of the world." The first medal was given to Hjalmar Gotfried Carlson as fulfillment of the award made on December 7, 1921, for his inventions and processes which made possible the timely production of drawn steel booster casings for artillery ammunition, thereby aiding victory in the World War.

The Junior Prize of $50 in cash was awarded to R. H. Heilman of the Mellon Institute of Industrial Research for his paper on Heat Losses through Insulating Materials.

The Student Prizes of $25 each were awarded to George Stuart Clark for his paper on Determination of the Gasoline Content of Absorption Oils, and jointly to L. J. Franklin and Charles H. Smith for their paper on The Effect of Inaccuracy of Spacing on the Strength

of Gear Teeth. All three of these students are members of the Student Branch of the A.S.M.E. at Leland Stanford University.

Charles T. Main Fund Award. The Council has approved an award of $150 for the year 1924-1925, to be financed out of the income from the Charles T. Main Fund, to be given to the member of any Student Branch of the Society for the best paper on The Influence of the Locomotive on the Unity of our Country.

STUDENT BRANCHES

There are now 79 Student Branches; these branches have held approximately 347 meetings during the year, some jointly with the Local Sections. This year a joint policy for meetings and handling of student activities has been worked out in the Founder Societies, on the same lines as for Local Sections.

EDUCATION AND TRAINING FOR THE INDUSTRIES

The work of this committee involves an active correspondence on matters of interest pertaining to education below the college grade. One of the principal sessions at the last Annual Meeting was a conference on Education and Training for the Industries. This committee is gathering valuable data and is confining its investigations to education for industry of non-college grade in contradistinction to the work of the Committee on Relations with Colleges.

LIBRARY

The Library of United Engineering Society is jointly administered by a Library Board composed of members from each of the four national Founder Societies of civil, mining, mechanical and electrical engineers. A most complete report of the work of the Library is issued each year in pamphlet form. The Library is administered by the Library Board of the United Engineering Society as a public reference library of engineering and the applied sciences. It contains 150,000 volumes and pamphlets and receives currently the important periodicals in its field.

The increase in the use made of the Engineering Societies Library, whose chief support comes from the Founder Societies, has made difficult its maintenance on an adequate scale. As much of this increase results from the enlarged interest of the industrial world in research, the situation of the Library was brought to the attention, early this year, of some of the industrial concerns that are making use of it. Many of them immediately recognized the value of such a storehouse of information to their engineers and research workers and the advantages of keeping its equipment complete, and as a direct result subscriptions have been received from a large number of important companies.

TECHNICAL COMMITTEES, STANDARDIZATION, AND RESEARCH WORK

Boiler Code

The interpretation service to the boiler industry by the Boiler Code Committee continues of great importance in the work of the committee. Nine meetings of the committee and several sub-committee meetings have been held during the year. Of major importance is the publication of the 1924 revised edition of the Boiler Construction Code; splendid progress is also being made in the formulation of rules for the construction of unfired pressure vessels and rules for the care of steam boilers and other pressure vessels in service.

Standardization

This standing committee is advisory to the Council in dimensional standardization work and in the Society's relations with American Engineering Standards Committee. Engineering and industrial standardization is of direct interest to our members and has made real progress during the year just closed. Fifty-five committees and subcommittees have been at work and have completed substantial parts of the projects assigned to them.

American Engineering Standards Committee

This committee aims to organize and systematize the formulation of engineering and industrial standards in the United States. The Society's three representatives on this committee are of necessity appointed from standardization committees. Quarterly meetings are held, with monthly meetings of the Executive Committee of A.E.S.C., of which one representative from the A.S.M.E. group is a member.

A full account of the year's activities of the A.E.S.C. will be found in its annual Year Book. Of the 152 projects having official status before the A.E.S.C. in 1924, 25 are in the mechanical engineering group. The Society is sponsor or joint sponsor for 14 of them, having this year accepted sponsorship for Pins and Washers and Scientific and Engineering Symbols and Abbreviations. Following is a record of some of the work carried on under the Procedure of the A.E.S.C.

Standardization of Shafting. The November, 1923, issue of *Mechanical Engineering* contained the first of the three complete reports of this sectional committee. The report covers Standards for Diameters of Cold-Finished Transmission Shafting, Diameters of Cold-Finished Machinery Shafting, Tolerances for These Dimensions, and Stock Lengths of Cold-Finished Shafting. It was approved by the Council in February, 1924, and has been presented for approval as a Tentative American Standard.

The second standard, published in the February, 1924, *Mechanical Engineering,* includes Standard Dimensions and Tolerances for Stock Key Sizes and Standard Tolerances for These Dimensions. At the suggestion of the American Petroleum Institute, the Committee will include in its final report standard dimensions for taper keys. Standard Formulas Used in Determining the Sizes of Transmission Shafting is in process. As a result of circulation to a selected list of engineers for criticism and comment, many valuable suggestions were received. The report has been submitted to the sectional committee for approval.

Standardization and Unification of Screw Threads. This sectional committee has completed two of the three tasks assigned to it: (1) Screw Threads for Bolts, Machine Screws, Nuts and Commercially Tapped Holes, based on the Progress Report of the National Screw Thread Commission published in January, 1921, was approved as an American Standard by the A.E.S.C. in May, 1924, and made available to the general public in pamphlet form in July. (2) Gages and Gaging of Screw Threads has been made ready for the approval of the sponsor societies and the A.E.S.C. The material forming the second report is to be incorporated in the revised report of the N.S.T.C. This Sectional Committee is now turning its attention to the development of international agreements on screw threads.

Standardization of Plain Limit Gages for General Engineering Work. The Society is the sole sponsor for this Sectional Committee, which has under consideration

1 Tolerances and Allowances for Machined Fits in Interchangeable Manufacture (report in Mech. Eng., vol. 45, Dec. 1923, p. 699.)
2 Methods of Gaging Manufactured Material
3 Gages, Their Limits, Manufacture and Use

Standardization of Gears. This Sectional Committee has reorganized into ten (10) working sub-committees, covering:

1 Program	6 Worm Gears
2 Editing	7 Bevel Gears
3 Nomenclature	8 Materials
4 Tooth Form (Spur Gears)	9 Inspection
5 Helical Gears	10 Horsepower Rating

Standardization of Pipe Flanges and Fittings. Standards for steel flanges designed for steam pressures of 250, 400, 600, and 900 lb. with superheat, will soon be ready for submission to the sponsor bodies and the A.E.S.C.

The Cast-Iron Flange Standards originally published by the A.S.M.E. in 1914 have been under revision and are nearing completion. During the year two working groups organized, one on Cast-Iron Flanges for Pressures under 100 lb. and one on Ammonia Flanges and Flanged Fittings. Both groups have made material progress.

Sub-Committee No. 2 on Screw Fittings completed standards for 125-lb. and 250-lb. cast-iron screwed fittings and 150-lb. malleable screwed fittings.

A fourth Sub-Committee deals with matters related to materials and stresses, working in close coöperation with the American Society for Testing Materials.

Standardization of Bolt, Nut, and Rivet Proportions. Standards for Carriage Bolts have been completed and tentative drafts of six others circulated. There are accordingly in various stages of completion tentative reports on:

1 Small Rivets, $\frac{7}{16}$-inch Diameter and Under
2 Tinners', Coopers', and Belt Rivets
3 Wrench-Head Bolts and Nuts
4 Slotted-Head Machine, Wood, and Cap Screws
5 Track Bolts and Nuts
6 Plow Bolts.

Standard Code for Identification of Piping Systems. The Sub-Committee on Identification by Color completed its report, which will be linked with reports in process from the Sub-Committee on Identification Markings other than Color and the Sub-Committee on Classification of Fluids.

Standardization of Small Tools and Machine Tool Elements. This Committee, half of whom represent the National Machine Tool Builders' Association and half The American Society of Mechanical Engineers, can report satisfactory progress this year. Its first Sub-Committee for a particular sub-project has been organized to develop Standards for T-Slots; it consists of 14 members, representing four organizations. A tentative draft of their report has been completed and distributed to interested firms and individuals and to members of the National Machine Tool Builders' Association for criticism and comment.

Standardization of Fire-Hose Couplings. For the purpose of establishing satisfactory tolerances and allowances for the National Standard Fire-Hose-Coupling Screw Thread which was originally developed by a committee of the N.F.P.A. (1905), subsequently endorsed by thirty-one (31) societies and associations of various kinds, and was printed by our Society in December, 1913, your committee reports the completion of a series of conferences with the National Board of Fire Underwriters, the National Screw Thread Commission, the Bureau of Standards, the National Fire Protection Association, the American Water Works Association, and manufacturers of fire-hose couplings, hydrant nipples, and special fittings.

While these negotiations have been in progress, material for three new pamphlets has been developed: (1) Complete Description of the

National Standard Fire-Hose-Coupling Screw Thread, (2) Production of National Standard Fire-Hose-Coupling Screw Thread, and (3) Field Inspection of National Standard Fire-Hose-Coupling Screw Thread.

Standardization of Scientific and Engineering Symbols and Abbreviations. In July, 1924, the Council approved the acceptance of joint sponsorship for the Sectional Committee on the Standardization of Scientific and Engineering Symbols and Abbreviations. A committee of eight was appointed by President Low to serve as the A.S.M.E. group.

Standardization of Ball Bearings. This Sectional Committee is sponsored jointly by the Society of Automotive Engineers and this Society. The project is of international interest and progress is being made in adjustments of opinions between the American committee and the international proposals.

Other A.E.S.C. Committee Representation. The Society has accepted invitations to appoint representatives on the following Committees organized by coöperating bodies: Steel Railway Bridges, Hose Specifications, Method of Testing Petroleum Products and Lubricants, Size of Publications for Standards, Methods for Testing Timber, Specifications for Special Materials for Use in the Manufacture of Trackwork, and Specifications for Movable Railway Bridges.

Power Test Codes

The procedure developed in 1918 for the revision of the A.S.M.E. Power Test Codes of 1915, has produced tangible results during this past year. Four more new codes have been issued: (1) Code on General Instructions, (2) Test Codes for Reciprocating Steam Engines, (3) Stationary Steam Boilers, and (4) Internal-Combustion Engines.

During the year four of the test codes were printed in *Mechanical Engineering:* Condensing Apparatus, Solid Fuels, Speed-Responsive Governors, and Gas Producers.

Good progress has been made in the development of the Test Code for Centrifugal and Rotary Pumps and the Test Code for Steam Turbines.

The Code on Definitions and Values and the Test Code for Evaporating Apparatus are about to be presented for adoption as standard practices of the Society.

The World Power Conference at Wembley, London, in June and July of this year, attracted a number of the members of the Power Test Codes Committee. Taking advantage of their presence in London, the Institutions of Civil and Mechanical Engineers arranged for meetings of their Joint Committees to consider preliminary drafts of the Test Codes for Hydraulic Power Plants and Boilers and Steam Engines. The American Committee directly connected with the development of the corresponding A.S.M.E. Codes was asked to frame criticisms and comments for transmission to the British Committee on the British: (1) Test Code on Hydraulic Power Plants, (2) Test Code on Definitions and Values, (3) Test Code on Boilers and Steam Engines, (4) Test Code on Internal-Combustion Engines, and (5) Commercial Trial of a Heavy-Oil Engine.

Safety Codes

The Safety Code activity has been concerned principally with the development and promulgation of three codes, as follows:

(1) The Safety Code for Mechanical Power-Transmission Apparatus was published in pamphlet form in December, 1923, and the first edition of 1000 copies was distributed during the year.

(2) The Safety Code for Elevators had its first edition of 8200 copies exhausted by July, 1923, and arrangements were made for

revision by a Sectional Committee under the American Engineering Standards Committee. This committee numbering 37 members, representing 21 organizations, completed a very thorough revision of the original code.

(3) The Safety Code for Machinery for Compressing Air has organized with 23 members representing 15 organizations. It is sponsored jointly by the American Society of Safety Engineers and The American Society of Mechanical Engineers.

In addition to the above the A.S.M.E. is officially represented on Sectional Committees to formulate safety codes for: Abrasive Wheels, Aeronautics, Conveyors and Conveying Machinery, Electric Power Control, Floor Openings, Railings and Toeboards, Forging, Industrial Lighting, Industrial Sanitation, Ladders, Laundries, Lighting Factories, Mills, and Other Work Places, Logging and Sawmill Machinery, Machine Tools, Mechanical Refrigeration, Paper and Pulp Mills, and Power Presses.

Research

In spite of our inability to appropriate more than $500 for research, the past year has been one of real progress by the Society's Special Research Committees. The Main Research Committee held three meetings during the year. In the intervals between meetings the work was carried on by correspondence. As a direct result papers were presented at the 1923 Annual Meeting on The Bending and Torsion of Multi-Throw Crankshafts on Many Supports, by S. Timoshenko; A Graphical Study of Journal Lubrication, by H. A. S. Howarth, and A Calorimetric Method of Surveying the Behavior of Steam, by N. S. Osborne. At the time of the Cleveland Spring Meeting, a Special Committee on Metal Springs was organized with 14 members. Researches in progress cover:

Lubrication. The experimental work has been carried forward at the Experiment Station of the Bureau of Mines, Pittsburgh, Pa., through a grant of $1000 from Engineering Foundation. The Committee was able to contribute $200 toward the fund being raised by the Advisory Board of Coal Operators and Engineers who asked coöperation with them in their research on Mine-Car Friction, which is to be carried on at the Carnegie Institute of Technology and the U. S. Bureau of Mines.

Fluid Meters. It is expected that Part 1 of the report, financed by contributions, will be soon available. Immediately following this publication the complete rewriting of Part 2 will be begun.

Strength of Gear Teeth. Negotiations through Dr. S. W. Stratton, President of Massachusetts Institute of Technology, secured coöperation and consent to carry on tests at the M. I. T. Laboratories on the effect of speed on the strength of gear teeth. The Committee is to supply the machine and the gears for test, and the Institute is to provide the power to run the machine, and the technical observers. The campaign for funds to cover the cost of the testing machine produced $4775. Of this sum the members of the American Gear Manufacturers Association contributed $1725, the members of the National Machine Tool Builders' Association contributed $500, $550 came from other users of gears, and $2000, available in two yearly grants of $1000 each, was subscribed by Engineering Foundation.

Bearing Metals. The Committee this year has devoted all its energies to the collection of funds to cover a five-year program of research in this field. The total amount sought is $50,000, which will be expended at the rate of approximately $10,000 per year. About one-third has been subscribed so far. The work requires the full time of trained observers and the Committee is accordingly laying out a five-year

program. Engineering Foundation has granted $5000, $1000 of which is to become available each year for five years.

Properties of Steam and Extension of Steam Tables. Throughout the year this investigation has been continued at Harvard University, at the Massachusetts Institute of Technology and at the Bureau of Standards, financed by a trust fund on deposit with the Society. A second progress report was presented at the December, 1923, Annual Meeting. These reports were published in the February, 1924, issue of *Mechanical Engineering* and were later distributed to the subscribers to this fund.

Cutting and Forming of Metals. A Progress Report on the Present Status and Future Problems of the Art of Cutting Metals was presented and discussed at the Machine-Shop Practice Session of the 1923 Annual Meeting and was printed in the January, 1924, *Mechanical Engineering.* As a first project a comparative study of "hardness" testing instruments was worked out by Prof. J. C. Keller, a member of the Committee. To assist it in fulfilling its clearing-house function this Committee is developing a permanent file of literature dealing with the arts of cutting and forming metals.

Metal Springs. The organization of this Special Committee was authorized at the December, 1923, meeting of the Main Committee with the understanding that no financial assistance incident to its organization could be borne by the Society. Its first meeting was held at the Cleveland Meeting for the consideration of its personnel and program of work. A preliminary Progress Report will be presented at the Research Session of the 1924 Meeting.

Properties of Metal at Extremely High and Low Temperatures. As a direct outcome of the activities of Sub-Committee No. 3 on the Standardization of Steel Flanges and Flanged Fittings of the Sectional Committee on the Standardization of Pipe Flanges and Fittings, a joint session with the American Society for Testing Materials was held during the Spring Meeting in Cleveland and four important papers were read on The Effect of Temperature on the Properties of Metals resulting in resolutions requesting the A.S.T.M. and the A.S.M.E. to organize a joint research committee for investigations in this field.

SOME COÖPERATIVE INTERSOCIETY AND PUBLIC RELATIONS

The major part of this work is reported under the technical committee activities. Provision for coöperative uniform action on matters of common interest between the four Founder Societies is being most fortunately and pleasantly carried out through the Joint Conference Committee. This committee consists of the presidents and secretaries of the four societies.

Engineering Foundation has been engaged during the year on the study of the problem of fatigue of metals, which is being carried out by Engineering Foundation and the National Research Council in coöperation with the University of Illinois. Valuable additions to knowledge and theory have been made and other important investigations in this country and abroad have been stimulated.

Engineering Foundation is also coöperating with research committees of the Founder Societies in investigations of concrete and reinforced-concrete arches, steel columns for buildings and bridges, mining methods, rock-drill steels, properties of steam, bearing metals, lubrication, and strength of gears. Besides the Foundation's appropriations for these researches, totaling $15,000, contributions from industries and other sources aggregate more than $100,000.

In appreciation of the assistance rendered by Engineering Foundation to the National Academy of Sciences in the establishment of the National Research Council, Engineering Foundation has been

assigned a room in the building of the National Academy of Sciences and the National Research Council at Washington, D. C.

By the will of Henry R. Towne, Past-President of The American Society of Mechanical Engineers, Engineering Foundation will receive the sum of $50,000 to establish the Henry R. Towne Engineering Fund.

American Engineering Council (formerly the Federated American Engineering Council). The President this year has been our own Past-President and ex-Governor of Vermont, Hon. James Hartness. Among the committees are the following: Reforestation and Timber Supply, Government Reorganization as Related to Engineering Matters, Public Affairs Committee, War Department Personnel, Elimination of Waste in Industry, Federal Power Committee, Committee on Patents, Committee on Contracts and Adjustments, etc.

A central feature of the Annual Meeting last January was the Public Works Conference. Delegates were present from more than 200 engineering and allied bodies. Cabinet Members directly connected with the Department of the Interior and the Department of Commerce have openly said that engineers would be looked to as the dominating influence in the consummation of this plan, especially that part of it which has to do with the allocation of technical and scientific bureaus.

Upon the request of Senator Norris, Chairman of the Senate Committee on Agriculture and Forestry, the American Engineering Council appointed a committee of engineers to make a study of the general economic phases of the Muscle Shoals problem.

Employment. The Employment Service of the four societies was entirely reorganized during 1923-1924. Approaching self-support in the New York office will release national funds for establishing employment service branches in other cities. The announcement of positions available in the special bulletins has become an income-producing activity. In the last fiscal year 1705 members were registered; 953 were placed; of 1313 advices of positions open, 873 were filled through the Bureau; 50 per cent of the total service rendered was to A.S.M.E. members. In all, $7507.72 was voluntarily contributed to the service by members securing positions. Negotiations are under way looking toward the establishment of branches at Chicago and San Francisco.

SOCIETY REPRESENTATION

Representation of the Society has been invited in many different functions, including the following:

- Rensselaer Polytechnic Centenary
- The Franklin Institute Centennial
- Purdue Semi-Centennial
- Annual Meeting of the American Academy of Political and Social Science
- Conferences on Traffic Problems and City Planning
- Annual Meeting of the Society for the Promotion of Engineering Education
- International Conference of Refrigeration, London
- International Mathematical Congress, Toronto, Canada
- Committee on Prevention of Accidents in the Textile Industry
- American Association for the Advancement of Science
- Fuel Conservation Committee of the U. S. Shipping Board
- 50th Anniversary of the University of Brussels School of Technology
- American Mining Congress
- Conference on Aeronautical Nomenclature of the National Advisory Committee for Aeronautics
- The Timber Conservation Conference called by the Department of Agriculture

At the Rensselaer Polytechnic Centenary celebration the Honorary Degree of Doctor of Engineering was conferred on President Low.

International

The World Power Conference held in London June 30 to July 12 was a most notable gathering.

The conference was organized by the British Electrical and Allied Manufacturers' Association in coöperation with numerous technical, scientific, and commercial organizations. The purpose was to consider the sources of world power by evaluating the resources of each country, by comparing experiences in the development of scientific agriculture, irrigation, and transportation, by engineering conferences, by consultations of power consumers and power-machinery manufacturers, by financial and economic discussions, and by conferences looking to the establishment of a permanent world bureau for the collection of data and the exchange of industrial and scientific information. The strenuous work of the technical sessions was relieved by a number of receptions, luncheons, banquets, and other social features arranged by British organizations. These formed a valuable part of the conference as they permitted the development of acquaintances, which should be one of the permanent results of the international meetings.

Coincident was the summer meeting of the Institution of Mechanical Engineers, July 7-9. The Institution conferred Honorary Membership on our President, Fred R. Low.

Kelvin Medal Award. The centenary of the birth of Lord Kelvin was commemorated on July 10 and 11 by a series of celebrations organized in London by the British Institutions of Civil, Mechanical, and Electrical Engineers. At this time the second triennial award of the Kelvin Medal was made to Elihu Thomson. The first award was made in 1920 to Dr. William Cawthorne Unwin, F. R. S., Hon. Mem. A.S.M.E. It is presented triennially as a mark of distinction to a person who has reached high eminence as an engineer or investigator in work applicable to the advancement of engineering. The fund for the medal is administered by the Institution of Civil Engineers and the committee is made up of the presidents of the principal British engineering societies. President Low and Ambrose Swasey, Past-President and Honorary Member, were the A.S.M.E. appointees on the Committee of Honor for the Kelvin Award.

The First International Management Congress, held in Prague, Czechoslovakia, July 20-24, was reported as a tremendous success. The official invitation was from the Government of the Republic of Czechoslovakia and the Masaryk Academy through the American Engineering Council to the four Founder Societies and to the member societies of the American Engineering Council. The Founder Societies in turn were represented by a general committee on American participation consisting of representatives of American Management Association, the Management Division of the A.S.M.E., the National Association of Cost Accountants, the Society of Industrial Engineers, and the Taylor Society.

The registration of over 800 from 15 countries and an attendance of 500 at the sessions, indicated the interest in management in central Europe. About 40 engineers from the United States participated in the Congress.

Pan-American Industrial Standardization. Important commercial, technical, and trade associations in this country interested in Latin-American commerce were represented at a conference on Pan-American standardization in Boston on June 3. The conference, called upon the request of the Inter-American High Commission and the United States

Department of Commerce, under the auspices of the American Engineering Standards Committee, was to enable American industries to arrange for participation in the official Pan-American Congress on Standardization, to be held in Lima, Peru, in December, 1924, and to give an opportunity to the technical industries of this country to formulate a general policy toward the undertaking.

The Chairman of the American Engineering Standards Committee was the official representative of the U. S. Government and of the joint standardization committees, and our Society appointed our members in Peru as the official Honorary Vice-Presidents to represent the A.S.M.E. at the Congress.

APPENDIX

REPORT OF ACCOUNTANTS

Wm. J. Struss & Co., certified public accountants, give the results of their examination of the books of the Society for the fiscal year ended September 30, 1924, in the following statements of assets and liabilities, and income and expenses.

STATEMENT OF FINANCIAL CONDITION, SEPTEMBER 30, 1924

Assets

Cash			
Cash on hand and in Bank (for Working Capital)	$37,328.91		
Cash on hand and in Bank (for Contributions) per contra	3,359.46		
Cash on hand and in Bank (for Advance Payments) per contra	1,548.09		
		$42,236.46	
Income Invested (for Reserve)			
Liberty Bonds 4¼s	$25,000.00		
Lawyers Mortgage Bonds 5½, 1929	41,000.00		
		66,000.00	
Accounts Receivable (Less Bad Debts—Depreciation)			
Set aside for Major Current Liabilities	$59,028.92		
Set aside for Surplus	66,589.62	125,618.54	
Total Assets			$233,855.00
Inventory			
Stores and Supplies (not merchandisable)			27,340.20
Deferred Charges (from surplus)			
Compiling Publications for Future Sale			20,625.73
Investments for Trust Funds (per contra)			
Lawyers Mortgage 5½—1927 and 1929	$50,000.00		
St. Louis, Peoria, and N. W. 5—1948	10,613.89		
Cash in Books	1,692.78		
Total			62,306.67
Fixed Assets (per contra)			
Building and Equipment (Book Value)	$496,642.38		
Library Books (Book Value)	13,000.00		
Engineering Index (Book Value)	10,000.00		
			519,642.38
Total			$863,769.98

Liabilities

Current Liabilities			
Major Current Liabilities			
Unfilled Obligations		$59,028.92	
Contributions (per contra)			
Taylor Society	$137.00		
Gear Research Fund	1,725.00		
Steam Table Research	638.71		
Standardization, Bolts, Nuts, Rivets	688.92		
Management Week Fund	169.83		
		3,359.46	
Dues Paid in Advance (per contra)		1,548.09	
Total Liabilities			$63,936.47
Trust Funds (per contra)			
Life Membership Fund		$48,534.42	
Library Development Funds		5,098.82	
Weeks Legacy		2,035.28	
Melville Fund		1,383.72	
Charles T. Main Fund		2,730.00	
Hunt Memorial		264.43	
Hess Prize—Juniors and Students		2,000.00	
Easby Trust Funds		260.00	
Total			62,306.67
Capital Invested (per contra)			
Invested in Building and Equipment		$496,642 38	
Invested in Library Books		13,000.00	
Invested in Engineering Index		10,000.00	
			519,642.38
Reserve			
Working Capital			
Inventory	$27,340.20		
Cash	37,328.91		
		$64,669.11	
Surplus			
Liberty Bonds and Lawyers Mortgage	$66,000.00		
Accounts Receivable	66,589.62		
Deferred charges (per contra)	20,625.73		
		153,215.35	
			$217,884.46
Total			$863,769.98

INCOME AND EXPENSES FOR THE TWELVE MONTHS ENDED SEPTEMBER 30, 1924

Income		Expenses	
		Administration and General	
		Secretarial Administration	$17,718.49
		Accounting	14,250.00
		Rent (our share)	9,822.33
		Upkeep of Rooms	404.40
		Insurance of Employees	656.72
		Council	11,718.03
		Nominating Committee, 1924	1,769.74
		Increase in Dues	2,383.09
		Unexpected Miscellaneous Expenses.	1,996.37
			$60,719.17
		Initiation Fees Account	
Initiation Fees	$30,327.92	Society Development	$2,839.27
		Membership Applications	9,933 38
		Student Branches	2,640.66
	$30,327.92		$15,413.31
		"Service to Member" Account (Class A) [1]	
Membership Dues	$218,504.15	Annual and Spring Meetings	$17,026.18
		1 Regional Meeting	754.32
		Professional Divisions	4,213.69
		Local Sections	34,551.92
		Mechanical Engineering Text	63,305.16
		A.S.M.E. News	10,776.82
		Transactions 44	26,000.00
		Transactions 45	39,555.15
		Year Book	7,838 34
		Library (our share)	8,006.50
		Constitution and By-Laws	785.44
	$218,504.15		$212,813.52
		Income Producing and Professional (Class B) [1]	
Mechanical Engineering Adv.	$186,669.31	Mechanical Engineering Adv.	$89,538.79
A.S.M.E. News	1,166.00	Condensed Catalogues	55,778.84
Condensed Catalogues	86,516.48	Publications Sales	34,600.30
Publications Sales	47,159.52	Miscellaneous Sales	4,002.37
Miscellaneous Sales	7,421.86	Transactions Bindings	5,250.00
		Journal Development	11,544.87
		Research	657.16
		Standardization	7,762.76
		Power Test Codes	5,280.07
		Boiler Code	7,093.85
	$328,933.17		$221,509.01
		"Service to Public" Account (Class C) [1]	
		Amer. Engineering Council	16,605.59
		Employment Service (our share)	3,711.60
		Amer. Engrg. Stds. Committee (our share)	1,500.00
			$21,817.19
		Miscellaneous	
Interest	$4,178.47	Provision for Bad Debts	$6,000.00
Meetings Registration Fees	1,476.00		
Boiler Code Royalties	1,000.00		
Unexpected Balance, 1922-23	4,640.03		
	$11,294.50		
TOTAL	$589,059.74	TOTAL	$538,272.20

[1] Refers to designations of 1923 Special Committee on Policy.

No. 1929

THE INCREASE IN THERMAL EFFICIENCY DUE TO RESUPERHEATING IN STEAM TURBINES

By W. E. Blowney[1] and G. B. Warren,[1] Schenectady, N. Y.
Junior Members

Experience has shown that as the ratio of the number of stages of a turbine operating in the superheated region to the number operating in the moisture region is increased, the actual thermal efficiency is improved at a rate far in excess of the theoretical value. The results of a large number of tests on turbines at different superheats have been analyzed to form a basis for calculating the gain in thermal efficiency due to resuperheating. In this paper, these results are given showing that the heat consumption of a turbine installation may be decreased from 6 to 7 per cent as a result of resuperheating the steam. There seems to be a rather broad pressure range at which the resuperheating may take place in order to show approximately the maximum saving. The probable gain due to more than one resuperheating has been obtained. These results indicate that, when taking into account the pressure drop in the resuperheaters, the gain possible by going to more than two stages of resuperheating would be very small. The effect of resuperheating upon the exhaust conditions and capacity of the turbine is discussed. Curves are presented to show that the gains due to the resuperheating and the regenerative cycles are very nearly independent of each other and that these features may be used in the same turbine, with the expectation that the savings due to both will be nearly additive.

THE primary purpose of the present paper is to set forth the analysis of the data available to the authors relative to the increase in turbine efficiency with increased initial superheat and to apply such analysis to a determination of the probable gain due to resuperheating in high-pressure turbine installations. In connection with this, the resulting resuperheat cycles have been combined in two instances with regenerative or steam-extraction feedwater-heating cycles and an analysis made of the combined gain in efficiency.

[1] Turbine Engineering Dept., Gen. Elec. Co.

Contributed by the Power Division and presented at the Annual Meeting, New York, December 1 to 4, 1924, of The American Society of Mechanical Engineers.

2 It has been recognized for a long time that whatever gain would result from resuperheating was in the main part due to the increase in the steam-turbine efficiency resulting from the greater superheat and reduced moisture content of the steam in the turbine rather than to the thermodynamic improvement of the heat cycle.

3 The data relative to the gain in turbine efficiency with increased initial superheat are, on the whole, not as complete as might be desired, particularly on large turbines, since it is difficult to vary the initial superheat over a satisfactory range in the average power station. In the main part, it has been necessary to base the present series of calculations upon tests on turbines of from 2500 to 10,000 kw. capacity, but very complete tests are available at low initial pressures and fairly complete tests are available at moderately high pressures at these capacities.

4 The data seem to point to the conclusion that the slope of a superheat-efficiency curve is almost independent of the type of turbine. This is borne out by published superheat-efficiency curves on other machines which agree very closely with the data that have been used.

5 The superheat-efficiency curves available to the authors have been plotted in comparison, graphically averaged to give curves of representative slope, and then these average curves have been analyzed on such a basis as to make the results available for determining the increase in the turbine efficiency due to resuperheating. This, when combined with the increase in the thermal efficiency of the heat cycle, gives the overall improvement in turbine heat consumption which can reasonably be expected.

Definitions

6 In this paper improvements in the heat consumption of a turbine will be spoken of as "percentage improvements in the thermal efficiency." For example: A 10 per cent change in a thermal efficiency which, let us say, is 30 per cent to begin with, would mean that the improved efficiency is 33 per cent. Percentage changes in the efficiency ratio of a turbine or turbine stage are taken in the same sense.

7 The following terms are used as defined:

Thermal Efficiency of Turbine: The ratio between the heat equivalent of the output of the turbine at a prescribed point and the total net heat input into the turbine.

Turbine Efficiency: The ratio between the heat equivalent of the output of the turbine at a prescribed point and the adiabatic available energy between the pressure and temperature limits through which the turbine works.

Turbine Output: Throughout this paper for the sake of simplicity, unless otherwise specified, the turbine output has been assumed to be the output at the turbine-wheel hubs and the

efficiencies have been based on this value. The losses due to the bearings, high- and low-pressure packings, generator, etc., have thus been excluded and the leaving velocity loss has been assumed to be zero, since these vary with each installation and design.

Fraction of Adiabatic Drop at Which Resuperheating Takes Place: This value is the ratio of the adiabatic drop which takes place before resuperheating as compared to the total adiabatic heat drop from the initial to the final pressures along the initial entropy line.

Assumptions Used in Analysis of the Gain in Turbine Efficiency with Increased Initial Superheat

8 Curve A-3, Fig. 1, shows a typical turbine-efficiency-initial-superheat curve. The improvement in the turbine efficiency as the superheat increases is due to an improvement in the individual stage efficiency and to an improvement in the reheat factor R with increasing initial superheat.

9 A modern turbine is made up of a number of individual stages, each stage taking a part of the pressure drop, and thus a part of the heat or energy drop. It would be thought at first that the efficiency of the complete turbine would be the weighted average (weighted according to the heat drop taken per stage) of the individual stage efficiencies. Such, however, is not quite true. Since each stage is less than 100 per cent efficient, a certain amount of energy is lost in each stage and is thus reconverted into heat which is used in "reheating" the steam. This reheating of the steam in each stage thus makes a greater amount of energy available in the succeeding stages, since the reheating increases both the entropy and the total heat in the stage. Therefore, the summation of the adiabatic energy drops of each stage is always greater than the adiabatic energy drop over the entire machine. Expressed in symbols, if

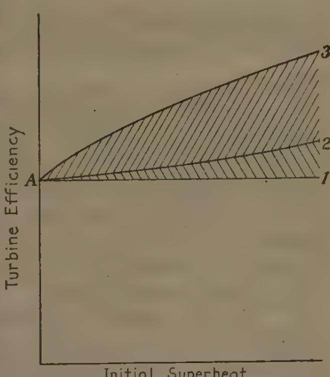

Fig. 1 Typical Superheat Turbine-Efficiency Curves

i_1 = initial total heat in the steam per lb., and if
i_2 = final total heat in the steam after adiabatic expansion from i_1 to the final back pressure, then
$i_1 - i_2$ = the available energy in B.t.u. per lb. of steam, and if
Δi = the fraction of the heat drop available for a single stage, then, on account of the reheating mentioned above,

$$\Sigma\Delta i = R(i_1 - i_2) \quad \ldots\ldots\ldots \quad [1]$$

where R is the so-called "reheat factor." Let

W = the work done in the turbine in ft-lb. per lb. of steam and

$A = \frac{1}{778}$ or heat equivalent of 1 ft-lb.

Then the turbine efficiency will be

$$\eta_t = AW/(i_1 - i_2) \quad . \; . \; . \; . \; . \; . \; . \; . \quad [2]$$

and we might define the average individual stage efficiency as

$$\eta_s = AW/\Sigma\Delta i \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [3]$$

But from Equation [1]

$$\frac{AW}{\Sigma\Delta i} R = \frac{AW}{(i_1 - i_2)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [4]$$

therefore,

$$\eta_t = R\eta_s \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [5]$$

R is always greater than unity since the return of heat to the steam due to the losses in the machine always make $\Sigma\Delta i$ greater than $i_1 - i_2$.

10 In general, for a given ratio of initial and final pressures the greater the proportion of the machine in the superheat region, the greater the value of R. (See Fig. 4.)

11 If, therefore, the stage efficiency η_s remains constant as the initial superheat increases, the turbine efficiency will follow some such line as A-2 in Fig. 1. The difference between the curve A-2 and the actual curve A-3 must then be due to an increase in the average stage efficiency.

12 There is reason to believe from data available that there is practically no change in the individual turbine stage efficiency with increase in the average superheat of the steam in the stage. The change in the average stage efficiency with superheat must then be due to the change in the stage efficiency with change in the moisture content of the stages in the moisture region. The method of analysis, therefore, to find the average correction to apply to the stage efficiency is to correct for the moisture content in the stage on the assumption that the expansion takes place in thermal equilibrium, and that the reduced efficiency with increased moisture content is due to the mechanical effect of the moisture in the steam. This method then involves no assumption regarding the probable degree of supersaturation of the steam during the expansion.

13 The authors are fully aware of the possibility, as brought out by H. M. Martin[1] and Prof. H. L. Callendar,[2] that a part of the improvement in turbine efficiency with increased superheat may be due to supersaturation, particularly since supersaturation or failure of the steam to condense and so give up its latent heat

[1] A New Theory of the Steam Turbine, *Engineering*, vol. 106, p. 1.
[2] Properties of Steam.

during expansion is known to exist to a certain extent in the extremely rapid expansion of steam in turbine nozzles. The exact extent of such supersaturation in turbines is, however, unknown to the authors at present; and, so far as can be seen, the results of the present analysis, when applied to the resuperheating problem, would not be different had the probable degree of supersaturation been considered.

Data Available and Analysis

14 Approximately twenty complete superheat-efficiency curves were available. These had been obtained on ten different turbine combinations. The machines were all of the multi-stage impulse type, the initial pressures ranged from 60 to 200 lb. per sq. in. absolute, and the back pressures were all one and a half in. Hg. absolute. These curves have been obtained at constant speed, but at many different points on the speed curves of the different machines. The tests were in many cases made practically under laboratory conditions.

15 In using these data in connection with a determination of the gain due to resuperheating, it was necessary to consider that in case a turbine was designed for a resuperheating installation, a greater amount of bucket speed would be put into the machine to take care of the increased available energy and to keep constant the ratio of the bucket speed to jet speed. Therefore, the superheat-efficiency curves had to be corrected to such a basis that the ratio of bucket speed to steam-jet speed was a constant. That is, it was assumed that the turbine was speeded up in proportion to the square root of the increased available energy as the superheat increased. This necessitated speed curves at constant superheat on the different machines, in order to make such corrections. These speed curves were available in most cases.

16 When these speed corrections had been made, the slopes of the curves with 60 lb. per sq. in. initial pressure and those with 200 lb. per sq. in. initial pressure were averaged separately. Then by a cut-and-try process, an average stage efficiency and a correction for moisture were obtained which, if applied together with the correct reheat factor, which had been previously calculated, enabled calculated superheat-efficiency curves to be obtained. These calculated curves coincided with the average test curves to within a fraction of 1 per cent throughout the entire range of superheat from 0 to 300 deg. fahr. superheat at both initial pressures. The correction for moisture came out in each case to be 1.15 per cent decrease in the turbine stage efficiency for each 1 per cent increase in the average moisture content in the stage. As an example, if the stage efficiency when operating with superheated steam were 85 per cent, with 10 per cent moisture in the steam it would be

$$0.85(100-1.15\times 10) = 75.2 \text{ per cent}$$

This value of 1.15 per cent decrease in stage efficiency with each 1 per cent increase in the moisture content has been used in the following analysis of the gain due to resuperheating.

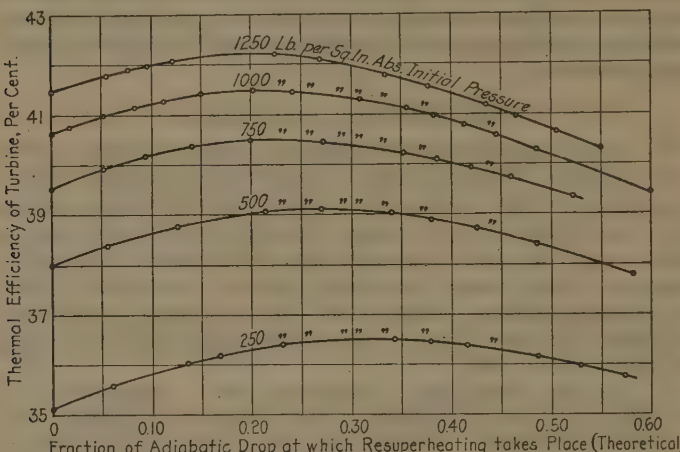

FIG. 2 THEORETICAL THERMAL EFFICIENCY OF HEAT CYCLES AT DIFFERENT INITIAL PRESSURES AND AT DIFFERENT POINTS OF RESUPERHEATING. (TURBINE EFFICIENCY 100 PER CENT)

(Initial and resuperheating temperature 750 deg. fahr. Back pressure 1 in. Hg abs.)

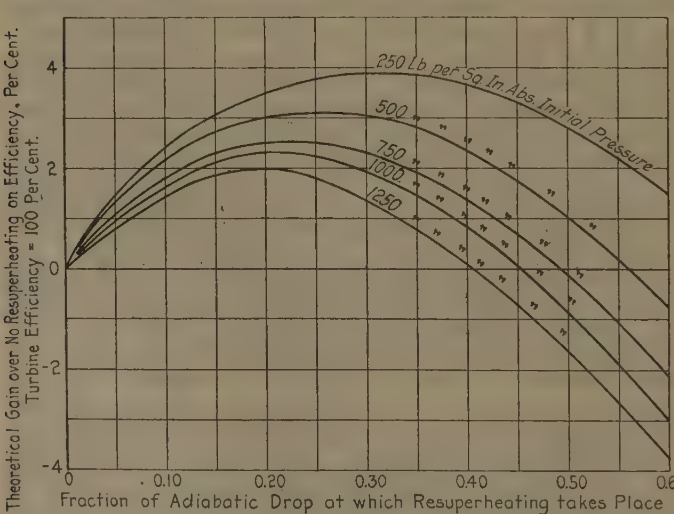

FIG. 3 THEORETICAL GAIN IN EFFICIENCY OF HEAT CYCLES DUE TO RESUPERHEATING AT DIFFERENT POINTS AND FOR DIFFERENT INITIAL PRESSURES

(Initial and resuperheating temperature 750 deg. fahr. Back pressure 1 in. Hg abs.)

17 The value of 1.15 per cent change in stage efficiency agrees very closely with the 1 per cent change per 1 per cent change in moisture content given by K. Baumann,[1] and differs in the right direction, since the value obtained here is on the basis of a change in turbine speed to correspond with a change in the superheat.

Application to Resuperheating

18 Before applying the preceding analysis to the calculation of the gain in turbine efficiency due to resuperheating it was necessary to calculate the theoretical thermal efficiencies of the resuperheating cycles which were to be studied and to determine

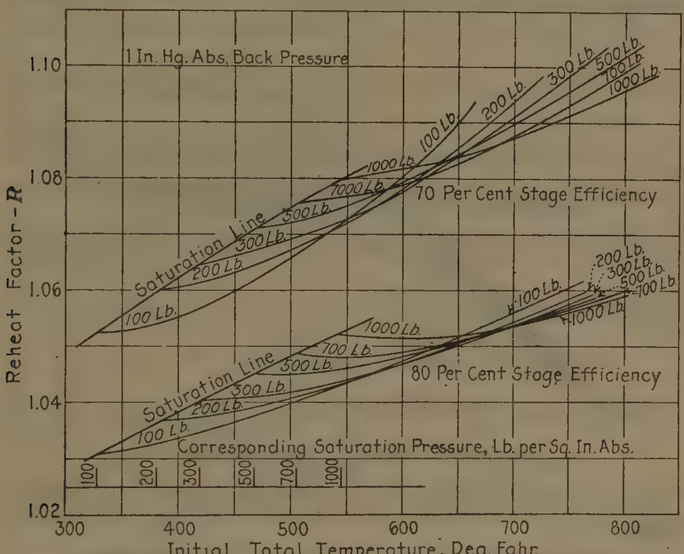

Fig. 4 Reheat Factors for an Infinite Number of Stages at Various Initial Pressures, Initial Temperatures and Stage Efficiencies

(Back pressure 1 in. Hg abs.)

the percentage gain in each case over the corresponding cycle without resuperheating. The resulting curves are given on Figs. 2 and 3. It will be noted that the theoretical savings shown on Fig. 3 are comparatively small, the maximum being 3.9 per cent on the 250-lb. curve, and the gain decreasing as the initial pressure increases.

19 After determining the correction of 1.15 per cent change in efficiency per 1 per cent in moisture content discussed in the preceding paragraphs, it was necessary to make certain assump-

[1] Some Recent Developments in Large Steam Turbine Practice, *Jour. Inst. E. E.*, vol. 59, p. 565.

tions regarding the variation in the dry-stage efficiency, and to determine the proper reheat factors to be used with the various steam conditions used in the calculations. By dry-stage efficiency is meant the efficiency of a turbine stage where the steam is dry or superheated.

20 In general the higher the pressure in a stage the lower will be its efficiency due to the necessarily smaller nozzles and buckets and to the higher rotation and packing losses. Inasmuch as the stage efficiency is largely a matter of design, two assumptions were made regarding its variation with pressure: (1) That the dry-stage efficiency decreased in a certain manner as the pressure increased, and (2) That the dry-stage efficiency was constant

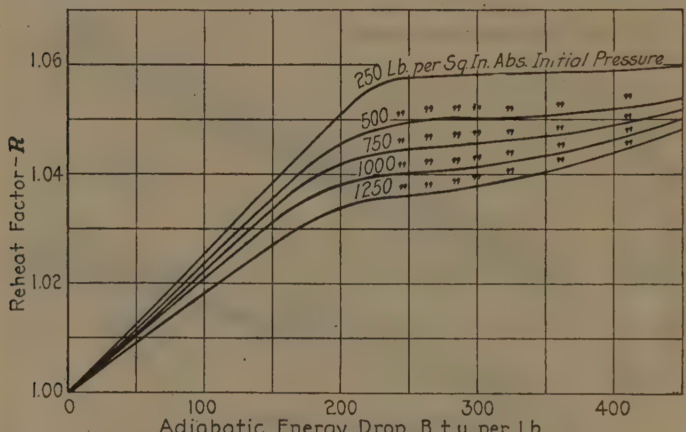

Fig. 5 Reheat Factors for an Infinite Number of Stages at Various Initial Pressures as Function of the Adiabatic Energy Drop

(80 per cent stage efficiency, 750 deg. fahr. initial temperature.)

throughout the turbine. As will be noted in later paragraphs, the results obtained using these two assumptions differ slightly.

21 Due to the fact that the curves given in this paper do not include last-stage leaving losses, bearing losses, high- or low-pressure packing losses, or generator losses, all thermal efficiency curves should be considered as being relative and not absolute in value.

22 The reheat factors which were used are given in Figs. 4 and 5. In both of these sets of curves the reheat factors are for a turbine having an infinite number of stages. A turbine having from fifteen to twenty stages will have a reheat factor in which the part greater than unity will have a value approximately 0.9 of the value with an infinite number of stages. This factor of 0.9 was used in all calculations.

23 It will be noted also that the reheat factor curves given in Fig. 5 are all on the basis of an 80 per cent stage efficiency.

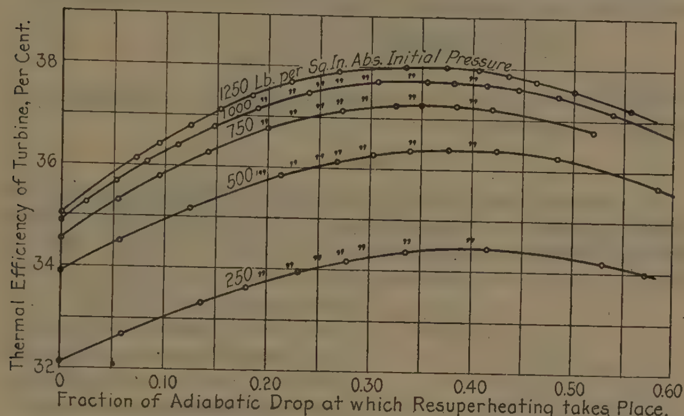

Fig. 6 Thermal Efficiency of Turbine for Various Initial Pressures as Function of Point of Resuperheating

(Initial and resuperheating temperatures 750 deg. fahr. Back pressure 1 in. Hg abs. *Variable stage efficiency as a function of stage pressure.* No pressure drop in resuperheater.)

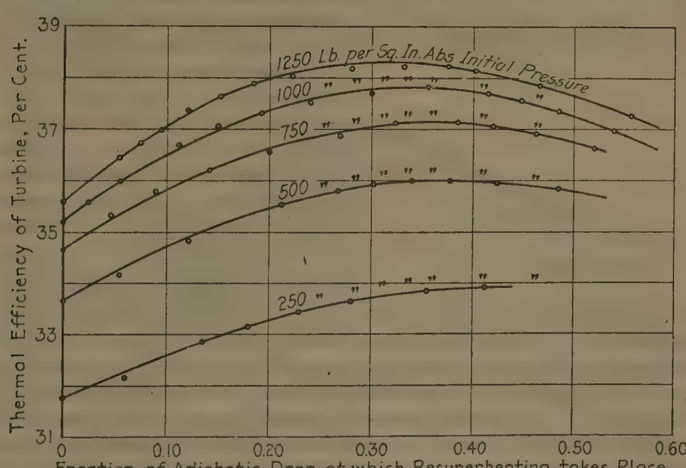

Fig. 7 Thermal Efficiency of Turbine for Various Initial Pressures as Function of Point of Resuperheating

(Initial and resuperheating temperatures 750 deg. fahr. Back pressure 1 in. Hg abs. *Constant stage efficiencies.* No drop in pressure in resuperheater.)

In order to obtain the reheat factor for efficiencies other than 80 per cent, the following approximation was used:

24 If $x = R' - 1$, where R' is the reheat factor for an infinite number of stages at 80 per cent stage efficiency, it was assumed

that the part x was proportional to the difference between 100 per cent and the per cent stage efficiency, which is very nearly true. These assumptions then make the reheat factor used

$$R = 1 + 0.9(R' - 1.0)\left(\frac{100 - \eta_s}{20}\right)$$

where R is the reheat factor used and η_s is the average stage efficiency in per cent.

Results

25 Figures 6 and 7 show the thermal efficiencies obtained for different points of resuperheating for the variable and constant dry-stage efficiency assumptions and with 250, 500, 750, 1000 and 1250 lb. per sq. in. absolute initial pressure and an initial and resuperheat temperature of 750 deg. fahr. These curves include the improvement in the turbine efficiency due to resuperheating and were obtained as follows:

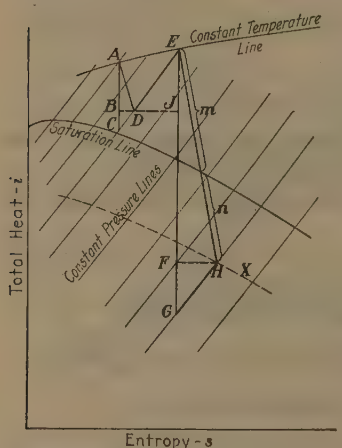

Fig. 8 Typical Expansion Lines on Mollier Diagram

26 Where the portion of the turbine under consideration was entirely in the superheat region, the end point D (see Fig. 8) was determined by multiplying the average stage efficiency by the reheat factor and subtracting the product of the result times the heat drop AC from the initial total heat A. Where a portion of the part of the turbine under consideration was in the moisture region, such as shown by the line EH, a "cut and try" process had to be used. An assumption was made regarding the location of point H and the per cent moisture at this condition was noted. It was assumed that the average moisture content per stage of the part of the machine in the wet region was one-half the final moisture content at H. Using the 1.15 per cent moisture correction previously described and the assumed dry-stage efficiencies under consideration, the efficiency of that portion of the machine in the moisture region was calculated. On the assumption that the state curve from E to H was a straight line and weighing the efficiencies in the wet and dry portion of the machine according to the energy drops n and m in the wet and dry portions of the turbine, respectively, the average stage efficiency from E to H was determined. The turbine efficiency was obtained by multiplying this by the proper reheat factor obtained from Fig. 4 and the formula previously given for R. Knowing this value, the closeness of

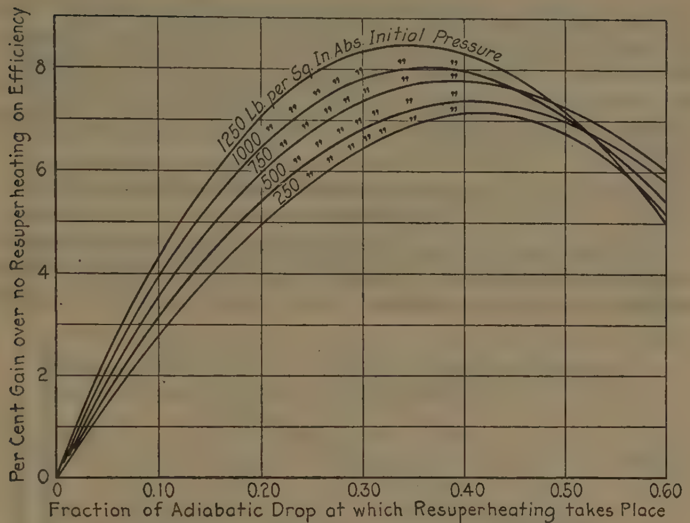

FIG. 9 PER CENT GAIN IN THERMAL EFFICIENCY OF TURBINE DUE TO RESUPERHEATING AS A FUNCTION OF FRACTION OF ADIABATIC DROP AFTER WHICH RESUPERHEATING TAKES PLACE

(Initial and resuperheating temperatures 750 deg. fahr. Back pressure 1 in. Hg abs. *Variable stage efficiency as function of stage pressure.* No drop in pressure in resuperheater.)

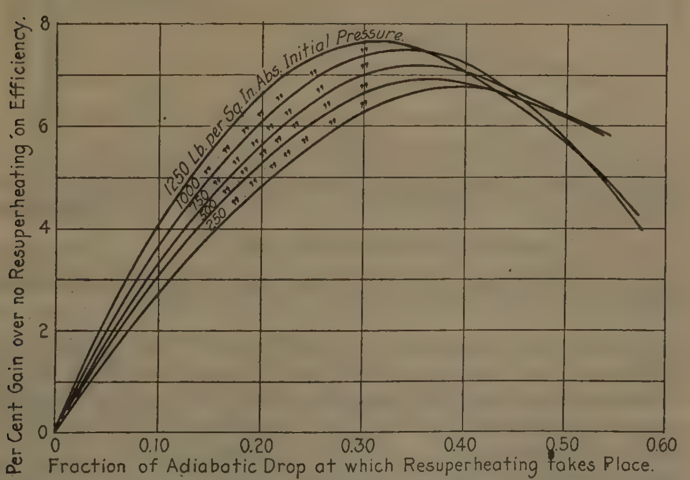

FIG. 10 PER CENT GAIN IN THERMAL EFFICIENCY OF TURBINE DUE TO RESUPERHEATING AS A FUNCTION OF FRACTION OF ADIABATIC DROP AFTER WHICH RESUPERHEATING TAKES PLACE

(Initial and resuperheating temperatures 750 deg. fahr. Back pressure 1 in. Hg abs. *Constant stage efficiency.* No pressure drop in resuperheater.)

the first approximation to the location of H was checked. Two or three approximations were usually all that were necessary to locate H. It was recognized that the assumption of a straight-line state curve between E and H was slightly in error, but it was not serious enough to affect the final results to any appreciable extent.

Increase in Thermal Efficiency of the Turbine with Resuperheating

27 Figs. 9 and 10 were obtained from Figs. 6 and 7 respectively by subtracting the thermal efficiency without resuperheating from the thermal efficiency with resuperheating at a given

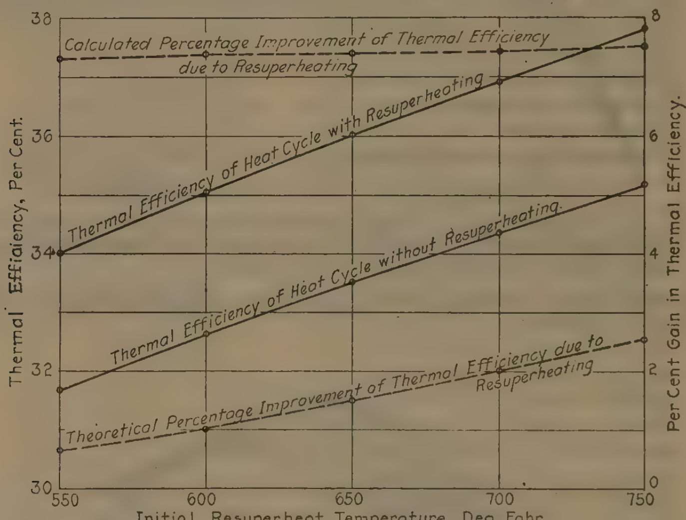

Fig. 11 Thermal Efficiency of Turbine and Per Cent Gain Due to Resuperheating as Function of Initial and Resuperheating Temperature

(Initial and resuperheat temperatures equal. 1000 lb. per sq. in. abs. initial pressure. Reheating at 150 lb. per sq. in. abs. Back pressure 1 in. Hg abs. No drop in pressure in resuperheater. Broken line at top of figure gives calculated percentage improvement of thermal efficiency of turbine due to resuperheating.)

point and at a corresponding initial pressure, and dividing the difference by the thermal efficiency without resuperheating. These curves show the order of magnitude of the percentage gain in thermal efficiency due to resuperheating. No account has been taken of the pressure drop in the resuperheater and connected piping, as this will, of course, depend upon the particular design. Such drop will probably reduce the gain by $\frac{1}{2}$ to $\frac{3}{4}$ of one per cent.

28 It is interesting to note that the gain, considering the change in turbine efficiency, is greater at the higher initial pressures, whereas the theoretical gain shown on Fig. 3, considering the heat

cycle alone, is lower at the higher pressures. This is due to the fact that a greater portion of the turbine is operating in the moisture region with the high initial pressures and so is susceptible of greater improvement.

29 It should also be noted by reference to Fig. 15 and in conjunction with Figs. 6, 7, 9 and 10 that the best point for resuperheating comes at approximately one-fifth to one-sixth of the initial pressure. This gives resuperheating pressures for initial pressures of 500 lb. per sq. in. and more, which do not require resuperheating piping of excessive size.

30 The greater gain shown by the assumption that the dry-stage efficiency decreased with increasing pressures in the stage

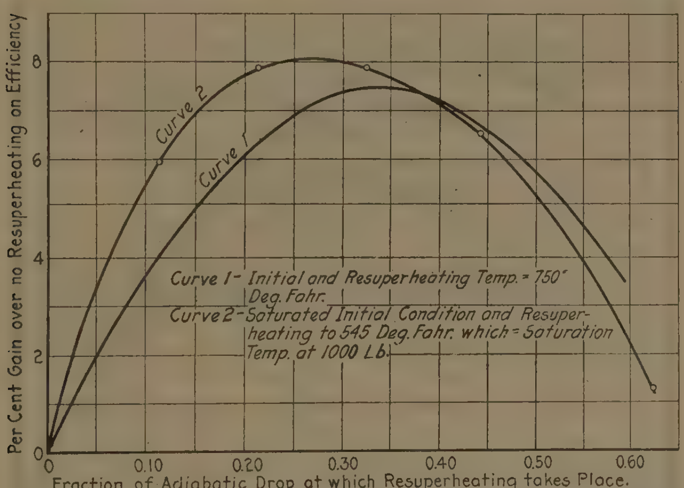

FIG. 12 COMPARATIVE GAIN IN THERMAL EFFICIENCY OF TURBINE DUE TO RESUPERHEATING WITH TWO DIFFERENT INITIAL TEMPERATURES AT 1000 LB. PER SQ. IN. ABS. INITIAL PRESSURE AND 1 IN. HG ABS. BACK PRESSURE

(No drop in pressure in resuperheater.)

is due to the fact that the gain in turbine efficiency from superheating is greatest in the low-pressure end where the efficiency is the highest under the variable-efficiency assumption. This results in a greater improvement than on the assumption of a constant stage efficiency. The variable stage efficiency is the more probable condition, considering the design of present-day machines.

31 In order to determine the effect of different initial and resuperheat temperatures upon the gain in thermal efficiency, a set of calculations was made on the assumption that the initial and resuperheat temperatures were varied from 750 deg. fahr. (corresponding to the other calculations) to 550 deg. fahr. total temperature. The initial and resuperheat temperatures were assumed equal in all calculations, and the point of resuperheating was taken

at a ratio of 0.355 of the adiabatic drop for the 750 deg. fahr. condition with 1000 lb. per sq. in. absolute initial pressure. This put the resuperheating pressure at 150 lb. per sq. in., absolute. The results of this calculation are shown on Fig. 11, and show that the percentage gain in thermal efficiency is approximately constant at this resuperheating pressure irrespective of the maximum temperature of the cycle so long as the resuperheating temperature equals the initial temperature. This is in marked contrast to the theoretical gain with increased initial and reheat temperature, as also shown on Fig. 11.

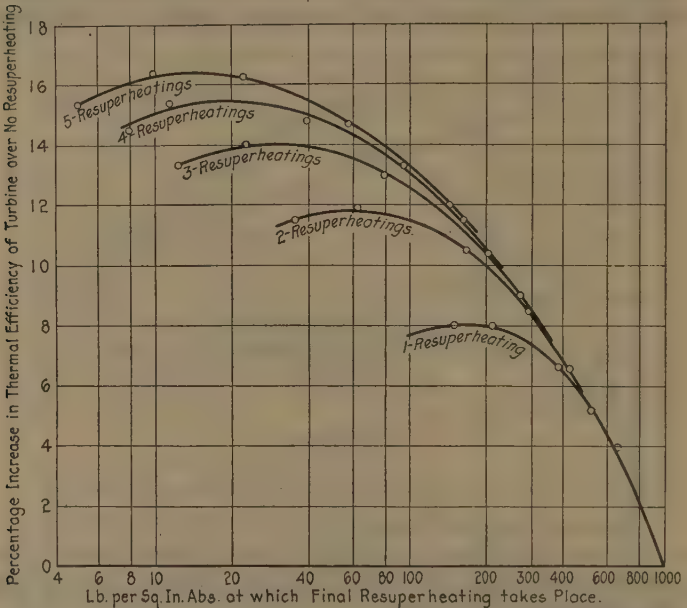

FIG. 13 COMPARATIVE GAIN IN THERMAL EFFICIENCY DUE TO USING ONE OR MORE STAGES OF RESUPERHEATING

(No drop in pressure in resuperheater. 1000 lb. per sq. in. abs. initial pressure. Each resuperheating to 750 deg. fahr. Final back pressure 1 in. Hg abs.)

32 Figure 12 shows comparative curves giving the per cent gain in thermal efficiency when reheating or resuperheating at various fractions of the adiabatic drop with initial temperatures and resuperheat temperatures of 750 deg. fahr. in one case and in the other case equal to the temperature of saturated steam at the 1000 lb. per sq. in. absolute initial pressure. These curves show that the fraction of the adiabatic drop at which it is best to resuperheat is slightly changed by the initial and resuperheating temperature, and that the maximum possible gain is somewhat greater in the case of the lower initial and resuperheating temperature. These

curves indicate that the best point for resuperheating is not necessarily the point at which the steam in the high-pressure turbine loses its superheat, as is quite commonly believed.

Increase in Thermal Efficiency of Turbine as a Result of Multiple Resuperheating

33 If it is advantageous to resuperheat once it immediately becomes apparent that it might be more advantageous to resuperheat more than once. The complication of such an installation would of course be very considerable, but the first thing is to

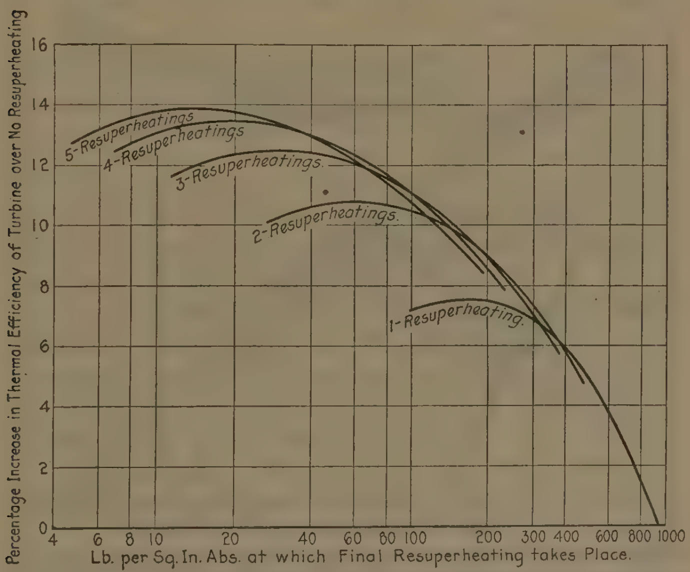

Fig. 14 Probable Actual Comparative Gain in Thermal Efficiency Due to Using One or More Stages of Resuperheating

(One-half of one per cent reduction in economy assumed for each resuperheating due to pressure drop in resuperheater and connected piping. 1000 lb. per sq. in. abs. initial pressure. Each resuperheating to 750 deg. fahr. Final back pressure 1 in. Hg abs.)

find out the probable gain which might result in order to determine whether such multiple resuperheating would pay economically considering the increased complication and investment involved.

34 The calculations necessary to a determination of the gain with multiple resuperheating taking into account the increase in turbine efficiency resulting from the reduced moisture content in the turbine become quite laborious, if made following the same system as that used in the single resuperheating calculations. A system was worked out, however, whereby it was possible to obtain the gain due to multiple resuperheating from the results

of the calculations for a single stage of resuperheating. The results of this calculation are shown on Fig. 13. If it is assumed that the drop in steam pressure in the resuperheaters and connected piping can be made so low as to reduce the available heat only one-half of one per cent for each resuperheating (this is somewhat lower than present practice) the net gains resulting from one or more resuperheating will be as shown in Fig. 14. It can then be seen that the gain in going from two to three stages of resuperheating becomes so slight as to offer slight inducement

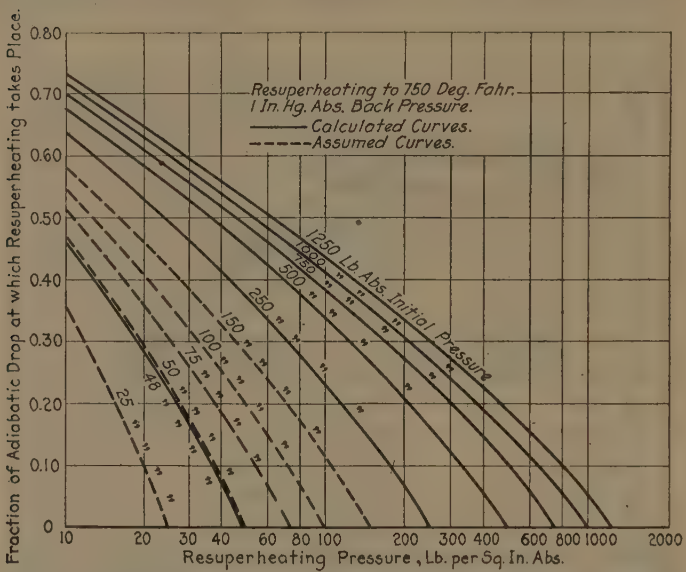

Fig. 15 Relation Between Pressure of Resuperheating and Fraction of Adiabatic Drop at Which Resuperheating Takes Place

to accept the increased complication and expense involved. The increase in efficiency resulting from two resuperheatings as compared to one might, however, warrant such an installation in the case of very large high-pressure base-load units.

35 Figure 15 shows the relation between the initial pressure, fraction of the adiabatic drop at which resuperheating takes place, and the resuperheating pressure, and will be of value in interpreting the results given in the preceding pages.

Effect of Resuperheating Upon Exhaust Conditions and Turbine Capacities

36 Fig. 16 shows several factors in the exhaust conditions of a turbine plotted as a function of the fraction of the adiabatic

drop before resuperheating. The particular curves of interest are those showing the variation of the total heat rejected to the condenser and the variation in the leaving loss of the turbine as the point of resuperheating is changed. The curves have been calculated for both the 1000 lb. per sq. in. and the 500 lb. per sq. in. initial conditions. Curves 2 and 6 on Fig. 16 show that per kw. of output the heat rejected to the condenser when resuperheating at 0.35 of the adiabatic drop is approximately 89 per cent of what it would be without resuperheating. This means that the condenser can be 11 per cent smaller for any given capacity.

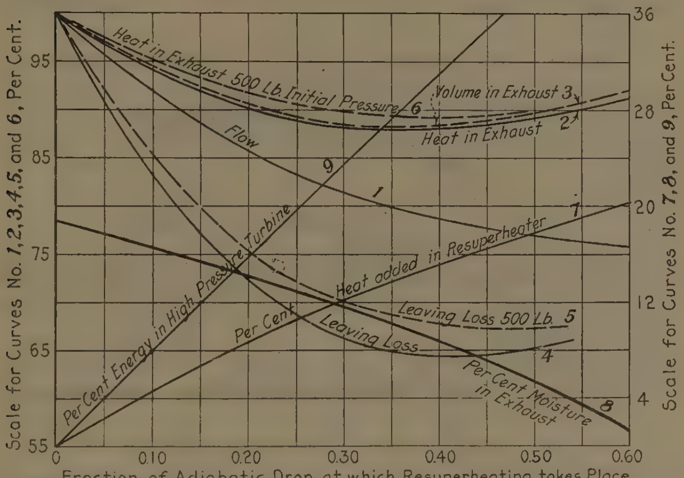

Fig. 16 Various Factors as Function of Point at Which Resuperheating Takes Place

(Unless otherwise noted on curve, initial pressure 1000 lb. per sq. in. abs.)

37 Curves 4 and 5 on Fig. 16 show the reduction in leaving loss of a turbine of any given capacity with any given last-stage wheel as the point of resuperheating is changed. These curves show that the exhaust loss with resuperheating becomes 65 to 67 per cent of what it would be at the same capacity and for the same last-stage wheel but without resuperheating. This is due to the reduced steam volume in the last stage per unit of output, as shown on Curve 3 on Fig. 16, and on account of the higher heat drop per pound of working fluid.

38 The reduction in leaving loss has not been credited to the machine in the analysis of the gain due to resuperheating, since in most cases advantage is taken of this fact to increase the capacity of the turbine. Since for any given capacity the percentage leaving loss is only 0.65 of what it would be without

resuperheating, the capacity of the turbine could accordingly be increased to $\frac{1}{\sqrt{0.65}}$ or 1.24 times what it would be with no resuperheating and with the same percentage leaving loss.

Gain Due to Combined Reheating and Steam-Extraction Regenerative Cycles

39 There appears to be no practical difficulty in the way of extracting superheated steam from a turbine for use in a closed or open heater for feed-heating purposes. It has been thought of value, therefore, to investigate whether both the resuperheating

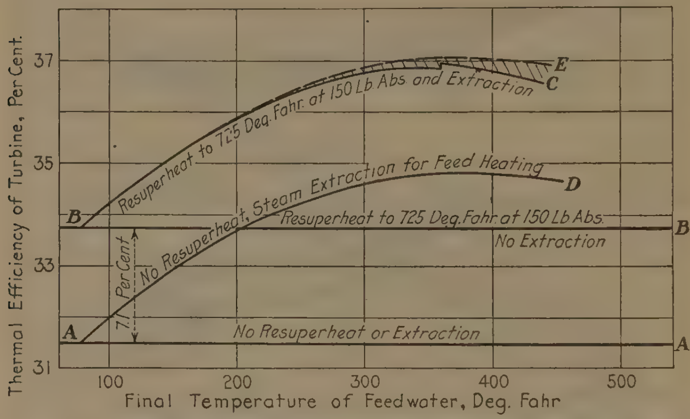

Fig. 17 Curves Showing Comparative Thermal Efficiencies of Turbines Without Resuperheating and with Steam Extraction for Feed Heating in Three Steps and of Turbines with Both Resuperheating and Extraction

(Initial pressure 515 lb. per sq. in. abs. Resuperheating at 150 lb. per sq. in. abs. Initial and resuperheating temperatures 750 deg. fahr. Back pressure 1 in. Hg abs.)

and regenerative cycles could be used together, so as to obtain the sum of the two gains of the cycles when used separately. It has been found that under the conditions which would probably obtain, the total gain in thermal efficiency of the combination is equal to practically 95 per cent of the sum of the gains in efficiency when used separately. This is on account of the very small proportion of the heat in the extracted steam which is due to the superheat.

40 Figs. 17 and 18 show such combinations. The curves in these two figures have been calculated under somewhat different turbine efficiency assumptions and steam conditions, and so are not mutually comparable. The curves in each figure are, however, strictly comparable.

41 Fig. 17 shows a resuperheating and a non-resuperheating cycle with steam extracted at three equally spaced points for heating the feedwater in both cases. The curves were made so as to show the gain in each case when heating the feedwater to different temperatures. The feed-heating calculations were made on an ideal basis assuming no temperature drop, no pressure drop and contraflow conditions in the heaters so that the drip from each heater left the heater at the temperature of the incoming feedwater. The drip was drained into the next lower heater in each case.

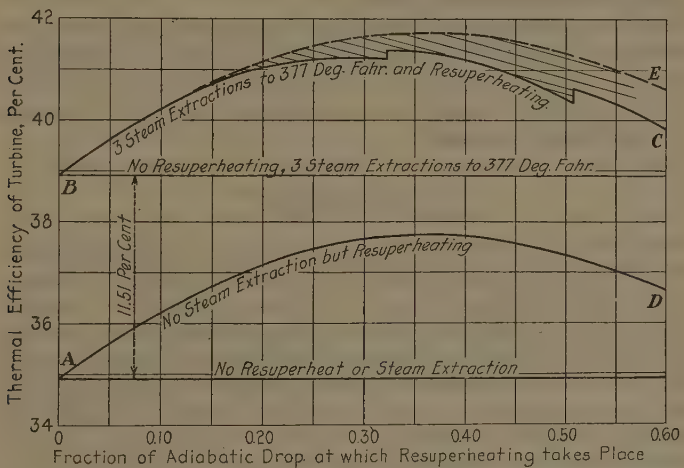

FIG. 18 CURVES SHOWING COMPARATIVE THERMAL EFFICIENCIES OF TURBINES WITH RESUPERHEATING AND NO STEAM EXTRACTION AND TURBINES WITH BOTH RESUPERHEATING AND STEAM EXTRACTION IN THREE STEPS FOR HEATING FEEDWATER TO 377 DEG. FAHR.

(Initial pressure 1000 lb. per sq. in. abs. Initial and resuperheating temperatures 750 deg. fahr. Back pressure 1 in. Hg abs.)

42 Curve *A-D* shows the thermal efficiency without resuperheating but with steam extraction for feed heating. Curve *B-C* shows the thermal efficiency with both resuperheating and steam extraction. Curve *B-E* is curve *A-D* translated up a distance *A-B*, thus showing that the two gains practically add one on top of the other. The cross-hatched area is the amount by which the total increase in efficiency, when using the combinations, fails to come up to the sum of the gains when used separately. The break in curve *B-C* occurs at the point where the upper heater reaches the resuperheating point. Curve *B-E* breaks away from curve *B-C* at approximately the point where the extracted steam becomes superheated.

43 Fig. 18 shows a similar set of calculations on the basis of resuperheating at various points in both a non-regenerative and a regenerative cycle, *A-D* being the non-regenerative cycle with resuperheating and *B-C* being the combined regenerating and resuperheating cycle. *B-E* is curve *A-D* moved up a distance *A-B*. It can be seen that the gains are nearly additive. The breaks in the curve *B-C* in Fig. 18 occur at the points where the resuperheating point coincides with the extraction points. These breaks show that wherever possible, it is more economical to extract at a pressure just above or at the resuperheating pressure and before the steam has been resuperheated, rather than at a point just below the resuperheating point.

44 The authors wish to express their appreciation of the help and advice of Mr. E. L. Robinson, Dr. A. Devaud and Mr. E. G. Swanson in the preparation of this paper. Fig. 4, which had been prepared by means of a method worked out by Mr. Robinson, was particularly helpful.

DISCUSSION

W. W. Johnson.[1] The magnitude of the gain possible due to resuperheating in steam turbines depends largely on the change in the stage efficiencies due to decrease of moisture. Tests on single turbine stages with different degrees of initial moisture are not available; in fact, such tests would be very difficult to make and their accuracy would be questionable. The authors have used the only practical and reliable method possible, namely, the analysis of actual turbine tests covering a wide range of superheat, in order to determine the influence of moisture on the efficiency of the stages in the wet-steam region.

In order to determine the performance of the wet-steam stages from tests with varying degrees of initial superheat, the stages which are entirely in the superheated region are separated by calculating their output. In this calculation the efficiencies of the "dry-steam" stages are fairly well known. The authors have assumed that the efficiency of a single stage does not change with initial superheat as long as the expansion in the nozzles does not cross the dry-steam line. This assumption is borne out by all data that the writer has seen.

It is well known that increase of initial superheat for a complete turbine is accompanied by a considerable increase in efficiency, the increase being greater than can be accounted for by diminished windage loss or other known effect. Since the "dry-steam" stages do not change, the increase in efficiency must necessarily come in the "wet-steam" stages, and must result from reduction of the average stage moisture.

[1] Turbine Engineering Dept., General Electric Company, Lynn, Mass.

The moisture correction finally deduced, of 1.15 per cent for each one per cent change in average moisture, when applied to tests with varying superheat gave calculated results which checked with the tests very closely for a wide range of superheat. The figure given by Baumann of one per cent per one per cent change in moisture checks very well. This effect determined, the remaining factors affecting the variation of efficiency with superheat, namely, the reheat factor and the thermal efficiency of the cycle, are well known and no great uncertainty exists regarding their magnitude. Previously published literature has dealt more with the steam-extraction regenerative cycle than with resuperheating. It is interesting to note that resuperheating in two stages gives very nearly the same gain in efficiency as three-stage extraction, and that both cycles may be used in combination without losing much of the individual gains.

Frank O. Ellenwood.[1] While the authors have presented many interesting and valuable relations, it is somewhat unfortunate that the results are based on data available to them only, and which should be given much more completely if their results are to be checked. Since resuperheating becomes of greater value as higher pressures are used, one is inclined to inquire why the maximum pressure considered was only 200 lb. per sq. in. abs. There surely must be data, now available to engineers of our large turbine manufacturers, in which turbines have been tested under various degrees of superheat with initial pressures much higher than 200 lb. per sq. in.

Except for Fig. 14 and Par. 27 the authors chose to draw their conclusions without consideration of the throttling losses through the reheaters and piping system, and without any consideration of the relative costs of these systems for the various reheating pressures. The title of the paper probably justifies the elimination of the item of cost, but it is the opinion of the writer that the paper would have been a more valuable one had various throttling losses been included in the study. For the central-station manager the selection of the proper reheating pressure certainly involves the cost of the reheating system and the space it occupies, as well as the increased thermal efficiency of the plant at various loads, when throttling is considered.

In Fig. 14 curves are given to show the relative advantages of multiple resuperheating when allowing one-half of one per cent reduction in economy for the drop of pressure in each reheating system. In the writer's opinion these curves should have been calculated with progressively larger percentage losses for the correspondingly lower pressures involved when more reheaters

[1] Professor of Heat Power Engineering, Cornell University, Ithaca, N. Y. Mem. A.S.M.E.

were considered, because the actual losses are bound to be greater with the lower pressures.

In Par. 9 the statement is made that in the actual turbine the "reheating of the steam in each stage thus makes a greater amount of energy available in the succeeding stages, *since it increases the entropy*." Applying this same reasoning a little further, one might conclude that the best way to obtain the maximum available energy from steam would be to throttle it at every opportunity as much as possible, since *throttling is always a sure way to increase its entropy*. The writer can hardly believe that the authors desire to have this statement stand as it is.

When dealing with reheating and the regenerative cycle in Par. 43 it is stated that "it is more economical to extract at a pressure just above or at the resuperheating pressure and before the steam has been resuperheated, rather than at a point just below the resuperheating point." The writer heartily agrees with this statement, but feels that more emphasis should be given to it when considering all the factors involved in any actual installation. It would seem to be the height of folly to heat feedwater by steam that has just been passed through a reheating system chiefly for the purpose of enabling this steam to improve the efficiency of the remaining portion of the turbine through which it passes.

Linn Helander.[1] The results obtained by the authors are somewhat inconclusive, since they neglect the influence on the best reheat pressure of losses in the reheater piping. Their data, however, appear to substantiate previous estimates, that the best reheating pressure for a non-regenerative cycle occurs after approximately one-third of the adiabatic heat drop when the initial pressure is 500 lb. per sq. in. and the initial and reheat temperatures are 750 deg. fahr. Fig. 18 of the paper also supports the opinion that higher reheating pressures can be used more profitably for a regenerative than for a non-regenerative cycle. This figure shows, when losses occurring in the reheater piping are ignored, that the best reheat pressure for a non-regenerative cycle occurs after 37 per cent of the adiabatic heat drop, while for a regenerative cycle the best reheat pressure occurs after 32 per cent of the adiabatic heat drop. Losses in the reheater piping will widen this difference. Other considerations, such as the distribution of the load between elements, assuming a compound turbine with reheating between elements, of course, may enter into the determination of the most suitable reheating pressure. But, particularly for the regenerative cycle, it appears best to use reheater pressures on the high pressure side of the peak of the efficiency curve.

Contrary to the opinions of the authors, Fig. 18 appears to lend weight to the argument that the gain in thermal efficiency due to

[1] Engr., Gen. Engrg. Div., Westinghouse Elec. & Mfg. Co., So. Philadelphia Sub-Station, Philadelphia, Pa. Mem. A.S.M.E.

the combined use of regeneration and reheating is appreciably less than the sum of the gains in thermal efficiency due to regeneration and reheating used separately. The authors point out that the gain due to the combined use of regeneration and reheating is five per cent less than the sum of the gains obtainable from the separate use of reheating and regeneration. Though this difference appears small, it cannot be ignored, since it is equivalent in actual value to one per cent of the total station fuel consumption. This is approximately of the same order as the gain usually creditable to the third-stage heater of a three-stage feedwater heating system. What is of particular interest, however, is not so much the amount by which the benefits from reheating and feedwater heating fail to be additive, as the influence that reheating has upon the economies of the higher-temperature feedwater heaters. It is probable that when reheating is incorporated in a power plant, the value of the third and fourth stages of feedwater heating will be considerably less than for a station not employing reheating. The argument holds with greater force when multiple-stage reheating is attempted. The authors' data indicate an appreciable improvement in economy from two-stage reheating with a non-regenerative cycle. The improvement would be less for a regenerative cycle. Practically, the difficulties in the way of reheating in two stages make two-stage reheating unattractive.

B. N. Broido.[1] This paper is practically the first publication showing the actual increase in efficiency of a turbine due to superheated steam or to the elimination of moisture in the steam. The authors give a factor of 1.15 as the percentage of decrease in efficiency of each individual stage for 1 per cent of moisture in the steam. It is generally accepted and proved by tests that the decrease in efficiency for 1 per cent of moisture is considerably larger, being somewhere between 1.5 and 2 per cent. It is probable that the factor found by the authors of the paper applies to laboratory tests where the blades of the turbine are smooth and of correct shape. When a turbine is working with saturated steam, the blades are subjected to erosion. After they lose their original correct shape the efficiency loss due to friction becomes greater, and the decrease of efficiency, due to the presence of moisture, is more pronounced. The general figure of from 1.5 to 2 per cent is the average for general conditions of a turbine.

The authors base their calculations for the increase in efficiency on the reheating being carried out to the final temperature of the initial steam in order to obtain the highest gain by reheating. This is thermodynamically correct. From a thermodynamic standpoint, the highest efficiency is to be obtained when the heat is added at

[1] Chief Engr., Industrial Dept., The Superheater Co., New York, N. Y. Mem. A.S.M.E.

the highest pressure. The authors have found that the most efficient point for resuperheating is at the ratio of about 0.35 of the adiabatic drop for a final temperature of 750 deg. fahr. This is theoretically correct. However, it will be difficult in practice to resuperheat the steam to 750 deg. With a pressure of 500 lb. per sq. in. and a final temperature of 750 deg. fahr. the superheat is 283 deg. Considerable space is required to accommodate a

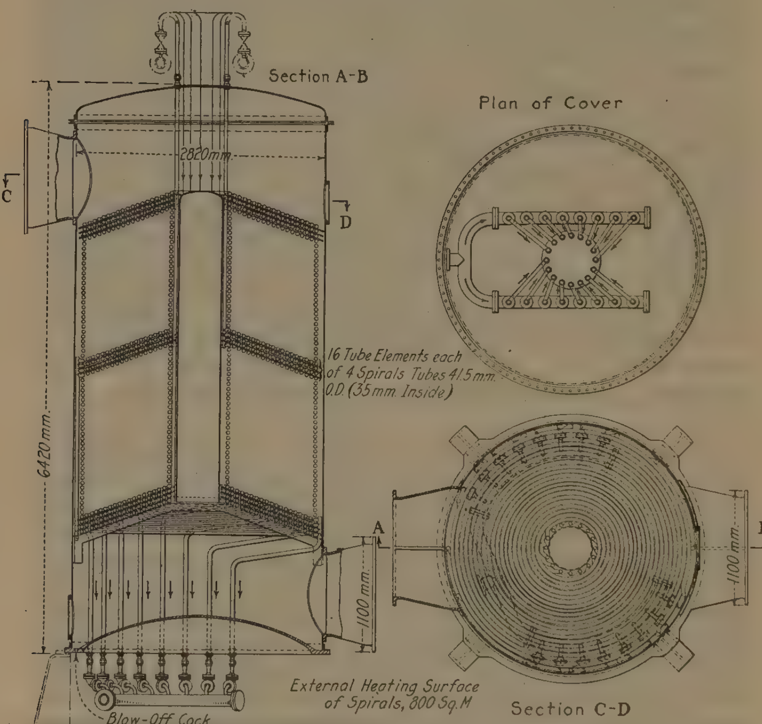

FIG. 19 GERMAN REHEATER FOR 200,000 LB. OF LIVE STEAM PER HOUR BUILT BY SCHMIDT SUPERHEATER COMPANY

superheater of sufficient area to get this temperature, even when it is in the ideal location near the furnace. Reheating the steam to 750 deg. after expanding it down to 70 lb. per sq. in. would result in about 450 deg. superheat. A very large superheater would be required for this and it could hardly be placed in the present standard boilers.

The writer believes that if we should depend for reheating upon the gases of the boiler, or if it should be necessary to conduct the steam from the turbine back to the boiler and then again to the

turbine, the gain to be obtained by resuperheating would hardly justify the complications. A much simpler method is to resuperheat the steam by live steam. While the final temperature of

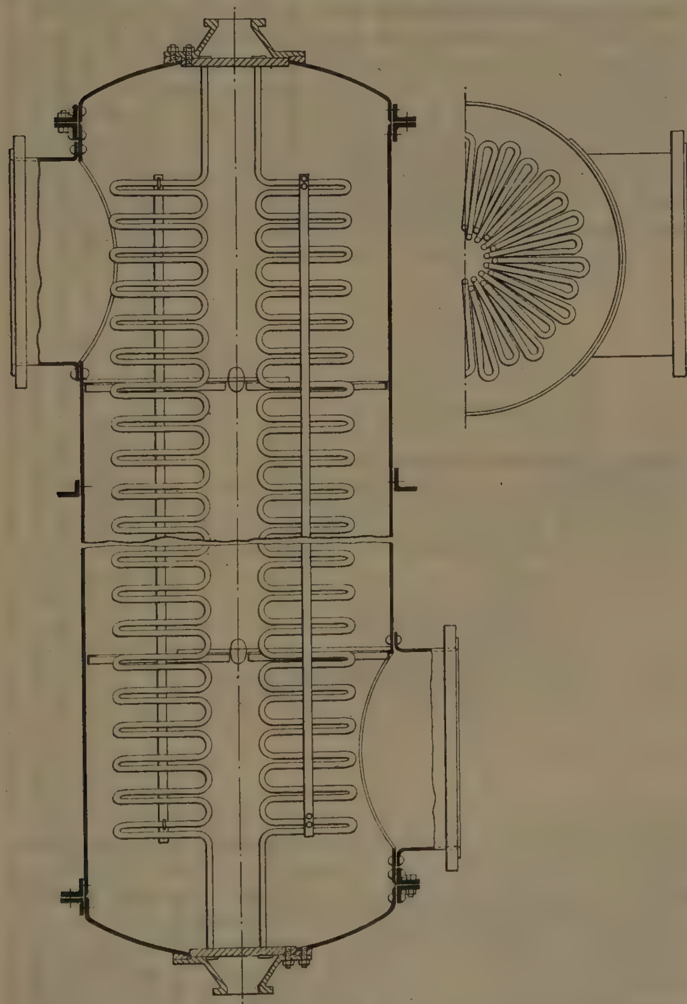

FIG. 20 REHEATER FOR 50,000 LB. OF LIVE STEAM PER HOUR DESIGNED BY THE SUPERHEATER COMPANY, NEW YORK, N. Y.

the resuperheated steam will be much lower than that of the superheated live steam, the arrangement is so simple that from a practical standpoint it is probably the most desirable. With 500 lb. per sq. in. pressure and 750 deg. fahr. final temperature, the

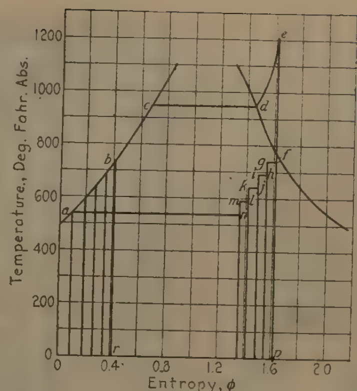

Fig. 21 Temperature-Entropy Diagram for Regenerative Cycle with Four-Stage Heating

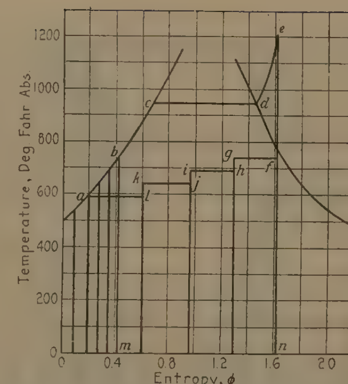

Fig. 22 Temperature-Entropy Diagram for House Turbine

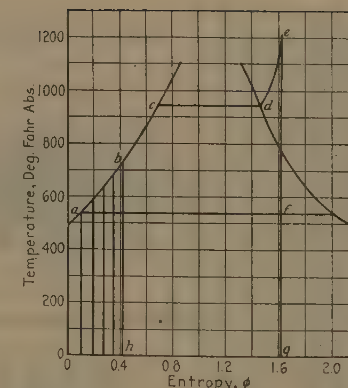

Fig. 23 Temperature-Entropy Diagram for Main Turbine

steam will expand down to 70 lb. per sq. in. before it loses its superheat. It can be reheated by superheated live steam to a temperature of about 550 deg. fahr. The reheater can be located near the turbine, so that no long piping is required. The regulation of temperature is ideal, as all that is necessary is to adjust a valve admitting live steam to the reheater. Fig. 19 shows a reheater for 200,000 lb. of steam per hour, designed abroad, where much consideration is being given to reheating by live steam.

Fig. 20 shows a reheater designed in this country for 50,000 lb. of steam at 5 lb. per sq. in. pressure to be superheated to 500 deg. fahr. It will be noted that on neither of these heaters are large flat surfaces exposed to high pressure. The live steam flows through the coils inside of the shells, the units of which are crowded together

into comparatively small flanges. The low-pressure steam is inside the shell around the tubes. The space occupied by such a reheater is small compared with the size of the turbine, condenser, etc.

It is true that both reheating and regenerative cycles can be applied at the same time on the same turbine and that the gains obtained from both are cumulative. There are, however, some objections to applying the regenerative cycle to a large turbine. The amount of water fed into the boiler is usually varying, which involves a varying amount of steam to be bled at the different stages. Further, it disturbs to a certain extent the relation between the stages, and this is necessarily accompanied by a decrease of turbine efficiency. A method has been suggested recently which overcomes this objection and also makes it possible to utilize the

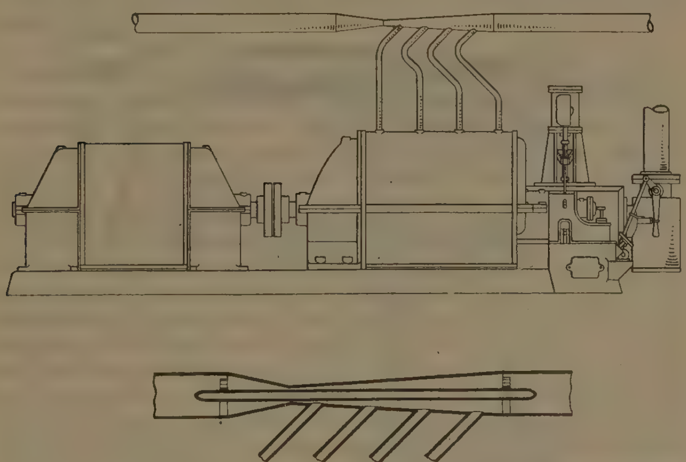

Fig. 24 New Method of Accomplishing the Regenerative Cycle

regenerative as well as the reheating cycle to the full advantage. Most large power plants are provided with a so-called house turbine to furnish power for all auxiliaries. If the steam for heating the feedwater is bled from a number of stages of the house turbine, while the reheating cycle is applied to the main turbine, the two above-mentioned cycles will be fully utilized without interfering with each other and without bleeding any steam from the main turbine.

Fig. 21 shows the temperature-entropy diagram for a regenerative cycle, with four-stage heating, applied to a turbine where enough steam is bled from it to heat the feedwater for the boiler supplying steam for this turbine. Figs. 22 and 23 are two diagrams showing the regenerative cycle applied as mentioned above. Fig. 22 is the diagram for the house turbine from which steam is bled to heat the feedwater for itself and also for the main turbine.

Fig. 23 shows the diagram for the main turbine alone, no steam being bled from it, but the feedwater being heated.

To accomplish the regenerative cycle, either a number of closed heaters or a number of pumps are required, together with the necessary piping, which is quite complicated. A new arrangement has been suggested recently which seems to have overcome the difficulties. With this arrangement, which is illustrated in Figs. 24 and 25, neither multiple pumps nor multiple heaters are required. The water to be heated is raised to a pressure enough higher than the boiler pressure to take care of friction losses through the device. This may be done by a standard feedwater pump. As shown, the feed line is gradually reduced in cross-section to a small

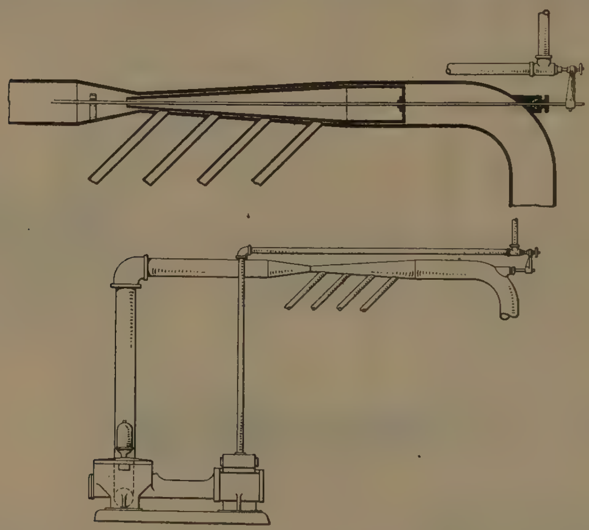

FIG. 25 DETAILS OF METHOD OF ACCOMPLISHING REGENERATIVE CYCLE

size and then gradually increased to the original size of the pipe, similar to the reduction and increase of area in a venturi meter. As in the venturi meter, the operation of the device depends upon the law of hydraulics that the sum of static head, pressure head, and velocity head at any point in a pipe line is equal to the sum at any other point, disregarding losses.

The section must be sufficiently reduced so that the pressure is less than the steam pressure of the lowest stage of the turbine from which steam is to be bled. The steam from this stage is introduced at this point and heats the feedwater to the temperature corresponding to the pressure. At a point further along the gradually increasing section, where the pressure, although higher than at the first point, is lower than the steam pressure of the next higher stage of the turbine from which steam is to be bled, steam is intro-

duced from this stage, raising the temperature of the water to the temperature corresponding to the steam pressure. Steam from as many stages of the turbine as desired is thus introduced at points along the increase in area. In each case the water pressure is less than the steam pressure, so that the steam from the turbine can enter and mix with the water. After the last point of introduction of steam, the area of the cross-section is still further increased so that velocity head is transformed back to pressure head sufficient for the water to enter the boiler. With this arrangement it is possible to heat the water by bleeding from many stages without undue complication. All that is necessary for each stage is a pipe connection provided with a check value between the heater and turbine, so that all advantages in connection with multiple-stage heating can be obtained in a comparatively simple manner.

The Authors. Reliable data on superheating curves at pressures higher than 200 lb. per sq. in. are not available so far as we know. These data are difficult to obtain in a regular power station. Our data were obtained in Schenectady by using a special battery of electric superheaters in order to vary the superheat before the steam passed to the throttle of the turbine. We have obtained a few isolated points on commercial machines in power stations and these data seem to substantiate those obtained in Schenectady and to justify the figure of 1.5 that we used in all calculations. Recent tests made in Schenectady on single wheel stages in which the steam conditions were varied from 300 deg. fahr. initial superheat to 15 per cent initial moisture content substantiate the assumptions upon which the calculations were based: viz., that the efficiency is approximately constant when the wheel is entirely in the superheat region and falls off approximately 1.15 per cent for each 1 per cent increase in the average moisture content.

Regarding the point raised by Mr. Broido, that the gain is apt to be 1.5 per cent for each 1 per cent reduction in moisture content when the blades are corroded, we have made a number of tests on machines at different superheats and with different conditions of blades, and in general the slopes of superheat efficiency curves were the same in all cases. Further investigations indicate that the loss in the turbine due to moisture is in the nature of internal friction in the steam itself rather than friction along the sides, and that this internal friction is independent of the other losses in the stage and only there by virtue of the properties of the fluid itself. We are inclined to believe that these curves will hold regardless of the condition of the turbine or the efficiency.

The superheat-efficiency curves referred to in Pars. 3 to 5, inclusive, in the paper appeared in the first draft and then were removed, as we believed that if they were included, the necessary

explanation would tend to destroy the unity of the paper rather than add to its value.

Regarding the cost of a resuperheating installation, a general statement giving the percentage excess cost of a resuperheating turbine over a unit of the same rating to operate between the same initial and exhaust conditions but without the resuperheating feature, cannot be made, since too many factors enter into the question. We have not made any high-pressure turbines without the resuperheating feature, nor have we built any low-pressure turbines with the resuperheating feature. In general, the resuperheating type of unit is inherently more expensive for several reasons. In order to utilize the additional energy properly, the equivalent of a larger number of stages is necessary, and the introduction of hot steam into a turbine at points widely separated makes necessary the use of steel to a greater extent than in the ordinary type of unit. In case a resuperheating unit should trip out for some reason, even though the main throttle valve might close, calculations show that there might be enough energy available in the steam in the turbine, in the resuperheater, and in the connected piping to accelerate the unit to a dangerous speed. For this reason, an intercepting valve should be furnished just ahead of the point where the resuperheated steam enters the turbine. This valve and the gear which operates it are of course not required on the non-resuperheating type of unit.

In regard to the difficulty of resuperheating steam to 750 deg. fahr. at low pressure, we have already supplied several turbines to use this resuperheating feature up to 725 deg. fahr. Apparently those who have designed the stations for these turbines have not considered this problem insurmountable.

We did not include the effect of throttling losses in the resuperheater and connecting pipes, because this is a factor beyond our control as turbine manufacturers or designers. This is a factor that enters into the design of the power station, the boiler, and the resuperheater. Furthermore, the determination of the correction to be applied to the curves in this paper in order to allow for this drop is comparatively simple. This correction makes the greatest difference in the case of multiple resuperheating, and in this case an assumed drop in pressure has been allowed for in Fig. 14.

Mr. Helander and Professor Ellenwood are undoubtedly right in pointing out that if the pressure drop in the resuperheater and connected piping together with the investment are taken into account, it would be more economical to resuperheat at a somewhat higher pressure than the peak of the curves given in the paper. The authors are inclined to believe that the variation in this pressure drop and in the cost of resuperheating equipment as a function of the pressure at which resuperheating is done, is a more important factor in determining the point at which it is most economical to resuperheat than the amount of the drop or

cost in itself. Not having these data available, it was impossible to take these factors into account properly.

In connection with the combined resuperheating and regenerative cycles, there are always two questions which apply when a regenerative cycle is to be used. First, to what temperature will the feedwater be heated? Secondly, how many heaters or stages will be employed to heat the water to the given temperature? The first depends primarily upon whether economizers or air preheaters or neither are used in the boiler installation, and also upon the method used for auxiliary drive. The second is primarily an economic question, since the gain due to adding an additional stage of heating when the final temperature is fixed is considerably smaller than the gain resulting from adding the preceding heater, and a point is soon reached at which the cost of additional stages of feed heating with any given final feedwater temperature does not justify the additional cost and complication involved. This situation applies in either a resuperheated or non-resuperheated installation with only slight modifications to meet the individual conditions.

Professor Ellenwood raised a question as to the correctness of the statement regarding the increase in the entropy of the steam resulting from the increased energy made available in the succeeding stages by the reheating. While the statement is correct as it stands, it would be perhaps somewhat clearer if changed to read, "since the reheating increases both the entropy and the total heat in the stage." The text has been so changed.

[illegible]

those who have designed the condensers [illegible] considered the problem [illegible].

We did not [illegible] the [illegible] in the recuperative boiler [illegible] is a [illegible] beyond our control as [illegible]. This is a factor that enters into the [illegible] the boiler, and the [illegible]. Furthermore, the [illegible] correction [illegible] to the [illegible] in order [illegible] for this [illegible] correction makes the greater difference in the [illegible], and in this case [illegible] assumed drop in [illegible] has been allowed for in [illegible] 14.

Mr. Holander and [illegible] are undoubtedly right in pointing out that if the [illegible] in the [illegible] together with the [illegible] it would be more [illegible] to [illegible] pressure [illegible] of the [illegible] are inclined to believe that [illegible] pressure drop and [illegible] of the [illegible] more important factor [illegible] the point most economical to [illegible] is [illegible]

No. 1930

A REVIEW OF RECENT APPLICATIONS OF POWDERED COAL TO STEAM BOILERS

By Henry Kreisinger,[1] New York, N. Y.
Member of the Society

This paper gives a brief statement of the trend of the development for the past two years of the application of powdered coal as a fuel for making steam. It includes treatment of developments in furnaces, driers, and mills. Test results are given from boilers and mills in six central stations using pulverized coal as fuel. The author also discusses mill capacities for various grades of coal.

THE last three years have shown a marked increase in the application of powdered coal to steam boilers. At present no important power plant decides on the coal-burning equipment without making a thorough investigation of the possibilities of powdered coal. For large stations and for large steam-generating units the indirect system seems to possess an advantage over the direct-firing system and is generally favored for this kind of service. On the other hand, direct firing seems to have some advantages in small industrial plants and small boiler units. This paper deals largely with the indirect system of firing.

2 The trend in the development in the various parts of the powdered coal equipment is outlined in the following paragraphs.

Furnace

3 Powdered coal gives high efficiency because the coal can be burned almost completely with very low excess air. However, low excess air causes high furnace temperature, which in turn causes fusion of ash and erosion of furnace lining. Many of the first attempts to burn powdered coal failed because of the excessive erosion of the furnace lining. Another cause of early failures was the difficulty of removing fused ash from the furnace. A large part

[1] Research Engr., Combustion Engineering Corporation.

Presented at the Annual Meeting, New York, December 1 to 4, 1924, of the American Society of Mechanical Engineers.

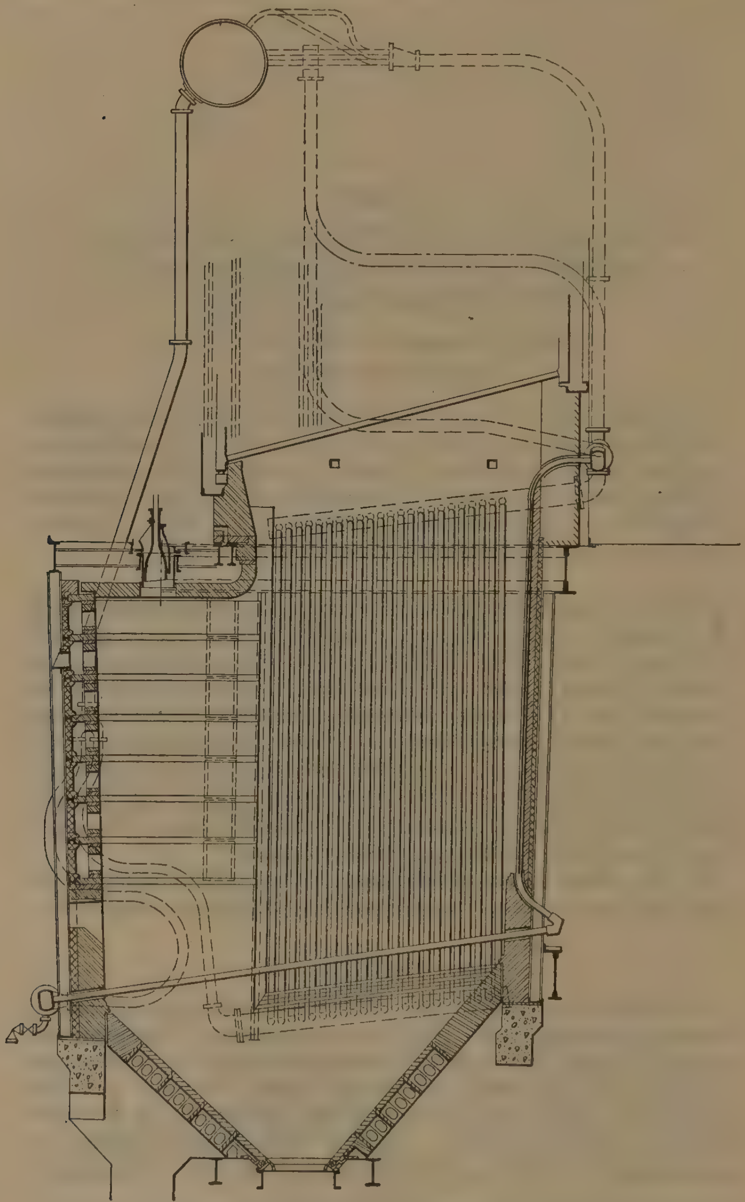

FIG. 1 VERTICAL SECTION THROUGH POWDERED-COAL FURNACE EQUIPPED WITH WATER SCREEN OVER THE BOTTOM AND REAR WALL, AND ALSO WITH FIN-TUBE SIDE WALLS.

(Babcock & Wilcox cross-drum boiler of 18,010 sq. ft. of heating surface, at Cahokia Station, Union Electric Light and Power Company, St. Louis, Mo. This is the latest design.)

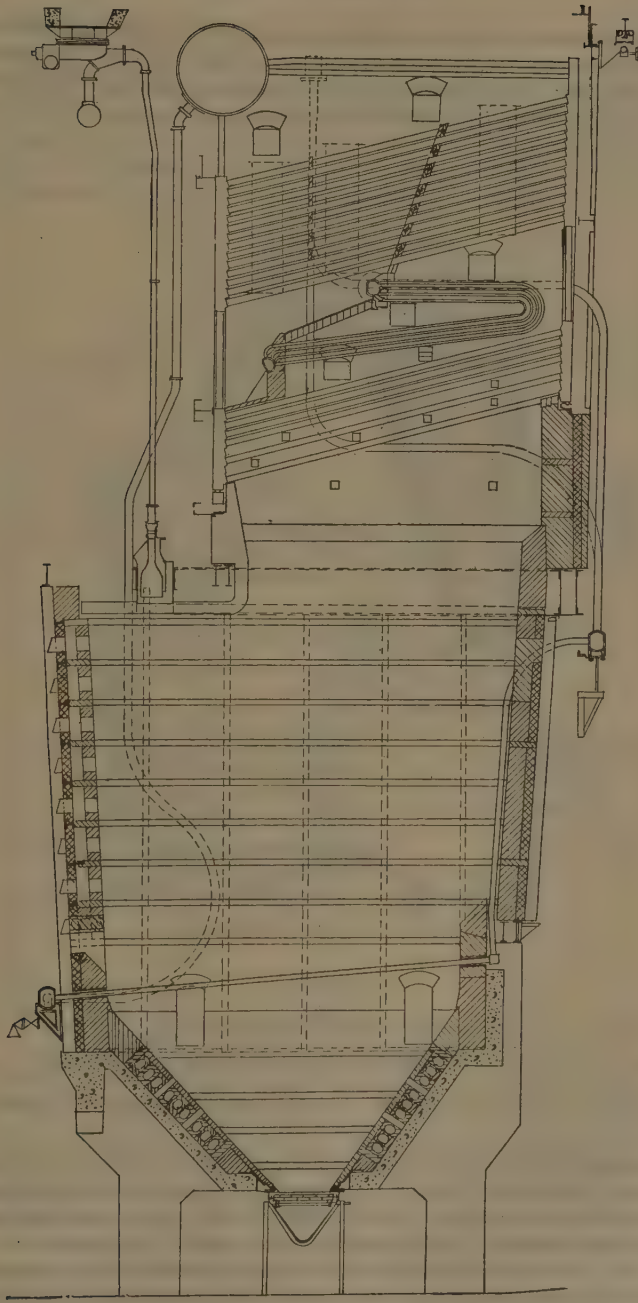

FIG. 2 VERTICAL SECTION THROUGH POWDERED-COAL FURNACE EQUIPPED WITH WATER SCREEN OVER BOTTOM AND REAR WALL OF FURNACE

(Cahokia Station. Tests presented in this paper were made with this type of furnace.)

of the ash was sprayed in a molten state on the walls and bottom of the furnace. The molten ash sprayed over the walls ran down, washing the brick along with it, and accumulated in a puddle of molten slag at the bottom. This slag could not be removed without

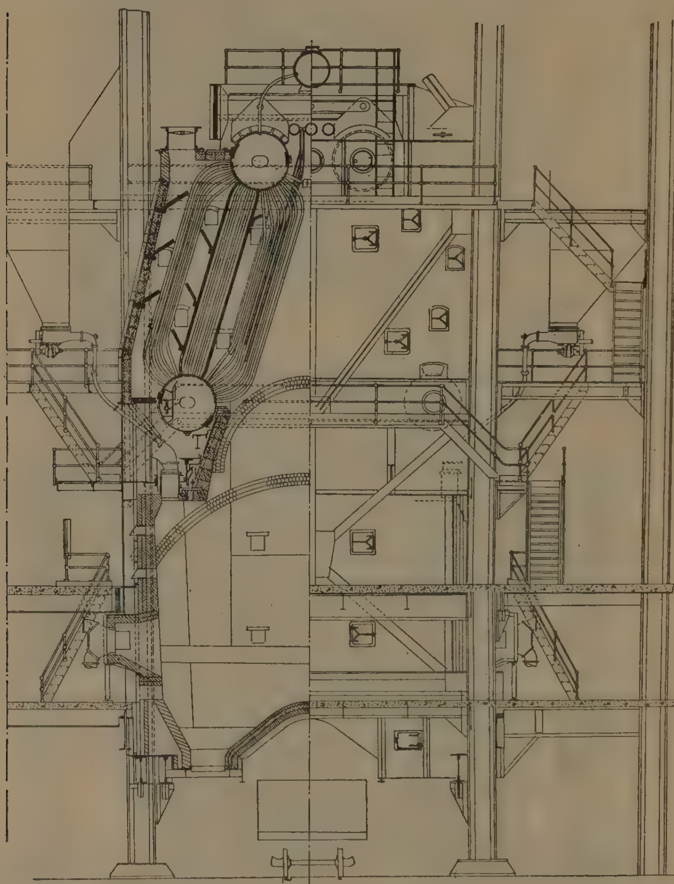

Fig. 3 Ladd Boiler. Heating Surface, 26,470 sq. ft. River Rouge Plant, Ford Motor Company. Twelve Lopulco Burners, Six on Each Side of Boiler. Solid Furnace Walls, no Water Screen

cooling the furnace and mining the slag out with picks. In designing a furnace for burning powdered coal there are two problems: (*a*) The prevention of the erosion of the walls; and (*b*) easy removal of the ash deposited at the bottom of the furnace.

4 At present, the trend of the development of the furnace is toward nearly complete water cooling. This apparently is a positive

solution for the above two problems. About four years ago a water screen was applied over the bottom of the furnace with the object of preventing fusion of the ash deposited on the bottom, and of making its removal easy. This water screen met with such success that it was soon applied to the back wall of the furnace, where the abrasion was great due to the turning of the flame. Water-cooled side walls are now used in addition to the screens

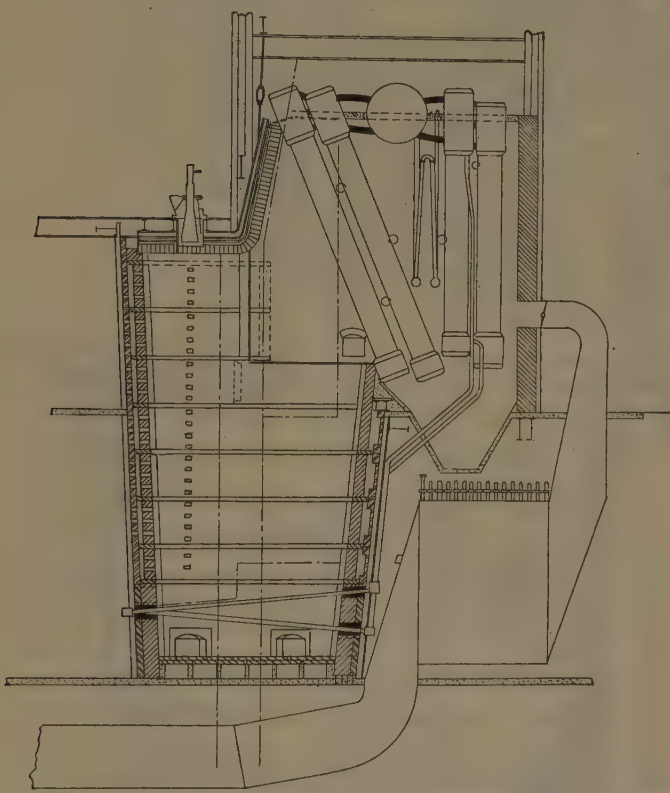

Fig. 4 Bigelow-Hornsby Boiler, 8750 sq. ft. of Heating Surface. Rochester Gas and Electric Company, Plant No. 3

over the bottom and back wall of the furnace. Fig. 1 shows a section through one of the last four furnaces now being installed at the Cahokia Power Station. This furnace has a water screen over the bottom and the rear wall, and fin-tube side walls. All these water-cooled tubes are connected into the circulation of the boiler, so that they really form a part of the boiler. This is a natural development in the design of furnaces. Powdered coal can never be a complete success until the boiler is built around the furnace.

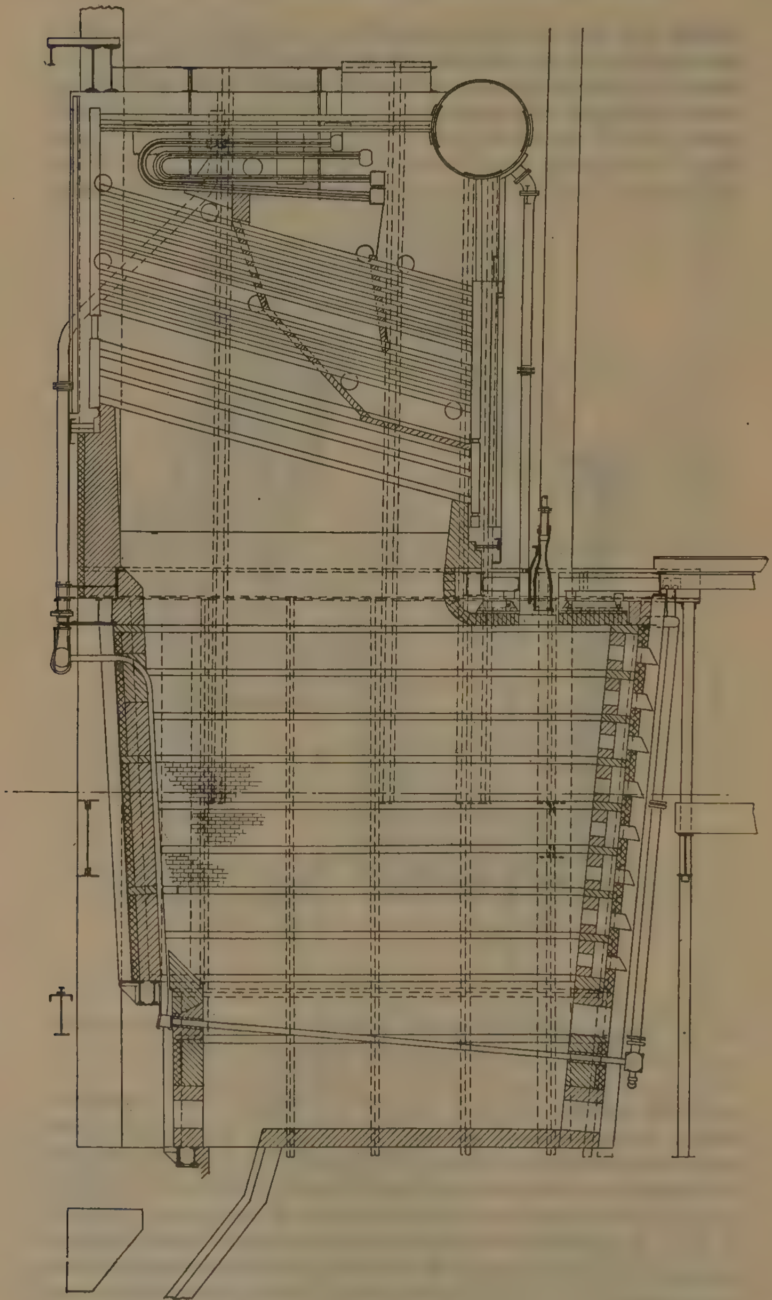

Fig. 5 Babcock & Wilcox Cross-Drum Boiler, 15,326 sq. ft. of Heating Surface. Springdale Plant, West Penn Power Company

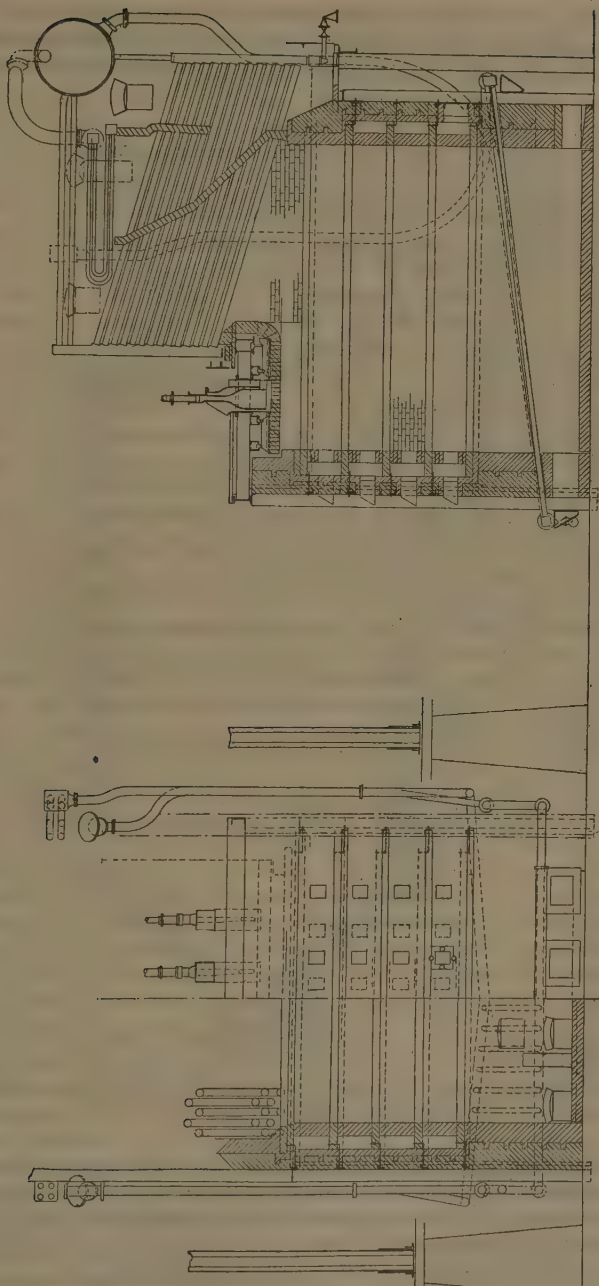

FIG. 6 BABCOCK & WILCOX CROSS-DRUM BOILER, 8220 SQ. FT. OF HEATING SURFACE. PENN SALT COMPANY, WYANDOTTE, MICH.

5 For greatly reducing the abrasion on the furnace walls, the hollow-wall construction has met with considerable success, especially with coal whose ash melts at a comparatively high tempera-

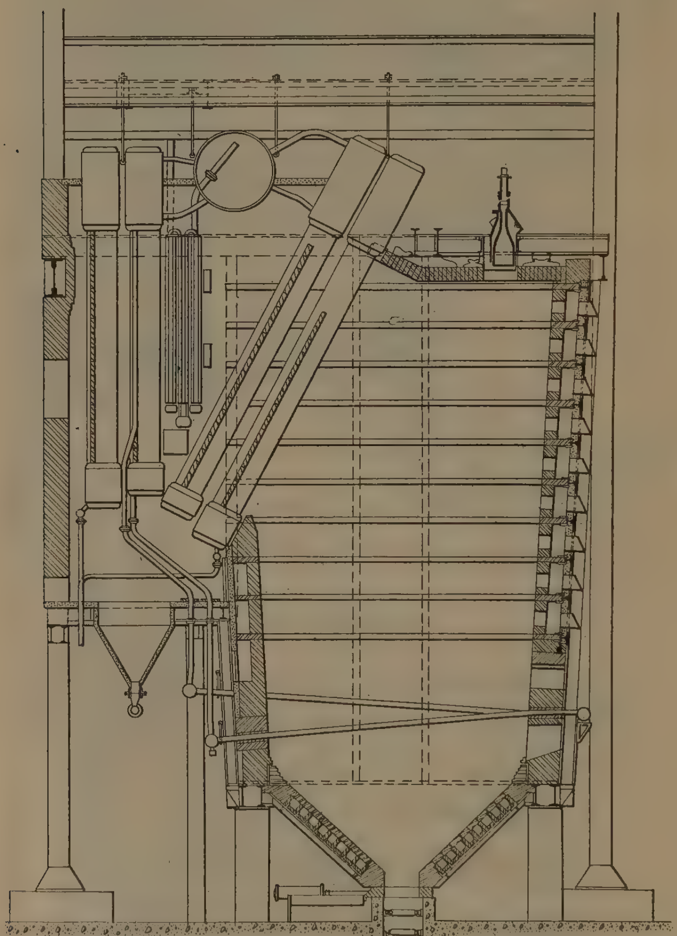

Fig. 7 Bigelow-Hornsby Boiler, 12,660 sq. ft. of Heating Surface. United Railways of Providence, Providence, R. I.

ture. In this hollow-wall construction the walls are built with channels between the furnace lining and the outer wall, and through these channels 60 to 80 per cent of the air needed for combustion is passed before it enters the furnace. The air passing through the hollow walls cools the furnace lining and greatly reduces its erosion

by the molten ash. Many furnaces of this design are in use and are meeting with considerable success. However, it appears that the water-cooled furnace such as shown in Fig. 1 is to be preferred where high ratings are desired and the coal has very fusible ash. The hollow-wall construction in its various forms is shown in Figs. 2, 3, 4, 6 and 7.

6 The hollow-wall construction has one commendable feature. No air inlets lead directly from outside into the furnace, through which a flame might puff back into the boiler room and ignite an accidentally caused dust cloud. In the hollow-wall construction the air ports supplying air for combustion open into horizontal air channels. These usually pass half-way around the furnace, so that the flame puffing out of the furnace would have to travel 30 to 40 ft. before reaching the outside of the setting. This feature makes powdered-coal furnaces safer, and hollow-wall construction also could be applied at least to some extent to water-cooled furnaces in order to make the operation of these furnaces safer.

7 It is frequently pointed out that pulverized coal requires large furnaces. This is undoubtedly true if the powdered-coal furnace is compared to the old-type stoker installations which usually have a furnace with a very small combustion space. However, when the comparison is made with the modern stoker furnace, the difference in size is not so great. In fact, in the past stoker furnaces were built much too small. In recent stoker installations attention is given to proper design of the furnace and we find stoker installations with 20 to 22 ft. between the stoker and the boiler tubes.

8 Another reason why the powdered-coal furnace seems large is that most of the powdered-coal furnaces have been installed under very large boilers which are to be operated at high ratings, and consequently the furnaces are made large. These large furnaces are apt to be compared with stoker or hand-fired furnaces of 10 years ago, when a boiler of 5000 sq. ft. of heating surface was considered large. In present-day power-plant practice steam-generating units of 30,000 sq. ft. of heating surface are quite common. It seems that for these large steam-generating units powdered coal is especially advantageous, because with powdered coal uniformly good combustion can be obtained throughout a large furnace and over a wide range of rate of driving.

9 Powdered-coal furnaces are made large for two reasons: First, to obtain complete combustion; second, to avoid impingement of the flame against the furnace walls, especially with the refractory-lined furnace.

10 Powdered coal is burned while in suspension in the air. The particles of powdered coal require from one to two seconds to burn almost completely. A large furnace must therefore be provided to permit these particles of coal to stay from one to two seconds in the combustion space. References are sometimes made to

locomotive furnaces burning pulverized coal successfully, and to the fact that locomotive furnaces are much smaller than furnaces used in the stationary boilers. However, it must be remembered that in the locomotive, powdered coal usually competes with hand firing, which is comparatively inefficient. In locomotives as much as 20 per cent of the coal fired on the grate may leave the stack in the form of sparks. Powdered coal has therefore a much larger margin in efficiency over the hand-fired locomotive. The locomotive furnace is entirely water cooled and contains little or no refractory to be destroyed by flame impingement. In the central-station practice, powdered coal has a smaller margin in efficiency over the well-operated stoker furnaces and therefore powdered-coal furnaces must be designed to get a better efficiency than the stoker to justify its use.

11 The second reason for larger furnaces is to reduce the impingement of flame against the refractories. More elbow room must be made for the flame so that it will not gouge into the walls and destroy them in a short time. It is possible that with water-cooled furnaces we may be able to reduce the combustion space to some extent.

Drier

12 Some coals require drying in order to make them pulverize easily, to facilitate conveying, and to make feeding of pulverized coal into the furnace more uniform. Other coals may be pulverized and fed to the furnace without drying. In general, coals from the Appalachian and Eastern coal fields may be pulverized and burned without drying. Coals from the Illinois and the Western coal fields must be partly dried before pulverization. The drier is always a nuisance, although sometimes a necessary one. In other cases it is not easy to decide between the difficulties of pulverizing and feeding undried coal, and the expense and trouble of drying it. In some other cases it is comparatively easy to decide to omit driers. There are a number of plants operating with powdered coal which never have had driers installed.

13 The trend in the development of coal driers is toward a small drier that will dry coal to a sufficient extent as the coal moves toward the mill. Such driers are built in the form of an enlarged coal chute, and the coal is dried as it passes through the drier either by waste gases from the boilers, or by exhaust steam. Such driers extract from two to four per cent moisture. Beside abstracting moisture, they preheat the coal, so that when it gets into the mill and is pulverized, it loses moisture readily, and part of the moisture is discharged from the mill system through the mill vent.

14 The Cahokia station is equipped with driers of this type, using flue gases from the boiler for drying. The gases enter the

driers at from 300 to 400 deg. fahr. and leave them at a temperature of 150 to 200 deg. The coal is heated to about 175 deg. fahr. and passes directly into the pulverizers which are located under the driers. Under these conditions of temperature, the driers may abstract 2 per cent moisture, while the mills abstract 6 per cent moisture from the coal. The coal entering the driers may contain 12 per cent moisture, while the coal leaving the mills may contain only 4 per cent moisture. As far as operating results are concerned,

TABLE 1 RESULTS OF BOILER TESTS AT CAHOKIA POWER PLANT, UNION ELECTRIC LIGHT AND POWER COMPANY APRIL AND MAY, 1924

(Babcock & Wilcox cross-drum boiler, 18,010 sq. ft. of heating surface)

Test No.	2	3	4	5	6	7	8
Duration, hr.	25.35	21.88	25.48	20.20	23.05	21.18	10.80
Rate of driving, per cent	149.2	208.4	130.4	261	180.1	214.9	271.1
Coal as fired							
Moisture, per cent	7.08	6.39	7.08	5.89	5.63	6.82	6.77
Ash, per cent	11.47	11.82	11.40	11.04	11.38	11.06	11.38
B.t.u. per lb.	11673	11553	11599	11802	11790	11713	11672
Fired per hour, lb.	8991	13267	8105	16187	11256	13161	17525
Per cu. ft. of combustion space per hr., lb.	0.77	1.13	0.69	1.38	0.96	1.12	1.49
Ash							
Combustible in furnace ash	0.2	......	......	......	......	......	......
Combustible in flue dust	4.8	3.4	4.0	5.1	1.8	2.8	3.0
Water, lb.							
Evaporated per hour	75948	104458	66286	129779	90454	107114	133763
Evaporated per lb. of coal	8.45	7.98	8.18	7.99	8.04	8.18	7.63
B.t u. per lb. of steam	1185.4	1203.8	1187.7	1213.3	1200.3	1210.6	1222.9
Temperatures, deg. fahr.							
Air to furnace	95	91	87	81	84	80	82
Air to feeders							
Gases leaving boiler	483	520	474	564	502	527	589
Feedwater	197	199	194	200	197	198	203
Superheated steam	670	707	668	727	697	718	750
Pressures, lb. per sq. in., abs.							
Superheater	330	332	328	334	330	331	332
Drafts, in. of water							
Furnace	0.13	0.15	0.08	0.34	0.14	0.18	0.22
Uptake	0.37	0.70	0.20	1.47	0.56	0.80	1.60
Pressure feeder air	9.2	12.2	11.3	17.8	13.4	13.5	19.8
Analysis of gases, uptake							
CO_2	14.7	14.7	14.4	14.1	14.5	14.7	14.0
O_2	4.1	3.9	4.4	4.7	4.3	4.3	5.0
CO	0	0	0	0	0	0	0
Heat account in per cent							
Heat absorbed by boiler and superheater	85.9	83.1	83.8	82.1	81.8	84.5	79.9
Loss in dry gases	9.1	10.2	9.2	11.7	10.0	10.5	12.5
Loss in water vapor	5.1	5.4	4.9	5.0	4.8	5.0	5.0
Loss in incomplete combustion	0.6	0.5	0.5	0.6	0.2	0.3	0.4
Radiation / Errors and unaccounted for	—0.7	+0.8	+1.6	+0.6	+3.2	—0.3	+2.2
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0

it really makes little difference where the moisture is abstracted, as long as the coal leaves the mills sufficiently dry to be handled by the air transport and fed uniformly by the feeders.

15 Driers using flue gases from the boilers for drying are adaptable to plants where the coal preparation room is so close to the boiler plant that the gas ducts are not long. The amount of air used in this type of drier is about 3 lb. of gas per pound of coal dried. In other words, about one-quarter of the boiler flue gases are used for drying purposes. These driers are also adaptable only

for plants where the flue-gas temperature is not lower than 300 deg. fahr. With plants having economizers which reduce the temperature of the gases below 300 deg. fahr., driers of this kind cannot be used.

16 In installations having the preparation room distant from the boiler plant, and in plants where economizers are used, a steam drier possesses an advantage over the flue-gas drier. This drier is of the same general design as the flue-gas drier, but the coal passages are made of steam-heated grids. As the coal passes through the drier it is heated, and the moisture is partly evaporated. A small amount of air is drawn through the coal, and this air carries the water vapor away. The amount of air used for this purpose is about one pound of air per pound of coal. This quantity of air is small, and only a very small pressure drop through the drier is necessary. The velocities are so low that no noticeable quantity of dust is drawn out with the air. In both types of driers more

TABLE 2 RESULTS OF BOILER TESTS AT RIVER ROUGE PLANT OF FORD MOTOR COMPANY, NOVEMBER AND DECEMBER, 1923

(Ladd boiler, 26,470 sq. ft. of heating surface)

Test No.	1	2	3	4	5
Duration, hours	30.58	31.00	32.07	23.82	24.03
Rate of driving, per cent	150.4	211.3	260.0	285.9	222.9
Coal as fired					
Through 60 mesh, per cent	97.8	96.4	96.4	97.2	98.0
Through 100 mesh, per cent	90.2	88.0	89.6	90.5	91.0
Through 200 mesh, per cent	76.2	72.8	73.3	76.0	77.0
Moisture, per cent	2.0	3.2	3.2	3.4	3.2
Ash, per cent	9.94	7.21	9.30	5.57	6.89
B.t.u. per lb.	13237	13299	13101	13467	13299
Fired per hour, lb.	12912	17723	21750	22900	18270
Per cu. ft. of combustion space per hr., lb.	0.81	1.11	1.36	1.44	1.15
Ash					
Combustible in furnace refuse	0	0	0	0	0
Combustible in flue dust	......	4.0	12.0	34.8	12.0
Water, lb.					
Evaporated per hour	106419	148406	183065	200176	156567
Evaporated per lb. of coal	8.22	8.38	8.42	8.74	8.57
B.t.u. per lb. of steam	1252	1263	1260	1267	1263
Temperatures, deg. fahr.					
Air to furnace	90.3	77.8	77.3	83.8	84.1
Air to feeders	90.3	77.8	77.3	83.8	84.1
Gases leaving boiler	558	611	635	654	615
Feedwater	121.4	116.6	119.6	105.3	111.0
Superheated steam	642	653	654	640	643
Pressures, lb. per sq. in., abs.					
Superheater	242.5	244.2	239.5	237.8	246.4
Drafts, in. of water					
Furnace	0.25	0.18	0.31	0.47	0.35
Uptake	1.28	1.75	1.89	2.15	1.63
Pressure feeder air	4.5	7.5	7.5	7.6	9.0
Analysis of gases, uptake					
CO_2	11.6	12.9	15.3	14.6	13.5
O_2	6.5	5.3	3.1	3.9	5.5
CO	0	0	0	0	0
Heat account in per cent					
Heat absorbed by boiler and superh'ter	77.8	79.6	80.9	82.2	82.0
Loss in dry gases	13.4	13.6	12.2	12.8	13.1
Loss in water vapor	4.0	4.4	4.3	4.3	4.2
Loss in incomplete combustion	0.5	0.5	0.9	1.6	0.6
Radiation / Errors and unaccounted for	+4.3	+1.9	+1.7	—0.9	+0.1
Total	100.0	100.0	100.0	100.0	100.0

moisture is driven from the coal if the coal is of a smaller size; that is, coal passing through $\frac{1}{4}$-in. screen will dry better than coal passing through a $1\frac{1}{2}$-in. and over a $\frac{1}{2}$-in. screen.

Mills

17 In the development of mills for large central stations the trend is decidedly toward higher capacity. Most of the mills at present used in central stations have a capacity of from 5 to 9 tons per hour. Some mills are now being installed which have capacity of 15 to 18 tons per hour. The demand seems to be for a mill of about 50 tons capacity. The mills used at present are of the rolling type; that is, the coal is pulverized by rolling a metal roller or ball over the coal and in that way crushing it. A real impact

TABLE 3 RESULTS OF BOILER TESTS OF BOILER NO. 18, PLANT NO. 3, ROCHESTER GAS AND ELECTRIC COMPANY, JULY AND AUGUST, 1924

(Bigelow-Hornsby boiler, 8750 sq. ft. of heating surface)

Test No.	5	[illegible]	7	8	9	10	11	12	13
Duration, hours	23.15	22.93	19.25	21.73	21.95	13.98	14.93	14.92	15.05
Rate of driving, per cent	157	170	199	178	180	163	173	177	166
Coal as fired									
Through 60 mesh, per cent	.....	.....	94.0	93.6	94.9	94.5	97.0	96.3	94.6
Through 100 mesh, per cent	83.0	86.8	88.1	87.6	88.2	89.0	89.3	89.5	87.7
Through 200 mesh, per cent	59.1	61.4	59.1	58.4	60.2	60.2	62.6	66.4	61.8
Moisture, per cent	2.6	2.9	3.2	3.0	3.1	3.6	2.9	3.4	4.6
Ash, per cent	8.5	9.8	9.2	8.4	10.3	10.6	11.0	10.3	9.7
B.t.u. per lb.	13535	13170	13240	13485	13120	13120	13110	13135	13235
Fired per hour, lb.	4212	4808	5588	4797	5020	4775	4912	4794	4536
Per cu. ft. of combustion space per hr. lb.	1.00	1.15	1.33	1.14	1.20	1.14	1.17	1.14	1.08
Ash									
Combustible in furnace refuse	0.3	4.5	.....	1.3	1.9	1.4	1.4	1.4	1.4
Combustible in flue dust	13.8	6.2	21.0	21.5	18.2	18.0	14.9	13.1	13.6
Water, lb.									
Evaporated per hr.	39969	43315	50747	45182	45856	41844	44379	45566	42670
Per lb. of coal	9.44	9.01	9.10	9.42	9.13	8.76	9.03	9.50	9.41
B.t.u. per lb. of steam	1210	1206	1204	1204	1206	1198	1194	1191	1198
Temperatures, deg. fahr.									
Air to furnace	86	88	77	76	71	86	87	84	73
Air to feeders	142	156	143	151	142	133	143	150	143
Gases leaving boiler	540	557	577	553	562	550	552	558	574
Gases leaving economizer	342	366	370	347	363	360	363	361	377
Water to economizer	87	93	85	85	87	95	94	95	94
Water to boiler	147	147	138	137	140	151	147	147	150
Superheated steam	496	499	481	481	487	490	481	477	487
Pressures, lb. per sq. in. abs.									
Boiler	220	221	219	217	218	217	217	220	217
Superheater	206	207	207	206	207	208	207	209	209
Drafts, in. of water									
Furnace	0.03	0.05	0.08	0.05	0.04	0.05	0.05	0.04	0.05
Gas leaving boiler	0.09	0.11	0.18	0.12	0.13	0.15	0.15	0.15	0.15
Gas leaving economizer	.....	.....	0.48	.....	0.46	.....	.....	.....	.....
Analysis of flue gases									
CO_2, gas leaving boiler	14.3	14.5	15.6	15.4	14.9	14.0	13.9	14.1	14.3
CO_2, gas leaving economizer	13.0	12.9	13.8	13.5	13.1	12.7	12.8	13.0	12.8
Heat account									
Heat absorbed by boiler and superheater	80.6	78.8	78.9	80.5	80.2	76.3	78.6	82.4	81.2
Heat absorbed by boiler, superheater and economizer	84.8	82.5	82.5	84.1	83.9	80.0	82.2	86.1	85.1
Loss in dry gases	6.4	7.0	6.8	6.5	6.6	6.8	6.9	6.9	7.6
Loss in water vapor	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3
Loss in incomplete combustion	1.3	1.0	3.1	3.1	3.2	1.8	1.8	1.8	1.8
Rad., errors and unaccounted	+3.2	+5.2	+3.3	+2.0	+2.0	+7.1	+4.8	+0.9	+1.2
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

mill is looked up to as having the possibility of being developed into a machine of high capacity, low power consumption, and small wear.

Results of Boiler Tests

18 Tables 1 to 7 give the results of seven series of boiler tests made at six different plants. The plants vary in size from an

TABLE 4 RESULTS OF TESTS AT SPRINGDALE STATION, WEST PENN POWER COMPANY, DECEMBER, 1923

(Babcock & Wilcox cross-drum boiler, 15,326 sq. ft. of heating surface)

Test No.	2	3	4	5	6	7
Duration, hours	24	24.07	23.37	22.58	45.92	23.05
Rate of driving, per cent	213.8	218.4	309.1	383.0	125.2	441.8
Coal as fired						
Through 60 mesh, per cent						
Through 100 mesh, per cent	82.93	92.74		86.72	85.16	87.67
Through 200 mesh, per cent	62.93	77.95		66.58	63.72	64.68
Moisture, per cent	1.37	1.12	1.47	1.84	1.51	1.51
Ash, per cent	11.26	12.23	11.51	11.35	11.85	9.45
B.t.u. per lb.	13246	13114	13356	13260	13244	13469
Fired per hour, lb	10183	10448	15076	18890	5865	22315
Per cu. ft of combustion space per hr., lb.	0.82	0.84	1.216	1.52	0.47	1.80
Water, lb.						
Evaporated per hour	89158	91516	127171	155314	53390	177383
Evaporated per lb. of coal	8.75	8.76	8.44	8.22	9.10	7.95
B.t.u. per lb. of steam	1232.3	1227.2	1248.5	1267.4	1205.5	1280.3
Temperatures, deg. fahr.						
Air to furnace	59	56	61	52	58	43
Air to feeders	141	141	133	127	135	104
Gases leaving boiler	555	556	608	665	494	691
Feedwater	94.	98.	95.	93.	94.	88.
Superheated steam	571	570	602	633	527	647
Pressures, lb. per sq. in. abs.						
Boiler	321	323	327	336	320	344
Superheater	318	320	322	331	319	337
Drafts, in. of water						
Furnace	.05	.05	.15	.24	.04	.30
Uptake	.42	.38	94	1.62	.10	2.22
Pressure feeder air	10.2	8.9	15.0	17.1	8.5	19.2
Analysis of gases, uptake						
CO_2	12.9	13.6	13.5	13.2	12.5	13.1
O_2	6.1	5.6	5.5	5.7	6.7	5.5
CO	.06	.05	.08	.08	.01	.16
Heat account in per cent						
Heat absorbed by boiler and superheater	81.4	81.9	78.9	78.5	82.8	75.5
Loss in dry gases	12.9	12.4	13.5	15.5	11.6	16.6
Loss in water vapor	4.4	4.4	4.5	4.7	4.3	4.7
Loss in incomplete combustion	1.0	1.0	1.5	2.0	1.0	2.5
Radiation	0.7	0.7	0.5	0.4	1.4	0.3
Errors and unaccounted for	—0.4	—0.4	+1.1	—1.1	—1.1	+0.4
Total	100.0	100.0	100.0	100.0	100.0	100.0

industrial plant with a unit of about 8000 sq. ft. of heating surface, to a central station with a unit of 18,000 and 26,000 sq. ft. of heating surface. These tests may therefore be considered as fairly representing the field of powdered-coal application for making steam. The description of the steam-generating units on which the tests were made and the method of making tests are given in Par. 25 et seq. The tests were made by trustworthy engineers and most of the results are accurate within ±2 per cent, and some of them

TABLE 5 RESULTS OF TESTS AT THE PLANT OF PENN SALT CO., WYANDOTTE, MICH., JULY AND AUGUST, 1923

(Babcock & Wilcox cross-drum boiler, 8220 sq. ft. heating surface)

Test No.	1	2	3	4	5	6	7	8	9	10	11	12	13
Duration, hr.	23.57	24.72	22.77	21.73	22.72	24.78	24.22	23.30	23.65	24.92	16.07	24.43	23.83
Rate of driving, per cent	174.2	174.2	125.0	125.0	164.3	164.3	205.8	201.0	201.0	201.0	221.0	189.5	211.6
Coal as fired													
Through 50 mesh, per cent	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.1	99.9
Through 100 mesh, per cent	95.3	94.9	96.1	96.8	97.2	97.3	97.7	97.8	96.7	96.7	95.5	93.7	96.1
Through 200 mesh, per cent	71.1	68.5	70.7	66.6	77.0	76.3	77.8	78.1	76.3	73.8	72.0	73.6	73.1
Moisture, per cent	2.4	2.1	1.8	2.1	1.9	2.4	2.1	1.8	2.0	1.9	1.6	5.6	2.6
Ash, per cent	15.1	13.9	14.2	13.4	12.9	15.5	14.0	13.9	13.6	13.1	13.3	14.3	13.5
Heat value, B.t.u.	12385	12518	12383	12530	12874	12276	12522	12702	12783	12889	12710	12116	12552
Coal per hr. lb.	4683	4560	3285	3122	4316	4182	5375	5585	5360	5390	5980	5270	5820
Coal per hr. per cu. ft. combustion space	1.39	1.36	0.98	0.93	1.28	1.25	1.60	1.66	1.59	1.61	1.78	1.57	1.73
Ash													
Combustible in furnace ash, per cent	5.0	1.4	2.5	2.2	1.2	4.0	3.8	3.7	4.0	4.8	9.15	......	9.4
Flue dust, per cent	10.2	10.2	10.5	8.7	11.8	11.8	......	......	15.8	15.8	15.2	......	16.9
Calculated combustible loss	1.3	1.1	1.1	0.9	1.1	1.5	1.7	1.7	1.7	1.6	1.8	2.0	1.9
Water													
Per hour, lb.	40700	41030	29810	29300	39650	37580	47950	46900	45950	47480	51350	44400	50800
Per lb. coal as fired	8.71	9.00	9.08	9.38	9.18	8.98	8.93	8.41	8.59	8.82	8.59	8.43	8.74
Temperatures, deg. fahr.													
Outside air	80.1	79.8	69.8	69.0	71.0	67.2	72.1	76.6	78.3	75.2	74.6	72.9	72.6
Air to furnace	89.9	90.1	81.5	80.1	83.1	80.2	83.0	89.8	90.0	87.2	86.7	86.6	86.0
Flue gas	459.3	446.3	409.4	414.6	476.9	475.2	524.5	545.2	546.0	555.4	563.6	550.7	504.2
Feedwater	102.0	101.7	101.4	100.0	100.5	99.7	100.2	102.9	103.0	102.2	101.4	103.1	145.0
Superheated steam	458.5	462.4	441.5	440.4	457.2	453.7	472.0	474.4	479.3	480.9	477.0	474.6	473.2
Pressure, lb. per sq. in. absolute													
Steam pressure	217.8	218.2	214.9	214.1	218.8	217.0	220.5	219.6	220.5	221.7	215.7	220.9	219.0
Draft and air pressures, in. of water													
Draft at damper	0.37	0.36	0.16	0.13	0.34	0.23	0.57	0.56	0.62	0.61	0.77	0.60	0.66
Draft at furnace bottom	0.26	0.25	0.12	0.09	0.23	0.18	0.29	0.31	0.35	0.34	0.40	0.35	0.44
Draft under arch	0.14	0.15	0.06	0.06	0.17	0.12	0.24	0.22	0.25	0.24	0.31	0.25	0.42
Pressure of air at feeders	4.8	5.5	5.6	5.4	6.0	5.5	5.00	12.7	10.7	11.1	14.2	14.0	15.3
Flue gas													
CO_2, per cent by volume	14.54	14.31	15.43	15.55	14.50	15.40	13.64	13.83	13.72	13.34	13.15	12.64	13.83
O_2, per cent by volume	4.50	4.82	3.62	3.37	4.51	3.80	5.53	5.40	5.52	5.92	6.02	6.76	5.97
CO, per cent by volume	0	0	0	0	0	0	0	0	0	0	0	0	0
Excess air, per cent	27.2	29.8	20.8	19.2	27.2	22.2	35.6	34.6	35.5	39.3	40.2	47.3	39.7
Heat account, per cent													
Heat absor. by boiler and superheater screen	82.5	84.1	85.3	87.1	83.6	85.8	84.3	78.2	79.4	80.9	79.8	82.1	79.8
Heat lost, per cent													
Dry flue gas	7.6	7.3	6.4	6.3	7.9	7.7	9.3	9.5	9.6	10.0	10.4	10.7	9.0
Hydrogen and moisture in coal	4.6	4.5	4.6	4.5	4.5	4.6	4.6	4.6	4.6	4.6	4.6	5.1	4.6
Combustible in ash and refuse	1.5	1.3	1.2	1.0	1.3	1.8	2.0	2.0	1.9	1.8	2.1	2.3	2.2
Radiation	1.7	1.0	2.4	2.4	1.7	1.8	1.4	1.5	1.5	1.5	1.3	1.5	1.4
Unaccounted for and errors	+2.1	+1.8	+0.1	−1.3	1.0	−1.7	−1.6	+4.2	+3.0	+1.2	+1.8	−1.7	+3.0
Total	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

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within ±1 per cent. The author witnessed practically all of the tests reported in this paper.

19 The accuracy of the tests is also indicated by the heat accounts, particularly by the last items — radiation, and errors and unaccounted for. In some cases the radiation is computed as a separate item, so that the line headed "errors and unaccounted for" really shows the errors made in collecting the data and the

TABLE [illegible] RESULTS OF BOILER TESTS AT THE PLANT OF THE UNITED RAILWAYS OF PROVIDENCE, AUGUST, SEPTEMBER AND OCTOBER, 1924

(Bigelow-Hornsby boiler, 12,660 sq. ft. of heating surface; New River coal)

Test No.	12	13	14	15	16	17	18	19	20
Duration, hr.	14.55	12.27	15.83	13.15	13.87	12.73	13.02	16.13	15.25
Rate of driving, per cent	178	324	188	229	241	228	190	262	175
Coal as fired [1]									
Moisture, per cent	3.0	2.9	2.6	4.8	6.4	5.5	3.1	3.1	2.6
Ash, per cent	6.0	6.8	7.2	6.7	5.6	5.4	8.1	7.3	6.3
B.t.u. per lb.	14075	13945	14000	13790	13330	13464	13732	13836	14171
Fired per hour, lb.	6153	11380	6181	7800	8509	8109	6562	9337	5996
Per cu. ft. of combustion space per hr., lb.	0.65	1.20	0.65	0.82	0.90	0.85	0.69	0.99	0.63
Water									
Evaporated per hr., lb.	65880	117060	68920	83560	88110	83080	68560	93120	64400
Per lb. coal, lb.	10.7	10.3	11.1	10.7	10.4	10.3	10.5	10.0	10.7
B.t.u. per lb. of steam	1147	1176	1156	1167	1162	1163	1176	1192	1159
Temperatures, deg. fahr.									
Air to furnace	90	93	89	96	94	100	97	84	87
Air to feeders	137	137	130	131	130	128	124	121	118
Gases leaving boiler	520	659	520	582	572	562	573	649	524
Gases leaving economizer	256	338	258	288	269	280	269	319	254
Water to economizer	190	201	190	194	193	189	191	192	195
Water to boiler	278	317	279	293	284	285	294	304	284
Superheated steam	572	647	589	615	605	600	629	660	604
Pressures, lb. per sq. in. abs.									
Boiler	231	259	239	234	234	234	231	237	228
Superheater	222	248	231	225	228	233	228	231	224
Drafts, in. of water									
Furnace	0.03	0.19	0.02	0.05	0.04	0.09	0.04	0.09	0.01
Gases leaving boiler	0.25	0.83	0.28	0.38	0.37	0.37	0.32	0.58	0.24
Gases leaving economizer	0.70	2.26	0.73	1.07	1.09	1.09	0.89	1.53	0.75
Pressure feeder air	11.2	12.1	12.4	12.8	11.3	12.1	11.8	12.6	11.8
Analysis of gas									
CO_2 leaving boiler	15.0	15.0	15.0	15.4	15.5	15.0	15.0	15.4	14.9
CO_2 leaving economizer	15.0	14.9	15.0	15.5	15.4	14.9	15.1	15.5	14.9
Heat account									
Heat absorbed by boiler and superheater	80.5	77.9	84.9	82.8	82.9	81.3	81.6	77.7	81.0
Heat absorbed by boiler, superheater and economizer	87.3	86.7	92.0	90.6	90.2	88.5	89.5	85.8	87.9
Loss in dry gases	3.7	5.4	3.8	4.2	3.9	4.1	3.8	4.1	3.7
Loss in water vapor	3.7	3.8	3.7	3.9	4.0	4.0	3.7	3.9	3.7
Loss in incom. combustion	2.6	2.5	1.2	1.3	1.6	1.5	1.3	1.7	1.1
Radiation / Errors and unaccounted for	2.7	1.6	—.7	0	.3	1.9	1.7	4.5	3.6

[1] Average fineness: Through 200 mesh, 75 per cent; through 100 mesh, 90 per cent

accuracy of the test. This method was applied particularly to the tests made at the West Penn Power Company's plant at Springdale, Pa., because at this plant the coal and water were weighed by Richardson automatic scales. These scales were carefully calibrated, and during the test were closely watched. The last item in the heat account indicates that the automatic scales did very well. The tests are presented in this paper because of the high rating that was obtained with the boiler, particularly on test No. 7. This test was started with a running start; that is, there had been

no fire in the furnace for six hours when the test was started, although the boiler was kept on the line during this period. When the test was started the burners were lit, and within less than five minutes the boiler was operating at 300 per cent of rating, and soon after that was brought up to above 400 per cent of rating. The average for the run, which lasted 24 hours, was 442 per cent.

TABLE 7 RESULTS OF TESTS AT THE PLANT OF UNITED RAILWAYS OF PROVIDENCE, OCTOBER, 1924

(Bigelow-Hornsby boiler, 12,660 sq. ft. of heating surface; Coal: Mixture of New River and Rhode Island coal)

Test No.		35	36	38	39	41	40	42
Duration, hr.		14.98	12.47	12.10	15.60	16.35	13.55	11.43
Rate of driving, per cent		157	227	272	192	261	185	220
Coal as fired								
Through 40 mesh, per cent		.....	.....	.....	99.6	.....	99.4	99.9
Through 100 mesh, per cent		.....	.....	.....	85.0	.....	87.6	89.9
Through 200 mesh, per cent		.....	.....	.....	65.4	.....	69.5	70.5
Moisture, per cent		4.7	4.5	5.9	4.4	4.1	4.5	4.6
Ash, per cent		17.4	19.3	18.7	17.8	17.7	21.1	20.7
B.t.u. per lb. of mixture		11543	11220	11147	11577	11424	10788	10808
Fired per hr., lb.	R. I.	3245	4923	5892	4103	6180	5506	6684
	N. R.	3327	4733	5861	4031	5274	2928	3371
Per cu. ft. combustion space per hr., lb.	R. I.	0.34	0.46	0.62	0.43	0.65	0.58	0.70
	N. R.	0.35	0.40	0.45	0.42	0.57	0.31	0.35
Water								
Evaporated per hr., lb.		58024	81265	95519	69731	92832	67180	79393
Evap. per lb. of coal, lb.		8.82	8.42	8.13	8.51	8.10	7.95	7.91
B.t.u. per lb. of steam		1155	1185	1206	1165	1192	1166	1171
Temperatures, deg. fahr.								
Air to furnace		79	83	78	79	82	83	88
Air to feeders		119	110	106	105	103	106	105
Gases leaving boiler		490	551	578	522	576	535	545
Gases leaving economizer		242	288	304	268	309	270	275
Water to economizer		188	189	188	189	197	194	195
Water to boiler		262	284	293	278	305	291	287
Superheated steam		583	639	680	610	669	615	626
Pressures, lb. per sq. in. abs.								
Boiler		234	235	239	237	239	233	237
Superheater		221	224	227	230	229	227	227
Drafts, in. of water								
Furnace		0.15	0.17	0.17	0.09	0.23	0.08	0.15
Gases leaving boiler		0.30	0.58	0.71	0.39	0.82	0.41	0.54
Gases leaving economizer		0.69	1.44	1.87	0.90	2.09	1.00	1.35
Pressure feeder air		14.1	14.4	14.6	14.4	15.2	14.1	14.8
Analysis of gas								
CO_2 leaving boiler		14.0	13.9	13.8	14.1	13.8	13.9	14.6
CO_2 leaving economizer		13.9	13.7	14.0	14.3	13.8	13.8	14.7
Heat account								
Heat absorbed by boiler and superheater		84.0	80.2	80.2	80.5	77.5	79.8	80.2
Heat absorbed by boiler, superheater and economizer		89.7	87.4	87.9	87.0	85.4	87.2	87.1
Loss in dry gases		3.9	4.8	5.4	4.3	5.4	4.1	3.8
Loss in water vapor		2.9	3.0	3.2	2.9	3.1	3.3	3.3
Loss in incomplete combustion, Radiation, Errors and unaccounted for		3.5	4.8	3.5	5.8	6.1	5.4	5.8

20 In the heat accounts, attention is called to the item "Loss in incomplete combustion." This item includes the combustible in the flue dust, which is by far the largest loss due to incomplete combustion. In many tests with powdered coal this source of incomplete combustion is entirely neglected. With improper furnace design and poor operation the flue dust may be as black as the powdered coal that is fed to the furnace. Usually only the flue gases are analyzed, and a sample of the deposit on the bottom of

the furnace may be taken and analyzed. Neither of these two may contain any appreciable amount of combustible, while at the same time a large amount of unburned carbon may be passing out of the boiler with the gases.

21 Table 7 gives the results of tests with Rhode Island graphitic coal, pulverized and burned in a mixture with New River coal. The tests were made at the same plant and on the same boiler as the tests presented in Table 6. The Rhode Island coal has a very low percentage of volatile matter, and the gases distilled as volatile

TABLE 8 RESULTS OF TESTS OF 6-ROLL RAYMOND MILLS

ILLINOIS COAL [1]

Test No.	1	2	3	4	5	6	7	8
Total mill hours	16.33	13.99	27.72	17.05	58.86	35.23	50.15	48.78
Coal per hour, tons	5.97	5.24	5.27	6.00	6.68	5.70	4.31	4.06
Moisture in coal, per cent	4.6	6.0	5.2	4.1	5.6	3.7	6.1	6.2
Kw. per hour	77.5	78.0	77.9	78.3	79.3	78.7	77.2	77.4
Power per ton of coal, kw-hr.	13.0	14.9	14.7	13.4	14.3	14.1	18.4	19.6

Average fineness of coal: Through 100 mesh 95 per cent, through 200 mesh 76 per cent.

Tests 1 to 6, dried coal; tests 7 and 8, undried coal.

PENNSYLVANIA COAL, NATRONA MINE, UNDRIED [2]

Test No.	1	2	3	4	5	6	7	8	9	10	11
Total mill hours	10.92	11.17	6.93	6.35	9.90	10.65	14.26	14.10	13.03	13.48	9.92
Coal per hour, tons	5.04	5.04	5.40	5.35	4.95	4.86	4.56	4.61	4.86	4.98	4.87
Moisture in coal, per cent	2.4	2.1	1.8	2.1	1.9	2.4	2.1	1.8	2.0	1.9	1.6
Kw. per hour	63.7	67.6	69.3	68.0	63.9	66.1	61.6	59.1	69.0	67.0	65.0
Power per ton of coal, kw-hr.	12.45	13.41	13.14	12.75	12.96	13.56	13.50	12.81	14.25	13.38	13.38

Average fineness of coal: Through 100 mesh 95 per cent, through 200 mesh 76 per cent.

NEW RIVER COAL, UNDRIED [3]

Test No.	1	2	3	4	6	7	9
Total mill hours	8.6	6.8	5.1	5.8	5.8	4.6	5.6
Coal per hour, tons	9.9	8.85	8.8	8.1	8.7	8.3	7.1
Moisture in coal, per cent	2.5	2.9	2.6	2.7	2.6	3.1	2.0
Kw. per hour	112	106	104	99	105	105	87
Power per ton of coal, kw-hr.	11.4	12.0	11.8	12.2	12.1	12.7	12.3

Average fineness of coal: Through 100 mesh 98.5 per cent, through 200 mesh 81.5 per cent.

[1] Tests at St. Joe Lead Co., River Mines, Mo., November, 1921.
[2] Tests at Penn Salt Co., Wyandotte, Mich., July and August, 1923.
[3] Tests at United Rys. of Providence, Providence, R. I., May, 1924.

matter are not combustible. For this reason it is practically impossible to start fires with it alone. However, in a mixture of half and half with New River coal, or even two parts of Rhode Island and one part of New River coal, the mixture ignites readily and is a practical fuel. The economic results obtained with these mixtures compare well with the results obtained when using New River coal.

RESULTS OF MILL TESTS

22 Table 8 gives the results of three series of mill tests made with three different coals at three different plants. The results

show that the output of the mill varies to a large extent with the quality of the coal. With Illinois and some of the hard Pennsylvania bituminous coal the capacity of a 6-roll Raymond mill is between 5 and 6 tons. With the coals having granular structure, such as New River and Pocahontas, the capacity is as high as 9 tons per hour. It is higher with dry coal and lower with moist coal.

Cost of Making Steam With Powdered Coal

23 Reliable data on the cost of making steam with powdered coal are still meager. It is to be expected that with many powdered-coal plants in operation more data on the cost of making steam soon will become available, so that it will be possible to make a fair comparison with the cost of making steam in stoker plants.

24 In the past, unfair deductions for powdered coal were often made by estimating the cost of preparing pulverized coal and then comparing it with the value of the coal saved by the higher thermal efficiency of powdered coal over the stoker. All comparisons should be made on the basis of total cost of preparing and burning pulverized coal, and the total cost of burning with stokers. However, such cost can be compiled best by the operating engineers. It is hoped that some such data will be presented in the discussion of this paper. The following method of presenting cost data is suggested:

COST OF BURNING ONE TON OF COAL

	Powdered-coal plant	Stoker plant
A. Operating labor	$............	$............
B. Power	....	....
C. Maintenance	....	....
D. Fixed charges	....	....

Description of Boilers and Furnaces and Method of Making Tests

25 The furnaces on which the tests were made were fired vertically downward, the flame turning up near the bottom, making a U-shaped path through the furnace. About 15 to 20 per cent of the air needed for combustion was supplied with the coal, another 10 to 15 per cent of the air was supplied through the burners around the nozzles, and 60 to 70 per cent through the hollow walls.

Test at Cahokia Power Plant of the Union Electric Light & Power Co., St. Louis, Mo.

26 The boiler shown in Fig. 2, was a Babcock & Wilcox cross-drum, 20 tubes high and 38 tubes wide, with 18,010 sq. ft. of heating surface. It was equipped with a Babcock & Wilcox super-

heater of 4070 sq. ft. of heating surface placed in an interdeck chamber above the sixth row of boiler tubes.

27 The furnace was of the hollow-wall construction with a steel casing. It was equipped with 10 Lopulco fantail burners and a water screen over the bottom and rear wall of the furnace, with 587 sq. ft. of heating surface exposed to fire. The water screen consisted of 4-in. tubes spaced 10.85 in. between centers. It was connected to the boiler drum by two 6-in. downcomers and two 8-in. risers. The combustion space above the water screen was 11,750 cu. ft. The average distance between arch and water screen was 22 ft.

28 The tests were made by the engineers of McClellan & Junkersfeld, Inc., and of the Union Electric Light & Power Co. Water was weighed in two special test tanks placed on a standard platform scale. Coal was weighed in the weighing tanks of the Quigley air transport as it was delivered to the bin of the test boiler. The results of the tests are given in Table 1.

29 The coal used was Illinois coal of the following typical composition:

Moisture	12.91	per cent
Ash	11.64	"
Carbon	60.74	"
Hydrogen	4.00	"
Nitrogen	1.15	"
Sulphur	1.32	"
Oxygen	8.24	"
Volatile	31.9	"
Fixed carbon	43.55	"

Tests at River Rouge Plant of the Ford Motor Company

30 The results of the tests are given in Table 2. The boiler shown in Fig. 3 was a Ladd water-tube boiler with 26,470 sq. ft. of heating surface. It was equipped with a locomotive superheater made by the Superheater Co. and placed between the boiler tubes in the first pass of the boiler. The heating surface of the superheater was 3140 sq. ft.

31 The furnace was of solid-wall construction equipped with 12 Lopulco burners, six burners on each side of the boiler. There was no water screen in the furnace. The total combustion space was 16,000 cu. ft. Coal was fired vertically downward. The distance between burner arches and the bottom of the furnace was 20 ft.

32 The tests were made by the engineers of the River Rouge Plant in coöperation with the test engineers of Combustion Engineering Corporation. Water was weighed in three special tanks placed on standard platform scale and fed to the boiler through a separate test line. Powdered coal was weighed as it was delivered to the boiler bins, in special weighing tanks suspended from standard scales.

33 The coal used on the tests was a mixture of two Kentucky coals from Banner Fork and Pond Creek mines, and had the following typical composition:

Moisture	2.0	per cent
Ash	9.94	"
Carbon	73.69	"
Hydrogen	4.50	"
Sulphur	0.53	"
Nitrogen	1.20	"
Oxygen	8.14	"
Volatile matter	33.4	"
Fixed carbon	54.5	"

Tests at No. 3 Plant of Rochester Gas and Electric Company

34 The boiler, Fig. 4, was of the Bigelow-Hornsby type, having 8750 sq. ft. of heating surface. It was equipped with a Foster superheater of 1055 sq. ft. of heating surface, and a Sturtevant cast-iron-tube economizer with 2390 sq. ft. of heating surface.

35 The furnace was of hollow-wall construction with a steel casing. It was equipped with four fantail Lopulco burners and a water screen over the bottom of the furnace having 157 sq. ft. of heating surface. The combustion space of the furnace was 4200 cu. ft. above the water screen and 655 cu. ft. below the screen. The distance between the burner arch and the water screen was 26 ft.

36 The tests were made by the plant test engineers in coöperation with the test engineers of the Combustion Engineering Corporation. Coal and water were weighed with standard scales. The results are given in Table 3.

37 The coal burned was Pennsylvania Lucerne Mine coal. The following is a typical composition:

Moisture	3.0	per cent
Ash	11.4	"
Carbon	71.75	"
Hydrogen	4.74	"
Nitrogen	1.27	"
Sulphur	2.15	"
Oxygen	5.73	"
Volatile matter	28.0	"
Fixed carbon	57.6	"

Test at Springdale, Pa., Plant of the West Penn Power Company

38 The boiler, Fig. 5, was a Babcock & Wilcox cross-drum, 15 tubes high and 42 tubes wide, the tubes being 20 ft. long. The heating surface was 15,326 sq. ft. The boiler was equipped with a Babcock & Wilcox superheater located above the first pass, and having 3860 sq. ft. of heating surface.

39 The furnace was of hollow-wall construction with a steel casing. It was equipped with eight Lopulco burners and a water screen over the bottom and rear wall of the furnace, and having 480 sq. ft. of heating surface. The water screen consisted of 4-in. tubes spaced 14 in. between centers and connected to the boiler

drum with two 6-in. downcomers and two 6-in. risers. The distance between the burner arch and the water screen was 23 ft.

40 The tests were made by the test engineers of the West Penn Power Company, and the results are given in Table 4. Water and coal were weighed with Richardson automatic scales. The coal used was Pittsburgh coal. A representative analysis of the coal is as follows:

Moisture	1.51	per cent
Carbon	73.44	"
Hydrogen	4.76	"
Oxygen	6.28	"
Nitrogen	1.45	"
Sulphur	0.71	"
Ash	11.85	"
Volatile matter	35.0	"
Fixed carbon	51.6	"

Tests at the Plant of Penn Salt Company, Wyandotte, Mich.

41 The boiler was a Babcock & Wilcox cross-drum boiler, 14 tubes high and 27 tubes wide, having 8220 sq. ft. of heating surface. The boiler was equipped with Babcock & Wilcox superheater. See Fig. 6.

42 The furnace was of hollow-wall brick construction, equipped with 4 Lopulco fantail burners and a water screen over the bottom of the furnace. The screen consisted of 4-in. tubes spaced 14½ in. centers, connected to boiler drum with two 6-in. downcomers and two 6-in. risers, and having 236 sq. ft. of heating surface. The combustion space was 3360 cu. ft. The distance between burner arch and the water screen was 14 ft.

43 The tests given in Table 5 were made by the engineers of the Penn Salt Company in coöperation with the test engineers of the Combustion Engineering Corporation. The coal was weighed on standard scales in lots of 1000 lb. as it was delivered to the pulverizing mill. The water was weighed in two special test tanks placed on standard platform scales. The coal used on these tests was Pennsylvania coal from the Natrona mine.

Test at the Plant of the United Railways of Providence, Providence, R. I.

44 The boiler tested, of the Bigelow-Hornsby type, had 12,660 sq. ft. of heating surface and is shown in Fig. 7. It was equipped with Foster superheater of 6060 sq. ft. of heating surface, and a Foster economizer of 7488 sq. ft. of heating surface.

45 The furnace was of hollow-wall construction with a steel casing. It was equipped with eight fantail Lopulco burners and a water screen over the bottom of the furnace having 320 sq. ft. of heating surface. The combustion space of the furnace was 9500 cu. ft. above the water screen. The distance between burner arch and the water screen was 26 ft.

46 The results of the tests are given in Tables 6 and 7. They were made by the test engineers of the United Railways of Providence in coöperation with those of Combustion Engineering Corporation. Coal was weighed in lots of 2000 lb. in a special hopper placed on standard platform scales. Water was measured in two special test tanks with conical bottom and top.

47 The coal used was New River coal and Rhode Island coal. The following are typical analyses of the coals:

NEW RIVER

Moisture	2.52	per cent
Ash	8.0	"
Carbon	78.71	"
Hydrogen	4.46	"
Nitrogen	1.31	"
Sulphur	6.8	"
Oxygen	4.32	"
Volatile matter	21.28	"
Fixed carbon	68.2	"

RHODE ISLAND COAL (CRANSTON MINE)

Moisture	9.71	per cent
Ash	25.71	"
Carbon	62.60	"
Hydrogen	0.29	"
Sulphur	0.68	"
Nitrogen	0.08	"
Oxygen	0.93	"
Volatile matter	2.60	"
Fixed carbon	61.98	"
B.t.u. as received	9770	per lb.

DISCUSSION

Walter C. Slade.[1] The 16 boiler tests made at the plant of the United Electric Railways Company reported by the author have been taken from a series of 39 tests which were carried to completion. All tests were made on the same boiler. They were started in May and completed in October, 1924. Three additional tests attempted with Rhode Island coal could not be finished on account of the difficulty of keeping the furnace temperature high enough to support continued combustion. On the other hand, seven tests in which a mixture of New River coal and Rhode Island coal was employed, were successfully completed.

Throughout the series of tests no changes were made in the boiler, except in the baffling in the front half. Altogether five different baffling arrangements were used. The tests reported in Table 6 were made with one, and the tests reported in Table 7 were made with another of these baffling arrangements. The changes in baffling were made to determine the effect upon furnace efficiency and superheat. It was found that superheat could be

[1] Vice-Pres., United Elec. Ry. Co., Providence, R. I.

fixed at any desired point within a comparatively wide range from 140 deg. to 210 deg. fahr. at 200 per cent of rating.

At the same time the temperature of the exit gases from the economizer assumed a corresponding temperature at a point within a range of not over 50 deg. This was due naturally to the capacity of the economizer to absorb heat from the gases leaving the boiler at the higher temperatures. The temperatures of the exit gas from the economizer ranged from 250 deg. up to 300 deg. fahr. for most of the tests where the unit operated at not over 250 per cent of rating. The highest temperature reached was in test No. 13, being 338 deg. at 324 per cent of rating. These figures were almost exactly duplicated in another test not reported.

The performance of the boiler, superheater, and economizer as a unit was therefore commendable. It is believed that the data which have been published are representative and are capable of duplication under test conditions at any time.

The baffling arrangement apparently also had a definite effect upon the percentage of unconsumed carbon carried over in the flue dust, and losses in this respect were noticeably higher in the case of one of the baffling arrangements while burning bituminous coal. They were also higher when burning a mixture of bituminous and Rhode Island coals. Unless attention is paid to furnace adjustments, considerable loss may occur in this manner.

The remark that powdered coal can never be a complete success until the boiler is built around the furnace, opens up a field for future development, if the burning of coals with high ash and low fusing point is to be thoroughly successful, particularly if such coals are to be burned at relatively high furnace temperatures for the higher rate of driving boilers demanded to-day. The three furnaces of the writer's company have the hollow wall construction and water screen, designed to reduce abrasion and the slagging of the refractories and the slagging of the ash from the coal. These three furnaces now have been in operation for almost exactly one year, each one being lined with a high-grade firebrick, made of a different type of clay in each case for the purpose of observation under practical operating conditions. Thus far it has not been necessary to make any repairs to the brickwork in any one of the three settings, although it is believed that in one setting some general repairs will be necessary on the front wall, probably within six months. Considering everything, the brickwork has stood up in a fairly satisfactory manner. It is still too early to draw any definite conclusions as to the merits of the refractories.

In none of the boiler tests was the coal dried, although some of it contained from five to six per cent moisture as fired. We have not yet attempted to dry coal in regular operation, although under certain conditions of excessively high moisture some trouble occurred on account of clogged feeders and wet coal in the pulverizers. If coal contains above five per cent moisture, opera-

tion will be made more smooth by removing the moisture by drying down to at least five per cent.

The six-roll Raymond pulverizer used for preparing coal throughout the tests has a nominal rating of six tons per hour, or one ton per roll. However, the Eastern coals from the New River or the Pocahontas fields, which we have been burning very generally, pulverized so readily that it was possible to prepare between eight and nine tons per hour per mill. It is surprising to find such a wide difference between the Eastern and the Western coals and some Pennsylvania coals as regards ease of pulverizing. With Rhode Island coal the capacity of the mill was reduced by 50 per cent as compared with New River coal.

As yet we have no definite data on the cost of maintenance of the pulverized-coal plant. This is due to the fact that we have not yet begun to renew some things which are subject to renewal, and further, to the fact that for a considerable period mill repairs did not amount to much, and even yet the normal level of expense has probably not been reached.

C. G. Spencer.[1] The erosion of furnace walls by molten slag and the ready disposal of this slag through the furnace bottoms are the two problems in burning pulverized coal at the highest efficiency, particularly coal from some of the Southern Illinois fields. It is expected that water-lining the furnace to a greater extent than heretofore will be a further step toward the final solution of these problems.

Some authorities, however, suggest the possibility of delayed combustion introducing additional problems. In the absence of definite operating experience, the water-lined furnace must still be considered somewhat in the twilight zone for pulverized coal with very high ash, sulphur, and iron content, if the highest attainable efficiency of combustion is to be attained continuously. With the admission of a little excess air and the sacrifice of one or two per cent in efficiency, the erosion of side walls and the slag in furnace bottoms are both greatly reduced and often eliminated.

Meanwhile, furnaces must be designed and constructed to meet the demand of the increasing load, and preferably at the highest attainable continuous efficiency. An outline of the development of the Cahokia furnaces may be of interest. Fig. 2 shows one of the eight original Cahokia furnaces on which the tests presented by the author were made. It has sloping walls on all four sides, and a relatively short rear-wall screen. The next three boilers installed, two of which are now in service, have vertical side and front walls, with consequently increased furnace width between side walls, thus adding furnace volume. The ashpit screen is lowered to increase flame travel. The rear water screen extends

[1] McClellan & Junkersfeld, Inc., New York, N. Y. Mem. A.S.M.E.

almost up to the lower boiler tubes, to improve rear-wall protection. Steel was substituted for concrete in the structure supporting and forming part of the ashpit, to lessen the possible damage from overheating, should conditions require that the flame be forced through the screen in the bottom of the ashpit. These changes increased the furnace volume by seven per cent and reduced the unit rate of combustion from 15,250 B.t.u. to 14,300 B.t.u. per cu. ft. of furnace volume at 275 per cent of rating. Tests have not as yet been made on these furnaces, and the operating experience has not been long enough to draw definite conclusions.

The third step in the furnace development at Cahokia is shown in Fig. 1, where fin tubes are being installed in the side walls. This furnace is scheduled to go into service late in 1924 or early in 1925. It is, in turn, 17 per cent greater in volume than the original design, with a corresponding reduction in the combustion rate to 13,100 B.t.u. per cu. ft. of furnace volume at 275 per cent of rating. This fin type of furnace protection has been reported as very effective in the stoker-fired furnaces at the Hell Gate station in New York, with the coals usual in this market.

Because of the unknown behavior of this furnace when burning pulverized Illinois coal, and the need for using the flue gases to operate driers from the two boilers yet to be erected, these furnaces are being designed with water-cooled side walls having tubes on 14-in. centers in vertical recesses, giving an intermediate step in furnace protection between the air-cooled firebrick side wall of Fig. 2 and the fin-tube side wall of Fig. 1.

The original eight furnaces are ordinarily operated until the side walls are eroded to partial destruction; that is, until a hole develops, admitting air directly from the horizontal air passages into the furnace. The 9-in. firebrick between the air passages and the furnace is reduced in some cases to one inch at the thinnest place before failure. This failure usually means a hole equivalent to a circle of 2 to 4 ft. in diameter.

Experience with this particular coal shows the necessity for larger rather than smaller furnaces, as indicated by this review of successive steps in design. This is a pertinent fact at a time when so much is being written adversely to the large volume of furnaces for burning pulverized coal. It emphasizes the necessity of proportioning the length of the flame travel and the furnace volume for the characteristics of the coal to be burned and the resultant ash.

The test results given in this paper are of great value. They apply to three different types of boilers and to coal from widely separated fields. A direct comparison of results, however, will be misleading unless corrections are first made for steam pressure and exit gas temperature by reducing them to a common basis. We must also take into account the limitations in efficiency

imposed by the effect of ash and iron content of different coals on furnace operation. The wide range of this effect for the same B.t.u. liberated for four different coals is illustrated by the following, assuming the same efficiency on a dry-coal basis:

	Good Pa. or W. Va. Coal	Poor Nova Scotia Coal	Poor Illinois Coal	Poor Montana Coal
Coal burned, tons	1000	1105	1275	1145
Total ash, tons	59	112	213	152.5
Iron content, tons	2.9	59.6	39.5	28.8
Ash per 1,000,000 B.t.u., lb.	4.0	7.7	14.7	10.9
Iron per 1,000,000 B.t.u., lb.	0.2	4.1	2.5	2.6

In addition to ash and iron content there are moisture, fusing point of ash, and other characteristics that further accentuate the difficulties of preparing and handling these poorer grades of coal, and of furnace operation and maintenance. The constant improvement in design, construction, and operation of combustion equipment, particularly for the poorer grades of coal, is; therefore, an ever-present public and private obligation.

Alfred Iddles.[1] The advisability of using pulverized coal in any new steam plant depends mainly on the increase in efficiency that may be obtained as compared with a standard stoker-fired plant. This gain in efficiency has been variously estimated at from 4 to 8 per cent, and the writer has attempted to determine definitely from which of the various losses this increase in efficiency is obtained.

The writer has made some comparisons of the tests given by the author with tests of boilers equipped with stokers. The boiler tests reported by Mr. C. W. E. Clarke[2] were used in making the comparisons. These tests were made on boilers almost exactly like those at the Cahokia plant upon which the pulverized-fuel tests in this paper were made. The boilers in each plant are Babcock & Wilcox cross-drum type, 20 tubes high. The furnace volume for the Cahokia boilers is 6½ cu. ft. per rated boiler horsepower, and at Colfax 3.3 cu. ft. per boiler horsepower. The flue-gas temperature at the exit of the boilers is almost identical in the two sets of tests, which fact indicates that the boilers themselves are very similar in their performance, and the difference in overall boiler and furnace efficiency should, therefore, result almost exclusively from the difference in the methods of burning fuel.

Fig. 8 shows the various items of the two sets of tests plotted with percentages of boiler rating as abscissas and percentages of heat in the fuel and other data plotted as ordinates.

[1] Chief Power Engr., Day & Zimmermann, Inc., Philadelphia, Pa. Mem. A.S.M.E.

[2] Boiler Test Results at Colfax Station of Duquesne Light Company, Trans. A.S.M.E., vol. 45 (1923) p. 567.

The stoker-fired-boiler tests at Colfax were plotted first by starting at the top line, representing 100 per cent of heat in the fuel, and measuring down to represent the percentage loss due to radiation and unaccounted for. The true picture of this loss seems to be shown by a straight horizontal line indicating an average loss due to radiation and unaccounted for of 1.1 per cent.

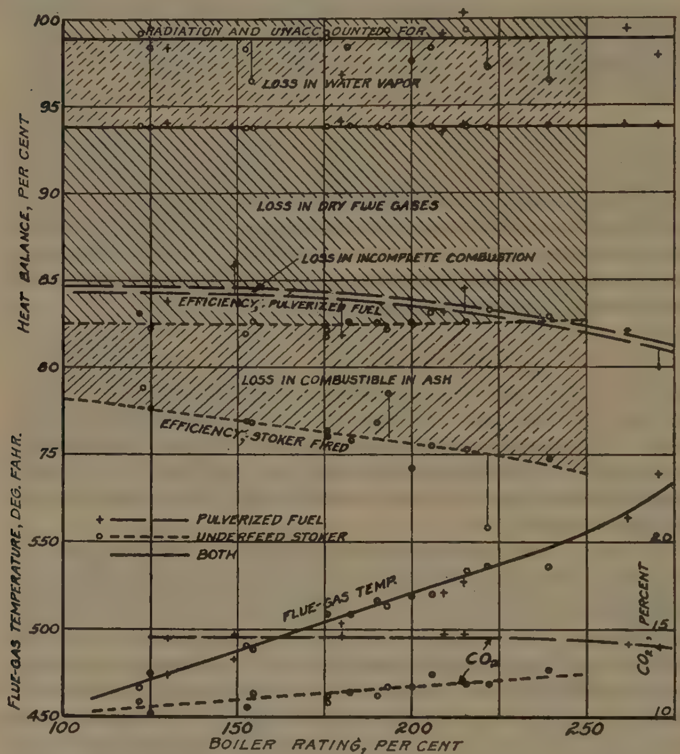

Fig. 8 Comparison of Pulverized Fuel and Underfeed Stoker in Two Plants

	Pulverized Fuel	Underfeed Stoker
Plant	Cahokia	Colfax
Type of boiler	B & W Cross-Drum	
No. of tubes high	20	20
Superheater	Interdeck	
Baffles	Similar	
Water screens	Yes	No
Furnace vol., cu. ft. per hp.	6.5	3.3
B.t.u. in fuel as fired	11,780	12,800

From this line as a base the losses due to water vapor were plotted. This results in a constant value of 5.1 per cent, which added to the loss due to radiation and unaccounted for, makes a total of 6.2 per cent, and is shown by the horizontal line at 93.8 per cent on the scale of ordinates.

Similarly, the losses due to dry flue gas and combustible in refuse were separately plotted. Each of these four items of losses is shown by a shaded area in Fig. 8.

The boiler tests reported by the author for the Cahokia plant using pulverized coal were plotted on the same chart. The average loss due to radiation and unaccounted for is the same as for the stoker tests. Likewise the loss due to water vapor in the flue gas is practically the same as for the stoker-fired boilers. The curve representing total heat absorbed by the boiler unit (or the efficiency curve) is about 8 per cent higher for the pulverized-fuel than for the stoker boiler.

The chart clearly shows that most of the increase in efficiency obtained by pulverized fuel is due to a saving of combustible in the refuse. This clearly indicates where the stoker manufacturer and user should seek improvement.

Frequently the advocates of pulverized fuel claim much for the possibility of burning coal in pulverized form with much less excess air and consequent smaller losses in the flue gas. In these two sets of tests it is evident that the gain due to less excess air with pulverized fuel was about two per cent at 150 per cent of boiler rating, one and one-half per cent at 200 per cent of boiler rating and nothing at higher rates. Apparently it is not practical to operate with the very small quantities of excess air which it is possible to get with pulverized fuel. Evidently this is because the furnace and refractory material develop trouble at the higher rates of driving, unless the excess air is increased to approximately the same as found in good stoker practice.

For this reason the use of water- and steam-cooled furnace walls is especially interesting. Without doubt combustion engineers are anxious to learn the results to be obtained with the Murray furnace installed in the addition to the Cahokia plant. It is hoped that these results will show that it is possible to operate pulverized-fuel furnaces with even less excess air and consequent lower flue gas losses and also, of course, with resulting lower furnace maintenance costs.

B. N. Broido.[1] The author shows results of a series of seven tests made in different plants. As the tests show the superheat, the writer was interested to see the variation of superheat at different rates of driving for these various plants, particularly as approximately the same kind of fuel and the same burners were used for all plants. Superheats have been plotted in Fig. 9.

With ever-increasing superheat, the question of constant temperature is of considerable importance. With high pressure and high superheat, constant temperature at various rates of driving is

[1] Chief Engr., Industrial Dept., The Superheater Co., New York, N. Y. Mem. A.S.M.E.

imperative in order to avoid an excessive final temperature which may prove detrimental.

From the test results given in the paper, the Elesco superheater installed in connection with the Ladd boiler at the River Rouge plant of the Ford Company seems to maintain a more constant steam temperature than any of the others. The superheat

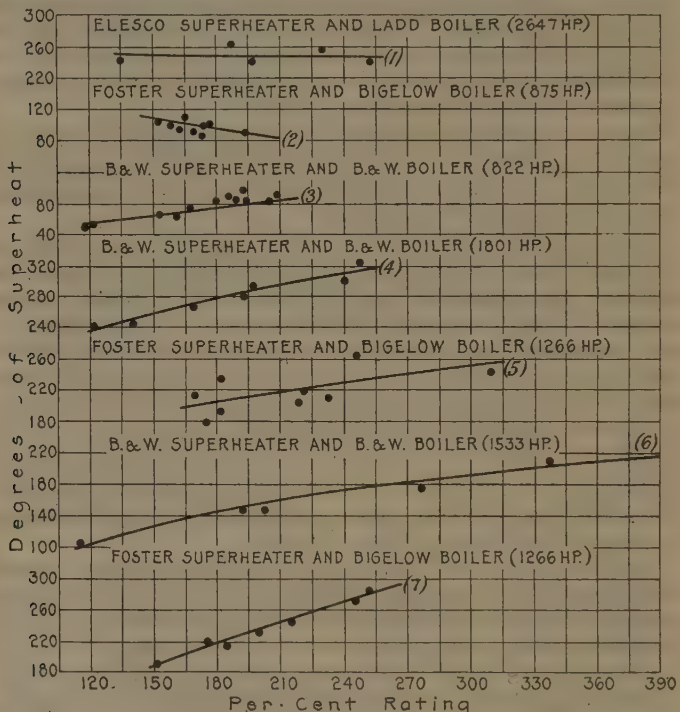

Fig. 9. Curve of Superheats of the Several Tests Reported in the Paper

(1) Ford Motor Co., River Rouge Plant; (2) Rochester Gas & Elec. Co.; (3) Penn Salt Co.; (4) Union Elec. Lt. & Power Co., Cahokia Station; (5) United Rys. of Providence, New River Coal; (6) West Penn Power Co., Springdale Station; (7) United Rys. of Providence, New River and R. I. coal.

between 135 and 255 per cent of rating is practically constant, and the line representing the average superheat is almost straight. The maximum variation above or below the average is less than 10 deg. In the other plants, the variation in superheat is considerable. The constant temperature in the Ford plant is due to the location of the superheater, the elements of which are spaced between the boiler tubes of the first pass.

Considering the fact that any load variation applies to both boiler and superheater, all the steam generated by the boiler pass-

ing to the superheater, the superheat will be constant if the heat absorbed by both boiler and superheater varies in the same proportion. It is a known fact that the first few rows of boiler tubes generate most of the steam, so that a superheater located between or near these tubes will be subjected to the same variations as the part of the boiler where most of the steam is generated, and in this way the ideal conditions for constant temperature are closely approached.

H. L. Smith.[1] A question that arises in connection with powdered coal is the effect of the water screen, when it becomes an appreciable percentage of the total heating surface, on the superheat. In one installation now under construction, the steam evaporated in the water screen is led away and superheated separately from the steam generated in the boiler proper. Since the steam generated in the water screen is generated by heat extracted from the gases largely by radiation, which would tend to reduce the superheat because of lowered temperature of furnace gases, it is pertinent to inquire as to the effect on the superheat when, as in the usual arrangement, this steam from the screen is passed through the superheater, diluting, as it were, the steam from the regular water tubes.

Henry B. Jones.[2] For over three years the writer has been operating a station of 66,000 sq. ft. heating surface, burning pulverized anthracite mine wastes. In this installation we have found it unnecessary to use either water-cooled walls or a water screen.

Our experience has made it apparent to us that ash fusing on the furnace floor is governed by four factors:

(*a*) The distance from the burner, or from the bottom of the flame loop to the furnace floor

(*b*) The rate at which combustion is proceeding

(*c*) The fusing point of the ash

(*d*) The angle at which radiant heat from the flame strikes the floor of the furnace.

The water screen undoubtedly is a successful solution of this problem, although introducing a regrettable complication into the construction of the boiler. A careful consideration of the principles just enumerated will undoubtedly frequently show a way to design a furnace so as to eliminate this complication.

The writer agrees with the author that great possibilities exist in completely surrounding the combustion space with water-heating surface, but believes that coöperation between boiler and furnace designers may well result in material simplification of

[1] New England Power Co., Worcester, Mass.

[2] Fuller-Lehigh Co., Fullerton, Pa. Mem. A.S.M.E.

the constructions consisting of various batteries of tubes connected into the boiler circulation.

The internally fired boiler is not new, and a locomotive type of firebox on a large scale is a possibility. Some development of the Bettington boiler also may offer a solution.

A. G. Christie.[1] The author refers to the action of the ash sprayed in a molten state on to the walls, and states that this molten ash, in running down, washes the brick along with it. This destruction of the furnace walls by the ash has three serious consequences: It reduces the overload capacity of the furnace; it reduces the efficiency at the heavier loads by requiring an increasing percentage of excess air to prevent increasing furnace temperatures; and finally it necessitates large furnaces.

The action of the ash is not the same in all cases, and it has been found to be worst with low-fusing ashes from high-sulphur coals. Such ashes naturally have high iron contents and on that account form a natural series of alloys or chemical compounds with silica and alumina. These compounds are characterized by much lower fusing points than the original constituents, and hence the resultant tendency of the wall to wash away.

This action can be prevented by two methods. Walls of a neutral brick containing neither silica nor alumina might be used, but so far no such satisfactory material has been demonstrated. Water-cooled metal side walls as shown in Fig. 1 form another alternative. A fear has been expressed that the presence of so much water-cooled surface in the furnace would cause such a rapid absorption of radiant heat that the furnace temperature would be seriously lowered and that the combustion would be incomplete. While our knowledge of furnaces and radiant-heat transfer is not complete, the writer's study of this subject has led to the same conclusions as that of the author when he states: "Powdered coal can never be a complete success until the boiler is built around the furnace."

However, should this all-metal furnace prove less economical than we hope, then there is a design under development, consisting of water-cooled reverberatory walls, that will meet every operating requirement. Engineers now recognize the tremendous influence of powdered-coal ash on successful furnace performance.

The author characterizes the coal drier as a nuisance, and this is true to a certain extent. A few plants are receiving reasonably dry coal from the mines and are operating satisfactorily without driers. However, the subject cannot be dismissed so easily, for combustion engineers must study more closely the action of powdered coal in the furnace and the effect of drying on combus-

[1] Prof. Mechanical Engineering, Johns Hopkins University, Baltimore, Md. Mem. A.S.M.E.

tion. Experiments indicate that dried coal yields its gases and volatiles more readily than moist coal, and also that coking of the former is an exothermic reaction, which does not seem to be the case when moisture is present. If these facts are true, then dried coal should burn with shorter and hotter flames than moist coal, and as a result smaller furnace volumes would be required, or greater capacity at higher efficiency could be developed with a given furnace volume. This subject is still open to question and requires further study. At the present time it would seem that the drier is a " necessary nuisance " which should be installed.

The author has stated the mill situation quite capably. He might also have added that lower power consumption for pulverizing is desired along with larger mills. It is unfortunate that no data were presented in the paper on the performance of the larger mills. The test data in the paper are very valuable and justify further study. These figures form ample justification for the installation of powdered-coal equipment in many modern power plants.

Frederick A. Scheffler.[1] The writer is glad to note that the author recommends drying of the raw coal before pulverizing. Although attempts have been made to demonstrate that the drying of the coal can be eliminated, even when moisture is as high as from 10 to 12 per cent, these have been unsuccessful, and have created much misunderstanding.

There is no difficulty in burning moist coal, if no trouble is experienced in getting the coal into the furnace. In moist condition the coal clogs the pulverized-coal bins and screw feeders to the burners, causing uncertain operation, and requiring constant attention from the operators. When the moisture in the raw coal is but two or three per cent, it can be bypassed across the driers to the pulverizers.

While the paper more particularly describes the use of pulverized coal in public-service plants requiring large boilers, furnaces, etc., it might be well to point out that pulverized coal is largely in use in much smaller plants. There is no difficulty whatever in applying this method of firing boilers to many of the existing industrial plants, and to new ones under consideration.

Many engineers undoubtedly will be startled by the expensive type of furnaces indicated by the author, as well as by their height, and by the suggestion that it is necessary to use water screens to take proper care of the slag at the bottom of the furnace. The Fuller-Lehigh Company has made probably over two hundred installations during the last few years, which are operating satisfactorily without water screens, and with all kinds of coal, from British Columbia in the West, to the Atlantic Coast in the East.

[1] Mgr., Power Dept., Fuller Engineering Co., Fuller-Lehigh Co., New York, N. Y. Mem. A.S.M.E.

It is entirely possible to apply pulverized coal to plants where the local conditions will not permit boiler furnaces over a height or 20 to 26 ft. and where it is possible properly to use horizontal firing. Many such installations are being operated without water screens and at fairly high rates of driving, such as 300 per cent. Some of them continuously by the month average 275 per cent of rating.

The method of firing horizontally is applicable in cases where the volatile content of the coal is about 18 per cent and higher, which means bituminous or semi-bituminous coal. This method also permits the use of smaller furnaces with multiple burners, the number being in proportion to the rate of driving the boiler, which should be predetermined before the installation is effected. Furthermore, numerous engineers have objected to the use of water screens in furnaces where the feedwater is not of the best and where scale is liable to form. Scale in such tubes is generally rather difficult to remove. The writer does not wish to be misunderstood as to his position in this respect. He believes that the water screens are of considerable advantage in a great many cases, if considered from the standpoint of additional boiler heating surface, provided that the feedwater is excellent.

Another point brought out very clearly by the author is that the high efficiencies shown in the tests with various kinds of fuel would indicate that it is not necessary to know all about the analysis of the coals before determining the efficiencies that should be expected when the coal is burned in pulverized form, assuming satisfactory boiler and furnace conditions, as well as the right kind of baffling. When the coal is pulverized, the boiler and furnace efficiencies are practically the same under such conditions, whether the coal is 9000 B.t.u. or 15,000 B.t.u. or anywhere between these limits. This is totally different from what may be observed in stoker-fired plants, in which it is always necessary to determine first the B.t.u. value of the coal, and also the physical analysis, before a statement can be made as to what the boiler and furnace efficiencies will be. The latter will be much lower in such cases with coal of low B.t.u. value than with coal of high B.t.u. value.

In regard to the discussion of Mr. Alfred Iddles, the writer believes that his comparison between the Colfax station efficiencies with stokers and those indicated in the author's paper are unfair because of the great difference in the two types of fuel used. If the St. Louis coal, which was pulverized and burned in the tests detailed by the author, had been burned on the stokers at Colfax, the results would not, in the latter test, show nearly as high an efficiency as in the actual tests referred to.

W. J. Wohlenberg.[1] The rapid growth of the art of burning pulverized fuel under large steam generators is well pictured in

[1] Asst. Prof., Mechanical Engineering, Sheffield Scientific School, Yale University, New Haven, Conn. Assoc.-Mem. A.S.M.E.

this paper. One is impressed by the magnitude of the activity, both mental and physical, which must have been required to bring about in such a short time the present highly developed systems.

The higher possible efficiencies and other advantages of such methods of burning coal must, of course, be charged with: (1) The increased cost due to a necessarily larger furnace, and (2) the extra cost of handling and preparation caused by pulverizing the coal. Both of these quantities are obviously influenced by the fineness of pulverization. Flame, and hence furnace volume and length, are also influenced by the intensity of mixing or turbulence which exists within the flame. In this discussion the writer will deal principally with the possible influence of fineness of pulverization on such conditions as furnace temperature and volume. It should be stated at the outset that the information contained is based very largely on an analytical method rather than on experimental results.

If we begin with conditions existing in the modern pulverized-fuel furnace as to flame length, furnace volume, and mixture, two limiting assumptions are possible with respect to the influence of finer coal particles: First, that the rate of combustion or energy liberation per cu. ft. per hr. is proportional to the increased surface of the particles held in suspension in the flame; or, second, that the combustion rate is dependent on turbulence and mixing conditions only, and is therefore uninfluenced by further fineness of pulverization.

If the first assumption is correct, an increase of fineness of pulverization of from a 200 mesh to a 300 mesh would result in a reduction of furnace volume to less than two-thirds of present requirements, and if pulverization to 400 mesh should be possible, then the furnace-volume requirements would be reduced to one-half of those contained in present furnaces.

The energy radiated from flames with the finer particles also, for a given flame temperature, would be much greater by reason of the increased amount of glowing carbon surface in suspension. The flame temperature, despite increased radiation, would remain constant because of the proportional increase in the rate of energy liberation. Consequently, for such conditions increased fineness of pulverization not only decreases the necessary furnace volume, but it likewise results in a greater rate of heat absorption by the steam generator, and hence higher steaming capacities and better overall efficiencies for any given steaming rate.

If the second assumption is correct, i.e., that the combustion rate beyond 200-mesh fineness is a function of the air-mixing process only, then finer pulverization of itself would obviously have no influence on furnace-volume requirements. For such a condition, however, the furnace temperature would be materially reduced by finer pulverization, because of the increased exposed glowing carbon surface for the same rate of energy liberation.

This condition might be desirable, as fusing of the ash particles could be avoided thereby.

Actually, both size of particle and mixture intensity or turbulence undoubtedly will have considerable influence on the possible rate of energy liberation, and it seems desirable to explore the field more thoroughly, to the end of devising a means for reducing the furnace volume.

Finer pulverization, of course, will entail a higher cost of preparation, which must be charged against any gains resulting therefrom. Such a condition is also probably combined with greater danger from dust explosions, as a curve plotted between energy-liberation rate and fineness of pulverization based on the first of the foregoing assumptions will show. The rise of the energy-liberation rate rapidly brings about an explosion condition as the particles become small enough to pass through a 1000-mesh sieve.

R. A. Foresman.[1] In the comparison given by Mr. Iddles between powdered-coal and stoker-fired furnaces, in which the Colfax station of the Duquesne Light Company was referred to as a representative stoker-fired plant, the statement was made that the difference in operating efficiency of the two systems was due mainly to the loss of combustible in the ash and refuse, and that that loss amounted to about eight per cent. At this station the sum of all other losses is roughly around 20 per cent, under normal operating conditions. The efficiency obtained on a yearly operating basis will permit only about one-half of the 8 per cent combustible loss in the refuse credited to this plant by Mr. Iddles to be charged against it. Furthermore, the Colfax station, which has been in service four years, has a stoker ratio of about 125 hp. per retort, as compared with a ratio of 100 hp. per retort where a similar stoker is used in more recent stations. The Colfax stokers, as compared to the more modern stokers, have a considerably higher combustible loss than the latter.

T. E. Keating.[2] One of the principal arguments against stokers and in favor of powdered coal has been the claim for higher efficiency obtainable with the latter. It is fair to ask where the difference comes in. The writer cannot follow the reasoning of Mr. Iddles to the same conclusions. Mr. Iddles concedes that the radiation, unaccounted for, and moisture losses are the same, but his differential of eight per cent in efficiency due to the combustible in the refuse does not seem reasonable, particularly with reference to the stations he has compared. These stations, we have been given to understand, have only about four per cent

[1] Chief Engineer, Stoker Dept., Westinghouse Elec. & Mfg. Co., South Philadelphia Sub-Station, Philadelphia, Pa. Mem. A.S.M.E.

[2] Syndicate Representative, Westinghouse Elec. & Mfg. Co., New York, N. Y. Mem. A.S.M.E.

differential in monthly operating efficiency in spite of the fact that the stokers are not designed to operate continuously at as high a rate of combustion as the powdered-coal plant. The difference may be due to the methods of operation employed in the two stations, and the powdered-coal stations may operate continuously under so-called test conditions, which is costly. It is fair to ask what the result would be if stokers were designed for similar rates of combustion and a corresponding amount of money were spent on the furnace design and operation. Automatic control is considered indispensable to powdered-coal installations, but it has not been applied so frequently on stokers. If the same refined control were applied on stokers as on powdered coal, the relative efficiency of the two should be nearly equal. Furthermore, if the same amount were to be spent in regularly maintaining the efficiency of the stoker plants as is spent on the powdered-coal plant, the prevailing monthly operating efficiencies of the former would undoubtedly be improved.

The statement was made that stokers are not able to burn the same type of fuel that was burned in the powdered-coal furnaces. In answer to this the writer presents a record of a run made with what is called Western Kentucky screenings on a 1512-hp. Babcock & Wilcox cross-drum boiler. The dry coal had a calorific value of from 11,832 B.t.u. up to 12,132 B.t.u. The moisture, as fired, ran from 2.3 up to 11.1 per cent; the ash ranged from 13.9 to 16.4 per cent, and its fusing point ranged from 1872 to 1915 deg. fahr. The rate of combustion ranged from 25.1 to 74.8 lb. of coal per square foot of grate surface on a 16-hr. test, and the combustible in the ash ranged from 9.2 to 13.9 per cent.

John H. Lawrence.[1] The writer thoroughly agrees with the author's statement that powdered coal cannot be a complete success until the boiler is built around the furnace. Our company has had some experience in the construction of water-cooled furnaces, having started our experiments to find a way to obviate the high cost of furnace-wall maintenance. The first boiler to be put into service was subjected to probably as complete a test as any boiler ever was, and showed on test an efficiency of as high as 84.5 per cent. This, the writer believes, proves that the side-wall construction does not at all affect the combustion. It also protects the front and rear walls. Recent tests to determine the temperature difference between the walls of the new boiler and the corresponding walls of the older boilers showed that the front and back walls on the new boiler were considerably cooler than the same walls on the stoker-fired boilers.

[1] Vice-Pres., & Engrg. Mgr., Thomas E. Murray, Inc., New York, N. Y. Mem. A.S.M.E.

When it was decided that six boilers at the Sherman Creek station should be fired with powdered coal, it was found possible to set the boilers only about 12 ft. above the floor. Notwithstanding dubious predictions of all kinds, such as that we would be unable to light the burners, and that the coal would not ignite, we have in service five of the six boilers, are getting very high efficiency, and have driven the boilers as high as 400 per cent of rating. Each one of these boilers has a water-cooled furnace, which is only as large as the ordinary stoker-fired furnace of ten or twelve years ago. This installation proves conclusively that powdered coal can be burned in a furnace that is almost completely surrounded by relatively cool walls. We know that high efficiency can be obtained on stoker-fired furnaces with water-cooled walls, and we believe that it also can be obtained with powdered coal even in a relatively small furnace volume, provided we can get the proper type of burner.

The powdered-coal installations that have been in service have had in most instances a very long flame. To obtain high efficiency with water-cooled side walls, the writer believes that a burner must be developed which will give a very short flame. Indications are that the tests now being made will develop a furnace, and possibly a burner, which will give as high an efficiency in a small space with water-cooled walls as is now obtained with the very large furnaces with firebrick walls.

One point that the author has omitted is operating costs. The tests, while illuminating, give no information in regard to yearly and monthly operating results. As high an efficiency can be obtained with the stoker as with powdered coal at certain rates of driving on tests, but whether these efficiencies can be carried throughout the year is not known. Information on this point would be helpful.

John Anderson.[1] At the Lakeside Station, Milwaukee, we have in the past four years consumed approximately one and a half million tons of coal in powdered form. We have complete data on the cost of operation, but a presentation of such data would be of no value in this discussion, because it would be necessary to make qualifications on account of the difference in unit cost of labor, fuel, and all other things that must necessarily go to make up the expenses of power-station operation.

The No. 2 boiler room just completed is more efficient than the No. 1 boiler room by a considerable amount. We are able to maintain the efficiency during steaming periods at about 90 per cent flat. A deduction of one per cent will cover banking and other miscellaneous losses.

[1] Chief Engr. of Power Plants, The Milwaukee Elec. Ry & Lt. Co., Milwaukee, Wis. Mem. A.S.M.E.

Mr. Lawrence stated that what was needed to improve on what had been done up to the present time in pulverized-fuel furnaces was a burner that would give a shorter, sharper flame than what he has up to now observed. We have given this matter a great deal of study and have as a result probably just the kind of burner that he is looking for.

With regard to the cost of maintenance, we find that in a comparison of our stoker-fired plants with the powdered-coal plants, the cost of operating supplies and expenses, together with maintenance material, is just about twice as much in the stoker plant as in the powdered-fuel plant. This takes into consideration all expenses other than labor and coal throughout the plant.

W. A. Shoudy.[1] While admitting the high efficiency that may be obtained with powdered coal, the writer would call attention to the fact that under certain circumstances this fuel cannot be used and that reliance must be placed on stokers. The point in which we are interested, as stoker users, is that the reported efficiency for long periods comes so close to the test efficiency. As a stoker user, we want to know, when a test efficiency of, say, 76 per cent is obtained, why we cannot obtain that same efficiency throughout the month. One reason may be that there are inaccuracies in the measurements. With powdered coal a higher annual efficiency can be obtained because of lower banking losses. Just what these amount to we do not know. Powdered coal also probably comes closer to the test efficiencies because of easier control of fuel and air, although a great loss may ensue if the carbon is not watched. In a powdered-coal installation carbon may be lost up the stack undetected, whereas in the stoker-fired plant it is visible in the ashpit.

While the stoker cannot be as closely regulated as the powdered-coal equipment, the question is more than one of mechanical regulation; the personal element enters to a large extent. Thus, practically all of the powdered-coal installations are large ones, and when the boilers are placed in operation, a series of tests is run to determine what can be done with that particular arrangement of furnaces. As the men who are to finally operate the furnaces probably have had no previous experience with powdered coal and have no traditions or precedents, they do as they are told and learn to operate the furnaces efficiently. On the other hand, in stoker-fired plants, men are always available who have operated stokers for 20 years or more. Such men are very hard to teach, since they believe that they know all there is to know about operating stokers. If a man who knows nothing about operating stokers is placed under a man who will consider its

[1] Supt., Steam Plants, Adirondack Power & Light Corpn., Schenectady; Assoc. in M. E., Columbia Univ., New York, N. Y. Mem. A.S.M.E.

operation from the purely scientific standpoint, the writer believes that we will begin to approach test efficiencies on continuous operation. If we attack the problem of stoker operation from the standpoint of eliminating the error due to the human element, we will get farther ahead in stoker practice.

Edwin Lundgren.[1] In the comparison made by Mr. Iddles, one factor was overlooked, namely, that it is incorrect to compare the performance of a stoker designed for a particular fuel with stoker performance using a fuel that is entirely unsuited for the stoker. In the statement of losses in a heat balance there are included certain items for which definite values can be fixed. For instance, in the comparisons given in this discussion the losses due to hydrogen, etc., were included. The hydrogen losses of Illinois coal are quite different from those of Pennsylvania coal. The losses due to the combustion of hydrogen would amount to far more than the loss of combustible if Illinois coal were burned in the installation designed for Pennsylvania coal, and furthermore there would be a greater limitation in the amount of coal that could be burned per square foot of grate. It is doubtful if the same rates of driving could have been obtained with low-grade Illinois coal of the Belleville district as have been obtained with powdered coal. These are only some of the factors that must be considered when comparing the performances of stokers with powdered coal.

Alfred Iddles. It may be permissible to answer some of the criticisms of the writer's written discussion of this paper (see p. 621).

The accuracy of the statement of the loss due to combustible in the ash has been questioned. As to this, the writer would say that the figures were taken from test results reported in the Society's Transactions. However, he is not acquainted with all of the conditions surrounding the tests.

At the low ratings on the stoker-fired boiler, results of which were analyzed in the written discussion, the loss due to combustible in the ash was 4½ per cent. This loss increased materially as the boiler rating was increased, being about 8½ per cent at 250 per cent of rating. It is evident, therefore, that the gain in efficiency with pulverized coal may be attributed to (1) the decrease of combustible in the refuse, and (2) the decrease in the quantity of heat lost in the dry flue gas. The difference in dry-flue-gas loss shows a material gain for powdered fuel at the lower rates of driving but this gain decreases from about 2 per cent at 150 per cent of boiler rating to 0 at 240 per cent of rating.

[1] Combustion Engineering Corpn., New York, N. Y.

The total difference in efficiency was 7 per cent at 150 per cent of rating and 8 per cent at 200 per cent of rating. It is obvious that the greater difference in efficiency at the higher rates of driving is due to the higher ashpit loss in the stoker-fired boiler. At another plant quite similar to the Colfax plant, the writer has found it very difficult to maintain a low percentage and this is a difficulty that is found in greater or less degree in all stoker-fired plants. The Hell Gate station has reported tests in which this loss was practically nil. These tests represent the very best which the stoker has been able to accomplish. Without doubt the older plants, such as Colfax, would show a much better performance in this respect if they were equipped with the very latest stoker installations.

The writer's object in making this summary was not primarily to compare the efficiency of the two systems of burning fuel but rather to determine wherein it was possible to make savings, due regard being given in each particular case to the operating conditions, quality of coal, etc.

The Author. A question has been asked as to whether or not the addition of a large amount of water-cooled surface in the furnace would affect adversely the superheat. Generally speaking, it would not. It makes very little difference whether the steam is made in the water-cooled furnace or in the boiler. As long as the superheater is of sufficient size to handle the required weight of steam produced by the steam-generating unit, the superheat should not be affected by the presence of water-cooled surfaces in the furnace. The location of a convection type of superheater with respect to the boiler heating surface is the most important factor affecting the superheat. If high superheat is required the superheater should be located close to the furnace. That is, there should not be too much boiler heating surface between the superheater and the furnace. With horizontal water-tube boilers, if the superheater is placed above the first six or seven rows of tubes, a superheat up to 700 deg. fahr. should be obtainable without much difficulty whether the furnace is water-cooled or not.

Higher efficiency is obtained with pulverized coal than with stokers because more complete combustion is obtained with powdered coal. The incomplete combustion is measured not only by the amount of carbon in the refuse deposited at the bottom of the furnace or in the ashpit and the analysis of the flue gases, but also by the amount of carbon in the flue dust and cinders going out of the stack. In the case of pulverized coal, the refuse deposited at the bottom of the furnace contains practically no carbon. The furnace can be operated with gases containing from 14 to 16 per cent CO_2 and practically no CO. If the furnace is properly designed the carbon in the flue dust is usually about five per cent. It may be at times as low as two per cent. With improper furnace

design or with poor operation the combustible in the flue dust may be 25 or 30 per cent. With stoker firing, besides the combustible in the ashpit refuse, there is a considerable amount of coal carried out of the furnace through the boiler into the stack in the form of cinders.

The amount of carbon carried away with flue dust in powdered-coal furnaces depends mainly on two factors, i.e., the length of flame travel in the furnace, and the amount of fixed carbon in the coal. If the flame travel is very short the combustible in the flue dust may easily exceed 25 per cent. This may occur in spite of the fact that the flame appears short. The combustible in the flue dust is usually in the form of coke. The visible flame in the furnace is caused by the burning of hydrocarbons in the volatile matter of the coal. Although the volatile matter may be burned completely, a large part of the fixed carbon may escape as particles of coke in the flue dust.

Coal of high fixed carbon when burned in pulverized form usually gives a very short visible flame and at the same time a high percentage of carbon in the flue dust. The flame is apparently short because there is a very small percentage of volatile matter in the coal to give visible flame. Most of the combustible is in the form of fixed carbon which has to burn as coke. Coke burns practically without any visible flame. If the furnace is not provided with sufficient flame travel some of this fixed carbon will pass out of the furnace unburned, and the flue dust will contain a large percentage of combustible. Burning coal of high fixed carbon is more difficult than burning high-volatile coal because the shortness of the visible flame with the former coal is deceptive. It gives the impression of complete combustion, whereas there may be a loss of several per cent from incomplete combustion of the carbon. A distinction must be made between long flame travel and long or short visible flame. Long flame travel is a property of furnace, i.e., the distance available for the flame travel within the furnace. Visible flame is the actual combustion of volatile matter as observed by the eye.

The objection is frequently made that large furnace volume and long flame travel are at present required for burning pulverized coal. It is possible that when we know more about the subject we can get complete combustion with a smaller furnace and shorter flame travel. Until that time, we must use large furnaces if we wish to obtain almost complete combustion without extensive furnace repairs.

Water-cooled furnaces practically eliminate the maintenance of refractories and facilitate the removal of ash. It is still a question whether the furnaces should be completely water-cooled or whether they should contain some refractories in order to help ignition and maintain good combustion. Some engineers fear that complete water cooling of the furnace will be detrimental to good com-

bustion. They may be right. It is better to proceed cautiously and increase the water cooling of the furnaces by small amounts until further increases cause incomplete combustion, or until the furnace is completely surrounded by water-cooled surfaces without interfering with combustion. The furnace shown in Fig. 1 is a procedure in the right direction. It is about 70 per cent water-cooled and 30 per cent refractory surface.

The question of the range of rating at which the furnace is to be operated should be given full consideration when designing a water-cooled furnace. If the furnace is to be operated continuously at comparatively high rates of driving the water cooling may be carried to completion. If, however, the furnaces are to be operated at very low rates part of the time, it may be advisable to have some refractory surfaces in the furnace in order to help ignition when burning coal at low rates.

The fineness of coal undoubtedly affects the completeness of combustion. Very fine coal can probably be burned completely in a smaller combustion space than coarsely pulverized coal. It is a question, however, whether it would be economical with the present pulverizing equipment to pulverize the coal much finer than it is done at present in order to reduce the required furnace volume. The increased cost of pulverization might offset the smaller cost of a smaller furnace.

Mixing of air and coal speeds up the chemical reaction, but the mixing must be done in the furnace itself. The mixing may be done by introducing the coal and air into the furnace in different directions so that there will be a relative motion between the coal particles and the air. Mixing can be also obtained by directing the mixture of air and coal against solid surfaces. However, it is difficult to obtain surfaces that will resist the abrasive action of the flame.

So far as comparison of test results with operating results is concerned, the author can present only test results because they are the only definite and reliable data he has. The operating results must be presented by the operating engineer who operates the plant. The results are not from selected tests, but from all tests that we made, showing poor as well as good results.

No. 1931

RECENT DEVELOPMENTS IN THE BURNING OF ANTHRACITE

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Member of the Society

and

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Non-Member

The anthracite referred to is that of which at least 95 per cent will pass through a $\frac{3}{16}$-in. round-mesh screen and less than 20 per cent through a $\frac{3}{32}$-in. round-mesh screen. The coal was burned on Coxe stokers.

The paper describes successive furnace designs made at the Amsterdam (N. Y.) steam station of the Adirondack Power and Light Corporation. The boilers are of Babcock & Wilcox design and of 1345 *hp. Each boiler has two Coxe stokers. Attempts were made to improve on the customary furnace design to which the following objections are made: (a) tendency to stratification of gases, (b) high carbon content of ash, (c) loss of fines to ashpit and stack, and (d) lack of flexibility.*

*The first experimental furnace had, in addition to the ignition arch, a very short arch over the rear of the grate. With this arrangement improvement, if any, was slight. A three-arch furnace was then built, with better results but with defects due to slagging. A third furnace of two-arch design was finally constructed. Tests reported in the paper show that with the final design stratification can be practically eliminated, as well as ignition troubles, even with low-grade coal. A decided improvement has also been made in the burning of undersizes. Greater capacity was obtained with low-grade coal (*60 *per cent undersize and* 33 *per cent ash) in the multiple-arch furnace, where* 300 *per cent of rating was easily reached, than in the single-arch furnace where* 190 *per cent was about the limit. The boilers and furnaces are described and illustrated, and a description of the tests with the principal data and results are given in the form of curves.*

[1] Superintendent of Steam Stations, Adirondack Power and Light Corporation.

[2] Test and Research Department, Combustion Engineering Corporation.

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BECAUSE of its cleanliness and smokelessness, anthracite has come to be recognized as a fuel to be reserved for domestic or household purposes almost exclusively. It is the low volatile content and high fixed carbon which give these advantages but at the same time make the coal difficult to ignite and very rock-like or "hard." If the coal is crushed to the smaller sizes and burned, considerable difference in pressure must exist to force the air for combustion through the fuel bed and the coal must be fired frequently in a thin layer. The coarser sizes, however, because of the larger air spaces, can be burned in a thick bed and fired only two or three times a day. These are therefore in great demand by the householder, which eliminates them as competitors with other fuels for large power requirements.

2 The crushing of the coal to bring it down to the domestic sizes naturally produces a varying percentage of smaller sizes which are not at present in demand for household use. The percentage of these smaller sizes varies with the structure of the coal. It is generally smaller in the northern districts of Pennsylvania and larger in the southern districts. Obviously, these sizes must be sold or the entire burden will be placed on users of the domestic sizes. Pea, No. 1 buckwheat and No. 2 buckwheat, although used for house heating to a very limited extent in special furnaces, are consumed principally in large furnaces where the employment of firemen is necessary and where greater draft can be obtained by higher chimneys or forced draft.

3 There still remains a percentage of the total, varying from 3 per cent to 10 per cent or more, that at present can only be used for power requirements. This coal, which we will discuss, is not accurately defined but is generally called No. 3 buckwheat, though sometimes "birdseye." The specifications of operators vary, but all of it passes through a 3/16-in. round-mesh screen. The authors use as their specifications the following sizing:

4 "Not less than 95 per cent to pass through a 3/16-in. round-mesh screen and not over 20 per cent through a 3/32-in. round-mesh screen." We have called all larger than 3/16 in., "oversize" and all smaller than 3/32 in., "undersize."

5 The proper sizing of anthracite plays a prominent part in producing satisfactory boiler efficiencies. There are in general use only two satisfactory methods of burning this fuel, viz., by hand firing and by chain-grate stokers. However, anthracite has been pulverized and burned satisfactorily. While the progress has not been so rapid as with other coals, pulverization promises a means of disposing of the "undersize."

6 The ignition difficulties inherent in anthracite have made it necessary to employ a refractory arch to shield the ignition end of the furnace from the cooling effect of the boiler tubes. This is an absolute essential with the chain-grate stoker and generally useful with hand fires also, unless enough bituminous

coal is mixed with the anthracite to supply the volatile constituent to support ignition. Fig. 1 illustrates this type of furnace. It is a well-designed furnace of the single-arch type, unusually high and with a large combustion volume. The front header is 18 ft.

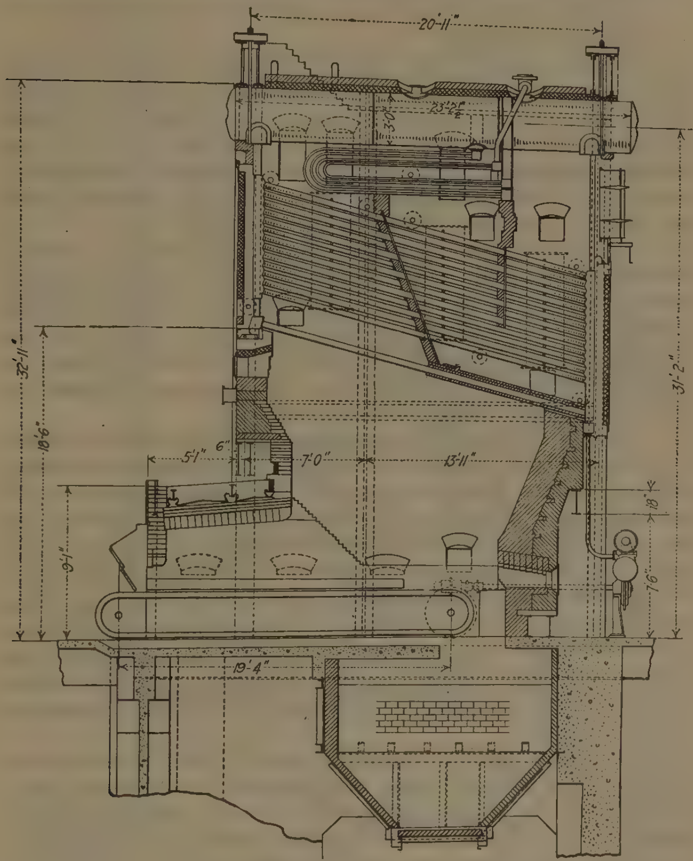

Fig. 1 Single-Arch Furnace for Burning Anthracite
(Original setting of boilers No. 1 and No. 2)

6 in. above the boiler-room floor and the grate extends under the full length of the furnace. Such a furnace will give as high a combined efficiency as can be obtained with a single-arch, the test results included in this paper proving this conclusion.

Objection to Former Design

7 This type of furnace was, until a few years ago, accepted as the approved design, and the low efficiencies (as compared with

bituminous coal) were accepted as inevitable. The objections to this type are:

a Tendency to stratification of gases
b High carbon content of ash
c Loss of fines (undersize) to ashpit and back connections
d Lack of flexibility.

8 Combustion on a chain grate is progressive. Ignition takes place at the front end and combustion extends over the center, with ash more or less completely burned at the rear. At the front there is generally found high CO, in the center high CO_2, and at the rear end considerable excess air. Unless these gases are well mixed, combustion is not complete. The alternative to operating with high excess air is high CO with its resultant loss.

9 Carbon cannot be completely burned from the ash without considerable excess air at the rear end. In the type of furnace under discussion this excess air usually follows a well-defined lane throughout the setting. Again the operator is faced with the alternatives of reducing the stack loss by reducing the excess air, thereby increasing the carbon loss to the ashpit, or reducing the carbon loss and increasing the stack loss.

10 Lack of uniformity in the sizing of the fuel results in closing up the air passages through the fuel bed. This necessitates carrying a higher pressure beneath the grate. The higher pressure breaks through the fire in spots and the velocity of the air picks up the finer coal and throws the unburned carbon back into the ashpit, or the gases carry these fines through the setting and deposit them on the tubes, in the back connection, and in the breeching. Some are carried off through the stack. An examination of these particles shows them to be coal, from which only the volatile content has been driven off. This is a direct loss, the extent of which varies with the care used in screening the coal and the rate of forcing the fires.

11 With this type of furnace the length of the fire varies with the load. The thickness is generally determined by the coal sizing. An increased load means lengthening the fire, and too rapid a change of speed tends to cause ignition troubles. Consequently the operator has still another set of alternatives to face if the load varies widely. He must carry a long fire and waste coal to the ashpit at light loads, or save this coal with shorter fires and take his chances on maintaining stream pressure.

The Experimental Furnace

12 For a number of years investigators have been studying this subject, and experiments with a second arch placed over the rear of the grate promised such satisfactory progress that the American Sugar Refining Company installed the boilers in its Baltimore refinery with a furnace embodying this principle. The authors' experiences with those furnaces convinced them that a distinct advance had been made.

13 The Amsterdam (N. Y.) Steam Station of the Adirondack Power and Light Corporation was started in October, 1921, with one 15,000-kw. turbine and two 1345-hp. boilers. The boilers are

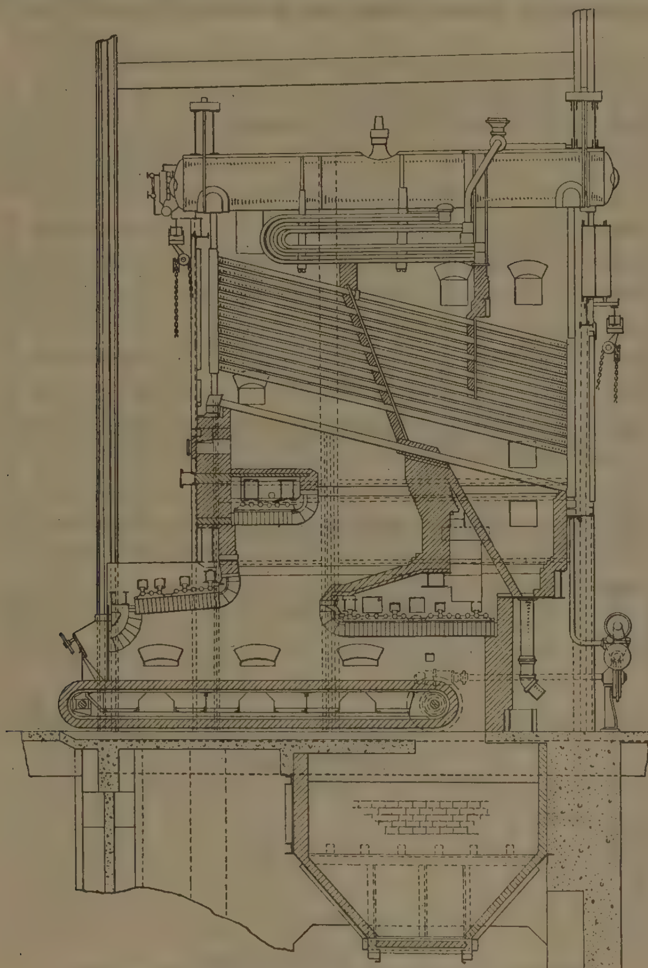

Fig. 2 Triple-Arch Experimental Furnace (Boiler No. 1) at Amsterdam

of Babcock & Wilcox design, 42 tubes wide and 14 tubes high, with 20-ft. tubes. Each boiler is set with two Coxe traveling-grate stokers 10 ft. 8⅝ in. wide by 17 ft. long (effective grate length). The fuel is anthracite. Since the station is a relay to the company's hydroelectric system the load is extremely

variable, and some difficulty was experienced in operating with but two boilers. The installation of another turbine and two more boilers somewhat improved the condition, but it was evident that improvement in furnace design was desirable. The second group

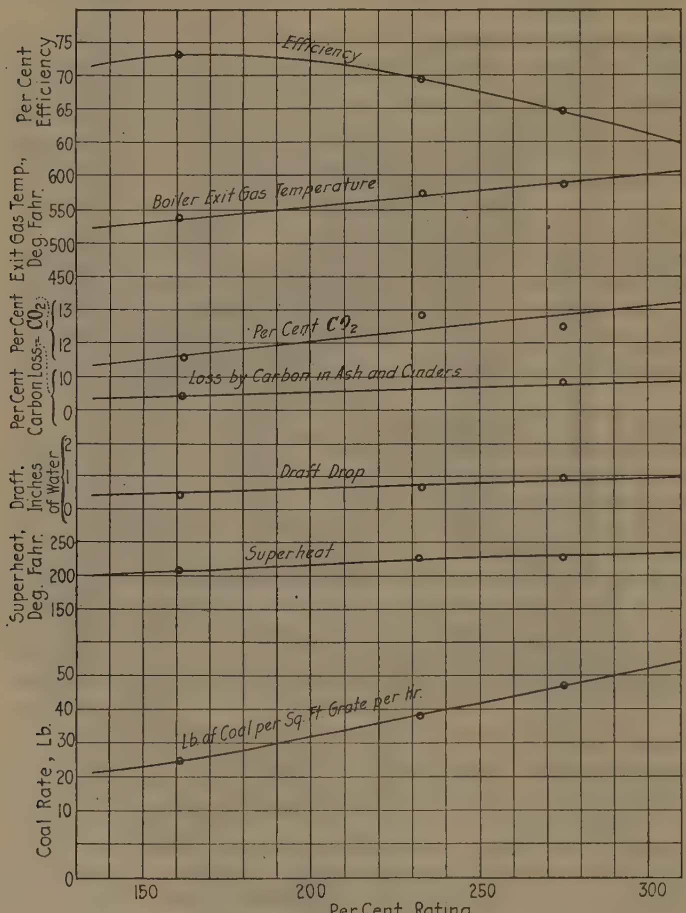

Fig. 3 Test Results with Single-Arch Furnace

of boilers was installed with short rear arches, but the improvement, if any, was slight.

14 An appropriation was authorized for the construction of an experimental furnace, and boiler No. 1 (Fig. 1) was used with the furnace shown in Fig. 2. It was decided to copy as nearly as possible the Baltimore furnace. The Baltimore boilers are of the

Stirling type and those at Amsterdam of the Babcock & Wilcox type; hence, as was recognized, the furnace was not directly applicable to the other boilers. However, the building of this furnace enabled the authors to study and arrive at the basic principles of the design.

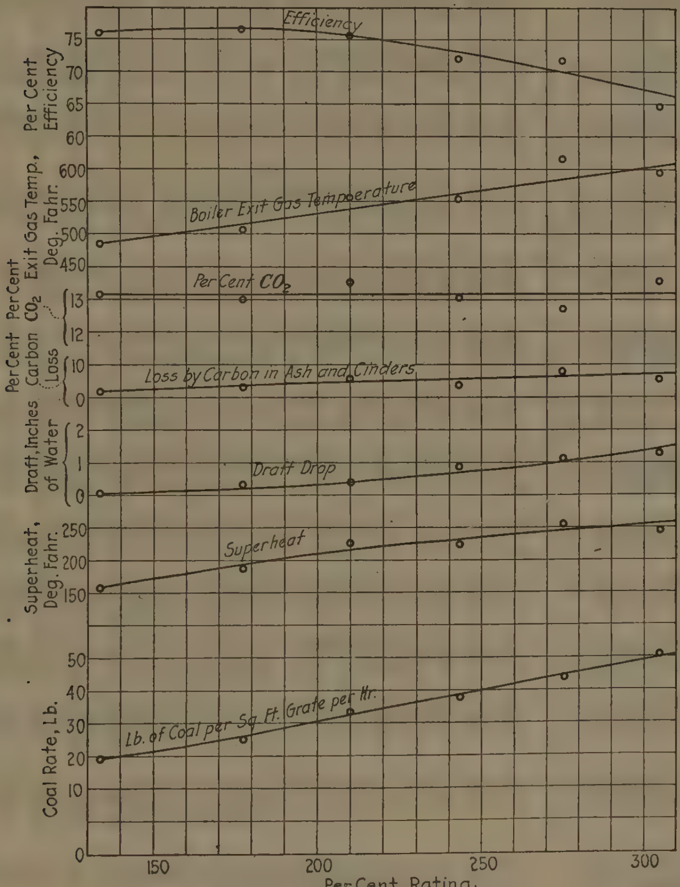

FIG. 4 TEST RESULTS WITH THREE-ARCH FURNACE

15 An extended series of tests was made of this furnace and of boiler No. 2, which is an identical boiler but with the old furnace shown in Fig. 1. The results of nine of these tests are given in Table 1 and plotted in Figs. 3 and 4. Six tests are included of the new furnace but only three of the old furnace, because the operating characteristics of the old furnace were so well known that only check runs were necessary.

TABLE 1 DATA AND RESULTS OF EVAPORATIVE TESTS

Test number		1	2	3	4	5	6	7	8	9	21	22	23	24	26	27
Boiler number		1	1	1	1	1	1	2	2	2	2	2	2	2	2	2
Grate surface,	sq. ft.	364	364	364	364	364	364	364	364	364	364	364	364	364	364	364
Water-heating surface,	sq. ft.	13,450	13,450	13,450	13,450	13,450	13,450	13,450	13,450	13,450	13,450	13,450	13,450	13,450	13,450	13,450
Superheating surface,	sq. ft.	3,863	3,863	3,863	3,863	3,863	3,863	3,863	3,863	3,863	3,863	3,863	3,863	3,863	3,863	3,863
Total heating surface,	sq. ft.	17,313	17,313	17,313	17,313	17,313	17,313	17,313	17,313	17,313	17,313	17,313	17,313	17,313	17,313	17,313
Ratio water heating to grate surf.		37-1	37-1	37-1	37-1	37-1	37-1	37-1	37-1	37-1	37-1	37-1	37-1	37-1	37-1	37-1
Ratio total heating to grate surf.		47.6-1	47.6-1	47.6-1	47.6-1	47.6-1	47.6-1	47.6-1	47.6-1	47.6-1	47.6-1	47.6-1	47.6-1	47.6-1	47.6-1	47.6-1
Volume of combustion chamber,	cu. ft.	2,690	2,690	2,690	2,690	2,690	2,690	6,140	6,140	6,140	....	....	......	......	......	......
Date		9/24/23	8/17/23	9/25/23	8/17–18/23	9/26/23	8/18–19/23	8/24/23	8/25/23	8/25–26/23	9/23/24	9/3/24	9/4/24	9/5/24	11/11/24	11/12/24
Duration,	hr.	8	8	8	9	8	9	8	8	8	8	8	8	8	8	8
Coal (anthracite):																
Oversize,	per cent	13.2	26.32	13.3	22.22	13.6	25.0	25.7	32.06	31.70	17.6	19.0	14.7	13.6	14.8	14.6
No. 3 Buckwheat,	per cent	51.2	57.87	50.5	59.28	59.3	60.0	48.6	41.52	46.34	48.5	49.7	48.0	46.0	52.0	52.6
Undersize,	per cent	35.6	15.81	36.2	18.50	27.1	15.0	25.7	26.42	21.96	33.9	31.3	37.3	40.4	33.2	32.8
Steam pressure by gage,	lb. per sq. in.	272.5	279.8	265.9	272.0	274.1	285.4	267.0	271.7	271.7	286.0	281.1	285.6	284.6	273	272
Barometer,	in. mercury	30.0	29.9	30.0	29.9	30.0	29.8	29.9	29.8	29.9	29.9	29.9	29.8	29.5	30.0	30.0
Steam temperature,	deg. fahr.	660	672	637	638	604	577	641	641	620	650	629	605	602	669	669
Saturated steam temperature,	deg. fahr.	413.6	415.8	411.5	413.4	414.4	417.5	411.8	412.9	412.9	417.8	416.2	417.8	417.8	414	413
Boiler-feed temperature,	deg. fahr.	164.0	146.4	154.0	156.3	152.0	156.3	141.0	146.0	145.0	185.0	191.0	190.0	197.0	180	179
Flue-gas temperature,	deg. fahr.	593	616	555	557	508	488	585	572	536	548	537	505	490	506	512
Draft betw. damper and boiler,	in. H_2O	1.30	1.16	0.90	0.38	......	0.09	0.97	0.67	0.48	1.13	0.94	0.80	0.79	0.255	0.194
Draft in main flue,	in. H_2O	2.61	1.60	1.24	1.05	0.32	0.50	1.80	1.42	1.72	1.50	0.95	1.37	1.42	0.370	0.470
Draft in furnace,	in. H_2O	0.00	—0.02	—0.01	0.00	+0.03	0.00	+0.01	—0.01	0.00	0.00	—0.026	—0.025	—0.135	+0.009	+0.020
External air temperature,	deg. fahr.	70	68	69	62	67	65	58	63	55	58	64	60	60	40	39
Entering air temperature	deg. fahr.	88	86	82	75	79	91	82	85	78	83	82	82	80	40	39
Superheat,	deg. fahr.	264.4	256.2	225.5	224.6	189.6	159.5	229.2	228.1	207.1	232.2	212.8	187.2	184.2	255	256
Total coal as fired,	lb.	148,021	129,227	108,000	107,736	74,022	62,895	137,524	111,341	72,404	154,489	144,796	107,409	82,738	69,572	63,046
Moisture as fired,	per cent	8.51	11.85	9.40	11.24	9.69	10.15	10.26	10.97	10.68	9.55	10.20	9.83	10.44	9.04	9.62
Total dry coal,	lb.	135,400	113,900	97,900	95,600	66,900	56,500	123,400	99,100	64,700	139,800	130,000	96,800	74,200	63,286	56,990
Ash from ashpit,	lb.	30,750	25,900	20,100	21,470	13,420	8,530	30,900	20,900	11,800	39,500	30,600	21,800	15,700	12,160	10,240
Cinders caught,	lb.	2,096	1,802	1,072	445	223	191	......	......	......	....	....	......	......	......	......
Total refuse,	lb.	32,846	27,702	21,172	21,915	13,643	8,721	30,900	20,900	11,800	39,500	30,600	21,800	15,700	12,160	10,240
Total combustible burned,	lb.	102,554	86,198	76,728	73,685	53,257	47,779	92,500	78,200	52,900	100,300	99,400	75,000	58,500	57,412	52,806
Total refuse based on dry coal,	per cent	24.2	24.3	21.6	22.9	20.4	15.4	25.0	21.1	18.2	28.2	23.5	22.5	21.2	19.2	18.0
Total water fed,	lb.	889,610	798,300	722,115	703,605	525,470	461,555	772,930	685,315	481,305	903,394	844,892	722,237	555,660	480,084	436,984
Factor of evaporation		1.253	1.277	1.251	1.249	1.252	1.218	1.267	1.261	1.246	1.225	1.205	1.194	1.185	1.242	1.243
Total equiv. water fed,	lb.	1,115,000	1,020,000	903,500	879,000	658,000	560,000	979,500	864,000	599,500	1,107,000	1,018,000	862,000	658,000	596,000	544,000
Dry coal per hour,	lb.	16,940	14,250	12,230	10,610	8,355	6,280	15,400	12,400	8,090	17,470	16,250	12,105	9,260	7,911	7,124
Dry coal per sq. ft. grate per hr.,	lb.	46.5	39.1	33.6	29.1	23.0	17.8	42.3	34.1	22.2	48.0	44.7	33.3	25.4	21.7	19.6
Water evap. per hr. corr. for quality,	lb.	111,201	997,888	90,264	78,178	65,684	51,284	96,616	85,664	60,163	112,924	105,612	90,279	69,458	60,011	54,623
Equiv. evap. per hour,	lb.	139,400	127,500	112,850	97,660	82,280	62,220	122,500	108,100	74,950	137,200	127,250	107,800	82,300	74,534	67,896
Equiv. evap. per hr. per sq. ft. h. s.,	lb.	10.37	9.48	8.39	7.26	6.12	4.63	9.11	8.04	5.57	10.2	9.47	8.02	6.12	5.55	5.05

Equiv. evaporation per hour,	lb.	139,400	127,500	112,850	97,660	82,280	62,220	114,500	108,100	749,500	137,200	127,250	107,800	82,300	74,534	67,896
Boiler hp. developed,	hp.	4,042	3,695	3,271	2,828	2,385	1,804	3,420	3,135	2,240	3,975	3,690	3,125	2,386	2,160	1,968
Rated boiler hp.	hp.	1,345	1,345	1,345	1,345	1,345	1,345	1,345	1,345	1,345	1,345	1,345	1,345	1,345	1,345	1,345
Percentage of rating developed,		300.5	275.0	243.1	210.0	177.4	134.0	254.0	233.0	161.0	295.0	274.0	232.0	177.0	161	147
Water fed per lb. coal as fired,	lb.	6.01	6.18	6.69	6.53	7.10	7.34	5.25	6.15	6.65	5.85	5.84	6.72	6.72	6.91	6.93
Water evap. per lb. dry coal,	lb.	6.57	7.01	7.37	7.35	7.86	8.17	5.85	6.91	7.43	6.47	6.49	7.46	7.50	7.58	7.68
Equiv. evap. per lb. coal [illegible] fired,	lb.	7.54	7.89	8.36	8.15	8.90	8.90	6.65	7.75	8.28	7.17	7.03	8.02	7.95	8.57	8.63
Equiv. evap. per lb. dry coal,	lb.	8.23	8.94	9.22	9.20	9.85	9.90	7.36	8.72	9.28	7.85	7.83	8.91	8.89	9.42	9.55
Equiv. evap. per lb. combustible,	lb.	10.87	10.62	11.77	11.92	12.35	11.72	9.90	11.04	11.33	11.02	10.2	11.5	11.3	10.38	10.28
Heat value per lb. dry coal (calc.)	B.t.u.	12,370	12,110	12,400	12,155	12,470	12,630	11,932	12,175	12,310	12,260	11,850	11,675	11,265	12,084	12,095
Efficiency—boiler—grate—furnace	per cent	64.6	71.6	72.1	73.5	76.7	76.0	64.5	69.5	73.2	62.2	64.2	74.0	76.6	75.6	76.6
Average fire thickness,	in.	4.4	4.6	5.5	4.8	4.6	3.9	4.5	4.5	4.4	5.0	4.5	4.5	4.5	4.5	4.25
Stoker speed,	ft. per hr.	44.3	36.7	31.7	29.8	21.2	18.9	40.5	33.0	21.6	44.8	41.0	30.3	24.5	20.7	19.7
Flue gas																
Carbon dioxide,	per cent	13.56	12.77	13.08	13.56	13.00	13.14	12.48	12.80	11.58	11.30	13.60	12.50	11.4	13.1	13.5
Oxygen,	per cent	6.10	5.81	6.80	5.73	6.58	6.34	6.34	6.08	7.62	7.67	5.27	7.05	8.0	6.3	6.5
Carbon monoxide,	per cent	0.00	0.02	0.00	0.23	0.00	0.06	0.23	0.45	0.07	0.69	1.23	0.00	0.0	0.16	0.0
Nitrogen,	per cent	80.34	81.40	80.12	80.48	80.42	80.46	80.95	80.67	80.73	80.34	79.90	80.45	80.6	80.44	80.0
Proximate analysis																
Moisture,	per cent	8.51	11.85	9.40	11.24	9.69	10.15	10.26	10.97	10.68	9.55	10.20	9.83	10.44	9.04	9.62
Volatile matter,	per cent	8.07	6.22	6.58	5.67	5.98	5.62	6.20	6.18	6.17	8.16	7.44	6.93	5.91	7.36	7.30
Fixed carbon,	per cent	65.77	66.26	67.55	66.44	68.48	71.95	67.02	67.62	69.97	66.63	65.91	67.20	66.68	67.04	67.64
Ash,	per cent	17.65	15.67	16.47	16.65	15.85	12.28	16.52	15.23	13.18	15.66	16.45	16.04	16.97	16.56	15.44
Sulphur (dry basis),	per cent	0.73	1.24	0.54	1.05	0.34	0.75	0.69	0.63	0.72	0.56	0.69	0.59	0.74	0.51	0.61
Ultimate analysis (calculated)																
Carbon,	per cent	73.09	75.52	75.32	74.69	76.55	79.57	74.93	76.11	78.42	75.97	75.05	75.58	74.59	74.86	75.92
Hydrogen,	per cent	2.99	2.55	2.73	2.36	2.61	2.44	2.54	2.52	2.47	2.76	2.97	2.76	2.60	2.74	2.79
Oxygen,	per cent	3.27	2.08	2.73	2.35	2.49	2.73	2.60	2.79	2.78	2.88	2.07	2.38	2.65	2.79	2.69
Nitrogen,	per cent	0.62	0.84	0.51	0.80	0.46	0.84	0.84	0.84	0.85	0.51	0.92	0.89	0.46	0.88	0.89
Sulphur	per cent	0.73	1.24	0.54	1.05	0.34	0.75	0.69	0.63	0.72	0.56	0.69	0.59	0.74	0.51	0.61
Ash,	per cent	19.30	17.77	18.17	18.75	17.55	13.67	18.40	17.11	14.76	17.32	18.30	17.80	18.96	18.22	17.10
Ash analysis																
Volatile matter + carbon,	per cent	15.00	21.85	11.45	16.50	12.50	9.55	26.40	18.85	19.15	32.30	22.25	21.00	10.53	5.10	4.82
Earthy matter,	per cent	85.00	78.15	88.55	83.50	87.50	90.45	73.60	81.15	80.85	67.70	77.75	79.00	89.47	94.90	95.18
Heat Balance—Dry Basis																
Heat absorbed by boiler,	per cent	64.60	71.60	72.10	73.50	76.70	76.00	64.5	69.50	73.20	62.20	64.20	74.00	76.60	75.6	76.6
Loss due to moisture in coal,	per cent	0.88	1.30	0.96	1.20	0.97	0.70	1.10	1.10	1.10	1.10	1.20	1.20	1.30	1.1	1.1
Loss due hydrogen burned,	per cent	2.77	2.50	2.50	2.20	2.34	2.10	2.40	2.40	2.30	2.60	2.80	2.60	2.60	2.6	2.7
Loss due to flue gases,	per cent	12.55	14.00	12.74	12.30	11.84	11.30	13.70	13.20	14.50	13.00	11.00	12.50	14.00	8.3	8.4
Loss due to carbon monoxide,	per cent	0.00	0.10	0.00	1.20	0.00	0.30	0.90	1.80	0.30	3.20	5.00	0.00	0.00	0.8	0.0
Loss due to combustible in ash,	per cent	4.01	6.00	2.77	4.40	2.96	1.70	*8.10	*4.80	*4.20	*9.80	*6.50	*5.90	*2.90	*1.2	*1.1
Loss due to carbon in cinders,	per cent	1.60	1.70	1.13	0.50	0.35	0.30	......	......	......	......	......	......	......	......	......
Unaccounted for,	per cent	13.59	2.80	7.64	4.70	4.84	7.00	9.00	7.20	4.40	8.10	9.30	3.80	2.60	10.4	10.1
Total,	per cent	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

* Includes cinder loss

16 The new furnace proved more than satisfactory from a combustion standpoint, but one operating difficulty made it necessary to redesign it to make it entirely satisfactory. No trouble

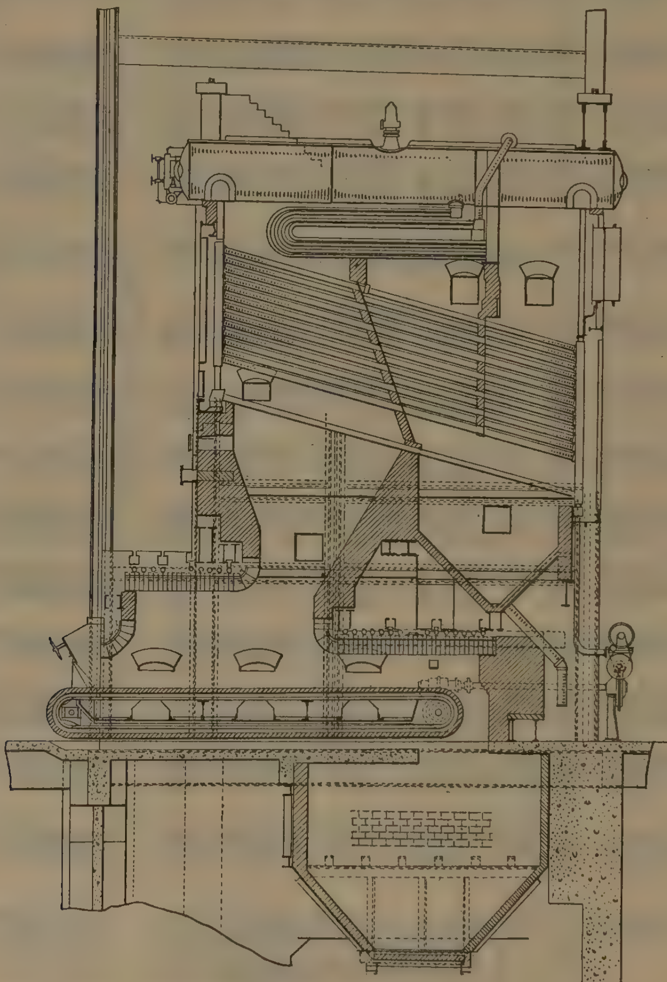

FIG. 5 TWO-ARCH FURNACE WITH RESTRICTED GAS PASSAGE (FINAL SETTING OF BOILER NO. 1)

whatever was experienced with the arches or the setting. The one trouble was the accumulation of a mixture of carbon and slag on the rear arch. The slag was deposited in a plastic state, and in about three weeks continuous operation so restricted the throat that the boiler had to be taken out for cleaning. This experience

condemned the design as a practical furnace, but pointed the way to what appears to be the solution of the problem.

An Improved Design

17 It was of course impossible to rebuild the furnace immediately because, this being a relay station, there is no spare boiler

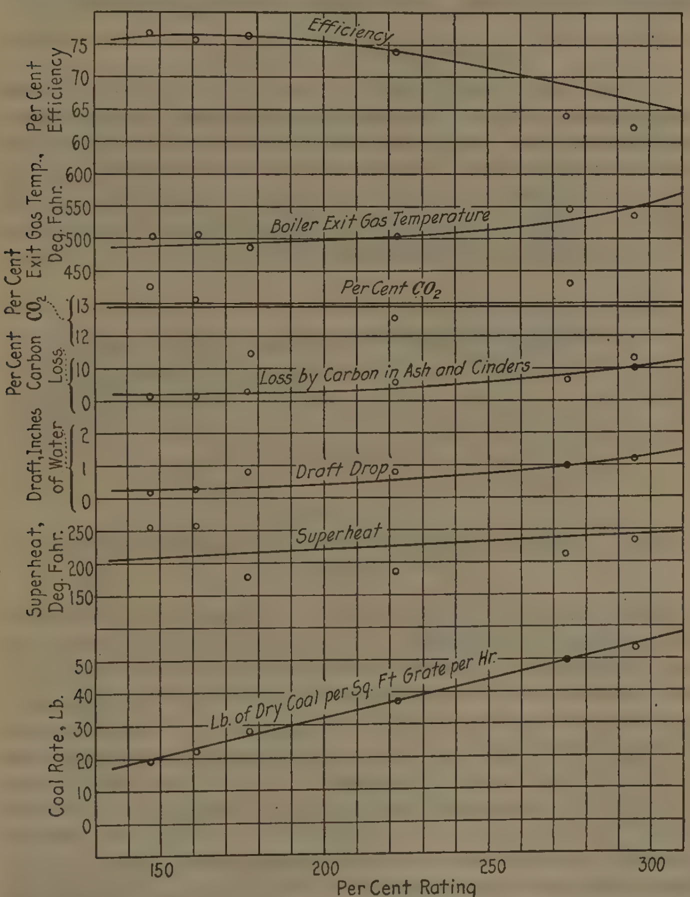

Fig. 6 Test Results with Two-Arch Furnace

equipment. Careful observations were made of the behavior of the furnace during this periodic building up of the slag, and it was found that there was little increase in draft drop with the reduction in gas passage. This deposit was allowed to accumulate until the opening between the upper arch and the slag was reduced to about a foot. Under this condition, the boiler was operated at

240 per cent of rating (measured by flow meter) with an increased draft drop of only 0.2 in. of water.

18 This experience pointed to the possibility of obtaining complete mixing of the gases by using a restricted throat, and boiler No. 2 was reset as shown in Fig. 5. This work was completed in March, 1924, since which time the boiler has been operated intermittently, but without operating difficulties.

Improvement in Efficiency

19 Six tests of this boiler are included, the principal results being plotted in Fig. 6. The combined efficiencies at ratings below 250 per cent check very closely with the tests of the three-arch furnace. The two tests at higher rates of driving show a marked falling off in efficiency. These two tests were made when it was difficult to regulate the station load. There were but two boilers in service, one under test and the other at light load taking the irregu-

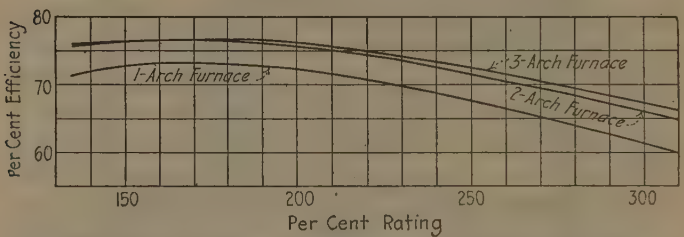

Fig. 7 Curves Comparing Efficiencies Obtained with Three Types of Furnaces

larities of the load. It was difficult to regulate combustion conditions at rates of driving below 170 per cent because of the very low rate of combustion. The two tests below this rate show high efficiency, but with a rather large unaccounted-for loss. They also show high superheat and high exit-gas temperatures. Only traces of CO were found, but probably some secondary combustion was taking place, since the low velocity of the gases undoubtedly made possible some stratification. Since these boilers are taken off the line at rates of driving below 170 per cent, and operated above 250 per cent of rating only during emergency conditions, this furnace satisfies the requirements of the station, but modifications in design will have to be made for boilers which will be operated at rates above or below this range.

20 The combined efficiency curves of the three furnaces are shown superimposed in Fig. 7 and illustrate very clearly the improvement obtained. The point of maximum efficiency for all furnaces is at about 170 per cent of rating, but the curves of the multiple-arch furnaces are flatter. The increase is slightly over three points at this rating, but rises to five points at 300 per cent

of rating. The boilers, apparatus and tests are described in the Appendix; the principal data and results are given in Table 1.

21 The reasons for the improvement in efficiency are apparent from a study of the numerous analyses of the flue gases taken on the three-arch furnace, which are shown plotted in Figs. 8, 9, and 10.

22 The influence on combustion of the three arches in the furnace is well exhibited in Fig. 8. Samples were taken through a

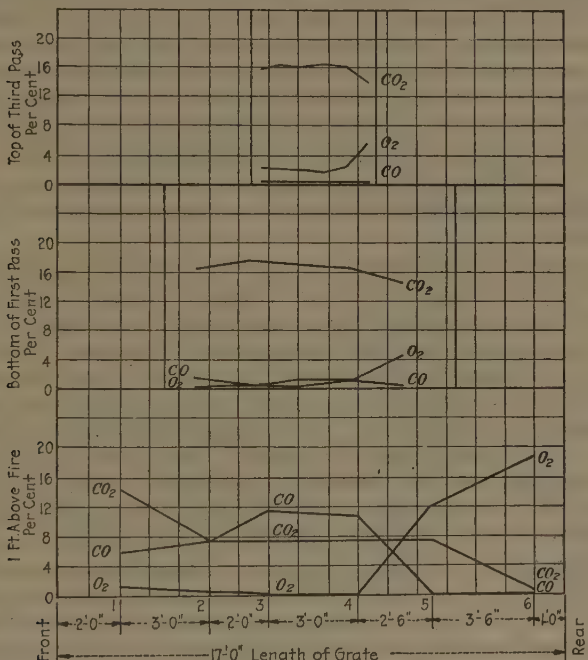

FIG. 8 VARIATIONS OF FLUE-GAS COMPOSITION IN THREE-ARCH FURNACE

(Special run, Aug. 21, 1923, 275 per cent of rating)

water-cooled sampling tube about 12 in. over the fire and about 30 in. from the side wall. The lower set of curves on this chart shows the 17-ft. length of grate and the various points at which gas samples were drawn. In general, these gas samples represent the products of combustion over the several air compartments of the stoker. The boiler was run at about 275 per cent of rating, with all conditions substantially the same as during the test of August 17.

23 Above 200 per cent of rating some air is used in the first five stoker compartments, but none is admitted to the sixth

compartment. Due to burning, the fuel bed decreases in thickness from the front of the stoker in a practically uniform slope down toward the rear. Air was admitted, not to secure any particular arrangements of draft readings by gage, but to burn the various sections of the fire in what seemed to be the most sensible manner.

24 As soon as the six analyses were made of the gas over the fire, the second set of five analyses was made in the bottom part of the first pass. The water-cooled tube was also used for this purpose. The location was not in the top of the combustion chamber just under the tubes but between the second and third row of tubes from the bottom, since that was the only place in which the sampling pipe could be introduced. As soon as these analyses were finished a set of six samples was drawn from the top of the third pass by means of one of the regular gas-sampling tubes.

25 These three sets of readings are in practically the same vertical plane through the setting and at practically identical firing conditions.

26 It will be noted that over the first four air compartments the fire is to some extent a gas producer. Contrary to the usual notion, there is only a moderate amount of CO produced over the first compartment, the figure being about 6 per cent. The CO_2 at this same point is a little over 14 per cent. It may be noted that the CO increases up to nearly 12 per cent over the third compartment, whereas the CO_2 decreases to about 7 per cent until the fifth compartment is reached. Here, under practically any condition of firing, excess air is admitted. This of course comes from the operation of burning out the fuel bed to form a reasonably clear ash. There is much less carbon burned over this compartment in proportion to the air blown through the fire. The CO_2 does not increase, but the free oxygen increases to about 12 per cent and the CO disappears entirely. Over the sixth compartment neither CO nor CO_2 is present in any appreciable quantity, the analyses showing practically pure air. The quantity represented by this last analysis is probably rather small since there is no direct air pressure in the sixth compartment.

27 The foregoing gas analyses are from a point about 12 in. above the fuel bed. Inspection of the curves in Fig. 8 for the analyses at the bottom of the first pass shows that combustion is practically complete. It is seen that the CO has been nearly all burned out and that there is very little free oxygen present except in the section of the pass nearest the baffle. The presence of this oxygen at this point is interesting as showing that stratification has apparently followed through the tortuous path of gas travel which the arches compel. The CO_2 at nearly all points in the first pass is above 16 per cent, dropping down to about 14 per cent near the baffle. The indications from these figures are that a

practically minimum amount of excess air has been used on this longitudinal section of the fire.

28 The curve of gas analyses at the top of the third pass shows substantially the same CO_2 but a uniformly low amount of CO. The free oxygen, amounting to around 2 per cent, is considerably higher at this point than it was at the bottom of the first pass. The inference is that some air has leaked into the setting between the two points.

29 The curves of Fig. 9 show a comparison of gas analyses over the fire at three different rates of driving. The difference be-

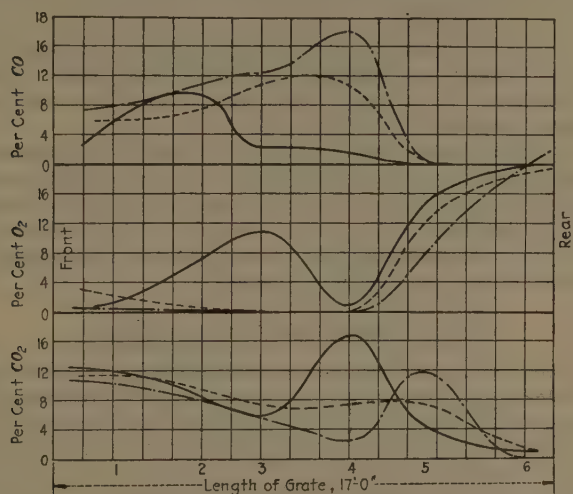

Fig. 9 Variations of Flue-Gas Composition Over Fire in Three-Arch Furnace

Full lines — 274 per cent of rating, Aug. 17, 1923
Dotted lines — 275 per cent of rating, Aug. 21, 1923
Dot-and-dash lines — 134 per cent of rating, Aug. 19, 1923

tween the full lines and the dotted lines, both of which are for about 275 per cent of rating, results from a difference in handling the fire on two different days. The dotted lines are the same curves that were studied in Fig. 8. In handling the fire on August 17 somewhat higher air pressures were carried on the second and third air compartments. This lowered the CO_2 over these compartments and increased the percentage of oxygen, but it greatly reduced the amount of CO over the third compartment. At the fourth compartment the air pressure was carried a little lower than in the other run and the excess air dropped down nearly to zero, while the CO_2 rose abnormally to over 16 per cent. Over No. 5 compartment there is a characteristic disappearance of the CO and a great increase in the free oxygen. Over the sixth com-

partment the results on all runs show substantially the same character of gas.

30 The low-rating run shown by the dot-and-dash line was made with much less air pressure under all the compartments. The penetration of the air through the fire was considerably less than in the high-rating runs, and the curves show exactly what might be expected. The CO rises from about 7½ per cent over the first compartment to about 17 per cent over the fourth compartment, after which it drops off to zero. There is practically no free oxygen anywhere in the fire except over the fifth and sixth compartments. The CO_2 runs a little less in the forward part of the fire, increases to 12 per cent over the fifth compartment, and then drops off to zero over the sixth compartment. These curves show to what extent an anthracite fire can be made to operate as a gas producer and still retain its properties of continuous operation.

31 All these results are valuable in showing to what extent gas mixing has occurred somewhere in the furnace. Without such a study we would know only that the three-arch furnace produced a result, without having any clear idea as to how the result was secured. The analysis is an important guide in undertaking the redesign of the furnace as shown in Fig. 5.

"Cross-Sectioning" Exit Gases

32 The arrangement of gas-sampling tubes and thermocouples for ascertaining the character of the exit gases in the top of the third pass of No. 1 boiler are fully described in the Appendix. It proved feasible to cross-section this area (24 ft. 4 in. by 4 ft. 4 in.) thoroughly with the thermocouples because the readings could be taken with the potentiometer almost instantaneously. To duplicate this performance for gas analyses with a hand-operated Orsat, involving 35 complete analyses for one cross-sectioning, was out of the question. It was, however, perfectly possible to get a fair average of seven simultaneous gas samples across the setting. There proved to be time during one of the tests to analyze the seven samples independently, and the curve of these analyses is shown in the lower part of Fig. 10.

33 It should be remembered that there are two stokers, with a three-foot center wall extending across the setting. That there is a serious amount of longitudinal stratification of gases is evidenced by the shape of the curve. Not only at 134 per cent of rating, but at various other rates, the low CO_2 at the two end pipes was frequently remarked. Had single gas-sampling tubes been used, extending in from either side to about the center of each stoker, the CO_2 recorded for the entire series of No. 1 boiler would have been in the neighborhood of 16 per cent. The evidence of this cross-analysis is that such a reading would be seriously misleading.

The curves on the upper part of Fig. 10 are the individual temperature readings across the setting at various ratings. These show the same characteristic shape as the CO_2 curve. The variation between high and low is much more marked at high than at low rates of driving. If flue temperatures are taken by thermometer or thermocouple at one point only in the setting width, there will

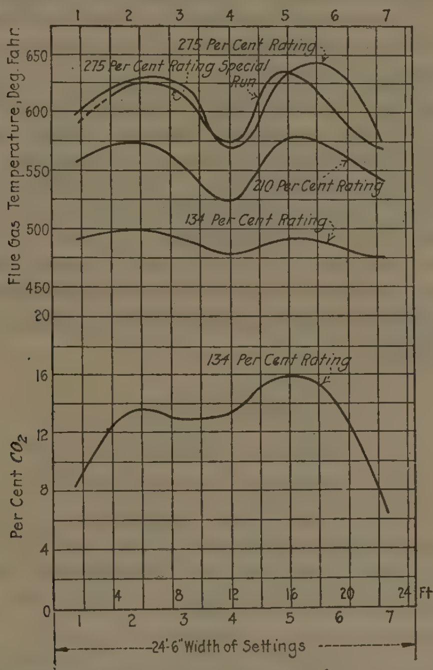

Fig. 10 Temperature and CO_2 Across Top of Third Pass of Three-Arch Furnace

evidently be the same kind of error as by taking the CO_2 at one point.

Conclusion

34 These tests are presented not as representing the ultimate that can be expected of this type of stoker and furnace, but rather as pointing a way to further development. That stratification can be practically eliminated and carbon loss to the ashpit reduced to a minimum, is shown clearly by the tests. Another accomplishment is the elimination of ignition troubles, even with low-grade coal. A decided improvement has been made in the burning of undersize at moderate rates of driving, but there is room for improvement at rates above 250 per cent.

35 All the tests were made with substantially the same quality of coal, for otherwise no just comparison of the furnaces could have been made. There has not been time to conduct a series of tests with coal containing larger percentages of undersize, but operation has been studied with varying qualities running down to river coal with over 60 per cent undersize and 33 per cent ash.

36 The maximum capacity obtainable with the latter coal and the single-arch furnace was about 190 per cent of rating, but with the two-arch furnace and the same coal 300 per cent of rating can be carried without difficulty.

37 The loss in efficiency with increasing undersize is pretty well known with the single-arch furnace, but it has not been measured for the multiple-arch design at Amsterdam. It has been estimated from the variations in coal per kilowatt-hour, and there is no doubt that the loss is less with the new type of furnace, particularly at the usual operating rates of about 225 per cent of capacity. The problem of burning the so-called undersize has already become an important one both to the mine operator and the consumer, and the development of these furnaces may help in the ultimate solution.

APPENDIX NO. 1

REPORT OF TESTS

Description of Boilers

38 No. 1 unit (Fig. 2) comprises a 1345-hp. Babcock & Wilcox boiler with six longitudinal drums and with superheater located above the tube bank, which is 14 tubes high and 42 tubes wide, and set with a three-arch furnace. This unit has two Coxe stokers each 10 ft. 8⅝ in. wide and 17 ft. long, making a total area of 364.5 sq. ft.

39 This furnace had been in operation for about five months. In this unit there is a soot chamber at the rear of the boiler underneath the second and third passes. Consequently it was possible to collect cinders from this back-connection chamber and measure them. Of course, some cinders must also have passed on to the chimney. There is no means of knowing what this quantity was, and the loss entailed by it is therefore in the "unaccounted for" in the heat balance.

40 Unit No. 2 was the regulation front arch usually furnished with Coxe stokers, as shown in Fig. 1, and later changed as shown in Fig. 5. This boiler is an exact duplicate of No. 1 and the stokers are also duplicates, the only difference being in the furnace arrangement. There was no soot chamber in the first setting, and the test results show no measurement whatever of cinders that have passed out of the furnace and into the boiler setting. Since the raising of cinders by the method of handling the fires was probably greater with this unit, there is some larger part of the unaccounted-for losses to be considered as due to unburned carbon in the cinders. The amount of this, however, is unknown.

Description of Apparatus

41 Feedwater for the boilers under test was taken from the regular feedwater system and passed through a specially built measuring

tank having a level overflow edge 6 ft. long. The quantity of water held by this tank at overflow was carefully measured and a calibration figure arrived at. During tests the temperature of each tank of water was taken and the weight of water per tank corrected for temperature. This method could scarcely be more than half of 1 per cent in error and provided a quick and positive means of measuring the water.

42 The measuring tank discharged into a large sump tank, from which a suction line was run to No. 1 feed pump. During tests this pump discharged through the auxiliary feed main to the boiler under test. All branches on the suction and discharge sides of the pump, as well as all branches on the auxiliary feed main, were double-valved with bleeders between the valves so as to insure no other water entering or leaving the test piping while the tests were under way. The temperature of the feedwater was taken in each case in the feed lines near the boiler.

43 For each of the boiler tests a good average grade of the coal on hand was provided, so that the results of tests should be on substantially the same fuel basis. The coal analyses show how nearly this condition was secured. Coal was fed from the bunker into hopper-bottom boxes on scales and weighed before being distributed to the stoker hoppers. Samples of coal were taken every half-hour from the weighing boxes.

44 The gas analyses of the setting, 24 ft. 4 in. wide, presented a considerable problem. In order to get these with reasonable accuracy, seven 1½-in. holes were made in the rear casing of the boiler so that seven individual gas-sampling pipes could be pushed into the space just above the bank of tubes in the third pass. These pipes were attached by rubber tubing to seven water bottles, which in turn connected to a common header leading to a small steam siphon. A pinch cock was provided on the tubing to each bottle. This arrangement permitted the drawing of gas samples from any one of the seven pipes individually or from any combination of these pipes simultaneously. The pinch cocks enabled the operator to regulate the quantity of gas flowing through each pipe, and the bubbling of the gas through the water in the bottles was a practical gage of the quantity flowing from each pipe. The gas sample reported in each test on No. 1 boiler was a combined sample from the seven pipes. On boiler No. 2 it was impossible to use this arrangement, so gas samples were taken only from a point in line with the center of each stoker. These, of course, were taken at the top of the third pass. The results of these analyses were averaged down to an equality with the average sample from No. 1 by a study of the curve of CO_2 across this boiler.

45 Exit flue-gas temperatures from No. 1 boiler were taken by means of seven thermocouples which were lashed to the seven gas-sampling tubes. These thermocouples were made with iron and constantan wire and the welded junction was located with a half-inch of the end of the gas-sampling pipe so that in all cases the temperature and gas analysis were of the same body of gas. The current from the thermocouple was measured with a Leeds and Northrup potentiometer. Other thermocouples were used to take temperatures throughout the setting. This method was successful in reading temperatures up to 1300 deg. fahr. without serious destruction of the thermocouples. After careful calibration with a mercury thermometer, it was found that the thermocouples were entirely satisfactory and accurate for reading the steam temperatures — which were around 600 deg. fahr. — and also for reading the feedwater temperatures. The wiring having been erected before the tests, this use of the potentiometer and the thermocouples proved a great time saver and convenience.

46 The various draft readings were taken with a variety of instruments. Inclined differential draft gages were used for the readings

of smaller quantities, such as the draft over the fire and the draft throughout the setting, so as to get these quantities accurately to a hundredth of an inch. Other draft and air-pressure readings were taken with the Bailey multi-pointer gages installed on the boiler-unit gage boards.

Method of Conducting Tests

47 For each test the boiler unit was operated for a number of hours in advance at the desired rate, and readings were taken during eight of nine hours of continuous operation under practically constant operating conditions. Since conditions were so steady, instrument readings were taken every 20 minutes. These show very little variation throughout any one boiler test and the averages are therefore of dependable accuracy. At each hourly interval the coal weights and water measurements were checked off, so that hourly evaporations and other results could be computed during the test.

48 The half-hourly coal samples were put into canvas bags and immediately weighed. These bags were put on top of the boiler over night and thus air-dried at about 150 deg. The next morning these dried samples were carefully weighed and the air-dried moisture was calculated. The entire amount of this air-dried coal was then split down into final quart-jar samples. One sample was sent to an outside laboratory for analysis and the other was analyzed by the plant test department. The results of these two sets of analyses checked in a very gratifying manner.

49 Ashes were not collected and weighed because of interference with operating conditions in the plant. Every effort, however, was made to secure representative samples of the ashes formed during the test runs. These ash samples amounted to about a barrelful per eight hours. This quantity was dumped on to the floor, pounded up to a small size, and put through a splitter so that the sampling process was as accurate as possible. The final ash samples were sent to an outside laboratory and were also analyzed by the plant test department.

DISCUSSION

A. R. Mumford.[1] The experiences of the New York Steam Corporation in burning the so-called steam sizes of anthracite is in exact agreement with the findings of the authors. The writer herewith offers evidence, independently collected and interpreted, which completely substantiates their conclusions. The authors point out that, with the former standard design of furnace, the operator had the alternative of reducing the stack loss by reducing the excess air, thereby increasing the carbon loss to the ashpit, or of reducing the carbon loss and increasing the stack loss. This condition is shown by Fig. 11, in which are plotted two sets of gas analyses which show the effect on the furnace gases of increased air pressure in the last compartment.

In this and in Fig. 12 the full lines represent the percentage of CO_2, the long dash line represents the percentage of O_2, and the short dash line represents the percentage of CO. The samples were collected simultaneously from all points shown, by means of water-

[1] Fuel Engr., N. Y. Steam Corpn., New York, N. Y. Assoc.-Mem. A.S.M.E.

cooled sampling tubes, whose open end was over the center line of the stoker. The samples were kept in glass bottles under water pressure until they were analyzed in a water Orsat.

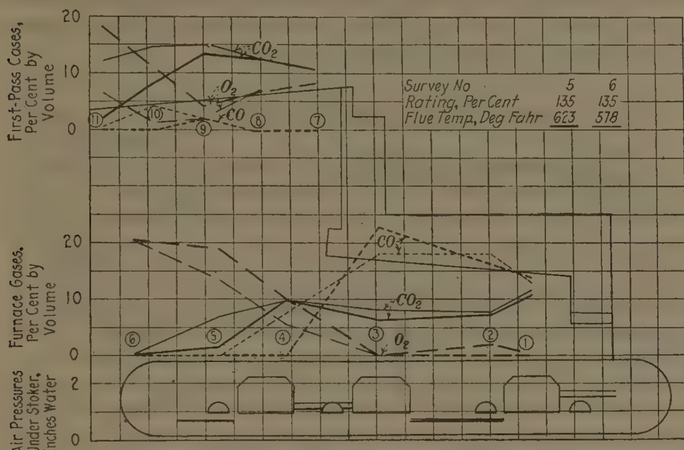

Fig. 11 Effect of Increasing Air Pressure in Last Air Chamber

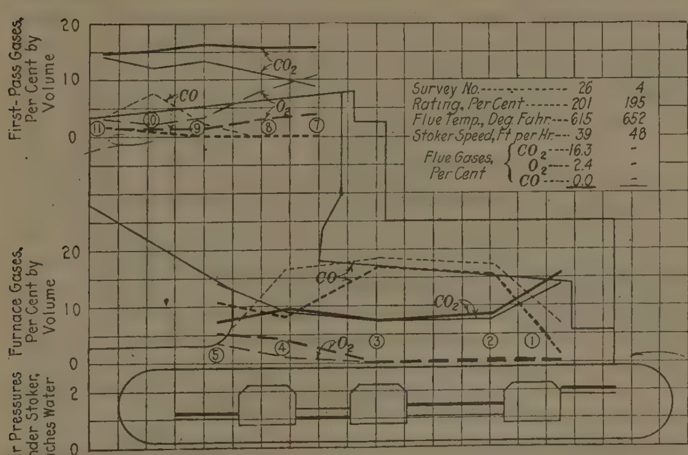

Fig. 12 Actual Effect of Rear Arch

The gas temperatures given on the charts are the average of the readings of five copper-constantan thermocouples read at 5-min. intervals during the 20-min. sampling period. The high temperatures shown are due to leaky baffles, as has since been proved by a considerable temperature reduction with new baffles installed.

As indicated by the rapid fall of the first-pass CO_2 curve near the bridgewall, it is seen that the air passing through the thin rear end of the fuel bed rises directly to the first pass and thence travels through the boiler. The condition under normal operation is shown by the light lines, while the heavy lines indicate the conditions with higher rear-end air pressure.

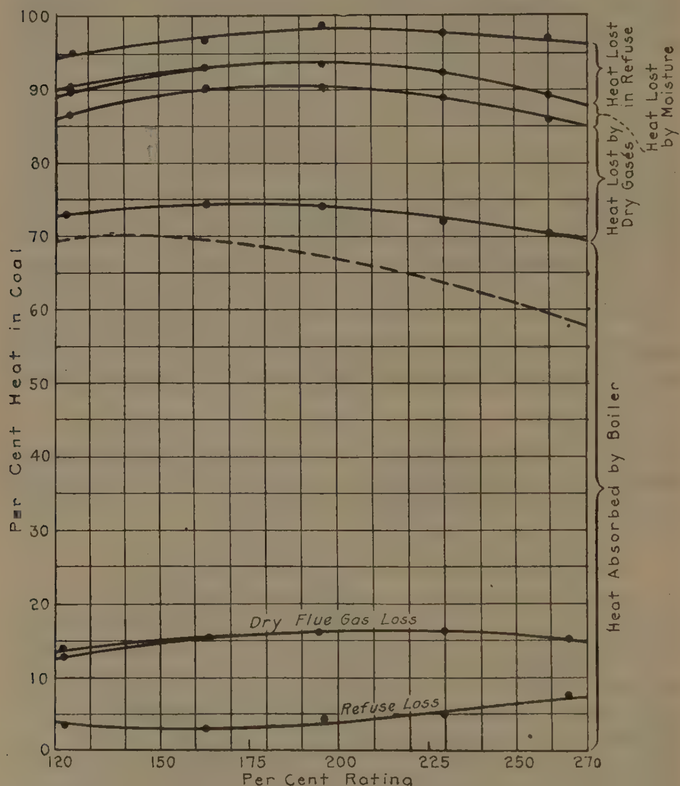

FIG. 13 HEAT ACCOUNT OF TESTS OF BOILER NO. 12 AT STATION A OF THE NEW YORK STEAM CORPORATION

The effect of the rear or reversing arch is shown in Fig. 12. Here the heavy lines represent the gas composition with the reversing arch installed, while the light lines represent the gas composition without the added arch. Three points stand out rather clearly: First, the larger percentage of oxygen rising from the fuel bed at the rear indicates a thinner and consequently a more completely burned fuel bed at this point; second, the almost complete elimination of CO in the gases entering the first pass shows

that the additional air has not increased the excess air but has been mixed thoroughly with the other gases in the furnace.

The opportunity which presented itself to install new baffles, in connection with other plant changes, made possible the alteration of the furnaces. At present the line of the top of the reversing arch is extended until it meets the tubes. It is impractical to raise the boilers in this plant to provide more vertical distance between the grates and tubes, although this change would probably be advantageous.

That these improvements in gas composition are actually translated into increased efficiency, the authors have shown in the tables of test results. That it was also true with the New York Steam Corporation is shown in Fig. 13. Here the solid lines represent the variation in the heat balance, with the rate of operating as shown by five complete thermal-efficiency tests in February, 1924. The broken line represents the efficiency as it was estimated to be at the various rates with the single-arch furnace by a competent engineer familiar with the installation. The improvement is gratifying and obvious. The greater improvement with increased rate of driving is also evident, as is the fact that the maximum efficiency was reached at from 170 to 180 per cent of normal rating. The engineers of the New York Steam Corporation have explained this by two facts. The reduction in gas loss by increase of CO_2 and elimination of CO is the first. The second is that the lower total gas quantity, at any given rate of combustion, lowers the gas velocity and consequently the weight of coal lost to the stack as fly cinder. This last effect is the one that we believe accounts for the flattening of the efficiency curve after the reversing arch was installed.

The solid line at the top of Fig. 13, however, representing the total per cent heat accounted for, drops away from the 100 per cent line after the rate of driving has passed about 200 per cent of rating, indicating that the quantity of cinders carried with the gases increases enough to offset improved combustion conditions. This is thought to mean that excess air must be kept at a minimum in any given furnace or that a furnace must be designed for a definite maximum gas velocity below which the quantity of cinders will not cause a serious loss.

The only point of difference noted between the authors' statements and the experience of the New York Steam Corporation is that there are 12 boilers in the latter's Station A which have been in service for one year, equipped with reversing arches, and in that time no trouble has developed with any of the arches and, under what we consider hard service, no boiler has ever had to be shut down on account of cinders on the reversing arch which is comparatively flat on top. It has been noted, however, that on reducing the rate of driving of a boiler, some cinders will flow

down from the top of the reversing arch and burn on the grates, thus again increasing the rate of driving.

HOWARD H. DALTON.[1] The authors show the necessity of a long rear arch for burning small-sized anthracite on chain-grate stokers and clearly establish the superiority of the two- and three-arch furnaces over the one-arch furnace. The information given does not so clearly demonstrate the superiority of the final, or two-arch over the three-arch design, as there was no trouble with the arches, and the efficiency of the latter was slightly higher. The only objection seems to be the formation of slag on the rear arch. The two-arch furnace has the disadvantage of relying entirely on a restricted passage for the mixture of the rich and lean gases, and naturally at low rates this mixture will not be as complete as at high rates of driving. If the passage is restricted sufficiently to get the best results at low rates, high rates will be impossible. For this reason the two-arch furnace, while it may meet the requirements of the Amsterdam plant, does not seem as good a design as the three-arch furnace.

Furthermore it does not seem that the removal of the top arch and the slight change in design of the rear arch would in themselves prevent slagging. It is possible that slight changes in operation might be responsible. The Baltimore furnaces mentioned by the authors have three arches almost identical in design with the boilers under discussion. These have shown absolutely no slagging up to rates of driving of 270 per cent, which is the maximum possible with the draft available. The Baltimore plant burns a much finer coal than Amsterdam, about 70 per cent passing through a 3/32-in. round hole and about 35 per cent through a 1/16-in. hole. This, if anything, should favor slagging at Baltimore.

The main operating difference (and it may be an important one) is that at Amsterdam the greatest pressure is carried in the wind boxes under the front end of the stoker, gradually reducing toward the rear end, while at Baltimore this condition is reversed. The Baltimore method of operation gives more excess air and lower temperatures under the rear arch and higher CO toward the front of the furnace, with the two gases mixing under the upper, or third, arch. That these gases do thoroughly mix, was proved by CO_2 readings taken at five points across the boiler at the last pass. The average CO_2 at the damper was over 14 per cent at rates of driving above 100 per cent, and over 15 per cent at rates above 140 per cent, with only a slight trace of CO. The efficiencies obtained were about the same as those at Amsterdam up to 160 per cent of rating, but somewhat lower on higher rates,

[1] Engr., Construction Division, American Sugar Refining Co., New York, N. Y. Mem. A.S.M.E.

as was to be expected when burning the finer coal. The difference can be accounted for entirely in the cinders carried into the back connections by the high air pressure.

The main arch over the front end of the stoker is absolutely essential, and if carefully designed to radiate and reflect a maximum of heat at the front end of the stoker, no trouble will be experienced with the ignition.

E. B. Powell.[1] It may be of interest to present the reasons which led to the design of the Baltimore furnace mentioned by the authors. We were at the time chiefly interested in determining the flexibility of a traveling-grate type of stoker in handling anthracite of very small size. The load conditions which we had to meet in the design of that plant involved rather sudden and large changes in boiler demand. Through the courtesy of Mr. H. M. Warren of the Glen Alden Coal Company we were enabled to make some tests at one of the anthracite collieries on an installation designed by Mr. Chris. Schillinger of the Coxe Stoker Sales Company which presented the basic principles of the two-arch furnace. While this installation had not much of a front arch, it still had all the fundamentals of a two-arch design and to Mr. Schillinger belongs the credit for applying that design to the burning of anthracite. In this furnace we were able to burn a very small size of anthracite, known as No. 4 buckwheat, at grate-travel rates changing almost instantly from about 20 ft. per hr. to something like 50 ft. or more per hr., without affecting the ignition. That experience led us to believe that there were great possibilities in the two-arch design, both from the standpoint of flexibility and from that of economy. The three-arch design, which was finally decided upon, was a development arrived at solely from considerations of economy. For our particular requirements the third arch seemed necessary to insure thorough mixing of the gases and proper utilization of the boiler heating surface.

Since the tests referred to by Mr. Dalton, we have had another occasion to check the effectiveness of the third arch. We have found, for example, that where the gases leaving the passage between the lower two arches would have a composition ranging from as high as two to six per cent CO and less than one-half of one per cent oxygen in the stream next to the front arch, to no CO and 12 to 14 per cent oxygen in the stream next to the rear arch, we were able to obtain beyond the third arch a practically uniform gas mixture, 14 to 16 per cent CO_2 with a total absence of CO, and practically complete cessation of flame.

The writer considers the work of the authors of the nature of a classic. It is a most valuable contribution to boiler-plant engi-

[1] Cons. Engr., Stone and Webster, Inc., Boston, Mass. Mem. A.S.M.E.

neering: it presents, in quantitative terms, the value of furnace design.

N. G. Reinicker.[1] The writer heartily agrees with the methods outlined by the authors for improving the overall economy of equipment using small sizes of anthracite. At the Hauto plant of the Pennsylvania Power & Light Company, we have been experimenting along similar lines to those described by the authors, over a period of several months on boilers of various sizes and furnace arrangement, the largest being a 12,000-sq.-ft. Stirling-type boiler, and have obtained results quite similar to those shown by the authors.

H. S. Colby.[2] The experiences of the authors parallel the writer's experiences to such a marked degree that a brief outline of his work may be of interest. In entering upon this investigation all precedent was set aside and it was decided to pioneer, and if possible to develop a new viewpoint, unhampered by prejudice or precedent. Prior to entering upon this investigation our experience had been limited to the burning of high-volatile bituminous coal and coke breeze.

In burning coke breeze in the conventional single or front-arch furnace, practically the same difficulties had been encountered as were encountered by the authors in the burning of anthracite. The nature of these difficulties suggested the use of an arch at the rear of the furnace. This arch was not installed on a coke-breeze-burning stoker installation, however, until after our initial contact with the burning of anthracite coal.

In the fall of 1921 and spring of 1922 several Harrington stokers were installed to burn fine-size anthracite. These initial anthracite installations were made, some with front arches only and others with front and rear arches. The arches were of several lengths and set at various degrees of inclination. The operation of these stokers made it apparent that the results obtained from furnaces with both front and rear arches were much more satisfactory than those with front arches only. The work of the last two years has been confined to establishing the proper proportioning of front and rear arches and throat (or space between the arches) for the various sizes of anthracite, and to determining the effect on arch design of height of setting or length of gas travel, and combustion rates.

The work to date indicates that the rear arch affords the means for obtaining the highest efficiency in the burning of all sizes of anthracite; that the throat area or space between the arches in

[1] Supt. of Operation, Pennsylvania Power & Light Co., Allentown, Pa. Assoc.-Mem. A.S.M.E.

[2] Asst. to Sales Mgr., Riley Stoker Corpn., Worcester, Mass.

with the gases are mixed should be modified with the fuel, rate of driving and setting height. Primarily, the area of the throat is a function of the maximum rate. Little or no improvement in quality of gases is obtained when the velocity of the gases passing through the throat exceeds 3000 ft. per min. The throat area can be increased as the setting height is increased, that is, as the length of gas travel from the throat to the tubes is increased. The rear arch should be set at a slight angle to the horizontal and so located with reference to the front arch that gas, comparatively rich in oxygen generated on the section of grate under the rear arch, will travel across the throat and mix with the gas lean in oxygen passing out from under the front arch.

While the front-and-rear-arch design was developed primarily for the more efficient burning of anthracite and coke breeze, it has also been found that the same general design of furnace with some modifications is more efficient for the burning of bituminous coal than furnaces with a front arch only.

The Authors. There is room for considerable difference of opinion regarding the relative merits of two-arch and three-arch furnaces. In the case of the particular problem at Amsterdam, the two-arch furnace appears to be necessary. The boiler and the stoker were both set, and changing the location of the stoker meant changing the ash hopper and some of the building steel, which would have been very expensive. A better furnace of the three-arch type might have been obtained if the boiler could have been changed or the stoker shifted, but the expense of such changes could not be justified. The problem we had was that of adapting a furnace to a boiler and stoker which were already installed. The two-arch furnace appeared to be the solution for this particular problem. It may not be the solution for another set of conditions. We are not sure that the two-arch furnace is quite so good for a wide variety of loads. It is possible that under very low rates of operation, with a low velocity of gas passing through this furnace, there may be a tendency toward stratification. The boilers described in the paper are taken off the line if the load drops below 170 per cent of rating. Below this rate we may get that action; above it, we do not. Hence, in that particular case, the furnace is satisfactory. For another installation it might not be. The furnace described is not a panacea. It is only a step in the solution of the problem.

Both Mr. Mumford and Mr. Dalton have expressed surprise that there should be any difficulty due to the deposit of cinder on the rear arch. We do not know the cause of our trouble. On the first experimental arch, the short arch that we did not describe, cinder did accumulate, but it accumulated in a comparatively cold state, and rolled off the arch as soon as the angle became sufficiently

steep. We know that happened at Baltimore and we believe that is what Mr. Mumford experienced. One might suppose that a difference in the fusion temperature of the ash is responsible, but the coal used in the first experiment was substantially the same coal as used later, and probably there was not much over 100 deg. fahr. difference in the ash-fusion temperature of the two coals. The coal came from exactly the same districts and had practically the same specifications.

No. 1932

PRODUCTION CONTROL

By George D. Babcock,[1] Peoria, Ill.
Non-Member

The conclusions arrived at by Taylor in his paper on Shop Management in 1903, *together with the methods he described, were based upon the thesis that every organized effort of human endeavor can be analyzed into its fundamental elements, and that these elements can be forecast and arranged in an orderly sequence that represents the best combination to attain the desired result. The analysis and arrangement of the elements of production brought about the two great divisions of productive effort that are characteristic of modern industrial management and production control—planning and performance.*

In this paper the author presents an outline of the subject of production control in manufacturing, taking up for consideration respectively actual output with given equipment; preplanning; the establishment of manufacturing programs; determination of lot sizes; establishment of the production schedule; operation analysis; stores systems; despatching of work; inspection; maintenance; and forms.

THE most efficient type of production management is based upon an application of the principles first laid down by Taylor in his classic paper Shop Management in 1903, and amplified and explained in his later writings and addresses. Taylor's great discovery in management was that scientific analysis should be substituted for rule-of-thumb and for traditional methods. Taylor's work also started the movement to put engineers in charge of the important functions in the supervision of industry, something that had up to that time been almost unknown. As a direct corollary of Taylor's discovery, the greater part of the burden of determining the best methods of doing work was shifted from the workmen to those charged with the management of the business. The reason for this shifting of the burden lay in the fact

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that such determinations involved many things that were beyond the control of the working force.

2 Among the items that are not within control of the workmen may be mentioned the following:

a The raw material used for production; its quality and the regularity of its supply

b The equipment provided for working this raw material

c The processes through which the raw material must pass to be converted into finished product

d The sequence in which the various parts entering into the final product are sent into the factory.

3 Each of these items requires detailed and extensive investigation before the best combination for the desired final result can be found. Some of these investigations may involve heavy expense and exact technical knowledge, and may consume a considerable amount of time. Obviously, it would be impossible for the workers on their own initiative to conduct these, even if they had the inclination to do so.

4 Taylor's conclusions and methods were based upon the thesis that every organized effort of human endeavor can be analyzed into its fundamental elements, and that these elements can be forecast and arranged in an orderly sequence that represents the best combination to attain the desired result. This analysis and arrangement of the elements of production brought about the two great divisions of productive effort that are characteristic of modern industrial management and production control — planning and performance.

5 The function of planning includes all of those elements that are beyond the control of the workmen, and some of those that formerly were regarded as within their sphere. It includes a decision as to the material that will be used in the processes of the factory, the exact equipment that shall be used, the method of handling this equipment, the sequence of individual operations on each part of the product, and in the highest development of managerial science, the time that shall be taken for each operation. It also includes provision for instructing the workmen in the methods of handling the equipment so as to insure that the performance of the workman will accord with the forecast of the planning department. Upon the degree to which an industrial establishment has developed and made use of these elements of planning, depends the efficiency of production.

Classification of Manufacturing Effort

6 Manufacturing can be classified into the following divisions, ranging in order of efficiency from the lowest to the highest:

a One order for one piece. The piece never to be reproduced

b One order for several pieces, never to be reproduced

c Repeat orders at irregular intervals for one or few pieces

d Repeat orders at irregular intervals for many pieces
e Repeat orders at uniform intervals for one or a few pieces
f Repeat orders at uniform intervals for many pieces
g Continuous or standing orders for the same piece.

7 Any industry may have in combination one or more of these classes of effort and in any analysis of the industry this should be recognized. The industry should be classified into the above classes and a control plan devised in such a manner as will secure the best results for each class. A general control plan will then be devised so as to relate the classes.

8 Irrespective of the class or combination of classes into which the work of a manufacturing plant falls, the efficiency of its operation will depend upon two things: (1) The efficiency in selection and use of the mechanical equipment of production; (2) The definite preplanning of every operation and event which takes place in the progress of the work through the factory. These may be considered in some detail.

The Mechanical Equipment

9 It is obvious that the best equipment that is available for a given purpose is the most desirable. High manufacturing efficiency does not depend upon this, however. Rather it depends on the best possible use of the equipment that is available.

10 If manufacturing efficiency be defined by the ratio

$$\text{Efficiency} = \frac{\text{Actual Output with Given Equipment}}{\text{Maximum Possible Output with Given Equipment}}$$

it will be evident that the efficiency of the plant does not depend upon the efficiency of the equipment *per se,* but upon the efficiency with which that equipment is operated.

11 Efficient operation depends on the following factors:

a Maintenance of equipment in perfect operating condition, so that there will be no failures during working hours to delay production

b Adequate power at the machine, so that it may be utilized to its full capacity

c Proper adjustment of machines and auxiliary equipment and, in the case of machine tools, properly formed cutting tools

d Determination of the best methods of operating the equipment, and insistence that these methods be followed

e An adequate supply of material upon which the equipment may work, so that there will be no idle time or delays due to lack of work for any machine

f Uniformity in the quality of raw material, permitting uniform operation at a predetermined rate. For example, excessive hardness in castings will compel slower operation of machine tools, and so decrease the predetermined production.

12 It follows from the foregoing that several departments are concerned in the utilization of equipment to its full capacity. Maintenance, power, and adjustment of machinery are within the province of a mechanical department. Determination of methods and an adequate supply of material at each machine are functions of preplanning. Uniform quality of material is a result of careful purchasing and rigid inspection of purchased material to insure conformity with specifications.

13 Maintenance can be insured by the establishment of a definite routine of inspection to discover defects, wear or misadjustment which will tend to cause failure or decrease the rate of production. When such are discovered, arrangements can then be made to effect the necessary repairs outside of working hours or at such times as will cause the least interruption to production.

Preplanning

14 Assuming that the equipment of the establishment is in first-class operating condition, and a routine has been set up that will maintain it in this condition, efficiency of production will then depend on the degree to which preplanning has been carried out.

15 Preplanning comprises:

a The establishment of a definite manufacturing program

b The purchase of materials and the insurance of their delivery in ample time to carry out this program

c The determination of the methods to be used in carrying out the program

d The sequence of operations to be performed on each component part of the final product (Routing)

e The establishment of definite schedules to fix the time at which each operation in the routing shall take place

f The despatching of the work in the factory in accordance with the schedule.

16 Another function of manufacturing, supplemental to but not necessarily a part of preplanning, is the inspection of the work, both while in progress and when completed, to make sure that it is up to the standard of quality and workmanship desired. Inspection has this relation to preplanning, that when defective work is discovered, it must be replaced, and the replacements must be fitted into the manufacturing schedule without disturbance or delay.

Establishment of Manufacturing Programs

17 Manufacturing may be divided into two classes as follows:

a Manufacturing for stock. Examples of this class are automobiles, food products, certain classes of electrical equipment, typewriters, etc.

b Manufacturing on orders to definite specifications furnished by the customer. Examples are hoisting and conveying machinery, locomotives, certain classes of textiles, etc.

18 A manufacturing plant may lie in either or both of these classes. The establishment of a manufacturing program is a radically different process in the two cases, and in the case of the plant manufacturing both for stock and to order the establishment of the program depends upon which class of work predominates.

Manufacturing for Stock

19 The establishment of a program of manufacturing for stock depends first upon the probable demand for the product. The analysis of this probable demand is a function of the sales department. The accuracy with which this department can forecast this demand is a measure of its efficiency. It is not within the scope of this paper to discuss the methods of sales analysis, but it may be pointed out that in general they should be based upon the same broad principles of preplanning and exact knowledge of the problem as form the foundation of production management.

20 The analysis of probable sales should show for the purpose of establishing a manufacturing program:

a The total demand for the product during a given period of time. This period should be as long as possible, and preferably not less than six months

b Whether or not this demand will be at a uniform rate. If not uniform it should show

c The variation in demand from month to month, or even for shorter periods if possible.

20 With these data before it, the management may then plan the productive activities of the plant so as to meet this demand in the most economical manner. Fig. 1 is a graphical presentation of a sales analysis and the resulting production program. Curve *A* represents the expected sales, month by month. Curve *B* shows these monthly sales accumulated to give the total sales at any time after the beginning of the sales program. Curve *C* shows the production program month by month, and curve *D* the monthly production accumulated to show the total production at any time during the program. Curve *E* is the inventory. The production program should be so laid out that the inventory will never fall to zero. Another curve might be added to show the actual total production in comparison with the planned production. If the production management, however, is efficient, this curve would fall exactly upon curve *D*.

21 It will be noted that the monthly production program does not follow the monthly sales program. The reason for this in the case at hand is that there is always a minimum size of lot that it is economical to put through the plant, and this minimum may

be larger than the expected sales at the beginning and end of the program. Furthermore, the expected sales at the middle of the program are greater than the manufacturing facilities of the plant, and provision must be made for these by building up the inventory during the early part of the program.

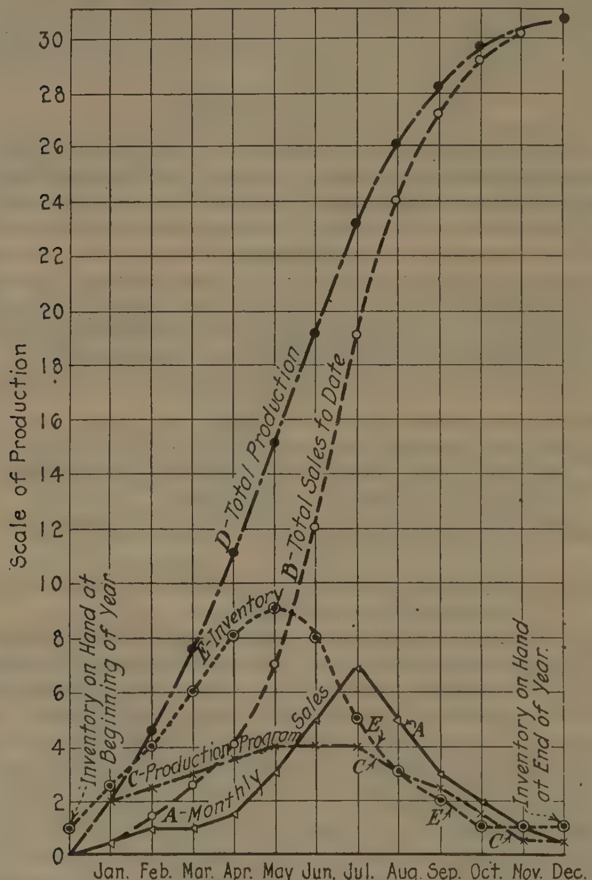

FIG. 1 DEVELOPMENT OF PRODUCTION PROGRAM

22 The establishing of uniform manufacture with a minimum amount of labor turnover, and at the least cost, is largely dependent, in an industry where sales vary from month to month, upon the starting of the manufacturing program for a given sales program at the proper time. This proper time is such that sufficient units of product will be built ahead of the sales peak, so that the sales peak will reduce the accumulated product to a normal or

predetermined inventory. There may be cases where this slow increase of inventory must be started several months in advance of the peak, for the further away from the peak the accumulation can begin, the less will be the disturbance created by the peak, and the maximum possible operating efficiency will result. A graphical analysis of this character will give a clearer idea of the manufacturing problem than any other presentation.

23 The rate of production having been decided, the next decision to be made is the size of the various lots of product that must be put through the plant, and the intervals at which these lots must be started in order to meet the production program. These decisions having been made, the production follows the routine described later.

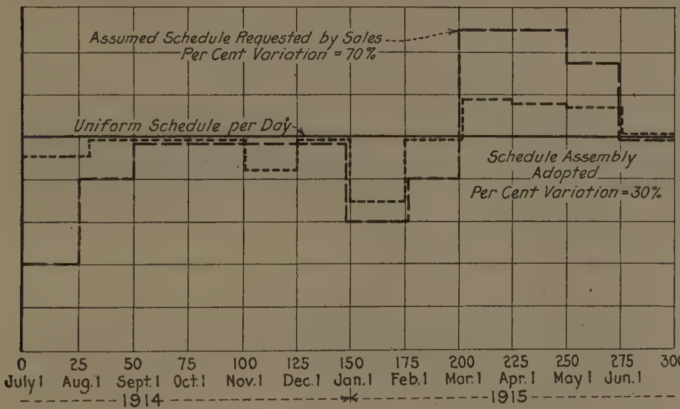

FIG. 2 AN AUTOMOBILE MANUFACTURING SCHEDULE

24 The production program should be analyzed with reference to economical utilization of personnel, as well as for production according to a sales program. If the final product is composed partly of parts made in the factory, and partly of outside purchases, the schedule of deliveries of the outside purchased parts may be scheduled to conform to the sales program, while the production of manufactured parts is maintained at a uniform or nearly uniform rate.

25 Fig. 2 shows such a schedule as laid out for automobile manufacture. The sales program shows a variation of 70 per cent. The schedule of assembly adopted to meet this variation shows a variation of 30 per cent. The schedule of production of manufactured parts is uniform and has zero variation. Analysis of the product developed the fact that 60 per cent of the cost of materials entering the final product was contained in 59 out of 1330 parts. Of these 59 parts, 44 parts, representing one-half of the cost of material, were purchased finished parts which did not enter into

the production schedule until the final assembly. The balance of the other parts entering the car represented such a small relative material value that it was possible to operate the productive labor manufacturing parts on a uniform schedule. The final assemblies, however, were made according to a variable schedule as shown in Fig. 2.

26 In arranging the variations of the assembly schedule, a variation in labor of not more than 30 per cent was permitted; i.e., not over one man in every three on the assembly force was allowed to be changed, two-thirds of the force thus consisting constantly of trained men. The cars were assembled according to the assembly, as shown in Fig. 2, and the high-cost materials noted above were ordered for delivery to agree with this schedule. The capital invested in inventory was thus kept at minimum, both for purchased and manufactured parts.

Determination of Lot Sizes

27 Several factors enter into the determination of the most economical size of lot to be used in manufacture. The first and most important factor is previous experience with similar product as a general guide. The following specific considerations must be taken into account:

28 Small lots are indicated by:

- *a* High cost of material
- *b* Large unit bulk of material
- *c* Long time required for various operations on a part
- *d* Probability of change of design
- *e* Reduction in inventory
- *f* Conservation of floor space
- *g* High unit weight of material
- *h* Use of perishable materials in processes
- *i* Early entry of part into sub-assembly.

29 Large lots are indicated by:

- *a* High average cost of machine set-ups
- *b* High despatching, inspection and trucking charges
- *c* Rapid production, even with elaborate set-up
- *d* Reduction of spoilage
- *e* In general, all preparation charges should be satisfied with largest possible lots.

30 A number of formulas have been prepared at one time or another for the mathematical determination of lot sizes. The more accurate of these are based upon material cost per piece, time required for operations, interest charges on material in process, rental charges for space occupied, wage rates, etc. Accurate formulas, taking all these facts into consideration, lead to cubic equations which are difficult of solution. The author has found it preferable to determine lot sizes by trial and error rather than by applying a formula.

Manufacturing to Order

31 A plant manufacturing to order obviously cannot plan its production in advance of the receipt of orders. Preplanning in this case involves only the analysis of the order to determine the material that must be purchased and the time required to procure it, and to determine the operations that are necessary, the sequence in which they shall be performed, and the time at which they shall start in order to meet the delivery date specified in the order. These determinations must be made with reference to orders already in process, and the new order must be fitted into the manufacturing schedule in such a manner as not to create disturbance. The purchasing is often simplified by the fact that most manufacturing concerns of the class under discussion operate in a somewhat limited field as regards product, and the raw material used for all orders is of the same character. Thus a company specializing in hoisting and conveying machinery will use for practically all of its orders pig iron, sheet and bar steel, alloys and certain standard manufactured products such as rivets, bolts, pillow blocks, chain belts, etc. These it will keep on hand in sufficiently large quantities to fill its requirements over a certain period, purchasing in large lots whenever the stock falls to a predetermined minimum. If the analysis of the order shows that there is sufficient raw material available to fill it, production can begin at the earliest date on which machine capacity is available.

32 Another class of manufacturing which is to order, but which has many of the characteristics of manufacturing to stock, is found in certain phases of the textile industry. In the manufacture of woolen fabrics for dress goods, orders are taken by means of samples for varying quantities of product of different grades and patterns from numerous customers. The small orders for each fabric are combined into single manufacturing orders, from which a manufacturing program is built up. Since the samples are sent out and orders taken before the manufacturing season starts, the mill is enabled to plan a production program in its entirety before actual production begins.

Manufacturing for Stock and Order

33 Where manufacturing for stock predominates, the stock program is laid out as heretofore described, and the product made on order is fitted into the stock program. This may be accomplished by ascertaining what proportion of the machine capacity is not utilized over a given period for stock manufacture, and then scheduling the special product through this excess capacity as if the balance of the equipment did not exist.

34 Where the stock production is of minor importance, it is scheduled on the basis of the excess machine capacity available. In doing this, if the stock program extends beyond the time for

which special orders on hand will utilize the equipment, care must be taken to reserve sufficient capacity to handle future special orders. The capacity so reserved should, in general, be equivalent to the capacity required for special work during some definite preceding period, say, from one to six months.

The Purchase of and Storage of Material

35 In purchasing material these considerations must be taken into account:

a Quality

b Price

c Time of delivery.

36 Quality is important, especially where the final product must meet specifications as regards strength, durability, etc. It is also important in that it may affect the time required for completing the processes in manufacture, and if of different quality than that upon which the manufacturing program is based may require much more time at each operation. The program then cannot be carried out as planned, except at the expense of overtime or the utilization of more machinery and men than were contemplated. It is poor economy to substitute material of unknown quality for that of proved quality to gain a slight advantage in price.

37 All things being equal, the lowest-priced material is to be desired. If, however, the quality is lower, or delivery cannot be insured at the desired time, price becomes less important than other considerations. The time required for delivery is extremely important in laying out a manufacturing program. Since production cannot begin until there is material upon which to work, ample time must be allowed in which to procure material. It is advisable to tabulate the maximum and minimum number of days required for each item of material that is commonly bought, as a guide in placing orders. The reputation of the seller for keeping delivery promises should also be considered, and preference given to the one who can be relied upon, rather than to one who will promise a prompt delivery but whose promises are unreliable. It is better to accept a later delivery date, with the knowledge that it will be met, than an earlier date where there is reason to doubt that delivery will be made on that date. Another factor to be considered is whether the material shall be bought from the mill or from a warehouse or jobber. Purchases from the mill usually cost less, but the time required for delivery is longer. Whenever possible, mill purchases are advisable.

38 It should be borne in mind that the shortest time required for the execution of a manufacturing program is the longest time required to procure any item of material plus the time required for the longest process through which that material must pass.

The importance of accurate forecasts of the time necessary for purchasing is thus evident.

39 In the routine of establishing a manufacturing program, then, the first item is to ascertain when the first orders for material must be placed. To do this it is necessary to work back through all the processes of production, starting from the time when the finished product must begin to leave the factory. The methods of doing this are explained later. This analysis of the operations fixes the time at which the material for each process must be available. To this time is added the length of time necessary to obtain prices and secure delivery. Having thus fixed the number of days that must elapse between placing the order for material and the completion of all the manufacturing operations, exact dates can be set for the beginning of each event in the production program. This is known as scheduling.

Establishment of the Production Schedule

40 In making a schedule for manufacturing, the completion of the final operation is the starting point. The number of days that

TABLE 1 PRODUCTION SCHEDULE FOR CYLINDER OF TRACTOR MOTOR

	Finished tractor	Assembly	Machine shop	Rough stores	Purchase	Engineering
No. of days in department	0	22	24	17	60	10
Working days ahead of completion.....	0	22	46	63	123	133
Date work should be finished...........	July 1	June 5	May 7	Apr. 7*	Feb. 7†	Jan. 25

* Date on which invoice should be paid.
† Date on which purchase order should be placed.

must be allowed for each standard lot or operation must be ascertained. The length of time that must be allowed between operations for movement of material, for inspection, for seasoning, or for any other purpose whatever also must be determined. These various intervals are then to be added progressively to show the number of working days prior to the date set for completion of the first lot of the product that each operation should begin. Then by means of a calendar, from which Sundays and holidays are omitted, the actual calendar dates for these operations can be fixed when the date of completion is known. Table 1 is an example of the fixing of the calendar dates for the principal events in the production of a cylinder for the gasoline motor of a tractor, deliveries of which are to begin on July 1.

41 This cylinder is a machine-shop product, on which many operations are performed. The actual working schedule would show in detail every operation, and the date at which that operation should begin. Several ways of doing this are in common use, and each one has its advantages. No one method can be said to be the best, for the reason that the method to be adopted should

depend on the circumstances surrounding its use. Two methods that have proved very successful may be cited.

42 The first uses what are known as route sheets. One route sheet is used for each item of product, and lists every operation in its order through which the item is to pass, together with the machine, and the date on which the operation is to be performed. Progress of the work is noted by checking off each operation as it is completed. When an operation is checked off, the orders for the next operation are issued on the date set opposite that operation. The route sheet may be used for large quantities or for single piece orders.

43 The second method is semi-graphical. Consider for a moment a single item of product that is made up of many parts. If a horizontal line is divided according to scale of working days, it can be made of such a length that the number of days represented by it is the total time from the completion of the design to the delivery of the final product. Then the schedule can be laid out by noting on this line, according to the scale of working days, the number of days prior to completion that each operation shall begin. The time required for this operation may be shown by blocking off on the line a distance equal in length to the space representing the number of days required. This space is laid down from the point denoting the beginning of the operation toward the point representing the completion of the product, known as the zero point. Another scale, also graduated to the same scale of working days, but reading from right to left in calendar dates, with Sundays and holidays omitted, is then laid along this line of operations, with the date set for completion placed on the zero point. The calendar date on which any operation should begin may be read directly on this scale. If for each item that enters into the final product a similar horizontal line of operations be laid out, there is presented a graphic picture of the production schedule for the entire product.

44 Such a graphical schedule may be used to control production by the addition of another scale, equal in length to the total time shown on the schedule for the part requiring the longest time. This scale is graduated to read in items of finished product, and is numbered from right to left, from zero to the total number of items in the program. This scale is movable, and at the beginning of the program is set with its zero at the extreme left of the schedule, and corresponding to the first operation shown. It is moved to the right each day a distance equal to one day on the horizontal scale. Whenever the zero of this moving scale passes over any operation posted on the schedule, it is an indication that that operation should begin. The necessary orders should then be issued for such operation.

45 Progress of the production program can be recorded by writing over each operation in terms of units of final completed

product, the quantity of product that has passed through it at the end of any period of time. If the number so written corresponds to the number immediately over it in the moving scale or schedule tape, the operation in question is in accord with the schedule. If the number is greater than that on the schedule tape, the operation is ahead of schedule. If it is less, the operation is behind schedule. The number of days that it is off schedule is ascertained by locating on the schedule tape the number corresponding to that written on the operation, and noting on the scale of work days the interval between the position of this number and the operation in question. (See Fig. 3.)

46 This method is the basis of control boards that have been used to control operation in repetitive work, where the product is put through in lots, as in the automobile industry. If it is desired at any time to increase the schedule, this may be done by decreasing the time interval at which lots are started in the factory.

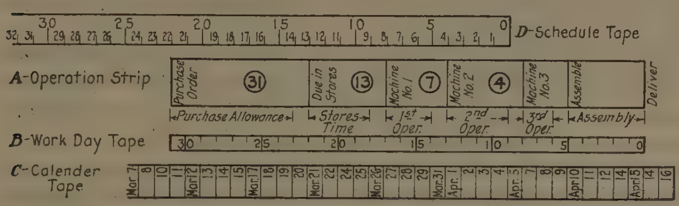

Fig. 3 The Principle of Graphic Control

47 The route sheet and the control board are adapted to industries where the product is made up either of single pieces or of a large number of parts that are assembled into a final product. Where the final product is not an assembly, such as cloth fabrics, or where it is an assembled product made in such large volume that the manufacture of each part is on a progressive or continuous basis, scheduling is determined by the speed of the final operation. This determines the amount of product that can be produced per day, and the purchase of material is arranged so that there will be daily supplied to the factory equipment an amount equal to the daily production. The only date to which much consideration must be given is the date on which the first purchase order must be placed. This is fixed by the time required for process up to the final operation plus the time required to procure material. After the flow has once been established, the operations become automatic in their relation to one another. In certain continuous processes, the speed of production is governed by the speed of conveyors which transport the work in process from one operation to the next. If it is desired to speed up the schedule in such a case, it is done by increasing the speed of the

conveyors, or by decreasing the intervals at which material is delivered to them for the first operation. Additional machine capacity must then be added at each operation to take care of the heavier flow of work.

Operation Analysis

48 It is evident from the discussion on scheduling that an important feature is exact knowledge of time required for each operation. It is not enough that the time for an operation by the method in common use be known. For the most efficient management, the time of the best method should be found. This includes both the best method of performing each individual operation and the best sequence of operations.

49 A prerequisite to efficient scheduling, then, is an exhaustive analysis of each operation to discover the best method and equipment for performing that operation. This also should include an investigation to discover and eliminate all causes that will hamper production, and to standardize the conditions under which production shall be carried on.

50 For instance, in machine-shop work there may be several methods available for performing a single operation. Boring and turning may be done, for example, in a lathe, a boring mill, or a turret lathe. Considerations that govern the choice of the particular machine in this case might include the size of the work, the number of pieces to be made, the quality of finish demanded, the relative time by each method, the number and cost of special tools, and the time required to set up each machine for the operation in question. The analysis should determine, first, the method, and next standardize the conditions of using that method. In the machine shop, the shape of the cutting tools has a marked effect on production. It is highly important, therefore, that these be standardized, and the same shape and method of heat treatment be used for the same work every time that it occurs. The best combination of speeds and feeds should also be ascertained for each job, and the workmen should be required to use these combinations. The machines themselves should be standardized so that identical work can be done in machines of the same class. If this is not done, there may be a congestion of work at some machines while others of the same class are idle because they are less efficient than the others. Or if the work is routed to these inefficient machines it may not be possible to do it in the time allowed in the schedule, which should be based on the best method. The schedule may thus be upset at these machines with disastrous results so far as controlled production is concerned. Standardization is a great aid to simplification of routing and scheduling.

51 Similar considerations obtain in other industries. In the textile field, for example, the speeds of spinning frames and looms on the same class of work should be standardized, otherwise the

production will become unbalanced due to one operation producing more than the next can handle.

Quality of Material

52 Uniformity in quality of material is an important factor in scheduling. In the analysis of a job a certain standard of quality is assumed, and all time estimates are based on this. Variations from this quality may seriously affect the time of operation. Thus in the machine shop, a certain grade of iron castings can be machined at a certain best combination of speed and feed with a certain depth of cut. If the casting actually furnished is harder than that used as a basis, the speed or feed must be decreased or more cuts taken than were contemplated. In any event, the time required will be greater and the schedule will suffer in consequence. In textiles, insufficient strength in the yarn due to improper blending of the raw material may cause frequent breakages of yarn, resulting in a shutting down of the machine with consequent loss of time, or in defective product.

53 Uniformity in quality can be insured as a rule by rigid inspection of raw material accompanied by tests to determine quality. All the material not up to the standard set should be rejected.

Determination of Operation Time

54 The time required for an operation can be ascertained in a number of ways. Time study is one. Estimation, based on similar work or on previous performance is another. In work where the machine has a definite output per unit of time, as in a spinning frame or loom in textile work, the time is a matter of calculation, based on the quantity of material to be handled and the speed of the machine, with a certain percentage added to cover unavoidable machine delays, and the setting up of the machine. In machine-shop work, where the time required for handling the work and for setting up the machine is relatively small compared to the time actually required for cutting metal, calculation based on feed and speed and the dimensions of the work, plus a percentage of flat time allowance for handling, etc., may be sufficiently accurate.

55 While it is desirable, of course, that the time allowed for an operation be as accurate as is possible, it is unwise to strive for absolute perfection when the advantage to be gained is small, and the cost of the last few per cent in the perfection scale is high. Thus, if the handling time on an operation is two minutes, and the machine time 30 minutes, an error of even 25 per cent in estimating the handling time will make an error of only about one per cent in the total time of the operation. The actual length of the job, however, must be considered, in deciding whether or not the error due to estimating will be serious. In the case just

considered, an error of 25 per cent or one-half minute per job, would amount to but 7 minutes per day, since less than 16 jobs are all that are possible in an eight-hour day, and the maximum difference in production would be one piece per week. If, however, a job requires but three minutes with a handling time of one minute, an error of 25 per cent in estimating handling time would be more serious. The difference in product would be one piece per hour, or eight per day.

56 Time study has not the same relative importance that it held at the beginning of the movement for better industrial management. Several reasons for this exist. In the first place, an enormous amount of time-study data has been accumulated and published. These data cover the smallest elements of the work to which they relate, and by combining these elements the time for the great majority of operations can be predetermined with considerable precision. Another reason is the standardization of equipment. Formerly hardly two machines designed for the same work were alike in their characteristics. At the present time there is a remarkable similarity in the characteristics and capacities of machines of the same class, even when built by different makers. Then, too, improvements have been made so that the time for adjustments and manipulation is much reduced, and as a rule, forms but a small fraction of the total time for operation. Machine capacity for a given job can be determined from tabular data or by calculation, aided by slide rules if desired, and in most cases the machine time is the most important factor. There are many things to be done in the line of standardization, establishment of material control, etc., before a factory is ready for time study. And as Mr. C. G. Barth has aptly said, " When these things have been done there will be little need for time study."

Stores Systems

57 The stores system is an important feature of production control. Unless there is an adequate supply of material available when and where it is needed, the most elaborate system and the finest equipment will fail of their purpose. The stores system includes the storage of material and the accounting for it. The accounting system should be so designed as to show at all times the quantity of material of each class and size actually on hand in the storeroom, the quantity on order but not yet delivered, the quantities apportioned to or reserved for various orders not yet in process and the particular orders to which these quantities are assigned, and the balance that is available for future orders. The stores-ledger sheets should also show the location in the storeroom of every item carried, and the unit value of these items. This last feature enables inventory to be taken directly from the stores ledgers without the necessity of a physical inventory in the storeroom.

58 In the storeroom, each commodity should carry a bin tag, on which is entered the quantity of material in storage. Additions to the quantity on hand should be entered on the tag, and added to the total already entered. Material issued should be deducted from the bin-tag total and a new total brought down. The total quantity shown on the bin tag should always correspond with the quantity shown as on hand in the ledgers, and frequent comparisons should be made to detect errors.

59 No material should be permitted to leave the storeroom, except on the authority of a stores-issue order, signed by some one with the necessary power. Neither should material be received unless accompanied by the necessary documents to show its origin. These, as well as the stores-issue orders, should pass through the hands of the stores-ledger clerks to insure that the ledgers will be kept up to date.

Despatching of Work

60 Except in continuous or progressive manufacturing, despatching is essential to make preplanning effective. Despatching is the assignment of jobs to machines or workmen in the sequence and at the times determined by the schedule established in advance. Under modern management, the foreman or overseer has no authority to dictate when or where any operation shall be performed. The time of its performance is fixed by the schedule, and the place by the job analysis. Work orders then are prepared for each operation shown on the schedule and filed according to the schedule dates. Each day the orders for that day are issued to the workmen concerned. If the work called for by any work order is not available, due to previous operations being incomplete, the order is marked in some manner to make it conspicuous, and pressure is exerted until the work is once more on schedule. Despatching thus may be performed by a well-trained clerical force and foremen relieved of routine. By noting on them the time of issue of work orders, and the time of completion of work, the despatching may be used as the time-keeping system of the shop and also used to accumulate data for the cost accounting.

Inspection

61 Inspection under modern industrial management is used to prevent defective work, as well as to discover it in completed product. Defective product results only from the two causes of defective material and poor workmanship. The inspection function can be arranged to prevent both of these causes. Inspection of incoming material has already been discussed. As a preventive of poor workmanship, inspection should take place at the beginning of a job. The inspector should ascertain whether the product at each operation is being made according to the specifications,

and if not, point out the cause of variation. He should not leave until the workman has produced and is producing perfect work. The despatching of work should include the directing of the inspector to operations that are just being started.

62 This inspection of work in process should not operate to dispense with final inspection of each part of the product after the last operation. In final inspection, the inspector should be advised as to just what faults or variations from standard are to be looked for, and the permissible amount of variation. The provision of a definite routine for inspection will render it more efficient and less costly.

Maintenance

63 Maintenance and repair form an exceedingly important function of production management. The duties of the maintenance department include the repair of equipment that fails or wears in service, and the establishment of a system of inspection and preventive repair that will remove causes of failure or make adjustments for wear before failure occurs.

64 Failure of equipment in service has many disadvantages. Not only is there the cost of repair, but also the wages of idle workmen while repairs are being made, and the loss of profitable production. Constant machine failures will prevent any schedule being laid down with the certainty that it can be met, and production control may become difficult if not impossible.

65 The maintenance department can avoid all or nearly all of these difficulties by establishing a routine that will insure inspection of the equipment of the plant at regular intervals. When conditions are revealed that require a remedy, arrangements can be made for the necessary repairs outside of working hours or at a time when they will cause the least interruption to production. It is worthy of note that those plants that have a highly developed maintenance system seldom have equipment failures.

Forms

66 To carry out the routine of any system of management, certain forms are necessary. The system of forms should not be confused with the system of management. The latter is an organized scheme for obtaining certain results, based on definite principles. The former is simply the mechanism provided for the latter. And let it be stated here that scientific management is not a particular system, nor a set of forms. These are but a means to an end. They represent only the easiest and most economical method of applying the principles of management to the case in hand. Two plants may have entirely different systems and their forms may be totally unlike, and still both plants may be truly representative examples of scientific management. Forms are

merely a permanent statement of the wishes of the management in regard to the matters to which they relate. They replace orders and instructions that in the early stages of an enterprise or program are given by written memoranda or verbally. When an order or instruction is repeated sufficiently often to warrant it, a form is devised to convey the same information, concisely and definitely, leaving only the necessity of writing in the figures, dates and symbols to make the form complete. Forms may also be considered a delegation of authority to the individual authorized to issue them, the authority, however, extending only to the subjects covered by the form.

DISCUSSION

J. P. Jordan.[1] There is no need for discussion or debate as to the desirability of the most detailed planning in the operation of companies manufacturing standard or repeat product. In fact, the only hope of success in such a business lies in the most intimate and detailed knowledge of all manufacturing processes, the use of this knowledge in planning and scheduling work, and an equal effort in connection with all material-handling operations. Granting this, would it not be of very distinct advantage to the movement for promoting better management of manufacturing institutions always to make it very clear that highly detailed methods of production control apply only to plants where repeat or continuous work is going on?

The writer's reason for suggesting this point is that he has seen so many cases where extreme detail has been attempted with disastrous results. Either the management of companies themselves, or some of the men they employ, have attempted detailed control methods that were impossible and should have been recognized as impossible at the start. Such failures have brought much discredit on production-control methods which are absolutely indispensable to concerns whose production is continuous or rapidly repeating.

This brief discussion, therefore, is a suggestion for every one interested in promoting good management and better profits to make very special efforts to differentiate between cases calling for extreme control and others calling for less control. It seems almost criminal for companies which ought to use control of the greatest detail to refuse to consider it on account of failures in companies which never should have attempted it. It is equally unfortunate that companies which should consider but specific portions or degrees of detailed control are constantly attempting the impossible, and largely on account of the failure of those who ought to know where to draw a reasonable line.

[1] Stevenson Corporation, G. Charter Harrison, J. P. Jordan & Associates, Mem. A.S.M.E.

Naturally, many hearing of the results which have been accomplished by the author would like to accomplish such results in their own business. But disappointment and failure are liable to follow unless the greatest care is taken to find out just how far to go and to what extent detailed production control can be safely attempted. Adequate and proper production control is absolutely indispensable to every business. The degree of detail and the extent to which it should go are questions which must be answered in each individual case if the integrity of control as a dependable mechanism is to be preserved.

Howard Coonley.[1] This paper gives the best outline of production-control principles, not applied to a specific industry, that has come to the writer's attention. A few points deserve special comment.

In Par. 25 the point is made that 60 per cent of the cost of materials entering into the finished product was contained in 59 of the 1330 parts. This is somewhat similar to our discovery that in one of our heavy "bread and butter" lines six out of 3680 items represent 37 per cent of the total tonnage produced.

We agree that some clever formulas have been developed for determining the economical lot size. Our own experience, however, indicates that they are usually too complicated for practical use. Concerning manufacturing for stock, evidently the author believes that the sales department should determine what the demand would be and that its estimate should be used as a basis for the establishment of a production program. Possibly he is looking for too high an efficiency in the sales department. While the tractor business may have been successful along these lines, we know that some of the large automobile manufacturers are considering the establishment of planning and statistical units that will weigh sales estimates against independent forecasts, based upon sound economic data. This procedure will enable the chief executive to decide upon a program that can be more nearly realized. We feel that forecasts will reach their maximum efficiency in the office of the chief executive, where all factors are taken into account.

Perhaps it was not considered wise to complicate a paper on production control with too detailed a description of the physical and accounting control over materials, miscellaneous stores, and finished stock. However, in an industry where 95 per cent of the orders are shipped from stock, the stores and stock systems are extremely important in controlling production. In fact, the remainder-of-stores record becomes the backbone of the whole system and from it are obtained the vital statistics so far as individual items are concerned. Where for each one of a number

[1] Pres., Walworth Mfg. Co., Boston, Mass. Assoc. A.S.M.E.

of classes of products sales can be estimated in units, such as tons, pounds, pieces, yards, etc., it is frequently possible to break down such class totals into items by applying predetermined ratios, so that a production program, expressed in convenient class units, is readily transformed into a definite quantity to order, for each size and kind of product.

In connection with the author's remarks on maintenance, we believe that many failures of equipment can be prevented by regular inspection, and that much maintenance work can be very profitably planned and scheduled. The next ten years will undoubtedly see great strides in the improvement of methods of handling this very important part of our production work.

Robert T. Kent.[1] There is one thing emphasized in this paper that is the keynote to all industrial management: the insistence that preplanning is the foundation stone. In the last few years we have found that preplanning is not merely a function of production, but applies to every activity of the business. The author hints at it when he says that we must preplan our purchases and preplan our sales, which leads us back to preplanning of finance. The fact that budgetary control in industry has come to be such an important factor in the last few years is an indication that preplanning is affecting every line of human activity. If we cannot preplan our sales, we cannot preplan our manufacturing accurately. If we cannot preplan our manufacturing, we cannot preplan our purchases, and we cannot finance a business unless we know what our manufacturing and purchasing schedules are to be.

It was the writer's privilege to visit the Peoria plant of the Holt Manufacturing Company last spring. As a result of that visit he is convinced that the Holt Manufacturing Company is the answer to all statements that scientific management does not work. The author has stated that they had at that plant 763 working days of sustained scheduled production, up to Dec. 1. The production was based on a schedule that was laid down at least five months previous to the date on which the product was delivered to the shipping department. That is, the company has had to determine, five months in advance, the exact quantity of work that had to be completed on any given day, the quantity varying from week to week and from month to month. For a period of over two years it has not failed, for a single day, to deliver the exact quantity of product called for by the production schedule. This has involved the scheduling and keeping on schedule of over 8,000,000 machine operations.

The product made is caterpillar tractors, which became familiar in the later years of the World War; these tractors must be built to stand the roughest usage, yet they must be constructed in

[1] Supt. of Prison Industries, State of New York. Mem. A.S.M.E.

accordance with the best standards of the best automobile practice. The fine work involved immensely complicates the problem. It is doubtful if that record has ever been equaled in this country or in the world.

Ralph E. Flanders.[1] The writer agrees with the statement in Par. 56 as to the relative unimportance of time study; its importance tends to diminish as the subject is developed and the results of time study are tabulated.

There is but one point on which the writer is inclined to disagree. He believes that a complicated mechanism, with a great many parts and sub-assemblies, passing through the shop at a relatively low rate of production, is susceptible of a manufacturing scheme and a production control that is as much standardized as is the continuous production at a high rate of the Ford automobile in the Ford plant. If such a plan is adopted, order and system can be introduced into the shop, leaving the purchasing department as the only place where worry will occur over quick changes of plan or program. Such changes should be relegated as far back toward the purely commercial end of the business as possible. It is because of this belief that such a policy was tried in the plant of the writer's company, and it is because it has worked out with a reasonable measure of success to the extent that it has been tried, that this plan and policy has been described in the writer's paper, The Design, Manufacture, and Production Control of a Standard Machine.[2]

Fred J. Miller.[3] With regard to the control board mentioned by the author, the writer hesitates to imply doubt as to the usefulness and excellence of this device, but these control boards do not appear to have the same value as the system of charts and records devised by H. L. Gantt. The Gantt charts, the writer believes, will give all the control that a control board does. They tell every executive when he ought to start work and when he ought to finish it. They also keep a record of progress during the course of the work. Furthermore, they are small and can be blueprinted as often as necessary. They can be placed on the desk of every interested executive every morning and will give him complete information as to the progress of the work.

The Author. The author planned to illustrate this paper with lantern slides, but believed that these would lead from principles to details and modify the intention of the paper, that is, the discussion of fundamentals.

[1] Mgr., Jones & Lamson Mch. Co., Springfield, Vt. Mem. A.S.M.E.
[2] See p. 691.
[3] Member, Public Service Commission of Commonwealth of Pennsylvania, Harrisburg, Pa. Mem. A.S.M.E.

Much confusion arises in a discussion of Dr. Taylor's principles because we draw comparisons of practices between different industries. In this paper were mentioned six different industrial forms, each of which would require largely different detailed treatment, but all within the one group of Taylor principles. If an industry in starting entered the group of lowest manufacturing efficiency, and slowly but surely expanded until it was manufacturing the same products in large output, it would slowly progress through each of these various stages of industrial activiy, each of which should, for the most efficient operation, be planned independently. Just as this industry increased from low efficiency to high efficiency in its manufacture, did the methods and plans of control change accordingly.

With respect to one of the discussions of the control boards, the author wishes to bring out that the control boards are but a mechanism in which it has been found possible better to adapt the principles to a practical application. No one can be acquainted intimately with the control boards, their purpose, nor the details of their operation, if he has not gone into detail in connection with one of them, either at the Franklin automobile plant or the plant of the Holt Manufacturing Company, because there are no others in existence. Furthermore, the author knows of no others which operate even on the same principle. The control boards are genuine control mechanisms, and provide for all authorizations of the requisitioning of material, its date of receipt, provision for its inspection, and so on through to the finished product, every detail shown being carried out by written orders (that is, forms), in time, rate, and quantity. It is a moving, live, mechanism. It is in no sense a bulletin board and in the author's opinion quite out of place for discussion here, since we are discussing fundamentals and not practices. We have endeavored to operate at this plant by every known means that would equal or improve over the control-board method. The manufacturing problem at the Holt Manufacturing Company was the same as that at the Franklin automobile plant, where the boards were developed; namely, a limited quantity of various-finished, complex assemblies having a large number of mechanical operations per unit, all of which must progress through a shop in an orderly, economical manner on time. The less the number of finished units, so long as the complexity remains the same, the more important becomes this problem of coördination which this control board gives. If our product were to be produced in four or five times its present output, the control boards would not be particularly useful to us.

The Gantt chart is an excellent controlling medium for progress of work, and has its own features for particular projects the same as has the control board, but the two are not at all comparable, and therefore it is impossible to say that one can be substituted for the other.

It would be our pleasure at any time to have any of those who have heard this discussion, or will read of it, to visit our plant and go into the many ramifications of detail which this mechanism provides for. It cannot be adequately described.

The author wishes to emphasize that not only do Taylor's principles still exist, but we have here exemplified their usefulness in one industry, to accomplish an exact control which is unusual in industrial performances. By far the largest percentage of the mechanisms and forms used were those introduced by Fred. W. Taylor in the plant of the Tabor Manufacturing Company, Philadelphia, where Taylor first developed a mechanism to apply his principles in their entirety.

The author wishes to bring out one other point, and that is the importance of eventually obtaining a machine-hour basis of cost finding. It is necessary wherever machine equipment or fixed equipment in any plant varies largely in value, space occupied, maintenance and power charges, etc. In any plant where the material, labor, and overhead method of cost finding is used, a machine-hour cost can be obtained for one of the largest and most expensive machines, and likewise for one of the smallest. These have been averaged to put them in the form of the average overhead basis. The charge has then been calculated based on the expense which each machine engenders. A wide discrepancy in the cost of the same part may be found, and it will be noted, in general, that that part which the producer believes he should purchase outside, is the very part which he should continue to manufacture; and the parts which he believes he should maintain in his plant are liable to be the very parts that he should buy outside. The machine-hour cost method is, so far as the author knows, the only one which will provide a fairly complete solution for this problem.

No. 1933

DESIGN, MANUFACTURE AND PRODUCTION CONTROL OF A STANDARD MACHINE

BY RALPH E. FLANDERS,[1] SPRINGFIELD, VT.

Member of the Society

The paper describes the methods by which difficulties in manufacture and production control were avoided by the Jones and Lamson Machine Company. The company having passed through a period of increasing speeds and feeds, improvements in methods of doing work and controlling it, with satisfactory results as to total machinery time and direct labor cost, directed its attention to overhead which had suffered a considerable increase due to foremen, clerical work, cost and production offices necessary for the methods which had been adopted.

Reorganization commenced with a segregation of the products in manufacture so that separate manufacturing organizations and equipments were provided for each product, i. e., the Hartness flat turret lathe, the Fay automatic lathe, and the Hartness opening die; while a fourth organization was established for repair and special work. A redesign of the product was then undertaken to eliminate as many parts as possible and to standardize parts to fit several types of machines. The shop was arranged on a basis of departments by products. The author describes the turret-lathe shop and the chief manufacturing processes involved.

The routing and stock-room control are described by the author. Each order is nominally for two months' production. Rough- and finished-parts stock rooms are kept supplied with half-lots in reserve to prevent hold up in production, but there is no stock-keeper assigned to them. Standard routing is made up from previous experience reckoning on 80 *per cent theoretical machine capacity. With bi-monthly starting dates, both rough and finished stock get a complete count six times a year, and adjustments of volume of production are easily made.*

Under the present system the foreman has full power within his territory and can measure the efficiency of his department by the

[1] Manager, Jones & Lamson Machine Co.

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schedule of hours for a given rate of output. There is little need for cost accounting, cost of machines being determined on overall operation of the shop, costs of material, labor, overhead and fixed charges being divided by output. An hourly rate basis is used for wage payment.

In a recapitulation, the author points out the following principles: Standardization of product; a separate manufacturing equipment and organization for the product; departmentalization by product rather than by process; a recurrent production schedule; concentration of plant; minimized transportation; disturbing and difficult factors confined to purchasing; visual control of work itself instead of remote control by records; control by orders instead of by records; automatic control of inventories; cost figuring by analyzing total rather than by totaling innumerable details; a plain job for every man and full responsibility with it.

A LITTLE over twelve years ago the author became connected with the Jones & Lamson Machine Company. About that time — or a little earlier — the management of the company began an intensive campaign for the increase of speeds and feeds, improvement in methods of doing work and controlling it, and development of all the other lines of progress which in general come under the head of "scientific management." This work was guided at times by members of the company's own force, and at times by an outside organization of specialists.

2 The net result of this several years of effort was decidedly profitable. Speeds and feeds were greatly increased, and the total machining time and total direct labor cost of the product noticeably reduced — and this in the face of considerably increased earnings to the men. All of this is, of course, the expected result of successful work of this kind.

3 About four or five years ago, however, we lost the first keenness of our interest in direct labor cost and became concerned with the "overhead." With only a moderate increase in output, there had been a considerable increase in the number of foremen — functional and otherwise — and a great increase in the amount of clerical work they were called on to do. In fact, it had become necessary to provide many of them with clerks.

4 There was also a violent increase in the personnel of the cost and production offices. The attempt to control the whole organization from a central point led also to a flood of written orders and reports, which was evidenced by the multitude, variety, and size of our printing bills.

5 It was possible, at any time and at any point in the shop, to make a rough estimate as to the percentage of the men in sight who were at that moment actually engaged in the processes

of profitable manufacture, and to find that percentage quite unsatisfactory.[1]

6 Finally, the general statistics of the plant showed that the direct labor cost of the product was very small — as had been hoped; it showed the material cost as much higher; but the manufacturing overhead was largest of all — so large that with a moderate selling expense the percentage of profit was somewhat precarious. The percentages were, in fact, such that if the item of direct labor cost had been completely eliminated, there still would not have been any extraordinary profits at the selling prices prevailing.

7 We are not the only manufacturers, or the first, who have made this interesting discovery. But whenever and wherever made, this discovery is a matter of the highest importance, and should lead to a radically changed treatment of the problem of management.

8 It was evident that in our case progress toward efficiency in direct labor cost had reached the point where the "law of diminishing returns" became effective, and further effort in that direction would be unprofitable. We therefore transferred our attention to the other elements of cost, and particularly to the largest of them — the manufacturing overhead.

9 The overhead cost had not grown to its preponderant dimensions suddenly or through carelessness. It was the result of slow accretions of small expenses, each added to take care of some real difficulty in management in what looked, at the time, like the simplest and least expensive way. An examination of the various details did not indicate that they could be successfully improved by individual treatment. A major operation seemed to be "indicated," as the doctors say.

10 To put it briefly, the major operation decided on consisted in rearranging all the elements of the business so that difficulties were *avoided* rather than *overcome*. These radical changes are detailed in the following paragraphs.

Segregating Products in Manufacture

11 The oldest and principal product of the company is the Hartness flat turret lathe. Other important products are the Fay automatic lathe and the Hartness automatic opening die. A beginning in segregation had been made some years before in developing the automatic die into a separate business, with its own manufacturing organization and equipment. The Fay auto-

[1] This visual estimate, it may be said, is a quite useful and revealing practice, and every shop manager who has not acquired the habit should contract it at once. It should be made with a view to criticizing the management rather than the workers.

matic was first assembled in a department of its own; it was now in turn provided with a practically complete equipment for making all its parts and given separate quarters.

12 In the turret-lathe division the work was of two kinds—the manufacture of new machines, and a rather large service business in the supply of replacement parts for the machines (numbering some 15,000) now in the users' hands. Many of these parts were obsolete, being for machines of a design established in 1893. The others were of the then current design, established in 1905.

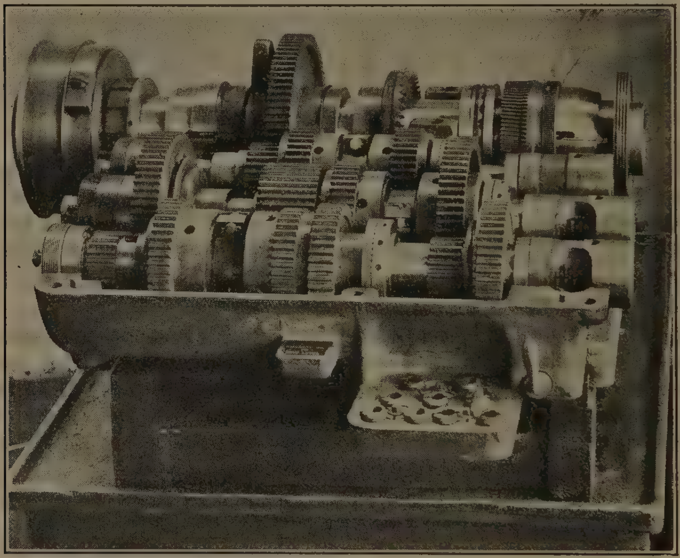

Fig. 1 Headstock of 3-in. Flat Turret Lathe

13 The final, and significant, act of segregation consisted in separating the service department completely from the turret-lathe production and putting it with all other special work into a division of its own. Here it was united with the manufacture of regular and special equipment for our customers, and the building of tools, fixtures, and special machines for ourselves.

14 Thus everything irregular, spasmodic, and unpredictable was assigned to this special department, while steady machine production, on a more easily predetermined program, was concentrated in the section devoted to machine building. By working toward regularity and routine we hoped to reach conditions under which a considerable element of the overhead expense would become unnecessary.

Redesign of Product

15 Meanwhile there was another element conducing to irregularity which had to be taken care of. While the department could be run under general orders to produce a given number of machines per week, there could be no foreknowledge far in advance as to how many of them should be "$2\frac{1}{4}$-in. machines," how many of them "3-in. machines" and how many of them "double spindles"—these being the shop terms for the three styles in which the turret lathe was built. These machines were designed on the same general plan, and many of the parts in the

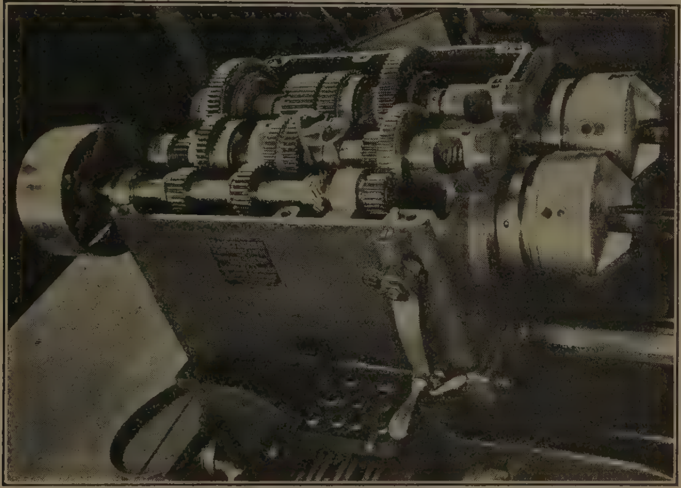

Fig. 2 Headstock of Double-Spindle Flat Turret Lathe

$2\frac{1}{4}$-in. and 3-in. machines interchanged, but this was not true to any extent of the double-spindle turret lathe.

16 A redesign of these three types was therefore undertaken to make them as nearly alike as possible, not only in their separate individual parts but also in their unit assemblies. This redesign was so successful that we are now able to assemble three basic sizes (or, with varying equipment, seven types in all) from the same parts with the following variety of major castings:

2 Headstocks	2 Turret Slides
2 Spindles	3 Turrets
2 Beds	

and a few minor parts.

17 Figs. 1 and 2 show respectively the 3-in. and double-spindle headstocks. In the elevations, Figs. 3 and 4, the same shafts are marked by corresponding letters. These shafts, with

their complete assemblies of gears, clutches, ball bearings, etc., as well as the spindles and their gears and boxes, can be interchanged in either headstock. Except for one or two minor points the man who assembles one of these shafts need not know into what style of machine it is going.

18 For a $2\frac{1}{4}$-in. headstock, with its higher spindle speed, the driving-gear ratio at A in Fig. 3 is altered, and a spindle with a different nose used. About the only criticism which could be directed against this redesign relates to using the powerful 3-in. drive on this $2\frac{1}{4}$-in. machine. It is much more powerful than is needed. *But it is cheaper to make it alike and heavier, than lighter and different.*

19 Besides bringing the design of the machines together, the variety of parts in a given machine was greatly reduced. For instance, the design of the friction clutches on the different shafts was made the same. These are of the multiple-disk type.

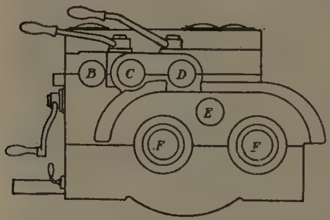

Fig. 4

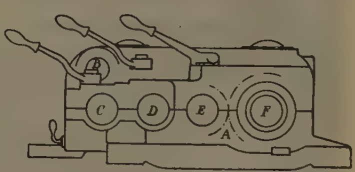

Fig. 3

Fig. 3 End Elevation of 3-in. Headstock
Fig. 4 End Elevation of Double-Spindle Headstock

On the fast-running shafts with light torque, thicker and fewer hardened disks are used. On the slow-moving shafts with heavy torque, more and thinner ones are employed. All of this work practically cut in half the total number of kinds of parts to be dealt with in making the full line.

20 The three basic sizes of machine having thus been unified in design, they became for all practical purposes one machine. The workmen scarcely know which size they are working on. They may be put through the assembly in groups of fifteen or twenty each, or mixed together indiscriminately, without making a break in the routine of manufacture.

21 This unification of the product was in reality a return, in a more developed way, to the first principles followed by Mr. Hartness, who confined the work of the shop from 1892 to 1905 to one machine and one size of machine. It is needless to say that now, as then, the range of the machines built was carefully determined to cover the largest practicable percentage of the lathe work of the world, in order to give the broadest possible market for our intentionally restricted line.

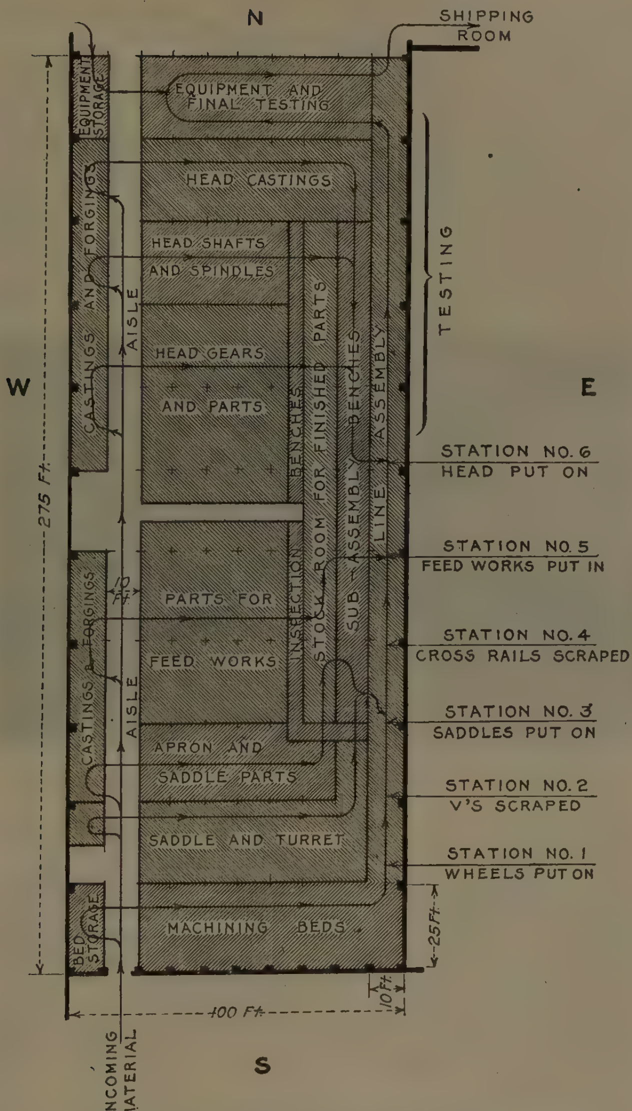

FIG. 5 GENERAL ARRANGEMENT OF TURRET-LATHE SHOP

Shop Arrangement

22 We now come to the most interesting physical feature of the problem — the arrangement of the shop.

23 There are two methods of arranging departments — by process and by product. By the former plan all milling is done in a milling department, all drilling in a drilling department, all turning in a lathe department, etc. The argument for this is that by specializing on a given operation the foreman and his assistants become highly skilled in that work, doing it better and more cheaply than if they were concerned with a wide range of operations. The disadvantages lie chiefly in the constant movement of work from department to department, with its consequent slowing up of the work flow, division of responsibility, and difficulty of control.

Fig. 6 Aisle of Shop Showing Bins for Raw Material

24 Arranging the departments by product means that any individual piece stays in a single department until it is completely finished, ready for assembly. Such a department may be arranged to complete all parts of a similar nature, no matter where found in the machine, or all parts of whatever nature belonging to a given assembly unit of the machine. Both arrangements are to be found in automobile manufacture, where departmentalization by product is the rule in the larger and more successful shops.

25 In this turret-lathe shop it was decided to departmentalize by product, primarily on the basis of the unit assembly, and only secondarily by similarity of parts. This arrangement was expected to prove strongest where the method by process was weakest; and its single disadvantage, lack of specialization by the foreman, was believed (and later found) to be largely illusory.

26 The space available was 275 ft. long by 100 ft. wide, divided by columns lengthwise, as shown in Fig. 5, into 10-ft. aisles, and crosswise into 25-ft. bays. Experience with this shop leads us to believe that about this width of shop and width of cross-bays are ideal for medium-sized manufacture. The columns should, however, be spaced to give 20-ft. aisles, as has been done in our later construction. The later buildings are also increased in width to 120 ft. Such shops may be built to any length required by the magnitude of the production. The building referred to is of steel-frame, sawtooth-roof construction and fireproof.

27 The general arrangement is indicated in Fig. 5. The incoming material passes north up a long aisle extending the full length of the shop. In the aisle on the left, next to the west wall, are the bins in which the raw material is stored (Fig. 6). The center

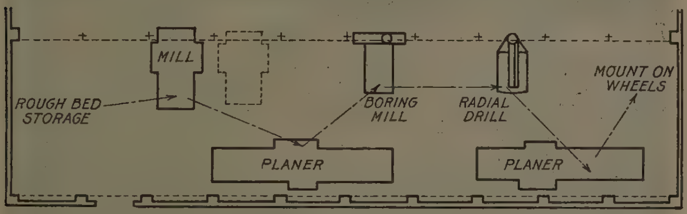

FIG. 7 LAYOUT OF DEPARTMENT FOR BED MACHINERY

of the shop is occupied by the various manufacturing departments which are, in order, reading from south to north:

Bed
Saddle (Turret Slide) and Turret
Apron and Carriage Parts
Feed Works
Head Gears and Parts
Shafts and Spindles for Head
Headstock Main Castings.

After that comes the Equipment and Final Testing of the machines, as will be explained.

28 In successive aisles toward the east, and running for practically the full length of the manufacturing section, come the Inspection Benches, Storage of Finished Parts, Sub-Assemblies and Line Assembly. From the Line Assembly the finished machines detour into the bay for Equipment and Final Testing, and out into the Painting and Shipping Room.

29 The flow of material is thus from the southwest corner north to the appropriate department, thence directly east across the width of the shop to final inspection and storage, thence to sub-assembly and line assembly. The line assembly moves north on the eastern side and out at the northeast.

30 Let us follow the progress of the bed. The arrangement of the machines for its department is shown in Fig. 7. The castings are stored at the west of the aisle. From there they are taken first to an Ingersoll mill, where the pads on the bottom are milled off, after which the bed is turned over and the cross-rails milled. Space

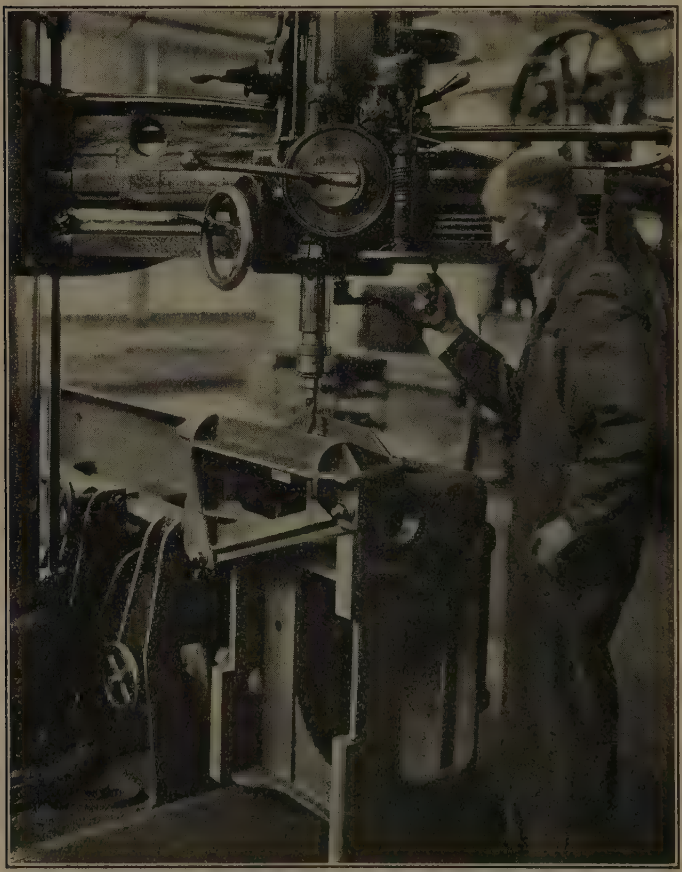

Fig. 8 Bed at Radial Drill

for a second mill is left as shown by the dotted lines in Fig. 7. The bed now goes to the roughing planer, and thence to the horizontal boring and milling machine. The next location is the radial drill, Fig. 8. Here the bed is held by the V's or special fixtures, and light sectional jigs, as shown, are used for the drilling.

31 The machining is now nearly completed, and the bed rests for a time on the floor. It then goes to the finishing planer to

scrape the V's — see Fig. 9. This is a final machining to remove all " wind " so that the hand scraper's job shall be one of finishing the surface rather than of removing material. The bed is now mounted on wheels instead of its regular legs, and is ready to start up the assembly line — of which more later.

32 The important things to observe are the permanently mounted fixtures (adjustable for the two styles of bed), the simplicity of the operations, and the unvarying routine of the procedure. The department is under the foreman of the line assembly, but as a matter of fact it is practically self-managing.

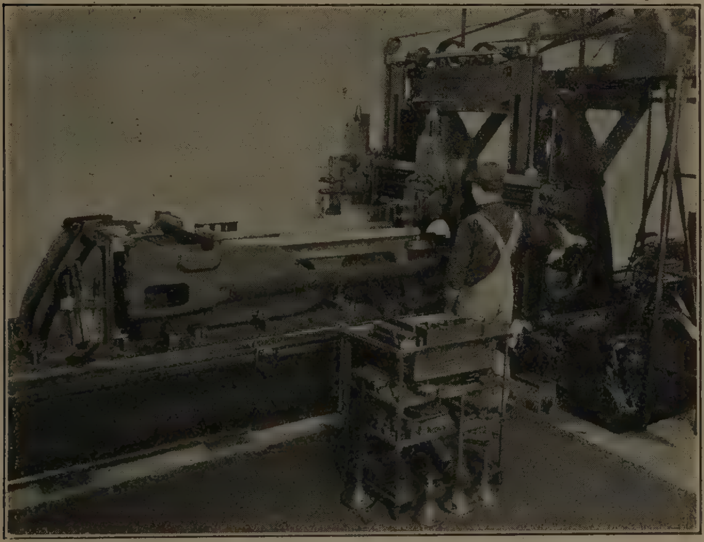

Fig. 9 Scraping V's of Bed in Finishing Planer

33 The " saddles " (turret slides) and turrets, machined in the next department, are also large pieces. The layout is shown in Fig. 10. When completed these parts shift north to the appropriate point in the sub-assembly aisle.

34 The apron and turret-slide parts in general comprise main castings, gears, shafts and studs, and milled steel parts. Fig. 11 shows a sub-assembly of apron parts. The layout for this department, Fig. 12, shows three lines of tools. The upper or northern line, comprising mills, driller, etc., takes care of the castings and milled steel parts. The lower or southern line, composed of turret lathes, machines the shaft, studs, and gears which, however, pass north at the end through the Fellows gear shaper and the Bryant internal grinder on their way to the inspection bench and the

parts storage. The central line is composed of miscellaneous machines common to the other two.

35 This arrangement exemplifies the subdivision of the departments on the basis of the character of the parts machined. A similar case is shown in Fig. 10, where the turrets go east along the south side, and the turret slides east along the north side. In this way the flow of every part can be made to approximate a straightforward progress from west to east, with only occasional and minor divagations. This makes for ease of control and orderliness in the department.

36 The layout of the department for machining feed-works parts is as shown in Fig. 13. Here again we have separate lines of machines for different classes of work, as indicated on the layout. From the machines the work is handed over to the inspection bench and thence to the stock room. From there it goes to the sub-assembly and line assembly.

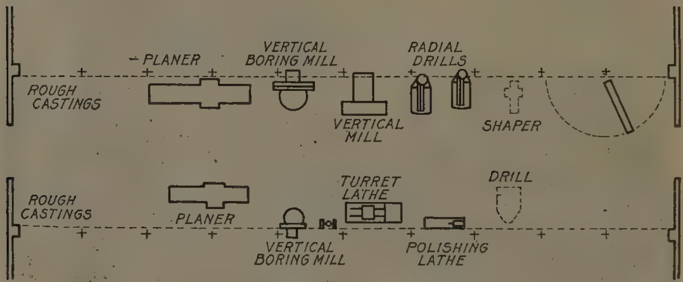

Fig. 10 Layout of Department for Turret and Saddle Machinery

37 The head-gear department is laid out as shown in Fig. 14, and the shaft and spindle department as in Fig. 15. A pair of Fay automatic lathes are kept set up with an adjustable, but standard, equipment for the shafts, as shown in Fig. 16. The succeeding spline millers, drills, and grinders are plainly indicated.

38 The head-castings department is interesting for its specialized equipment. The bases are planed on the bottom, then caps and bases are milled for the joint. After drilling together, the united headstock goes to a special boring machine, shown in Fig. 17, when all the shaft and spindle holes are rough and finish bored, reamed, faced, and tapped at one setting. The machine for this work costs much less than a standard boring machine and jig for the same purpose, and finishes all the holes at once instead of one at a time. This is an example of the economies possible in continuous production.

39 The head then goes to the radial drill, etc., for finishing operations, and thence to the sub-assembly, on a trolley beam around the north end of the finished-parts storage.

40 Let us now return to the assembly line, following the order shown in Fig. 5. The different operations are so divided as to give about equal lengths to each.

41 At the first station, see Fig. 18, the wheels are put on. At station 2 the V's are scraped. At station 3, the turret slide is put on, scraped to the V's, and the turret rescraped to the slide. At station 4 the cross-rails for the cross-sliding head are scraped, using special devices to bring them at right angles with and parallel to the V's.

42 At station 5 the oil pump and piping and the feed work are put in. The machine is now in a condition shown in the fore-

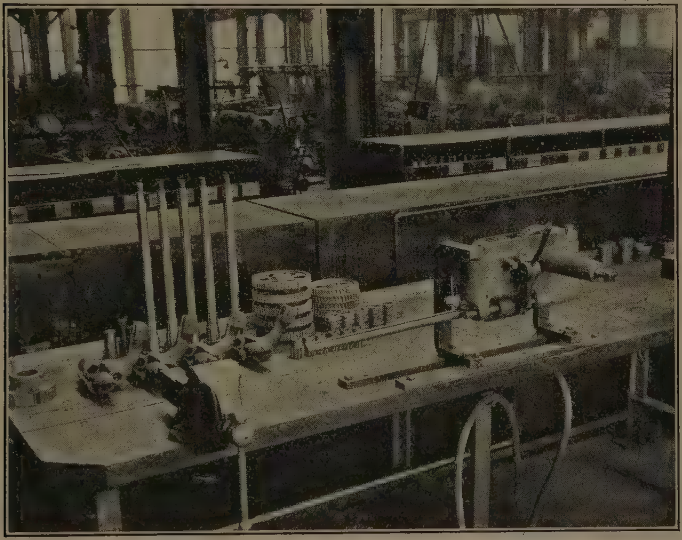

Fig. 11 Sub-Assembly of Apron Parts

ground of Fig. 19. Next the headstock is placed on the machine, as shown in the same illustration, and it is now complete. There are still, however, five stations remaining before reaching the department for equipment and final testing. Advantage is taken of this space to give the machines a final "run-in," they being belted to the motors shown overhead.

43 This running-in is opposite to the head assembly and is under the care of that group. Any difficulties or needed adjustments are taken care of there.

44 There is nothing special to be said about the department for equipment and final testing. The machines are mounted on their legs there, equipped with motors if required, and then with regular or special equipment as may have been ordered. Finally they are put through standard alignment and operative tests.

45 There are certain departments and operations not included in this layout. Steel storage and cutting-off are taken care of in one department for the whole plant. So are casting, snagging, cleaning, and filling. These come before the parts arrive in the turret-lathe shop. After the machines leave the equipment and testing department they go to a painting, boxing, and shipping room common to all of the products.

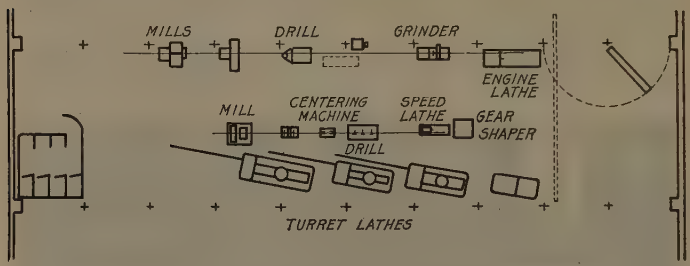

FIG. 12 LAYOUT OF DEPARTMENT FOR MACHINING APRON AND TURRET-SLIDE PARTS

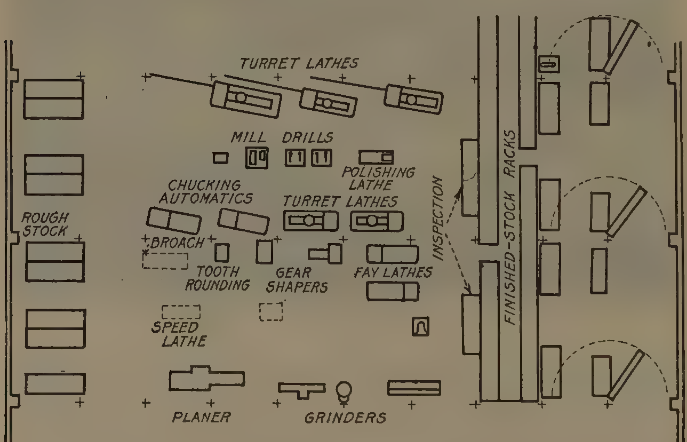

FIG. 13 LAYOUT OF DEPARTMENT FOR MACHINING FEED-WORKS PARTS

46 Automatic-screw-machine products are taken care of in one department for the whole shop; and so, finally is the hardening and heat treating. This latter operation is the only one making a break in the orderly flow of work through the shop.

47 With this description of the shop arrangement in mind, we can now return to a discussion of the way in which this arrangement and its accompanying methods of management operate to reduce the overhead.

Routing and Stock-Room Control

48 The orders for parts are routed through this shop in a predetermined and effective course, but without any machinery of control. The rough- and finished-parts stock rooms are kept supplied with an adequate store of parts to prevent any hold-up in production, but there is no stockkeeper assigned to them; nor does this lack of special supervision result in unbalanced or undesirably large inventories or in unexpected shortages. These desirable features are attained by a "program" scheme of ordering and routing all parts except the main castings.

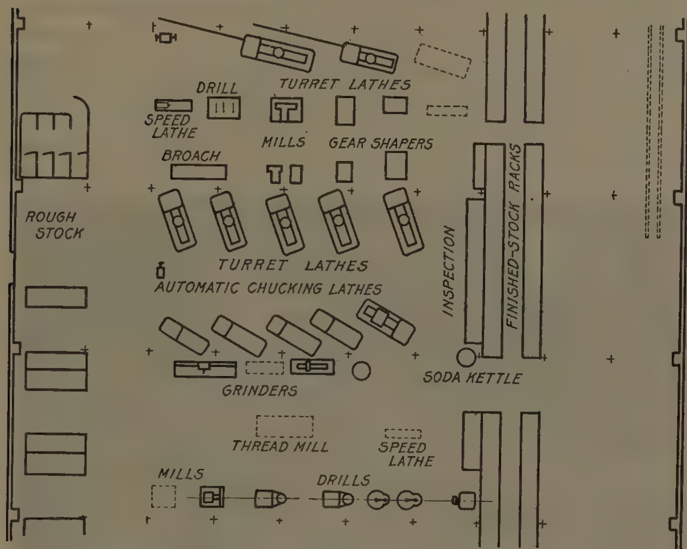

Fig. 14 Layout of Head-Gear Department

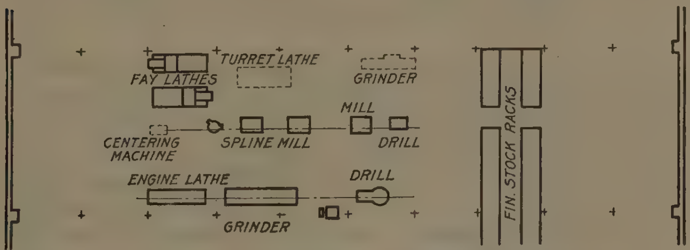

Fig. 15 Layout of Shaft and Spindle Department

49 Each order is nominally of a sufficient size to furnish enough for a two-month's production. An order for a given part

is thus placed in the shop every two months, or six times a year. The second period of the year (March and April) is an exact duplicate of January and February. A standard routing of all the parts passing through a given department may thus be laid out, from machine to machine, and duplicated for the next period and the next, and so on for the six periods of the year.

50 The routing was originally laid out on the basis of full production of five machines per day, on special sheets provided for this work. Fig. 20 shows sample pages from this routing book.

51 This standard routing is made up on the basis of our previous years of experience in speeds and feeds, and is arranged

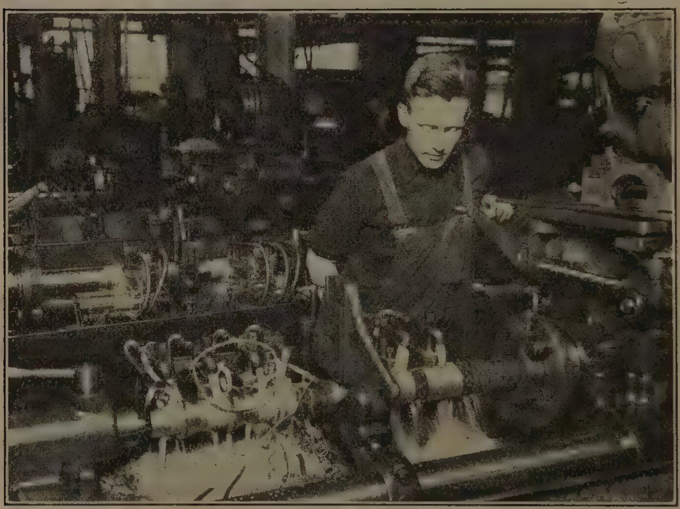

Fig. 16 Automatic Lathes with Adjustable Equipment for Machining Shafts

to put the orders through in the minimum time, with sufficient waits between operations to insure against trouble. All long waits, however, are eliminated, and with them the expensive items of storage space and idle capital for inactive stock. The ideal aimed at has been that of a small, fast-flowing stream of work instead of a large, sluggish one. Further insurance against trouble is offered by reckoning on only about 80 per cent of the theoretical machine capacity. The margin takes care of breakdowns, breaking in new operators, poor stock, etc. There is the additional possibility of running overtime in an emergency.

52 The original laying out of the schedule is a matter of juggling, judgment, and experience. A successful solution implies a minimum of equipment, a minimum of floor area, a minimum

inventory of work in process, a maximum rate of progress of the work, and *a simple and easily comprehended standard of achievement for the department.*

53 The finished routing schedule gives a starting date, repeated every two months, for each part. These starting dates, with other data, are recorded on a card shown in Fig. 21. If castings or forgings are required, the date on which they are to be ordered is also given, and on that date the foreman is held responsible for making a requisition on the purchasing department.

54 Note that a wide range of rates of production is provided for — from 3 to 30 per week. The rate of production is determined by the general management, and the shop takes its information from the corresponding line on the card.

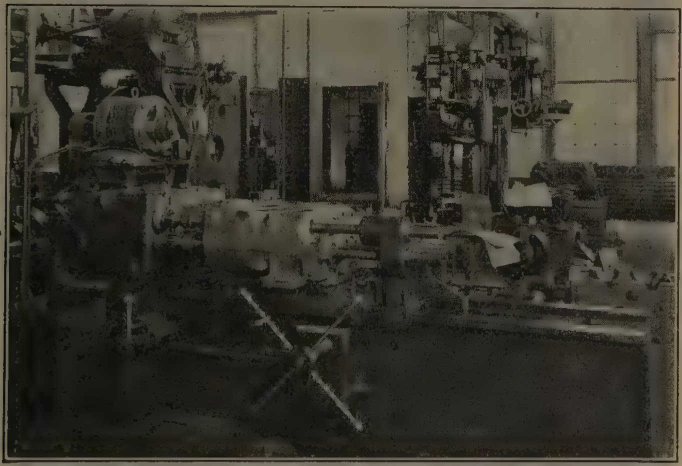

Fig. 17 Special Machine for Boring and Finishing All Shaft and Spindle Holes in Headstock at One Setting

55 Now there is supposed to be carried in the rough-stock bins a full half-lot of castings or forgings in reserve. The first step of the foreman is to count this reserve stock. If it is below normal for the scheduled production, he adds to the normal order enough to make up the deficit. If it is above normal, he decreases his normal order by this amount.

56 This half-lot in reserve is insurance against trouble. It takes care of poor castings, delayed deliveries, spoiled work, etc. It is particularly important from the standpoint of starting the orders on time. If only half of a lot has arrived, it is still possible to put the work into the shop by drawing on this reserve. The *sine qua non* of this method of production control is starting on time. Nervous tension, frantic struggling, and despairing clutch-

ing at straws must be eliminated from the production schedule and concentrated on the problem of getting started on time. If necessary, we must be willing to start out by dog team on a winter's night over the ten-foot snows of our Northern winter, to get from a distant foundry the castings needed to preserve the schedule! But there is no place for such desperate measures within the four peaceful walls of the shop.

FIG. 18 SCRAPING THE V'S OF BED

57 The amount of rough stock ordered is thus determined by the scheduled rate of production and an actual count of the reserve supply. The finished-stock room is run on the same principle. When a new lot arrives in the stock room it is supposed to find a half-lot in reserve waiting there. At the earlier period of starting the lot there should be a correspondingly larger amount, and this amount is set down on the card. If the reserve stock is low, the foreman increases the normal lot to make up the deficit. If it is in excess, he correspondingly decreases his order.

58 *There are no stock-room records.* Both rough stock and finished stock get a complete count six times a year, and a better knowledge is obtained than is possible with all the chances for error in the clerical work of the perpetual inventory.

59 *The stock is under continuous control.* Orders go out from the main office to work on a production basis of 30 or 75 machines a month, or whatever it may be, and automatically the stock adjusts itself to this production. Change in scale of production is made easy by the provision of a reserve of half a lot in both rough stock and finished parts. The scheduled production on the card of Fig. 20 may be increased a step a month, or decreased at the same rate. If the proper changes in labor capacity are made

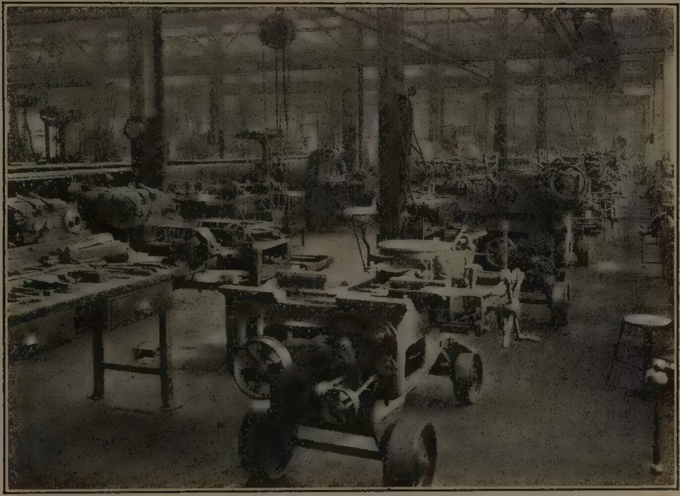

Fig. 19 Final Steps in Assembly of Machine

to correspond, the adjustment of size of lots and size of stock takes place easily with the half-lot reserve as a balance wheel.

60 This is startingly different from the old order of things with us, in which shortened production left us with a large stock, and increased production with an insufficient one. There is the accompanying financial advantage that the amount tied up in rough stock, work in process, and finished parts is automatically adjusted to the scale of production. It leaves for special decision only the major financial problem of how much shall be tied up in finished machines.

61 As there are no stock records there is no stockkeeper. The foreman of each department oversees the putting into and taking out of the rough stock. The inspector, located beside the stock room, O. K's the finished work and places it in the proper bins.

The bins are located opposite the point where the parts are finished and close to the point where they are assembled. One of the assembly men, detailed for the purpose, steps in and lays a group of parts on the top of the rack opposite the man who needs them when he needs them. It is possible to look down the aisle of the stock room twenty times and see a man in it only once.

The Duties of the Foreman

62 This scheme of management is essentially a decentralized one. By keeping the work completely within the department from

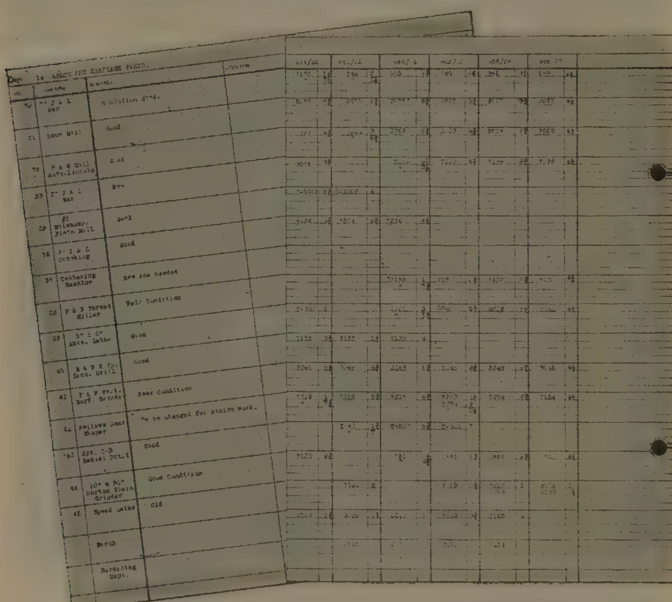

Fig. 20 Sample Page from Routing Book

rough stock to finished stock, we avoid the necessity for detailed control of its movements, particularly with reference to interdepartmental relations. These are minimized, relating almost entirely to hardening operations. Roaming about the shop becomes unnecessary, the telephone falls into disuse, clerks disappear, and a hundred chances for confusion are dissolved.

63 The foreman is bounded only by his production orders and his schedule. Within these limits he is king of his territory, with full power given to him — and full responsibility demanded of him. His job is his own in a way that is never possible under the other plan of departmentalization and other schemes of management.

64 Mention was made of the fact that only about 80 per cent of full production per machine was reckoned with in laying out the schedule. This will ordinarily leave some idle time. When such idle time appears imminent on a machine, the foreman goes shopping for work in the special or other departments. The service parts furnish convenient "knitting work." Since it is difficult to predict the call for repair parts, this branch of the business is handled in the old-fashioned way. The foreman can usually pick up something here to fill in his idle time.

65 He is provided, finally, with a definite yardstick with which to measure the efficiency of his department. The schedule requires a certain total number of hours for a given rate of output. If he requires more, he is not up to standard. If less, his

LIST NO. 7233 NO IN 2^{IN} 2 3^{IN} 2 2^{SP} 0

ORDER STOCK 1 odd START 10 even FINISH 28 even

MACHINES PER WEEK	NORMAL ROUGH STOCK	NORMAL FINISHED PARTS	NORMAL ORDER	TOTAL HOURS
3	10	16	20	68
4	14	22	28	92
5	18	28	36	116
7	24	38	48	152
9	32	51	64	200
12	40	64	80	248
15	52	82	104	320
19	66	105	132	404
24	84	134	168	512
30	104	166	208	632

Fig. 21 Card Used in Routing

management may be highly commended. In general, he will have to stick very closely to the definite routing prescribed by the schedule to maintain full production. On lessened production, with fewer men but a full complement of equipment, more latitude is allowed.

Cost Accounting

66 There is little need for detailed cost accounting. The cost of the machines can be determined sufficiently for price-setting purposes by reckoning on the overall operations of the shop — so much spent for steel, castings, forgings, etc., per month, so much for labor, and so much for overhead and fixed charges. This, divided by the output, gives a good cost per machine, *because the department is run on an even basis.* This simplified method of cost keeping is not the least of the advantages of this method of management; and the author believes it to be far more

accurate — as well as far less expensive — that the elaborate addition of the labor, material, and overhead for every one of the several hundred separate parts.

67 It should also be noted that the standard schedule presupposes standard costs with a given average labor rate and market cost of material. This standard cost derived from the standard schedule, set against the actual cost obtained as above, gives another gage of achievement.

Methods of Wage Payment

68 We are now working on a straight hourly rate basis. The effectiveness of workmen and foremen can in general be gaged by their ability to keep to schedule, and their rate of pay can be adjusted accordingly. The plan would, however, seem to be adaptable to premium payment, and perhaps still more so to piece-work or bonus payment. It would seem to be especially suited to the group-bonus scheme, as the whole organization is divided up into small groups, each of which has a definite section of the work assigned to it and within its control.

Recapitulation of Principles

69 In the foregoing mass of details some of the principles involved are brought out clearly, others only incidentally. It may be well to recapitulate them here.

1 *Standardization of Product.* This is the first necessary step. It is the prime requirement of successful manufacture that a design of the product be settled upon which will give a wide market over a considerable period of years. This is the definite predetermined simplification of the problem which makes the solution possible.

2 *A Separate Manufacturing Equipment and Organization for the Product.*

3 *Departmentalization by Product Rather Than by Process.* There is nothing new in this. It is the plan of the high-production automobile shop. The novelty lies in adapting it to a low scale of production — in our case to an extreme low point of fifteen per month.

4 *A Recurrent Production Schedule or Program.* It is this feature which permits the adaptation to small production, and also makes possible the measurement of efficiency and the simplification in control and cost accounting.

5 *Concentration of Plant.* Principles 3 and 4 can operate to cut down machine and building investment for a given production, or to increase production for a given

investment. It is worth while to decrease "the great open spaces" in a shop, if only to keep the work from taking root and going to seed in them.

6 *Minimized Transportation.*

7 *Disturbing and Difficult Factors Confined to Purchasing.* So far as possible, strenuous effort is made to have material ready to start on time. Then confusion is banished from the shop.

8 *Visual Control of the Work Itself, Instead of Remote Control by Records.* It is less than one hundred feet from raw material to finished machine. Within this short distance the work is to be found (except while being heat treated). It cannot escape. The foreman has it under his eye until it is assembled and rolls into the shipping room. In control by records some superexecutive functionary sits at a desk and pores over cards and slips of paper. A few hours of this and the eyes glaze and the judgment falters. It is better to deal with things rather than with the records of things.

9 *Control by Orders Instead of by Records.* Records are always late. Direct the thing before it happens and save some tons of paper and miles of scribbling. And yet success in giving proper orders was only possible because we had kept records for years. Perhaps it is best to consider records as a necessary evil, to be discarded as soon as possible.

10 *Automatic Control of Inventories.* This is important. It is the very commonest of errors to put into inventories resources which belong in dividends.

11 *Cost-Figures by Analyzing Total rather than by Totaling Innumerable Details.* Cheaper and surer.

12 *A Plain Job for Every Man and Full Responsibility with it.* A man can do more, and prove that he has done it, thus establishing a basis for increased earnings.

13 And lastly: *Don't Try to Overcome Difficulties — Avoid Them.*

Acknowledgments

70 The general shop layout here described was devised by Mr. Henry S. Beal, assistant general manager of the Jones & Lamson Machine Company. The details of the machine arrangement and the arrangement of the production schedule are the work of Mr. Louis Hronek, assistant superintendent. Mr. George A. Perry, chief engineer, was responsible for the redesign of the product on which this method of manufacture is based, and for the mechanical details of the production scheme. Credit is also due to the personnel of the Fay lathe department, who tried out many of these methods under more difficult conditions.

DISCUSSION

Robert P. A. Taylor.[1] No attempt will be made in this discussion to criticize any of the features of management set forth by the author. The writer's object is only to clarify a few points by asking questions and to point out some of the limiting conditions.

A cardinal principle of design is economical manufacture. This is no new feature. One of the oldest machine-tool builders in this country had the draftsman indicate on the detail drawings the various machining operations, thus in effect routing from the engineering department. The author designs and builds up his scheme of manufacture and control at the same time that the product is designed. This scheme of control is based entirely upon: (*a*) a standard product; (*b*) the assurance of a wide market over a considerable period of years; and (*c*) the assurance of a constant and fairly steady demand.

If these conditions cannot be met, the whole plan becomes inoperative. If the author will give us answers to several questions, a little more light may be thrown on the various features of his plan. These questions are:

1 In preparing the routing schedule, was the time in hours for completing the scheduled number of parts or units arrived at by estimation, by former records of performances, or by time study?

2 Would it not become necessary to have time or production studies made in the event that there is a radical change in design? If not, how would we go about making up the hour requirements or time schedules on, say, a new design of longitudinal feed mechanism?

3 Would the author recommend this type of management if there were more than three basic designs of machine tools involved? Does he feel that it would be applicable to a plant making six models of milling machines and three models of grinders?

4 In the hours necessary for a given rate of output of units or assemblies the management has a yardstick by which to measure the foreman; who knows whether this measure is correct, or what the possibilities are without a time-study unit? The same remarks apply to the men under the foreman.

5 "A plain job for every man and full responsibility with it." This is true of management's relation to the foreman, but how about the management's relation to the men under the foreman?

6 Upon what are we going to base increases and promotions if there are no records of performance? There hardly can be intelligent judgment without performance records.

The problem of changing models should not involve any especial difficulties. The designs of turret and automatic lathes no doubt change so slowly that it will not be difficult to realign and retool

[1] Crompton & Knowles Loom Works, Worcester, Mass.

the various departments and assemblies without making any great changes from the original plans and layouts, and consequently without great expense. The likelihood of change of design is an important consideration if the type of management described is to be used.

Cost accounting serves not only as a basis for determining selling price; it should also serve as another means of analyzing the product to locate excessive costs, either in material, design, or manufacture. In the case of so small an item as a handwheel handle, some being purchased and others made in the shop, the two costs varied only a few cents, but the usage was 15,000. A cost department need only point out a comparatively few such discrepancies per year to pay its way.

There is no apparent reason why the following features could not be included in the plan of management outlined by the author:

1 Bonus plan of wage payment
2 Time study (made necessary by 1)
3 Maintenance of record of performance
4 Maintenance of cost records.

These should be accepted only on a pay-as-you-go basis and they will much more than pay for themselves if properly handled.

A word of caution should be added. Before a department is cut off, a certain function eliminated, or short cuts (so-called simplifications) are taken, it is a good plan to ask: Will this function ever be needed? If so, provide for it somehow in the new scheme. The inertia of the old carries us along for a time, but sooner or later, if the function has not been provided for in the new organization, there will be trouble. We cannot throw our whole production mechanism, as it relates to the individual worker, out of the window, and expect to get the same results from him. We may coast for a time, but all the while production will be slowing down.

The author is to be congratulated on conceiving and carrying into effect the type of management described. The machine-tool industry as a whole would do well to follow his example, where conditions and designs permit.

Ralph G. Wells.[1] The paper illustrates the value of simplification of control methods, and is indicative of the present trend toward less elaborate practice. Engineers have been urging simplification of product and operation processes, but many have failed to apply this same principle to the control systems installed. Control methods are frequently too elaborate. There is a need for simplification of systems and a reduction of administrative overhead.

Much of the existing antagonism and prejudice against scientific methods results from installations which have been too complicated

[1] Research Service for Manufacturers, Boston, Mass.

and too expensive. Such installations do more harm than good. Sooner or later they are discarded, and one more obstacle is placed in the path of progress. Each instance of this kind adds to unreasoning prejudice, the most difficult handicap in bringing about better control methods in a plant.

Par. 6 of Mr. Flanders' paper is worthy of emphasis. In seeking ways to reduce expenses and unit costs, the larger possibilities for saving may be found in manufacturing and administrative overhead. The increase in these items during recent years has been out of proportion to the savings effected. In spite of the improvements in management methods, the amount of burden which must be added on to costs in many plants is not offset by a corresponding reduction in other items. It is encouraging to have this point emphasized and to have such an excellent illustration of what can be done. Instead of becoming disgusted with all control methods, this firm had the vision and courage to work out from under too elaborate a system in a most effective manner.

It would be interesting to know whether it would be possible to obtain as good results with the author's present simplified production-control methods, if the plant had not previously had a more elaborate system of planning and scheduling by means of which the organization and the men in each department acquired the habit of working to a definite schedule and keeping work flowing smoothly and speedily with the minimum of delay. In other words, is it not necessary sometimes to put in a more elaborate control system at first than is permanently necessary, in order to get the organization out of the rut and to train it to work in accord with prearranged plans?

In connection with the method of cost keeping, the question may be raised as to whether or not the present method gives sufficiently detailed information for comparing and analyzing costs in a search for further reductions. Not infrequently, under such methods as are described, it is possible for waste and idleness to creep in without its being detected as promptly as if detailed costs were kept and compared with predetermined standards. Perhaps the standards of material and of working time give a sufficient gage to check up waste of time or material if it occurs.

A third question would be as to whether, if for any reason the morale of his organization and working force were to break down, the author's present system would be sufficiently effective to keep the organization in line or to bring order out of chaos.

Charles Meier.[1] The excellent paper of the author suggests several questions as follows:

[1] Treasurer and Works Mgr., The Acme Machine Tool Co., Cincinnati, Ohio. Mem. A.S.M.E.

1 What arrangement is made for storage and testing of jigs after the operator has finished using them? The system generally in use is that jigs are checked as soon as the operator has finished with them and before they are put back into the tool crib.

2 How many assembly foremen are necessary for the arrangement described, or is each department foreman held responsible for the unit, from the rough storage to the finished assembly? Also, is there an inspector appointed for each group or department?

3 How many inspectors and how many foremen are there with the present system, as compared with the former system of manufacture?

4 In the event of a decline of business, which might necessitate a reduction of the working force, would it not be difficult to find men who would be versatile enough to perform all operations, in order to keep the various groups balanced?

5 What number of parts in raw stock and finished stock would constitute the half-lot if the production basis varied from 30 to 75 machines a month.

If the author would supply answers to these questions, his paper would have much additional value.

J. O. Keller.[1] The system of management described by the author, Mr. Flanders, conforms in principle to the known fundamentals of scientific management. Nevertheless, such management might be considered rather daring. Many persons would not consider this type of management unique, as perhaps 50 per cent of the plants in this country are similarly operated, in that they have no cost system, no central storeroom, no stores records or clerks, etc. The management system described by the author differs from these plants, in that there is conscious control. The management knows exactly what it is trying to accomplish, and approaches its problems in the mental attitude essential to scientific management.

The 1912 majority report of the sub-committee on administration of the A.S.M.E., entitled The Present State of the Art of Industrial Management, would lead us to believe that the so-called Taylor System contained three essential principles, namely, (1) the division of labor; (2) the transfer of skill; and (3) a certain mental attitude, without which no form of management is scientific.

Roughly speaking, division of labor means the dividing of mental and physical effort into portions and in such a manner that unskilled men can do their portions with increased skill, where before such men could not be utilized.

Transfer of skill, in turn, means the creation of a device or devices which will permit unskilled men to do work which other-

[1] Professor of Industrial Engineering, Pennsylvania State College, State College, Pennsylvania. Jun. A.S.M.E.

wise would require skilled help. The use of jigs, fixtures, templets, patterns, automatic machines, adding machines, slide rules, and, in fact, all forms of machinery, illustrates this principle.

The application of these two principles in any plant tends toward economy, but at the present time our knowledge of this fact is mostly qualitative, and what we now need is more quantitative knowledge. We do know that the extent to which either of these principles may be applied in a given plant is limited by the quantity of similar goods to be produced at any one time. Hence, standardization is employed to increase the quantity of similar goods.

Dean Kimball's illustration of this point is excellent. If a skilled man is required to drill parts one at a time without the use of a jig, the labor cost may be one dollar per piece. With a jig, the labor cost might drop to four cents per piece. If the jig costs ten dollars, and there are only four parts to drill, it would be cheaper not to make and use a jig, but for 100 parts the jig would be economical. At some point between four and 100 either method might be employed.

The division of labor is limited in the same manner, but many plants fail to recognize this point. They probably assume that if it is good to subdivide jobs at all, it is better to do it fully. These plants soon find themselves burdened with so many special clerks that all economy from employing division of labor is lost. The economist would say that both division of labor and transfer of skill operate under the law of diminishing returns. Mr. Flanders, with common sense and good judgment, has eliminated much unnecessary indirect labor, labor which may have been necessary at one time during a period of change in methods.

The majority report of the A.S.M.E. sub-committee, above mentioned, showed that all of the devices in the Taylor System were simply applications of the above two principles. For instance, the Barth slide rule represents transfer of skill, functional foremen represent division of labor, etc. It stated, however, that the Taylor System evidently contained a new element which was designated as a "certain mental attitude." This attitude realizes that experimentation and research are essential to success, not only with regard to materials and equipment but also to the human side of industry, which up to that time had been neglected. This mental attitude further realizes that no matter how perfect present methods and devices may be, they will need improvement sometime in the future. This attitude is represented in the management of the Jones and Lamson Machine Company, as reported by the author.

In 1914 Frederick W. Taylor stated that most people, including his own followers, had mistaken his devices for his principles. By his devices he meant the stop watch, the functional foreman, the slide rule, the graphic chart, etc., and he predicted that the

stop watch would be considered obsolete in 1920. The writer at the time did not believe this statement, and many then believed that Taylor was merely trying to be sensational. He further stated that his devices were his own crude method of carrying out his management "principles" which, he claimed, would be true forever.

The writer now believes that Mr. Taylor was right, and that the particular mental attitude which believes in experimentation for any plant is essential to successful management. But like the other two principles, the amount of experimentation and research is also limited by the quantity of similar products to be produced. Perhaps a very small plant cannot afford to spend any money on research as such; but at least it can be on the lookout for improvements in methods, can keep up to date, and can make conscious efforts to continually improve the weak spots as they are discovered. It is this attitude of the management, as interpreted from the author's paper, that makes one believe that the Jones and Lamson plant is operated under a real system of scientific management.

WALTER M. KIDDER.[1] The case of the Jones and Lamson Machine Company is of unusual interest. It is a typical case, in the course that eventuated through the stages of perfecting, along modern lines of design and general organization, raising operative efficiency, and finally the development of oversystematized routine. A second phase of especial interest lies in the ultimate reaction following a realization that "system" had become top-heavy.

Because the solution adopted as a result of that reaction is attractively simple, and has worked out satisfactorily, there is a strong possibility that similar methods may be attempted by others who seek relief from excessive overhead or insufficiency of profits. The warning must be given that the methods described will not work profitably in all cases; they surely will fail to give the hoped-for favorable results except under conditions to which they are fully adapted.

The excellence of the principles of design, of shop arrangement, and of conferring both authority and responsibility upon able foremen, as described by the author, Mr. Flanders, is beyond question. So also is the objective of seeking to avoid difficulties rather than having to overcome them.

That the company has been able to accomplish this latter purpose in such large measure may be due in part to:

(*a*) Ample floor space
(*b*) Adequate plant facilities
(*c*) Able foremanship
(*d*) Workmen of native intelligence, and above the average in disposition toward team work.

[1] Consulting Production Engineer, New York, N. Y.

Any one who is familiar with the contrast in each of these points between a representative Vermont machine shop and the majority of those situated elsewhere, particularly in cities, will understand why Jones and Lamson succeeded and why copyists of their simplified and emasculated routine are liable to fail.

Analysis of the description given by Mr. Flanders suggests the following friendly critical comments:

Shop organization *by product* involves the employment of more machinery and more floor space than shop organization *by process*, assuming equally effective control of movement of product in both cases. Hence, greater capital investment is required in the first instance, and the idle hours of machinery will be greater. To some extent this will offset the saving in clerical labor achieved through substituting *visual control* of production in place of *control through records.*

Payment of producers on a straight wage basis fails to furnish incentive to high individual output and quality of workmanship. Both quality and quantity therefore depend, first, upon the disposition and skill of workmen, and, second, upon pressure exerted by the foremen. The temptation to foremen to employ the discredited " driving " methods of the past is increased.

Visual control of production with a sufficiently frequent check by actual count seems ideal, but if product is varied, and complex rather than simple, and with average foremen, almost certain failure is invited. The method will work in conditions duplicating those of this Vermont shop, but may not work under conditions which are materially different.

If a foreman is negligent or overlooks an item, tardy starting of process work it not automatically prevented. Neiter is sluggish movement, nor congestion; nor is an equilibrium of parts, nor progress in their processing, assured. Actually the method described seems to fall short of " avoiding " these difficulties, and if they are not encountered, it is not through the adequacy of the method but because of the vigilance of the foremen.

Dependence upon the uncontrolled, uncoördinated judgment and independent action of individual foremen creates a condition saturated with risk. That it does not do so with the Jones and Lamson Company is a tribute to their foremen, and to their harmony of purpose and effort with each other and with the management.

Par. 51 states that it " is arranged to put orders through in minimum time." Tangible means to insure this apparently are lacking. It seems to depend upon the alertness, diligence, and shirt-sleeve activity of the foremen. If the foremen are not of that sort, the method might yield only correspondingly poor results.

Par. 52 states that " successful solution implies a minimum equipment, minimum floor area, minimum inventory of work in

process, and maximum rate of progress of work." But shop organization by product and visual control of movement of product inevitably entails the exact reverse.

Par. 58 stresses the fact that there are no stock-room records. If there are permanent, comparable, and cumulative records of output of parts, they are not mentioned.

Apparently no classified records of spoilage are kept. Dealing with such occurrences on the instant has certain advantages, but even more effective in rooting out and permanently dealing with causes are adequate records which enable analysis and the determination of both causes and personal responsibility.

Abolishing cost records of parts and disregarding their fluctuations seem to constitute an omission to utilize a vital means for reducing cost and for consistently holding it to the lowest practicable level as to every part. Present knowledge of costs appears to be limited to that required to fix selling prices. Unit reductions cannot possibly be controlled effectively merely through knowing total machine costs. This opportunity for enhancing profits by direct action is apparently neglected.

In conclusion, it may be observed that this case of the Jones and Lamson Machine Company is a perfect example of success, up to an unstated point, with methods which in themselves are unsafe when applied to most manufacturing establishments. The reaction which abolished records for control of production and for gaging current fluctuations in unit costs, was carried to an extreme wholly unsafe to copy unless exceptional conditions are 100 per cent favorable.

The problem in organization, after the point of accumulating essential data for establishing standards of performance under known conditions has been reached, is to formulate methods adequate to control, without yielding to the temptation of excessive elaboration.

Few executives are so well informed in the field of methods for control and for enhancing profits by analytic methods, that they can formulate methods without falling into the errors of either too much, too little, or the wrong kind of routine relating to control of production and of costs.

The Jones and Lamson Machine Company was fortunate in having fundamental conditions of comparatively simple product, relatively small volume, and exceptional quality of personnel, and has therefore shown a certain success. But it may be quite possible and, in fact, seems probable, that their profits can be further enhanced by conservative application of means which, at small cost, will add to both *amount* and *certainty* of net profits.

Copyists will be prudent to content themselves with emulating their thoroughness in developing productive efficiency in the shop, in standardizing and simplifying the design of product, and in the splendid "straight-line" layout of the shop floor. In the rest of

the practices described by Mr. Flanders there lies too great possibility of trouble for shops located outside of the state of Vermont with its superior labor and foremen.

Wendell S. Brown.[1] It is refreshing to note in this paper, among other things, wholesome reference to the wisdom of scrutinizing the cost of management itself. Many are the industrial concerns which have at one time or another been confronted with the realization of the gradual — sometimes almost insidious — but nevertheless alarming and disproportionate accretion in management expense. Only by eternal vigilance can it be avoided.

In order that the term "scientific management" may become synonymous with common-sense management, cost relationships must be clearly analyzed and broadly gaged into at least the four fundamental factors making up the right-hand side of the equation:

Value of product − margin = capital charges + cost of raw material + cost of management + cost of labor and other miscellaneous service such as power, light, and heat.

J. N. Heald.[2] The writer believes that the plan outlined by the author will be efficient in reducing overhead costs, which have increased noticeably during the past few years. However, it involves separate shops for each different type of machine produced, and almost involves duplicate sets of tools and jigs for producing repair parts. These complications, to a degree, would offset the gains otherwise secured. However, there is great value in the author's suggestion and in many lines of work it could be adopted to great advantage.

Luther D. Burlingame.[3] The question as to whether a plan such as that suggested by the author will prove of advantage in any particular factory depends on the conditions and on the nature of the work. There is much to be said in favor of such a plan where conditions permit of its application. All will agree as to needs for bringing down and keeping down overhead expense. This plan suggests methods which, if they cannot be adopted as a whole, may by partial application prove of value in keeping down such expenses. The company whose organization is described,

[1] Mgr., F. P. Sheldon & Sons, Providence, R. I. Mem. A.S.M.E.

[2] Treasurer, Genl. Mgr., The Heald Machine Company, Worcester, Mass. Mem. A.S.M.E.

[3] Industrial Supt. and Patent Expert, Brown & Sharpe Mfg. Co., Providence, R. I. Mem. A.S.M.E.

by reason of its limited line of machines, is especially well adapted to show up to advantage the use of such a system.

At the works of the Brown and Sharpe Manufacturing Company certain lines of manufacturing are carried on in almost identically the same way as described in this paper. An illustration of quite a different product, where such a method is used successfully, is the manufacture of micrometer calipers. In the manufacture of milling machines, the plan of having individual units so designed as to be used in as many sizes and kinds as possible has also had consideration. Thus, the speed case used on seventeen sizes and kinds of machines is made in five sizes. The question as to how far the plan of using a larger size of unit for smaller machines could be economically adopted depends on the number required; it might be plotted as a series of curves showing, for small quantities, a greater economy in the plan suggested by the author, but reaching a point where the lines would cross and the expense of manufacture for large quantities would, when a certain quantity production is reached, show that economy would result from having each design suited to its own size.

The extra cost resulting from moving the work to various departments for successive operations is largely avoided when all work is done on a single floor, regardless of the particular plan.

The question of the need of keeping detailed costs would be somewhat different in a new organization where carefully kept detailed costs are not always available.

Although the method of paying workmen would not appear to be a necessary feature of the plan, it is thought that paying as far as possible by piece or premium rates stimulates increased production and gives the fairest possible return to the workman, dependent on his efforts. In the matter of having certain lines of work done in separate departments, for example, screw-machine work, gear cutting, planer work, etc., both for machines in process and also for parts made for sale, much can be said for having these as organized departments rather than having such machines separated for each line of manufacture.

Generally speaking, the disadvantages of the plan suggested by the author would be rapidly multiplied as the variety of work is increased, and while it is believed that the proposed plan has much to commend it for use under conditions such as outlined, it is believed that the author himself appreciates many of the limitations above pointed out and would advocate its adoption only in such cases as the conditions warrant.

Henry P. Dutton.[1] The author's description of the methods used in the reduction of overhead expense, and the philosophy

[1] Professor of Factory Management, Northwestern University, Evanston, Ill.

which he expresses, constitute a clear statement of a situation which has demanded attention for some time. The practice of management passed through a period of development in which the emphasis was upon mechanism. In this period there was a tendency to tie up every possible contingency by means of a form. This involved a multitude of clerks, with other clerks to check them and supervisors to check the checkers. This swing to an extreme of mechanical control, of course, represented an attempt to organize a situation hitherto unorganized, and to get away from an undue dependence upon employees of low-grade ability.

Clerical detail cannot always be avoided. Wherever numerous small and varied orders must be handled, wherever all parts of a complex program are so related that an intelligent decision cannot be made locally, some sort of system must be used to reinforce the powers of the executive by devices for digesting facts and visualizing them, or for supplementing the memory of the deciding executive.

But in numerous cases, such as the situation in the production of the standard lathe, so clearly described by Mr. Flanders, there is a better solution than the use of elaborate control. Wherever possible it is certainly advisable to secure control rather by simplifying the situation so that decisions can be made by ordinary men unaided. The writer would expect such a system to be effective not only in reducing clerical expense, but by making most effective use of the initiative and intelligence of the man in the ranks, a force we have been prone to neglect. Methods such as those described by Mr. Flanders seem to offer one of the practicable remedies for the condition of monotony and lack of incentive in industry, of which we hear so much.

The shop layout outlined by the author is apparently almost ideal in its directness of flow. Perhaps too general an assumption is made as to the possibility of using the product layout, since in many plants such product departmentalization would be secured at the cost of considerable overequipment.

Henry H. Farquhar.[1] The results arrived at by the Jones and Lamson Machine Company seem to represent the highest development of common-sense management.

The writer is particularly impressed with the extent to which the best current practice has been followed, without the overelaboration and superfluous refinement which is often found as part of an attempt to develop advanced principles of manufacturing control. Emphasis in this company's development is in accord with that which must exist in any manufacturing establishment

[1] Assistant Professor of Industrial Management, Graduate School of Business Administration, Harvard University, Cambridge, Mass.; Cons. Management Engineer, Belmont, Mass.

which hopes to continue in the lead of business in the future, that is:

1 A careful analysis of the particular needs of the particular industry as regards the general policy to be followed; the determination of the part which each particular department of the business is to play in the execution of the policy, and the selection of particular methods and mechanisms, which under local conditions will produce most satisfactory results.

2 The recognition that certain management functions may be and should be thoroughly standardized to the end that more attention may be devoted by the manager to those features which cannot be so automatically controlled.

3 The rigid standardization of machines, of tools and equipment, of organization or the relationships between the various members of the personnel, of production schedules, and of material programs as dictated by manufacturing requirements.

4 As a result of the above, elimination of detailed records as regards materials, follow-up of work in process, costs, and similar activities in every case where standard operating ratios may be determined and used as a simpler measure of performance.

As the job of the manager grows more and more complex, more and more thought must be given to simple yet effective means of control, and to means by which every department of the business may be made to function for the good of the business as a whole.

G. E. Hagemann.[1] This paper is an important contribution to the growing volume of evidence showing that scientific management, as developed in principle by Frederick W. Taylor, and applied by him and a large number of engineers, has reached the healthy period of conserving its important industrial gains. In the development of a new science, as in the conducting of an extensive campaign in warfare, there comes a period during which the advances must be definitely secured by careful but vigorous measures. In the few years which have followed the world war this tendency has been especially marked in the field of scientific management.

Broadly speaking, the problem before the Jones and Lamson Company was to apply to the large overhead which grew out of the installation of a management system, the same principles which were originally applied to the installation itself, namely, doing the work better, more quickly, at lower cost, and with more direct application to the problem at hand. The conservation of the gains already accomplished by the installation of a management system in production operation, in this case has been essentially the application of motion study to overhead. It is an elimination of red tape, and a short cut to direct results. The term

[1] Assoc. Editor, *Management and Administration*, Ronald Press, New York, N. Y. Mem. A.S.M.E.

"motion study" is here used according to the comprehensive scope given to it by its originators, Frank B. Gilbreth and Lillian M. Gilbreth, to include mental processes as well as actual physical operations.

Perhaps the modifications introduced by the author and his associates of the Jones and Lamson Company do not give as firm a grip or control upon manufacturing operations as the more elaborate system which they have supplanted. But the problem of manufacturing is not so much one of complete control over all operations, as it is getting the most economical returns from the investment after it is put into the business.

The impression created by careful reading of the paper is that a slight sacrifice has been made in the absolute control of manufacturing, but that as a result a much more satisfactory and profitable method of operation has been developed. This is bound to work far more successfully in accomplishing the real objects of scientific management, which are to save time, confusion, energy and motion, and put more wages into the pockets of workmen and more profits into the coffers of the company.

From the experience that the writer has had along the lines of management in other companies, it would seem that the work of the author and his associates should be credited with other advantages in addition to those which he correctly and justly claims in his paper. One of these advantages is the restoration of at least a part of the old element of personal contact between management and foreman, and foreman and workers, which scientific management in its first developments naturally tended to lose. As the Jones and Lamson plant is operated at present, there is undoubtedly a greater feeling on the part of the workmen that they really have some share in the management of the enterprise, and that work which they do in saving the company money by any means is sure to be directly understood, appreciated, and eventually rewarded. One of the greatest savings in manufacturing is always produced when the workers are brought to this feeling by a sincere and easily understood policy on the part of the management.

Another advantage evidently results from the ability of the foreman to control the operation of his own department directly. Most of us have been making the mistake of giving the foreman too little credit for intelligence, and too much credit for knowing facts we think he should know about, but concerning which we have never enabled him either to get complete information, or fully to understand what he does get. The Taylor form of functional organization has seemed to work best in cases where the functions themselves were clearly understood and applied, but where two or more of these functions were combined in the person of an individual foreman, well founded in the functional idea, and fulfilling a double duty.

At first thought it might appear that weaknesses exist at various points in the system which the Jones and Lamson Company now operates. One of these weaknesses might lie in the failure of the foremen to exercise proper supervision over the reserve stock of parts. They are charged with this duty instead of leaving it to a perpetual inventory system.

It appears from the author's description that in connection with standardizing the product, shop operations have also been standardized and probably some form of time study applied to most of the individual jobs. The method of manufacture now followed undoubtedly not only gives complete freedom to further development work along this line, but also may show up inconsistencies, wrong methods, undue time allowances, etc., even more prominently than under a rigid control system. Therefore such losses are probably now more quickly detected and more effectively remedied. The coöperation of the workman in bringing about such savings also seems to be more definitely assured under the present plan.

One feature has been omitted from the paper which would strengthen the presentation, namely, the savings in actual dollars and cents brought about. Although intangibles are hard to measure, the books of the company will surely show the savings in due course of time.

Altogether, the author's statement of Principle 13, "Don't try to overcome difficulties — avoid them," sums up the entire story. In other words, eliminate useless work. The preponderance of overhead over direct labor costs and material costs is probably a common situation in all industries. The work done at the Jones and Lamson plant marks, not what appears on first thought to be a backward step in scientific management, but rather a long step forward in conserving its gains, and a strict adherence to Taylor's fundamental principles. Thus one more case is added to the already innumerable examples proving the absolute soundness of Taylor's principles when applied fully and energetically, but adapted to the need of each particular manufacturing plant.

E. P. Bullard, Jr.[1] Overhead has a faculty for creeping in unexpectedly in many disguises. In this paper it appears, first, in the title, which smacks strongly of overhead of a high degree. It also appears in the standard drawings with established tolerances, material specifications, jigs, tools, and fixtures for production of this kind, which imply planning, time study, costs, and records, all of which call for overhead.

Indeed, the proper shop arrangement for a plan of this kind can be provided only when certain essentials, such as the methods and costs of manufacture, have been used in their correct relation to each other, as obtained from previous records of each and every operation.

[1] Pres., Bullard Machine Tool Co., Bridgeport, Conn. Mem. A.S.M.E.

Placing raw material in bins, available to the various departments, requires control of a high order, if no check is to be made of its outgo. Replenishment without accounting would be optimistic, to say the least. The flow of material through a department, from the raw to the finished state, requires an established sequence of operation so timed that all parts come together at a predetermined time and place for assembly. The rate of flow through a department will depend upon the time required for the longest operation, and all other operations must conform thereto.

For the machine under discussion, the bed casting probably represents the longest time required for machining an individual piece. Assuming the longest operation on this piece to require two hours, such time would represent the rate of flow. The time for centering the headstock spindle should not exceed two minutes. Assuming this to be the shortest operation, such time would represent the maximum flow rate. Therefore, it would be possible to produce four beds and to center 240 headstock spindles in an eight-hour day. However, only four headstock spindles are required, leaving unused centering capacity for 236 spindles. The above illustration merely shows the unbalanced condition which is occasioned by such methods of manufacture. There are many other operations which would show similar conditions, and overhead would appear here in the form of idle equipment, excessive floor space, and inefficient operation.

In Par. 58 reference is made to the fact that no stock room records are kept. Is not the stock room your bank? Hence, should not similar precautions be exercised, i.e., no withdrawal without authorized instruments? Are not the contents of storerooms a deciding factor of business and essential to the preparation of financial statements? Does the elimination of stores records assist in keeping a balanced stock? Admittedly, no.

Apparently the author presupposes that inventories of raw, in process, and finished parts remain constant. Thus he argues that the cost of completed machines is represented by total payroll, material purchases, and expenses for that division. These conditions, however, do not obtain. Therefore any costing of a period of varying manufacture would neither be reasonably accurate nor give comparable results. Inaccuracy would obtain in the separation of costs of the three types of machines made.

From an accounting standpoint, it is necessary to begin with the values of finished machines, finished parts, raw materials, and work in process. These values must be determined from actual costs or from estimates.

Starting with a fixed value, as above, the process is to charge to the department producing $2\frac{1}{4}$-in. machines, 3-in. machines, and double-spindle machines, all the labor, overhead, and material; and then to divide this total by the number of completed machines for a given month; i.e., if the total expenditure was $50,000 and

50 machines were completed, the average price would be $1000 each. The cost of all machines would be the same unless some provision was made whereby a comparative cost might have been predetermined on the basis of, say, 10, 15, and 25, thus making the 50.

It is assumed that parts in stores, materials, and work in process shall be in exactly the same condition at the beginning of each period, a condition which is rather unnatural, for there is certainly a variation in costs, owing to fluctuations in labor, prices, overhead, and materials. It might happen that the sum of $50,000 expended would place into stores an increase of 25 per cent over what there was at the beginning, or even 25 per cent less than at the beginning; but there would be no variation whatever in the price of the units, for all of the elements are poured into a funnel and spread over the completed machines.

It is the general custom to take a period of 12 months for actual closing of books and figuring of profits, and, in order properly to determine the value of the inventory at the end of the period as compared with the beginning of the period, it would seem necessary actually to count and price each individual piece of finished parts and raw material, and to estimate the value of the work in process, all of which would be extremely difficult and expensive.

By a funnel-spread, it would be a very simple thing to build up an inventory of a very large sum and far in excess of what one would be able to eventually realize for the product. A paper profit would thus be shown, on which it would be necessary to pay a tax to the Federal Government.

The scheme, as outlined, places the burden upon the foreman. It is he who must determine the quantity to be produced, ascertainable only after an actual count of the number of pieces of any given machine in the storeroom, and of raw material in the stock room, and he also must oversee production. As it would be impossible for any one man to do all that, he must have assistants — and if assistants are necessary, it would certainly require but little additional time to compile an accurate record of the findings, which could be referred to at all times, and a perpetual inventory of items could be maintained without any additional cost.

It would be possible, of course, to have a predetermined price for each item and if the inventory were absolutely maintained, piece for piece, the general spread over completed machines would be quite correct. However, there would have to be a bulk debit or credit to provide either for the greater or lesser cost of the parts produced, which would be handled in exactly the same manner as we now take care of our unabsorbed overhead.

While a monthly statement could be made by taking the predetermined or estimated costs of the various machines completed,

it would be so unreliable that it might indicate a handsome profit, whereas there actually was a big loss.

The idea might be successfully executed, provided that each piece could be produced repeatedly for the same identical material, labor, and overhead costs, and that, at the start of each period, every piece of work then in process were in identically the same position as were similar parts at the beginning of the preceding period. If it had been planned to complete five machines each day, then these machines must actually have been completed.

It would seem that the cost of obtaining any figures by which it would be possible to show the gain or loss of the year would be just as high in determining the estimated value as it would in determining the true value, with the further disadvantage that, while with the ordinary method we are constantly in touch with the situation, with the physical inventory at the end of the year nothing is known until it is all taken and computed.

The real point, as we understand it, is an effort to control the overhead. This is certainly very desirable, for there is no doubt whatever that overhead is a more fixed item in a plant than direct labor. We believe, however, that the author's plan would not easily eliminate overhead to the extent which he assumes, as a careful consideration of the issues would show.

Richard A. Feiss.[1] Management, in the sense that it is set forth both in this paper and in Colonel Babcock's paper, Production Control,[2] for complete control requires big men and big leaders. Big things always require big men, and management is a big thing. The development of management along scientific lines at the present time is particularly difficult by reason of extraneous conditions. These conditions have affected the minds of management superiors, that is, the administrators. They often cannot conceive of the things that management does in the same sense as do those who have studied it as a profession and a science. The great differences that seem irreconcilable, are not so by nature, but are made so by our background. For this reason progress must be slow. We must go ahead, but at the same time wait for the coming generations to see our point, and to make it possible to accomplish what we have started.

In management, as in other things, we are accustomed to make comparisons of things that exist with others, or with things as we think they should be, as distinguished from the scientific approach, which makes comparisons only with the kind of thing or kind of performance that is scientifically determined attainable. Such an attitude is fundamental in considering the paper under discussion.

[1] Vice-Pres., Mgr. of Mfg., Joseph & Feiss Co., Cleveland, Ohio.
[2] See p. 667.

R. H. Lansburgh.[1] The writer is in substantial agreement with the general principles of the author's paper, but believes that the author did not mean what he implied in the statement that in time a point is reached where direct labor costs are relatively negligible and overhead costs are the only ones to be considered. In many organizations this doctrine, if pursued to its logical conclusion, could only lead to what happened in one organization with which the writer is familiar. In this particular case an overhead ratio of one to six had been established, and in order to put on one clerk charged to overhead it was necessary to add six machinists that were not needed. It should be emphasized that principles applicable to certain conditions in a small factory in a small Vermont town, may be entirely inapplicable to manufacturing organizations that are larger and situated in wholly different environments. If an attempt is made to blindly copy and apply these principles, it may result in disaster.

A point that the author did not emphasize sufficiently is the fact that in his plant raw material is less than 100 ft. distant from the finished machine. This is one of the most important factors in the scheme of management described by the author, but there are many industries, such as paper, linoleum, and steel, in which it is not possible to have less than 100 ft. between raw material and final product.

The author further states that cost figures obtained by analyzing totals rather than by totaling numerous details, are cheaper and surer. This procedure is not applicable to most products, and furthermore, cost figures are meant to provide more than a selling price.

John H. Williams.[2] The paper and the discussion have largely centered around the question of the extent of paper records and the extent of management control. A means of determining the desirable amount of paper work is to realize that the only possible purpose paper work can serve in management is to augment the memory and to facilitate the coöperation between different persons who must exercise a controlling function. Any paper work that goes beyond those two requirements, is, and deserves to be called, red tape.

The Author. The author would emphasize the fact that the particular scheme described in the paper should not be regarded as a panacea for all kinds of manufacturing problems. It does not apply to cases where the product cannot be controlled as to design, or where it is subject to the whims of fashion. Neither

[1] Secretary of Labor and Industry, Commonwealth of Pennsylvania, Harrisburg, Pa.
[2] Consulting Management Engr., New York, N. Y. Mem. A.S.M.E.

does it apply to businesses made up of a large number of separate items never assembled into one uniform mechanism. It does apply, however, to a large range of manufacture and particularly to the manufacture of machinery of ordinary size in all sorts and varieties. It is only on the particular work to which it does apply that the author urges its adoption in any form.

He also desires to remove any impression that the paper implies criticism of management engineering as it is known today. He distinctly wishes to imply that the management engineer has provided control schemes for businesses which ought not to exist at all in their present form; the expert has often been asked to provide for the control of a business where the product ranges through from 50 to 500 sizes, varieties, kinds and types, when there should have been but one, two or three. The difficulty lies with the original management of the company itself.

In such a case the main effort of the company should not be expended in systematizing its business in its present form, but in fundamentally changing the business itself in the direction of specializing on a smaller number of products with a corresponding constructive effort on their sale. Such a policy, generally followed, will reduce the cost of manufacture, and tend to reduce the retail cost to the public of a very wide range of useful and necessary articles. Of course, such standardized production does not always apply to objects of pure beauty as distinguished from objects of usefulness, nor is the policy proposed always consistent with the progress of change and invention; but the author is strong in the belief that a very large area of the productive field is ripe for this simplification and standardization. It is as a contribution to this particular area of industry that this paper has been presented.

It should also be especially observed that the needless complication of business has demanded an unusual mental capacity on the part of the management engineer, and an unusual and rather rare type of personality, of which Mr. Taylor, Mr. Gantt and Mr. Gilbreth stand as shining examples. It is the rareness of this type that has held back the development of management. It is only by deliberate simplification of the problem that we of less outstanding genius can carry on the work which they have left undone.

Before giving detailed answers to the various criticisms and suggestions raised in the discussion of this paper, fairness requires that a statement should be made of the exact present condition of the system of production control here described. So far it has been applied to a very small scale of production. This is one of its most encouraging features. It has resulted in an overhead cost that is only about 25 per cent greater, measured in cents per hour, than the average overhead costs of a period when our output was three or four times its present volume. We feel, therefore, that at least one of the desired results has been effectively obtained.

The program method of scheduling has been in effect on the work over a period of several months with the exception of one feature. We have not yet in this department definitely predetermined on what particular machine a given job shall be done when several machines are available, but since we have carried out this practice in previous years we have no doubt but that in the future we shall be able to preplan easily and simply the machines on which the work shall be done.

While the author has read over the discussion with care and will in many cases refer to specific criticisms and give the name of the writer presenting them, yet the same objections are in some cases presented by so many people that this cannot always be done.

In answer to the questions put by Mr. Charles Meier, it may be said that small jigs are kept in the tool room and cared for as in any ordinarily well-run manufacturing plant. Large, heavy jigs are stored on the floor of the department.

One foreman takes care of the machining of the beds and of the assembly line. A second foreman takes care of all the subassembling except that for the headstock, which is in charge of the foreman of the headstock department. There is one head inspector with as many assistants as necessary, and they are used all over the department as the head inspector may direct. The single exception is that the duty of inspecting and testing the finished machine is a special one devolving upon men trained for the purpose.

The number of inspectors is about the same as with the previous system, and the number of foremen is cut about one-third. Two of the foremen also are working foremen.

In the event of a decline of business necessitating a reduction of the working force, we keep those who are versatile or whom we wish to train to be versatile.

With regard to Mr. Meier's last question, the author tried to make this situation clear in Par. 59 of the paper. Whether the production basis is thirty a month or seventy-five a month, the raw stock and finished stock normally in reserve is supposed to be one-half of the normal lot for that rate of production. We would not go at one jump from thirty to seventy-five machines a month. As stated in the paper, we would go only one step a month under any normal changes in business. That is, referring to the card in Fig. 21, if we were building seven machines a week and wished to increase our capacity, we would give orders the next month to step it up to nine machines a week, and the following month to twelve machines a week, and so on. These changes in output are not so rapid but what the rough and finished stock in reserve will absorb them.

Answering Mr. Taylor's questions, in preparing the routing schedule, the time in hours was arrived at by estimates based on former records of performance. Radical changes in design seldom

require parts of a radically different character from those previously made, but where they do require some completely unfamiliar operation, a new time study would have to be made.

In the event that the author were considering a plant for making six models of milling machines and three models of grinders, he would be inclined first to see if it were not possible to bring a large number of units of the milling machines to a common design, and try to do the same thing on the grinders. If this were possible, he would make a grinding-machine department and a milling-machine department, each with practically its own complete equipment, and endeavor to run them on the basis described in the paper.

With regard to the yardstick with which to measure the foreman and the output, this yardstick is based in our case on time estimates derived from past experience. The important part about this whole thing is that the rate of production attained in the past, whether 100 per cent or not, was very creditable as compared with the topheavy overhead expenditures which accompanied it. Also, our reasons for not worrying very much as to whether we have gotten down to the last few fractions of a cent in time-study refinement are that the efforts are not worth while as compared with the very large reduction possible in the overhead. The real measure, after all, of the usefulness of the foreman and the workman under this scheme is his ability to stick to the schedule and thus promote the easy, orderly, and profitable progress of the work through the shop, irrespective of whether that progress in certain details is quite as fast as it ought to be. The schedule as a whole makes it fast enough to be profitable.

The author fails to see that there is any less responsibility devolving on the individual foreman, as indicated in Mr. Taylor's question, than there is in any other system of management which endeavors to plan things ahead of time; and so far as increases and promotions can be based on records, the good record in this system of management is that of the man or of the foreman who keeps up to schedule and thus contributes to the orderly production of so many machines per week at a reasonable cost.

Mr. Taylor, and a number of others, have been troubled about the comparatively rudimentary nature of the cost figures. Getting detailed cost figures on operations where such figures have been kept over long periods of years is a sterile, fruitless operation so far as making large savings in cost of manufacture is concerned, particularly when compared with the immensely larger savings to be made from attacking the overhead directly. Why worry about the cents when the dollars are melting away?

Above all, the author would not hesitate to cut out a function even though it might sometime be needed, as Mr. Taylor suggests. He would cut it out on suspicion, and only put it back later if it were found impossible to get along without it. This does not mean

throwing our whole production mechanism out of the window. It means retaining that which has proved to be essential, and discouraging the rest.

We are often deceived by our success in putting the time and effort of a high-grade man on some detail of cost accounting, and thereby making a definite saving in that area of investigation. That saving will only be maintained so long as the original high-grade and highly paid investigator is on the job. It is the inevitable tendency of such detailed investigations to degenerate into rust and rot as soon as the original high-grade man has turned his attention to something else. Do not get too enthusiastic over the savings made by the skilled man. See if they can be maintained over a period of years by the work of the ordinary every-day clerk on whom routine duties finally devolve.

Mr. Wells' suggestions are interesting. It is probable that we could not have had as good results as we have had with our simplified control methods if we had not previously gone through a more elaborate system of planning and scheduling.

As mentioned in connection with other criticisms, the present method of cost keeping will not ordinarily give sufficient detailed information for comparing and analyzing costs in search for further reductions, the reason being simply that the rich field for further reductions lies not in these direct costs, but in the overhead. However, every time card, every material card, and every element of cost is recorded and saved, so that at any time, if detailed cost studies need to be made, the material is all there. We are not, however, at the present time working it up as a matter of routine.

Mr. Kidder seems to fear that the plan of production control suggested will require more floor space and machinery. In our own case the result has been surprisingly opposite to this. We found that the laying out of a standard production program as a more or less academic job, fitting the whole production schedule together like a picture puzzle or working it out like a game of chess, produced such an effective use of the machinery we had that each element of it could be used for a very much larger percentage of the total working hours than had ever been the case previously. Our standard schedule, therefore, calls for a higher number of machines per week produced from our present equipment than we have ever been able to get in practice.

The only conditions under which this scheme called for added equipment were in the highly specialized tools like broaching machines, tooth-rounding machines, spline millers and other such tools, where one machine had previously been sufficient for the whole plant. The dividing up of the plant into different departments for different products would require that in general each department should have its own machines of these highly special types.

With regard to the quality of our foremen and of our workmen, it is certain that we are proud of both, but if the author were manager of a plant under similar conditions in regions which are supposed to be less Elysian than his native soil, he would expect this scheme of production to make things as much easier with unpromising material as it does with that of a higher grade.

With regard to floor space occupied, it should be noted that the rapid flow of the work, made possible by the scientific production program, requires much less floor space than under the old scheme. If we had gone into this plan five years ago, we would have saved ourselves many tens of thousands of dollars on now unneeded shop structures, built when costs were at their highest.

Any method of wage payment is possible with this scheme, as stated in the paper.

There is no automatic means to prevent the foreman from falling behind. However, his dates are given him in plain black and white, and it is easier for the superintendent, and the foreman himself, to see that he is falling behind his schedule than it is where the instructions do not stare him in the face. It is the cases where the foreman is failing that will mostly engage his attention, and that of the production superintendent. That is, in fact, the work for which they are hired: to look after the points of weakness and failure.

There are no stock-room records, and we have been running very successfully without them. Classified records of spoilage are, as a matter of fact, being kept. Sometimes we look after this item of cost and sometimes we do not, depending upon whether or not we are passing through a period where it is serious.

Mr. Hagemann is right in laying stress on the restoration in part of the old element of personal contact between management and foreman, and foreman and workers. There is still another immensely important element in the scheme, and that is the personal contact between the worker and his work. The importance of having individual workmen acquainted with the history of a piece from raw material to finished machine, and having it all under his eye, cannot be overestimated from whatever angle of the problem of management it is viewed.

The author hopes at some future time to remedy the omission that Mr. Hagemann notes: namely, the actual savings in dollars and cents brought about. The best that could be done on that subject at the time the paper was written is the more or less vague comparison given in the fifth paragraph of this closure. Since the time when we began scientific production control, we have never worked under such a low schedule of production as we have during the year or so past. For that reason direct comparisons are difficult.

Mr. Bullard's criticisms are interesting, but it is probably impossible to answer them to his satisfaction. It can only be said

with regard to the storage of raw material, for instance, that the placing of the reserve of forgings and castings in the department which uses them has not resulted in any terrible loss. No one concerned would now care to go back to the old clerical way of looking after this matter, which now appears to look after itself so well.

His criticisms as to the varying rates of flow are not quite clear. They are perhaps based on the idea of having everything come together at a given assembly point at a given time. As a matter of fact, however, a careful reading of the paper will show that our assembly is continuous, so the problem is not that of having every part done and complete at a given point at a given time, but of keeping a minimum supply of all parts continuously on hand. In doing this, of course, some lots have to be started many weeks before they are needed, while others can be started only a few days before the reserve supply is expected to become low. Furthermore, in a part like the headstock spindle, which has some very long operations and some very short ones, it is perfectly advisable to do as we have done and to overlap the long operations; that is to say, the rough turning of a lot of spindles can start shortly after the boring operation has started on them. It does not need to wait until all the spindles are bored out. A vast amount of telescoping of the longer operations was involved in laying out the standard program. The fears of large inventories, burdens on the foreman, and the necessity for clerical assistants are entirely groundless. These are difficulties which have been cured by this scheme.

With regard to the monthly statements, they have a quite high degree of reliability simply because manufacturing and assembling operations are continuous rather than cyclic. Otherwise his criticisms would be justified. Finally, with regard to Mr. Bullard's fears, whatever a careful theoretical consideration of the issue may show, the actual practice has resulted in a reduction in overhead.

Mr. Lansburgh's reference to the overhead ratio of one to six is interesting. While ratios have been mentioned in the paper, no real account is taken of them. The fact is that the absolute results of our campaign to reduce labor were satisfactory, and the absolute overhead costs were exceedingly unsatisfactory irrespective of any ratio between the two.

In conclusion, the author realizes that this problem and solution has been presented in this paper as though it were a problem in statics which was solved once for all, and required no further change or development. That, of course, is not true. In the days after the paper was written and before it was presented minor changes and adjustments had taken place. In the period since the presentation of the paper and the final editing of this discussion, still other adjustments have had to be made. These changes and adjustments are not accidental and occasional features, but a part

of the business of management; but they have not as yet appeared to us to affect the usefulness of the fundamental principles on which this scheme has been based.

If these changes in adjustments should ever become sufficient in volume or interesting enough in kind to warrant presentation, it might be wise for the author to offer to the Society a second paper which would not be concerned with the statics of management, but rather with its dynamics.

No. 1934

THE TEMPERATURES OF EVAPORATION OF WATER INTO AIR

AN EXPERIMENTAL DETERMINATION OF THE LAWS GOVERNING THE DEVIATION OF THE ACTUAL TEMPERATURE OF EVAPORATION FROM THE THEORETICAL

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Member of the Society

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Non-Member

This paper contains a report of experiments carried on at Cornell and at the Case School of Applied Science to show that under ideal conditions permitting true adiabatic saturation the wet-bulb temperature is actually the temperature of equilibrium represented in the heat-equilibrium equation, and also to show how and to what extent the wet-bulb temperature observed under ordinary conditions varies from the theoretical equilibrium temperature.

Appendices deal with the process of adiabatic saturation, an approximation of the effect of radiation in raising the wet-bulb temperature above the theoretical, a description of the adiabatic saturator and the experimental methods, and a description of the apparatus and experimental methods used in determining the variation of error in the wet-bulb temperature with variation in air velocity and wet-bulb temperature.

Introduction

One of the authors, W. H. Carrier, has previously presented, in a series of papers[3] before the several American engineering societies, discussions relative to the theory of evaporation. The

[1] President, Carrier Engineering Corporation.
[2] Physicist, Carrier Engineering Corporation.
[3] Rational Psychrometric Formulas, Trans. A.S.M.E., vol. 33, 1911, p. 1005. The Temperature of Evaporation, Trans. Am. Soc. Heat & Vent. Engrs., vol. 24, 1918, p. 25. The Theory of Atmospheric Evaporation; With Special Reference to Compartment Dryers, *Jl. Ind. & Eng. Chem.*, vol. 13, no. 5, May, 1921, p. 432.

Presented at the Annual Meeting, New York, December 1 to 4, 1924, of The American Society of Mechanical Engineers.

matter contained in these papers was in part a thermodynamic interpretation or explanation of the common phenomena of drying and cooling by evaporation. It was first shown in the paper entitled, Rational Psychrometric Formulas, presented before this Society at the Annual Meeting of 1911, that a fundamental equation of heat balance may be established between any evaporating liquid and the surrounding atmosphere into which it is evaporating. Specifically, the law deduced from this equation of heat equilibrium stated that a liquid evaporating freely into an atmosphere and receiving heat only from that atmosphere, eventually reaches a constant equilibrium temperature, which can be altered only by changing the total-heat relation between the liquid and the atmosphere.

2 This equilibrium temperature is based entirely upon the known physical properties of the gas and the evaporating liquid, and is commonly termed the *wet-bulb temperature* due to the fact that the extent of cooling by evaporation and the resultant temperature may be measured by a thermometer, the bulb of which is kept wet by the evaporating liquid. It has been found that the wet-bulb temperature throws a very interesting light on the fundamental laws involved in all commercial evaporative processes such as the drying of materials, the cooling of liquids with air, as in cooling towers, the cooling of air or gases by means of an evaporating liquid, and in certain chemical processes; and perhaps of the greatest popular interest is the effect of the wet-bulb temperature upon human physical comfort.

3 The experimental data herein presented prove conclusively that the temperature of evaporation is not an accidental or indeterminate temperature, as some have supposed, but has a very precise and definite limit which in practical application is very closely approximated and which is dependent upon the ultimate equilibrium of heat conditions as brought out in the papers previously mentioned, and as subsequently presented in this paper.

4 Although it has been recognized that observations of wet-bulb temperatures are in error with respect to the theoretical temperature of equilibrium, exact data as to the amount of this error and its variation relative to temperature and effective velocities of air have heretofore been only approximated from observations made over rather narrow ranges of conditions. During the past four years the authors have conducted exhaustive experiments with special and carefully designed equipment at Cornell University. Also, during the early part of the present year, special apparatus was sent to the Case School of Applied Science where the actual experimental observations were made by senior engineering students, Messers. Worley and Wilcox, under the general supervison of Professor Stark and the authors. The purpose of the experiments has been; first, to show that under ideal conditions permitting true adiabatic saturation, i.e., maintaining a

constant total-heat relation during saturation, the wet-bulb temperature is actually the temperature of equilibrium represented in the heat-equilibrium equation; second, to show how, and to what extent, the wet-bulb temperature, observed under ordinary conditions, varies from the theoretical equilibrium temperature.

5 The results of the experiments have verified the equation for equilibrium temperature and have shown the error in wet-bulb temperature, observed under ordinary conditions, to decrease with increased wet-bulb temperature and with increased effective velocity over the wet-bulb. It is believed that the results now obtained not only have supplied exceedingly valuable and exact information for use in psychrometry but also have been analyzed in such a manner as to throw a very interesting light on the physical processes involved in evaporation.

6 The general effect of both increased velocity and increased wet-bulb temperature in decreasing the discrepancy between the theoretical and actual wet-bulb temperature was shown in the original papers on this subject previously cited (Par. 1). The deviations at various velocities, as presented in these papers, were based on experiments over a small range of comparatively high velocities, while the deviations at various temperatures were computed from theoretical considerations. Both of these effects are now shown experimentally to be correct in direction but quite in error quantitatively. In other words, the general theory is upheld but the values have been considerably altered. The direction of these errors undoubtedly holds true for any liquid evaporating into any gas. Quantitatively they must necessarily depend upon the properties of the liquid and the atmosphere into which it is evaporating.

7 It was also originally assumed by Carrier that the errors observed in wet-bulb depression were due entirely to radiation from surrounding objects and conduction along the thermometer stem. This more recent study of their magnitude and manner of variation would seem to show that while at high air velocities this is approximately true, and it is apparent that radiation errors undoubtedly play a part at all temperatures, yet the major effect at low air velocities over the wet surface is due to the limiting rates of diffusion of the vapors through the surface film and the diffusion of heat from the atmosphere through the film to the liquid surface. (See Table 7, Appendix No. 2.)

The Equations of Heat Balance and Relative Rates of Diffusion

8 Two general methods have been proposed for the study of laws underlying the equilibrium temperature which is established in an evaporating liquid which obtains its heat of vaporization from the surrounding atmosphere alone, in other words the wet-

bulb temperature. The first of these methods may be termed the *statistical* method which was first proposed by James Apjohn[1] in 1836 and was first quantitatively and mathematically analyzed by Carrier[2] in 1911. The second method may be termed the *historical* method which was adopted by Regnault[3] and first formally analyzed by W. K. Lewis.[4] The first method involves the equation expressing the increase in the heat of vaporization as equal to the decrease in the sensible heat of the atmosphere from which the heat is extracted. This may also be termed the method of *heat balance.* The equation is:

Latent heat absorbed = Sensible heat lost

$$r'(w'-w) = (C_{pa}+C_{ps}w)(t-t') \quad . \; . \; . \; . \; . \quad [1]$$

where

r' = latent heat of vaporization per unit weight of vapor at the resultant temperature t'

w' = the final weight of vapor per unit weight of gas into which the liquid is evaporating

w = the initial weight of vapor per unit weight of gas

C_{pa} = the mean specific heat of the gas between temperatures $(t-t')$

C_{ps} = the mean specific heat of the vapor between temperatures $(t-t')$

t = the initial temperature of the gas

t' = the final temperature of the gas, vapor and liquid.

From this the theoretical temperature of equilibrium may be definitely located if the initial temperature and moisture content and the final moisture content of the gas are known.

$$t' = t - \frac{r'(w'-w)}{C_{pa}+C_{ps}w} \quad \text{(See Appendix No. 1.)}$$

9 The second method analyzes the rates of heat flow — the sensible heat flow to the wet bulb from the surrounding atmosphere — due to temperature difference, and the corresponding rate of outward heat flow, due to the evaporation of the liquid and its diffusion into the surrounding atmosphere. In this method the inward rate of diffusion of heat is balanced against the outward rate of diffusion of the vapor. These equations were written by Lewis (Par. 8) as follows:

[1] Irish Academy Trans., vol. 18, 1838, pp. 1-17.

[2] Rational Psychrometric Formulas, Trans. A.S.M.E., vol. 33, 1911, p. 1005.

[3] Mémoires de L'Académie des Sciences de L'Institute de France. Tome XXI, 1847.

[4] The Evaporation of a Liquid into a Gas, Trans. A.S.M.E., vol. 44, 1922, p. 325.

Rate of moisture diffusion from unit area of liquid surface per unit time

$$\frac{-dw}{Ad\theta} = k'(e'-e) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [2]$$

where
dw = weight of liquid evaporated per unit time $d\theta$
A = area of evaporating surface
k' = coefficient of moisture diffusion per unit difference in vapor pressure of liquid and vapor pressure in the atmosphere
e' = vapor pressure of liquid
e = vapor pressure in atmosphere.

Rate of heat diffusion through film to liquid surface:

$$\frac{dq}{Ad\theta} = h(t-t') \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [3]$$

where dq = heat diffused per unit time $d\theta$
A = area of surface
h = coefficient of heat diffusion per degree difference in temperature of liquid and atmosphere

but $dq = r'dw$

where r' = latent heat of vaporization corresponding to temperature t',

therefore

$$\frac{dw}{Ad\theta} = \frac{h}{r'}(t-t')$$

and

$$(e'-e) = \frac{h}{k'r'}(t-t') \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [4]$$

According to the law of gaseous mixtures the weights of the two component parts of a mixture are in proportion to the products of their respective pressures and their specific weights. So in a mixture of water vapor and air:

$$w = \frac{Se}{P-e}$$

where w = the weight of moisture per pound of air
S = the specific weight of water vapor compared with air
e = the vapor pressure
P = the barometric or total pressure of the mixture.

$$e = \frac{Pw}{S+w}$$

Substituting for e in Equation [4],

$$\frac{Pw'}{S+w'} - \frac{Pw}{S+w} = \frac{h}{k'r'}(t-t')$$

At temperatures below 150 deg. fahr. where w and w' are small, this equation may be written[1]

$$\frac{P}{S+w}(w'-w) = \frac{h}{k'r'}(t-t')$$

$$r'(w'-w) = \frac{h}{k}(t-t') \quad . \; . \; . \; . \; . \; . \; . \; [5]$$

where

$$k = k'P/(S+w)$$

By comparing Equations [1] and [5] established by the two methods presented, it will be seen that the two equations are of the same form and that h/k must be theoretically equivalent to $(C_{pa}+C_{ps}w)$, which is the heat involved in a change of one degree in the temperature of one pound of the gas plus the initial weight of vapor which it contains. If the gas were initially free from vapor, the value would represent its specific heat. This relation was pointed out by Lewis[2] and Grosvenor[3] as well as by Carrier,[4] and is of particular importance and interest. A further analysis of Equation [1] is contained in Appendix No. 1.

10 As mentioned in a previous paragraph, the results of experiments have shown the deviation of the observed wet-bulb temperature from the equilibrium temperature as given in Equation [1]. The causes of this deviation are apparently as follows:

1 Incomplete saturation in the film immediately in contact with the wet surface; in other words an inward flow of *sensible* heat, from the atmosphere, greater than the outward flow of *latent* heat from the liquid at the theoretical temperature of equilibrium.

2 By direct radiation from surrounding objects which are normally at the temperature of the atmosphere, and by transmission through the stem of the thermometer, if it is exposed to the temperature of the atmosphere.

11 It would seem obvious that the actual temperature of evaporation must be limited by either one of two effects. First, the capacity of a given weight of air, under given conditions of initial temperature and initial moisture content, to give up heat and to hold a definite quantity of saturated vapor within the space it occupies at the temperature it assumes as a result of the process. This limiting condition is described by Equation [1] and is the true temperature of adiabatic saturation. Second, the resultant temperature (of the liquid) may be limited by the rates of diffusion with which the latent heat may pass outward through the gas film at the surface of the liquid and the inward flow of heat through the film from the surrounding atmosphere. If the

[1] The necessity of this approximation does not occur in the heat-balance equation [1].

[2] See Par. 8. [3] W. H. Grosvenor, Proc. Am. Inst. Chem. Engrs., vol. 1, 1908. [4] See Par. 1.

rate of diffusion of latent heat outward is greater than or equal to that required for maintaining the temperature of adiabatic equilibrium then the theoretical temperature of Equation [1] would be reached but no lower temperature. If, however, this rate of heat flow outward, i.e., the diffusion of the vapor through the film, is less than the normal rate of heat flow from the outside atmosphere to the surface, at the theoretical temperature, then the actual temperature of evaporation will rise until the two rates, namely, the rate of inward flow of heat and outward flow of heat, are balanced. This rise of temperature above the theoretical will be more marked with increased thickness of the gas film in contact with the liquid, as occurs at very low velocities, and becomes exceedingly small at high velocities where the thickness of the film is negligible.[1] Specifically, at ordinary room temperature, the error in the wet-bulb depression, in air moving only at ordinary convection velocities, has been found to be from 14 to 18 per cent, while at transverse air velocities over the wet bulb of 1000 to 4000 ft. per min. the error is from 1.3 to 0.2 per cent.

12 The effect of increased wet-bulb temperature in decreasing the deviation from the theoretical temperature of adiabatic saturation is explained as follows: Under ordinary conditions of observation, the wet bulb or wet surface is exposed to radiation from surrounding objects presumably at the temperature of the atmosphere. The effect of radiant heat received by the wet surface is to raise the temperature of the surface above the true temperature of adiabatic saturation, in other words heat is being received from a source other than the gases involved in the conversion of sensible heat to latent heat. At low wet-bulb temperatures this effect is appreciable. However, as the wet-bulb temperature is increased the change in total heat per degree is rapidly increased (see Table 7, Appendix No. 2) and the total heat transformation from sensible to latent heat at the evaporating surface becomes larger relative to the heat received by radiation. Observations herein presented have shown, for instance, that at an air velocity of 1000 ft. per min. over the wet bulb, an observed wet-bulb temperature of 40 deg. fahr. is in error by 1.72 per cent of the observed wet-bulb depres-

[1] The possible variation of the diffusion coefficients with vapor concentration, that is, the inconstancy of h/k (Equation [**5**]) during saturation, was pointed out by W. K. Lewis (*Mechanical Engineering*, vol. 44, 1922, p. 325). He states that the relation expressed in Equation [**1**] is merely a limiting case. This statement is correct; for Equation [**1**] does express the limiting theoretical temperature of adiabatic saturation. That this temperature is not attained in practice has been shown in papers previously cited. It is the purpose of this paper to show wherein and to what degree evaporative temperatures, as attained in common practice, deviate from the theoretical.

sion, while at a wet-bulb temperature of 100 deg. fahr. the error in per cent of the observed depression is but 0.86 per cent. A mathematical analysis of the heat conveyed to the wet bulb by radiation is contained in Appendix No. 2.

13 We may now proceed to mention briefly the experiments conducted and to present the established quantitative deviation of observed wet-bulb temperatures from the theoretical temperature of adiabatic saturation.

The Verification of the Equation of Heat Balance

14 For the purpose of demonstrating experimentally that the true wet-bulb temperature is the temperature of adiabatic saturation and that this temperature remains constant throughout the process of saturation so long as the total heat of the mixture remains constant, it was necessary to construct apparatus by which actual adiabatic saturation might be accomplished. Specifically this requirement was that during the process of saturating air passing over a wet surface, no heat should be received from or delivered to outside sources, but that the process should consist entirely of a conversion of the sensible heat of an unsaturated atmosphere to latent heat of evaporation. If the equation of heat balance is correct, the wet-bulb temperature should remain unchanged throughout the process. The fact that the wet-bulb temperature does remain practically unchanged throughout the process of adiabatic saturation is approximated and commonly observed in a commercial air washer or humidifier. In this case, providing the spray water is not heated or cooled by artificial means, the water eventually assumes the wet-bulb temperature of the entering air and the air leaves the washer in practically a saturated condition and at a wet-bulb temperature very nearly the same as that of the entering air. A purely adiabatic process is, of course, impossible in an air washer due to the transmission of heat through the casing and from other sources. Carrier also demonstrated experimentally the identity of the wet-bulb temperature and the temperature of adiabatic saturation in his original paper on Rational Psychrometric Formulas.[1] In those experiments, however, the apparatus used lacked certain refinements which have later been developed, and the quantities of air used were too small to assure accuracy in observations.

15 For the purpose of obtaining the data here presented, an adiabatic saturator was constructed on the basis of the requirements outlined above. This was used in connection with a very elaborate apparatus designed primarily for the purpose of the determination of psychrometric coefficients to be used in dew-point determinations by psychrometric methods, which are to be

[1] Trans. A.S.M.E., vol. 33, 1911, p. 1005.

presented in a subsequent paper. The equipment was placed in the laboratory of physics at Cornell University and the experiments conducted there. A description of the adiabatic saturator and the experimental methods is given in Appendix No. 3.

16 Table 1 shows the results of adiabatic saturation as accomplished with the equipment. The temperature t' is the wet-bulb temperature of the entering unsaturated air. The difference between the wet and dry-bulb temperature of the entering air is given in column 2. The temperature t'_2 is the wet-bulb temperature of the air leaving the adiabatic saturator and is, as predicted, essentially identical with the wet-bulb temperature of the entering

TABLE 1 SHOWING CONSTANCY OF WET-BULB TEMPERATURE THROUGHOUT PROCESS OF ADIABATIC SATURATION

Velocity of air over thermometers = 2000 ft. per min.

(1)	(2)	(3)	(4)	(5)	(6)
t' Wet-bulb temperature entering adiabatic saturator	$t - t'$ Wet-bulb depression entering adiabatic saturator	$t_2 - t'_2$ Wet-bulb depression leaving adiabatic saturator	t'_2 Wet-bulb temperature leaving adiabatic saturator	$t' - t'_2$	Error in per cent of entering depression $\frac{t' - t'_2}{t - t'}$
47.0	9.8	0.68	46.97	0.03	0.30
53.5	11.7	0.17	53.45	0.05	0.43
53.0	11.5	...	52.97	0.03	0.26
54.7	13.5	...	53.99	0.08	0.59
54.8	21.3	1.69	54.66	0.14	0.66
55.1	19.4	...	54.99	0.11	0.57
61.5	20.8	...	61.45	0.05	0.24
64.0	10.6	...	63.86	0.14	0.71
69.25	23.3	0.34	69.09	0.16	0.69
69.90	23.1	1.08	69.78	0.12	0.52
75.30	19.7	0.38	75.23	0.07	0.35
75.30	20.0	...	75.23	0.07	0.35
79.70	34.4	1.12	79.49	0.21	0.61
78.40	41.4	4.75	78.25	0.15	0.36
80.00	22.0	...	79.97	0.03	0.14
82.50	24.0	...	82.44	0.06	0.25
82.70	25.5	...	82.65	0.05	0.20
84.30	44.7	3.49	83.14	0.16	0.36
94.85	50.0	3.52	94.58	0.27	0.53

air, while the dry-bulb temperature of the entering air has dropped almost to the wet-bulb, as shown in column 3, wherever the dry-bulb temperature of the leaving air was observed.

17 There is a slight difference between the observed t' (entering wet bulb) and t'_2 (leaving wet bulb). It will be noted that t' is, in all cases, slightly higher than t'_2. This is due to the fact that the thermometer, from which the values for t' (the entering wet bulb) were observed, was exposed to radiation and conduction from the surrounding surfaces, which were at the temperature t (of the entering dry bulb). The thermometer from which the values of t'_2 (the leaving wet bulb) were observed, was free from this effect, since the dry-bulb temperature of the leaving air and consequently the temperature of the walls of the surrounding duct was but slightly higher than the wet-bulb temperature. Complete protection from radiation to or from outside sources was also

afforded by the construction of the apparatus as detailed in Appendix No. 3. The difference between t' and t'_2 is shown in column 5; and the mean of these observed differences at a given wet-bulb temperature is taken to be the deviation of the wet bulb, due to radiation and conduction, from the true wet bulb as represented by the values of t'_2. This deviation is expressed, in column 6, as per cent of the wet-bulb depression of the entering air. In the light of these observed percentages of deviation from the true wet-bulb temperature and from a mathematical extrapolation of the deviations observed at lower velocities, the absolute errors in wet-bulb depression were established at the various temperatures and at an air velocity over the wet bulb of 2000 ft. per min.

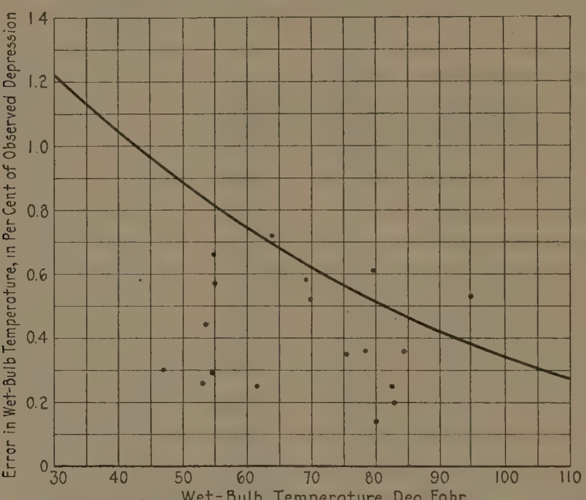


FIG. 1 WET-BULB ERROR IN PER CENT OF OBSERVED DEPRESSION
(Transverse air velocity over wet bulb = 2000 ft. per min.)

18 Fig. 1 shows graphically the conclusions reached and is consistent with the succeeding data showing the errors at various velocities. It will be noted that the errors from Table 1 plotted in Fig. 1 for the most part lie below the curve drawn to represent the errors over the complete range of temperature. The curve was drawn through points determined by the final equation which was found to be most consistent with all of the experimental determinations. It is likely that a very small quantity of heat was admitted to the adiabatic saturator from the outside sources and this would reduce the observed error, particularly at the lower temperatures. Placing the curve as shown seems justified, and, in any case, the maximum mean departure of the observed points from the curve is less than 0.15 per cent in wet-bulb error.

Quantitative Determination of the Variation of Wet-Bulb Error with Velocity and Temperature

19 To accomplish adiabatic saturation and thereby produce the true wet-bulb temperature for a given heat condition is difficult. We believe that in the results presented in the preceding paragraphs and the appendices referring thereto we have approached the limit of experimental accuracy. It is not difficult, however, to show the degree of variation in wet-bulb temperature with variation in velocity and temperature, if this variation is referred to a fixed temperature at a fixed high velocity instead of to the theoretical temperature. For instance, it is not difficult to observe the difference in temperature between two wet-bulb thermometers subjected to the same heat conditions but one being subjected to an air velocity of 15 ft. per min. over the bulb while the velocity over the other bulb is 2000 ft. per min. Neither is it difficult to compare two sets of such observations taken at different temperatures and thus show the variation of the difference with variation in the wet-bulb temperature. This was the object of a second series of experiments which were conducted at Case School of Applied Science and to which reference was made in Par. 4.

20 The apparatus used in these experiments and the methods of observation are described in detail in Appendix No. 4. Briefly the apparatus consisted of a fan supplying air to a metal pipe which was about 17 ft. in length and which was divided into four velocity stages. Each successive stage was so reduced in area that the velocity of air through that stage was four times greater than the velocity through the preceding stage. The outlet of the pipe was 3 in. in diameter. As an example, approximately 100 cu. ft. of air per minute is supplied to the duct to produce an outlet velocity of 2000 ft. per min. Under this condition the velocity at the successive stages is as follows: Stage 1 = 31.25 ft. per min.; Stage 2 = 125 ft. per min.; Stage 3 = 500 ft. per min.; Stage 4 = 2000 ft. per min. Wet- and dry-bulb thermometers were placed in suitable positions at each stage and simultaneous observations of the wet-bulb depression were made between any two stages under comparison. With the proper corrections for the slight temperature change due to pressure change between the stages, a comparison of the wet-bulb depression at any stage with that of the final stage gives the difference in wet-bulb error due to the known difference in velocity. Similar sets of observations taken at different wet-bulb temperatures permit a comparison of the difference in error due to the variation in wet-bulb temperature.

21 Eight complete sets of observations were taken in this series of experiments. Table 2 is a sample sheet of observations showing the corrections applied and the computed difference in

TABLE 2 OBSERVATIONS OF THE VARIATION OF THE WET-BULB ERROR WITH VARIATION IN VELOCITY

(Run No. 1. Barometric pressure = 29.43 in. Hg.)

Run No. 1		Station No. 1[1]				Station No. 4[2]				Correction applied to wet-bulb temperature at Station 1, 2 or 3 to compensate for dry bulb temperature change between stations							
	Room temp., deg. fahr.	Observed		Corrected		Observed		Corrected		(1)	(2)	(3)	(4)				
		t_1 D. B.	t_1' W. B.	t_1 D. B.	t_1' W. B.	t_4 D. B.	t_4' W. B.	t_4 D. B.	t_4' W. B.	$t_1 - t_4$	$\frac{Dt'}{Dt}$	(1)×(2)	t'_{1c} [t_1'−(3)]	$t'_{1c}-t_4'$	$t_4-t'_4$	$\frac{t'_{1c}-t_4'}{t_4-t_4'}\times 100$	
Mar. 31, 1924, 2:37 to 4:15 p. m.		Ther. 3	Ther. 1			Ther. 5	Ther. 4										Avg. 12.56 per cent
	55.0	54.65	45.00	54.60	45.04	54.20	43.55	54.27	43.50	0.33	0.5	0.16	44.88	1.38	10.77	12.80	
		54.70	45.00	54.65	45.04	54.30	43.50	54.37	43.45	0.28	0.5	0.14	44.90	1.45	10.92	13.30	
		54.80	45.00	54.75	45.04	54.50	43.40	54.57	43.30	0.18	0.5	0.09	44.95	1.65	11.27	14.60	
		54.80	45.00	54.75	45.04	54.50	43.45	54.57	43.35	0.18	0.5	0.09	44.95	1.60	11.22	14.30	
		54.80	45.00	54.75	45.04	54.60	43.45	54.67	43.35	0.08	0.5	0.04	45.00	1.65	11.32	14.60	
	55.0	54.90	45.00	54.85	45.04	54.60	43.50	54.67	43.42	0.18	0.5	0.09	44.95	1.53	11.25	13.60	
		Ther. 5	Ther. 4			Ther. 3	Ther. 1										
	55.5	55.10	44.50	55.15	44.49	55.00	43.80	54.95	43.85	0.20	0.5	0.10	44.39	0.54	11.10	....	
		55.35	45.10	55.40	45.09	55.20	43.50	55.16	43.45	0.24	0.5	0.12	44.97	1.52	11.71	13.00	
		55.50	45.05	55.53	45.05	55.30	43.40	55.26	43.36	0.27	0.5	0.14	44.91	1.55	11.90	13.05	
		55.45	45.05	55.49	45.05	55.30	43.20	55.26	43.15	0.23	0.5	0.12	44.93	1.78	12.11	14.70	
		55.45	45.05	55.49	45.05	55.20	44.00	55.16	44.05	0.33	0.5	0.16	44.89	0.84	11.11	....	
		55.45	45.05	55.49	45.05	55.30	44.00	55.26	44.05	0.23	0.5	0.12	44.93	0.88	11.21	7.75	
		55.45	45.05	55.49	45.05	55.20	43.80	55.16	43.85	0.33	0.5	0.16	44.89	1.04	11.31	9.20	
		55.45	45.05	55.49	45.05	55.30	43.80	55.26	43.85	0.23	0.5	0.12	44.93	1.08	11.41	9.50	
		55.45	45.05	55.49	45.05	55.30	43.75	55.26	43.80	0.23	0.5	0.12	44.93	1.13	11.46	9.90	
		55.45	45.05	55.49	45.05	55.30	43.80	55.26	43.85	0.23	.0.5	0.12	44.93	1.08	11.41	9.50	
		55.50	44.60	55.53	44.60	55.30	43.80	55.26	43.85	0.27	0.5	0.14	44.46	0.61	11.41	....	
		55.50	44.55	55.53	44.55	55.30	44.00	55.26	44.05	0.27	0.5	0.14	44.41	0.36	11.21	....	

April 1, 1924, 1:45 to 4 p.m.		Station No. 2[3]				Station No. 4[4]											
		Ther. 7	Ther. 1			Ther. 3	Ther. 2										
	52.5	52.50	42.70	52.45	42.68	52.30	41.90	52.26	42.06	0.19	0.05	0.06	42.62	0.56	10.20	5.50	Avg. 5.61 per cent
		52.50	42.70	52.45	42.68	52.30	41.90	52.26	42.06	0.19	0.05	0.06	42.62	0.56	10.20	5.50	
		52.50	42.70	52.45	42.68	52.30	41.95	52.26	42.11	0.19	0.05	0.06	42.62	0.51	10.15	5.02	
		52.60	42.70	52.55	42.68	52.40	42.00	52.36	42.15	0.19	0.05	0.06	42.62	0.48	10.21	4.60	
		52.60	42.85	52.55	42.85	52.40	42.05	52.36	42.20	0.19	0.05	0.06	42.79	0.59	10.16	5.81	
		52.60	42.80	52.55	42.80	52.40	42.08	52.36	42.23	0.19	0.05	0.06	42.74	0.51	10.13	5.03	
		Ther. 3	Ther. 2			Ther. 7	Ther. 1										
		52.60	42.40	52.55	42.56	52.55	41.90	52.50	41.90	0.05	0.05	0.06	42.50	0.60	10.60	5.66	
		52.60	42.40	52.55	42.56	52.60	41.90	52.55	41.90	0.00	0.05	0.06	42.50	0.60	10.65	5.64	
		52.60	42.50	52.55	42.69	52.55	42.00	52.50	41.99	0.05	0.05	0.06	42.63	0.64	10.51	6.08	
		52.60	42.50	52.55	42.69	52.55	42.00	52.50	41.99	0.05	0.05	0.06	42.63	0.64	10.51	6 08	
		52.60	42.50	52.55	42.69	52.60	42.00	52.55	41.99	0.00	0.05	0.06	42.63	0.64	10.56	6.06	
	52.5	52.65	42.60	52.60	42.80	52.61	42.10	52.56	42.07	0.04	0.05	0.06	42.74	0.67	10.49	6.38	
		Station No. 3[5]				Station No. 4[6]											
		Ther. 6	Ther. 8			Ther. 7	Ther. 2										
	52.5	53.00	42.75	52.97	42.65	53.20	42.20	53.14	42.34	—0.17				0.31	10.80	2.87	Avg. 2.84 per cent
		53.10	42.80	53.08	42.70	53.20	42.20	53.14	42.34	—0.06				0.36	10.80	3.33	
		53.10	42.65	53.08	42.54	53.20	42.10	53.14	42.25	—0.06				0.29	10.89	2.67	
		53.10	42.70	53.08	42.60	53.20	41.90	53.14	42.06	—0.06		No effect		0.54	11.08	4.86	
		53.10	42.60	53.08	42.49	53.20	42.00	53.14	42.15	—0.06				0.34	10.99	3.09	
		53.10	42.55	53.08	42.44	53.20	42.00	53.14	42.15	—0.06				0.29	10.99	2.64	
		Ther. 7	Ther. 2			Ther. 6	Ther. 8										
		53.60	42.10	53.54	42.56	53.50	42.40	53.52	42.29	+0.02				0.27	11.23	2.40	
		53.65	42.40	53.59	42.56	53.50	42.40	53.52	42.29	0.07				0.27	11.23	2.40	
		53.65	42.40	53.59	42.56	53.50	42.40	53.52	42.29	0.07				0.27	11.23	2.40	
		53.65	42.40	53.59	42.56	53.50	42.40	53.52	42.29	0.07				0.27	11.23	2.40	
		53.65	42.40	53.59	42.56	53.50	42.40	53.52	42.29	0.07				0.27	11.23	2.40	
		53.65	42.50	53.59	42.69	53.40	42.50	53.41	42.39	0.09				0.30	11.02	2.72	

[1] Velocity 14.3 ft. per min. [2] Velocity 916.8 ft. per min. [3] Velocity 57.1 ft. per min. [4] Velocity 913.6 ft. per min.
[5] Velocity 229.6 ft. per min. [6] Velocity 889.7 ft. per min.

depression between the last stage and the three preceding stages. Table 3 is a skeleton table showing the complete results of the eight sets of observations. In the record of the observations, the successive stages are designated as: Stations 1, 2, 3 and 4.

22 An additional experiment was performed to determine the difference in the wet-bulb temperature registered by a wet-bulb thermometer subjected only to the air velocity produced by natural convection and that of a sling psychrometer subjected to the same air conditions but whirled at an approximate bulb

TABLE 3 AVERAGE OF OBSERVATIONS MADE TO DETERMINE EFFECT OF VARIATION OF VELOCITY AND TEMPERATURE UPON WET-BULB TEMPERATURE

Experiment number	Number of readings made	Station number	Velocity at sta. 1, 2 and 3 equiv. to mass vel. at 70° F.	Average W. B. temperature at station 1, 2 and 3	Velocity at sta. 4 equiv. to mass velocity at 70° F.	Average W. B. temperature at station 4	Average W. B. depression at station 4	Average error at stations 1, 2 and 3 in per cent of depression at Station 4
	18	1	14.3	45.0	917	43.58	11.3	12.56
1	12	2	57.1	42.7	914	42.12	10.4	5.61
	12	3	229.6	42.5	919	42.18	11.1	2.84
	16	1	32	46.9	2001	45.35	16.3	9.52
2	13	2	....	....	2195	....	....	....
	13	3	547	46.5	2159	46.27	16.4	1.44
	13	1	31.2	64.8	1995	63.29	19.5	7.73
3	12	2	124.7	64.4	1995	63.83	19.5	2.99
	12	3	498.5	64.2	1994	....	....	....
	12	1	30.8	81.9	1970	80.61	21.3	5.31
4	12	2	120.5	82.0	1928	81.41	25.9	2.27
	12	3	471.9	82.5	1887	....	....	....
	12	1	33.0	100.2	2109	99.44	22.5	3.40
5	13	2	131.6	99.5	2105	99.08	25.10	1.68
	12	3	536.0	100.4	2147	100.31	21.60	0.42
	12	1	10.6	104.2	677	103.09	22.1	5.00
6	11	2	42.4	102.7	679	102.15	22.1	2.48
	12	3	169.6	101.5	678	101.35	22.5	0.68
	13	1	35.9	82.0	2299	....	21.8	....
7	12	2	143.7	80.5	2299	80.02	20.7	2.30
	12	3	574.6	79.0	2299	78.79	21.5	0.97
	12	1	13.9	82.4	890	....	....	....
8	12	2	55.6	81.0	890	80.26	20.7	3.59
	12	3	222.5	80.0	890	79.78	21.6	1.01

velocity of 1350 ft. per min. The observations of the stationary wet-bulb temperature were made with the bulb shielded on four sides by a muslin screen. This permitted free diffusion of the atmosphere about the bulb and allowed vertical convection but protected the bulb from lateral drafts. The results of these observations are presented in Table 4.

23 As a result of the three series of experiments described we have data covering, first, the total error in wet-bulb temperature at an air velocity of 2000 ft. per min. and over a considerable temperature range as shown in Fig. 1, as well as points showing

the absolute error at velocities of 970 ft. per min. and 1580 ft. per min., the observed values for which are plotted in Fig. 4. Next we have in Table 3 data showing the *difference* in error at a velocity of 2000 ft. per min. and velocities as low as 10 ft. per min. at four different temperatures; finally we have shown in Table 4 the difference in error between 2000 ft. per min. and zero transverse velocity at one temperature. It now remains to assemble these data and derive an equation which will cover the entire useful range of velocity and temperature.

24 The observed errors presented in Table 3 are corrected to correspond to absolute errors and are plotted against velocity in Fig. 2.

TABLE 4 OBSERVATIONS OF THE ERROR IN THE DEPRESSION OF A STATIONARY WET-BULB THERMOMETER, PROTECTED FROM DRAFTS

Observed stationary			Observed whirling 1350 f.p.m.			$t - t_0'$ Corrected for difference in error between 1350 f.p.m. and 2000 f.p.m.	$(t-t_0')$ minus $(t-t_s')$	Error of stationary W. B. in per cent of $(t-t_0')$
Dry bulb	Wet bulb	Depression $t - t_s'$	Dry bulb	Wet bulb	$t - t'$			
67.2	59.3	7.9	67.2	58.1	9.1	9.118	1.218	13.38
67.2	59.5	7.7	67.0	58.0	9.0	9.018	1.318	14.61
67.3	59.7	7.6	67.4	58.4	9.0	9.018	1.418	15.72
68.0	60.2	7.8	68.0	59.0	9.0	9.018	1.218	13.50
68.0	60.0	8.0	68.0	58.6	9.4	9.419	1.419	15.06
68.0	60.3	7.7	68.4	59.0	9.4	9.419	1.719	18.25
68.3	60.0	8.3	68.3	58.7	9.6	9.619	1.319	13.71
68.8	59.7	9.1	69.1	58.3	10.8	10.822	1.722	15.91
70.2	61.1	9.1	70.0	59.3	10.7	10.721	1.621	15.12
69.5	60.6	8.9	69.5	59.2	10.3	10.321	1.421	13.76
69.6	61.0	8.6	69.6	59.5	10.10	10.120	1.520	15.02
							Average 15.82 per cent	

25 In Appendix No. 2 it is shown that the error in the wet-bulb temperature must vary as an inverse function of the total velocity of the air over the bulb.

$$E = \frac{C}{(V+V_0)^a} \quad \ldots\ldots\ldots\ldots [6]$$

where

E = the absolute error in per cent of the theoretical wet-bulb depression

C = a constant

V = the transverse velocity of the air over the wet bulb

V_0 = the velocity of natural convection over the wet bulb

α = a constant exponent.

This may be written

$$\log E = \log C - \alpha \log (V + V_0)$$

The convection velocity was taken to be, $V_0 = 20$, the value having been determined by solving for V_0 from observations and selected as the value which best satisfied the equation throughout the velocity range.

26 In Fig. 3 the logs of the values of E taken from the curves in Fig. 2 and the logs of the observed values of E as recorded in Fig. 2 are plotted against the logs of corresponding total velocities $(V+20)$. The result is a set of four parallel straight lines each having the equation

$$\log E = \log C_t - 0.675 \log (V+20)$$

Thus the exponent of velocity is found to be

$$\alpha = 0.675$$

27 It is also shown in Appendix No. 2 that the error, E, varies

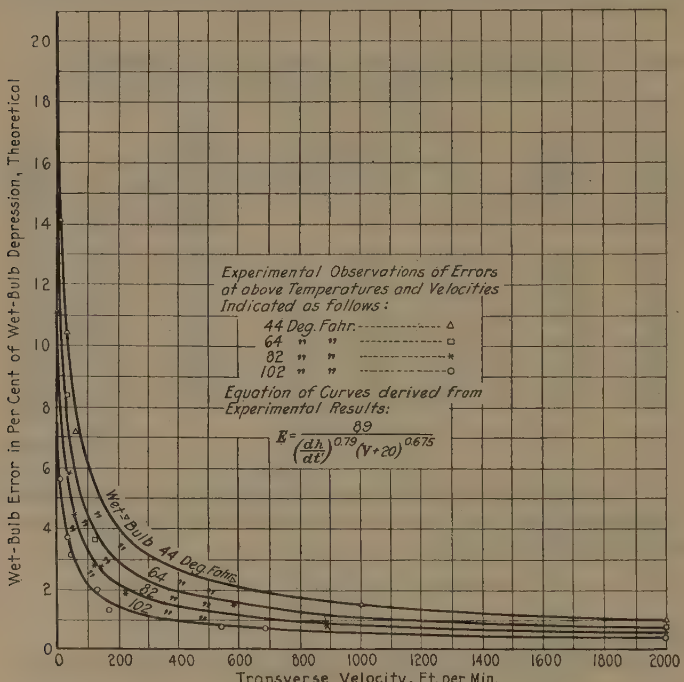


FIG. 2 ERROR IN WET-BULB DEPRESSION AT TRANSVERSE AIR VELOCITIES FROM 0 TO 2000 FT. PER MIN., AND AT TEMPERATURES OF 44, 64, 82, AND 102 DEG. FAHR.

as an inverse function of the change in total heat per degree change in the evaporating temperature of the mixture of gas and vapor.

$$E = \frac{C}{\left[\frac{dh}{dt'}\right]^{\beta}} \qquad \ldots\ldots\ldots\ldots \quad [7]$$

where C = a constant and

$\frac{dh}{dt'}$ = change in total heat of mixture per degree change in wet-bulb temperature,

or

$$\frac{E_{t1}}{E_{t2}} = \left[\frac{\frac{dh}{dt_2}}{\frac{dh}{dt_1}}\right]^{\beta}$$

which may be written

$$\log E_{t1} - \log E_{t2} = \beta\left(\log\frac{dh}{dt_2} - \log\frac{dh}{dt_1}\right)$$

But from Fig. 3 the differences in the logs of errors for any

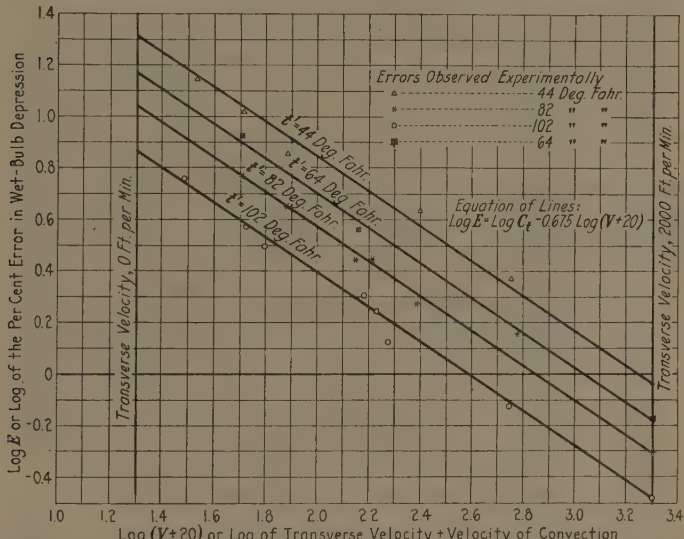


FIG. 3 LOG OF PER CENT ERROR IN THE THEORETICAL WET-BULB DEPRESSION VS. LOG OF THE TRANSVERSE AIR VELOCITY PLUS THE VELOCITY OF CONVECTION

two temperatures is found to be the constant distance between two parallel lines. Thus from Fig. 3

$$\log E_{82} - \log E_{102} = \log C = 0.176$$

So the exponent of change in total heat is

$$\beta = \frac{0.176}{\log\frac{dh}{dt_{102}} - \log\frac{dh}{dt_{82}}} = 0.79$$

Equations [6] and [7] may now be combined and we have

$$E = \frac{C}{\left[\frac{dh}{dt'}\right]^{0.79} (V+20)^{0.675}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [8]$$

Writing this

$$\log\left[E\left(\frac{dh}{dt'}\right)^{0.79}\right] = \log C - 0.675 \log (V+20)$$

and plotting the values of

$$\log\left[E\left(\frac{dh}{dt'}\right)^{0.79}\right] \text{ taken from Fig. 2}$$

and from computed changes in total heat, vs. log $(V+20)$, we obtain a composite line of observed errors as shown in Fig. 4. Fig. 4 shows the rather remarkable consistency of all observed

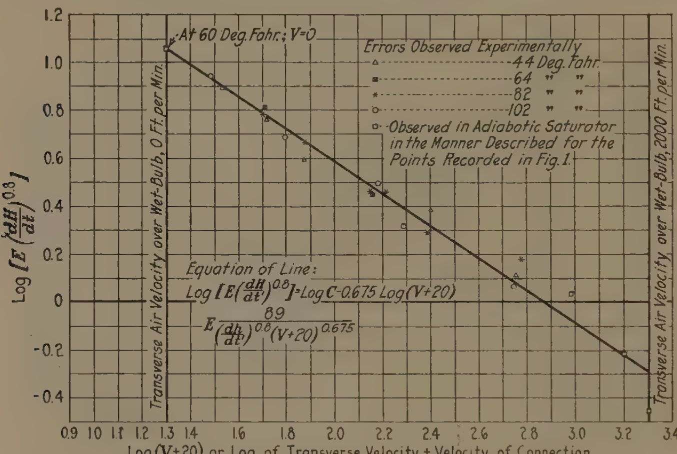


FIG. 4 $\text{Log}[E(dh/dt)^{0.79}]$ vs. $\text{Log}(V+20)$

values of E over the entire range of velocities and at wet-bulb temperatures from 44 to 102 deg. fahr.

28 Finally the constant C in Equation [8] may be determined by substituting known values of E and V. The equation then becomes:

$$E = \frac{89}{\left[\frac{dh}{dt'}\right]^{0.79} (V+20)^{0.675}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [9]$$

29 Using Equation [9], values of E have been computed over the entire useful range of temperatures and velocities. These values are recorded in Fig. 5 which gives the values for the error in the wet-bulb temperatures in terms of per cent of the true, *theoretical* wet-bulb depression.

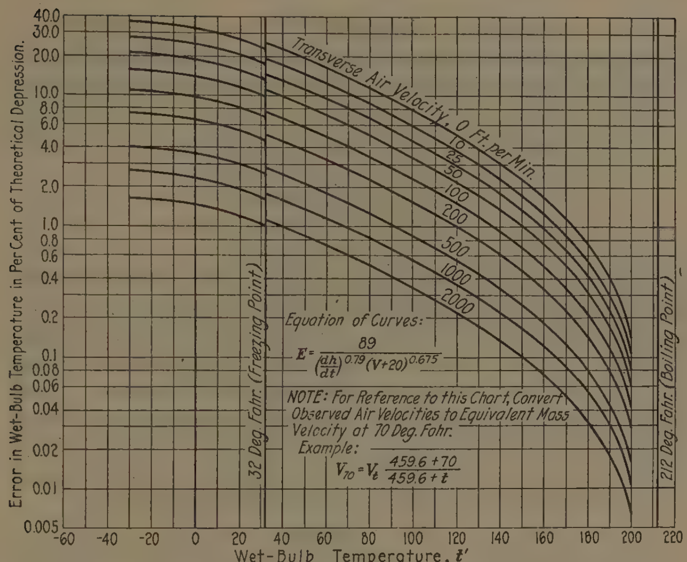


FIG. 5 ERROR IN WET-BULB TEMPERATURE EXPRESSED IN PER CENT OF THE THEORETICAL WET-BULB DEPRESSION AT WET-BULB TEMPERATURES FROM −30 TO 200 DEG. FAHR. AND AT TRANSVERSE AIR VELOCITIES OVER THE WET BULB RANGING FROM 0 TO 2000 FT. PER MIN.

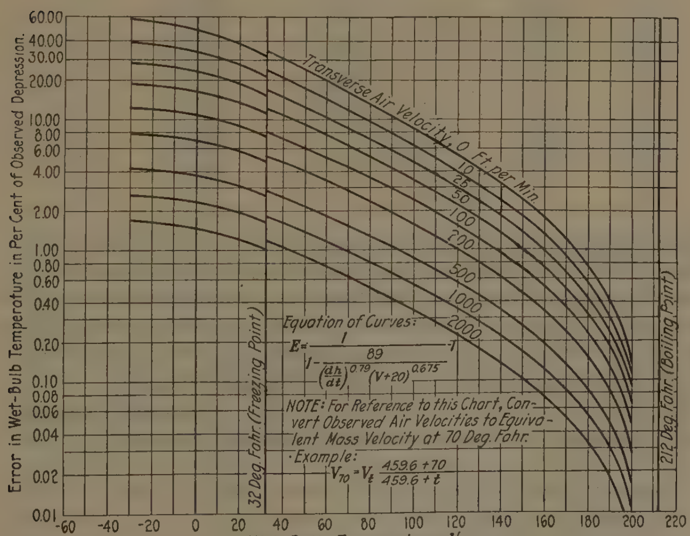


FIG. 6 ERROR IN WET-BULB TEMPERATURE EXPRESSED IN PER CENT OF THE OBSERVED WET-BULB DEPRESSION AT WET-BULB TEMPERATURES FROM −30 TO 200 DEG. FAHR. AND AT TRANSVERSE AIR VELOCITIES OVER THE WET BULB RANGING FROM 0 TO 2000 FT. PER MIN.

30 For psychrometric observations it is more convenient to state the error in per cent of the *observed* wet-bulb depression. This is done in Fig. 6, and the equation of the curves becomes

$$E = \left[\frac{1}{1 - \dfrac{89}{\left(\dfrac{dh}{dt'}\right)^{0.79} (V+20)^{0.675}}}\right] - 1. \quad . \quad . \quad . \quad . \quad [10]$$

31 The wet-bulb errors, as recorded for the various temperatures and velocities in Fig. 5 and 6, are computed for a barometric pressure of 30 in. The wet-bulb error varies with the barometric pressure since the factor dh/dt' contained in Equations [9] and [10] varies with the barometric pressure. Thus, as shown in Table 5, dh/dt' for a wet-bulb temperature of 60 deg.

TABLE 5 dh/dt' CHANGE IN TOTAL HEAT IN B.T.U. PER POUND OF AIR PER DEGREE CHANGE IN WET-BULB TEMPERATURE

Deg. t' fahr.	Barometric pressure 30 in. Hg	25 in. Hg
(Over ice)		
—20	0.257	0.261
—10	0.271	0.278
0	0.291	0.300
+10	0.322	0.335
20	0.368	0.390
30	0.434	0.471
(Over water)		
30	0.391	0.426
40	0.455	0.501
50	0.543	0.604
60	0.660	0.743
70	0.814	0.932
80	1.02	1.20
90	1.30	1.55
100	1.68	2.03
110	2.18	2.66
120	2.83	3.50

fahr. observed at a barometric pressure of 30 in. is 0.660, while dh/dt' for 60 deg. fahr. at 25 in. is 0.743. On the other hand, a wet-bulb temperature of 60 deg. fahr. at a barometric pressure of 25 in. has the same dh/dt' as 66 deg. fahr. observed at 30 in. So at like air velocities the value for E from Equation [10] would be the same as for 60 deg. fahr. at 25 in. and 66 deg. fahr. at 30 in.

32 Table 6 is for use in referring to Fig. 6 wet-bulb temperatures observed at barometric pressures other than 30 in.

EXAMPLE: Observed wet-bulb temperature, 50 deg. fahr.
Transverse velocity of air, 1000 ft. per min.
Barometric pressure, 25 in. Hg.

For reference to Fig. 6, we find from Table 6 that 50 deg. at 25 in. has the same error as 55.9 deg. at 30 in., namely, 1.25 per cent of the wet-bulb depression, at an air velocity of 1000 ft. per. min.

Conclusion

33 Perhaps the most important deduction to be drawn from this investigation is that under most practical conditions the temperature of evaporation of a free water surface approximates the theoretical temperature of adiabatic saturation, as very simply expressed in the fundamental equation of heat balance between sensible-heat and latent-heat interchange. (Equation [**1**])

34 The variations observed were insignificant, from an engineering standpoint — certainly at velocities above 200 ft. per min. In still air, or at inappreciable velocities, an allowance has to be made and the magnitude of this allowance can now be fairly estimated, at least more closely than is required for practical engineering purposes.

TABLE 6 WET-BULB TEMPERATURES (DEG. FAHR.) HAVING THE SAME PER CENT ERROR AT VARIOUS BAROMETRIC PRESSURES (SEE EXAMPLE, PAR. 32)

	Barometric Pressure, In. Hg					
	30	29	28	27	26	25
Over ice	—28	—28.4	—28.8	—29.3	—29.6	—30.0
	—20	—20.7	—21.4	—22.1	—22.8	—23.5
	—10	—10.8	—11.5	—12.2	—13.0	—13.8
	0	— 0.8	— 1.6	— 2.3	— 3.1	— 4.0
	+10	+ 9.2	+ 8.4	+ 7.7	+ 6.9	+ 6.0
	20	19.2	18.4	17.6	16.8	16.0
	30	29.2	28.4	27.6	26.8	26.0
	32	31.2	30.4	29.7	28.8	28.0
Over water	32	31.9	29.9	28.8	27.6	26.5
	40	39.0	37.8	36.8	35.6	34.4
	50	48.8	47.7	46.5	45.3	44.1
	60	58.8	57.7	56.5	55.3	54.1
	70	68.7	67.6	66.3	65.1	63.8
	80	78.7	77.4	76.1	74.8	73.5
	90	88.7	87.3	86.0	84.7	83.3
	100	98.6	97.2	95.8	94.4	93.0
	110	108.6	107.1	105.7	104.2	102.8
	120	118.5	117.0	115.6	114.0	112.6
	130	128.4	126.8	125.3	123.6	122.1
	140	138.4	136.8	135.3	133.6	132.0
	150	148.3	146.7	145.1	143.4	141.7

35 It also follows, from this study, that the determination of relative rates of diffusion in the film of the vapor and of the gas can, for practical purposes, be entirely ignored and the relationship simplified. In other words, for practical purposes the engineer may treat problems of evaporation from a liquid to a gas, whether the heat be supplied from the liquid or from the gas, purely as a problem of conversion from sensible to latent heat. The assumption can always be made, as this study indicates, that the surface film is both approximately at the temperature of the liquid and at the moisture content corresponding to saturation at the liquid temperature.

36 It is because of this state of the film in contact with the liquid that the heat removed by evaporation must parallel, in its amount, the heat absorbed from the air by convection. The same

effect holds where the liquid is heated, except that the amount of heat removed from the liquid directly by convection, as sensible heat, is in direct proportion to the amount of latent heat removed simultaneously from the film. It is also for this reason that the rate of evaporation is substantially proportional to the difference of vapor pressure between the liquid and the gas. The rate of evaporation is not caused by the fact that there are, theoretically, two different vapor pressures, one in the gas and one at the surface of the liquid, but by the fact that the density of vapor in the film is greater than the density of vapor in the surrounding air. That is to say, the weight of vapor per pound of air is greater in the film, and any mixture of the outside air with the film, as caused by velocity, whether of forced or natural convection, tends to carry away an amount of moisture which is directly proportional to the difference of moisture content between the films and the moving air. In other words, the evaporation of moisture from the liquid into the gas takes place exactly as heat transfer to air from a dry surface. There is really no difference in the problem. It is purely a problem of mixture in both cases. The problem of evaporation, as well as the problem of heat transfer, under forced convection, is primarily one of mass displacement rather than molecular activity, as the latter plays only a minor part. At very low velocities, or in practically still air, it is true that diffusion plays a relatively more important part in evaporation, just as conduction through gases, as distinguished from convection, become a considerable factor in heat transfer under similar conditions.

37 These statements are made in view of the trend of experimental evidence, both in heat transfer and in evaporation, and it is appreciated that they are, in some respects, contrary to the general view. In problems of evaporation, for instance, it is customary for engineers and physicists to think of the differences of vapor pressure existing as active forces in themselves, causing diffusion or transfer of moisture, as comparable, for example, to osmotic pressure. It is true that, in so far as these effects exist, they affect diffusion of moisture and conduction of heat, but these quantities are insignificant, from an engineering standpoint, as compared with the far greater effects of mass action ordinarily found in engineering practice. To illustrate the effect of convection and diffusion of water vapor as caused purely by molecular activity, consider a tall glass partially filled with water free to evaporate into room atmosphere. The temperature of the water will remain at a slightly lower temperature than that of the room. There will be no convection currents from the cooler water to the air above, as the lower layer of air will be slightly colder than the upper layer. The heat transfer and evaporation

will then occur purely by conduction and diffusion, and the effect of either is relatively insignificant, as compared with the magnitudes employed in most engineering problems.

38 It has been shown that the law of wet-bulb error, from causes outside of the assumptions in the equations of heat equilibrium, varies substantially in accordance with the laws of heat transfer, that is, the exponent of velocities is substantially the same in this case as in the case of heat transfer. See Appendix No. 2.

39 It has also been shown, experimentally, that the effect of radiation upon wet-bulb temperature is directly related to the total heat of the air, that is, to the sum of sensible and latent heat in the air. Further, it is shown that the equation determined by experiment for this relationship is substantially the same as the theoretical equation as determined by the assumption of radiation as the sole cause of wet-bulb deviation. In the first case, it varies inversely as the 0.79 power of dh/dt' while, theoretically, it varies inversely as the 0.7 power of dh/dt'. (Appendix No. 2, Fig. 8-*a*.)

40 It should be noted from the theoretical equation as given in Appendix No. 2 that the error should vary slightly with the moisture content. In other words, the per cent of error will vary slightly with the vapor pressure, as well as with the wet-bulb temperature, in as much as it is a direct function of $(C_{pa}+C_{ps}W)$. In absolutely dry air this term will become constant at a given temperature since there will be no w, while in saturated air w will correspond to moisture content of saturation at the wet-bulb temperature. This variation, with the variations in wet-bulb depression, however, is slight. For example: At a wet-bulb temperature of 100 deg. fahr. the theoretical coefficient of error, when no moisture is present in the air, would be about 93 per cent of the coefficient at saturation. This assumes an extreme example of 180 deg. fahr. wet-bulb depression. It will also be observed by reference to a psychometric chart that this per cent of effect due to wet-bulb depression is in all cases substantially proportional to the depression, regardless of the temperature at which it occurs.

41 From a standpoint of exact measurements in psychrometry, the foregoing experimental determinations and deductions therefrom are believed to be valuable, since they determine for the first time with fair precision the corrections to be applied to wet-bulb observations, in order to rationalize them with the true temperature of adiabatic saturation, as determined by the physical properties of water vapor and air. It is also possible by the data obtained to rationalize the wet-bulb observations of a stationary hygrometer. It may be said, however, that without unusual precautions the stationary hygrometer is an exceedingly unreliable instrument for even the most ordinary work, since the drafts and convection currents existing in a heated room are uncertain,

and consequently the corrections to be applied are uncertain. This is particularly to be emphasized, since the errors on a stationary wet-bulb thermometer occur on the most sensitive portion of the correction curves, i.e., at very low velocities. It is shown, for instance, that the error of the stationary wet-bulb thermometer will vary from about 5 per cent at 120 deg. fahr. to 33 per cent at 32 deg. fahr. In ordinary room conditions, velocities from zero to 50 ft. per min. may exist without being sensible to the observer. or in other words, be noticed as a draft. At a 50-deg. wet-bulb temperature, the error will vary from 24 per cent in still air to 9 per cent at 50 ft. per min. This effect of extreme variability of the stationary wet bulb at normal velocity conditions entirely invalidates Glaisher's Hygrometric Tables, which have been commonly used and which were compiled without any observations being taken as to the velocity over the wet bulb or any extraordinary precautions being taken to prevent same. On the other hand, the results have shown that the corrections to be applied to the sling-psychrometer observations, taken at a whirling-bulb velocity above 1000 ft. per min., are relatively insignificant, except where extreme accuracy with precision thermometers is desired.

42 In the light of the results of the experiment presented in this paper, some further investigation would seem to be desirable. The values found for the deviation in the observed wet-bulb temperature from the theoretical temperature of adiabatic saturation show definitely the law of their variation with velocity and temperature. Inasmuch as it has been found, however, in the mathematical investigation presented in Appendix No. 2, that a large, if not the larger, portion of this deviation is due to radiation, and not to differences in diffusion rates, the heat emissivity of surrounding walls or ducts assumes considerable experimental importance. Judging from accepted coefficients of heat emissivity for various materials, the effect of radiation upon a wet-bulb thermometer in a dull metal duct would be considerably less than that which would be found in taking psychrometric observations in an ordinary room. Further investigation to determine thoroughly the effect of variation in the emissivity of surrounding walls upon the absolute value of the wet-bulb error is desirable. This might easily be accomplished with the present equipment, by coating the interior of the air ducts with paints of known coefficients of emissivity.

43 In an extensive investigation closely allied to the matter presented in this paper, the authors have made determinations of the psychrometric coefficients to be applied to sling-psychrometer observations in dewpoint and relative-humidity determinations. Apparatus of considerable precision was used, part of which was also employed in the determination of absolute wet-bulb error. The determinations show the present psychrometric tables of the United States Weather Bureau to be sufficiently accurate for

usual commercial work at normal outdoor temperatures. However, from a standpoint of precision measurements, the dewpoints and moisture contents given in these tables are found to be considerably low. The results of the authors' investigation and causes of the errors in the present psychrometric tables will be presented in a subsequent paper.

APPENDIX NO. 1

44 The process of adiabatic saturation may be visualized graphically in Fig. 7. Assume 1 lb. of air at temperature t containing a weight w lb. of superheated moisture which has a dewpoint or saturation temperature t_0. Curve AB is the density-temperature curve for the moisture at saturation. If now a quantity of water dw is evaporated into the air, the moisture content of the air rises along curve AB,

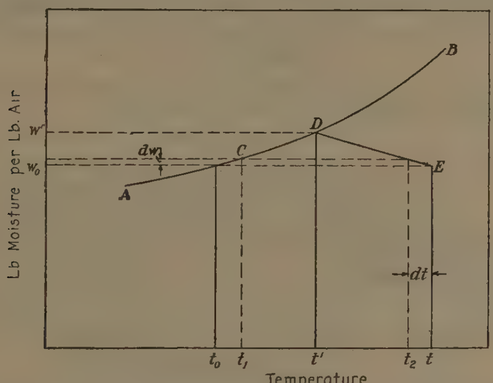


FIG. 7 MOISTURE-TEMPERATURE CHANGES IN PROCESS OF ADIABATIC SATURATION

say to point C, and the air and initial vapor from which the heat necessary for evaporation is taken is cooled by an amount dt. The dew-point temperature has risen to t_1 and the air temperature has been reduced to t_2 and their difference has been reduced to (t_2-t_1). If evaporation is continued, the weight of vapor will continue to increase along curve AB and the air temperature will continue to fall along some such curve as ED. Eventually curve ED will intersect curve AB at D at which state saturation has been reached with a final vapor content w' and a resultant temperature t'. Thus t' is the temperature of equilibrium or, as has been defined, the temperature of adiabatic saturation.

45 It seems obvious, with the present knowledge of adhesive-film effects,[1] that a body of liquid, perfectly insulated from all heat sources other than that of heat transfer from the atmosphere, would immediately be surfaced by a film of gas and saturated vapor at some equilibrium temperature such as t' above. As a result of this, the liquid must eventually assume the temperature of the film and remain at

[1] Langmuir, *Physical Review,* vol. 34, 1912, p. 421.

this temperature throughout the process of saturation regardless of the rate of evaporation so long as the total heat content of the surrounding atmosphere remains unchanged. This is, of course, assuming that the rate of heat diffusion to the liquid from the air is equaled by the heat diffusion from the liquid in the form of latent heat of evaporation. The close approximation to this condition as applied to water evaporating into air was demonstrated experimentally and the results have been shown in Table 1.

46 Adiabatic saturation has been defined as purely a transformation of sensible heat to latent heat of evaporation as expressed in Equation [1]

$$r'(w'-w) = (C_{pa}+C_{ps}w)(t-t')$$

47 Verification of this equation may be made, in light of the above remarks, by equating the total heat of the atmosphere in its initial state, plus the heat of the liquid which it evaporates, to the total heat in the final state of adiabatic saturation.

48 The total heat in a mixture of 1 lb. of pure air at temperature t and initial moisture content w_0 corresponding to saturation temperature t_0 is

$$\Sigma_1 = C_{pa}t+[H_0+C_{ps}(t-t_0)]w_0 \quad \ldots\ldots \quad [11]$$

49 It has been shown in a previous paper[1] assuming steam to be a perfect gas at the temperatures considered that

$$H_0 = r'+q'-C_{ps}(t'-t_0)$$

Substituting for H_0 in Equation [11]

$$\Sigma_1 = C_{pa}t+r'w_0+q'w_0+C_{ps}w_0(t-t') \quad \ldots\ldots \quad [12]$$

50 We must add to Equation [12] the heat of the liquid that will be added from the water which will be evaporated, in which case Equation [12] becomes

$$\Sigma_1+q'(w'-w_0) = C_{pa}t+r'w_0+q'w_0+C_{ps}w_0(t-t')+q'(w'-w_0) \quad [13]$$

51 The total heat of the mixture in the final state is

$$\Sigma_2 = C_{pa}t'+r'w'+q'w' \quad \ldots\ldots \quad [14]$$

52 Equating Equation [13], the total heat in the initial state plus the heat of the liquid evaporated, and Equation [14], the total heat of the final mixture we have

$$C_{pa}t+r'w_0+q'w_0+C_{ps}w_0(t-t')+q'(w'-w_0) = C_{pa}t'+r'w'+q'w' \quad [15]$$

or

$$r'(w'-w_0) = (C_{pa}+C_{ps}w_0)(t-t') \quad \ldots\ldots \quad [16]$$

which is identical with Equation [1]. The following notation is used in the above equations:

C_{pa} = mean specific heat of air
C_{ps} = mean specific heat of steam
H_0 = latent heat plus heat of the liquid at t_0
H' = latent heat plus heat of the liquid at t'
r' = latent heat corresponding to t'
q' = heat of the liquid
t = initial air temperature
t_0 = saturation temperature of initial moisture
t' = final wet-bulb equilibrium temperature.

[1] Rational Psychrometric Formulas, Trans. A.S.M.E., vol. 33, 1911, p. 1038.

53 The original paper,[1] presents a derivation essentially the same as that just given, as well as another complete derivation based upon the integration of the heat interchange between the atmosphere and the liquid. In the discussion following the same paper Prof. G. A. Goodenough presented a very enlightening interpretation of the same equation by the use of a temperature-entropy diagram to show the vapor phases and resultant temperature during the process of adiabatic saturation.

APPENDIX NO. 2

APPROXIMATION OF THE EFFECT OF RADIATION IN RAISING THE WET-BULB TEMPERATURE ABOVE THE THEORETICAL

54 No direct experiments have been conducted to determine what portion of deviation from the temperature of adiabatic saturation is due to radiant heat received by the evaporating surfaces. It is possible, however, to approximate closely by computation the heat thus received.

55 The heat absorbed by a surface surrounded by another surface at a higher temperature is by the Stefan-Boltzmann Law

$$Q = \frac{\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4}{\frac{1}{a} + \frac{1}{b} - \frac{1}{c}} \quad \ldots\ldots\ldots\ldots [17]$$

where Q = B.t.u. absorbed per sq. ft. per hr.
T_1 = absolute temperature of radiating surface
T_2 = absolute temperature of absorbing surface
a, b and c = radiation coefficients respectively for radiating surface, absorbing surface and absolute black body.

56 For the computation in Table 7, the following radiation coefficients were used

a = assumed value for wall surface = 0.0972
b = white-cloth wet-bulb covering = 0.0745
c = absolute black body = 0.1618

56 An equation of ratios may be written expressing the temperature rise effected at a surface due to the addition of given quantity of heat Q above that received from the air in an adiabatic process

$$\frac{\Delta t'}{t - (t' + \Delta t')} = \frac{Q(C_{pa} + C_{ps}w)}{H\, dh/dt'} \quad \ldots\ldots\ldots [18]$$

where

$\Delta t'$ = degrees rise in t' due to the addition of Q
$C_{pa} + C_{ps}w$ = initial specific heat of one pound of air containing w pounds of water vapor upon entering the process
dh/dt' = the change in total heat per pound of air per degree change in t', the saturation temperature
H = B.t.u. heat transmission per hr. per sq. ft.

57 The following is the derivation of the theoretical effect of radiation in producing an error in wet-bulb temperature:

58 If Q represents the radiant heat absorbed per sq. ft. per hr. at an absolute wet-bulb temperature T' with a surrounding temperature T, then, according to Equation [17]

$$Q = x[(T)^4 - (T')^4]$$

[1] Rational Psychrometric Formulas, Appendix No. 3, Trans. A.S.M.E., vol. 33, 1911, p. 1037.

where x is a coefficient of radiation depending on the absolute emissivity of the wet bulb and its surroundings.

59 However, if $T-T'=\Delta T'$ is small compared with T', we may write the equation

$$Q = x[(T'+\Delta T')^4 - (T')^4]$$

hence

$$Q = 4x(T')^3\Delta T' + 6x(T')^2(\Delta T')^2 + 4xT'(\Delta T')^3$$

or

$$Q = 4x(T')^4\left[\frac{\Delta T'}{T'} + 1.5\left(\frac{\Delta T'}{T'}\right)^2 + \left(\frac{\Delta T'}{T'}\right)^3\right]$$

TABLE 7 COMPUTED DEVIATION FROM THEORETICAL WET-BULB TEMPERATURE DUE TO HEAT RECEIVED BY RADIATION EXPRESSED IN PER CENT OF OBSERVED WET-BULB DEPRESSION

Equation: $\dfrac{\Delta t'}{t-(t'+\Delta t')} = \dfrac{Q\ (C_{pa}+C_{ps}w)}{H\ dh/dt'}$

t Dry-bulb temp., deg. fahr.	t' Wet-bulb temp., deg. fahr.	P Bar. pres., in. Hg.	V Transverse air velocity, ft. per min.	Q Radiant heat B.t.u./sq. ft./hr.	H Heat transmission, B.t.u./sq. ft./hr.	$(C_{pa}+C_{ps}w)$ Specific heat of atmosphere, initial state	dh/dt' Change in total heat per degree t'	Calculated deviation in per cent of depression $\frac{\Delta t'}{t-(t'+\Delta t')}\times 100$	Total deviation observed experimentally
80	60	30	0	6.77	36	0.244	0.66	6.85	19.50
"	"	"	10	"	38.5	"	"	6.40	14.20
"	"	"	25	"	43	"	"	5.74	10.20
"	"	"	50	"	50	"	"	4.94	7.50
"	"	"	100	"	34.5	"	"	3.82	5.10
"	"	"	200	"	89	"	"	2.77	3.40
"	"	"	500	"	155	"	"	1.59	1.82
"	"	"	1000	"	219.5	"	"	1.12	1.17
"	"	"	2000	"	290	"	"	0.85	0.75
120	100	30	0	8.43	36	0.2592	1.67	3.63	8.40
"	"	"	10	"	38.5	"	"	3.40	6.30
"	"	"	25	"	43	"	"	3.04	4.65
"	"	"	50	"	50	"	"	2.61	3.40
"	"	"	100	"	64.5	"	"	2.03	2.40
"	"	"	200	"	89	"	"	1.47	1.55
"	"	"	500	"	155	"	"	0.84	0.87
"	"	"	1000	"	219.5	"	"	0.60	0.54
"	"	"	2000	"	290	"	"	0.45	0.34

Since the second and third terms are negligible, we may write:

$$Q = 4x(T')^3(t-t')\ \text{(approx.)} \quad . \; . \; . \; . \; . \; . \; . \; . \quad [19]$$

Where $(t-t')$ is the wet-bulb depression. This permits a convenient expression for Q in terms of wet-bulb temperature and wet-bulb depression.

60 If we assume the temperature t' is the limiting temperature of adiabatic saturation, unaffected by radiation, we may write the fundamental equation of heat balance

$(C_{pa}+C_{ps}w)(t-t') = r'(w'-w)$ (See Eq. [1] and Appen. No. 1) . [20]

But if $K = f(v)$ is the coefficient of convection for any mass velocity and H = B.t.u. transmission per sq. ft. per hr., we have from [20]

$$H = K(t-t') = \frac{K}{(C_{pa}+C_{ps}w)}\ r'(w'-w) \quad . \; . \; . \; . \quad [21]$$

61 If a quantity Q of radiation is added, it is obvious that the equilibrium of sensible and radiant heat plus radiation, with latent heat, can be attained by a rise $\Delta t'$ in wet-bulb temperatures and an increase of $\Delta w'$ in corresponding moisture content at saturation, therefore, we may write from [21]

$$K[t-(t'+\Delta t')]+Q = \frac{K}{(C_{pa}+C_{ps}w)} r'[(w'+\Delta w')-w] \quad . \quad . \quad [22]$$

Subtracting Equation [21] from [22]

$$-K(\Delta t')+Q = \frac{K}{(C_{pa}+C_{ps}w)} r'(\Delta w')$$

or

$$K(\Delta t') + \frac{Kr'(\Delta w')}{(C_{pa}+C_{ps}w)} = Q \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . [23]$$

substituting

$$4x(T')^3[t-(t'+\Delta t')] = Q$$

and dividing both terms by

$$\frac{K(\Delta t')}{C_{pa}+C_{ps}w}$$

we have

$$\frac{(C_{pa}+C_{ps}w)\Delta t'+r'(\Delta w')}{\Delta t'} = \frac{4x(T')^3(C_{pa}+C_{ps}w)}{K} \frac{t-(t'+\Delta t')}{\Delta t'} \quad . \; [24]$$

But the first term represents the change in *total heat* per degree change in wet-bulb temperature, i.e., dh/dt'.

Therefore, by substituting and rearranging

$$\frac{\Delta t'}{t-(t'+\Delta t')} = \frac{4x(T')^3}{K} \frac{(C_{pa}+C_{ps}w)}{\frac{dh}{dt'}} \quad . \; . \; . \; . \; . \; [25]$$

62 That is, the ratio of wet-bulb radiation error to the observed wet-bulb depression is the product of the two ratios; first, that of the radiation coefficient to the convection coefficient; second, that of the true specific heat to the apparent specific heat when accompanied by a change of moisture content corresponding to the temperature.

63 In Table 7 are recorded the computed values of $\frac{\Delta t'}{t-(t'+\Delta t')}$, the deviation from the theoretical wet-bulb temperature due to radiant heat expressed in per cent of the observed depression, which, as has been stated, is calculated on the basis of an assumed emissivity value for wall surfaces of 0.0972.

64 For the sake of comparison, the total deviations as determined by experiment are included in the table in the last column. The deviations, as given in the table, obviously include both radiation and diffusion effects. The radiation effect in the actual experiments is undoubtedly considerably lower than those obtained in the calculations for wall surfaces, owing to the probability of the duct having a considerably lower constant of emissivity than that assumed for the wall. It should also be taken into consideration that the values for H, as calculated from known transmission values, are undoubtedly somewhat higher at zero and very low velocities than those actually occurring on the wet-bulb surface. The reason for the convection factors being lower for a wet-bulb surface than for a dry surface is that at a dry surface the density of the air is reduced in direct proportion to the reduction of the absolute temperature at the surface and the convection velocity, assuming viscous flow, will be in direct proportion to the density. With an evaporating surface, however, the density of the air

does not decrease in proportion to the reduction in absolute temperature of surface, owing to the increase of the moisture content, which causes a slight reduction in the specific weight which is accompanied by reduction in the density of the mixture.

65 It will be noted that while the calculated values for the error correspond very closely to the observed error between 200 and 1000 ft. per min., they do not agree with the observed values at lower velocities. Attention is also called to the fact that the ratio of computed values to the experimental is not greatly affected by the temperature range. The difference in the rate of change of error with velocity as computed and as observed, it would seem probable, is due to the effect of diffusion. The indication is that at low velocities and in still air, transfer of heat through moisture diffusion is at a noticeably lower rate than

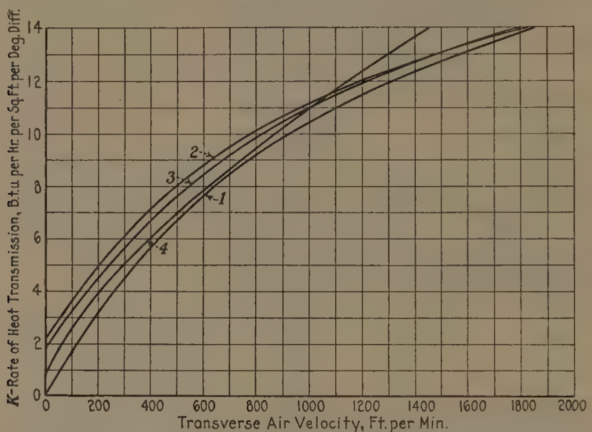


FIG. 8 COEFFICIENT OF HEAT TRANSMISSION (K) FOR VARIOUS VELOCITIES

(Curve 1, transmission without natural convection or radiation; Curve 2, transmission including natural convection and radiation; Curve 3, transmission including natural convection only; Curve 4, form of curve for transmission to a wet-bulb thermometer, including natural convection only.)

the transfer of sensible heat through conduction. The values used for rate of heat transmission are taken from Curve 3, shown in Fig. 8, which is computed from the equation

$$K = \frac{1}{0.0447 + \dfrac{50.66}{V+100}}$$

This may be compared with curve 1, giving experimental values of heat transmission for pipe coils, as determined by Carrier,[1] in which there was neither a material amount of convection nor radiation and with curve 2, as determined on the katathermometer [2] in which there was both free convection and radiation.

[1] Air Conditioning Apparatus, W. H. Carrier and F. L. Busey. Trans. A.S.M.E., vol. 33, 1911, p. 1055.

[2] Temperature, Humidity and Air Motion, O. W. Armspach and Margaret Ingels, Jl. Am. Soc. of Heat. & Vent. Engrs. Mar., 1922, p. 173.

66 Curve 4 would represent the heat transfer as computed from the empirical formula derived from the observed error at various velocities, the equation for which would be:

$$K = m(V+V_0)^{0.675}$$

67 It has been shown that through certain ranges of velocity, transmission of heat may be expressed approximately by the following equation [1]

$$K = m(V)^{0.67}$$

68 It should be observed that this approximate exponent of velocity corresponds very closely to the exponent of velocity derived from the observations in this paper.

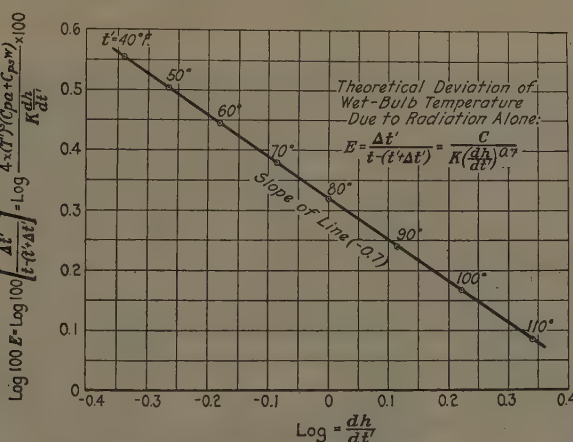


FIG. 8-*a* THEORETICAL DEVIATION OF WET-BULB TEMPERATURE DUE TO RADIATION ALONE

69 Referring again to Equation [25], a study may be made of the variation in radiation effect with variation in temperature. It is evident that dh/dt' is a function of the absolute temperature at any given barometric pressure and value of K. Owing to this relationship, it is possible to plot the entire right-hand side of this equation as a function of dh/dt' when K is constant. Plotting in Fig. 8-*a*, the logarithms of the computed per cent of error against the logarithms of the values of dh/dt' in this equation, we find that the radiation error should vary theoretically as approximately the (-0.7) power of dh/dt', as compared with the (-0.79) power determined by these experiments.

APPENDIX NO. 3

THE ADIABATIC SATURATOR AND EXPERIMENTAL METHODS

70 The adiabatic saturator is shown in photograph Fig. 9 in conjunction with a large apparatus used in determining psychrometric

[1] The Design of Indirect Heating Systems, F. L. Busey and W. H Carrier, Jl. Am. Soc. Heat. & Vent. Engrs., Jan., 1913.

coefficients. A cross-section of the adiabatic saturator and temperature observation stations is also detailed as Section *E*. Figs. 10 and 11.

Path of Air

71 Air is supplied to the adiabatic saturator by fan *A*. (See Figs. 10 and 11.) The air passes through Section *B* where it is saturated, thence through Section *C* where it is heated to the desired temperature, thence through Section *D* and finally to Section *E*, the adiabatic saturator.

72 The wet- and dry-bulb temperature of the air entering the adiabatic saturator are measured at temperature station III. The air then passes through the saturator where it evaporates moisture, to a point approaching saturation, by passing over wetted cloth strips wound over and under pipes lying parallel to the air flow, as shown in Section *AA*, Fig. 11. The cloth strips and inner walls of the saturator

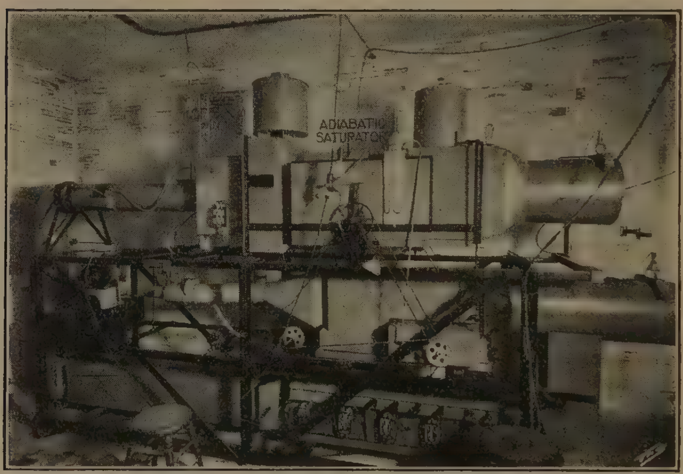


Fig. 9 The Adiabatic Saturator

are kept wet by a small gear pump recirculating water from the bottom of the saturator to the headers and distribution pipes at the top, from which it flows down over the cloths. Water evaporated is replaced by water supplied from an air-tight tank on top of the saturator, which allows a small amount of water to enter the saturator whenever the level of the liquid in the bottom of the saturator is low enough to permit an air bubble to pass through the connecting pipe into the tank. (See Fig. 12.)

73 Upon leaving the adiabatic saturator, the air is usually within one or two degrees of saturation, that is, the entering dry-bulb temperature has been lowered nearly to the wet-bulb temperature, due to the conversion of sensible heat to latent heat. At temperature station IV, the wet- and dry-bulb temperatures are again observed. From here the air is returned over the entire inner casing *K*, (Fig. 11) of the saturator and is finally discharged at *M*.

Quantity and Velocity of Air

74 The diameter of each duct where temperature observations were made is 3 inches. Like air velocities were produced over each set

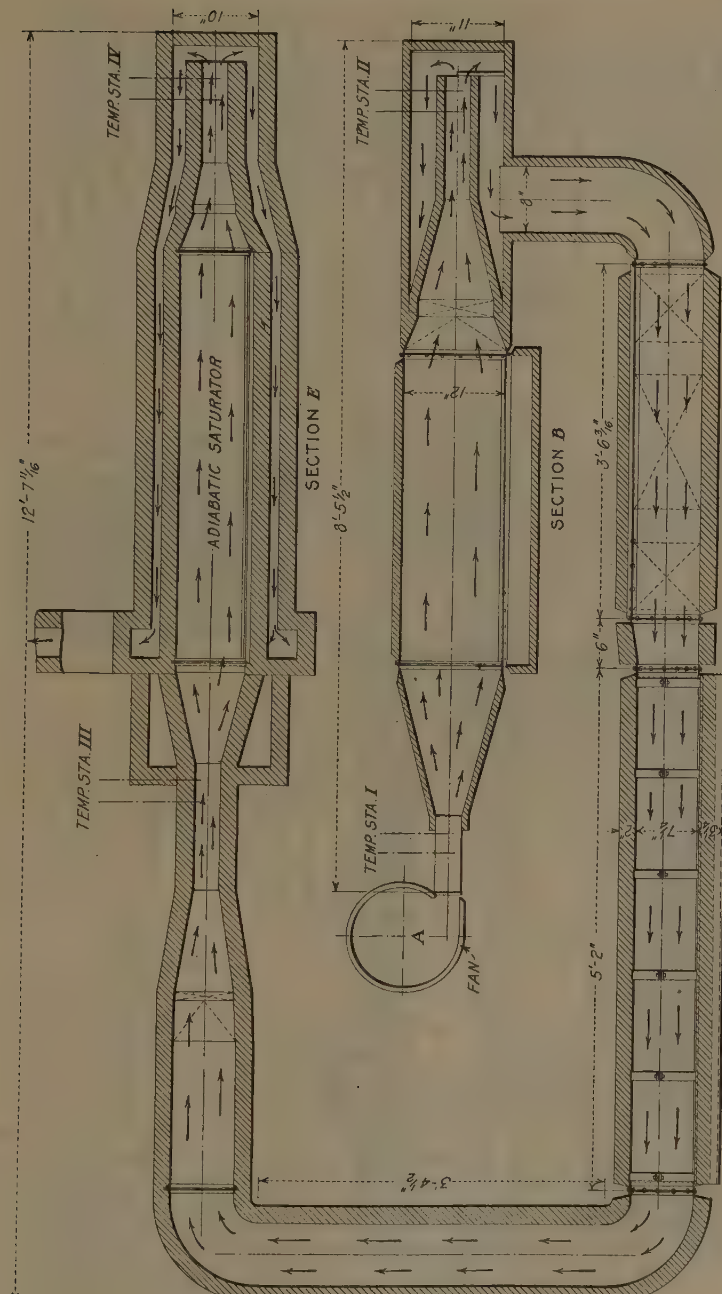


Fig. 10 Elevation of Apparatus Including Adiabatic Saturator (Sec. *E*)

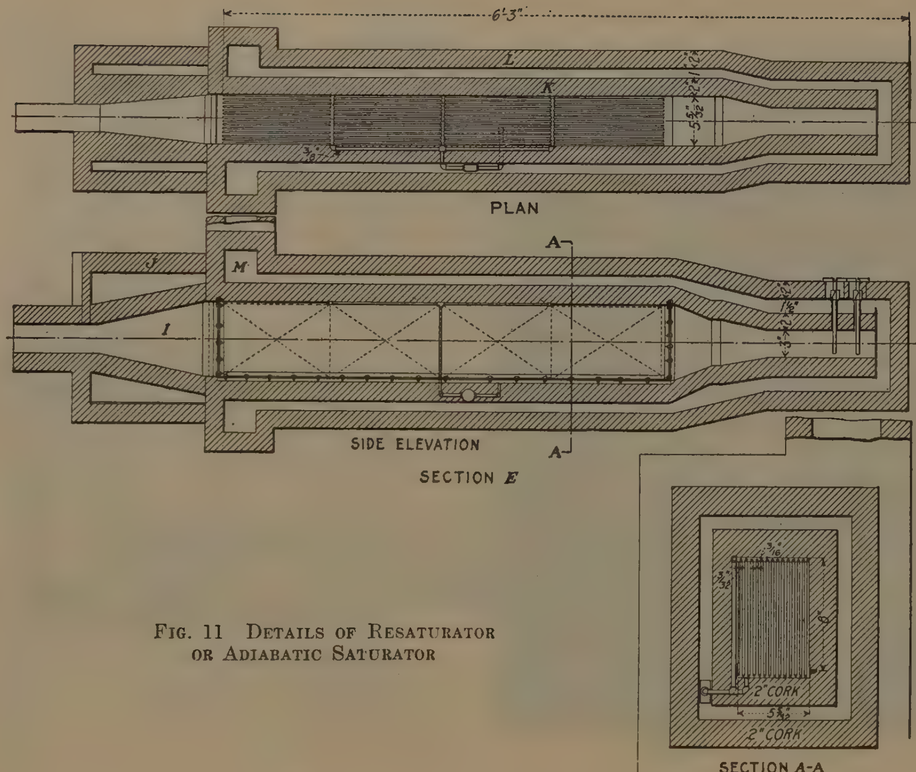


Fig. 11 Details of Resaturator or Adiabatic Saturator

of thermometers. At a mass velocity equivalent to 2000 ft. per min. at 70 deg. fahr., the condition under which the values in Table 1 were observed, approximately 100 cu. ft. of air per min. passed through the adiabatic saturator.

Temperature Control

75 The temperature of the air supplied to the adiabatic saturator was controlled and maintained at the desired point in the following manner: The air in the room from which the fan draws its supply was controlled within practical limits as to temperature and humidity by a small air-conditioning apparatus supplying air to the room. The dewpoint of the air passing through the saturator, Section *B*, Fig. 10, was fixed by saturation and fluctuations in the air temperature largely eliminated, by the heat capacity of the large quantity of water used in Section *B*, which, of course, assumes the practically constant temperature of the entering wet bulb. The heater, Section *C*, is made up of a series of electric resistance grids and the current supplied thereto was main-

tained at a desired constant state by very accurate, automatic regulation of the voltage on the laboratory supply lines. However, to further eliminate any temperature fluctuations that might occur, Section *D*, Fig. 10, was provided. This is known as a heat accumulator and is made up of about 1000 lb. of copper plates, stacked one on the other, with space between for free air passage. The effect of this is, of course, to furnish heat capacity and a resultant temperature lag, tending to eliminate any fluctuations that might take place. The result was prac-

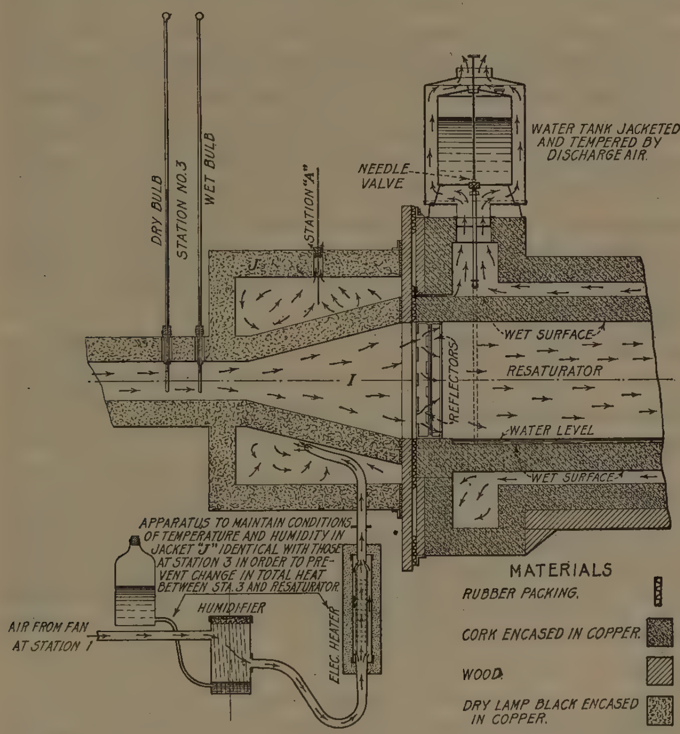


Fig. 12 Details of Construction of Adiabatic Saturator

tically constant wet- and dry-bulb temperatures at Station III, the entrance to the adiabatic saturator.

How Change of Total Heat is Prevented in the Adiabatic Saturator

76 *Conduction Through Casing Prevented:* Referring to Fig. 11, the casing marked K is built of cork encased in copper, while a second and similar casing marked L is built entirely around the first. The air passes from the saturator into casing L and is returned over the entire outer surface of K. The entire exterior surface of K, over which the air is returned, is kept wet. Thus the exterior and interior surfaces of the saturator casing are at the same temperature, i.e., the

wet-bulb temperature of the air and conduction through casing K is impossible.

77 *To Prevent Loss of Heat between Temperature Station III and Entrance to Saturator:* The temperature of the air measured at Station III, at entrance to adiabatic saturator, Fig. 12, was in most instances higher than that of the room. In order to prevent loss of heat by conduction and radiation between Station III and the saturator entrance, a double casing marked J, Fig. 12, was built around the main duct marked I, forming an air space as shown. By means of the small, auxiliary saturator and heater, as shown in Fig. 12, air was circulated within casing J at exactly the same condition, as to temperature and moisture content, as the air passing into the saturator. Thus, the interior and exterior surfaces of the duct between Station III and the saturator are at the same temperature and heat transfer through inner duct I is impossible. Since the air in J has passed through the same process of saturation as that in I, the vapor pressure in the air passing to the saturator and that circulated in J are alike and moisture diffusion through possible leaks into J is also prevented.

78 *To Prevent Entrance of Heat to Saturator by Conduction from Duct I and Casing J:* The connection between the saturator and duct I is constructed of waterproofed wood and rubber packing (both of which are poor heat conductors), in such a way that conduction of heat to the inner casing of the saturator is negligible. Details of this construction are shown in Fig. 12.

79 *To Prevent Entrance of Heat to Saturator by Radiation from Duct I:* In order to prevent entrance of heat into the saturator by radiation from duct I, a highly polished metal screen, made up of strips so arranged as to allow the passage of air, but to serve as a complete reflector of radiant heat, was placed at the entrance of the saturator as shown in Fig. 12.

80 *Temperature of Water Supply:* The small quantity of water admitted periodically to the saturator, to replace that removed by evaporation, was approximately at the same temperature as the water in the saturator, namely, the wet-bulb temperature of the air. This was accomplished by causing the air discharged from the saturator at M to pass over the surface of the tank containing the supply water, as shown in Fig. 12. The result was eventually to bring the supply water to the temperature of the discharge air, which we have shown to be approximately at the wet-bulb temperature.

Experimental Methods

81 *Observations and Corrections:* Preparatory to taking each set of observations of temperatures entering and leaving the saturator, the apparatus was in operation for four or five hours, in order to establish constant conditions. Temperature observations were, in all cases, made simultaneously at Stations 3 and 4, that is, at the entrance and discharge of the saturator. The mean results of the several series of such observations have been shown in Table 1.

Thermometry: The precision mercury thermometers used in the observations were manufactured especially for the work and donated by the Taylor Instrument Companies of Rochester, New York. Each thermometer was 30 in. in length and each graduated over a range of 30 deg. fahr. The divisions were approximately 0.75 in. per degree and each degree was divided in 1/10 degrees. Eighteen such thermometers were furnished, covering a wide range in the temperature scale. All thermometers were carefully calibrated in the laboratories of the Taylor Instrument Companies against their standard thermometers, which, in turn, had been recently calibrated against primary

standards at the United States Bureau of Standards. Calibration curves were made and utilized in correcting all observations.

83 All thermometers were calibrated for the seven-inch immersion at which they were to be used in the observations. The positions of the thermometers in the ducts are shown in Figs. 10, 11, and 12. Each thermometer in position was surrounded by a metal tube leading from the duct to the atmosphere. Leakage was permitted through each of these tubes, thus surrounding the entire immersed portion of the thermometer with air at the same temperature as that in contact with the bulb. (See Figs. 11 and 12.)

84 To insure accuracy of thermometer readings a peep sight device was employed which made it possible to sight along a plane perpendicular to the mercury column. All readings were made to an estimated 1/100 degree, which was not difficult, since 1/10-degree divisions were approximately 0.08 in. apart.

85 Finally, in order to detect and eliminate any constant discrepancy between the sets of thermometers used at the intake and discharge of the saturator, the thermometers were interchanged in position during each set of observations.

86 *Mechanical and Thermodynamic Corrections:* Two corrections applied to the resultant wet-bulb temperature at the outlet of the saturator remain to be described.

87 *Mechanical Heat Conveyed to the Liquid by the Pump:* One known source of external heat entering the saturator rested in the mechanical heat delivered by the pump to the water in the form of frictional heat and the potential energy dissipated from the water in flowing from the distribution pipes to the bottom of the saturator. It was assumed that if we could cause all of such heat to be delivered to the water and allow none to escape to other sources, that the external heat entering the saturator would be a determinate amount, namely, the total power input to the pump.

88 To establish this condition, the pump and all the piping were located in the air space between casing K and L. (See Fig. 11.) The pump casing and piping were then insulated with an inch of hair felt, covered with waterproof tape and finally covered with wicking which was kept wet during operation. The air leaving the saturator and passing back over casing K also passes over the wet insulated surfaces of the pump and piping, thus maintaining the surfaces at the wet-bulb temperature of the air, which is the ultimate temperature of the recirculated water. With this treatment, any transfer of heat to the exterior of the pump and piping is negligible and it seems safe to consider the total power delivered to the pump as heat added to the air passing through the saturator. The rise in the wet-bulb temperature of the air leaving the saturator, due to this source of heat, may be computed from the following equation:

$$D = \frac{h}{(C_{pa}+C_{ps}w')\,A}\,\frac{dt'}{dt} \quad \ldots\ldots\ldots\ldots \quad [26]$$

where:

D = temperature rise in wet bulb due to pump heat

h = B.t.u. per minute delivered by pump, or

h = watts to operate pumps exclusive of all external shafting belts, etc. $\times 0.0569$ B.t.u.

C_{pa} = specific heat of the air

C_{ps} = specific heat of steam

w' = pounds of water vapor per pound of air when saturated at wet-bulb temperature t'

A = pounds of air passing through saturator per minute

$\frac{dt'}{dt}$ = change in wet-bulb temperature per degree change in dry-bulb temperature.

89 The rise in wet-bulb temperature, computed from this equation, ranges from 0.01 to 0.03 deg. fahr. according to the temperature, and such values were deducted from the observed values of the wet bulb.

90 *Change in Wet-Bulb Temperature due to Change in Total Air Pressure between the Inlet and Outlet of the Saturator:* In passing through the saturator, the total pressure of the air is reduced and in case no water is being evaporated into the air, the partial pressure of the vapor in the air is reduced by a proportionate amount, that is

$$\frac{P_1}{P_2} = \frac{e_1}{e_2}$$

However, the effect upon the wet-bulb temperature must be described by a somewhat more complicated equation which is presented without the steps of derivation in the following form:

$$\Delta t' = \frac{\left(\dfrac{P_1-P_2}{P_1}\right)e_1 + \dfrac{(P_1-P_2)\,(t_1-t'_1)}{2850}}{\dfrac{de'}{dt'}\left[1+\dfrac{(t_1-t'_1)}{2850}\right] + \dfrac{(P_2-e'_1)}{2850}} \quad \ldots\ldots\ [27]$$

where

$\Delta t'$ = reduction in wet-bulb temperature due to reduction in pressure at outlet of saturator

P_1 = total pressure of air entering saturator

P_2 = total pressure of air leaving saturator

t_1 = dry-bulb temperature of entering air

t_1' = wet-bulb temperature of entering air

e_1 = vapor pressure corresponding to dewpoint temperature of entering air

e'_1 = vapor pressure corresponding to the wet-bulb temperature of entering air

$\dfrac{de'}{dt'}$ = increment in vapor pressure per degree change in wet-bulb temperature.

91 The values of $\Delta t'$, due to change in pressure, computed from the above equation, at wet-bulb temperatures from 40 to 90 deg. fahr. at an air velocity of 2000 ft. per min., range from 0.02 to 0.05 deg. fahr. Such values were added to the observed wet-bulb at Station IV, the discharge of the adiabatic saturator.

92 In Table 1 a summary of the observations, with all corrections applied, has been presented.

APPENDIX NO. 4

APPARATUS AND EXPERIMENTAL METHODS IN DETERMINATION OF THE VARIATION OF ERROR IN THE WET-BULB TEMPERATURE WITH VARIATION IN AIR VELOCITY AND WET-BULB TEMPERATURE

92 The general method and purpose of this experiment have been described in Pars. 19 and 20.

Apparatus

93 Fig. 13 is a general view of the apparatus and the room in which it was situated. The apparatus consists of a fan supplying air to a long duct which is divided into four area-reduction stages as shown. The areas of the duct are so proportioned that the velocity of the air is multiplied by four at each successive stage. For example, 100 cu. ft. of air per min. at 70 deg. fahr., supplied by the fan, is dis-

charged at Station 4, the end of the duct, at a velocity of approximately 2000 ft. per min. Thus the velocities at the three preceding stations, numbered Stations 1, 2 and 3, are respectively 31.25, 125 and 500 ft. per min. Wet- and dry-bulb thermometers are located as shown at each of these stations to permit the simultaneous observation of wet-bulb depression at like air conditions and different velocities. Uniform distribution of the air over the entire cross-sectional area of the duct is accomplished by the baffle and two layers of cheese cloth, located as shown near the discharge of the fan.

94 The air supplied to the duct is drawn in from the room by the fan and discharged to the room. Hence temperature conditions within and without the full length of the duct are alike except for the small amount of mechanical heat delivered to the air by the fan. Corrections for this change of temperature between stations or a change of temperature due to lag within the duct are applied as shown, in Table

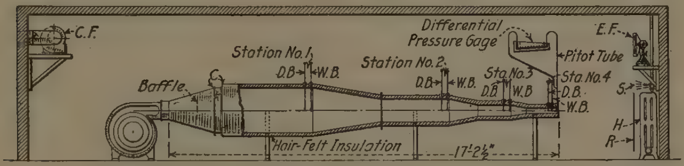


Fig. 13 Apparatus for Determination of Error in Wet-Bulb Temperature Due to Variation in Transverse Velocity over the Wet-Bulb Thermometer

(D.B., dry-bulb thermometer; W.B., wet-bulb thermometer; C., two cloth distributors giving uniform velocity over entire area of duct; C.F., small conoidal fan to deliver outside air, controlled by hand damper; E.F., electric fan to produce uniform room-air circulation; H., heater, hand controlled; R., radiation shield; S., hand-controlled steam spray for humidity control.)

2. To prevent local changes of temperature in the duct, the entire outer surface is insulated with two inches of hair felt.

The Room

95 The room in which the apparatus is located is constructed of studding upon which is tacked an inner and outer wall of composition roofing paper. Temperature and humidity conditions in the room were maintained at a satisfactorily constant state by the equipment shown in the illustration; a fan to supply fresh cool air from the outside in required amounts, a disk fan situated to circulate the air within the room, a direct steam radiator under hand control and a hand-controlled steam jet for humidifying.

Temperature Observation

96 Observations were made in three sets for each temperature run, as shown in sample Table 2. For instance, operating at a velocity of 2000 ft. per min. at Station 4 with dry- and wet-bulb temperatures of 100 and 80 deg. fahr., a series of readings were taken simultaneously at Stations 1 and 4. The comparative velocities in this case are 31.25 and 2000 ft. per min. Similar series of readings were taken at Stations 2 and 4 and Stations 3 and 4. In some cases a like set of readings was made at the same temperature but at a velocity of 1000 ft. per min. at Station 4 with resulting velocities of 15, 125 and 250 ft. per min. at the preceding stations. Two such sets of observations provided seven points of deviation in depression with respect to the depression at 2000 ft. per min.

97 Fig. 14 is an example of two sets of observed deviations in wet-bulb depressions with respect to the depression at 2000 ft. per min. In Fig. 2, previously given, the points are plotted from similar observations but the absolute errors at the various temperatures for 2000 ft. per min., as taken from the curve Fig. 1, have been added to each corresponding observed error.

Thermometry

98 The thermometers used in these observations were a good-grade, mercury, precision type having 1/10 deg. cent. graduations. Each thermometer used was carefully calibrated against a recently calibrated

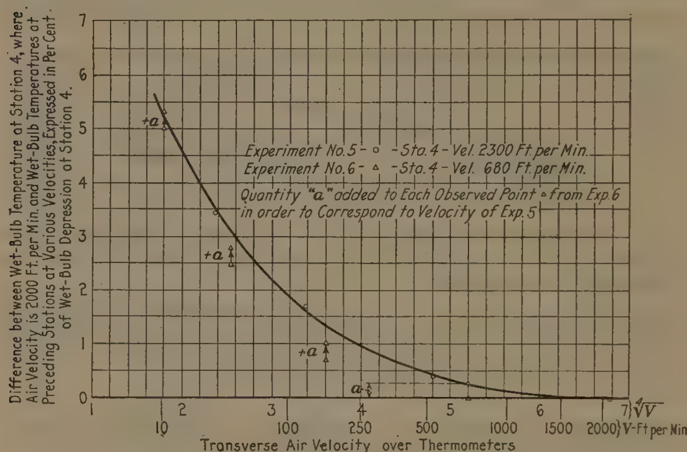


Fig. 14 Observed Deviations in Wet-Bulb Depressions with Respect to the Depression at 2000 ft. per min.

(Observations at wet-bulb temperature of 120 deg. fahr. taken from Table 1.)

standard and was intercompared with companion thermometers used in the observations. The precaution of reversing thermometers between stations being compared was also taken in these observations, as described in the discussion of thermometry in Appendix No. 3.

Velocity Measurements

99 All velocity measurements were made at Station 4 using a pitot tube in conjunction with a differential oil pressure gage. All velocity observations were reduced to equivalent mass velocities at 70 deg. fahr.

Correction in Observed Wet-Bulb Temperature for Change in Absolute Pressure between Stations

100 Static-pressure observations were made at each station for each velocity run. The difference in absolute pressure between each station and Station 4 was observed and corrections computed from Equation [27], Appendix No. 3, were added to the observed wet-bulb temperature at Station 4.

PRECAUTIONS OBSERVED IN READING WET-BULB THERMOMETERS

101 *Covering of Wet Bulb:* Clean, close-fitting white silk or muslin free from sizing or other soluble matter should cover entire immersed portion of bulb and stem. Covering should be frequently renewed.

102 *Liquid:* Liquid for wetting bulb should be distilled water at approximately the wet-bulb temperature or slightly cooler.

103 *Observations:* Observations of the temperature at which the mercury column reaches a stationary point are preferably to be made on a rising column rather than a dropping column, which is the reason for having the liquid somewhat cooler than the approximated wet-bulb temperature. For precision, care should be taken to sight the meniscus of the mercury column along a plane perpendicular to the column.

DANIEL C. LINDSAY. The writer wishes to emphasize a point, and perhaps add something to the paper prepared by Mr. Carrier and himself. Mr. Carrier has pointed out that our researches have shown that for practical engineering purposes the computed or theoretical temperature of evaporation, based on the heat-exchange equation, may be assumed to be the temperature that will actually result wherever the gas movement, over the evaporating surface, is above 200 ft. per min. Below this velocity, reference should be made to Figs. 5 and 6 of the paper. It might be inferred from this that we have shown only a negative result, or that we have pursued trouble only to find it non-existent. What we have done, however, is to establish definitely the limits within which the simple laws of heat transfer apply and at what point we must give attention to corrections.

For the purpose of accurate measurement of dewpoint temperatures and relative humidity the correction curves assume greater importance. It is explained in the body of the paper how, by the use of these curves, wet-bulb observations, taken under any condition, may be rationalized and corrected to conform to the humidity chart or table to which the observations are to be referred. Although it has long been common knowledge that the stationary hygrometer is subject to considerable error, the manner of making observations and their reference to tables, without regard to air velocities, has been most varied. The result has been a complete lack of accuracy and uniformity in recording this important property of the atmosphere, namely, humidity with reference to its effect upon persons and materials. For this reason alone the wet bulb has been discredited by many as an accurate instrument for dew-point observations.

With the use of coefficients involving the corrections shown in Figs. 5 and 6 of this paper, the writer ventures to state that the sling psychrometer is the most highly accurate instrument available for such determinations, aside, perhaps, from laborious, almost atomic, weighings of the moisture present in the atmosphere. The Regnault dewpoint hygrometer, in which the dewpoint is measured by observing the temperature of an evaporating liquid within a silver thimble at the instant condensation occurs upon the outer

surface of the thimble, is subject to serious error which many seem to have overlooked. Though the metal is a good conductor of heat, it is certain that the liquid is colder than the outer surface of the metal when condensation occurs. Some have suggested that the dewpoint temperature be taken as the mean of the liquid temperature at the instants of appearance and disappearance of condensation. A little thought will show, however, that unless the heat for causing the disappearance is applied from within the liquid, which has not been the practice, the observation of the liquid temperature at the instant of disappearance will be lower than the outer metal surface, just as at the instant of appearance, though perhaps not as much lower. The authors have definitely shown the amount of this error and its effect upon the present accepted Weather Bureau dewpoint tables. This will be presented in a subsequent paper.

Since the importance of velocity in its effect upon evaporative temperatures has been mentioned, the writer wishes to take this opportunity to again correct a fallacy which we see in print with surprising frequency. That is, the supposition that the amount of evaporative cooling or the wet-bulb temperature is dependent upon the rate of evaporation. This is true to just the extent shown in Figs. 5 and 6 of this paper. It is true because the constant sources of heat, other than those involved in the process of adiabatic saturation, become relatively less important as the velocity increases the rate of evaporation. It will be seen from the data therein presented that the difference between the wet-bulb temperature, at a velocity of 1000 ft. per min. and that at 2000 ft. per min. is negligible, while the rate of evaporation has almost doubled between these two velocities.

Anticipating a query as to the reason for the breaks in the curves in Figs. 5 and 6 at 32 deg. fahr., the reason is as follows:

At 32 deg. the latent heat of vaporization takes on an added 144 B.t.u.—the heat of fusion of ice. There is, of course, a resulting change in dh/dt, the slope of the total-heat curve involved in the equation of the curves in Figs. 5 and 6. It is an interesting point in thermodynamics to note the cause of this sudden change in latent heat. If we plot the observed values of vapor pressure over water and over ice, we find the two the same at 32 deg. fahr. However, at this point there is a sudden change in the slope of the vapor-pressure curve. By reference to the Clapeyron equation for latent heat, which is, $r=A(v''-v')T(dp/dt)$, it is readily seen that the increased slope of the vapor-pressure curve below freezing will cause the elevation of the values for latent heat.

No. 1935

THE DEVELOPMENT OF A MODERN HOSIERY PLANT

By Sanford E. Thompson,[1] Boston, Mass.

Member of the Society

and

H. T. Rollins,[2] Des Moines, Iowa

Non-Member

This paper describes the development of a large hosiery plant in Iowa during the thirty-two years of its existence.

From nearly the beginning the scientific method of approach is evidenced, culminating in the most recent development of personnel service, production control, standardization of methods and incentives, cost control, and marketing.

Each of these is discussed in detail with a view to indicating, not simply the rather notable results attained, but the means employed for their accomplishment. These are presented with the purpose of giving inspiration to the reader who desires to develop his own plant and organization along modern lines and in a fashion broad enough to include the human element as a major factor.

MANUFACTURERS are familiar with the facts of the growth of scientific management in the automobile, metal trades, and similar industries. Such application in the textile industry, particularly in the knit-goods division, has been more rare. Perhaps the best example of all-round accomplishment is afforded by the recent development in the Rollins Hosiery Mills, the largest hosiery mill west of the Mississippi River. The following paragraphs describe the methods adopted, which contributed to the increase of business shown in Fig. 1 and the decrease in labor turnover shown in Fig. 2. The developments described cannot be properly evaluated in terms of dollars and cents, because the chief benefits consisted in gaining smoothness of operation and procedure

[1] The Thompson & Lichtner Co., Engineers.
[2] President, Rollins Hosiery Mills.

Contributed by the Management Division and presented at the Annual Meeting, New York, December 1 to 4, 1924, of The American Society of Mechanical Engineers. Abridged.

and adjustment of inequalities which make not only for economy, but for the training and the well-being of the individual. It may be said, however, that the yearly savings accruing from this recent intensive work approach six figures. The small increase in overhead required to carry on the more scientific methods of management is evident from the fact that only three additional executives

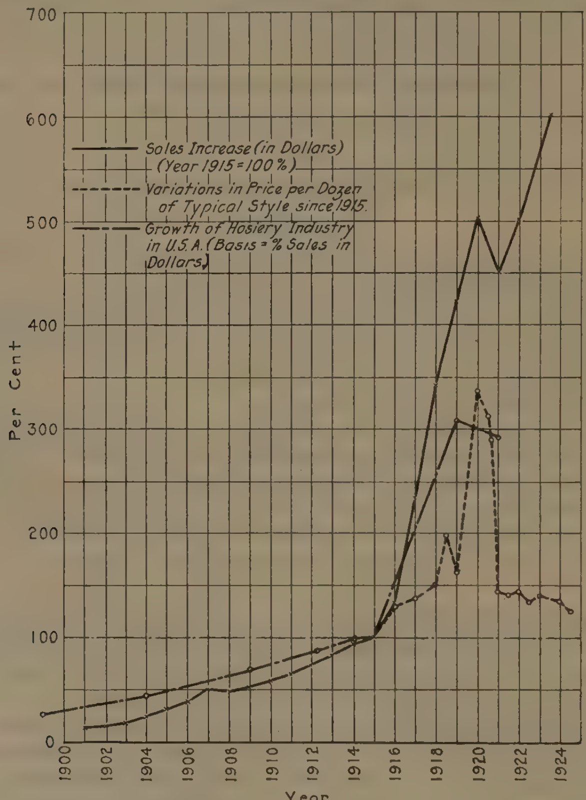


FIG. 1 GROWTH OF BUSINESS AS COMPARED WITH GROWTH OF INDUSTRY AS A WHOLE

and clerks are needed to handle the planned control, the standardization, and the cost accounting. The officers and executives of the company were assisted in much of this development — including the planned control, the cost accounting, the job analysis, and adjustment of methods and rates — by The Thompson and Lichtner Company, Engineers, of Boston. The company organization furnished the knowledge of process and the engineers their experience in control development in various lines of industry.

Organization

2 *Purpose.* " The purpose of organization is to correlate human effort for a mutual accomplishment. It is the function of management to develop equipment and method only as a means to that greater accomplishment which can be obtained alone through harmonious and coördinated human effort. The problem of human relationship, therefore, is the main problem of management."

3 In order to facilitate the working of its organization, the Rollins Hosiery Mills set down in writing, with the aid of the engineers, the chief functions of the organization and defined

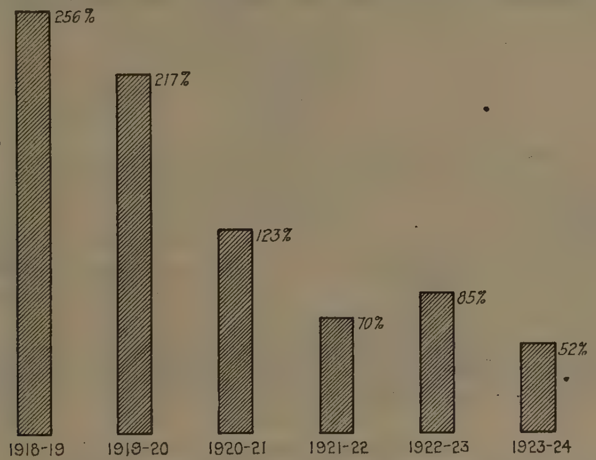


Fig. 2 Labor Turnover, 1918-1924, Des Moines Hosiery Plant

(Average in Des Moines plants estimated by H. C. Pfund, statistician of Des Moines Employment Bureau, to be 200 per cent at present time.)

plainly these functions and their correlations. This written analysis consisted of

(*a*) Organization chart
(*b*) Responsibility layouts for executives and supervisors
(*c*) Handbooks of standard practice.

These have proved their value in plant operation and practice.

4 *Organization Chart.* A partial organization chart is shown in Fig. 3. All of the departments are subdivided into the proper subsidiary divisions. The officers and directors act as the administrative board to formulate business policy and the two principal officers head the three departments responsible for carrying out this policy, namely:

(*a*) Manufacturing
(*b*) Financial and
(*c*) Sales Management.

5 In the manufacturing division the production superintendent and the director of service are staff assistants, the former handling

planning and control, processing and maintenance, the latter directing industrial relations and motivating the organization to carry out the policies decided upon. The details of planning and control devolve upon the planning supervisor. The other functions of production management are supervised respectively by the cost-department supervisor, methods engineer, payroll supervisor, chemical engineer, and mechanical superintendent. Two process superintendents handle the actual processing, one in charge of the knitting, the other of the finishing departments. Each of these in turn is assisted by various supervisors in charge of divisions of the processing, such as the rib-knitting supervisor, the string-

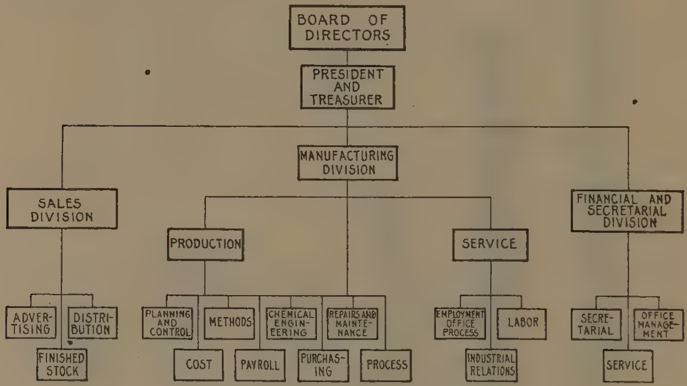


Fig. 3 Partial Organization Chart

knitting supervisor, etc. Responsibility for carrying out the production orders lies with such a supervisor, and he is assisted by

(*a*) An assistant supervisor in charge of the training and discipline of the personnel. If the department is large, one or two instructors are given the function of training and reporting to the assistant supervisor

(*b*) A head machinist in charge of the equipment and group of assisting machinists.

6 The director of service reports to the head of the manufacturing division, but the department's activities apply to the entire organization, plant and office, and cover a wide range of effective service. Besides the usual employment function, for example, the department handles the personnel-development program, betterment work including food service, first aid, employees' benefit association, recreational activities, and community-service work. The department coöperates in matters of plant policy, working conditions, and wage setting, and acts as an intermediary between management and workers through the Employees' Representative Committee of Service. This committee is composed of

representatives from each major division of plant and office, who meet regularly to discuss all general matters affecting the personnel: Hours, plant improvements, wage policies, activities. With advisory powers only, this shop committee acts as a clearing house for what is on the workers' minds as well as what is intended by the management.

7 *Responsibility Layouts.* A written analysis, called a "responsibility layout," has been prepared for each position from process superintendent to operative. This gives in brief the line organization and the general responsibilities of the job. Following this are described, step by step, detailed responsibilities. For the supervisors the headings cover:

(*a*) Operations and processes
(*b*) Equipment
(*c*) Stock
(*d*) Personnel
(*e*) Coöperation.

8 For the assistant supervisors, records and instructions are treated instead of equipment and stock. Particular care has been taken in these layouts to point out the proper staff coördination of each function, a coördination not shown on the organization chart. Frequent reference is made also to the Supervisors' Handbook of Standard Practice.

9 Standard practice includes the definition of plant policies in the supervisors' handbook, the instruction sheets for each job, and the employees' handbook. Not only are these direct aids in the plant operation, but also uniform practice is assured throughout the organization.

10 *Supervisors' Handbook* has two parts:

I — Duties of Supervisors
II — Regulations and Information.

Under Duties are treated such subjects as coöperation with management, personnel, quality, and efficient production. Under Regulations and Information are treated the details of plant practice as to attendance, wages, employment, safety, conduct, and plant services. For example, the exact procedure for transferring an employee is set down, so that a supervisor knows how far he is to proceed in the matter, his responsibility to his immediate superior — the process superintendent — and to the proper staff executive, who in this instance is the director of service.

11 The section on wages is also of particular value. Here are treated such subjects as the procedure of wage payment, bonuses and premiums, rate setting, wage adjustment and correction, change of rate, payment in exceptional cases such as National Guard and jury service, overtime, absence, workmen's compensation, temporary illness, idle-machine time, committee service, plant tours, and lastly, the individual-earnings charts.

12 A complete index makes the information in the handbook instantly available to the supervisor.

13 *Job Instruction Sheets* are followed by the instructors in training learners, and for reference. Each production job has been analyzed from an instructional standpoint and the steps in it set down, not in the order of performance necessarily, but in the proper order for presentation to the beginner. The instruction sheets, which are revised from time to time in the light of experience and time-study data, include the correct cycle for performing the job elements, with the standard time for each element. They give the minimum, expected, and maximum production for each machine per style and size.

14 *Employees' Handbook,* a printed booklet entitled "Workers Together" and prepared for the employees, summarizes the privileges and practices at the Rollins mills. Upon being hired an employee is required to study this handbook.

15 The purpose of the management in preparing these charts, outlines, handbooks, and instruction sheets was not to make the organization formal or cumbersome, but to give an understanding of the proper functionalization and coördination of line and staff executives, and thus develop a spirit of teamwork.

Service Department

16 An outline of the progressive development of the Service Department — functioning as it does with only a director and one assistant — is of interest not merely from the standpoint of hosiery mills but also as illustrating modern and up-to-date methods applicable to any industry.

17 *Securing the Labor Supply.* The main objective of the first year's work of the Service Department was the securing of a sufficient supply of labor. Des Moines, the capital of an agricultural state, has no industrial population. A careful study of the situation showed, first, that an ample supply of desirable labor existed, and second, that if proper publicity were used it could be attracted to the mills.

18 The publicity decided upon was designed to "sell" the workers as well as the general public on the institution and the opportunities it offered. Every old employee, as well as each new employee, was conducted through the plant and each job was carefully explained. Not only was the entire process made clear but a new sense of responsibility, interest, and job pride was created as a result of these tours. Instead of envying May Jones, transfer knitter, who was said to make high wages at a very easy job, the girls in the boarding department came back to their jobs rejoicing that they didn't have to work with all that noisy machinery. Threats to quit and requests for transfer diminished. Instead, the workers began to bring in their friends, seeking jobs for them. Visitors from the outside were made welcome, especially friends of employees or classes from the schools and colleges. The use of the want-ad columns was discontinued. Instead, a series

of pictures appeared in the Sunday rotogravure section of the leading newspaper, depicting interesting phases of the work. Pages appeared regularly in the neighboring school magazines, the copy, institutional in character, calling attention to vocational opportunities offered by the mills. A series of letters sent to every home in the community were of a similar nature.

19 Textile exhibits were prepared and sent to the local high schools, talks were given on vocational education before schools and clubs, the chamber of commerce and other interested groups, and exhibits of an educational and institutional character were given at the annual industrial exposition.

20 All these efforts bore fruit; today the organization is held in the highest repute, it has a large waiting list of progressively superior applicants, and for the past three years the labor supply has taken care of itself without special attention.

21 *Selecting the Workers.* The next objective was the proper selection of the working force. During the first year the director of service studied each job in the plant in detail. The number employed on each job, the labor turnover, performance records of different types and ages, training methods, length and cost of training period, method of wage payment, etc. These studies resulted in the drawing up of written job specifications to be used in selecting and assigning workers, and disclosed the need of proper training methods.

22 It was found advisable to test the eyes of all prospective knitters, loopers, inspectors, and menders. The examination of those already on these operations resulted in the transfer of many to other less exacting jobs. One woman who had been an inspector for more than three years was found to have less than half normal vision. A change to double sole cutting, where vision is of minor importance, enabled her to increase her earnings and relieved her of chronic headaches to which she had long been subject.

23 With the substitution, in the boarding department, of metal forms in place of the old-type wooden forms, women workers replaced men boarders. This is one of the most difficult departments in a hosiery mill, because the work requires constant standing and walking about between steam-heated metal forms over which the wet stockings are drawn for shaping. The less experienced operator's hands sometimes blister in handling dainty silk and chiffon stockings. After a year's trial it was found that the women did better work but objected to boarding ladies' styles, asserting that the extra reaching caused unusual fatigue. It seemed impossible to overcome this prejudice. Several men, especially selected for their careful workmanship, were assigned to ladies' style forms. Soon after some of the girls asked for assignment to ladies' styles, and thus another tradition had been disposed of.

24 Turning the stockings right side out is one of the low-end jobs in a hosiery mill. Young boys had always been used on this job, but they were careless, inattentive, miscounted the lots, and otherwise created a disturbance. At the same time it had been a problem to find enough jobs to absorb the girls who failed on knitting and looping. It was decided to substitute girls for boys on turning and thus use the girls who proved unadapted for the more skilled operations. The change has proved most satisfactory, particularly from the standpoints of neatness and accuracy.

25 *Training the Operatives.* A preliminary study of training methods in existence disclosed that in most cases the machinist or foreman did the teaching, or the newcomer was assigned to an older operator to "pick up" the job. Turnover records showed that 20.8 per cent of those who left did so before one week had elapsed, 45.5 per cent under one month, and 71 per cent before three months. Of those who survived, very few attained what is today termed "70 per cent efficiency." The worst feature of the old method was that failure due to lack of standardization was attributed to stupidity, carelessness, or indifference. In tackling this problem, three points were considered:

(*a*) The standardization of equipment and method of operation

(*b*) The selection and training of competent instructors in the correct use of both materials and methods

(*c*) The standardization of the length of the training period and compensation during such period.

26 It was decided not to set up a separate training department, but to isolate certain machines on the factory floor and train beginners there on regular work in the atmosphere of regular production. The theory that "anything will do to learn on" was rejected. The beginner was given the best equipment, her chair or table was adjusted to her individual requirements, and she was drilled on the simplest elements of the job for short periods at a time. With rest periods and early dismissal throughout the first week, the fatigue of new work and unaccustomed positions was successfully combated.

27 The foreman, in work requiring exceptional skill, should not be expected to include teaching among his responsibilities. To dub a good operator "teacher" is nearly as bad, as all too frequently a good operator may become a poor teacher. The people with teaching ability were chosen first and taught the job if necessary. Then everything the learner should know was set down step by step, arranged in the order of instruction. Coöperating with the Federal and State Board for Vocational Education, a specialist in trade training was brought in to develop the instructors.

28 A learner's bonus decreasing in proportion to the increase in efficiency had been paid, in addition to the piece rates, on all

jobs requiring over two weeks to learn. Such a plan offers an incentive to progress, with the total earnings gradually but constantly on the increase. It was soon found that owing to the rapid increase in skill the amounts of these bonuses could be materially decreased.

29 Under the present system, the moment an applicant is hired, her training begins. A preliminary talk gives her a good idea of the advantages and disadvantages of the new work she is to undertake, the starting wage, and opportunities for advancement, and a manual of information sets her right on many points. When she reports for work she is personally taken to the department and introduced to the supervisor and instructor. Further lessons include the history and organization of the company, its policies, a study of all processes in the making of hosiery, the character and source of raw materials, trade terms, the why of inspection standards, elementary economics concerning costs, including exact figures, wage setting, production records, shop hygiene, and general plant practices. She is informed as to her committee representative and the channels for hearing of grievances, as well as of social and recreational opportunities; she is shown through the entire plant; in brief, she is taught how to become a functioning part of the organization. Most of this training is given on the production floor to the individual beginner, though frequently group meetings and discussions are held.

30 By posting schedules for accomplishment for each day and week of the training and by graphic charts showing just how each girl exceeds or falls below the schedule, great interest and enthusiasm have been aroused. Daily reports are sent to the Service Department showing each beginner's work for the previous day. A definite amount of production for each day's and week's progress has been worked out by the Methods Department, and those failing to attain it are given individual attention to see if transfer to some other work would help.

31 *Training the Supervisors.* At the beginning of this development a series of round-table conferences was arranged on the primary problems of a supervisor, followed by a series of talks by various department heads on their particular departmental work. Then the study of a correspondence course in production methods was undertaken, during which bi-weekly conferences were held on the text and problems of each unit of the course. Next, more specialized problems of the organization were analyzed and studied, with speakers from the outside alternating with the company's own speakers. Group conferences were utilized to present the new systems of production control and cost accounting, to explain the organization charts and standard practice. Individual conferences were resorted to in special instances to develop certain supervisors.

Production Control

32 The plan of control adopted with the coöperation of The Thompson and Lichtner Co., Engineers, provided means for planning the work in advance, both as a whole and in the various departments, and in some cases for the operators themselves. This control of production made it possible to place goods on the shelves in the shipping department in the right proportion of styles, sizes, and colors to meet the date of delivery furnished with the orders.

33 In order that the reader may have as a background a picture of the manufacturing process, a brief description of this follows:

34 *The Product.* Four classes of stockings are made, namely, men's, ladies', misses', and infants', the sizes in each class ranging respectively, by half-sizes, as follows: 9 to 13, 8 to 10½, 5 to 11, and 4 to 6½. Approximately forty styles are represented in these four classes, each style differing chiefly in kind of yarn and number of needles used in knitting. Six different kinds of yarn are used, such as thread silk, cotton, etc., the same kind of yarn varying in count of number of threads to a strand. The yarns are white, and each style, designated by a style number, after knitting, is dyed into two or more colors—in ladies' styles, for example, as many as twenty-five different colors making up the line.

35 *The Manufacturing Processes.* In planning the production the manufacture is divided into two major processes. The first process consists primarily of knitting operations and completes the hosiery for Worked Material Stores known as Gray Stock. The second process is chiefly dyeing and finishing the hosiery for shipment.

36 The twenty-two possible operations through which the stocking must pass are in sequence as follows:

A—Knitting
- Rib knitting (all men's, children's, and misses')
- Rib Cut and Inspect (All men's, children's, and misses')
- String Knitting (Ladies' only)
- Double Sole Cutting
- Inspection
- Seaming
- Looping
- Inspect and Mend
- Turning

B—Bleaching and Finishing
- Singeing
- Dyeing or Bleaching
- Beating
- Boarding
- Pairing
- "O.K." ing
- Mending
- Stamping
- Toe Sticking
- Ticketing
- Pressing
- Folding
- Boxing

37 *Development of Control.* A tabulation and study of sales was the first requisite. From this study it was possible to work

out the correct volume of production needed by styles, and the correct proportion of sizes and colors for each style.

38 A comparison of Total Sales, At Once Sales,[1] Orders to Knit, Total Knit, as well as Sales records for previous seasons, is maintained, corrections being made to this record weekly. From this comparison, total orders are determined, these orders being in turn broken down by departments and individual sets of machines. From these individual orders, Knit production is deducted daily and additions made from time to time as sales increase, thus maintaining a correct balance of orders at all times. A weekly accumulation of sales statistics is maintained and delay in presentation of sales figures is eliminated.

39 The Production Control Board, with its movable tapes and colored pins, visualizes the status of sales and production. Fig. 4, is a photograph of the board itself and does not show details clearly. Its use is described in Appendix No. 1.

40 Studies were made to determine the possible production from each department. Using these figures as a norm, actual production is checked against them, taking into consideration, of course, losses due to idle equipment. Idle-machine cards are used in each department, on which machine time lost is recorded, together with the reason for the loss. Production in all manufacturing departments, both at the Des Moines and the Boone, Iowa, plant, which is controlled from Des Moines, is recorded daily, thus enabling the Production Control Department to keep in closer touch with actual operations and to anticipate more easily the changes necessary from time to time.

41 By starting a record from actual inventory of goods at each process of manufacture and posting to this record daily the work passing through each operation, a daily check is secured on the accumulation of goods at any point in the plant. Permanent inventories by style and size are set up for goods in Gray Stores, also by style, size, and color for all finished goods in warehouse. Orders for dyeing are then made from a close study of the sales, with the gray stock and finished stocks in mind, together with the capacity of the Dyeing and Finishing Departments considered. A careful accounting for all goods sent to the Dyeing and Finishing Departments is required, with an explanation of any goods

[1] "At Once" sales are divided into two classes: those shipped on date of receipt, and those considered "At Once" from a manufacturing standpoint. In both cases shipments are made according to specified delivery date. As orders are received they are tabulated, and at the end of each week accumulated and entered on either the "At Once" or "Future Sales" record. At the end of the last week in each month, "Future Sales" for two months ahead are considered as "At Once" sales and entered on the "At Once" Sales Record. This plan enables the Production Office to maintain sufficient stock in warehouse to meet shippers' demands. The tie-in of these sales figures with the balance of goods in process and finished stock is discussed later.

running over or short of the original amount issued from the Gray Stores.

42 Stocks in correct proportion and an evenly balanced production are emphasized throughout the processing departments. Stocks are cleared out in all departments as orders are issued to discontinue the manufacture of any style or size, thus preventing the accumulation of obsolete merchandise.

43 Route sheets are maintained for following the goods through the Dyeing and Finishing Departments. In case they do not proceed through the various operations in these departments according to the schedule provided, delayed-batch reports are dispatched to the supervisor calling for an explanation of the delay.

44 A back-order sheet is furnished daily by the Shipping Department, showing all items which have been delayed in shipment due to lack of finished stock. This report is closely watched

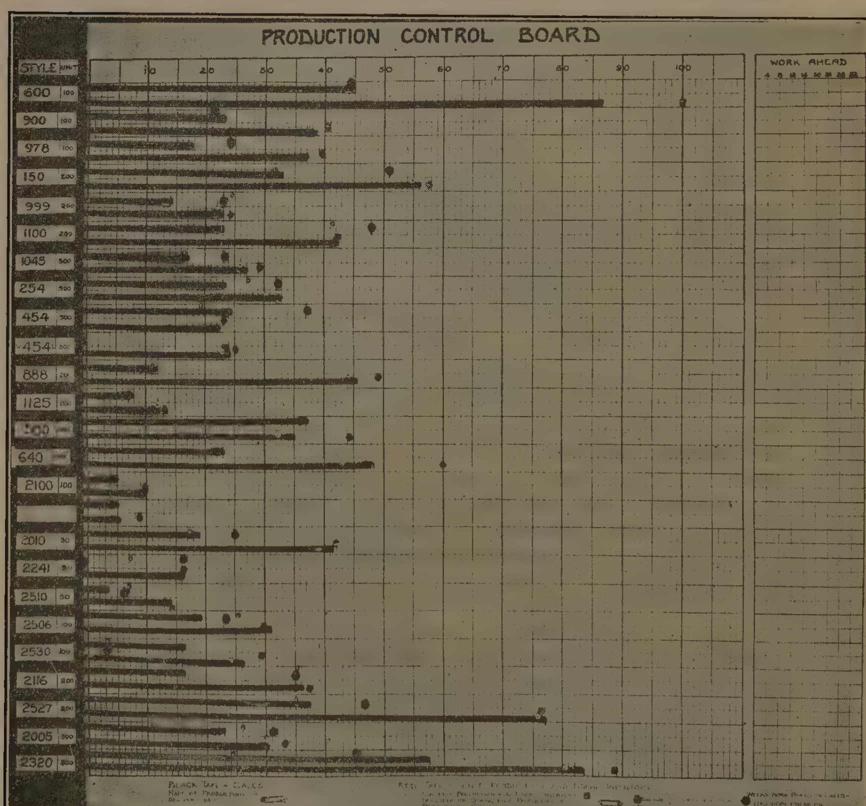


Fig. 4 The Production Control Board

as the prevention of back orders is, in the final analysis, one of the main functions of a production control plan.

45 There are two seasons in the hosiery industry, spring and fall. Gross orders to make are based chiefly on estimated requirements and present inventories. The percentages of average sales of each style are derived from sales statistics. In determining the quantities to make for new styles, estimates are based entirely on sales probabilities. The same method is pursued in determining the amount to dye of any new color.

46 The actual operation of the plan in practice may be visualized better by following an order for a certain style through the various steps of production control to the point where the goods are placed in the finished stock. This procedure is given in detail in Appendix No. 1.

47 *Results of Planned Control.* Of the various accomplishments through Planned Control, the most important, probably, were, first, those that made possible a uniform flow of the goods in process through the plant, eliminating the occasional congestions and deficiencies; and, second, the balancing of the styles and colors, both in process and in finished stock, so as to maintain a known and required proportion of each and thus truly tie together the manufacturing and sales divisions and give what is wanted at the time it is wanted.

48 The study of methods of sales analysis and tabulation resulted in the adoption of uniform methods and an adequate plan of work to meet requirements. This analysis of sales requirements and production schedules also resulted in reducing to an extremely small percentage the amount of stock which was not perfect in every respect and available for shipping. This also permitted an appreciable reduction, amounting to nearly 50 per cent, of the stockings in Gray Stock — that is, completed ready to dye. It also, because of the better-balanced quantities of the various styles and colors, permitted an appreciable reduction in warehouse inventory of finished goods without affecting the ability to fill orders promptly. This relieved congestion and reduced the amount of invested capital. Incidentally, also, the quantity of "reboards" — that is, the stockings which had to be sent back to the Finishing Department to be reboarded — was reduced, accumulation of redyes was eliminated and further accumulation prevented by the control from the Production Office. The routine established for handling "seconds" reduced the number and eliminated excessive work on day pay in the Finishing Department.

49 The Production Control not only smoothed the operating but also controlled the Maintenance Department through proper issuance of instruction and follow-up of work done. This also permitted proper distribution of departmental expenditures into the Cost accounts, and concentration of machine parts and supplies

to insure having supplies on hand when wanted, and reduced waste in machine parts.

50 The fundamental principle in this production control is the functional development that caused the various plans to operate, not merely at the start but on a permanent basis. The planning function is carried out with this clear-cut plan by the Production Office instead of by individual foremen, thus assuring coördination through the entire plant, utilization of capacity of all machines, control of materials and supplies, and immediate knowledge of manufacturing progress. The concentration also of pay roll, cost, methods, and production under one production superintendent permitted better functioning of each and a closer exercise of supervision.

51 Essential also to the permanent development of the control has been the standardization which made accurate time schedules possible, the cost accounting which tied in with the control and furnished valuable figures, and the coöperation of the Service Department in training the workers and developing the organization as a whole.

Standardization

52 *Establishing Base Rates.* To determine correct base rates for each job in the factory, a study was made of wages in the community, wages in the industry, supply of labor available, amount of job instruction necessary, and such factors as the disagreeableness of the work and its hazards. The character of each job was weighed in its relation to its starting wage and it was found possible, with 75 cents added for good attendance, to choose a highly selective group of workers in the locality.

53 Base rates were thus established for twenty different operations, these rates applying to operatives when on day work, and also formed a guide to the amount that should be earned on piece work in these operations. The hourly rates on the more skilled operations such as Transfer Knit, Looping, and Boarding were in themselves graded and were substantially double the hourly rates on the less-skilled operations such as Raveling and Turning.

54 *Standardization of Methods.* Several examples of the changes and improvements in methods and the results obtained through the work of the Methods Department are of interest as showing what can be accomplished in an established industry and a well-managed plant.

55 "Rib Knitting" is an automatic operation in which the machines knit continuous tubes of ribbed fabrics which later are cut into ribbed legs for children's stockings, ribbed tops for men's half-hose, and ribbed tops for ladies' stockings. An operator runs twenty to thirty machines. The method of driving machines was changed from an overhead belt drive to a drive from a floor shaft. With the elimination of the belt hazard and the simpler method

of operation, girls of less ability replaced men who were transferred elsewhere, with a reduction of 15 per cent in the base rate and with an appreciable annual saving.

56 "Rib Cut and Inspect" is a hand operation in which the tubes of ribbed fabric are cut with shears into the single ribs, the lengths and places for cutting being indicated by welt markings knit into the fabric. Previously one operator cut the ribs and another inspected. By job analysis with time study it was found possible to make this a single operation and eliminate part of the labor. Now, the cutter inspects one side as she draws the tube to position for cutting, turns it over just before cutting, and inspects that side almost simultaneously with cutting. Also the tables at which the cutting was done were raised so that, with high chairs and foot rests, the operators could change from sitting to standing at their work at any time, reducing fatigue. The number of style changes in a day were reduced by better planning, and production was greatly increased. The changes made, resulting in no harder and yet more diversified work, reduced the operating time more than 50 per cent.

57 "String Knitting" is an automatic operation in which a complete ladies' stocking is knitted, leaving only the toe open — which is later closed by "looping." One operator was handling thirteen machines in a line. This was changed to a double line so that the operator could tend the machines more easily and effectively and she was enabled to run sixteen without really adding to her labor. The machine's capacity was increased 14 per cent while the production increase was 20 per cent. In this, as in all knitting operations, the proper speeds were determined and machine speeds made to conform to these standards.

58 "Looping," while a semi-automatic operation, is essentially a hand operation from a time-study viewpoint, because the machines can be run at any speed desired, depending on the handling and feeding of the stocking.

59 The operators had been sitting in chairs close together and so low as to cause undue fatigue. The machines were raised to permit chairs of comfortable height; the machines were tilted up at a small angle to make a better angle between the operators' eyes and the points; the spacing between machines was increased; trays were provided for the work which had been in the way about the floor; and the machines were rearranged so that the finer machines received the best lighting.

60 The greatest improvement, however, was a change to the "counter method," whereby the operator sits erect and puts the stocking on to the moving points of the circular disk from left to right instead of from right to left, thus working toward the advancing points of the dial instead of striving to overtake them. By this method of working and regular instruction, the time of

training a looper was very markedly decreased. The output, because of these various changes, was increased over 15 per cent.

61 The study of this operation, which had formerly required 24 weeks to learn, indicated that the learning period could be reduced to 12 weeks. The cost of training was reduced as shown in Fig. 5 from $100.83 to $55.35. The learners' bonus was set in such a way that, when added to the piece-work earnings expected, the operator would receive constantly increasing weekly earnings, with no drop after cessation of the bonus. Fig. 6 indicates both expected and actual results. Any operator falling below the line of minimum earnings is given special instructions, and if this fails to remedy the situation, the operator is transferred to some other operation to which she is more adapted. Careful selection of operators by the Service Department reduces the number of such transfers required. Out of a total of fourteen started in eight months, two were transferred. Of the other twelve the curve for *B* shows the lowest record of the twelve, and *A*, one of the two highest. Each curve, notwithstanding its uniform slope, records more than one class of work by the operator.

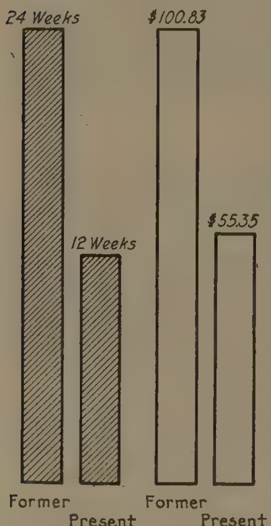


Fig. 5 Reduction in Cost of Training Loopers through Changes in Method and Training

62 " Boarding " is a hand operation in which the dyed stockings, wet, are dried smoothly in shape by being placed on steam-heated forms.

63 The operation requires physical energy under conditions of heat and humidity above normal. Ventilation was solved by putting in a fan to introduce air near the floor at each set of forms. After considerable study the mat selected to be placed on the concrete floor for the operators to stand on was an Ozite base with rubber matting upon it. Shadowless lighting, important to enable operators to detect wrinkles in the stockings on the forms, was obtained by Cooper-Hewitt mercury lamps. Recesses of fifteen minutes were established morning and afternoon. Operation of the actual work itself was studied, and the method standardized. All of these things added to the physical comfort, and tended to reduce the labor turnover.

64 *Incentives.* The various operations in the mill are of so varied a character as to require, for best service, various forms of incentives, each adapted to the individual operation.

65 In " Rib Knitting " a piece rate is paid plus a waste bonus, the relative amounts of bonus earned being greater as the amount

of waste is lower. The waste can be kept low by careful, regular inspection by the operator and immediate stopping for adjustment of any machine producing waste.

66 In "Transfer Knitting" four-tenths of the regular piece rate is paid for all imperfect work. Absolute perfection, however, is not expected, as the imperfects may be due either to the opera-

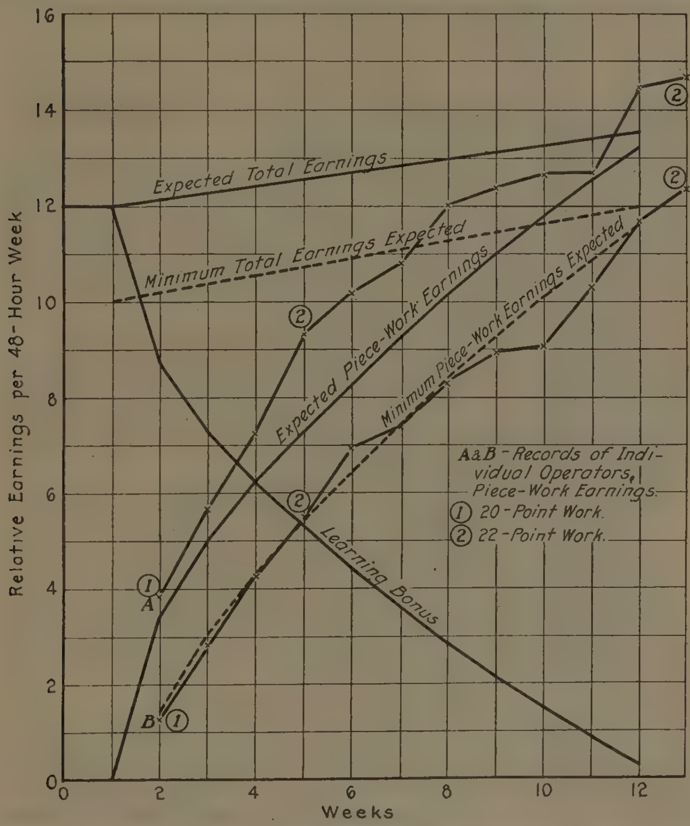


FIG. 6 DIAGRAM ILLUSTRATING THE GAIN IN EARNING CAPACITY OF LOOPER LEARNERS

tor or the machine. The piece rates therefore make allowances so that the operator receives only enough less for imperfects as to induce her to seek prompt adjustment of any machine producing imperfect work.

67 In "Seaming and Hemming," imperfects or waste are due almost entirely to careless work of the operator; therefore an operator is classed in one of three grades, A, B, or C, according to the percentage of imperfects found in her work. Piece rates for

grades B and C are less than the A rates by greater percentages than the imperfect percentages limiting the classification of the operator.

68 In "Looping," each stocking can and should be inspected immediately after it is looped. A small percentage of imperfects is allowed; if over this, the operator receives lower pay in proportion to her imperfect percentage. The effect of changes made, increasing unit production, is shown in Fig. 5.

69 In "Boarding," poor work is returned to the boarder and reboarded without pay. Only a careless boarder has any but a negligible amount returned.

70 *Maintenance of Piece Rates.* Several new styles are added and many changes in styles are made each year. The Methods Department is apprised of these changes by seeing the knitting specifications, which authorize the departments to make the changes, before they are issued to the Knitting Department. The time studies subdivide the operations into small elements to facilitate the rating of new styles.

71 For example, in analyzing transfer knitting in which the ribbed top of the stocking is placed on a quill ring — a circle of points — transferred to the knitting machine, and then the rest of the stocking knit, the operation was divided into six elements:

(*a*) Top (place rib on quill ring)
(*b*) Ravel (excess rib fabric outside of ring)
(*c*) Carry quill ring with rib on it from table to machine
(*d*) Transfer rib to machine and start machine
(*e*) Inspect stocking previously knit by machine
(*f*) Return with quill ring and stocking to table.

72 The standard times for each of these elements for each style were determined. When a change is made, it may change, for example, only the times of elements 1 and 2. These can then be timed, all other element times serving as check times.

73 *Time Ratings.* To provide for contingencies likely to arise, each piece worker has a "Time Rate" to be applied as a minimum hourly guaranteed rate when learning, when failing to earn more on piece work due to management's fault (accidental), or when transferred to day work temporarily. These time ratings are maintained in general about 20 per cent less than the operator's piece-work earnings. This time rating also applies to time lost due to causes beyond the operator's control and for which the company holds itself liable.

74 *Operators' Earnings Charts.* An individual earnings chart, shown in Fig. 7, is kept for each piece worker. This entails a small amount of clerical work, but since, in addition to the weekly entries, only one point in a curve is required, the clerical labor is very small and incommensurate with the value which the charts have for reference purposes.

75 The charts not only graphically picture the progress of the individual operators in comparison with the line showing the base rate for the operator, but serve the Methods Department as a check on the proper functioning of the various piece rates, the amount of day work in the various departments, and as a basis for determining the operators' time ratings.

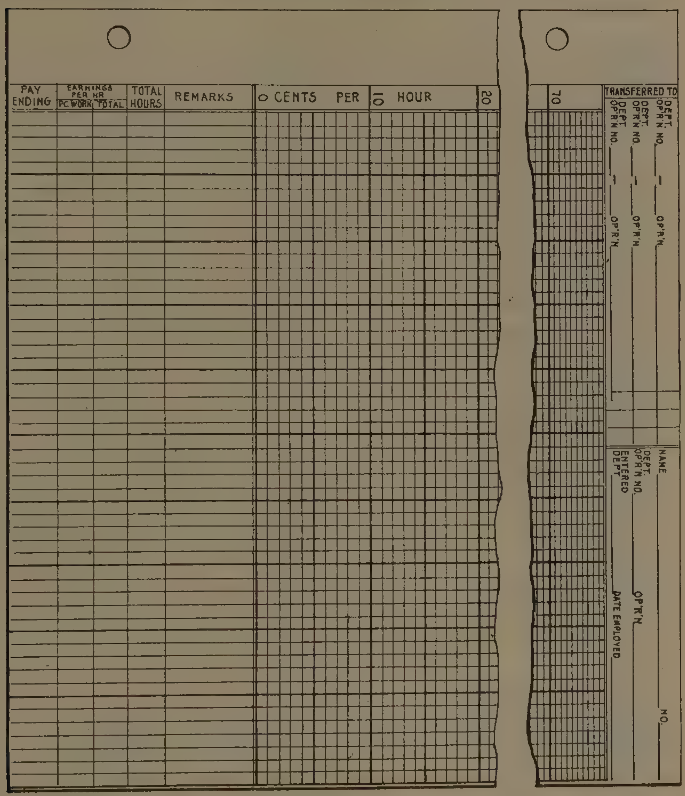


FIG. 7 EARNINGS CHART

76 *Results.* The standardization of methods and rates has resulted in more uniform and higher average earnings to the operators and exceptionally satisfactory working conditions, as well as savings amounting to thousands of dollars per year to the company.

COST ACCOUNTING

77 The cost installation developed by the engineers followed the general line suggested in the Uniform Hosiery Cost Report as prepared by the National Association of Hosiery and Underwear

Manufacturers. A uniform cost system, however, even as applied to a particular industry, must be adapted to the individual plant. The uniform system was followed to an extent sufficient to com-

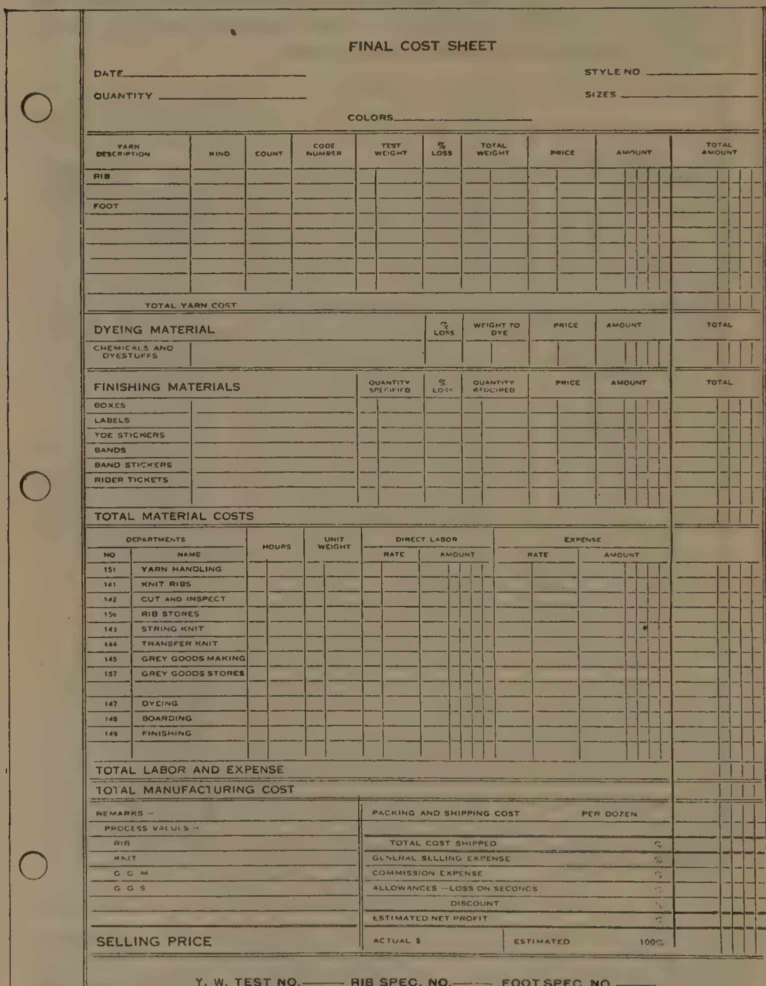


Fig. 8 Final Cost Sheet

pare fundamental data with those of other plants having the uniform system. The entire cost accounting is done by one man with part-time assistance from another.

78 In the scheme followed previous to the new installation, the total overhead for the year was secured from the general books

and this was broken down into five groups representing the principal operating departments. The percentage of each to the total sales was found and applied to the selling price of the particular style being figured. As a result of this the higher-priced goods were receiving a larger share of the overhead than the lower-priced goods, and consequently the lower-priced goods were showing a profit that was erroneous in amount. Such discrepancies as these are found in most plants at the present time.

79 One of the most important changes made was the introducing of "normal" rates for material, labor, and overhead. By this means the expenses are averaged in such a way that during a normal season no changes have to be made in the selling prices on account of fluctuating overhead. The normal costs have proved valuable in times when production is low and overhead rates have increased. The figures for the items of material, direct labor, and expense are set up on a basis of past costs, and these are maintained as standards for a period of six months or a season's condition, and not changed unless there is some relatively permanent change in the conditions of manufacture. The variations between the actual and the normal are considered as abnormal gains and losses as applied to manufacturing profit and loss. The difference between them is determined period by period and adjusted. In this way the common error of showing huge overhead costs when business is poor, resulting in a desire to increase prices, is avoided. Variations in overhead and other varying expenses are thus balanced — as they should be — through the entire year, giving the proper indication of true average costs.

80 The departments of a mill may be divided into two classes:

(a) *Productive* — Those departments which actually perform manufacturing operations upon the product:

Goods in Process A
Goods in Process B.

The individual accounts in each of these correspond to the departments scheduled in the section on Production Control.

(b) *Contributory* — Those departments which aid the Productive:

111 — General Factory
112 — Manufacturing Office
113 — Power
114 — Boiler
116 — Maintenance and Repairs
117 — Trucking
151 — Yarn Handling
155 — Waste and Winding
157 — Gray Stock
158 — Box Making
159 — Printing.

81 Aside from these two manufacturing groups are the departments of distribution of which Shipping and General Selling are the principal ones.

82 The cost periods are of four weeks each, giving thirteen closings in a year.

83 Although the details of the cost-accounting plan are of little interest in themselves a brief outline of the more important steps is valuable as illustrating modern principles and a practical use of "normals." This outline is presented as Appendix No. 2.

84 The general scheme as there outlined is to lead up to a final thirteen-period cost sheet for each line manufactured, the figures being designed for comparison with previous months, study of overhead expenses, and relationship of selling price to cost. The information on materials used, both in the manufacture and boxing, is of interest. The Final Cost Sheet is shown in Fig. 8.

85 When the system was being adopted the department supervisors were called in separately and the figures gone over with them minutely. So much interest was shown that it was decided to give them a copy of the analysis sheet each period — after it had been O.K.'d by the superintendent — to examine and use for comparison with previous months.

86 An illustration of the value of this is shown in the case of a certain surpervisor who, after examining the report, was surprised that the indirect labor in his department was so high. He immediately set about to combine duties of various employees and eliminate day work so that there was a proportionate decrease in this figure on the next month's report.

87 The Cost Control is so tied in with the Production Planning that they function almost as a single unit, the one supplementing the other. The benefits derived particularly from this uniform cost system may be summarized as follows:

(*a*) Unbalanced prices, showing losses, were found on an appreciable percentage of the styles. This was rectified either by increased prices or discouragement of sales. Since relative prices are based largely on competition, the remarkable fact is evident that there is lack of knowledge on the part of the entire hosiery industry of the true costs of their product.

(*b*) Styles could be picked out upon which it was safe to cut prices in times of depression or where necessary to meet a competitor's price.

(*c*) Manufacture of certain lines showing a loss was discontinued and the equipment was utilized preferably for the manufacture of profitable lines.

(*d*) Lines showing fairly large profits could be handled in such a way as to stimulate sales.

(*e*) The cost record showed that it is cheaper to sell certain classes of defective hosiery as "waste" instead of putting them through as "seconds."

(*f*) Decisions upon purchase of new types of machinery are based upon facts showing whether savings can be made through their installation.

(*g*) Through costing of sales the Sales Department can determine the relative worth of the salesmen to the company.

(*h*) A monthly profit-and-loss statement is provided instead of a semi-annual statement.

(*i*) Proper distribution of direct and indirect labor to departmental expenses has resulted in reduction of departmental labor costs.

Marketing

88 Features of sales development contributing toward the growth in business shown in Fig. 1 have been the selling direct to retailer, national advertising, sales promotion and assignment of quotas to each salesman, which furnished also a logical program for the Production Department.

89 In making up the quotas each territory is analyzed separately, considering the general conditions as compared with the previous year as to crops, local industrial conditions, general business status, number of new accounts sold, and mail orders received. Also the characteristics of the individual salesman are reviewed, keeping in mind his temperament and trend of sales ability. Each man is given a chart of his last year's business and total territorial sales for each week. This chart also indicates the percentage of gain he is expected to make. A duplicate of each chart is kept in the central office and each week's new business is registered in a column opposite the record for the same week in the preceding year, thus keeping definite tabs upon the salesman. It has been found that whether or not the salesman makes his quota, he invariably is moved to better effort than without the mark to aim at.

APPENDIX NO. 1

FACTORY PROCEDURE IN PLANNING

90 The actual operation of production planning may be illustrated best by following through the various steps of Production Control to the point where the goods are placed in the finished stock.

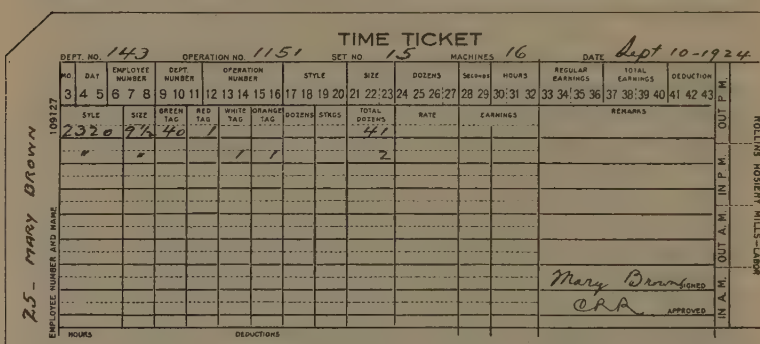
91 A sales order is received bearing an item, say, for style 2320, size 9½, in beige, which is designated as color number 18, for November 1 delivery. After this order receives the usual checking as to prices, extensions, credit, approvals, etc., it passes to the Tabulating Department, where a Hollerith card is punched for the item shown.

92 After the card is punched the punchings are verified, and by means of the Hollerith automatic sorting machine it is sorted along

with cards for other items to secure whatever information is desired. In this case the style, size, color, dozens, and date of delivery are wanted. By putting the sorted cards through the tabulating machine, this information is secured and recorded. These records then constitute the sales figures, including the item of style 2320. From these summaries the sales tape is run out on the Production Control Board, a photograph of which is shown in Fig. 4.

93. On this board are shown various quantity items of sales and production to assist in scheduling production and give a graphical view of conditions of the more important styles. Opposite each style number are two tapes which can be extended or distended at will by the action of springs at the left. Pins of different colors provide other information for reading quantities figured on the basis of the unit number of dozens at the left, designed to utilize a single quantity scale.

94 The upper (black) tape for each style represents the total sales for the season up to the last day of the monthly period, in this case August 30. A green pin shows for comparison the total sales for the



TIME TICKET

DEPT. NO. 143 OPERATION NO. 1151 SET NO. 15 MACHINES 16 DATE Sept 10-1924

MO.	DAY	EMPLOYEE NUMBER	DEPT. NUMBER	OPERATION NUMBER	STYLE	SIZE	DOZENS	SECONDS	HOURS	REGULAR EARNINGS	TOTAL EARNINGS	DEDUCTION
3	4 5	6 7 8	9 10 11	12 13 14 15 16	17 18 19 20	21 22 23	24 25 26 27	28 29	30 31 32	33 34 35 36	37 38 39 40	41 42 43

STYLE	SIZE	GREEN TAG	RED TAG	WHITE TAG	ORANGE TAG	DOZENS	STKGS	TOTAL DOZENS	RATE	EARNINGS	REMARKS
2320	9½	40	1					41			
"	"			1	1			2			

Mary Brown SIGNED
CRA APPROVED

HOURS DEDUCTIONS

109127 EMPLOYEE NUMBER AND NAME 25 - MARY BROWN

OUT P. M. IN P. M. OUT A. M. IN A. M.

ROLLINS HOSIERY MILLS—LABOR

Fig. 9 Time Ticket

previous six months season ending April 1, while a red-and-white pin shows total sales for the season ending September 30 of the preceding year.

95 Knit-production figures — that is, the quantity of hosiery knitted and ready to dye — are represented by the lower (red) tape. These figures are secured by summarizing the production shown on individual operative's daily time tickets. This ticket, which gives a typical day's production from one set of machines handled by the operator, is illustrated in Fig. 9.

96 This red tape, during the month of September, shows the total knit for the season including the period ending August 30. A white pin near the end of the tape shows the number of dozens of this style during the last week of the period. A black pin opposite the red tape represents total orders authorized on the basis of sales shown.

97 A record is maintained in the Production office showing the division among the individual operatives of the total orders authorized, and from this record is deducted daily the knit production of each. As the division of orders is made, or any changes are desired affecting the individuals, a knitting order is issued.

98 An important feature of the control is the individual operator's card, which also serves as a balance of work. When planning the distribution of the work to the knitters and machine numbers, the Production Clerk proceeds with the idea of completing the order on the

different sizes at approximately the same time. This is influenced by such factors as style, size, and operator's efficiency, and the number of machines handled by operators. Information concerning machine capacities and operators' performance is carefully defined and maintained in the planning room. In apportioning the work, for example, silk styles are given to the better operators and the machine times are consulted and compared with the operators' production record so as to give each operator a size which will pass through her machine in a period of time which corresponds to her working speed.

99 Special planning is required in knitting men's, misses', and infants' hosiery because it must pass through three successive operations of rib knitting, rib cut and inspect, and transfer knitting. The ribs are manufactured for Worked Material Stores. In the planning of these operations, which is done in the Production Office — which is governed by the production of the transfer knitting operations — the work is scheduled so as to insure an even flow of work to the ribbing machines. The number of ribs necessary to have on hand varies with the amount of knit orders on hand, so that careful analysis of stock orders of ribs compared with the knit production of each size and style is necessary to prevent excessive inventory of ribs.

100 A graphical layout of ribbing machines, whether running or idle, assists the Production Clerk in making machine changes. Complete records are maintained of machine capacities and the different styles that can be produced on any given machine.

101 In a similar way the progress of production is regulated on successive operations such as seaming, looping, and turning.

102 After passing through these various processes in the Knitting Department, the goods arrive at what is known as the Gray Stock, a stock of goods completely knit but not dyed nor finished. The permanent inventory set up for this stock, called the Gray Stores Stock Record, with a card for each style, having columns for each size, has been a decided advantage in planning the work for finishing operations to follow and in keeping production in proper proportion as to style and size.

103 From this stock the goods are transferred to the dye, the amount of each style, size, and color being determined from a study of the record known as Position of Stock and Orders. This form gives for each style and with separate columns for each color the number of pairs of a given size: in Warehouse; in Process No. 2 (that is, quantity dyed); Sales (several lines); Surplus or Shortage; To the Dye this Period; and Total Sales to Date. Opposite "Warehouse" is given the permanent inventory of the Finished Stock. The sales secured from the Tabulating Department are shown under "Total Sales to Date," with that portion known as "Future Sales" separated according to dates of delivery.

104 The Position of Stock and Orders is really the key form in controlling the stock of finished goods, as the information secured from this record together with the record of gray stock enables the Production office to keep Finished stocks in proper proportion as to style, sizes, and colors.

105 One of the most important features of Production Control is the originating of Gray Goods orders. This covers goods to be sent from Gray Stores to the Dyeing Department, and shows what is needed as reflected by the "Position of Stock and Orders." Certain limitations must be considered, however, such as the quantity of goods in gray stock, approved dyeing formulas, the groupings of styles permitted under each formula, and the capacity of both the Dyeing and Boarding Department.

106 For each item appearing on the gray goods order a coupon batch ticket is made out. It accompanies the batch of goods through

all the finishing processes. Stubs are removed as the various operations are performed, the top or major portion of the ticket being used to show the various classes into which the finished work fell and to account for the exact number of dozens issued from the Gray Stores Department. The batch ticket not only is the means for following the batch through the operations of process B, but it also serves to analyze the breakdown of the batch, i. e., the number of perfect stockings obtained, the number of rejects, and the reasons for rejecting. Furthermore, the quantities finished by each operator are noted from the stub of this ticket and accumulated on Hollerith cards in order to compute earnings. The progress of each batch is thus followed by the Production Office. This ticket as now made out serves its purpose very well and has been the means of securing a closer check on the finished products while passing through the finishing processes.

107 Before batch tickets leave the Production Office they are summarized on a route sheet, which is used for checking the progress of the work through the Dyeing and Finishing Departments. As the stubs from batch tickets covering the operations of dyeing, boarding, pairing, and boxing are received, they are checked off on this record. While this sheet does not cover all operations in these two departments, the four mentioned are sufficient to serve the purpose.

108 In case any batch does not pass through the above operations according to the schedule of time allowed for the various processes, a Record of Delayed Batches is forwarded to the supervisor calling for an explanation of the delay. This record works out very nicely and benefits both the Production Office and the Processing Departments.

109 On Monday each week a report is made by the Shipping Department of the styles, sizes, and colors which were back-ordered on that day due to lack of finished stock. This report is forwarded to the Production Office where it is used as a guide to the goods needed. Each day during the week this sheet is returned to the Shipping Department for revision. In addition to this report, the back orders themselves are routed through the Production Office daily and once each week a summary, or Back Order report, is made showing the number and amount of the back orders for one day.

110 The entire procedure of production planning and control is handled in the Production Office by two clerks, no increase over the number previously employed for similar duties. The supervisors in the plant are relieved so that they may attend to their regular executive duties and delays are practically eliminated.

APPENDIX NO. 2

COST-ACCOUNTING DETAILS

111 A brief outline of the important details of the cost-accounting plan is presented in this appendix to illustrate the practical use of "normals" and modern methods of distributing expenses.

112 Goods in process are in two groups. "Goods in Process A" includes all the knitting operations and all other operations necessary to complete the stockings in the gray, that is, before dyeing. Completed stockings in the gray are credited at a normal value to the "Goods in Process A" account and charged to the "Gray Goods Stores Inventory" account.

113 "Goods in Process B" covers all the finishing operations prior to entry into the Finished Goods Warehouse. As the product is sent to the dye, "Goods in Process B" is charged with the total "Goods in Process A" cost plus a charge for the Gray Goods Stores' department expense, and the "Gray Goods Stores Inventory" account is credited.

114 When the product is sent to the Finished Goods Warehouse, "Goods in Process B" is credited and "Finished Goods" is charged.

115 By this subdivision the value of goods in process can be ascertained in the knitting departments, the Gray Goods Stores, and the goods in process of finishing.

116 The three elements of Costs are Raw Materials, Direct Labor, and Expense.

117 *Raw Materials.* This includes yarns, dyestuffs and chemicals, boxes, labels, bands, etc.

118 All materials and supplies are bought by the Purchasing Department, received in the storeroom, and requisitioned by the several departments as needed, using a Hollerith stores-issue card. A minimum and maximum figure is observed to insure a constant supply of all materials.

119 These Hollerith cards, showing the department to which issued, the kind of yarn and count, the code number, the case number, the net weight and the price, are sent to the Cost Department at the close of each week to be checked and figured. On the closing day of each cost period and after the knitters have quit work, the storekeeper takes a physical inventory of all yarns on the knitting floors, by departments. This does not include such yarns as are on the knitting machines, but it does include all backwound and rewound, keeping all three separate. Hollerith stores credit cards are made out for each kind of yarn on this inventory and deducted from the amount originally issued. The storekeeper is advised to allow the stock of yarns in the department bins to run down to a minimum which will not hamper production; this facilitates the taking of these inventories. Hollerith stores issues are then made out for this inventory and added to the amount issued for the following period.

120 All yarns are charged to the knitting departments (or Goods in Process A) at a normal value; therefore, if the actual amount used is greater than the normal amount, the actual amount is a credit to the inventory account and the difference is a debit to Manufacturing Gain or Loss, as a variation in yarn prices. If the actual is less than the normal, then the difference is a credit to Manufacturing Gain or Loss.

121 Physical bi-weekly inventories of dyeing materials are taken in order to secure the actual amount of dyestuffs used. Issues are so small and frequent that running inventories are impracticable. The normal value of dyeing materials used is secured by multiplying the number of dozens drawn from Gray Stores by the "weight to dye" (that is, the average test weight of a dozen stockings) of each style and size and multiplying this product by the normal price per pound of dyeing materials. The normal material consumption for the period is thus secured. The differences between actual and normal are handled in the same manner as mentioned above.

122 Through Amount of Inventory and Stores Requisitions, the finishing department materials used are secured and the difference between actuals and normals are handled in the same manner.

123 *Direct Labor.* Most of the machine operations are on a piece-rate basis and likewise the hand operations that are constant. In determining labor cost for day-work operations the normal production of the particular operation is divided into the earnings of the number of employees required to turn out this normal production. In most operations the unit of production is a dozen. In the dyeing, however, labor rate is applied on a per-pound basis. The normal rate in dyeing is arrived at by dividing the total direct labor earnings by the total number of pounds dyed during the period.

124 Hollerith daily time cards are used for both the recording of the individual employee's "In and Out" time and their production

(by styles and sizes) for the day. The name of the operator, department number, the operation in which the employee is normally engaged, and the method of payment are all filled in with the use of an addressograph machine, all properly coded. Through this coding and with the aid of the Hollerith machine the payments made for the various operations are grouped and thus charges are made either to "Goods in Process A" or "Goods in Process B."

125 *Expense.* The various items that compose the item of overhead and the manner in which it is to be distributed to the various departments is shown on a Chart of Expense Analysis procedure.

126 The Master Expense Analysis Sheet, used by the executives, is filled in month by month up to six months. From this are copied, on to separate expense analysis sheets each month, for the supervisor of each department, the expenses for the monthly period in his department.

127 The Total Direct Expense covers all items that are directly under the control of the supervisors; the other charges are beyond their control.

128 *Final Cost Sheet.* Final costs are the complete costs that have been developed for all lines manufactured. They are made up from the usual elements, viz., material, labor, and expense for consideration of manufacturing cost, and Manufacturing Cost, Shipping Cost, Selling Expense, Commission Expense, Allowance for Loss on Seconds and Discount, and Estimated Net Profit (Administrative Items) as related to the Selling Price. A copy of this Final Cost Sheet is shown in Fig. 12 in the section on Cost Accounting.

129 Under the Materials section the test weight of the yarns in the particular style is given, plus an allowance for losses due to style, plus an allowance for losses due to breakage, shrinkage, defects, etc. This cost is extended using the normal price of the yarns for the period.

130 "Dyeing Material" is the weight to dye as taken from the Yarn Specification cards, multiplied by the normal price per pound to dye.

131 "Finishing Materials" are the actual quantity of boxes, labels, toe stockers, bands, band stickers, rider tickets, etc., necessary, plus a percentage of loss, to give the normal or required amount and each is priced at a normal figure.

132 This completes the total Material Costs. Next come the Direct Labor and Expense sections.

133 "Labor Rates" for each operation are taken from the Labor Specification Sheet, the Methods Department notifying the Cost Department of any changes in rates. The expense rates are taken from the departmental expense-analysis sheets and distributed to the product.

134 The total Labor and Expense is added to the Total Material Costs to secure the Total Manufacturing Cost. Packing and Shipping Cost is added (on a per-dozen basis) to secure the Total Cost Shipped. This figure represents the difference between 100 per cent and the total percentages to be added for General Selling Expense, Commission Expense, Allowances for Loss on Seconds and Discount, and the Estimated Net Profit (Administrative Items). The estimated Selling Price represents 100 per cent.

135 The process values for rib, knit, gray-goods making, and gray goods stores are used to make the transfers from Goods in Process A to Gray Goods Stores, Gray Goods Stores to Goods in Process B, and from Goods in Process B to Finished Goods Stores.

136 Benefits due to the installation of this uniform system are outlined under the section on Cost Accounting.

No. 1936

A GRAPHICAL STUDY OF JOURNAL LUBRICATION (PART II)[1]

By H. A. S. Howarth,[2] Philadelphia, Pa.

Member of the Society

The present paper continues the investigations of journal lubrication originally reported to the Society a year ago in a paper of the same title covering the case of a journal completely surrounded by its bearing. The case of a partial bearing is considered in the following paper. The method of analysis leads to the formation of charts for the study of partial-bearing design problems, and some typical examples are solved with the use of these charts.

SINCE the presentation of the first section of this paper dealing primarily with full bearings, that work has been carefully reviewed and the study of partial bearings continued. The fundamental equations when more closely examined, with special reference to partial bearings, have revealed a vast series of such bearings with lower capacities than the series already described. The influence of end leakage has again been neglected below, just as it was in the first section. This phase of the subject will be studied later in connection with the examination of experimental data.

2 In order that the lubrication of partial bearings may be more thoroughly examined it is necessary to go back a step and make use of some additional equations, [13] and [14], used by W. J. Harrison, which are given below with comments:

$$h^3 \frac{\partial P}{\partial x} = 6\mu U(h-h_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [13]$$

μ = the absolute viscosity of the lubricant in the film
U = the surface speed of the journal = $2\pi aN$.

[1] Part I was presented in December, 1923, and published in Trans. A.S.M.E., vol. 45 (1923), p. 421. Figs. 1 to 19, inclusive and Equations [1] to [12] inclusive, appeared in Part I.
[2] Genl. Mgr., Ch. Engr., Kingsbury Machine Works.

Contributed by the Machine Shop Practice Division and the Special Research Committee on Lubrication and presented at the Annual Meeting, New York, December 1 to 4, 1924, of The American Society of Mechanical Engineers.

Formula [13] is identical with one of those first developed by Osborne Reynolds. In order to apply it to a journal bearing Harrison assumes that distances x are measured along the surface of the journal and that film thicknesses h are measured normal thereto. The symbols used in the equations are the same as used by both Reynolds and Harrison. These with other references are illustrated in Fig. 20.

3 The film thickness where the pressure is maximum or minimum is designated by h_1. Substituting in [13] the following formulas for x and for film thickness in terms of θ, c, and μ,

$$dx = a d\theta \qquad h = \eta(1+c\cos\theta) \qquad h_1 = \eta(1+c\cos\theta_1)$$

where η = radial clearance when the journal and bearing are running concentric

c = percentage of eccentricity between axis of the journal and bearing for any running condition

P = pressure within the oil film at any point,

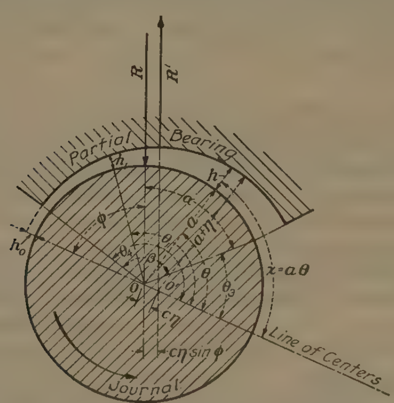


Fig. 20 Section of Journal and Partial Bearing Illustrating Notation Used in Formulas

Harrison's equation [14], is obtained, with which the graphical work of this second section of the paper begins.

$$\frac{dP}{d\theta} = \frac{6\mu Ua}{\eta^2}\left[\frac{c(\cos\theta - \cos\theta_1)}{(1+c\cos\theta)^3}\right] \quad . \; . \; . \; . \; . \; [14]$$

Equation [14] applies to all journal bearings, whether partial or full, in which the radius of curvature of the bearing is greater than that of the journal. The case of partial bearings in which the radii of curvature of the journal and bearing are equal will be discussed in a subsequent paper.

4 The bracketed portion of the right member of [14] is proportional to $\frac{dP}{d\theta}$ and can be plotted against θ for assumed values of

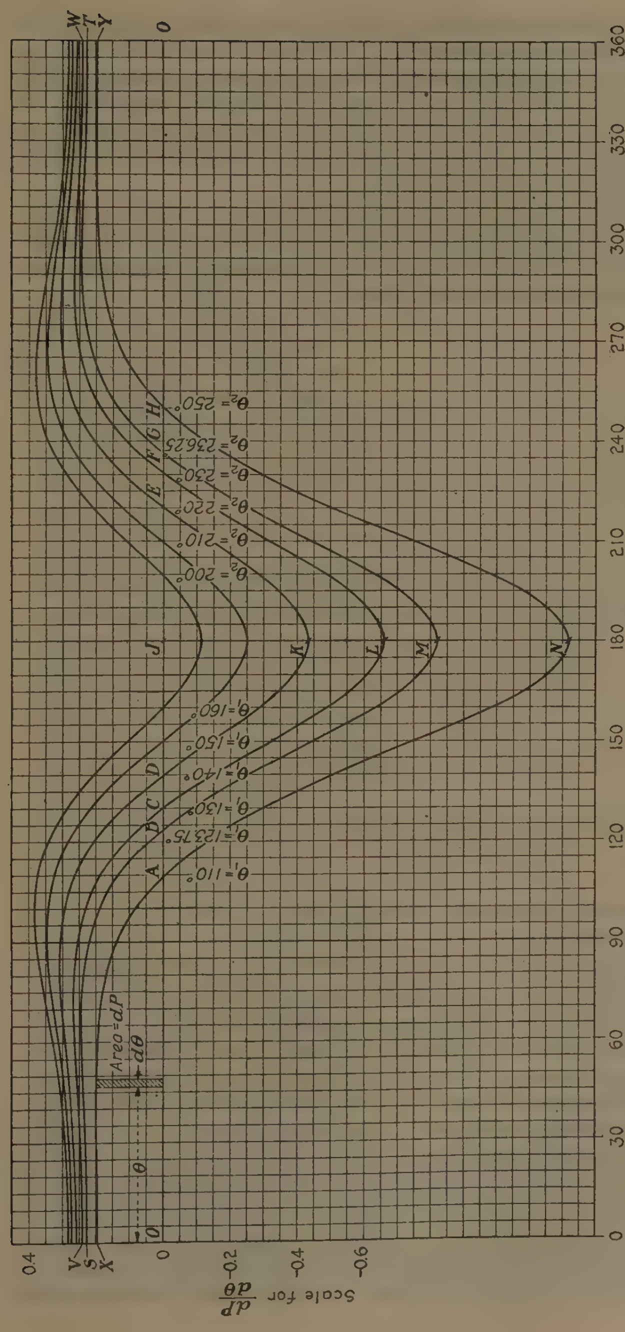


FIG. 21 GRAPH OF EQUATION $\frac{dP}{d\theta} = \frac{6\mu Ua}{\eta^2}\left[\frac{c\,(\cos\theta - \cos\theta_1)}{(1+c\cos\theta)^3}\right]$

ASSUMING $\frac{6\mu Ua}{\eta^2} = 1$ AND $c = 0.4$

c and θ_1. A series of these curves is illustrated in Fig. 21 for $c = 0.4$ and several values of θ_1.

5 The area under any one of these curves for a length $d\theta$ is therefore proportional to dP. Hence a mechanical integration of dP can be performed by a planimeter between one value of θ and another. The area so obtained will be proportional to the difference in pressure between those points.

6 Harrison integrated Equation [14] and, in order to get a definite solution in which for each value of θ there would be but one value of P, he found the cosine of the angles of maximum and minimum pressure to be as follows:

$$\cos\theta_1 = \cos\theta_2 = -\frac{3c}{2+c^2} \quad \ldots\ldots \quad [2]$$

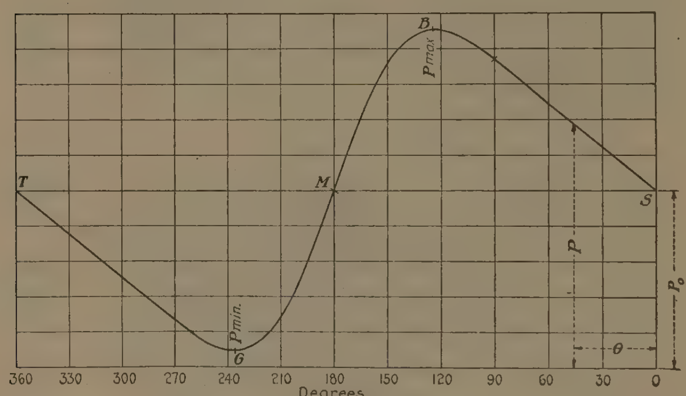


Fig. 22 Variation in Pressure within the Oil Film of a Full Bearing when the Pressure is Laid Off Perpendicularly to the Developed Circumference of the Journal (c=0.4)

This is the same relation as expressed in Equation [2] in the first section of this paper.

7 Assuming $c = 0.4$ it is found from Equation [2] that $\theta_1 =$ 123.75 deg. The curve through B in Fig. 21 is drawn for $\theta_1 =$ 123.75 deg. The other similar curves are drawn for $\theta_1 = 110$, 130 and 140 deg. respectively, passing through points A, C, and D.

8 Examinations of curve $SBMGT$, drawn for $\theta_1 = 123.75$ deg., shows that the sum of the areas above the axis OO' equals the area below, and that positive area $OSBO$ equals negative area $BJMB$. From this curve, by mechanical integration as explained above, a series of pressures is obtained for $c = 0.4$. These may be laid off normal to the developed circumference of the journal as in Fig. 22 or normal to the projected position of the angles on a diameter as in Fig. 23. These curves $TGMBS$ of Figs. 22 and 23 could, by assuming $c = 0.4$, also have been plotted directly from pressure

values obtained from Equation [3] of the first section of this paper, reproduced below for convenience. They are the only ones that can be obtained from Formula [3] for $c = 0.4$.

$$P = P_0 + \frac{6\mu U_a}{\eta^2}\left\{\frac{c \sin\theta(2+c\cos\theta)}{(2+c^2)(1+c\cos\theta)^2}\right\} \quad \ldots\ldots \quad [3]$$

9 The conditions surrounding this curve *SBMGT* of Figs. 21, 22 and 23 are such that if the bearing is complete ($\beta = 360$ deg.) the pressure in the film will increase from some value P_0 at S ($\theta = 0$ deg.) to $P_{max.}$ at B, ($\theta_1 = 123.75$ deg.). It will then decrease to P_0 at M ($\theta = 180$ deg.), and will continue to decrease to $P_{min.}$ at G ($\theta_2 = 236.25$ deg.). From this point it will increase to P_0 at T ($\theta = 360$ deg.), thereby completing the circuit with the same pressure with which it began.

10 For all other curves than *SBMGT* in Fig. 21 the areas above and below OO' are not properly balanced for a complete (360 deg.) bearing. The pressure after completing the circuit of 360 deg. would be higher or lower than at the start. Those curves are useful however for the study of possible proportions of partial bearings. Points A, B, C and D, i.e., intersections lying between O and J, mark the angles θ_1 where the pressures are maximum. Points E, F, G and H, between J and O', similarly mark the angles θ_2 where the pressures are minimum.

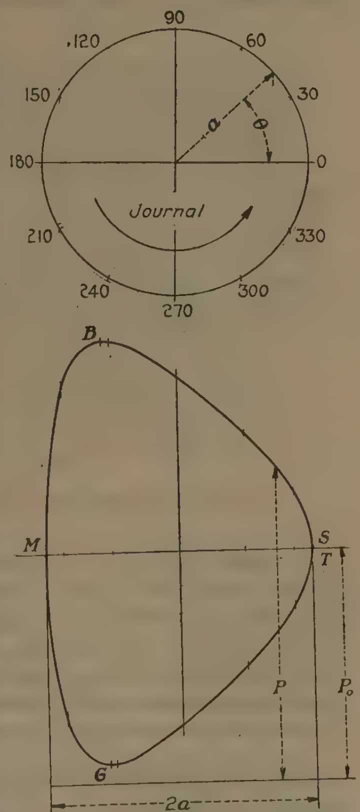


FIG. 23 PRESSURE DIAGRAM FOR JOURNAL WITH 360-DEG. BEARING (c=0.4)

11 Curve *VCLFW* of Fig. 21 is illustrated alone in Fig. 24 for convenience of study. Its use may be demonstrated by solving a problem. For example: By using this curve can the pressures be determined for a partial bearing, terminating at $\theta = 190$ deg. when $c = 0.4$? The solution is obtained as follows: Lay off vertical line AB at $\theta_4 = 190$ deg. Then measure area *CABC*. Then lay off vertical line *DE* in such

a position that area $DCED$ equals area $CABC$. Measuring the angle from O to E we find $\theta_3 = 17.2$ deg. This bearing starts at $\theta_3 = 17.2$ deg. with atmospheric pressure. Its pressure increases with θ until $\theta = 130$ deg. It then falls off till the initial pressure is reached when $\theta = 190$ deg. The bearing length is therefore $\beta = 172.8$ deg. This partial bearing is illustrated in Fig. 25, together with pressure diagrams. Points on the pressure curves in these diagrams are obtained from measurements of areas on Fig. 24. The pressure at C ($\theta = 130$ deg.) equals the initial pressure plus the increase in pressure from E to C, i.e., from $\theta = 17.2$ deg. to $\theta = 130$ deg. The increase is proportional to area $EDCE$. The pressure at J ($\theta = 180$ deg.) equals the initial pressure plus

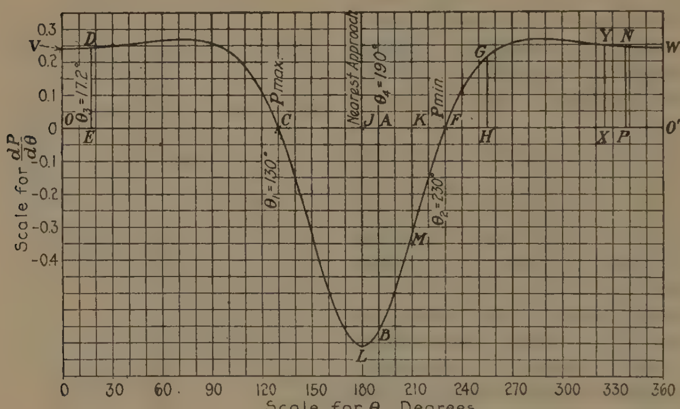


FIG. 24 REPRODUCTION OF CURVE $VCLFW$ OF FIG. 21

the increase from E to J. This increase is proportional to area $EDCE$ minus area $CJLC$. Intermediate pressures are found in the same way. When these pressures are plotted as in Fig. 25, the areas under the resulting curves are proportional to the total components of the film pressure upon the journal in the two directions chosen. By combining these components the magnitude and direction of the resultant pressure R can be obtained. This resultant divides the bearing angle β in the ratio $\frac{\alpha}{\beta} =$ 0.568, in which α is the angle from the leading edge of the bearing to the line of action of the resultant pressure.

12 It may next be assumed that a partial bearing terminates at the right of F as at H ($\theta = 255$ deg.) in Fig. 24. It is assumed that $c = 0.4$ as before. There would then have to be a negative pressure in the bearing equivalent to area FGH because the mini-

mum pressure would occur at F and the pressure would increase from F to H. A line KM must therefore be located at left of F so as to give an area KFM = area FGH. At K the pressure will now be the same as at H. Then a line NP must be so located that the positive area, or area above line OO' will equal area $CKMLC$. This positive area will have to consist of area $NWO'P$ plus area

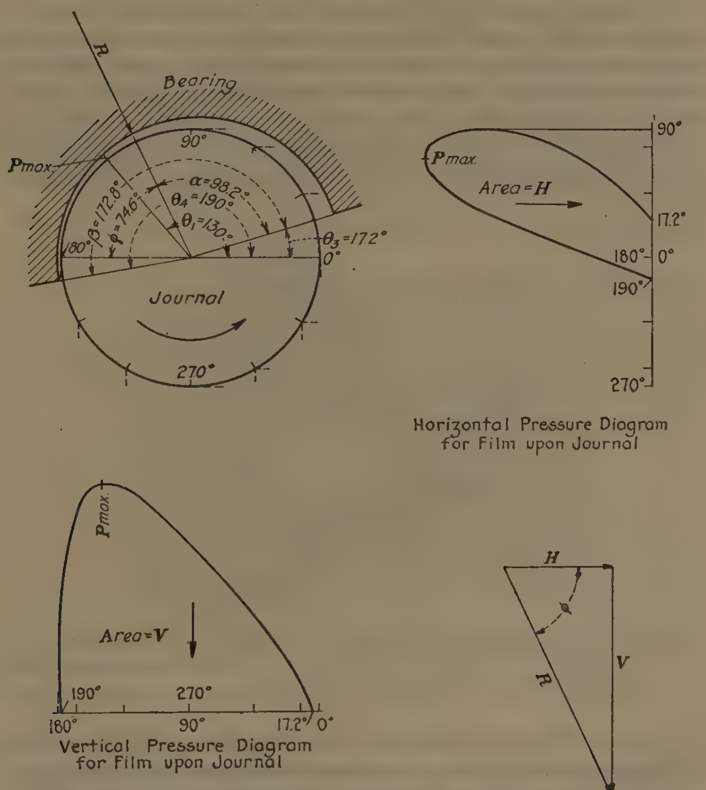


FIG. 25 CAPACITY DIAGRAMS FOR 172.8 DEG. PARTIAL BEARING (c=0.4)

$VCOV$. A partial bearing to satisfy the assumed conditions will therefore start at P (θ = 337.6 deg.) and extend through O' (θ = 360 deg.) which is the same as O (θ = 0 deg.), to H (θ = 255 deg.). The maximum pressure will occur at C (θ_1 = 130 deg.), the minimum at F (θ_2 = 230 deg.), and the initial pressure will recur at P, K, and H. The total arc of this bearing will be β = 277.4 deg. Its resultant, found as explained in previous paragraph, would divide the surface so that α/β = 0.472.

13 It will be noted that of the two cases examined in two previous paragraphs one gave $\alpha/\beta = 0.568$ which is greater than 0.5, while the other gave $\alpha/\beta = 0.472$ which is less than 0.5. Evidently there is an angle β between $\beta = 172.8$ deg. and $\beta = 277.4$ deg. for which the line of action of the resultant pressure R will bisect β when the conditions are such that $c = 0.4$ and $\theta_1 = 130$ deg. (See Par. 19.)

14 Referring to Fig. 21, the area $CFLC$ is less than the sum of the areas $VCOV$ and $FWO'F$. It is therefore evident that so long as the curves of Fig. 21 lie above curve $SBMGT$, i.e. so long as θ_1 is greater than 123.75 deg., those curves will apply only to bearings less than 360 deg. long, i.e. to partial bearings. This is true because the terminal pressure in the film must equal the initial pressure. To make this perfectly clear let it be assumed that a bearing starts

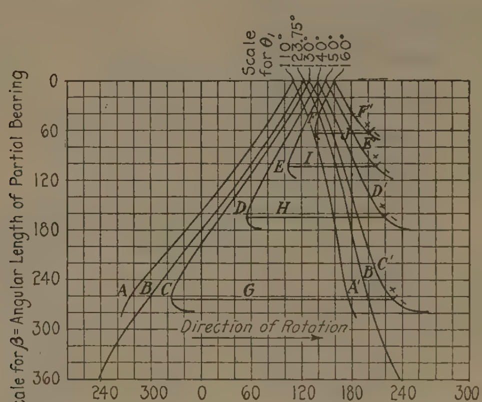


FIG. 26 PARTIAL-BEARING DIAGRAM FOR $c=0.4$. (LEFT BRANCH OF EACH CURVE SHOWS ANGLE TO LEADING EDGE. RIGHT BRANCH OF EACH CURVE SHOWS ANGLE TO TRAILING EDGE.)

at F ($\theta_2 = 230$ deg.) (Fig. 24.) The pressure would build up from that point as the bearing angle extends through O' ($\theta = 360$ deg.), [O ($\theta = 0$ deg.),] till it reaches a maximum at C($\theta = 130$ deg.). The pressure would then fall off as the bearing angle increases *toward* F. But when F is reached the pressure will not have fallen to its initial value because area CLF is too small to bring this about, being less than area FWO' + area VCO. The bearing must therefore start at some other point beyond F, as at X ($\theta = 325$ deg.) if it is to end at F, the conditions being that area $YWO'X$ plus area VCO equal area CLF. This angle is obviously less than 360 deg. For Fig. 24 the angle $XO' + OCF$ does not represent the maximum β, because the unbalanced area $FYXF$ can have a shorter base than FX. This is obvious from the form of the curve between F and W.

15 The greater the value of θ_1, between 123.75 and 180 deg., the less will be the total angle β of the maximum partial bearing. This is well illustrated in Fig. 26 by curves CC', DD', EE' and FF'. Lines G, H, I and J respectively show the maximum β for bearings in which only positive pressures are found. For larger bearings negative pressures will be found near the trailing edge of the bearing, as marked along curves C', D', E' and F'.

16 If we examine bearings derived from the curve SMT and those below it, in Fig. 21, the areas below OO', such as ANH will become progressively larger than the corresponding positive areas XAO plus HYO', as θ_1 is decreased below 123.75 deg. Hence the maximum β for each successive set of derived bearings, as indicated by curve AA', Fig. 26, will fall farther and farther below 360 deg. and in *none* of these bearings will a *negative* pressure be possible.

17 It is evident from the above discussion of Equation [14] that for a given value of c and each distinct value of θ_1 a series of

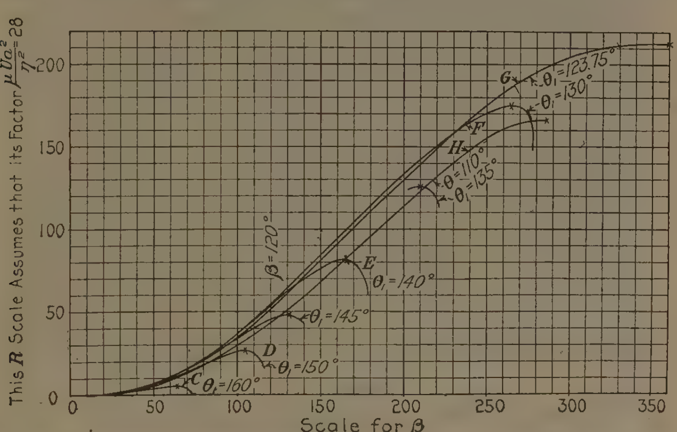


Fig. 27 Capacity Chart for Partial Journals ($c=0.4$)

partial bearings is possible. For the shortest one, $\beta = 0$ deg., and for the largest one, $\beta < 360$ deg., except that for one only value of θ_1 for each value of c can $\beta_{max} = 360$ deg.

18 Curves may be drawn as in Fig. 27 to show the carrying capacities of all bearings for a given value of c. They show that the value of β for the bearing with maximum carrying capacity for each assumed θ_1, is less than β_{max}, except that for $\theta_1 \gtreqless 123.75$ deg., the maximum capacity coincides with the maximum β. A set of charts like Fig. 27 drawn for a wide range of values of c, will show the laws of the carrying capacities of all partial bearings.

19 If a vertical line be drawn through the curves in Fig. 27 at the point where $\beta = 120$ deg., it will cut curves D, E, F, G and H giving the capacities of the bearings determined by a similar line in Fig. 26. These several 120 deg. bearings will have different

capacities R, and the resultant will lie in a different direction with respect to each one. For only one such bearing (when $\theta_1 = 143$ deg. as explained below) will the resultant bisect the bearing angle β. This will be readily seen from Fig. 28 in which α/β is plotted against β for $c = 0.4$, a curve being drawn for each assumed value of θ_1. Points on these curves for small values of β were difficult to obtain graphically. Hence the auxiliary curves A and B were drawn on same chart showing the relation between θ_1 and β for bearings in which $\alpha/\beta = 0.5$. The two curves will terminate at $\beta = 0$, $\theta_1 = 180$ deg. From them the characteristics of the α/β curves are better understood. The line drawn vertically for $\beta = 120$ deg. cuts curve B where $\theta_1 = 143$ deg.

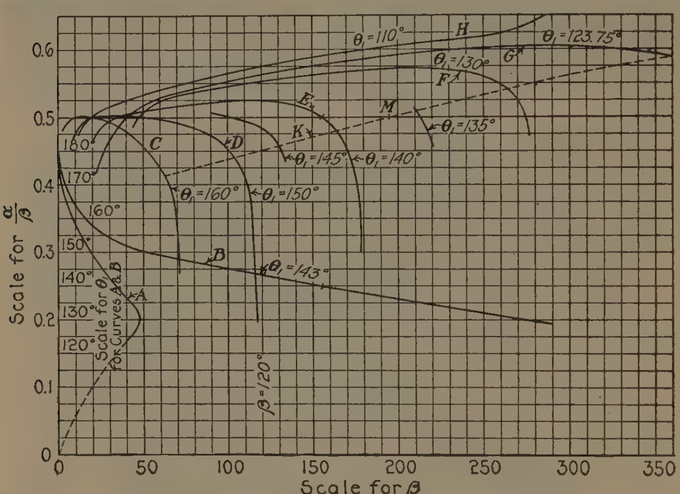


FIG. 28 RESULTANT POSITION CHART (c=0.4)

20 The curves of Fig. 28 are especially interesting. The horizontal line showing values of $\alpha/\beta = 0.5$ is drawn heavier than the others. Curves C, D, E and F cross it twice. This means that for each value of θ_1 greater than 123.75 deg, when $c = 0.4$, there are two bearing angles β in which the resultant R will pass through the center of the bearing surface. The magnitude of the resultant is much greater in one case than in the other. Curves G and H cross the line $\alpha/\beta = 0.5$ at only one point each. Curve G is the same as shown for $c = 0.4$ in Fig. 19[1] in first section of this paper. Curve K in Fig. 28 passes through points on curves C, D, E, F and G where the bearing carrying capacity is maximum for each value of θ_1, these maximum points being found from the curves in Fig. 27. It crosses $\alpha/\beta = 0.5$ at M where $\beta = 195$ deg., the correct value when $c = 0.4$. Attention is called to these features of the curves in Fig.

[1] Reproduced on page 825.

28 because of the subsequent use to be made of them in connection with similar data for other values of c, in the preparation of the final working curves that can be used for the solving of partial-bearing problems.

21 Having thus obtained from Fig. 28 a set of angles β for which $\alpha/\beta = 0.5$ the corresponding capacities are obtained from Fig. 27 and the two plotted on Fig. 29 giving a capacity curve for $c = 0.4$. Similar curves are plotted in Fig. 29 for several other values of c. This Fig. 29 shows the same data for central partial bearings (bisected by the resultant) as Fig. 18, first section, showed for offset partial bearings. The scales in the two figures are not the same however.

22 The loading chart, Fig. 30, for bearings in which $\alpha/\beta = 0.5$, is plotted for useful partial bearings, 20 deg. apart, from $\beta = 180$ to

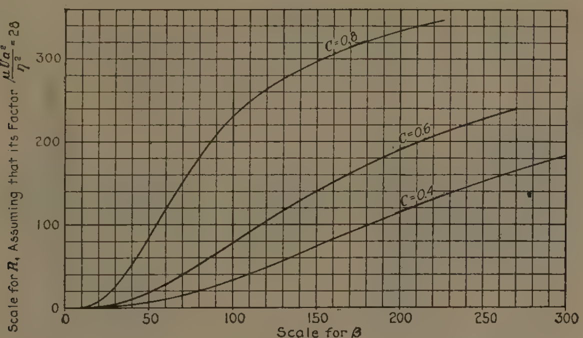


Fig. 29 Capacity Chart for Central Partial Bearings ($\alpha/\beta=0.5$)

$\beta = 60$ deg. For this purpose a definite viscosity $\mu = 3.4 \times 10^{-6}$ (150 S. U. V.) and clearance ratio $\eta/a = 0.001$ were chosen. The resultant pressure R was divided by $2a$ to obtain w, and the velocity U was replaced by its equivalent $\dfrac{2\pi aN}{60}$. By this means the load scale is made to give values of w/N. Additional clearance scales η/a were then added just as in Fig. 10[1] of the first section.

23 Most partial bearings are made with the surface divided into two equal parts by an axial plane perpendicular to the bearing joint. If the resultant pressure against the bearing lies in that plane the loading displacement characteristics of such a bearing, when $\beta \lesseqgtr 180$ deg. would be found from Fig. 30. In many cases however the bearing pressure does not act at the center of a partial bearing surface. It is therefore necessary to know how the carrying capacity of such a bearing compares with those in which $2\alpha = \beta$. Obviously from Figs. 28 and 27 using any desired ratio

[1] Reproduced on page 824.

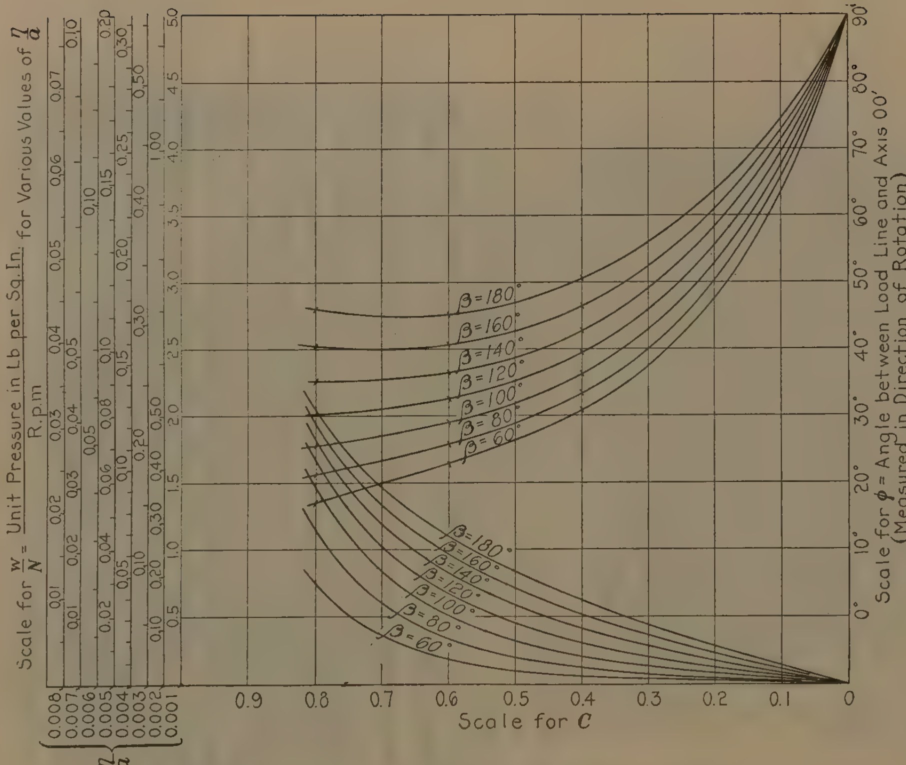


FIG. 30 LOADING CHART FOR CENTRAL PARTIAL BEARINGS
$\mu = 3.4 \times 10^{-6}$ (150 sec. Saybolt with density 0.84)
(Use left-hand vertical scale with lower set of curves)
(Use right-hand vertical scale with upper set of curves)

of α/β it is possible to obtain the corresponding carrying capacities of bearings with different angular lengths β. This would entail much work of doubtful present value. A special case however should be considered.

24 In Fig. 31 five partial bearings are illustrated, all 120 deg. long. Each is drawn in the position it would occupy if the load upon it is vertically downward through the center of the journal. The eccentricity is assumed the same in every case; $c = 0.4$. The diagonal line drawn through the journal axis in each case passes also through the center of curvature of the partial bearing surface. This diagonal makes angle ϕ with the load line R. Where the diagonal cuts the partial bearing surface the distance from journal to bearing $= h_0 = \eta(1-c)$. The resultant pressures on these bearings were

found to be proportional to the lengths of arrows R drawn just above the bearings. These lengths were obtained from Fig. 27 by drawing a vertical through $\beta = 120$ deg., cutting curves E, F, G and H. The value of R for bearing M, for which $\theta_1 = 143$ deg. was obtained by interpolation. The value of $\theta_1 = 143$ deg. for bearing M was obtained from curve B in Fig. 28.

25 The value of ϕ that will give the greatest carrying capacity for $\beta = 120$ deg. and $c = 0.4$ can be obtained from Fig. 32 in which curve B shows R plotted against ϕ. The high point is evidently at E for which $\phi = 52$ deg. approximately. From the characteristics of the bearings illustrated in Fig. 31 it appears that the bearing would have its greatest capacity when the diagonal line through the centers passes through the terminal edge of the partial bearing surface.

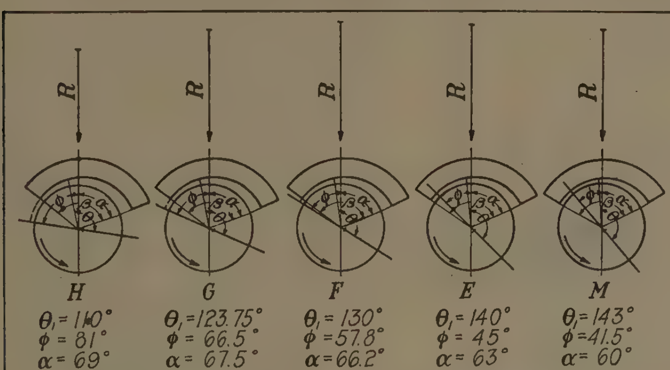


FIG. 31 FIVE 120-DEG. PARTIAL BEARINGS (c=0.4)

26 The carrying capacity of a bearing, determined by the pressures generated within the lubricating film, naturally increases as the mean film thickness decreases, other conditions being favorable. The capacities of the bearings of Fig. 31 may therefore be referred to the accompanying film thicknesses. The thicknesses are determined by measuring the areas between the vertical lines HH, GG, etc. in Fig. 33, laid off to correspond with the bearings shown in Fig. 31. The areas have the same base (120 deg.) and the values obtained (0.606, 0.551, 0.516, 0.472, 0.455) are therefore proportional to the mean film thicknesses. Multiplying these mean thicknesses into the corresponding values of R gives the capacities relative to film thicknesses as plotted in curve A in Fig. 32. The high point on this curve lies at F for which $\phi = 66.25$ deg. This point corresponds exactly with bearing G of Fig. 31, whose capacity was obtained from curve G in Fig. 27. It will be recalled that curve G was drawn for $\theta_1 = 123.75$ deg. obtained from Formula [2] by

assuming $c = 0.4$. This same conclusion was reached by an investigation of bearings in which $c = 0.8$. Hence it may be stated as a general rule that *partial bearings* derived from Equation [2] and the resulting pressure Equation [3] will have the *greatest carrying capacities compared with the mean film thicknesses.*

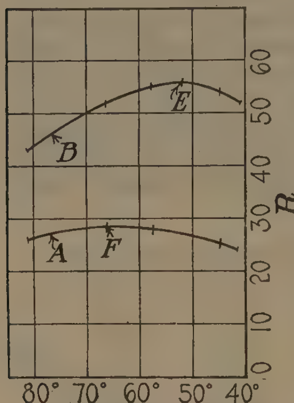


FIG. 32 RELATION OF ϕ TO LOAD

27 It has therefore been found that the 120 deg. bearing with the greatest carrying capacity relative to its mean film thickness has a leading angle α that is more than one-half of β if $c = 0.4$. In a similar manner it can be shown that this statement is true for nearly all values of β and c. In a few cases only is $\alpha \gtreqless \beta/2$ for the bearing with the greatest carrying capacity relative to its mean film thickness. Therefore it is desirable that curves for the characteristics of these best partial bearings be drawn in the same form and as fully as for those particular partial bearings, bisected by the resultant, illustrated in Fig. 30. These so-called best partial bearings relative to mean film thickness may be called *offset partial journal bearings.* They have the thickest possible oil films for the loads they carry. They are the same partial bearings as described in the first section of this paper which was presented to the Society in December, 1923. Load-displacement characteristics of these *offset partial journal bearings* are given in Fig. 34.

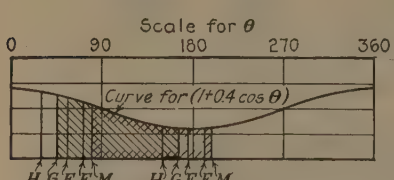


FIG. 33 CURVE FOR DETERMINING MEAN FILM THICKNESSES

28 In order that the exact position of a journal relative to its partial bearing may be known for all running conditions it is necessary that the angle ϕ of Fig. 20 be charted with relation to c preferably in the same chart that shows the relation between w/N and c. This has been done for central partial bearings in Fig. 30 and for offset partial bearings in Fig. 34, by means of the curves terminating in the upper right corner.

29 The lower set of curves in Fig. 30 were plotted from data that located the points for $c = 0.4$, $c = 0.6$, and $c = 0.8$. Additional work is necessary before the points determined by smaller and larger values of c can be located with precision. For Fig. 34

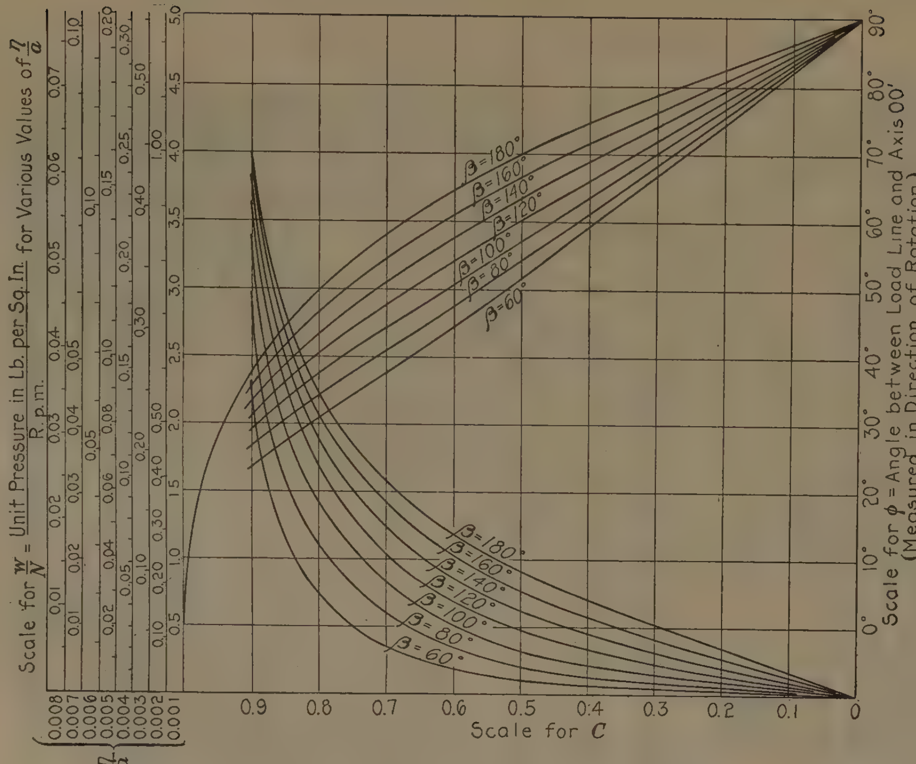


FIG. 34 LOADING CHART FOR OFFSET PARTIAL BEARINGS
$\mu = 3.4 \times 10^{-6}$ (150 sec. Saybolt with density 0.84)
(Use left-hand vertical scale with lower set of curves)
(Use right-hand vertical scale with upper set of curves)

however the curves were drawn through points determined for $c = 0.1$, $c = 0.2$, $c = 0.4$, $c = 0.6$, $c = 0.8$, and $c = 0.9$. Very close approach of the bearing surfaces, i.e., when $c > 0.9$, will be made the subject of a special study, for which time has not yet been available.

30 The journal friction coefficient curves have not yet been determined for partial bearings. The plan for their determination for offset partial bearings is quite simple and those curves will be next prepared. Following that the friction curves for central partial bearings will be investigated. When these friction curves are ready combined charts will be prepared like Fig. 10, of the first section, that will show w/N, ϕ, θ_1, and λ with relation to c, for several viscosities. It will be advisable to have a chart for each partial bearing, i.e., $\beta = 60$, 80,

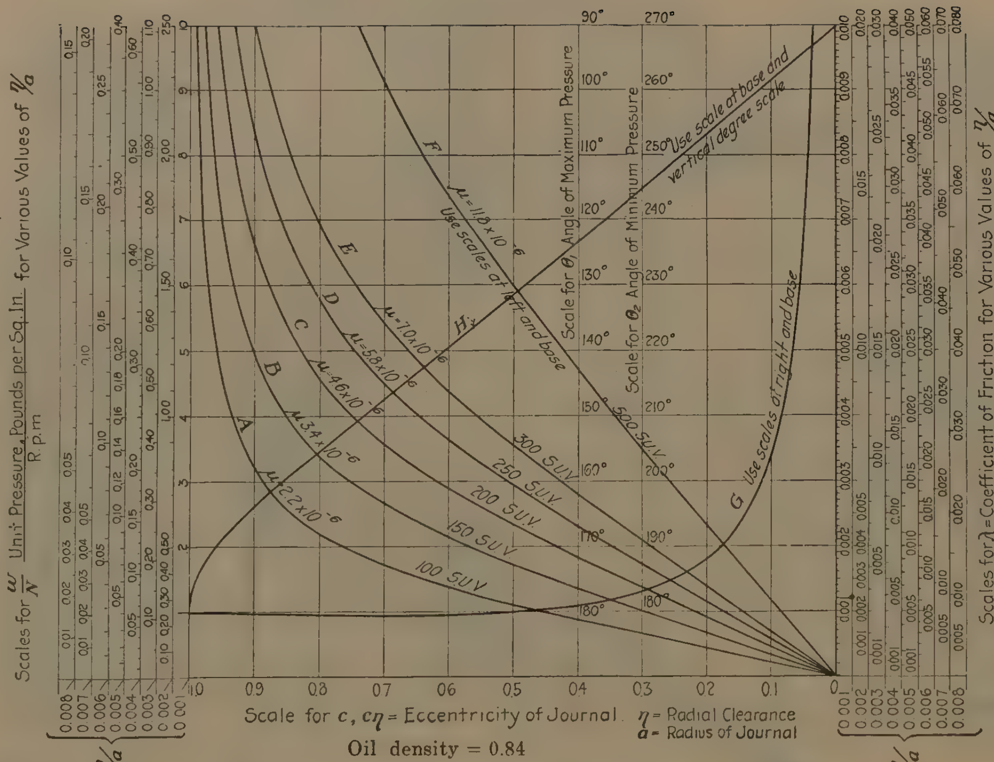


Fig. 10 Characteristics of Journal Surrounded by Bearing

100, 120, 140, 160, and 180 deg. for both the central and the offset bearings making fourteen charts in all. The region of close approach $c \geqq 0.9$ should be covered by these charts. The author will have them prepared at the earliest opportunity.

31 The bearing friction coefficient differs from the journal coefficient and it is hoped that they can both be shown on the same charts. When this is done and the friction coefficient for the full bearing is drawn on Fig. 10, it will be possible to check experimental results for partial and full bearings and learn how closely theory and practice agree, and what influ-

ence end leakage exerts upon the friction and displacement, taking into account the relation of bearing length to journal diameter.

32 Now that this graphical analysis has been carried as far as available time to the present would permit it will be well to examine the results obtained to find what they teach about bearing problems. In order that full and partial bearings may be compared Figs. 10 and 19 of Part I are reproduced. Fig. 19 applies to offset partial bearings. A shorter method than Fig. 19 for obtaining the leading angle α will be available when the additional charts outlined above are completed.

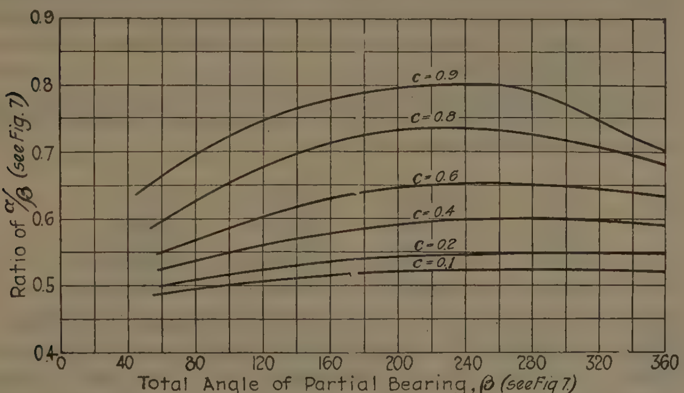


FIG. 19 LOCATION OF APPLIED LOAD WITH RELATION TO BEARING ARC β

33 It will be interesting to study the displacements of full and partial bearings under similar load-speed conditions. The following problems are devised for this purpose.

Example 3. A 2-in. journal revolves at 300 r.p.m. in a full bearing bored 2.004 in. ($\eta/a=0.002$). The unit pressure on the projected area of the journal is 60 lb. per sq. in. and the oil viscosity at operating temperature is 150 sec. Saybolt. Find the eccentricity, angle of maximum pressure, coefficient of friction, and nearest approach of the surfaces.

$\omega/N=0.2$. Hence from Fig. 10, curve B, it is shown that $c=0.235$. Hence eccentricity$=0.00047$. From curve H it is found that $\theta_1=110$ deg. and from curve G that $\lambda=0.00325$. The nearest approach for a full bearing is always $h_0=\eta\ (1-c)$. Hence for this case $h_0=0.002\ (1-0.235)$ $=.00153$ in.

Example 4. Same data as Ex. 3 except that bearing is partial and central and $\beta=120$ deg. Find c, ϕ and nearest approach.

$\omega/N=0.2$. From Fig. 30 the lower curve for $\beta=120$ deg. gives $c=0.642$ while the upper one gives $\phi=31.8$ deg. Since ϕ is less than $\beta/2$ the nearest approach is $h_0=\eta\ (1-c)$ which in this case equals 0.000716 in.

Example 5. Same data as Ex. 4 except that bearing is offset. Find c, ϕ, and nearest approach.

$\omega/N=0.2$. From Fig. 34 the lower curve for $\beta=120$ deg. gives $c=0.638$ while the upper curve gives $\phi=51.9$ deg. From Fig. 19 it is

found that $\alpha/\beta=0.63$. Hence $\alpha=75.6$ deg. The line of centers does not cut the trailing edge of the bearing because $\beta-\alpha>\phi$. Hence the nearest approach will be $h_n=\eta\ [1-c \cos (\beta-\alpha-\phi)]$. Substituting in this gives $h_n=0.002\ [1-0.638 \cos 7.5°]=0.000735$ in.

34. Examination of the answers to Examples 3, 4 and 5 will show that the full bearing has about twice the minimum film thickness found in the two 120-deg. partial bearings. Of the latter the offset bearing has the greater minimum and a still greater mean film thickness. This is obvious from the fact that the line of centers lies wholly outside the offset partial bearing whereas it cuts the trailing edge of the central partial bearing. Just what influence this has upon the friction coefficients of the two cannot be forecast with accuracy at this time but will be discussed when the friction curves have been drawn.

Example 6. Full bearing same[1] as in Ex. 1. Solve for $\eta/a=0.001$, $\eta/a=0.003$, $\eta/a=0.004$ and $\eta/a=0.005$. The results are tabulated below together with those obtained from Ex. 1.

η/a	c	λ	h_o
0.001	0.056	0.0057	0.00094
0.002	0.235	0.00325	0.00153
0.003	0.520	0.0031	0.00144
0.004	0.780	0.0038	0.00088
0.005	0.905	0.0051	0.000475

35 The above results are particularly illuminating. Small clearance gives high friction accompanied by close approach. Moderate clearance gives low friction accompanied by thick film. Larger clearance gives higher friction accompanied again by closer approach.

36 It should be remembered that the clearance that will give the best running conditions for one combination of load and speed will not give the best for another unless w/N is the same in both cases. There is however a fair range to the clearance ratios that give low friction and fair film thicknesses. Hence one ratio could be chosen with good results over a fair range of w/N.

DISCUSSION

E. O. Waters.[2] Whenever a highly theoretical analysis of physical conditions is made, the question always arises as to whether all the factors pertaining to the case have been considered, so that calculations from theoretical formulas will tally with the results of tests. For example, is it fair to assume that Harrison's differential equation for the rate of pressure change can be inte-

[1] See first section of this paper, Trans. A.S.M.E., vol. 45, p. 421.
[2] Asst. Prof. of Machine Design, Yale University, New Haven, Conn. Jun. A.S.M.E.

grated between any limits whatever, from 360 to 0 deg., or that the component of flow of lubricant parallel to the axis of the bearing (so-called endwise flow) may be disregarded?

The experiments carried out at the National Physical Laboratory in 1922, and reported briefly by Stanton in his book on Friction, shed some light on this subject. In these tests the journal was completely surrounded by a bearing, but the radial clearance ratio $\frac{n}{a}$ was so large that an actual oil film was built up only for an arc of the circumference of 30 degrees or less. Pressures were measured at every degree of this arc, and a curve plotted which resembles somewhat the right-hand half of Fig. 22. By noting the maximum point on this curve, and the point of inflection, the value of θ_1 was at once obtained. Stanton then scaled off two values of $\frac{dp}{d\theta}$ from this curve, substituted them with the corresponding values of θ in

$$\frac{dp}{d\theta} = \frac{6\mu Uac}{\eta^2}\left(\frac{\cos\theta - \cos\theta_1}{(1+c\cos\theta)^3}\right) \quad . \; . \; . \; . \; [14][1]$$

and solved the two equations simultaneously for the viscosity and eccentricity c. The viscosity as thus calculated agreed tolerably well with the observed viscosity of the lubricant at the temperature of the bearing, after correcting for the effect of high pressure. The eccentricity was used to compute the distance of nearest approach between journal and bearing, and apparently was not checked against any direct measurement of this distance.

Using the values for μ and c that Stanton obtained for one of these tests (that in which sperm oil is the lubricant), the writer has plotted a curve for $\frac{dp}{d\theta}$ between the leading and the trailing edges of the oil film, taking ordinates at 10-deg. intervals from $\theta = 150$ degrees to $\theta = 190$ degrees, (Fig. 35). Then by measuring successive areas under this curve with a planimeter, the differential equation was integrated and a curve of pressure vs. angle plotted (Fig. 36). This curve shows a decided agreement, as regards general form, with the pressures that were actually observed, and affords corroborative evidence of the soundness of the hydrodynamic theory and its applicability to partial bearings.

It is interesting to note whether a partial bearing of this type, in which the length of actual bearing surface is determined automatically by the oil, is in the class of the author's so-called "offset" bearings. According to Stanton's analysis of the pressure curve in the above reported test, $\theta = 177° \; 15'$ and $c = 0.9946$. For an offset bearing, $\cos\theta_1$ should equal $-\frac{3c}{2+c^2}$, or, for this

[1] See Par. 3, page 810.

value of c, θ_1 = about 176° 35′ . . . . a discrepancy of two-thirds of a degree.

The author's argument, in Pars. 26 and 27, that the "best" partial bearings are those in which the angle θ_1 is determined by the relation

$$\cos \theta_1 = -\frac{3c}{2+c^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [2] ^1$$

the same as for 360-deg. bearings, appears to be plausible but unconvincing. "Best" bearing is taken to mean a bearing which

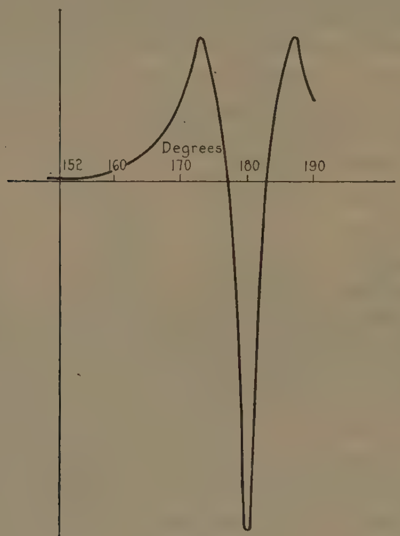


Fig. 35

has the maximum load-carrying capacity for a given set of conditions, including film thickness. In this connection, the five 120-deg. bearings in Fig. 31 are especially misleading, because the so-called "best" bearing G shows almost exactly the same load capacity as the central bearing M. A hasty reading leaves the impression that a bearing between F and E would be even better than any of the five shown.

The writer believes that a figure showing five 120-deg. bearings with varying degrees of eccentricity c, but all having the same film thickness, would be a stronger argument. One of these bearings would have the line of maximum pressure determined by Equation [2] and would thus fall in the author's "offset" class. If this

[1] See Par. 6, page 812.

bearing showed a higher loading than any of the others, its superiority would be more easily accepted.

Such an illustration might require much extra graphical work in connection with bearings lying between the "central" and "offset" classes. The writer has made an attempt to find the various properties and dimensions of the 120-deg. offset bearing which would have the same mean film thickness as the central bearing M, using the loading chart in Fig. 34 to obtain the values of ϕ for assumed values of c. As a result of this work, an offset bearing was evaluated having an eccentricity of 0.7, load capacity of 164.7 units, and a relative mean film thickness of 1.550. The central bearing M, Fig. 31, has an eccentricity of 0.4, load capacity

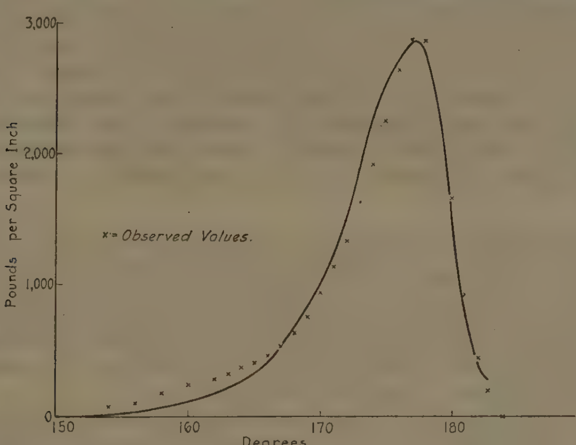


FIG. 36

of 49.6 units, and a relative mean film thickness of 1.576. This indicates that the load capacity of the offset bearing is about three times as great as that of the central bearing having the same mean film thickness. It is not claimed that this comparison is accurate; but by careful interpolation and the use of large-scale diagrams, an offset bearing could doubtless be designed having the same mean film thickness as the central bearing, and very obviously superior as regards load capacity.

DANIEL P. BARNARD.[1] The author has carefully stated that his investigations apply to those conditions under which "end leakage" is negligible, and has refrained from emphasizing the direct, practical application of his work. However, the writer's opinion, based on the results of a certain amount of experimental work,

[1] Research Associate, Dept. of Chem. Engrg., Mass. Inst. of Technology, Cambridge, Mass.

is that the author's results have far greater immediate value than one would at first sight suppose.

It is not necessary to eliminate end leakage in order to render these formulas valid. They should be regarded, rather, as determining the path of flow of the lubricant through the bearing. This is one of the most important points in connection with bearing design. Thus, when a journal runs in a bearing under conditions such that the oil makes a great many revolutions with the shaft before finally escaping at the ends, serious heating of the bearing surfaces may occur by reason of the oil failing properly to carry away the heat. Certain types of high-speed bearings have actually been damaged in this way.

On the other hand, if the oil escapes too rapidly from the ends of the bearings, the surfaces will be insufficiently separated by the small amount remaining in the film. There seems to be no particular obstacle to the application of the author's work in its present form to the solution of problems of oil flow through actual bearings of limited length — even where the length is not greater than the diameter and the "end effects" are extremely large.

M. D. Hersey.[1] It has been stated that the leakage out of the journal bearing would be negligible if the bearing were long enough. It is the writer's impression that the effect of end leakage cannot in this way be made negligible as regards eccentricity, nor, therefore, as regards the friction of the bearing. This seems to be a question that requires further explanation.

What specific gravity does the author assume for the oil when giving 150 seconds Saybolt as the standard value for which the curves are drawn? This information is desired since it would be possible for two oils of somewhat different viscosities each to read 150 seconds Saybolt, if their specific gravities are different.

The Author. Mr. Hersey has pointed out that when Saybolt viscosities are converted into absolute units the density of the oil must be taken into account. Each curve in Fig. 10 was drawn using the absolute viscosity marked along it. This is expressed in English units — inches, pounds, seconds — for example, $\mu = 3.4 \times 10^{-6}$ for curve B. The Saybolt universal viscosity (S.U.V.) corresponding with this was assumed to be 150 seconds, which requires a density of about 0.84. This same density was assumed for all curves in Figs. 10, 30, and 34, in which the viscosity is a factor. The absolute viscosity for the lower set of curves in Figs. 30 and 34 is 3.4×10^{-6}. For convenience of users of the curves this absolute-viscosity value has now been added to the data under these figures.

[1] Physicist, U. S. Bureau of Mines, Pittsburgh, Pa. Mem. A.S.M.E.

The question has been asked, "In what classes of service would partial bearings be superior to full bearings?" A complete answer to this question cannot be given until the friction characteristics of partial bearings are available. The author expects to complete that part of the work during the summer of 1925.

There are comparatively few cases in which a pure full bearing is used with adequate lubricant to produce a complete film. There are many bearings that are neither pure full nor pure partial. For example, a full-bearing bushing with two axial grooves, diametrically opposite, extending the whole length, must for purpose of analysis be assumed to consist of two opposed partial bearings fixed with respect to each other. Confining our attention, however, to pure full and pure partial bearings, it was found in Examples 3, 4, and 5 that for the same load and speed the minimum film thickness was about twice as great in the full bearings as in the 120-deg. partial bearings. The advantage therefore lies with the full bearing in the matter of film thickness. Even though the full bearing has a thicker film than the partial, it also has a greater area of film under shear. It is therefore not obvious which bearing will run the cooler.

Answering the question, "How does journal speed influence the choice of running clearance?" in general it favors an increase of clearance for an increase of speed.

For a full bearing this problem may be studied very well in Fig. 10. The minimum friction is nearly reached when the eccentricity is 30 per cent. Thirty per cent may therefore be chosen as the standard of comparison for this particular discussion. The unit load may be taken as 100 lb. per sq. in. and the oil viscosity as 150 S.U.V. The speed may be varied from 200 to 1000 r.p.m. At 200 r.p.m., $w/N = 0.5$. Assuming $\eta/a = 0.001$, we have $c = 0.15$, and $\lambda = 0.00235$. If $\eta/a = 0.002$, then $c = 0.57$, and $\lambda = 0.0019$. It therefore appears that with an eccentricity of $c = 0.3$ the friction coefficient would be about $\lambda = 0.00215$ with a clearance of about $\eta/a = 0.0014$. If the speed is 1000 r.p.m. then $w/N = 0.10$. If $\eta/a = 0.002$, then $c = 0.13$ and $\lambda = 0.0054$. With $\eta/a = 0.003$, $c = 0.27$, and $\lambda = 0.0042$. With $\eta/a = 0.004$, $c = 0.48$ and $\lambda = 0.0040$. It therefore appears that with an eccentricity of $c = 0.3$ the friction coefficient would be about $\lambda = 0.00417$ with a clearance of about $\eta/a = 0.0031$.

Summarizing these results, we find that for 1000 r.p.m. the specific clearance should be 0.0031, whereas for 200 r.p.m. it should be 0.0014, provided that the unit pressure on the bearing is 100 lb. per sq. in. in both cases, and the same oil (150 S.U.V.) is used. It may therefore be stated that, other conditions being equal, higher speed requires greater specific clearance, i.e., greater clearance per inch of diameter of the journal. This applies to pure full bearings and also to all cases where the journal is confined between two or more partial bearings.

The question of leakage from the sides of the lubricating films in journal bearings, i.e., at the ends of the bearing, is very important when interpreting charts based on the two-dimensional theory. Certain corrections from the tabular results must be made to cover the effect of this leakage, usually spoken of as "end leakage." The shorter the bearing is, in terms of its diameter, the greater will be the end leakage. This leakage would probably reduce the minimum film thickness below the value indicated by the charts, Figs. 10, 30, and 34. The point of nearest approach would also probably move nearer to the line of the load as end leakage in increased. A quantitative statement of these deviations must await the results of careful experimenting.

In the experiments referred to by Professor Waters, Dr. T. E. Stanton employed very large clearances for the purpose of producing very short films. In one case in which $\eta/a = 0.020$ the film covered only 30 deg. of a journal 1 in. in diameter by 2.5 in. long. In another case $\eta/a = 0.060$ and the film covered 15 deg. of the journal surface. The lengths of these films in the direction of rotation were 0.26 in. and 0.13 in., respectively, which, compared with the width of 2.5 in., were roughly equivalent to ordinary partial bearings 20 diameters and 10 diameters long, respectively, as far as the effect of end leakage was concerned. It was therefore to be expected that the test results would agree well with calculations based on a correct two-dimensional theory.

In conclusion, attention is drawn to the need of carefully conducted experiments on journal bearings of various lengths and diameters, and to the desirability of plotting the test results on charts like Figs. 10, 30, and 34. Such plots should have plainly marked on them the ratio of film width to film length. When the bearing is four or five diameters long and the clearance small, ($\eta/a = 0.002$ or less) the test results should correspond closely with those indicated by the two-dimensional theory from which Figs. 10, 30, and 34 are plotted. The heavier the oil used, the closer will be the agreements. The greatest deviation is to be expected with relatively short bearings.

No. 1937

HIGH-PRESSURE-BEARING RESEARCH

By Louis Illmer,[1] Cortland, N. Y.

Member of the Society

This paper presents a synopsis of some research investigations into the laws of friction and deduces a practical method for determining friction coefficients as based upon a wide range of high-pressure-bearing practice. The author's previously published study relating to perfect oil-borne lubrication is briefly reviewed, and it is now found that the identical underlying principles lead to a solution of friction problems where the oil film has been partially or wholly broken down under extreme pressure.

The paper is more particularly directed toward the intermediate field of friction that lies between perfect film and dry metallic friction. The analysis arrives at basic values for the corresponding friction coefficients and introduces a number of modifying constants for use under any given set of operating conditions.

The various kinds of bearing practice have been coördinated to secure results which will harmonize with expectations in practice. Suitable values are presented for determining the coefficient of friction for oil-borne journals; it is shown that the transition from a perfect film to a partial oil-film lubrication causes the friction coefficient to undergo a gradual change rather than an abrupt increase, depending primarily upon the ratio of the actual working pressure to the critical pressure. The friction coefficient applying to high working pressures is also found to vary inversely as the rubbing velocity and the temperature assumed by the bearing.

The oil supply, attendance, location, and structural conditions affect friction losses. Coefficients are furthermore dependent upon the method of working the lubricant between the rubbing surfaces since parallel flat faces do not have the same capacity for building up an oil film that is possessed by rotating journals. Certain factors have therefore been introduced to make due allowances for such differences in the bearing type.

The matter of pressure limits for slow-moving bearings has also been investigated and it is shown that the ultimate load capacity is partly fixed by the composition and the abrasive limits of the materials in rubbing contact.

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The friction coefficient applying to partially oil-borne journals is found to be substantially independent of the materials in contact, but when such journals operate at slow speeds under extreme pressures, the character of materials used assumes a greater significance.

THE present paper supplements an earlier one by the author relating to high-speed bearings[1] working under moderate pressure. This previous research included a critique of friction literature as applied to perfect unbroken oil-film lubrication, and compared the results ascertained by experimental inquiry with oil-borne bearing design practice.

2 The underlying principles thus deduced are now found to have a far wider application in that they also lead to a solution of high-pressure friction problems where the oil film has been partially or wholly broken down and allows of direct metallic contact between the rubbing surfaces. This aspect of the problem has been investigated and some new deductions relating to high-pressure bearings have been formulated.

3 It is not the purpose of this paper to advocate any particular friction theory, but rather to make a survey and explain a simple, practical method for predetermining friction coefficients. Different kinds of bearing practice have been coördinated along comprehensive lines in order to arrive at a working basis that will harmonize with such practice. The equations presented have been empirically deduced and are intended to connect up the available design and experimental data in a usable manner for engineering purposes. The method of procedure laid down fixes the minimum friction losses that may be expected under favorable operating conditions and provides an ample margin of safety against abrasion for slow-moving parts working at high unit pressure.

4 The discussion will be especially directed toward the intermediate field of friction that lies between perfect film lubrication and dry metallic friction. A rational solution of friction problems falling within the designated region requires the finding of a suitable basic value for the corresponding friction coefficient. As compared with the friction coefficient for perfect lubrication, the breaking down of the oil film introduces a number of additional variable factors that affect such coefficients, and these change with conditions of service. Certain constants are therefore presented to serve as a guide in appraising the various modifying factors, appropriate for any given set of operating conditions under partial lubrication.

[1] Bearing Design Constants, *Power,* Feb. 22, 1916, p. 251. Abstracted in Kent's M. E. Handbook, 10th ed., p. 1710, and in Leutweiler's Elements of Machine Design, 1st ed., pp. 530-532.

Review of Perfect-Film-Friction Formulas

5 The effect of partially breaking down the oil film can more readily be traced by taking up first the case of completely oil-borne journals as applied to the commercial type and finish of bearings.

6 In such bearings the rubbing-surface velocity must be sufficiently high to allow of lifting the rotating journal and establishing a perfect oil film beneath it. The paper referred to in Par. 1 shows that the resulting coefficient of friction may be determined with a fair degree of accuracy and that it is dependent upon four principal factors, which may be summarized as follows:

7 (*a*) *Quality of Oil Used,* especially as regards its viscosity characteristics. The change in viscosity is largely controlled by temperature; for ordinary grades of mineral oils a suitable allowance for this change may be made by introducing the temperature factor T or T' as defined below.

8 (*b*) *Quantity of Oil Used.* Experiments clearly show that when any bearing is operated with a stinted oil supply, the coefficient of friction will be materially higher than when the same bearing is served with a copious and evenly distributed oil supply.

9 In the case of fast-running oil-borne bearings the minimum of friction is assured when at least one gallon of lubricant is fed for each 350,000 sq. ft. of journal surface passed over the rubbing area. A supply in excess of this stipulated amount will be termed flooded lubrication.

10 (*c*) *Radiating Capacity.* This will vary under different temperature heads and it is also dependent upon the massiveness of the bearing construction. The rate of heat offtake may be checked by means of the following simple relation which takes the radiating capacity directly proportional to the projected area of the bearing and temperature head:

$$Ht_h = \mu PV \quad \text{. . . . . [1]}$$

where μ = coefficient of friction for perfect oil-film lubrication as applied to a flooded rotating journal

P = mean specific or unit pressure carried by an oil-borne bearing, lb. per sq. in. of projected area

V = mean surface or rubbing velocity of a rotating journal, ft. per min.

t_h = temperature head of rubbing surface with respect to the cooling medium or atmosphere, deg. fahr.

(Allowances: 40 to 60 deg. fahr. rise for heavy-duty bearings; 60 to 90 deg. fahr. rise for high-speed bearings.)

H = specific radiating capacity per sq. in. of maintained projected area of bearing, which factor increases

approximately as $\sqrt{t_h}$, ft-lb. per min. per 1 deg. fahr. temperature head.[1]

11 (*d*) *Pressure and Temperature Factors.* When a rigid journal is worn to a proper fit and copiously supplied with a suitable

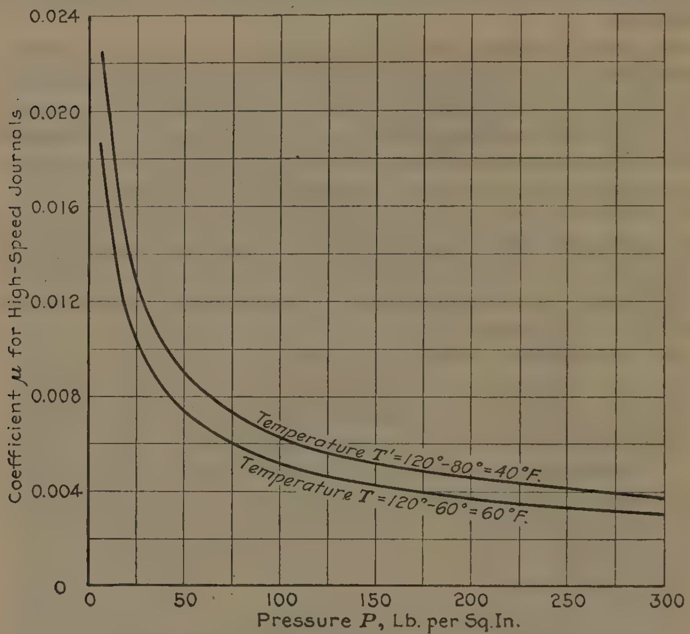

FIG. 1 COEFFICIENT OF FRICTION FOR PERFECT OIL FILMS AS DETERMINED BY EQUATION [2]

lubricant, the friction coefficient is primarily determined by the unit working pressure and the temperature attained by the rubbing

[1] Allowances for natural radiation and still air convection into atmosphere for journals at about $t_h = 80$ deg. fahr.:

H = about 2 for babbitted lineshaft-hanger bearings
= about 3 for light outboard or generator bearings without oil pockets
= about 4 for small ring-oiler motor bearings with oil pockets
= about 5 for heavy-duty outboard or generator bearings without oil pockets which value may be considered as a standard of reference for high-pressure work
= about 6 for heavy-duty engine main bearings without oil pockets

Ht_h = normal or usual still-air heat-offtake allowance, about 150 to 200 ft-lb. per min.
= maximum still-air allowance, ordinarily about 300 or 400 ft-lb. per min.

For bearings in movement, e. g., crankpins, use 1½ to 2 times the still-air allowance.

For liquid cooling or forced ventilation use 1½ to 3 times the still-air allowance, depending upon the relative effectiveness of the cooling medium.

surfaces. As applied to an oil-borne journal rotating at relatively high speeds, its value, where $P<P_c$, is given by the following simplified formula as derived in the paper previously referred to:

$$\mu = \frac{2}{5\sqrt{PT}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [2]$$

where $T = t_1 - 60° =$ virtual temperature for the more common grades of mineral engine oils, deg. fahr.

$t_1 =$ temperature of the rubbing surface which should be limited to about 200 deg. fahr.

$P_c =$ critical pressure as defined below.

12 The given temperature factor T allows for change in viscosity of ordinary grades of mineral engine or machine oils. This simple method of compensating for the oil body satisfies present requirements when the bearing temperature t_1 does not exceed the usual values allowed in good practice.

13 Equation [2] shows the coefficient μ to be independent of the velocity factor V. In the case of high-speed oil-borne bearings this factor was previously found to have no appreciable influence upon μ above 500 ft. per min. Furthermore the relatively small decrease in friction that should occur at slower speeds is largely lost under ordinary conditions of operation.

14 As stated, the above deductions apply particularly to a rotating journal properly fitted into rigid bearing shells flooded with mineral oil. A series of values for the perfect-film-friction coefficient μ as determined from Equation [2] are plotted in Fig. 1.

Critical Pressure

15 In order to lift the journal against a given unit bearing pressure and permit of fully establishing an oil film, it is necessary to run the rubbing surface above a certain critical velocity. The relation between the rubbing velocity and the resulting lifting power or counter pressure which a rotating journal is capable of exerting against the working pressure was also explained in the author's previous paper. The speed needed for this purpose when using the more common grades of mineral engine oils, was found to be approximately that given by the equation

$$P_c = 140 \sqrt[3]{\frac{V}{T}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [3]$$

where $P_c =$ critical pressure at or below which a perfect oil film may be maintained under a journal when running at a given rubbing velocity V, lb. per sq. in. of projected area.

Imperfect Film Friction

16 Stanton[1] defines two essentially different kinds of lubrication: (1) surfaces separated by a film of oil whose rubbing friction is largely determined by the viscosity of the oil, and (2) "boundary" or partial film lubrication, where the friction is more largely dependent upon the nature of the surfaces in rubbing contact and the chemical constitution of the oil.

17 The subject-matter of the present paper is especially concerned with the latter kind of friction and the effects produced upon the coefficient μ when working with a unit pressure that is in excess of the critical pressure P_c. This results in a partial breaking down of the oil film, and according to the author's method its effect may be analyzed thus:

18 When the unit working pressure P assumes a value equal to P_c, the coefficient μ will then be reduced to the minimum value expected in good practice, namely,

$$\mu_{\min} = \frac{2}{5\sqrt{P_c T}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [4]$$

19 This value serves as a standard of reference and will be so used in the discussion that follows. It will be apparent that when the average working pressure is allowed to exceed the critical pressure P_c, the friction losses will thereby be augmented because the journal can no longer ride on a perfect film of oil. When operating under such imperfect conditions, the working pressure will be denoted by P_{av} and the corresponding friction coefficient will be designated as μ_{exc}.

20 Contrary to prevailing opinion however, it is found that the transition from a perfect oil film to a partial oil film does not produce an abrupt rise in the friction coefficient. Instead, the breaking down of the oil film causes a gradual increase in friction, depending largely upon the ratio P_{av}/P_c, as will be more fully explained presently.

21 It is pointed out that the oil film may readily be broken down by lack of the necessary amount of lubricant to keep the journal flooded, and that in practice the so-called perfect oil film is itself subjected to certain disturbances which in some respects are similar to those produced in the case of partial oil films. Such a condition is apparent from Equation [2] as deduced from experimental data. This equation shows the coefficient μ actually to vary inversely as the square root of P, while the theoretical value for the perfect film lubrication might be expected to drop at a faster rate and vary inversely as P. This difference in the rate of drop with pressure may be partly accounted for by the

[1] Lecture by T. E. Stanton before The International Air Congress, London, June 20, 1923. Abstracted in *Sibley Journal of Engineering*, Nov., 1923.

combined effects of end leakage, surface irregularities, lack of rigidity, and like imperfect structural conditions.

22 In this connection, attention is further directed to some important experiments that have been recently conducted by the A. S. M. E. Research Committee on Lubrication.[1] These tests prove that when thin mineral oils are put under a pressure of about 10,000 lb. per sq. in., their viscosity will be raised to more than double that determined under atmospheric pressure. As regards the variation of viscosity with temperature, this may still be allowed for by the method embodied in Equation [2].

23 In the case of partial film lubrication there are, in addition to the noted increase of viscosity, other important factors that must be taken into account when fixing upon the friction coefficient under extreme pressures. Of these may be cited the need for heavier grades of lubricants to maintain the required thickness of oil film; also the increasing effect of unevenness in the rubbing surfaces, since this becomes more pronounced as the thickness of the oil film is reduced under high pressures.

24 These and like uncertain elements can best be incorporated into the following equations in terms of the pressure factor by multiplying the coefficient μ_{min} by the fourth root of the ratio P_{av}/P_c. This method not only affords the advantage of simplicity but allows for discrepancies that are otherwise difficult to trace.

25 Such factors may be brought into fair agreement with practice when the reference Equation [4] is empirically modified for excess pressure in the following manner:

$$\mu_{\text{exc}}=\mu_{\text{min}}\sqrt[4]{\frac{P_{\text{av}}}{P_c}}=\frac{2}{5\sqrt{P_cT}}\sqrt[4]{\frac{P_{\text{av}}}{P_c}} \quad . \ . \ . \ . \ . \quad [5]$$

where μ_{exc} = coefficient of friction under conditions of partial oil film lubrication as applied to a journal bearing flooded with suitable mineral oil

P_{av} = average dead-load pressure on the rubbing surface when $P_{\text{av}}>P_c$, lb. per sq. in. of projected area

26 Equation [5] assumes the pressure P_{av} to remain reasonably constant; under variable or peak loads satisfactory results may still be expected when the maximum unit pressure in the cycle does not exceed $1\frac{1}{2}$ to $2P_{\text{av}}$, provided the maximum lies below the abrasive pressure limit.

Basic Equation for High-Pressure Friction Coefficients

27 Substituting in Equation [5] for P_c its value as given by Equation [3], the coefficient of friction applying to a flooded

[1] Third Report on Viscosity of Lubrication Oils at High Pressure, *Mechanical Engineering*, vol. 45, May, 1923, p. 315.

rotating journal after the oil film has been partially broken down where $P_{av} > P_c$, then becomes approximately equal to:

$$\mu_{exc} = \frac{1}{100}\sqrt[4]{\frac{P_{av}}{VT}} \quad \ldots\ldots\ldots\ldots \quad [6]$$

Where V is the mean velocity or average rate of movement between the rubbing surfaces in ft. per min.

28 This basic expression for high-pressure friction coefficients has been largely deduced from the identical principles that underlie

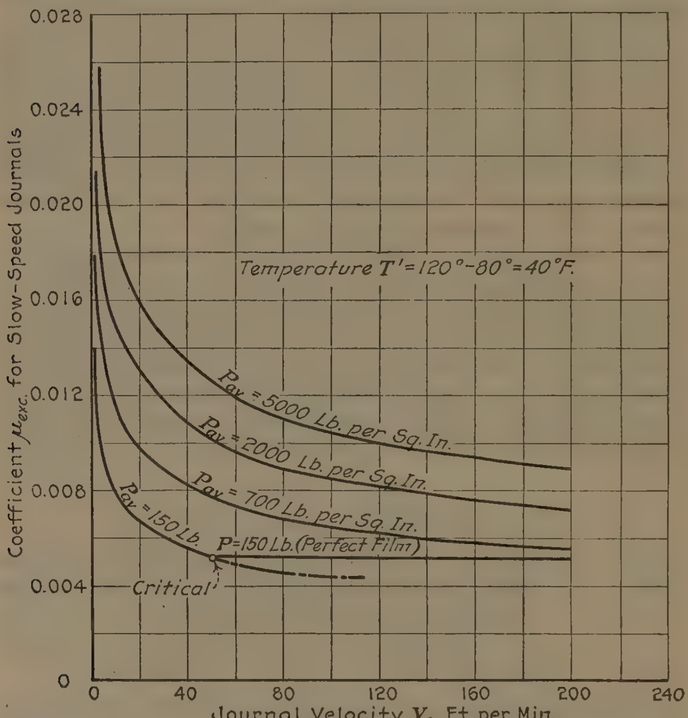

FIG. 2 COEFFICIENT OF FRICTION FOR IMPERFECT OIL FILMS AS DETERMINED BY EQUATION [6]

perfect film lubrication; it is believed that such procedure ought ultimately lead to a complete solution of the high-pressure friction problem.

29 Some values derived from Equation [6] have been plotted in Fig. 2. It is evident that the coefficient μ_{exc} may still be reasonably low, although the use of an excessive working pressure has broken down the oil film. The value of μ_{exc} can, however, in no case assume a value less than the corresponding coefficient μ. Under ordinary service conditions the friction resulting from

imperfect lubrication is usually several times greater than that which applies in the case of completely oil-borne journals.

30 For moderately heavy mineral oils such as are needed at higher pressures, the corresponding increase in viscosity may be compensated for by substituting $T' = t_1 - 80°$ in the place of the previously given $T = t_1 - 60°$. Extreme pressure may require still heavier lubricants such as cylinder oil, and for this a value of $t_1 - 100$ deg. fahr. may be used.

31 Equation [6] is only applicable within a normal temperature range and should t_1 exceed 200 deg., the friction coefficient is likely to increase rather than decrease with a further temperature rise. This equation embodies the basic relations sought for determining high-pressure friction coefficients for rotating journals; the remainder of the paper will center largely about this deduction.

32 In the case of imperfect film lubrication, the coefficient μ_{exc} is shown to vary inversely as the velocity factor. The velocity limits applying to Equation [6] range between 1 to 500 ft. per min., provided the upper limit lies below the critical velocity as fixed by Equation [3]. Chattering of the journal and uneven oil distribution are likely to augment friction at the lower speed limit.

33 At speeds still slower than 1 ft. per min. the given friction coefficient tends to rise abruptly and finally reaches starting values. For well-lubricated bearing metals this starting coefficient does not usually exceed 1/8 to 3/16, but such values might be doubled or tripled in the case of dry surfaces, particularly so when the rubbing surfaces have not been worn to a reasonably smooth fit.

34 Equation [6] further shows that the coefficient μ_{exc} increases at a rather slow rate with an augmented pressure until the abrasion point is reached. For perfect film lubrication, the coefficient μ as given by Equation [2] was shown to act in a contrary manner and to drop with increased pressure The turning point is reached when the unit working pressure P_{av} becomes equal to the critical pressure P_c.

35 The temperature factor T in Equation [6] bears a relation similar to that of Equation [2], except that the effect of temperature change is now less pronounced.

36 In arriving at a normal or expected value for the coefficient μ_{exc} under ordinary conditions of high-pressure service, no appreciable error will be involved by fixing the bearing temperature at a constant value $t_1 = 120$ deg. fahr., i. e., $T' = 40$ deg. fahr. Equation [6] then takes the following form which may be used for estimating the friction coefficient under high pressures when working with a moderate temperature rise and without perceptible abrasion:

Where $P_{av} > P_c$: $$\mu_{exc} = \frac{C_1 C_2}{250} \sqrt[4]{\frac{P_{av}}{V}} \quad \ldots\ldots\ldots \quad [7]$$

in which C_1 and C_2 are respectively circumstance and bearing-type constants taken from Tables 1 and 2.

37 The effect of temperature change upon the value of μ_{exc} is practically negligible within the restricted limits permissible in good bearing practice. The elimination of this minor variable serves to simplify the further consideration of the high-pressure friction problem.

38 Equation [7] presupposes the use of the most suitable mineral oil for the purpose intended. If the lubricant is not properly selected or if the shell is not suitably equipped for proper distribution of the oil, the friction coefficient is likely to be materially higher as a consequence.

Circumstance Constant C_1

39 In order to arrive at a correct appraisal of the coefficient μ_{exc}, the quantity of oil supplied to the journal must still be taken into account. High-pressure bearings are generally run on a

TABLE 1 VALUES OF CIRCUMSTANCE CONSTANT C_1

C_1	Lubrication	Workmanship	Attendance	Location
1	Oil bath or flooded	High-grade	First class	Clean and protected
2	Oil, free drop (constant feed)	Good	Fairly good	Favorable (ordinary conditions)
4	Oil cup or grease (intermittent feed)	Fair	Poor	Exposed to dirt or grit or other unfavorable conditions

stinted rate of oil supply that falls short of the standard set for flooded lubrication. Such cutting down in the rate of oil feed does not generally raise the friction losses to a dangerous degree until the minimum allowable oil requirement is approached.

40 Furthermore, rigidity of the bearing structure, workmanship, attendance, location, and similar conditions are also likely to exert considerable influence upon the resulting friction. The term "workmanship," as used in Table 1, is intended to serve as a measure of the character of design and machine work, alignment and fitting of bearings, and such other structural items as may be expected to affect friction losses. Allowances for these modifying factors have been incorporated into Equation [7] by means of the circumstance constant C_1. Suitable numerical values may be interpolated in accordance with Table 1 by modifying the given values to fit any other combination of circumstances.

41 When the bearing structure lacks rigidity or is otherwise such as to make it difficult to maintain a uniform distribution of lubricant or load, this condition might tend to double the given value of the constant C_1. For brake bands, clutch disks, friction

gears and like parts operating with substantially dry metal rubbing surfaces, the value of C_1 appears to lie between 10 and 30 with an average value of about 20.

Bearing-Type Constant C_2

42 All of the equations thus far deduced are intended to apply to rigid bearings provided with rotating journals. A different method for working the lubricant between the rubbing surfaces is found to modify the resulting coefficient of friction.

43 Parallel flat surfaces, for instance, do not have the same capacity for building up partial oil films that is possessed by rotating journals. Suitable allowance must therefore be made for the fact that rotating journals are more effective in keeping the rubbing surfaces separated than parallel flat surfaces such as the common type of thrust bearings, crosshead shoes, and the like.

44 The data available are insufficient to establish a value for the critical pressure P_c applying to flat parallel rubbing surfaces.

TABLE 2 VALUES OF TYPE CONSTANT C_2

C_2	Type and example of bearing
1	Rotating journals, such as rigid bearings and crankpins
1	Oscillating journals, such as rigid wristpins and pintle blocks
2	Rotating journals lacking ample rigidity, such as eccentrics and the like
2	Rotating flat surfaces lubricated from center to circumference, such as annular step or pivot bearings, etc.
2-3	Sliding flat surfaces wiping over the guide ends, such as reciprocating crosshead shoes. Use 2 for relatively long guides and 3 for short guides
3-4	Sliding or wiping flat surfaces lubricated from the periphery or outer wiping edge, such as marine thrust bearings and worm gears
4-6	Long power-screw nuts and like wiping parts over which it is difficult to effect a uniform distribution of lubricant or load.

Based upon Tower's experiments with flat-end step or pivot bearings lubricated through a central hole and radial grooves, the critical pressure P_c for any given velocity V lies between one-third and one-fourth of that prescribed by Equation [3] for flooded journals. Carrying this new relation through the various equations, the corresponding coefficient μ_{exc} is thereby increased from two to three times the value given by Equation [6].

45 The advantage of the Kingsbury type of thrust bearing resides in the fact that its tilted pivoted shoes provide for the formation of a wedge-shaped oil film similar to that in a rotating journal; hence it may be treated as such by taking the velocity factor on a mean-diameter basis.[1]

46 A further factor that tends to augment friction in the case of flat parallel surfaces has to do with the method of applying the lubricant to this particular type of bearing. When flat rotating

[1] See Slow-Speed Tests of Kingsbury Thrust Bearings, H. A. S. Howarth. Trans. A. S. M. E., vol. 41, p. 685. The more favorable results attained with cylinder oils check closely with values given herein for journals. See also A. Graphical Study of Journal Lubrication by the same author in *Mechanical Engineering*, February, 1924, p. 77.

surfaces are lubricated from the center outward, the friction coefficient is found to be considerably lower than when the lubricant is fed in a reverse direction.

47 The introduction of the constant C_2 as given in Table 2 makes an allowance for differences in the type of bearings and also for the method of applying the lubricant to flat surfaces. The values given are purposely rounded off since this phase of the art has received but scant experimental investigation.

Heating Effects

48 Equation [7] is intended to apply to both rotating and sliding flat surfaces, also to journals. To obviate disastrous heating

TABLE 3 VALUES OF MATERIAL CONSTANT P_n

Materials in Contact	P_n lb. per sq. in.	Remarks
Hardened tool steel on lumen or phosphor bronze	10,000	The given values apply to rigid polished and accurately fitted rubbing surfaces.
0.50 C machine steel on lumen or phosphor bronze	8,000	When not worn to a fit or well lubricated, reduce values to about $\frac{3}{4}P_n$.
Hardened tool steel on hardened tool steel (common grades)	7,000	
0.50 C machine steel or wrought iron on genuine hard babbitt	6,000	
Cast iron on cast iron (close grained or chilled)	4,500	
Case-hardened machine steel on case-hardened machine steel	4,000	
0.30 C machine steel on cast iron (close grained)	3,500	
0.40 C machine steel on soft common babbitt	3,000	
Soft machine steel on machine steel (not case-hardened)	2,000	
Machine steel on lignum-vitae (water-lubricated)	1,500	

effects and ultimate abrasion in continuous service, the rubbing velocity should be kept within the following limits:

$$Ht_h = \mu_{\text{exc}} P_{\text{av}} V = \frac{C_1 C_2}{250} \times P_{\text{av}} V \sqrt[4]{\frac{P_{\text{av}}}{V}} \quad . \; . \; . \; . \; [8]$$

49 In the above equation the factor of safety that underlies the pressure P_{av} with respect to the abrasion limit is approximately 2 to 3, provided the temperature is kept within the range prescribed. It will be apparent that the maximum allowable dead-load pressure which a bearing can safely carry is largely dependent upon the rubbing velocity and the effectiveness of the cooling medium.

50 For any fixed values of the constants C_1 and C_2, Equation [8], can be transposed to take the following simplified form:

$$P_{\text{av}} V^{0.6} = P_{\text{av}} \sqrt[3]{V^2} \text{ approx.}, = \text{Constant} \quad . \; . \; . \; [9]$$

51 This relation may be used to advantage in proportioning new sizes of high-pressure bearings in any given line of machinery, especially where certain other sizes have already been worked out.

Slow-Moving High-Pressure Bearings

52 When the rubbing velocity is quite low the heating effects as determined by Equation [8] are apt to be negligible, and a bearing may then carry extremely high unit pressures without necessarily robbing the factor of safety. The ultimate load-carrying capacity of such slow-speed bearings is in part fixed by the nature of the surfaces and composition of the materials in rubbing contact. The allowable pressure at minimum rubbing velocity must therefore be restricted in accordance with the abrasive-pressure limits of the respective materials used.

53 Approximate material constants, designated as P_n, are given in Table 3, and these apply to the various combinations of bearing surfaces commonly employed in practice.

54 The rounded value of P_n has in part been interpolated from design data and is taken at about four times the usual allowances for well-lubricated engine-wristpin pressures. Of the materials listed, only babbitt and bronze-lined shells are suitable for higher speeds, but the first-named tends to squeeze out under high pressure.

55 If is further necessary to make an allowance for the character of load carried by a high-pressure bearing, since the ultimate load-carrying capacity of a bearing on a full or partially reversing load is approximately double that of its dead-load capacity. For the intermediate condition such as periodically variable or peak load acting in one and the same direction, the capacity is approximately $1\frac{1}{2}$ times that of a dead or uniform load. This difference in favor of a reversing load may be ascribed in part to the more favorable distribution of the lubricant, which keeps the rubbing surfaces separated to better advantage when under slow movement. The load constant C_3 given in Table 4 is intended to take care of such load characteristics.

TABLE 4 VALUES OF LOAD CONSTANT C_3

C_3	Load Characteristics	Example of Bearing
1	Dead or steady load	Such as generator main bearings
3/2	Variable or peak load	Such as punch and shear crankpins
2	Reversing loads	Such as steam-engine wristpins

Ultimate Load Capacity

56 In determining the ultimate load capacity of a slow-moving bearing when working without appreciable heating effects, the oil supply constant C_1 is no longer a factor of importance, except that the rubbing surfaces must still be reasonably well lubricated. The following empirical formula may be used for finding the

allowable limit pressure for well-fitted heavy-duty bearings running on a partial oil film, the underlying factor of safety with respect to abrasion being about 2 to 3 when the constant $C_4 = 1$:

Where $V \gtreqless 1$ ft. per min.,

$$P_{\lim} = 0.6 C_3 C_4 \frac{P_n}{\sqrt[3]{V_e^2}} \sqrt[3]{H} \quad . \quad . \quad . \quad . \quad . \quad . \quad [10]$$

Where $P_{\lim}$ = limiting average dead-load pressure allowable under negligible heating effects, lb. per sq. in. of projected area

C_3 = load constant as given in Table 4

C_4 = intermittency factor; for continuous service = 1; for intermittent service, $1\frac{1}{2}$ to 2 as noted below

P_n = material constant as given in Table 3

V_e = equivalent rubbing velocity, ft. per min. For slow-moving oscillating loads this may be taken as inches of movement actually under load × (r.p.m.÷6)

H = specific radiating capacity of bearing or rate of heat dissipation as defined under Equation [1].

57 The ultimate load-carrying capacity $P_{\lim}$ is dependent upon the velocity V_e. When an oscillating load acts only in one direction of movement, the heating effects are thereby reduced to an equivalent rubbing velocity basis of about $V_e/2$. On the other hand, the corresponding friction coefficient as fixed by Equation [6] should be determined by the actual mean rate of movement V with which the lubricant is being worked between the rubbing surfaces. The pressure $P_{\lim}$ is also found to increase at a relatively slow rate with an enlarged radiating factor H. The specific radiation may be promoted by bringing up fresh rubbing surfaces into engagement and this serves to increase the unit load carrying capacity of the maintained or actual contact area in proportion to the cube root of the factor H. In the event that the bearing surfaces are not adequately lubricated, this would tend to restrict the given value of $P_{\lim}$ to the extent indicated in Table 3.

58 In the use of Equation [10] for variable or peak loads, reasonably satisfactory results at the expense of wear may still be expected when the maximum value in the cycle does not exceed $1\frac{1}{2}$ to 2 times average pressures. That $P_{\lim}$ may safely be carried to very high pressures when the rubbing velocity is sufficiently low, is made evident in Fig. 3, which shows the allowable mean pintle-block pressures as derived from Equation [10] for a normal value of $H = 5$. The given load capacities would naturally have to be reduced when the bearing should show signs of considerable heating when working under such high pressures, which case is treated under Equation [11].

INTERMITTENCY FACTOR C_4

59 In machinery intended for intermittent use, it is permissible to reduce the factor of safety to some extent and subject the working parts to a greater maximum load than is allowable under conditions of continuous service. Machinery in intermittent

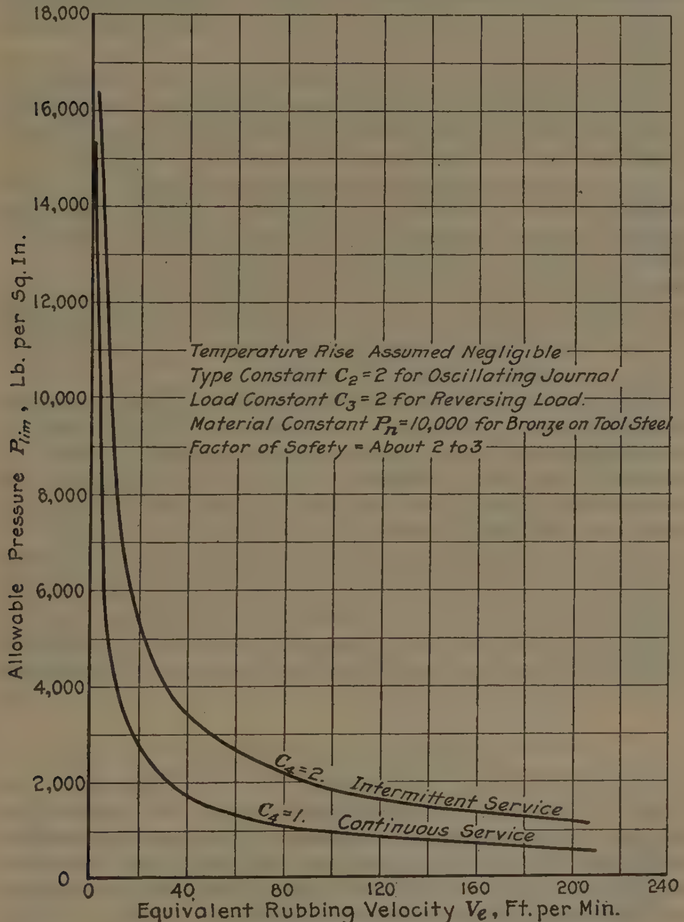

FIG. 3 LIMITING AVERAGE PRESSURE FOR PINTLE BLOCKS AS DETERMINED BY EQUATION [10]

service usually operates at a lower bearing temperature and partly for this reason may be expected to withstand a higher momentary peak load without abrasion. The given factor C_4 has been incorporated into Equation [10] to take care of such contingencies, since without this factor bearing constants applying to different kinds of practice would not be comparable.

60 For machinery put into service occasionally or intermittently used at maximum loads or such for which a fairly rapid wear is permissible, the limiting pressures prescribed in Equation [10] may be raised by taking the value of the factor C_4 equal to $1\frac{1}{2}$ to 2, depending upon the degree of intermittency and the purpose intended. The factor C_4 is not unlike the allowances usually applied to stress relations when working under similar conditions.

61 In certain cases, such as intermittently-used slow-moving pivots, pintle blocks for punches, toggle pins, power screws, thrust blocks, and like parts, the factor of safety may be still further reduced and the extreme pressures carried up to correspond to $C_4 = 2$ or 3. Such measures may entirely eliminate the factor of safety, and to avoid serious abrasion at starting speeds the maximum working pressure P_{lim} as determined from Equation [10] for $V = 1$ ft. per min. should not exceed $2P_n$.

62 At very high pressures a certain amount of abrasion is unavoidable, but when the rubbing surfaces are kept properly lubricated and cooled, this need cause no great concern provided a rather rapid rate of wear is permissible.

63 It is not generally realized that in certain classes of machines it is often necessary or desirable to subject their bearings to more or less rapid wear, either because of lack of room when carrying abnormally high total pressures, or because the results can be more cheaply accomplished even after making due allowance for frequent replacement of working parts.

64 In the case of wristpins, pintle blocks, and other slow-moving pivot bearings it is often more convenient to take their rubbing surfaces in terms of the corresponding crankpin areas as fixed by Equation [8]. The pivot areas should then be checked for maximum unit pressure as allowed by Equation [10], provided the heating effects are negligible.

Effects of Materials in Rubbing Contact

65 Referring further to Equation [8], it will be noted that the coefficient μ_{exc} is taken independently of the material constant P_n. Accordingly, any combination of materials working within their load-carrying capacities and worn to a smooth fit may be expected to work with substantially the same friction coefficient under otherwise similar conditions.

66 Equation [8] also shows that the value of μ_{exc} does not rise abruptly, although the specific working pressure may greatly exceed the critical pressure P_c. Even a slight oil film under extreme pressure must therefore be quite effective in keeping the rubbing surfaces adequately separated, at least sufficiently so to obviate abrasion. This no doubt explains why the friction coefficient μ_{exc} applying to partially oil-borne journals is found to be substantially independent of the materials in contact.

67 Under extreme pressures on surfaces that are but slightly lubricated and also at starting speeds, the material factor assumes a greater significance. Furthermore, in the case of certain bearing materials which do not readily take or maintain a smooth polish, such a rough surface condition may materially raise the given coefficient values.

68 Abrasion between suitably lubricated bearing metals should not occur when the working pressure is kept within the limits prescribed for a value of $C_4 = 2$. In the event of abrasion under still higher pressure and temperatures, the friction coefficient is apt to undergo a rather abrupt rise and finally approaches that of dry friction.

69 The present investigation indicates that the chief advantage afforded by bearing metals having a high material constant P_n lies in their greater heat offtake and load-carrying capacities rather than in any inherent anti-friction attributes. Rubbing surfaces which carry a high P_n value are apparently smoother and capable of running in closer rubbing contact, and therefore less likely to abrade with a rise of bearing temperature.

70 High-pressure-bearing practice clearly shows that the allowable temperature head t_h is usually raised in a direct relation to the material constant P_n. At excessive temperatures the lubricant loses its viscosity or body; rapid abrasion and final seizing is likely to occur whenever the partial oil film is no longer capable of keeping the high spots of the uneven rubbing surfaces out of direct metallic contact, since this leads to a tearing off of the gliding surface projections. In using a heavy lubricant its greater body or viscosity better maintains the required thickness of film against high pressure; hence in actual slow-speed service the heavier oils frequently show a lower friction loss than do thinner oils under such severe conditions.

The Final Bearing Formula

71 When working with partial oil films the following modification of Equation [9] holds approximately within rather wide limits, and may be used to advantage in making an estimate for the allowable load-carrying capacity of a slow- or moderate-speed bearing operating under accumulative heating effects. This equation summarizes the substance of the foregoing discussion and applies with a fair degree of accuracy to non-cooled high-pressure bearings of all kinds.

When $P_{av} > P_c$,

$$\frac{P_{\max}}{m}\sqrt[3]{\left(\frac{V}{n}\right)^2} = P_{av}\sqrt[3]{V_e^2} = \frac{0.6C_4}{C_1C_2}P_n\sqrt[3]{H} = C_0 \quad . \quad . \quad [11]$$

where $P_{av} = P_{max}/m$ = average unit bearing pressure, lb. per sq. in. of projected area. For repeated peak loads such as in punches and shears, the bearing pressure is best restricted to that portion of a full revolution which corresponds to the cutting period as taken for the entire thickness of the sheared plate

P_{max} = unit peak pressure on the bearing, lb. per sq. in. of projected area

m = ratio P_{max}/P_{av}, which in the case of punches and shears may roughly be taken at about 2

$V_e = V/n$ = equivalent velocity applying while the mean load P_{av} acts on the bearing, ft. per min.

n = actual rubbing velocity V divided by V_e, which in the case of punches and shears may be taken at 3 to 4, depending upon the length of time required to cut the plate relative to the period of one revolution of the journal. For dead or uniform load, $n = 1$.

TABLE 5 HIGH-PRESSURE-BEARING CONSTANTS

Average values in practice		Type of bearing	Kind of machine	P_n (Table 3)	Circum. const. C_1	Type const. C_2	Intermittently factor C_4	$P_{av}V_e^{2/3} = C_0$ by Eq. [11]
P_{max} lb.	V/n ft./min.							
850	10	Main roll	Sugar mach.	8000	2	1	1	4000
650	10	Main roll	Sugar mach.	6000	2	1	1	3000
220	50	Gear shaft	Sugar mach.	6000	2	1	1	3000
85	200	Hangers	Line-shaft	3000	1	1	1	3000
70	100	Hangers	Line-shaft	3000	2	1	1	1500
1800/5	38	Wristpin	Oil engine	8000	2	1	1	4000
10,000/2	2/4	Pintle blocks	Punches and shears	4500	4	1	3	3400
4400/2	40/3	Main bearing	Punches and shears	8000	2	1	3	12,000
2000/2	50/4	Main bearing	Punches and shears	3500	2	1	3	5200
3300/2	60/3	Crank pins	Punches and shears	8000	2	1	3	12,000
5300/2	40/4	Crank pins	Punches and shears	8000	2	1	3	12,000
240/2	125	Counter-shafts	Punches and shears	3000	3	1	3	3000
2700	9	Main roll	Bending roll	8000	2	1	3	12,000
2000	5	Pin bearing	Guide rollers	8000	4	1	3	6000
500/4	65	Pin bearing	Cam rollers	8000	4	1	1	2000
45	200	Collar bearing	Marine thrust	6000	1	4	1	1500
55	400	Collar bearing	Marine thrust	6000	1	2	1	3000
1400	10/2	Thrust collar	High-pressure	8000	2	2	2	4000
850	10	Power screws	Bending rolls	8000	2	2	2	4000
60/4	650	Crosshead shoe	Engine work	4500	2	2	1	1100
850/2	10/2.5	Slide blocks	Punches and shears	4500	4	3	3	1100

72 It will be seen that Equation [11] is not unlike Equation [10] except that the first-named embodies the factor C_2 while the factor C_3 has been eliminated. Equation [10] applies to the

maximum pressure that can be tolerated for a brief period and if continued, such extreme loads would seriously injure the rubbing surfaces. Equation [11] takes account of the loss in load carrying capacity which results when the viscosity of the lubricant is reduced by accumulative temperature effects under a continuously applied load. The relation given above provides a simple check for the maximum average pressure P_{av} that can safely be carried upon different kinds of materials at slow velocities in continuous service. Equation [11] is also applicable at higher velocities when the service conditions do not allow of maintaining a perfect oil film between the rubbing surfaces.

73 An important advantage afforded by Equation [11] lies in the fact that the bearing constants used for various kinds of high-pressure practice can readily be compared with each other as shown in Table 5, which is based upon a normal radiation of $H = 5$.

74 The intended combination of bearing materials is indicated by the given P_n values and may readily be traced by reference to Table 3. The application of Equation [11] to any particular bearing problem will no doubt be apparent from the various factors listed under headings C_1, C_2, and C_4. Under continuous load the constant C_0 lies close to 3,000 for babbitted shells at $C_4 = 1$, while for intermittently loaded bronzed-lined punch and shear bearings this constant is raised to approximately 12,000 when working with a C_4 value of 3. As regards sliding and wiping surfaces as in high-pressure thrust collars and power screws, the table shows that such parts usually operate at a lower value of about $C_0 = 4{,}000$ when $C_4 = 2$.

75 A common method of fixing high-speed oil-borne bearing areas is to work with a plain PV constant. As applied to slow-speed sugar machinery, the corresponding $P_{av}\, V_e$ product would be a relatively low one, amounting to less than 10,000 for continuously loaded main bearings. In the case of high-speed main bearings such as engine work, the PV constant ordinarily used lies between 30,000 and 90,000, with 60,000 as a fair average value. This difference in favor of engine bearings may be accredited to the more perfect method of lubrication and the resulting lower friction coefficient inherent in oil films.

76 From the foregoing it will also be evident that at slower speeds perfect films can at best only carry low pressures, and that the ordinary moderate-speed journal is not likely to be oil-borne unless special precaution is taken to limit pressure and otherwise guard against film breakdown.

Conclusion

77 The basic Equation [6] for determining the coefficient μ_{exc} has been taken as a function of the fourth root of the factors P_{av}, V, and T. A still closer approximation could no doubt be

arrived at by further discriminating in the selection of a particular root for each of the controlling friction factors. This, however, leads to complications without effecting any material change in results.

78 The friction coefficient for partial oil films was also shown to be dependent upon a number of other important conditions such as those having to do with constructive details, oil supply, attendance, and the like. The probable error in appraising these modifying circumstances as embodied in the constant C_1 is apt to be considerably greater than that involved in the use of Equation [6]. The constructive factors may produce a rather wide departure from ideal conditions and must be taken into account in properly fixing pressure limitations.

79 In this connection it should be remembered that the friction coefficient as applied to any dry or partially lubricated surface is not really a physical constant but merely an arbitrary ratio that varies widely when determined under different conditions of service. It is therefore unsafe to rely wholly on theoretical deductions as to the friction and resulting heating effects that may be expected to occur between imperfectly lubricated surfaces. In fact, it appears quite doubtful whether any simple rational equation for predetermining the coefficient μ_{exc} can be evolved from theoretical considerations alone.

80 It is thought that the present study affords a suitable basis for further experimental investigation in this art. Research work directed along the lines indicated should lead to a better understanding of the rather perplexing principles that underlie imperfectly lubricated surfaces.

81 The author desires to thank Mr. G. Harry Case, chief engineer of the Southwark Foundry and Machine Company, for furnishing data relating to high-pressure bearing practice, and to acknowledge the helpful suggestions received from Prof. William N. Barnard, of Cornell University.

DISCUSSION

M. D. Hersey.[1] The author might tell us more about the basis of some of his equations, such as Equation [10] involving the cube root of H, and Equation [5] involving the fourth root of the pressures. To what extent are those experimentally determined relations?

In the writer's paper of 1915,[2] an attempt was made to take account of the heating effects by allowing for the loss of viscosity; this led to equations for permissible pressure and for variation of

[1] Physicist, U. S. Bureau of Mines, Pittsburgh, Pa. Mem. A.S.M.E.

[2] On the Laws of Lubrication of Journal Bearings, Trans., A.S.M.E., vol. 37 (1915), p. 167.

friction as the bearings heat up, which are substantially consistent with the equations given in the earlier part of the author's paper. This seems interesting in that the writer's results were arrived at deductively while those of the author are apparently offered as a coördination of values found satisfactory in practical experience.

A. E. Flowers.[1] There is a possible explanation of the question of temperature functions. Batschinski published in the 1900 Proceedings of the St. Petersburg Academy a series of measurements of the change of absolute viscosity with absolute temperature and for a large range of materials. He showed that the absolute viscosity — not the reading but the viscosity — varies inversely as the cube of the absolute temperature.

The Author. Replying to the request for more detailed information pertaining to the derivation of certain given equations such as [**5**] and [**10**], the basis underlying the first-named equation has been rather fully analyzed in Pars. 24 and 25.

With respect to the derivation of Equation [**10**], the given relations were obtained in a similar fashion. A comprehensive check of appropriate data shows that the limit pressure may be augmented by an increase in the radiating factor H. In this connection, reference is made to the definition of factor H as given in Par. 10, which states that the radiating capacity per unit bearing area is assumed to be directly proportional to the temperature head t_h; in reality, radiation increases at a somewhat faster rate. In case the radiation factor H were to be more accurately compensated for change in temperature head it would tend to complicate the given equations, but it is not unlikely that the simplification resorted to in this paper has led to the introduction of the factor $3\sqrt{H}$ as presented in Equation [**10**].

As stated in the concluding remarks, particularly Pars. 78 and 79, it is thought that in the present state of the art a more dependable method for predetermining friction coefficients may be derived from an analysis of a wide range of bearing practice than would be the case when basing conclusions upon a limited range of experimental data. It will be noted, however, that a theoretical basis underlies the given equations, but that the particular roots used for the various factors have been empirically evaluated to make them best fit in with practical results.

[1] Engr. in Charge of Development, The DeLaval Separator Co., Poughkeepsie, N. Y. Mem. A.S.M.E.

No. 1938

AN INVESTIGATION OF THE CRITICAL BEARING PRESSURES CAUSING RUPTURE IN LUBRICATING-OIL FILMS

By Leonard Noel Linsley,[1] U. S. N.

Non-Member

The problem involved in the paper is to determine the critical or breakdown pressure in the oil films formed in a bearing, using straight mineral oils and also to determine the influence on this critical pressure of the admixture of definite amounts of oleic acid. The author first presents a résumé of the work of other investigators. He describes the apparatus with which his own experiments were conducted and the characteristics of the oils used in his tests. Tests were run with cast-iron and with babbitt bearings, and the results of the final trials, which were run with two of the oils, first untreated and later treated with oleic acid, are given in tabular and graphic form. Efforts to determine the breakdown pressure electrically and the method of measuring film thickness are described. The paper concludes with suggestions for the extension of the experimental work and the redesign of the test apparatus.

In discussing his conclusions the author remarks that the actual breakdown pressures of oils are several hundred times greater than have hitherto been accepted as possible. Although not definitely established, a relationship between breakdown pressure, speed, and viscosity was shown to exist. The actual breakdown pressures at which the film is completely dislodged and the bearing seizes the journal should not be considered as the critical pressure but rather the point at which metallic contact first makes itself manifest. With reference to oils treated with oleic acid the inference cannot be avoided that oils having the same absolute viscosity but differing in chemical consistency do not have the same resistance to rupture and hence, from this point, differ in lubricating value.

[1] Lieutenant Commander, U. S. N., U. S. S. *Wright*.

Contributed by the Machine Shop Practice Division and the Special Research Committee on Lubrication and presented at the Annual Meeting, New York, December 1 to 4, 1924, of The American Society of Mechanical Engineers.

THE problem involved in the present investigation can be stated as follows: To determine the critical or breakdown pressure in the oil films formed in a bearing, using straight mineral oils, and also to determine the influence on this critical pressure of the admixture of definite amounts of oleic acid.

Previous Work

2 Little experimental work has been done with the definite object of determining these breakdown pressures. The usual method of attack follows the lead of Sommerfeld [1] who made use of the experimental results of Stribeck.[2]

3 An examination of the curves in Sommerfeld's paper [1] discloses the fact that when pressures are plotted as abscissas and coefficients of friction as ordinates on rectangular coördinates, the coefficient of friction decreases with increase of pressure until a certain minimum value is reached, but beyond this point it increases. This minimum value of the coefficient of friction has the same magnitude for all velocities of bearing face. The curves, however, present distinctly different trends with various values of velocity. At low speeds the slopes are very abrupt but flatten out as the speeds are increased. This minimum, or "transition point," as it is called, is regarded as being at or near the breakdown point of the oil film. This seems reasonable, if the notion is valid that there is a gradually increasing area of metallic contact after the critical pressure is reached. In that case it would certainly tend to increase the friction above that which could be attributed to viscous friction, in proportion to the increasing area of the contact surfaces.

4 Sommerfeld gives the following equations for the condition at the transition point for a bearing completely surrounding the journal

$$\mu = \frac{50P}{\pi^2 N}\left(\frac{\Delta}{d}\right)^2 \quad \ldots\ldots\ldots\ldots [1]$$

where μ = viscosity in poises
P = pressure in dynes per sq. cm. of projected area
N = revolutions per minute
Δ = difference in radii of bearing and journal in cm.
d = diameter of journal in centimeters.

5 If the viscosity is expressed in poises, the pressure must be stated in dynes per square centimeter, or 68,965.5 times the pressure in pounds per square inch, so that if it is desired to

[1] Zeitschrift für Mathematik und Physik, 1904, p. 597.
[2] Mitteilungen über Forschungsarbeiten, Heft. 7, 1903.

use English units throughout, Equation [1] may be written thus:

$$\mu = \frac{349{,}556.5P}{N}\left(\frac{\Delta}{d}\right)^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [2]$$

or

$$P = \frac{\mu N}{349{,}556.5}\left(\frac{d}{\Delta}\right)^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [3]$$

where μ = viscosity in poises
P = pressure in lb. per sq. in. of projected area
N = revolutions per minute
Δ = thickness of oil film in inches
d = diameter of journal in inches.

Equation [3] is a very simple relationship and would be very convenient if it were not for the difficulty in measuring the thickness of the oil film. This difficulty is a very real one and nullifies this line of attack, especially in view of the fact that after actual rupture of the film has started it becomes zero at some points and of course it is impossible to say where contact occurs or what is the extent of this area of contact.

6 Sommerfeld's equation was deduced on the assumption that the bearing completely surrounded the journal and it would be hazardous to contend that the same relationship would hold good with a bearing having an arc of contact of 240 deg. or even less, as is usually the case in practice.

7 Dr. J. T. Nicolson[1] in a very able paper following a similar method of deduction, after a very careful analytic procedure arrived at the following equation for various cases usually occurring in practice:

$$P = 40(dN)^{\frac{1}{3}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [4]$$

where P = pressure in lb. per sq. in. projected area
N = r.p.m.
d = diameter of journal in inches.

Formula [4] is intended for the design of bearings which completely surround the journal and, of course, in that respect the same limitations as that of Sommerfeld apply, but nevertheless, if the premises for which it is derived are correct it should yield a value which is near the critical pressure.

8 The actual measurement of lubricating-film thicknesses has been reported by Kingsbury,[2] Green[3] and Stoney[4] but the only report of a serious attempt to determine the actual breakdown

[1] Proceedings, Manchester Association of Engineers, November, 1907, p. 65.

[2] Journal, American Society of Naval Engineers, vol. 9, 1897, p. 267.

[3] Journal, A. S. M. E., vol. 39 (1917), p. 320.

[4] *Engineering,* March 3, 1922, p. 249.

pressure of the film in a quantitative sense is that of Prof. Herbert S. Moore.[1]

9 Professor Moore's experimental apparatus consisted of a test journal having a diameter of 1.286 in. arranged so that the lower half would revolve in an oil bath. To this journal was fitted a half bearing so contrived that the load could be varied by applying known weights to a hanger attached to the bearing. The journal and bearing with the oil film between them formed part of an electric circuit. When the machine was in operation and the oil film formed, this film would offer a great resistance to the flow of current, while on the other hand if the film were broken this resistance would fall off to zero or thereabouts. A voltmeter was connected across the line to indicate the line potential and in this way the pressure causing rupture of the film could be determined.

10 With the data obtained in this way Professor Moore plotted a curve of speed in feet per minute as abscissas and breakdown pressure in pounds per square inch of projected area as ordinates. He found that the following expression fitted this curve very well over the range of his observations:

$$P = 7.47\sqrt{V} \qquad [5]$$

where P = breakdown pressure in lb. per sq. in. of projected area
V = rubbing velocity in ft. per min.

11 This equation has been largely used in design. Mr. Axel K. Pedersen writing in the *American Machinist*[2] says: "This equation is fundamental and is now considered as a very close approximation by the best authorities, and should, therefore, be used in all cases where a perfect oil-film lubrication is desired."

12 Dr. Sanford A. Moss, chief engineer of the Mechanical Research Division of the General Electric Company, in a communication to the writer referred to Professor Moore's work as "a classic in the matter" and suggested that a check and extension of Professor Moore's work would be highly desirable.

13 It was with this end primarily in view that the present investigation was undertaken. It was thought desirable, however, to devise some mechanical means of determining the breakdown pressure and using the electrical method as a check. The first method contemplated was to fit a solid bearing with known clearance to a journal supported by two pedestals and load the bearing by means of a lever and jack. It was proposed to fit a torsion meter to the shaft and determine the breakdown of the film by the sudden increase of torque indicated on the torsion meter. This plan was abandoned principally on account of the time required to build the apparatus and also because an optical torsion meter, which was thought essential to this arrangement, was not im-

[1] *American Machinist,* vol. 26, September 10, 1903, p. 1281.
[2] *American Machinist,* vol. 37, October 10, 1912, p. 599.

mediately available. The apparatus finally decided upon can best be illustrated by the following detailed description.

Description of Experimental Apparatus

14 In the following description of the apparatus it will be convenient to refer the numerals to the diagram shown in Fig. 1.

15 The experimental apparatus was constructed in the university shop and was made entirely of miscellaneous equipment found about the laboratory. The design was influenced largely by what was available rather than by what was thought to be desirable.

16 The machine consists essentially of the test journal (16) which is made of Sanderson's tool steel, manufactured by the Crucible Steel Company of America and is described by that company as a straight carbon tool steel, containing about one per cent of carbon, with a phosphorus and sulphur content under 0.025 per cent. The journal was turned oversize, hardened, and then ground and lapped to a diameter of exactly two inches.

17 The journal is supported in its pedestal by two bronze bushed bearings (10). The pedestal is firmly bolted to a heavy oak laboratory table and this table in turn is bolted to the heavy timbered frame that supports the driving mechanism. The machine is belt-driven and actuated by a 3-hp., 1200-r.p.m., variable-speed shunt motor which is controlled by the starting panel and controller (23). Close graduation of speed is secured by the armature resistance (26). Alignment of the driving shaft is maintained and transmitted vibration from the motive element is reduced by a Fast flexible coupling (8) manufactured by the Bartlett-Hayward Company of Baltimore.

18 The test bearing consists of a split bearing (13), the two halves of which are pressed together on the journal by means of a hydraulic loading device (11). This part of the apparatus is shown in Fig. 2. A suitable steel strap surrounds the jack and bearing so that the thrust of the jack will be applied to both halves of the bearing. This loading device is known as a Loadometer and is manufactured by the Black and Decker Company of Baltimore. It is normally used to determine the weight of loaded motor trucks. The total load applied by the jack is indicated by the pressure gage (12) which is graduated to read in pounds total load. The load is applied by rotating the worm gear by means of the handwheel (14).

19 The bearing with the jack, loading mechanism and supporting strap and other appendages, constitute a pendulum, the details of which are shown in Fig. 3. The displacement of this pendulum through any angle ϕ indicated by the pointer (20) on the scale (21) will be a measure of the friction involved. The

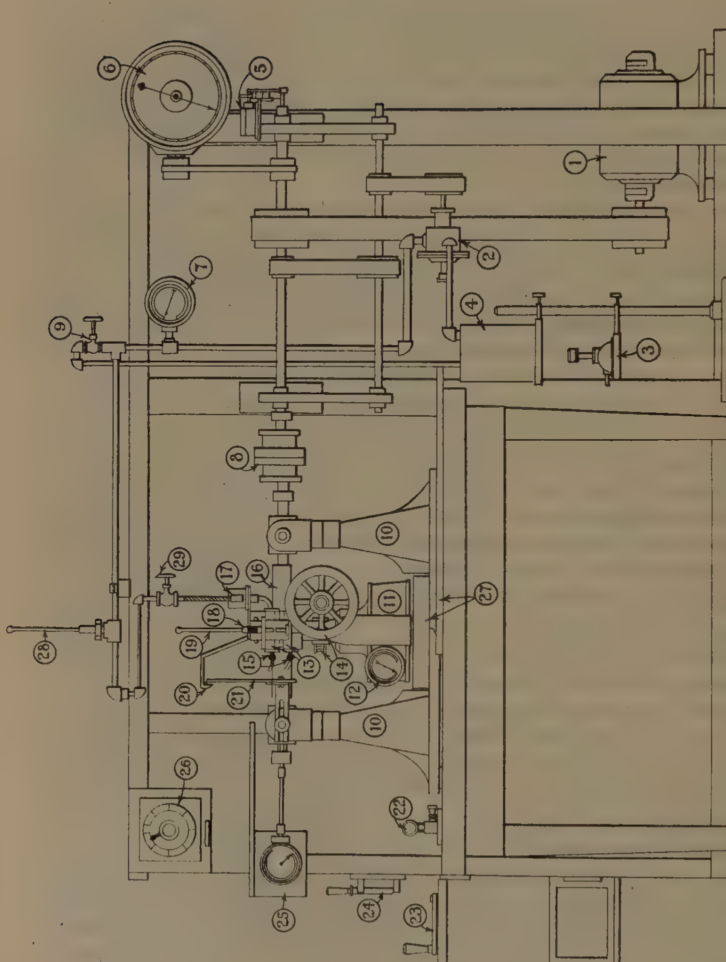

FIG. 1
(Caption on following page)

total friction and the coefficient of friction can be determined as follows:

Let W = weight of pendulum (114 lb.)
R = radius from center of journal to center of gravity of pendulum (6 in.)
r = radius of journal (1 in.)
P = total pressure on bearing
ϕ = angular displacement of the pendulum from the perpendicular
f = coefficient of friction
F = total frictional resistance.

The total pressure P is the pressure indicated by the gage, plus the loadometer calibration correction and plus the weight of the pendulum. This is sufficiently accurate in this case as the total loads encountered are large in proportion to the weight of the pendulum, but it should be borne in mind that the load on the upper half of the bearing is greater than that on the lower by an amount equal to the weight of the pendulum.

20 When $\phi = 90$ deg.,

$$WR = Fr$$

Then for any other angle

$$WR \sin \phi = Fr$$

The expression for the coefficient of friction, which has been defined above, may be written

$$f = F/P$$

but

$$F = \frac{WR \sin \phi}{r}$$

then

$$f = \frac{WR \sin \phi}{Pr} = \frac{114 \times 6 \sin \phi}{P}$$

$$f = \frac{684 \sin \phi}{P} \quad \ldots\ldots\ldots\ldots \quad [6]$$

With this, a table of coefficients of friction could be calculated for a wide range of deflections and pressures.

Fig. 1 Apparatus for Determining Critical Pressures in Lubricating-Oil Films

1 Variable-speed motor
2 Rotary pump
3 Bunsen burner
4 Oil reservoir
5 Revolution counter
6 Centrifugal tachometer
7 Oil pressure gage
8 Flexible coupling
9 Oil pressure vent valve
10 Supporting bearings
11 Loadometer
12 Hydraulic pressure gage
13 Test bearing
14 Loading wheel and worm gear
15 Thermocouple inlets
16 Test journal
17 Sight feed cups
18 Micrometer heads
19 Bearing thermometer
20 Deflection indicator
21 Deflection dial
22 Cut-out switch
23 Main speed controller
24 Line switch
25 Chronometric tachometer
26 Auxiliary speed controller
27 Oil drip pan and return
28 Oil inlet thermometer
29 Oil supply control valve

21 It should be noted that the radius R in the above expressions is the same at all conditions of load, as the movement of the piston is very small indeed, so small, that R is not disturbed under varying conditions of load. This feature is unique and is considered a notable improvement over the spring-loaded machines in which the center of gravity changes with every change of load.

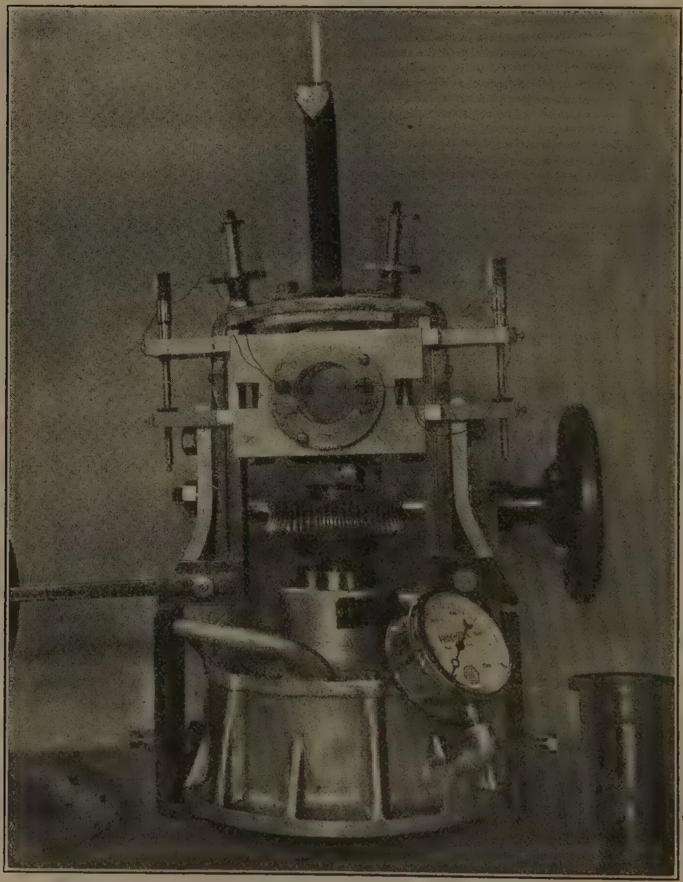

Fig. 2 Hydraulic Loading Device

22 The actual bearing surface is made in the form of a split sleeve which is bolted in the cast-iron holder. The cast-iron sleeve is shown in Fig. 2. This sleeve carries the bearing surface which on the first trials was made of close-grained cast iron and on the latter of high-grade babbitt metal, manufactured by the United American Metals Corporation of Brooklyn, N. Y., and described by that company as Syracuse Government Genuine Babbitt, con-

taining approximately 90 per cent tin, 3 per cent copper and 7 per cent antimony.

23 The bearing block which carries the bearing sleeve is split in the same manner as the sleeve and provided with spherical sections at the top and bottom that fit into similar female sections attached to the strap at the top, and the head of the jack at the bottom. This feature insures the self-alignment of the bearing under all conditions of operation.

24 The oil supply is maintained by a rotary pump (2) which takes its suction from the reservoir (4) and discharges into the sight-feed oil cups (17) and by this means a copious supply of lubricant is assured. The oil inlet line is vented by the valve (9) so that entrained air will be discharged through it, and not carried into the oil supply. This valve also serves to regulate the pressure on the oil line which is indicated by the gage (7). The oil is carried to the sight-feed cups by means of the flexible metallic tubing so there will be no resistance to the movement of the pendulum at this point. The oil return is accomplished by means of the drip pan and trough (27) which act also as an oil cooler.

25 The speed is measured by an Elgin chronometric tachometer (25) supplemented by a centrifugal tachometer (6) and a positive counter (5). The Elgin tachometer functioned perfectly and proved to be highly satisfactory for the work. No irregularity could be detected by comparing it with the counter.

26 The temperature of the bearing was determined by a mercurial thermometer (19) which was supported by a fiber bushing in a recess drilled into the bearing within an eighth of an inch of the bearing surface. Thermocouples made of copper and " advance " wire were fitted into recesses right at the entering and leaving edges of the actual bearing area so that the oil temperatures at both could be determined. The cold junctions of the thermocouples were immersed in melting ice in a vacuum bottle. The current generated was determined by a highly sensitive galvanometer so that small differences in temperature could be determined.

27 Vernier micrometers (18) that could be read to ten-thousandths of an inch were fitted to each half of bearing supporting blocks so that the vertical movement of the two halves of the bearing could be measured in relation to each other and in this way an approximate measure of the film thickness determined. The method of fitting these micrometers is shown in Fig. 2.

Operation of the Machine

28 The method of operation is as follows: The machine is started by closing the line switch (24) and bringing the motor up to speed by the controller (23). The load is then applied by the handwheel (14) which actuates the hydraulic loading device.

The application of the load is accomplished by pressing together the two halves of the test bearing (13) which are free to move in the supporting strap but are constrained to move vertically in relation to each other by hand-fitted dowel pins which run through both halves of the bearing supporting block.

29 As the pressure increases the oil film thins out until small points of contact occur. Before this point is reached the pendulum remains perfectly steady and the pointer (20) indicates a definite deflection which is a measure of the fluid friction; beyond this

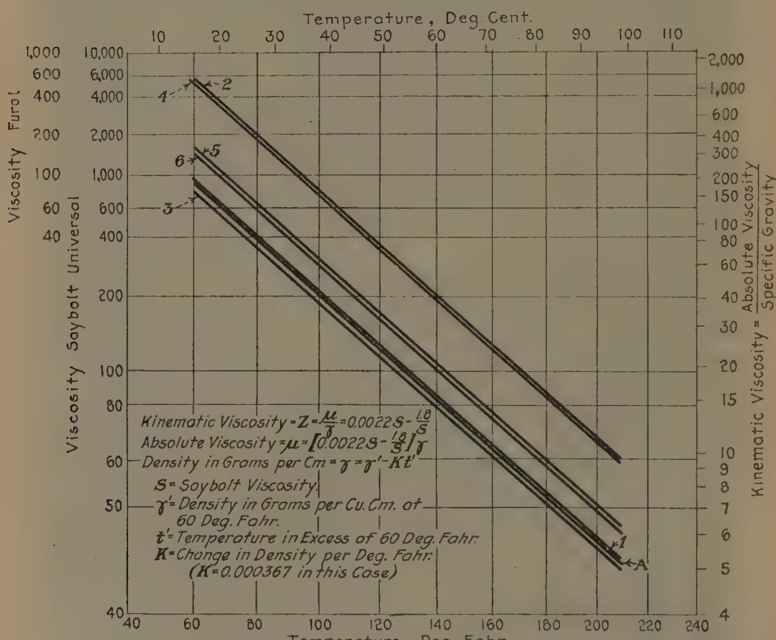

FIG. 3 SAYBOLT AND KINEMATIC VISCOSITIES OF OILS IN TABLE 1

point, however, the pendulum grows increasingly unsteady until the critical point is reached, and the bearing grips the journal. When this occurs the pendulum is suddenly deflected through a large arc until its weight is sufficient to cause the driving belt, which is loosely fitted, to slip. The operator then opens the cut-out switch (22) and shuts down the machine.

30 In getting the true breakdown pressure any irregular mechanical application of force is avoided by bringing the pressure up to a value just below the critical point and then allowing the increase of temperature to cause the viscosity to decrease until rupture occurs.

Oils for Test

31 The oils for the test were supplied through the courteous coöperation of Dr. Raymond Haskell of the Texas Company, and were procured direct from the refinery at Port Arthur, Texas, so there would be no chance of them becoming contaminated by the admixture of unknown ingredients.

32 The characteristics of these oils, as determined by the company's chemist, is given in Table 1. The viscosities and gravities were carefully checked in the laboratory at the university.

TABLE 1 CHARACTERISTICS OF OILS USED IN TESTS

	A	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
Gravity, deg. B.......	21.6	21.4	20.2	26.0	24.8	20.8	20.4
Specific gravity	0.9235	0.9247	0.9321	0.8974	0.9044	0.9284	0.9309
Flash point, deg. fahr..	340	330	385	375	410	355	550
Fire point, deg. fahr...	400	385	460	430	470	410	400
Saybolt viscosity at 100 deg. fahr.	202	205	763	182	760	308	290

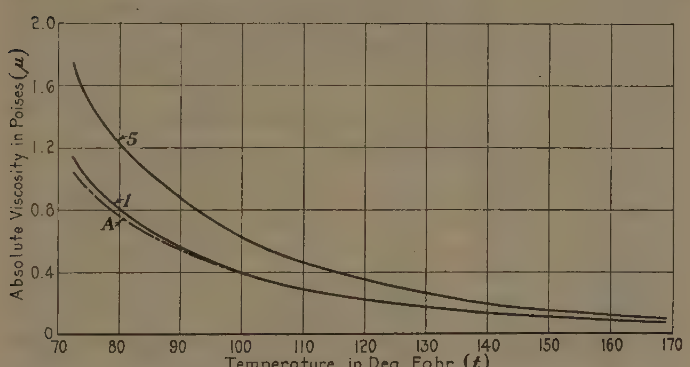

Fig. 4 Absolute Viscosities in Poises of Oils "A" and Nos. 1 and 5 Used in Final Runs

33 Oil "A" is a commercial red oil (200 Saybolt) and oils 1, 2, 5 and 6 are of Gulf Coast origin, while 3 and 4 are from Mid-Continental fields. All of these oils are straight mineral oils without compounding or blending.

34 The Saybolt and kinematic viscosities of all these oils covering the range of temperature used in the experiments are plotted in Fig. 3. The form of this chart was devised by Mr. MacCoull, of the Texas Company, and is very useful for the purpose. The absolute viscosities of oils "A," 1 and 5 which were used in the final runs, are plotted in Fig. 4.

35 Commercial oleic acid was used in the fatty-acid treatment of these oils and contained 91 per cent oleic acid, the rest being fixed oils, principally lard oil.

Preliminary Trials

36 In the preliminary trials a cast-iron-bearing having a length of four inches was used with oil "A" of Table 1. It was apparent immediately that the apparatus would function as intended, but just as soon as the bearing began to seat itself the pressure necessary to break the film became so great at high speeds that it taxed the capacity of the machine, so it was decided to reduce the bearing length to two inches.

Trials with Cast-Iron Bearing

37 The original bearing was modified by milling off the excess bearing surface to a length of two inches. It was first carefully

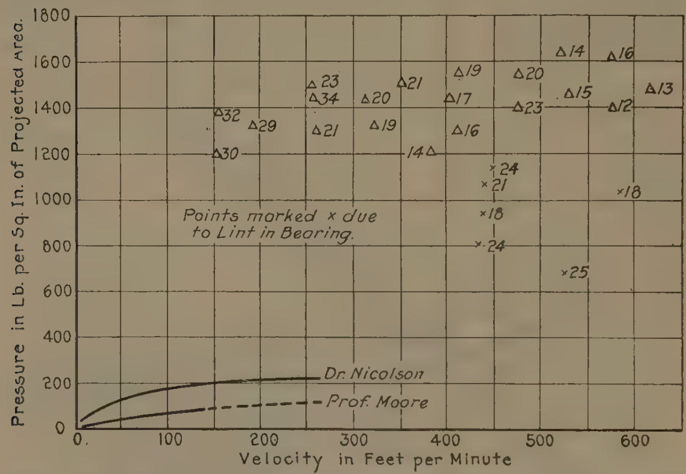

Fig. 5 Cast-Iron Bearing, 2 sq. in. Projected Area, Oil "A," S=202
(Points marked with crosses due to lint in bearing. Numerals refer to absolute viscosity in poises.)

fitted by hand then lapped in on an arbor with a lapping compound composed of 46 per cent silica, 40 per cent white lead, 7 per cent sodium carbonate, 4 per cent black lead and 3 per cent fullers' earth. This method produced a very fine bearing surface, but in addition to this the bearing was "run in" under some pressure before the actual test runs were started.

38 The final trials with the cast-iron bearing were also run with oil "A." The results are given in tabular form in Table 2 and graphically in Fig. 5. The numerals near the plotted points indicate the absolute viscosity of the oil in poises. On this chart is also given the curves determined from Dr. Nicolson's and Professor Moore's equations.

39 Three important points were brought to light in these trials, namely:

a That the pressure causing rupture of the oil film was much greater than that calculated either by Dr. Nicolson's [4] or Professor Moore's [5] equations

b That with each refitting of the bearing the pressure required to break the film became higher and even without a refitting, after the bearing seated itself better, the rupturing pressures grew appreciably higher

c That in spite of great care taken to remove it, the lint from the rags which were used in wiping out the drip pans, return trough and reservoir would collect in a solid wad at the entering edge of the bearing surface, partially interrupt the flow of oil, and cause in consequence very low and discordant breakdown points. This fault was remedied by fitting 50-mesh copper-wire strainers to the end of the oil supply tubes.

40 Although the results of these trials were surprising, no great importance was attached to the value of the breakdown pressures

TABLE 2 TEST WITH CAST-IRON BEARINGS USING OIL "A"

Speed, ft. per min.	Bearing pressure, lb. per sq. in. projected area	Absolute viscosity, poises	Speed, ft. per min.	Bearing pressure, lb. per sq. in. projected area	Absolute viscosity, poises
150	1200	0.30	430	800	0.24x
153	1380	0.32	435	940	0.18x
190	1320	0.29	440	1060	0.21x
255	1440	0.34	445	1140	0.24x
255	1500	0.23	475	1400	0.23
260	1300	0.21	475	1540	0.20
310	1440	0.20	520	1640	0.14
320	1320	0.19	525	680	0.25x
350	1500	0.21	530	1460	0.15
380	1200	0.14	575	1400	0.12
400	1440	0.17	575	1620	0.16
410	1300	0.16	585	1040	0.18x
410	1540	0.19	615	1480	0.13

Points marked "x" due to lint in bearing.

obtained, as it became apparent later that the real relationship was obscured by mechanical crudities of operation and by the lack of permanence in the fit of the bearing.

41 It was found after some runs at high speeds that the seizing was causing injury to the journal and it was decided to go no further with the cast-iron bearing but to install the babbitt bearing at once. The abrasion of the journal was very slight and it was successfully lapped out with the lapping compound previously mentioned. This lapping process reduced the diameter of the journal about four ten-thousandths of an inch. After this had been done, no unevenness could be detected along the bearing surface on the journal.

Trials with Babbitt Bearing

42 A special cast-iron sleeve was made, turned one-sixteenth inch large, recessed and undercut to hold the babbitt-metal liner, having a finished length of two inches. The babbitt was poured in the usual manner, peened, and then turned a little oversize. It was then fitted by hand and lapped in the same manner as the cast-iron bearing, but in addition was touched up finally with powdered Sapolio on the test journal. This method produced a very fine bearing surface which would become highly polished

TABLE 3 TEST WITH BABBITT BEARING USING OIL NO. 5

Speed, ft. per min.	Bearing pressure, lb. per sq. in. projected area	Temp. of bearing, deg. fahr.	Absolute viscosity, poises
Untreated			
155	3500	121	0.32
160	3750	120	0.33
180	3250	130	0.26
200	3100	148	0.17
205	3600	129	0.27
230	3150	154	0.15
250	3650	133	0.24
270	3850	130	0.26
285	3200	175	0.10
290	3450	154	0.15
310	4200	124	0.30
325	4050	129	0.27
325	3700	142	0.19
330	3300	175	0.19
330	3950	133	0.24
335	3500	160	0.13
340	4200	126	0.29
365	3750	145	0.18
370	3900	136	0.22
390	4200	127	0.28
395	4350	127	0.28
425	3750	151	0.16
435	3850	148	0.17
435	3650	157	0.14
445	4350	129	0.27
450	4050	138	0.21
400	3450	170	0.11
475	4100	129	0.27
495	3600	165	0.12
495	4050	145	0.18
500	4200	136	0.22
530	4400	132	0.25
530	3650	165	0.12
535	3950	145	0.18
550	3750	157	0.14
555	3900	154	0.15
565	3500	189	0.08
575	4150	139	0.20
575	4050	142	0.19
Treated			
200	2900	118	0.36
215	2450	121	0.32
260	3200	117	0.37
275	2900	139	0.20
335	2900	151	0.16
375	3200	145	0.18
400	2750	157	0.14
410	3100	151	0.16
460	3050	170	0.11
475	3600	138	0.21
490	3300	157	0.14
500	2800	165	0.12
535	2950	160	0.13
555	3250	157	0.14

when in use a short time, but it was found to be impossible to make this polished contact surface occupy the entire bearing surface available. The actual bearing area would grow larger with each refit and with wear, but even when the experimental work was stopped only about 80 per cent of the bearing surface had come into play. It is to this fact that the progressively higher pressures with each refit is attributed. Entirely concordant results could only be expected after an unchanging, highly polished bearing surface had been produced. It is thought that to produce such a surface, covering the entire available area, would take weeks, perhaps months of "running in" the bearing under considerable pressure.

43 The babbitt used was evidently of a very superior quality, as it was found that no abrasion of the bearing surface or the journal occurred, even under the most severe conditions of operation.

44 The preliminary trials with the babbitt bearing and oil "A" indicated immediately that the breakdown pressures would be much higher than those encountered with the cast-iron bearing. After the second refitting of the bearing the breakdown pressures became so high that the ⅜-in. steel tap bolts which held the strap together were repeatedly sheared, and in fact the strap itself was

TABLE 4 TEST WITH BABBITT BEARING USING OIL NO. 1

Speed, ft. per min.	Bearing pressure, lb. per sq. in. projected area	Temp. of bearing, deg. fahr.	Absolute viscosity, poises	Speed, ft. per min.	Bearing pressure, lb. per sq. in. projected area	Temp. of bearing, deg. fahr.	Absolute viscosity, poises
Untreated				Untreated			
150	3650	107	0.34	480	3700	144	0.14
155	3000	123	0.22	500	4550	114	0.28
155	2900	132	0.18	525	4250	123	0.22
170	3400	115	0.27	530	3850	140	0.15
195	3750	108	0.33	540	3750	135	0.17
195	3300	119	0.24	555	3600	152	0.12
240	3300	137	0.16	570	4300	120	0.23
245	3850	114	0.28	570	3950	132	0.18
265	4050	112	0.30	575	3500	162	0.10
280	3700	123	0.22				
305	3350	148	0.13				
310	3250	162	0.10	Treated			
315	4050	119	0.24				
315	3750	125	0.21	155	2250	104	0.37
325	3550	140	0.15	175	2800	117	0.26
350	4350	113	0.29	225	2750	105	0.36
360	4250	113	0.29	240	3150	104	0.37
365	3450	144	0.14	275	2600	130	0.19
380	4050	120	0.23	350	3050	137	0.16
390	4200	118	0.25	350	2750	132	0.18
395	4100	118	0.25	450	3250	125	0.21
400	3900	130	0.19	450	2750	152	0.12
400	3650	140	0.15	500	3250	130	0.19
420	3500	162	0.10	500	2900	162	0.10
460	4150	119	0.24	510	3650	118	0.25
470	4450	115	0.27	550	3400	127	0.20
470	3850	137	0.16	550	3100	157	0.11

distorted to such an extent that it looked as if it might carry away at any minute.

45 At this point it became obvious that as the bearing seated itself better the breakdown pressures would go quite beyond the strength of the strap, so it was decided to forge a new strap in a solid piece, thus eliminating the source of weakness in the tap screws. It was also decided to reduce the projected area of the bearing to 2.764 sq. in. so as to make sure the capacity of the machine would not be exceeded.

46 When these two modifications had been made the bearing was again refitted and lapped as previously described and then "run in" with oil "A."

47 A systematic series of runs were then begun, using in succession in the order named, oils "A," and Nos. 1, 2, 3, 5, 4 and 6.

Oils Nos. 2 and 4 were so viscous and the flow conditions with them so poor that the tests with them were abandoned. Repeated runs with the other oils confirmed the fact that the breakdown pressures were many times greater than those which had heretofore been regarded as probable. It also developed as the tests progressed, that in each succeeding run the breakdown pressure

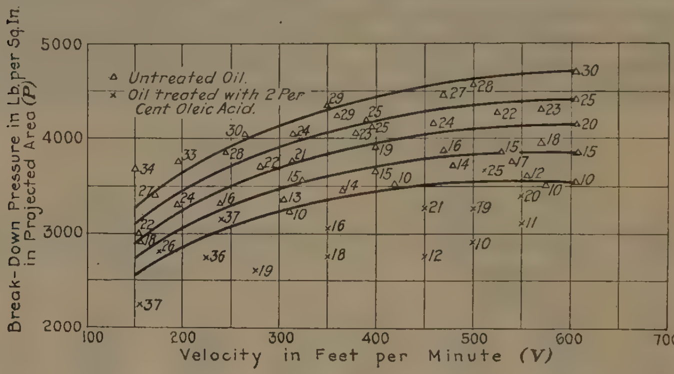

Fig. 6 Babbitt Bearing, 2.764 sq. in. Projected Area, Oil No. 5, S=308

(Numerals refer to absolute vicosity in poises. Circles refer to untreated oil Crosses refer to oil treated with 2 per cent oleic acid.)

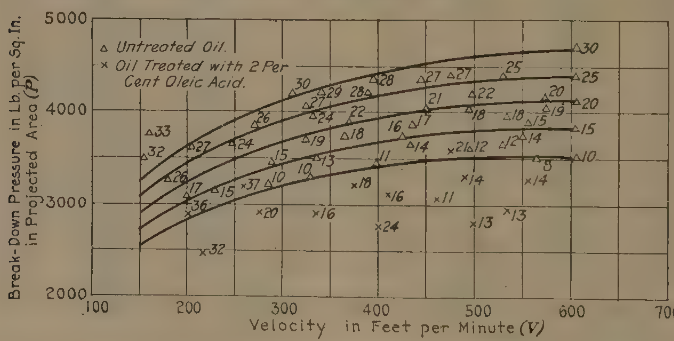

Fig. 7 Babbitt Bearing, 2.764 sq. in. Projected Area, Oil No. 1, S=205

(Numerals refer to absolute viscosity in poises. Circles refer to untreated oil. Crosses refer to oil treated with 2 per cent oleic acid.)

grew higher, confirming the conclusion that had already been reached that the fit of the bearing was the controlling factor influencing the results. In any period, however, fairly consistent results were obtained and one could predict quite closely at what pressure the rupture of the film would occur.

48 Unfortunately, when the bearing was reassembled after examination and a new trial started, the former relationship would be considerably disturbed.

49 No very close agreement with the former values could be attained, but as the work progressed it became more evident that as the fit improved the results would become more consistent.

Final Trials

50 After the trials had progressed to such an extent that the results grew consistent to a fair degree, it was decided to proceed with the final trials. Oils Nos. 1 and 5 were selected for the final trials as their viscosities covered the range ordinarily encountered in practice.

51 The procedure followed was to run a series of trials with the straight mineral oils until enough data had been determined to establish the curve, then shift over to the treated oils and repeat the process.

52 The results of these trials are given in Tables 3 and 4, and shown graphically in Figs. 6 and 7.

Comment on Curves

53 The curves drawn in Figs. 6 and 7 are of the same form and are not intended to be curves that represent the characteristic behavior of the breakdown pressure. They are drawn merely to aid the eye in following the trend of the data. It would be too much to say that the curves represent what really happens, because it is recognized that the machine was mechanically deficient, the degree of fit not perfect, and the temperature observed was probably not that which really obtained in the oil film itself. It is believed, however, that the data recorded does sensibly represent the magnitude of the breakdown pressures with that particular degree of fit. With another refitting of the bearing the pressures would certainly go still higher and the slope of the curve might have to be modified to meet the new condition.

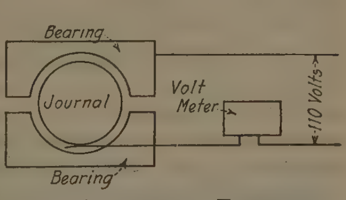

Fig. 8 Diagram of Electrical Method of Determining Breakdown Pressure

54 Contrary to expectation the break-down pressures obtained after the oils had been treated with the two per cent oleic acid were materially less than with the untreated oils. The breakdown point was not so sharply defined as with the straight mineral oils. The unstable period which indicated contact was much longer and in consequence the actual critical points were very erratic. This apparent discordance was no doubt due in a large measure to the fact that the real temperature of the film was unknown as in this case the contact friction caused the temperature to rise very rapidly and no doubt it was not properly registered by the method used.

55 The plotted data indicate that the breakdown pressure does depend largely on the viscosity, as an inspection of the curves shows that although there is no great degree of concordance, there is certainly a marked similarity between the two sets of data.

Electrical Method of Determining Breakdown Pressure

56 The electrical method of determining the breakdown pressure was tried during the trials with the cast-iron bearing. The arrangement was similar to that used by Professor Moore. It consisted simply in connecting the bearing and journal in a circuit as shown in Fig. 8.

57 The theory of this arrangement is that as long as the film is maintained, the voltage, indicated on the voltmeter, will be zero, but when the film is ruptured metallic contact occurs and the voltmeter will indicate full-line voltage.

58 In the first attempts with this method the apparent breakdown pressures were so far below those indicated by the mechanical method, that it was thought that perhaps metallic particles were being stripped from the bearing and carried along in the oil. The attempt was discontinued at that time but was tried again on the final trials.

59 These second attempts were not very much more successful than the first, the apparent breakdown occurring at a few hundred pounds when it was obvious from the mechanical indicator that such was not the case. No serious attempt was made to get exact results by this method as the load could not be determined accurately at the low pressures involved, and also because study on the subject indicates that this method of attack is fallacious for the following reasons:

a When the critical condition is approached the film thickness is very small, so small in fact that its dielectric strength is very uncertain; and further, this uncertainty of dielectric strength is not a matter of the oil thickness alone. Mr. J. L. R. Hayden and Mr. W. N. Eddy[1] conclude that "The disruptive breakdown of oil under dielectric stress is not due to the voltage exceeding the dielectric strength of oil, as is the case in air, but is due to something being carried into the dielectric field, or being produced in the dielectric field, which weakens the dielectric strength so as to cause a premature breakdown."

b Metallic particles which are always present in the oil and which are being constantly circulated in the system would be fatal to this method if they were carried into the bearing. These particles, although invisible to the unaided eye, are still large enough to bridge the film and cause current to flow when the oil-film thickness is small.

[1] Journal, American Institute of Electrical Engineers, vol. 41, February, 1922, p. 138.

c It was demonstrated repeatedly in the trials that when the breakdown pressure was approached, contact would occur at points and would evince itself by an unsteadiness of the pendulum. These periods were of short duration, without exception, and the conclusion is that they were caused by small excrescences of the bearing surface that would be smoothed off after a few moments. These false points would often occur when the pressure had reached only a small fraction of the real breakdown pressure.

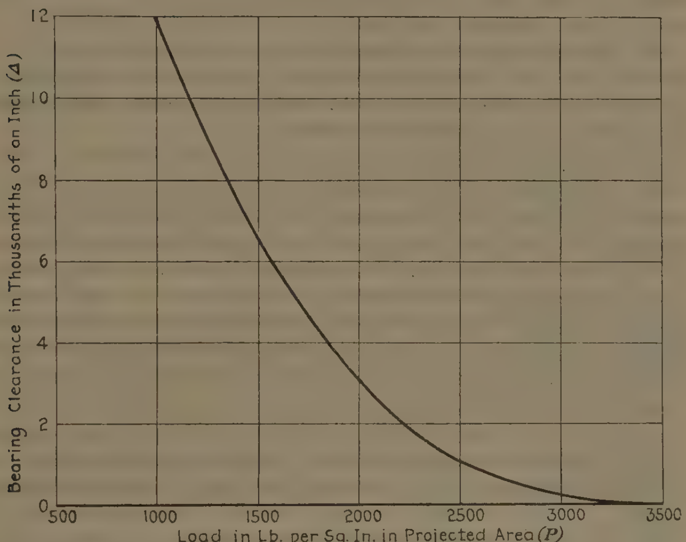

FIG. 9 BEARING CLEARANCE, OIL NO. 1, AVERAGE OF ALL OBSERVATIONS

For these reasons, it is evident that if the electrical method were depended on it would give entirely erroneous results.

DETERMINATION OF FILM THICKNESS

60 During the final trial a careful determination was made of the relative position of the two halves of the bearing by means of the micrometers. The average of all observations made while using oil No. 1 are shown in Fig. 9.

61 In view of the fact that the clearance in the bearing was maintained entirely by the pressure of the oil film, it is thought that the values determined are a fair approximation of the oil-film thickness. The method did not admit of closely measuring the movements of the bearing blocks near the critical point and consequently the magnitude of the oil film in this region could not be determined with great exactitude.

Conclusions

62 The most important fact brought out in this work is the astonishing intensity of pressure that the oil film will stand before being displaced. Even under the most adverse conditions of operation in the experimental work, it is seen that the actual breakdown pressures are several hundred times the values which have hitherto been accepted as possible. No indication of this has been recorded in the work of previous experiments with cylindrical bearings, so it is thought that this point can be claimed as an original contribution to the knowledge on the subject.

63 The relationship between breakdown pressure, speed, and viscosity was not definitely established, but enough was accomplished in this direction to indicate that there is such a relationship and to develop its general character.

64 The actual breakdown pressures at which the film is completely dislodged and the bearing seizes the journal should not be considered as the critical pressures but rather the point at which metallic contact first makes itself manifest by the unstableness of the pendulum. This point should mark the limit, at least as far as economic loading of the bearing is concerned. It was deemed impractical if not impossible to determine this point exactly with the present apparatus, because the mechanism recording deflections of the pendulum was not delicate enough to indicate these first contacts with any degree of accuracy, so in order to get an unmistakable reference point the actual seizing pressure had to be the criterion. It should be noted, however, that the pressures at which unstableness was first noted were always near the seizing points.

65 The results with the oils treated with oleic acid indicate that the breakdown pressures with the oils so treated are less than with straight mineral oils. The experimental results in this direction were meager and it would not be safe to claim that this is invariably true without further confirmation. The influence from this evidence cannot be avoided, that oils having the same absolute viscosity but differing in chemical consistency do not have the same resistance to rupture and hence, from this point of view, differ in lubricating value.

66 In the original design of the experimental apparatus the necessary devices were developed for determining the essential elements for the complete exposition of the hydrodynamical theory of lubrication and with suitable modifications and refinements in a redesign of the apparatus every premise in the theory could be checked. In order that these modifications and refinements can be definitely fixed, they will be dealt with in detail in the following discussion.

Extension of the Experimental Work and Redesign of the Test Apparatus

67 Lack of time prevented carrying the investigation to an entirely satisfactory conclusion and, in fact, it is believed that further work would prove abortive with the apparatus as it stands, due to its limitations. It is thought, however, that these could be entirely overcome by a redesign.

68 The inherent defects of the machine will be pointed out so that if it is considered desirable to continue this line of attack, these could be corrected when the machine is rebuilt.

a The entire structure is too flimsy. This is particularly true of the structural elements of the pendulum. In the original design a maximum pressure of 1500 lb. was contemplated but as a matter of fact, in the actual runs, pressures of 12,000 lb. were not unusual.

b The speed control was poor and it was found impossible to operate satisfactorily at speeds lower than 300 r.p.m., as the power of the motor was so feeble at these slow speeds that small increases in the friction was enough to make a large change in the speed and in consequence observations taken at these low speeds were unreliable. This could be readily overcome by installing a motor of ample power fitted with the proper kind of speed control. A wide range of speed could be obtained by step pulleys such as are fitted to lathes, and very low speeds could be realized by back gears. An hydraulic friction cone or some other similar variable-speed control could be substituted for the back gears.

c The self-aligning feature of the bearing should be carefully considered, and made in a manner to insure the proper functioning of this feature under all conditions of load.

d A more accurate value of the actual temperatures existing in the oil film should be obtained. This could be accomplished by imbedding several thermocouples or resistance thermometers in the actual bearing surface.

e The bearing block should be cored and arranged with a water-cooling service, so that the temperature of the bearing could be controlled. This could be accomplished by using connections of flexible metallic tubing, as the effort of the pendulum is so great that any friction involved would not affect its displacement.

f Another desirable feature would be a quick-releasing device for the loading mechanism so that the load could be quickly removed. This could be done by tapping into the space under the piston and fitting the jack with a small auxiliary piston that could be worked with the fingers very quickly to release the pressure, and such a device would serve also for making fine load adjustments.

g A cut-out switch could be provided so that when the pendulum is displaced beyond a certain limiting angle the machine will automatically shut itself down.

h A multiplying device to indicate small changes or unstableness in the deflection of the pendulum should be installed so that the pressure at which the film first begins to rupture can be detected with accuracy.

i In order to determine the effect of change of diameter, and length of bearing surface, a series of test journals could be made in the form of sleeves that would slip on the spindle and arranged so that they could be secured to a collar on the spindle by means of tap screws. A number of test bearings of various diameters and lengths could also be provided so that the diameter and length of the journal and bearing could be varied at will.

j With the apparatus modified as described, a systematic method of testing could be followed, establishing definitely the various factors necessary to the complete solution of the hydrodynamical theory, viz.: (1) Diameter and length of journal and bearing, (2) rubbing velocity, (3) intensity of pressure on unit area, (4) temperature and pressure of film and consequently the absolute viscosity of the film and (5) the film thickness.

Acknowledgment

69 This investigation was undertaken under the direction of Professor Alexander C. Christie, to whom I am greatly indebted, not only for many valuable suggestions regarding the experimental work, but also for his advice, instruction, and inspiration during the whole period of my graduate work.

70 I am also grateful to Associate-Professor Myrick W. Pullen, who contributed many ideas and who was every ready to assist in any way possible. Mr. Carl E. Cummings rendered very essential assistance in taking data, perfecting the mechanical and electrical devices and tabulating results.

71 Mr. Samuel Styers and Mr. Frank Skrivan, assisted by Mr. Duncan Bryant, made very skilful use of the miscellaneous material available at the university and produced, under my direction, the experimental apparatus described.

72 Dr. Robert B. Roulston, Associate-Professor Thompson and Messrs. Medaugh and Comber, under whom I pursued very interesting and valuable courses, were especially considerate, and this acknowledgment would not be complete without recording my appreciation of the interest shown by the entire faculty.

DISCUSSION

H. F. Moore.[1] The author has referred to the work of the writer at Cornell University in 1903, and has shown by his experiments that it was possible for an oil film to carry much higher

[1] Research Professor of Engineering Materials, Univ. of Ill., Urbana. Ill. Mem. A.S.M.E.

loads without rupture than had been shown by the results of the Cornell experiments. The Cornell tests, however, covered a very narrow range, and the formula deduced was expressly stated to be tentative, in the absence of any other information available, but it gave results very markedly on the safe side of the breakdown pressures.

While on the faculty of the University of Wisconsin, the writer had further tests made, under his direction, on the breakdown strength of oil films, using the electric contact method to detect breakdown. The results of these tests have never been published. The bearings and the journals used were finished with a much greater degree of care than in the Cornell tests, and the breakdown loads in the Wisconsin tests were about three times those found in the Cornell tests. It may be noted that in the tests at Cornell University the bearing was a one-side bearing extending half-way round the journal, while in the Wisconsin tests load was applied to bearings on both sides of the journal, as was the case in the author's experiments.

In the opinion of the writer, the breakdown strength of the oil film is very markedly affected by surface conditions of journal and bearing. The author's explanation that electric contact between the bearing and journal under pressure indicates not the actual breakdown of the film but the thinning of the film to a point where small metal particles cause current to flow, as they come to the section of nearest approach between bearing and journal, seems a reasonable one. On the other hand, the location of the breakdown of the film at the point where well-marked " seizing " occurs would seem to give a value of pressure higher than that which could be safely used in practice. The writer believes that the breakdown of the oil film is a gradual affair. It may be roughly analogous to the phenomena observed in a tension test of steel. The occurrence of electric contact may correspond to the minute inelastic action which, by the use of very delicate apparatus, can be detected under low stress in the tension test of steel. Under higher loads, breakdown of the film occurs over appreciable areas of the bearing, but this breakdown is not complete, and the bearing still functions fairly well. This stage in a very rough way corresponds with the yield point of the steel in the tension tests. Under still higher pressure, the oil film breaks down completely and " seizing " occurs. This, in our rough analogy, corresponds to the ultimate tensile strength in the test of the steel.

Following this analogy a little further, the writer believes that an attempt to locate the very first point of breakdown of film over a minute area will lead to results similar to those found in an attempt to locate the absolute elastic limit in metals; i.e., the value determined will depend largely on the sensitiveness of the apparatus used. It may be worth while to develop some standard

way of locating the breakdown point, even if the value thus determined is somewhat arbitrary.

J. M. Lessells.[1] The pendulum feature of the author's testing machine, which insures no displacement of the center of gravity with change in load, is, in the writer's opinion, the only true method of designing bearing-testing apparatus. This method is similar to that of Martens. The defect in the machine used by the author lies in the fact that pressure is applied to the top and bottom halves of the bearing simultaneously. This never occurs in practice. The machine designed by P. L. Irwin in the Westinghouse research laboratory overcomes this defect while still retaining the pendulum feature. A second point of criticism of the apparatus is the small size of the bearing used, namely, 2 in. diam. Since such data are to be applied to large bearings, it is unfortunate that this experimental bearing was not at least $3\frac{1}{2}$ to 4 in. in diameter.

With regard to the results obtained, it is interesting to note that the values of the critical bearing pressures are much higher than was previously believed possible. It must be observed, however, that our previous knowledge of this phenomenon was based largely on the results obtained by Professor Moore. These were obtained on an even smaller bearing, namely, 1.289 in. in diameter and $1\frac{1}{2}$ in. long. The results obtained by the author seem to invalidate those obtained by Moore. Probably these latter results were influenced to a great degree by metallic particles breaking off and being carried round by the oil, thereby establishing electrical contact. This belief is strengthened when it is known that Moore did not seem to make any provision for filtering the oil, but used the same oil continuously. Moreover, the fact that no perceptible rise in bearing temperature was observed after the film was broken down, may support this argument. On the other hand, in justice to this previous investigator, it should be noted that the maximum pressure observed was 85 lb. per sq. in. at a velocity of 140 ft. per min. These conditions are far removed from those obtained in the present paper, and the interpolation shown in Fig. 5, therefore, does not appear justifiable.

With regard to Nicolson's equation, this was based on Sommerfeld's work. The relation $P = 40(dN)^{1/4}$ was deduced by him on the assumption that the bearing was running cool and without wear, and was, moreover, limited to speeds of 450 ft. per min. Since wear does actually occur in service, and since bearings do not always run cool, we can conclude that Nicolson's results are conservative. The value of the constant on the right-hand side of the above equation will be subject to modification, depending on the type of bearing employed.

[1] Engr., Research Laboratory, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa. Mem. A.S.M.E.

M. D. Hersey.[1] The author refers to Sommerfeld's formula for permissible pressure and to the difficulty of applying that formula, Equation [3], on account of the difficulty of measuring the oil-film thickness. Sommerfeld's formula is not concerned with the actual film thickness, but only with the clearance between journal and bearing, and the bearing is considered to be rigid. Thus the formula does not apply to the particular kind of bearing used in this experiment; and for rigid bearings all that has to be measured is the clearance, or difference in radii.

Professor Moore in 1915 published a statement that he afterward found values considerably higher than the Cornell values of 1903.[2] The writer in 1909 also made a study of the electrical method, deriving this idea from Professor Moore's publication of 1903. The work was carried out on a full bearing, whereas Moore in 1903 had used partial bearings. It was found impossible to get a short-circuit or film rupture with the full bearing. Loads as high as the author had were not available, but the results are in agreement with his so far as they went. The higher the velocities, the heavier the loads that could be carried with a given film thickness. Further details may be found in a recent paper.[3]

One final criticism may be made with the reservation that, in any event, the author's work is a most important step forward in the study of permissible bearing pressures. This criticism is that the bearing is not rigid; it has two halves that are separately pushed against the journal. That device may be valuable in comparative tests, and was used by Kingsbury in his experiments at Worcester and is also familiar in the Thurston machine. Does the author consider it possible to predict results for rigid bearings from the tests made on this bearing with its two independent parts? The difficulty seems to be important. In an ordinary rigid bearing the unloaded half which has the thicker oil film, develops a still thicker film with increase of load; whereas in the author's experiment the unloaded or lower half develops a thinner film with increase of load. The same is true with varying speed. In a rigid bearing the film gets thinner on the unloaded half as the speed increases, whereas in this experiment the film gets thicker on the less loaded half with increase of speed.

On the question of end leakage, a paper worthy of study may be found in the *Monatsblätter,* Berlin Bezirksverein deutscher Ingenieure for May and June, 1914, pp. 87-104, 109-120. The author was the late Professor Gümbel of the Technical School at Charlottenburg. His experiments and calculations give some evidence on the effect of end leakage in relation to Sommerfeld's theory.

[1] Physicist, U. S. Bureau of Mines, Pittsburgh, Pa. Mem. A.S.M.E.
[2] Trans. A.S.M.E. vol. 37 (1915), p. 199.
[3] Jour. Am. Soc. Naval Engrs., vol. 35 (1923), p. 652.

A. E. Flowers.[1] A factor that should be further emphasized is the immense importance of the fit of journal and bearing. The running-in of bearings to obtain a fine fit is well known. The paper emphasizes the value of fine machine work to produce an initially good fit without waiting for it to be produced by operation. The paper also emphasizes the importance of selecting such materials for bearing and journal as will flow into a perfect concentric pair of cylinders. The fit should be obtained by flow of metal, not by wear.

The author states that his results show a difference due to the viscosities of the oil and in each one of the two curves in Figs. 6 and 7 this fact seems to be shown for that particular oil. However, a comparison of values in Fig. 6 with those in Fig. 7, which are for two different oils, shows that the values are exactly the same for the two oils. This leads at once to the conclusion that the differences found, which were obtained by bringing up the temperature, may have been due to another effect caused by temperature rather than to change in viscosity. The inference from this is that a change of surface tension rather than a change of viscosity caused the change in critical load.

Attention should be called to the title of the paper. Some confusion has been caused by the use of the term "critical bearing pressure." This has previously been applied to the condition of minimum friction. As the friction coefficient is plotted against speed, it goes from a large value at or near the stationary condition down to a minimum, and then rises again. Heretofore this minimum value has been considered the critical bearing pressure. Critical bearing pressure is not used in this sense in the paper, but in the sense of what is usually called "seizing pressure." This fact should be considered in contrasting the results reported by the author with those reported previously by Professors Moore and Nicolson. The earlier results applied to the old or minimum pressure as the critical pressure rather than to these conditions of seizure. The seizing conditions were determined while running and by continually increasing the temperature until seizure occurred. They do not apply to starting such a bearing from standstill. The bearing might have seized at a given pressure when starting and not seized when running. The paper does not imply that these pressures can be used in a machine that must start from standstill.

The effect of the viscosity deserves attention. Higher viscosity is not necessarily more suitable than low viscosity, but under certain conditions of high pressure and low speed, higher-viscosity oil may be necessary. If the pressures are higher, a higher-viscosity oil is needed in order to get the best working conditions. In connection with the relation of seizing pressures and viscosity

[1] Engr. in Charge of Development, The DeLaval Separator Co., Poughkeepsie, N. Y. Mem. A.S.M.E.

Sommerfeld showed that the starting static friction was always the same, irrespective of the viscosity of oil assumed to be used in the complete bearing, the coefficient being of the order of 18 per cent. The minimum friction coefficients again were always the same, approximately 1 per cent or less, but occurring at different and higher values for the pressure for the higher viscosities. Carrying this same line of argument to the other end of the scale, where the pressures have gone to the extreme maximum, it is reasonable to expect the viscosity effect to again drop out at that end just as it does at the starting end.

C. H. Bierbaum.[1] The author has failed to give the coefficients of friction immediately preceding his "breakdown" pressures. It would have been highly desirable to have had this information included.

The paper shows the "breakdown" pressures to be over 4000 lb. per sq. in., while the composition of the babbitt used — 90 tin, 3 copper and 7 antimony — is such that it begins to flow at from 1000 to 1500 lb. per sq. in. Consequently these tests were actually made under conditions of flowing babbitt. The journal was of Sanderson's tool steel, a pure carbon steel hardened, which is the hardest and highest possible microscopically homogeneous steel formation; therefore, it was absolutely impossible for the babbitt to seize or cut the journal. The point the author was actually determining was where partial oil lubrication failed and babbitt lubrication began.

It is evident, though the results are not given, that the amount of surface wear occurring in the author's tests was so great that no machine designer would dare to use these values as permissible pressures.

H. A. S. Howarth.[2] The author has given more than usual attention to the accuracy and finish of the journal and bearing surfaces. Their importance is paramount for such tests as he has described.

When studying bearing test data it is essential to know the geometrical form and relation of the surfaces. For this purpose cylindrical journal bearings may be divided into two main classes, *full* and *partial*. A full bearing, by its very nature, must have a greater radius of curvature than the journal it surrounds. A partial bearing whose angular length of fit is less than 180 deg. can either fit the journal perfectly or have a greater radius of curvature.

[1] Vice-Pres. and Cons. Engr., Lumen Bearing Co., Buffalo, N. Y. Mem. A.S.M.E.

[2] Gen. Mgr., Ch. Engr., Kingsbury Mch. Wks., Philadelphia, Pa. Mem. A.S.M.E.

Bearings and journals with different radii of curvature have been covered mathematically and graphically by various analysts, but, so far as the writer is aware, no analysis of *fitted partial bearings* has ever been published. Osborne Reynolds has given us the fundamental equation from which to start this work. In his study of Tower's experiments, however, he assumed a difference of curvature in the surfaces.

Fitted partial bearings can be studied by the graphical method used by the writer in Part II of his paper entitled A Graphical Study of Journal Lubrication.[1] It appears, however, from an unpublished preliminary mathematical study by S. J. Needs that this mathematical study would be simpler than the graphical.

It is obvious that for extremely thin lubricating films, such as produced by high pressure, light oil, and low rotative speed, the partial bearing (<180 deg.) should fit its journal as closely as possible. The closer this fit is made and the more accurately cylindrical the surfaces are first formed and then maintained during test, the more heavily the bearing can be loaded before metallic contact of the surfaces will take place.

If both surfaces are perfectly cylindrical and practically inelastic and without difference of curvature, the bearing arc being under 180 deg., then perfect fluid lubrication could be maintained, without seizure, up to a pressure whose limit would be the strength of the materials and of the lubricant. These pressures would be far in excess of those found by the author.

The load that a bearing can be expected to carry, in such tests as the author has described, depends, therefore, on factors under control of the *designer*, the *builder*, and the *operator* of the testing machine. The results obtained show that these three factors have been intelligently and carefully handled by the author.

He estimated, from the appearance of his bearing after its removal from the machine, that 80 per cent of its surface had been in contact with the journal. It is quite possible that this is an excessive figure, because the surfaces were fitted in the first place by lapping them together. It is practically impossible to lap a partial bearing to an exact fit with its journal. The lapping compound will act as a lubricant and produce in the partial bearing a slightly greater curvature. A slight elasticity in the yoke above the bearing might help to counteract this difference, but it would also permit the lubricating film to shape itself to suit its environment. The gradual increase in area of apparent contact, which the author reports, is probably caused by the slight wear that took place when the lubricating film broke down at the end of each test run.

The matter therefore comes down again to the practical question, "How well should bearing surfaces be finished in practical

[1] See page 809.

applications?" We now *know* the conditions under which bearings can be made to carry loads greatly in excess of usual practice. Designers may therefore be expected to apply bearings more intelligently, making use of high pressures or low, depending upon the other requirements of their problems.

In the author's work the tests were stopped when the friction was such as to lift the pendulum beyond a certain limit. If the tests had been continued beyond that point some additional interesting information would have been obtained. Would the bearing have seized or burned out? The ability of a bearing to dissipate heat is one of the factors determining the amount of pressure it will sustain. Hence bearings that are water-cooled close to the babbitt can carry heavier loads at high speed than those that have less effective provision for cooling. Some of the oil applied to a bearing works out at the ends, and some passes all the way through the film. There is a limit to this quantity which requires its constant renewal or cooling in high-speed bearings. When no provision is made for removing heat, the bearing will not stand as great a pressure as it otherwise would.

Some one asked why the author used two partial bearings pressing against the journal from opposite sides, this condition not being encountered in practice. In the design employed no serious stresses are set up within the journal. If the journal was loaded on one side only the possible applied load would be limited by the deflection of the journal and the stresses within it.

Closure to Discussion.[1] Professor Moore's deduction that the breakdown of the film is gradual was corroborated by observation during the author's tests. The pendulum was observed to become unstable with a load less than the seizing load, indicating a partial rupture of the film at this point. Further application of load caused increased unsteadiness of the pendulum, indicating an increasing metallic contact, until the point of seizure was reached. The apparatus was not sensitive enough to accurately determine the initial point of failure of the film, the true "critical pressure," but it was estimated that this occurred at a load proportionate to the seizing load in all tests. This is not to be confused with an instability of the pendulum, occurring occasionally at much lower loads, after which the pendulum would become stable at its frictional deflection for that load. A partial metallic contact due to an isolated high spot in the bearing surface undoubtedly caused this action, and as the metal was worn away, the condition of perfect film lubrication returned.

The final instability observed might result either from the thinning of the film to a point where a comparatively large portion

[1] Prepared by Mr. C. E. Cummings, Engineer, The Texas Company, Long Island City, N. Y., who presented the paper for Commander Linsley.

of the bearing was in metallic contact, or the "yield point" of the lubricant was reached, as suggested by Professor Moore, or both. Since a perfect bearing surface would be necessary to eliminate the factor of a gradually increasing area of metallic contact, it is doubtful whether the establishment of an elastic theory for oil could be accomplished by an apparatus of this type.

Attention is again called to the fact that the viscosities shown in Fig. 6 and Fig. 7 are in absolute units, computed from the initial viscosity of the oil and the temperature of the film just preceding rupture. That the pressures in Fig. 6 and Fig. 7 are the same is attributed to the fact that the absolute viscosities of the oil are the same for given journal velocities, due to the greater temperature rise of the oil with the greater initial Saybolt viscosity. This leads to the conclusion that the breakdown pressure is dependent on journal velocity and the absolute viscosity of the lubricant at the time of seizure. That the surface tension of the lubricant can play any part in this relation is hard to conceive.

It is unfortunate that the time and facilities available during the author's experimental work did not permit the extension of the investigation to include data on larger bearings and rigid bearings as has been suggested in the discussion. The data submitted in the paper are not proposed to be the exact answer for all classes and sizes of bearings. It is believed, however, that the oil-film formation in the rigid bearing is sufficiently similar to that in the partial type used here, to give values of breakdown pressure of similar magnitude. Until a standard of bearing fit can be established for any given set of experiments, no exact comparison can be made between breakdown pressures for partial and rigid bearings, or for the effect of bearing sizes. The influence of the fit of the bearing upon breakdown pressure has been repeatedly demonstrated by experiment and its importance cannot be overemphasized.

Though the author's data are not set forth for practical application without further investigation, they should prove valuable in the design of apparatus for subsequent experiments, and, it is hoped, prove a stimulus for much-needed research in the subject.

No. 1939

THE EFFECT OF INACCURACY OF SPACING ON THE STRENGTH OF GEAR TEETH

By Lloyd J. Franklin [1] and Charles H. Smith [2]
Junior Members of the Society

In a paper presented before the Society in 1912, *Professor Guido H. Marx reported results of an extended series of tests to determine the strength of gear teeth at pitch velocities from* 0 *to* 500 *ft. per min. During the discussion of this paper it was suggested that further tests be made in order to obtain definite data as to the effect of inaccuracy of spacing on the strength of the teeth at high speeds. At the instance of Professor Marx the authors undertook such a series of tests, the results obtained and a description of the apparatus and procedure employed being given in the present paper.*

Among other things the authors found that, in a broad way, at pitch velocities of 1000 *ft. per min. and upward, gears whose inaccuracies of spacing do not exceed* 0.001 *in. will carry twice the load of those having inaccuracies of spacing of* 0.006 *in.; and that the strength of gears having inaccuracies of spacing of the order of* 0.002 *in. is about half-way between the two. An error of* 0.006 *in. in the size of teeth tested is much more than will ordinarily be found in first-class commercial cut gears.*

PREVIOUS to the year 1911 there had been very little experimental investigation of the strength of gear teeth — at least if such tests were made there are no records of them available.

2 During 1911 and 1912, Prof. Guido H. Marx of Stanford University, Cal., performed quite extensive tests dealing with the strength of gear teeth. The report of these tests [3] shows the results obtained by him in testing gears for the strength of the teeth at pitch-circle velocities from 0 to 500 ft. per min. These tests

[1] Draftsman, San Bernardino Ice and Precooling Plant, San Bernardino, Cal.
[2] Instructor in Mechanical Engineering, Stanford University, Cal.
[3] Trans. A.S.M.E., vol. 34, 1912, p. 1323.

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were supplemented[1] by Professor Marx assisted by Prof. L. E. Cutter in 1914, at which time they conducted tests to determine the strength of gear teeth at pitch velocities as high as 2000 ft. per min.

3 In the discussion following the first of these papers, R. E. Flanders says: "It is also important to know how much the accuracy of the cutting affects the strength of the gears at high speed. All grades of accuracy are used in commercial work. To investigate this matter it might be possible to try two or three sets of gears; one made with the cutter set central, the next with the cutter off center 0.002 in. and the others with the cutter set off center 0.003 or 0.004 in. The chances are that an investigation of this kind would show that a high premium is put on accuracy of cutting from the standpoint of strength. If this is so it should be

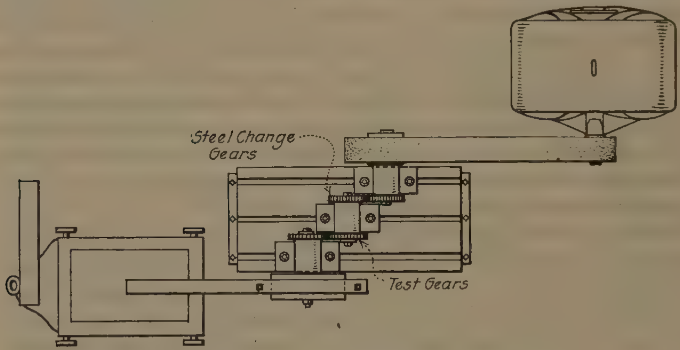


Fig. 1 Plan of Gear-Testing Apparatus

definitely known, though it is not practicable to include the factor in a formula."

4 At the suggestion of Professor Marx the authors decided to undertake a series of tests which would, it was hoped, give some definite data as to the effect of inaccuracy of spacing on the strength of gear teeth. Accordingly, the apparatus used by Professors Marx and Cutter in 1914 was restored to working order and set up as shown in Figs. 1 and 2. As several changes had been made in the mounting of the motor needed, it was necessary to devise a somewhat different arrangement of the apparatus from that used by the authors of the previous paper. The brake had to be reconstructed to operate in the direction reverse to that for which it had been originally designed, and other alterations of the set-up were found necessary. These changes may be seen readily by comparing the plan view of Fig. 1 with that of the apparatus used by Professor Marx in his first tests.

[1] Trans. A.S.M.E., vol. 37, 1915, p. 503.

Description of the Apparatus

5 The motor, a 50-hp., three-phase, induction type, capable of carrying a heavy overload momentarily, was connected by a Morse silent chain to a shaft on which were mounted a large sprocket and one of a pair of steel change gears on opposite sides of a pedestal bearing. The second of these two 8-pitch steel change gears was keyed to an intermediate shaft on which was also mounted the driving gear of the test pair. The remaining shaft, mounted in a similar manner, held the driven test gear and the brake wheel. The brake used was the same design as shown by Professors

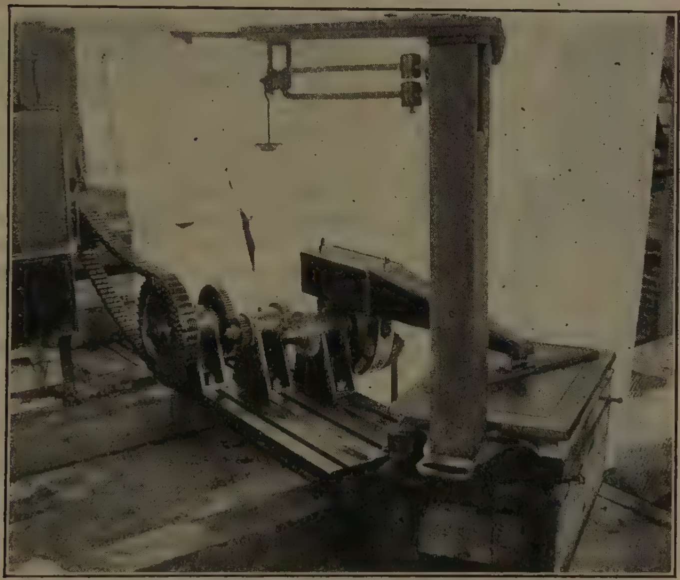


Fig. 2 Gear-Testing Apparatus

Marx and Cutter in the report of their tests. A platform scale supported the brake arm by means of a plate and knife edge, the latter being mounted on the arm. This arrangement is shown clearly in Fig. 2. When the apparatus was in operation, guards, not shown in Fig. 2, were placed over the gears. The change gears were lubricated by dipping in a bath of heavy steam-cylinder oil, and the chain and test gears were lubricated by a mixture of graphite, grease, and oil applied at the start of each run.

Test Procedure

6 The tests were all made in the laboratory at Leland Stanford Junior University. For each run the motor was started at zero load and the brake gradually tightened until the gears ruptured. A

tachometer was observed at each increment of load, the final speed and brake load being recorded at the conclusion of each test. A calibration of the scales showed them to be correct throughout the range of the tests, and the corrections to the tachometer were made by calibrating the instrument. Increments of load of 5 lb. were used until the gears showed signs of labor, when the increment was reduced to 2, and finally to 1 lb.

Types of Gears Tested

7 The gears tested were all 10-diametral-pitch, 60-tooth, cast-iron, $14\frac{1}{2}$-degree involute type furnished by Pratt & Whitney Company especially for the tests. The width of face of the gears was $1\frac{1}{16}$ in. and the bore $1\frac{5}{16}$ in. A $\frac{5}{16}$-in. key was used to secure the gear to the shaft. Extra long hubs were used on all the gears, as it was found by an examination of the test previously made by Professors Marx and Cutter that several of their tests had been disregarded due to the keyways failing. A drawing of the gears used in the tests is given in Fig. 3.

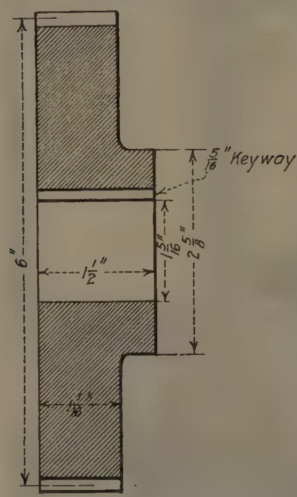


Fig. 3 Dimensions of 10-Pitch, $14\frac{1}{2}$-Deg. Involute Cast-Iron Cut Gear Used in Tests

8 The gears were divided into three classes according to the accuracy of the spacing. In one class the ordinary milled gears were cut with a Brown & Sharpe milling cutter; another class was composed of what were called "standard Maag gears"; and the third class was made up of a series of "mismated" gears purposely constructed with inaccuracy of spacing. The error of the spacing of these three classes is about as follows:

Maag gears, from 0.0005 to 0.001 in.
Ordinary milled gears, 0.002 in. ±
Mismated Maag gears, 0.006 in. ±

9 These errors may be seen on the Saurer gear charts that were furnished with the gears. In the charts a departure of 0.030 in. from a perfect circle represents an error of approximately 0.001 in. Three of the actual charts are shown on a reduced scale in Fig. 4. Upon conclusion of the tests an effort was made to determine whether individual tooth errors as shown by the diagrams were responsible for rupture of the gears. It was shown clearly by marking several of these charts that this could not be

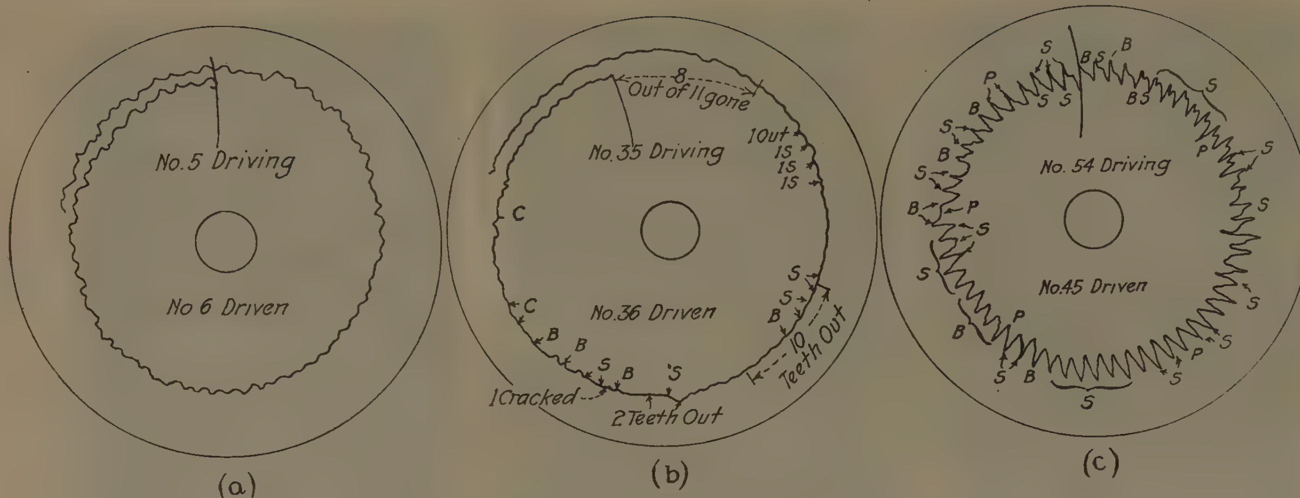


FIG. 4 SAUER GEAR CHARTS (REDUCED IN SIZE)

(*a* Gears 5 and 6, B. & S. milled, used in test A 3; *b* Gears 35 and 36, Maag, used in test C 10 and C 11; *c* Gears 54 and 45, "Mismated," used in test C 7.)

depended upon, for at times what appeared as a good portion of the gear on the chart would be stripped, and vice versa. This may have been due to slightly dirty gears when the charts were made, or to improper manipulation of the machine. On the marked charts the letter *S* means tooth sheared; *B*, broken out; *P*, partially broken; and *C*, cracked. Notation outside of the diagram is for the driver, notation inside the diagram for the driven gear.

Description of the Tests Performed

10 The tests were conducted in three series. The first series was made at a pitch-line velocity of approximately 1000 ft. per min.; the second at about 1500 ft. per min.; and the third close to 2000 ft. per min. Three runs were made with each type of gear in each series in order that a check might be made with the conditions of loading and testing as closely identical as possible. The gears were set up accurately so that the correct teeth as shown by the Saurer diagrams were in mesh with one another, and the adjustments for fit, alignment, and backlash were made very carefully in all cases. Tables 1, 2, and 3 contain the data observed during these tests.

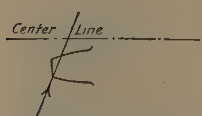


Fig. 5 Diagram Showing Method of Making Static Test

11 Static tests of the gears were made with the teeth in the weakest position as shown in Fig. 5, in order that experimentally complete curves might be drawn. These tests were made by using a single-toothed steel gear in mesh with the cast gears. Care was taken in testing to see that the load came on the tooth at the point shown, but due to the spring in the apparatus there is some doubt as to the accuracy of some of the results. This spring was noticed by Professors Marx and Cutter in their tests.[1] It is fairly safe to assume a value of 3200 lb. as the breaking load for the ordinary and "mismated" milled gears. The Maag gears showed a noticeably higher breaking strength in the static tests, the average being 3300 lb.

12 At the conclusion of the tests of the gear teeth, test specimens $\frac{1}{4}$ by $1\frac{1}{16}$ by 4 in. were cut from all gears in order that a check on the material might be made. For the purpose of comparison all values of breaking loads were reduced to correspond to a flexural strength of 56,320 lb. per sq. in. — found as the average strength of all specimens tested.

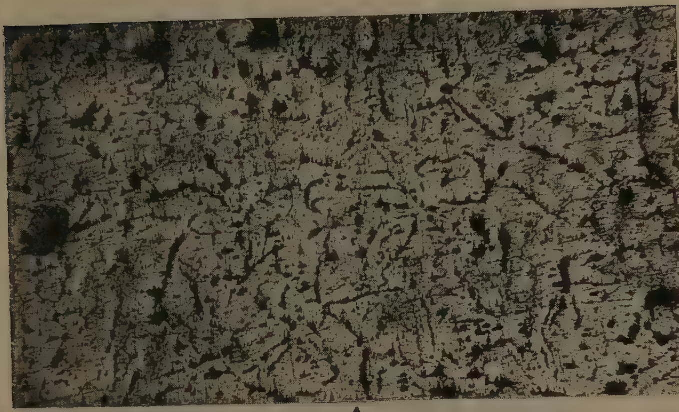
13 In order to determine definitely the material and structure of the metal used in the various types of gears, samples of each type were analyzed in the University metallurgy laboratory by Mr. Samuel E. Vaughan, a graduate student interested in mate-

[1] Loc. cit., Appendix No. 1, notes.

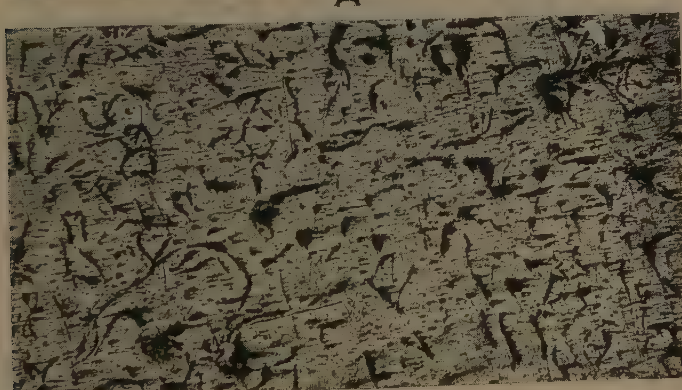
Fig. 6 Photomicrographs of Cast Iron Used in Gears Tested

Dark areas are graphite plates imbedded in pearlite matrix

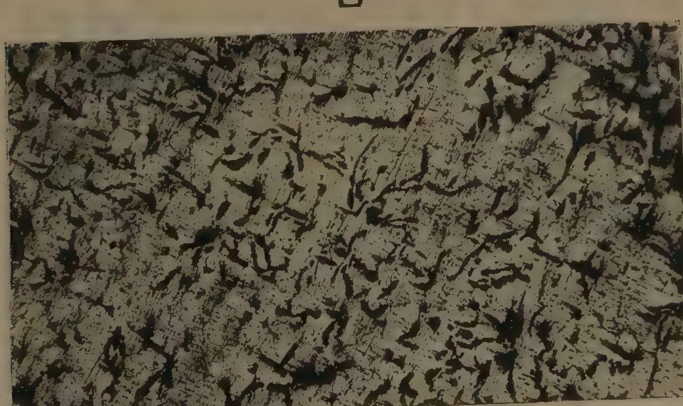
A Specimen No. 17 × 50 B. & S. Milled Gear — *B* Specimen No. 29 × 50 Maag Gear — *C* Specimen No. 49 × 50 Mismated Gear



A



B



C

(See caption at bottom of preceding page.)

TABLE 1 TEST OF 10-DIAMETRAL-PITCH MAAG 15-DEG. INVOLUTE CAST-IRON GEARS

(Motor sprocket, 19 teeth; change-gear sprocket, 37 teeth; brake arm, 30 in.; zero brake load = 3 lb.)

Test No.	Date, 1924	Steel gears 8-P., B. & S. No. of teeth		Cast gears. Gear number		Gross load, lb.	Motor r.p.m., observed, tachometer	Notes Made at Conclusion of Each Test
		Driver	Driven	Driver	Driven			
A 4	Feb. 20	40	30	19	20	182	890	Gears showed no sign of breaking. At this point the apparatus began to vibrate. An examination showed the steel pinion badly abraded and the teeth bent at base. Used 80 tooth and 60 tooth to continue at this speed.
A 8	Feb. 27	80	60	21	22	245	860	7½ teeth broken out of No. 21. 10 teeth and 2 pieces (about ¾ tooth) broken out of No. 22.
A 9	Feb. 27	80	60	23	24	230	875	17 teeth sheared out; 1 broken out; 1 cracked in No. 23. 15 teeth broken out; 2 cracked in No. 24.
A 10	Feb. 28	80	60	19	20	208	875	Repetition of Test No. A 4. 4 teeth sheared out; 14 teeth partially broken; 2 teeth cracked in No. 19. 8 teeth sheared out; 2 broken out; 2 cracked in No. 20.
B 7	Mar. 3	80	40	25	26	223	800-830	Motor stalled; fuse blown. Run to be repeated with heavier fuses.
B 8	Mar. 3	80	40	25	26	230	860	15 teeth sheared out; 4 broken out; 4 partially broken; 6 cracked in No. 25. 12 teeth sheared out; 2 partially broken; 2 broken out; 2 cracked in No. 26.
B 9	Mar. 4	80	40	27	28	239	860	Driver No. 27, 8 teeth sheared out; 5 broken out; 1 half broken. Driven No. 28, 10 teeth sheared out; 1 tooth almost completely broken out. Both gears noticeably abraded.
B 10	Mar. 5	80	40	29	30	230	860	Brake wore out and load could not be increased further. Shows gears are stronger than this.
B 11	Mar. 5	80	40	29	30	235	860	Repetition of Test No. B 10. Driver No. 29, 6 teeth sheared out; 3 broken out; 3 partially broken; 3 cracked. Driven No. 30, 5 teeth sheared out; 7 broken out; 1 tooth slivered; 1 cracked. Brake badly worn again.
C 8	Mar. 12	80	30	31	32	165	855	Driver No. 31, 25 teeth broken out; 28 partially broken. Driven No. 32, 11 teeth broken out; 9 partly broken; 2 cracked. 30-tooth pinion is nickel steel, 80-tooth change gear is 18-point carbon.
C 9	Mar. 12	80	30	33	34	175	850	Driver No. 33, 22 teeth broken out; 22 partially broken. Driven No. 34, 7 teeth broken out; 7 partially broken; 1 tooth cracked.
C 10	Mar. 13	80	30	35	36	197	840	Brake failed at this point. Impossible to increase load. Large steel gear badly abraded.
C 11	Mar. 13	80	30	35	36	200	830	Repetition of Test No. C 10. Driver No. 35, 18 teeth broken out; 5 partially broken; 3 cracked. Driven No. 36, 10 teeth broken out; 3 partially broken; 4 cracked. Large steel gear practically useless. Doubtful if it would make another run.

TABLE 2 TEST OF 10-DIAMETRAL-PITCH BROWN & SHARPE $14\frac{1}{2}$-DEG. INVOLUTE CAST-IRON GEARS

(Motor sprocket, 19 teeth; change-gear sprocket, 37 teeth; brake arm = 30 in.; zero brake load = 3 lb.)

Test No.	Date, 1924	Steel gears 8-P., B. & S. No. of teeth		Cast gears, Gear number		Gross brake load, lb.	Motor r.p.m., observed, tachometer	Notes Made at Conclusion of Each Test
		Driver	Driven	Driver	Driven			
A 1	Feb. 15	40	30	1	2	174	895	A decided knock was noticed at a load of 164 lb., but the load was increased until the gears ruptured. Evidently one tooth out. 12 teeth cleanly sheared from driver. 6 teeth cleanly sheared from driven gear. Gears slightly abraded.
A 2	Feb. 16	40	30	3	4	141	900	Gears failed at load shown. There was no initial knock in this case. Driver No. 3, 1 tooth cleanly sheared; 1 tooth $\frac{1}{8}$ broken out. Driven No. 4, 9 teeth cleanly sheared; 1 tooth $\frac{3}{4}$ broken; 1 tooth feathered at face.
A 3	Feb. 20	40	30	5	6	140	900	Gears failed without a knock. Driver No. 5, 13 teeth cleanly sheared; 2 teeth $\frac{3}{4}$ broken out; 4 teeth wholly broken out; 11 teeth cracked. Driven No. 6, 6 teeth cleanly sheared; 2 teeth broken; 1 tooth $\frac{3}{4}$ broken.
B 1	Feb. 28	80	40	7	8	155	880	Driver No. 7, 15 teeth sheared; 20 teeth broken, 4 of which were half taken out, the rest broken off; 1 tooth cracked. Driven No. 8, 4 teeth sheared; 1 tooth $\frac{3}{4}$ broken; 1 tooth broken out.
B 2	Feb. 28	80	40	9	10	172	875	Driver No. 9, 13 teeth sheared; 19 partially broken; 10 broken out. Driven No. 10, 2 teeth sheared; 2 broken out; 5 partially broken out.
B 3	Feb. 28	80	40	11	12	184	867	Driver No. 11, 17 teeth sheared; 7 partially broken; 2 cracked. Driven No. 12, 8 teeth sheared; 2 partially broken.
C 1	Mar. 7	80	30	13	14	155	865	Brake started flaming. Shut down to cool off. Shows gears are stronger than this.
C 2	Mar. 7	80	30	13	14	135	870	Repetition of Test No. C 1. Gears evidently weakened by previous run. Driver No. 13, 7 teeth sheared out; 12 partially broken; 1 tooth broken; 1 cracked. Driven No. 14, 20 teeth sheared; 5 broken out; 2 partially broken; 3 cracked.
C 3	Mar. 7	80	30	15	16	142	860	Driver No. 15, 14 teeth sheared out; 2 slivered; 1 tooth broken out; 1 cracked. Driven No. 16, 23 teeth sheared out; 6 broken out; 8 partially broken; 10 cracked; 3 slivered at top.
C 4	Mar. 11	80	30	17	18	135	870	Driver No. 17, 23 teeth sheared out; 10 completely broken out; 13 partially broken. Driven No. 18, 6 teeth broken out; 2 half broken out; 17 teeth sheared.

TABLE 3 TEST OF 10-DIAMETRAL-PITCH MAAG (MISMATED) 15- AND $18\frac{1}{2}$-DEG. INVOLUTE CAST-IRON GEARS

(Motor sprocket, 19 teeth; change-gear sprocket, 37 teeth; brake arm = 30 in.; zero brake load = 3 lb.)

Test No.	Date, 1924	Steel gears 8-P., B. & S. No. of teeth		Cast gears, Gear number		Gross brake load, lb.	Motor r.p.m., observed, tachometer	Notes Made at Conclusion of Each Test
		Driver	Driven	Driver	Driven			
A 5	Feb. 26	80	60	46	37	134	905	Driver No. 46, 16 teeth cleanly sheared. Driven No. 37, 15 teeth cleanly sheared; 1 tooth $\frac{3}{4}$ broken; 1 tooth cracked; 1 tooth broken off on driving edge.
A 6	Feb. 26	80	60	47	38	127	905	Driver No. 47, 1 tooth cleanly sheared. Driven No. 38, 13 teeth cleanly sheared; 1 tooth cracked.
A 7	Feb. 27	80	60	48	39	120	900	Load was up to 128 lb., then dropped to 115 lb., and in building up again, ruptured as shown. Brake seized. Driver No. 48, 1 tooth cleanly sheared; 1 tooth $\frac{1}{4}$ broken out. Driven No. 39, 9 teeth cleanly sheared; 4 teeth cracked; 5 teeth broken out.
B 4	Feb. 29	80	40	49	40	75	910	Driver No. 49, 1 tooth sheared. Examination of tooth shows a slight defect in tooth at root. Driven No. 40, 5 teeth sheared.
B 5	Feb. 29	80	40	50	41	102	895	Driver No. 50, 6 teeth cleanly sheared. Driven No. 41, 33 teeth cleanly sheared; 12 teeth cracked at root.
B 6	Feb. 29	80	40	51	42	102	895	Driver No. 51, 8 teeth sheared; 4 teeth partially broken. Driven No. 42, 20 teeth sheared; 3 teeth partially broken; 1 tooth cracked.
C 5	Mar. 12	80	30	52	43	94	890	Driver No. 52, 11 teeth sheared out; 2 partially broken; 1 tooth cracked. Driven No. 43, 27 teeth sheared out; 13 broken out; 7 partially broken out; 1 tooth cracked.
C 6	Mar. 12	80	30	53	44	45	905	Gears both show indications that some foreign substance dropped into them, causing the rupture. Driver No. 53, 6 teeth sheared out; 4 teeth partially broken out. Driven No. 44, 9 teeth sheared out; 10 teeth broken out; 11 teeth partially broken out.
C 7	Mar. 12	80	30	54	45	78	895	Driver No. 54, 6 teeth sheared out; 1 tooth broken out; 3 teeth partially broken out. Driven No. 45, 34 teeth sheared out; 11 teeth broken out; 3 teeth partially broken.

rials. Chemical analyses and photomicrographs of the gears were made and included in his report, a copy of which follows:

SIRS.—I beg to present the following report with reference to certain specimens of graphitic iron delivered to me for microscopic and chemical analysis:

1 The samples received were labeled 10, 17, 29, 30, 45, 49. The specimens for microscopic examination were in the form of blocks about 1″ x ½″ x ¼″, evidently taken from fracture test bars. Samples for chemical analysis were in the form of drillings.

2 The results of the examination would indicate that all six specimens were of the same material. The physical and chemical compositions were as nearly alike as may be expected from this class of material, even when poured from the same heat of metal.

3 Microscopic examinations show the material to be iron of the class commonly known as semi-steel. Small plates of graphite are well distributed in a matrix of pearlite. Some free ferrite is visible at the boundaries of the original austenite grains. There is no cementite present in the massive form. The material is exceptionally free from inclusions. Microphotographs are appended.[1]

4 All six specimens showed a uniform Brinell hardness number of 229. A 10-mm. ball was used with a load of 3000 kg.

5 The chemical analyses are as follows:

Specimen No.	10	17	29	30	45	49
Total carbon	3.05	3.20	2.99	3.00	3.03	2.98
Graphitic carbon	2.21	2.26	2.32	2.31	2.22	2.17
Combined carbon	0.84	0.94	0.67	0.69	0.81	0.71
Silicon	1.77	1.65	1.84	1.75	1.78	1.64
Manganese	0.41	0.49	0.48	0.51	0.43	0.44
Sulphur	0.112	0.118	0.110	0.103	0.118	0.096
Phosphorus	0.502	0.491	0.552	0.478	0.536	0.517

Respectfully submitted,
(Signed) S. E. VAUGHAN.

Stanford University, Cal.
May 22, 1924.

14 From this report it can be seen that the metal (really semi-steel) in the different types of gears tested is as uniform as could be expected for cast iron, both as to structure and chemical constituents.

EXPLANATION OF CURVES

15 The graphical results of the tests taken from Table 4 are shown in the curves of Figs. 7 to 11, inclusive. Figs. 7, 8, and 9 show a comparison between the actual breaking loads and the breaking loads as corrected for variability of material by reducing the modulus of flexure for each set of gears to the value of 56,300 lb. per sq. in. found as an average for the entire set of gears.

16 An examination of these curves shows that the reduction of these values to a uniform modulus greatly improves their smoothness, as of course such a correction should.

17 In Fig. 8 there will be noticed a dash-line dip in curve 1. This dip passes through the mean of the three points of breaking for the 1000-ft.-per-min. run. However, it is thought that the points located by testing gears 3-4 and 5-6 are too low. This was due to faulty operation of the brake at the beginning of the tests because of failure to lubricate it with oil as was done in all succeeding runs to prevent burning and seizing, and to produce smoother operation.

18 Fig. 10 shows the actual test curves drawn for a comparison of the strengths of the different types of gears at the various speeds.

[1] Three of these are shown in Fig. 6.

It will be noticed that smooth curves have been drawn as near the mean of observed results as possible in all cases. From these curves it is obvious that the Maag gears have a much greater strength than either the ordinary milled gears or those erroneously

TABLE 4 RESULTS OF TESTS AND REDUCTION OF BREAKING LOAD TO UNIFORM FLEXURAL STRENGTH

Test No. and series	Actual flexural strength, average of 2 gears	Actual breaking load at pitch line, lb.	Actual velocity at pitch line, ft. per min.	Equivalent breaking load with flexural strength of 56,300 lb. per sq. in.	Remarks
			MAAG GEARS		
A 4	52454	1790*	946.42	....	See A 10, gears not broken.
A 8	57352	2420	915.23	2375	
A 9	53802	2270	930.24	2380	
A 10	52454	2050	930.24	2205	
B 7	60926	2200*	?	....	See B 8, motor stalled.
B 8	60926	2270	1373.9	2100	
B 9	58912	2360	1373.9	2255	
B 10	57868	2270*	1373.9	....	See B 11, brake failed to hold.
B 11	57868	2320	1373.9	2260	
C 8	61322	1620	1819.7	1488	
C 9	51542	1720	1809.0	1880	
C 10	58655	1940*	1787.5	....	See C 11, brake failed to hold.
C 11	58655	1970	1766.0	1894	
			B. & S. MILLED GEARS		
A 1	58054	1710	949.64	1660	
A 2	59710	1380	955.12	1300	
A 3	58869	1370	955.12	1312	
B 1	54105	1520	1403.5	1582	
B 2	65366	1690	1395.5	1458	
B 3	61940	1810	1384.2	1648	
C 1	51369	1520*	1841.3	1655	See C 2, brake began to flame.
C 2	51369	1320	1849.9	1446	
C 3	59172	1390	1830.5	1322	
C 4	47446	1320	1849.9	1570	
			"MISMATED" MAAG GEARS		
A 5	56053	1310	960.4	1316	
A 6	60494	1240	960.4	1155	
A 7	56570	1170	955.12	1165	
B 4	55914	720	1448.3	726	
B 5	52244	990	1424.5	1068	
B 6	48661	990	1424.5	1145	
C 5	58293	910	1892.9	878	
C 6	52325	420	1920.1	452	
C 7	51343	750	1899.3	824	

* Run does not count.

milled. This would tend to show that the more accurately a gear is milled the more power it will be able to transmit for any given speed. The Maag gears were noticeably less noisy than the erroneously milled ones, and also somewhat quieter than the ordinary B. & S. milled gears.

19 Fig. 11 shows a comparison of the curves for the three types of gears all reduced to the common modulus of rupture. From

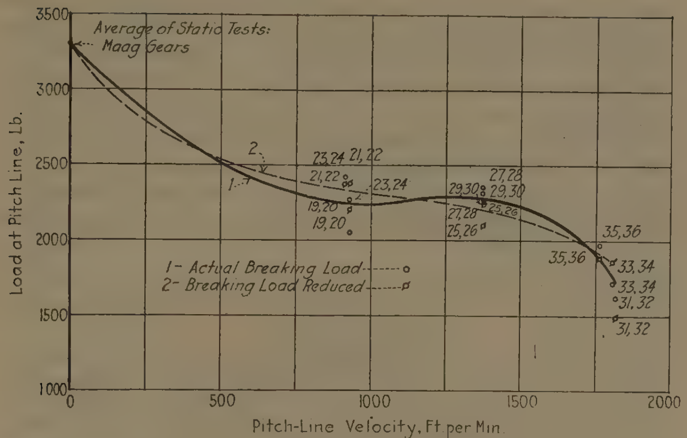


FIG. 7 CURVES FOR MAAG GEARS, SHOWING RELATION TO PITCH-LINE VELOCITY OF ACTUAL BREAKING LOAD (1) AND BREAKING LOAD. REDUCED TO UNIFORM MODULUS OF 56,300 LB. PER SQ. IN. (2)

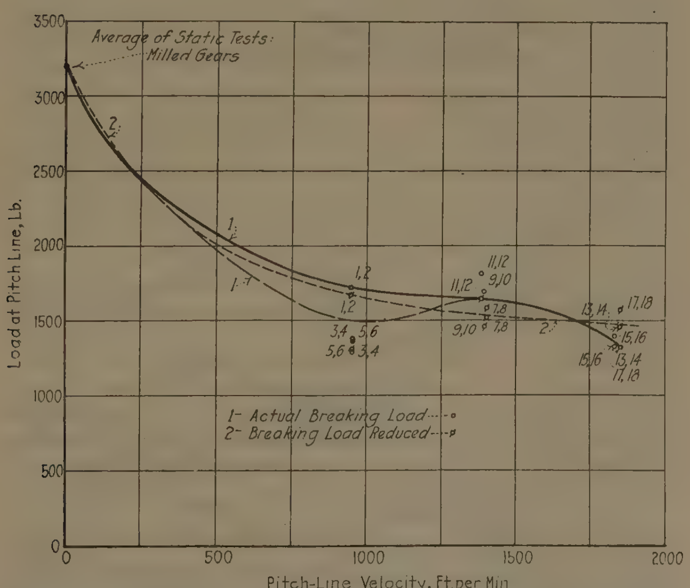


FIG. 8 CURVES FOR B. & S. MILLED GEARS, SHOWING RELATION TO PITCH-LINE VELOCITY OF ACTUAL BREAKING LOAD (1) AND BREAKING LOAD REDUCED TO UNIFORM MODULUS OF 56,300 LB. PER SQ. IN. (2)

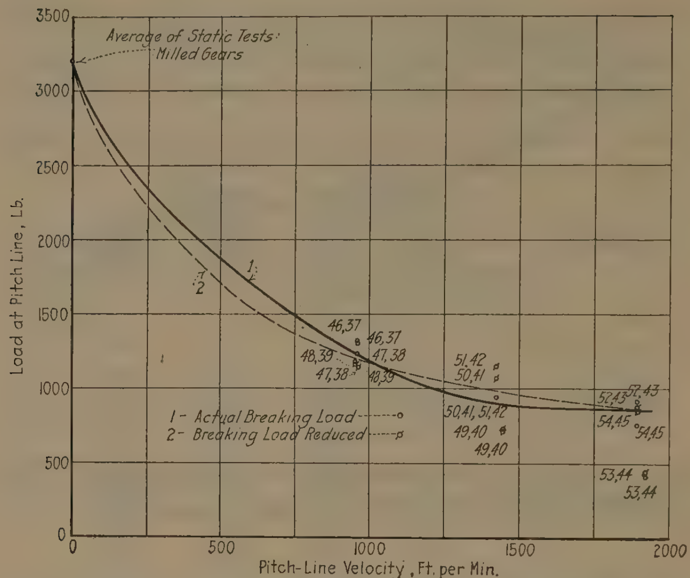


Fig. 9 Curves for Mismated Gears, Showing Relation to Pitch-Line Velocity of Actual Breaking Load (1) and Breaking Load Reduced to Uniform Modulus of 56,300 lb. per sq. in. (2)

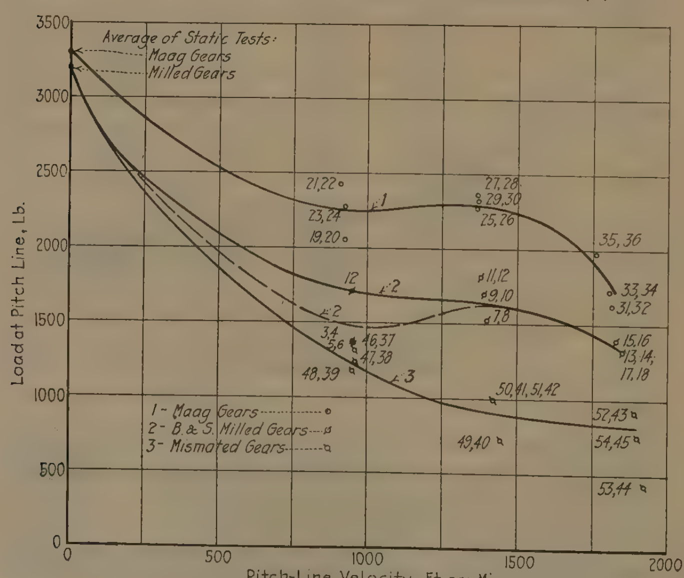


Fig. 10 Curves Showing Relation of Actual Breaking Load and Pitch-Line Velocity

(1) Maag gears, (2) B. & S. milled gears, (3) Mismated gears

these curves it may be seen that the results obtained check very closely with the results obtained by Professors Marx and Cutter in their tests of 1915. That is, all curves are of the same general shape, and follow each other as closely as experimentally obtained data could be expected to check. This is especially noteworthy because of the great difference in flexural strength of the gears used in the two investigations (40,909 as against 56,320). It will be seen that the static loads of the Maag gears and the two types of milled gears do not check. It was not thought advisable to take

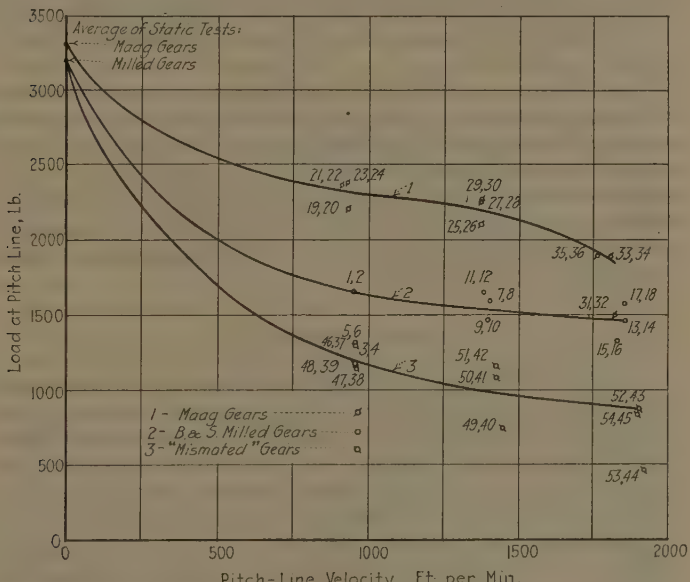


FIG. 11 CURVES SHOWING RELATION TO PITCH-LINE VELOCITY OF BREAKING LOAD REDUCED TO STANDARD MODULUS OF 56,300 LB. PER SQ. IN.

(1) Maag gears, (2) B. & S. milled gears, (3) Mismated gears

the average of the three types in this case as the milled gears were very close together and the Maag gears higher, as shown.

CONCLUSIONS

20 These tests show that inaccuracy of spacing of gear teeth materially affects their carrying capacity, particularly at speeds of 1000 ft. per min. and upward. Broadly, it may be said that at such speeds gears whose inaccuracies of spacing do not exceed 0.001 in. will carry twice the load of those having inaccuracies of 0.006 in.; and that the strength of gears having inaccuracies of

the order of 0.002 in. is just about half-way between the two. An error of 0.006 in. on this size of tooth was the extreme that the makers were willing to provide, and is much more than will ordinarily be found in first-class commercial cut gears.

21 It is to be noted that although the strength ratio of the most accurately to the most inaccurately cut gears is two to one (at a speed of 2000 ft. per min.), this is by no means as great a difference as some writers had expected to be shown; and it is particularly to be noted that the falling off of strength does not follow the law Mr. Wilfred Lewis[1] thought he had deduced from Lasche's discussion,[2] and which led him to the obvious error that the allowable load becomes zero at speeds of 2600 or 2700 ft. per min.

22 It is to be regretted that more gears were not available to test, as further checks on the tests would of course give greater accuracy. It would also be very interesting to continue the investigation above the speed attained in the present tests, but a larger motor and a more effective brake would be necessary to continue as the apparatus used was taxed to its capacity in performing the final tests: namely, those of Maag gears at a speed approaching 2000 ft. per min.

Acknowledgment

23 The authors wish to extend their thanks to the professors and assistants who so kindly gave advice and aid by the loan of apparatus for the tests. Especial thanks are due Professors Marx and Cutter for time so freely given in conferences and for the use of apparatus formerly used by them in similar tests, and to Professor Moser for the use of motor and space in his laboratory for the performance of the tests. All gears used in the tests were kindly donated by the Pratt & Whitney Company, to whom the authors are greatly indebted.

DISCUSSION

G. H. Marx.[3] The authors of this paper have done a very conscientious and valuable piece of work, adding positively to our knowledge of the subject. The criticism may be raised that they have not specifically differentiated the dynamic factors entering into the problem; but it is the writer's judgment that, for the present at least, such a differentiation is impracticable. After all, what the designer and user of gears wish to know is their integrated performance, and to this the present paper offers a positive and

[1] *Mechanical Engineering*, vol. 44, no. 12, p. 813.
[2] *Zeit. des Vereines deutscher Ingenieure*, vol. 43.
[3] Prof. Mch. Design, Stanford University, Cal. Mem. A.S.M.E.

adequate answer. It corroborates the results found in similar tests by Professor Cutter and the writer.[1]

Only those who have carried out similar work know the difficulties encountered, the amount of painstaking labor and patience necessary to complete such an investigation as this, and the appreciation due those who advance the boundaries of the positively known by direct experimentation.

Luther D. Burlingame.[2] A review of the paper raises a number of questions to which a study of it does not appear to give answer. These questions are such that without an answer the value of the deduction made by the authors is left in doubt.

While the name " Brown & Sharpe " is made prominent in the comparative tests, it is apparent that the gears tested were not made by the Brown & Sharpe Mfg. Co., but rather by another who may have used a cutter made by the Brown & Sharpe Mfg. Co., as the paper states in Par. 8 that " in one class the ordinary milled gears were cut with a Brown & Sharpe milling cutter," which may mean it was made by Brown & Sharpe Mfg. Co., presumably a No. 2 cutter from 55 to 135 teeth, or that although made by some other manufacturer it was so called because it was made according to a system known as the " Brown & Sharpe Gear System " for the reason that it was introduced by Joseph R. Brown in connection with his invention of the formed cutter which could be sharpened on its face without changing its form, and thus took as its name that of the company of which he was the senior partner.

The question might be raised, however, why gears made by two different methods are compared, especially when the gears cut by one method have an error of plus or minus 0.002 in., or a total error of 0.004 in., and those that were cut by another system with the spacing so much more accurate. The first thought would be that if gears are going to be inaccurately spaced and then tested, they should at least be all made by the same method so that the same shape of tooth would be used throughout.

While the authors are to be commended for the care with which their tests were conducted after the receipt of the test gears, a doubt is left in the mind of one desiring to secure data of value from these tests as to whether the results might not have been quite different if, for example, the " milled gears " had been furnished to the same degree of accuracy as the others.

There is no evidence in the paper as to whether the outside diameter or depth of tooth is the same in both sets of gears. The difference here would in itself have a marked influence on results. Neither is there evidence as to whether the gear cutter used for

[1] See Trans. A.S.M.E. Vol. 37 (1915), p. 503.

[2] Industrial Supt., and Patent Expert, Brown & Sharpe Mfg. Co., Providence, R. I. Mem. A.S.M.E.

the " ordinary milled gears " was of the correct form for the number of teeth in each gear, or was a stock cutter made to cover a range. It may be assumed that the authors of the paper did not make it a part of their investigation to check up on these details, which, however, are important, not only as regards strength, but also as affecting quiet running.

It would seem that in order to make a test for strength, giving a comparison between accurate and inaccurate spacing of gear teeth, the test should be made between sets of gears of the same type and using the same method of manufacture. Instead, the authors have taken one type of gear made by one method as a sample for inaccurate spacing, and gears of another type as made by another method for accurate spacing.

It would seem that such interesting tests as those that have been made should be continued by comparing gears of the same type and using the same method of manufacture for both accurate and inaccurate spacing. Deductions could then be drawn as to the effect of inaccuracy of spacing which would accord with the title of the paper.

G. M. Eaton.[1] The authors make little or no reference to certain fundamentals of vital importance in connection with the transient forces set up by errors of pitch spacing. These omissions entirely invalidate the broad conclusions drawn by them.

The most important fundamentals are the inertia and rigidity of connection of the associated moving parts. On the input side is a comparatively massive rotor. This, however, is connected, through a Morse chain, which is quite flexible tangentially under the high frequency and small amplitude of the existing disturbing forces. The change gears, shafts, and test gears have inertia characteristics which, though small, are an appreciable percentage of the totals involved.

On the output side is a brake drum. The output load is composed of a large frictional component, which is quite constant at any given set-up (except perhaps at the failure point), and a small, but quite rigidly connected, inertia component.

The characteristic curve of tooth pressure is, therefore, at any running position, a line parallel with the zero line, at a distance from zero determined by the frictional component, and showing a ripple of frequency and amplitude determined by the spacing and contour of the teeth. There will also be a still higher frequency of ripple determined by the character of the working surfaces of the teeth, which we find of very great importance in its effect on the quietness of operation. The importance increases rapidly as effectiveness of lubrication decreases.

[1] Chief Mechanical Engineer, Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa. Mem. A.S.M.E.

Thus under uniform average speed and load conditions the difference between the characteristics of tooth pressure corresponding to various degrees of pitch error is practically all due to the inertia component of the load if lubrication is first class, since the frictional component is not a material variable. Therefore, when the designer is faced with the quite usual conditions of heavy and comparatively rigidly connected moving parts, he can make no use of the conclusions drawn by the authors.

The writer recommends that, in any further tests of the kind conducted by the authors, moving parts of known and varying inertia be introduced, and the effect of inertia be isolated as accurately as possible.

The next apparent omission is the analysis of the errors of pitch and contour, etc., of the change gears. It seems unfortunate to introduce in a research test, the variable undergoing test, in series with the test objects. The transient changes in velocity produced by the test gears are no more vital in producing ultimate failure than those produced by the change gears, except that a very slight cushioning exists by virtue of the elasticity of the change-gear connection with the test gears.

This furnishes a plausible explanation for the fact that the authors could not always associate failure with test-gear pitch error. Of course it is to be expected that non-uniformity of test gear material also entered into this phase of the analysis. In any further tests electric speed control should be employed, and the test gears should be the only gears involved in the set-up.

If phase unbalance was present in any marked degree in the motor supply, there was a torque pulsation of double the supply frequency. The amplitude of this pulsation might vary as some function of the load. This would invalidate the comparison of the Maag and other gears.

If the distribution of mass and elasticity were such as to introduce or approach resonance, wide discrepancies would exist in the findings. We find that the first approach toward resonance cannot be detected by feeling or by the ear, and the use of the torsionmeter is recommended in any further tests. We are finding the torsionmeter invaluable in the analysis of gear structures, as it enables our analysts to trace to their source many disturbances which we were unable to explain without this instrument.

Attention should also be directed to the fact that the charts of gear error are a combination of the effects of both pitch and contour, and that it is exceedingly questionable whether they can be quantitatively separated. We have recently conducted some tests on much larger gears of radically different pitch, design, and mass association, and we find it essential to consider pitch and contour independently.

It is possible to derive from tests certain semi-empirical values that will give designers more insight than they now have con-

cerning the quantitative effect of errors of tooth spacing, but it is impossible to dissociate from these values the effect of inertia and elasticity, resonance, etc.

Henry J. Eberhardt.[1] This paper records tests, which it is assumed were to be a logical step in the investigations of Lewis, Marx and others, on the strength of gear teeth. The authors have evidently started out with the desire thoroughly to explore the effects of accuracy and inaccuracy on gear-tooth strength. For this high aim, as well as for numerous tests made, and the candor with which the various phenomena are recorded, the authors deserve commendation and thanks.

It is hoped that the following criticisms will prove constructive. The title of the paper is ambiguous. In Par. 3, Mr. Flanders is quoted as suggesting testing gears for strength as influenced by the "accuracy of cutting," caused by the cutters being "off center." In Par. 4, the phrase "effect of inaccuracy of spacing" is used. Whether "inaccuracy of spacing" refers to lop-sided teeth, due to cutting off-center, or to error of division, such as can be caused on any gear-cutting machine or process by gross carelessness, is not disclosed. The reduced Saurer charts appear to indicate simply errors of tooth form, and not errors of division. In Par. 9, the authors mention evidence which leads them to discount the truth of the chart records, where they say "This may have been due to slightly dirty gears when the charts were made, or to improper manipulation of the machine."

This investigation would have been highly scientific if but one type of gear, of various degrees of accuracy, both of tooth form and division, had been tested. In the present state of the art, no one type or make of cut gear holds exclusive superiority over other types. The accuracy of all types depends upon accurate machines, in which the work is accurately mounted. Furthermore, when testing such gears, they must be as accurately mounted in the testing apparatus.

In Par. 8, reference is made to one of the types tested as "standard Maag gears." The phrase calls for some defining, since to date this particular product has been extensively advertised as being "free from any and all standards." "The basic idea in the Maag system is to disregard the requirements of standardization as now understood." It hardly appears fair or scientific to test this type of gear by using one pair of such gears of determined accuracy with gears of another make and predetermined inaccuracy. In tests of this nature, the same type should be used for the error tests also.

Referring to the apparatus shown in Fig. 1, the first driver and the second driver should both be on the same side of the inter-

[1] Secy., Engr., Newark Gear Cutting Machine Co., Newark, N. J. Mem. A.S.M.E.

mediate shaft, to obtain effects more nearly parallel, or non-kinking, with the short bearings used. The gears should have a bearing on each side. The bearings should have a top bridge or connecting piece. These refinements will greatly increase the efficiency of the test. It is suggested that this type of apparatus be discarded in future investigations along this line. Professor Marx had motor and fuse trouble, and brake trouble. The authors also report much of the same trouble.

For all comparative tests it should be sufficient to keep records of power input only. The gears under tests can be put to work destroying themselves, as in the Lewis machine, or better, coupled to a generator to charge storage batteries, or to an air compressor, etc. For very heavy load tests the testing apparatus may be connected to a metal-cutting machine tool, as a lathe or boring mill. Loads can be varied over a wide range by changing cutting speeds and depths of cuts. The 100 per cent more strength shown in the authors' tests for accurate gears over inaccurate ones is easily accounted for by the fact that, due to elasticity, two teeth of well-made gears pick up and carry a load almost immediately, under static loads or loads at any speed.

It seems advisable to make future tests of steel gears rather than of gray-iron gears, as gray iron is comparatively more erratic a material. Steel gears do not have to be broken to be tested for strength, as deformation and wear are also measures of strength, and data of these factors will be of first importance to the engineer in producing gears that will run quietly and have long life.

EARLE BUCKINGHAM.[1] As far as the writer is aware, this paper makes public the results of the first definite tests on the strength of gear teeth of different degrees of accuracy. In many respects, it may be taken as a progress report of the Special Research Committee on Gears of this Society.

At the annual meeting of the Society in 1922, Wilfred Lewis presented a paper[2] describing a proposed gear-testing machine which would determine the effects of various inaccuracies on the strength of teeth by measuring the "increment load" developed at various speeds by different errors of known amounts. In an informal discussion in the special committee above mentioned, two different opinions as to the possible results of error were developed. One opinion was that extremely small errors would have a very appreciable effect on the strength of the teeth. The other opinion held that when the errors were reduced to about one or two one-thousandths of an inch, the flexing of the teeth under load and the compression of the metal at the point of contact would probably absorb errors of such small amounts that no appreciable difference in their strength would be found by actual test.

[1] Engr., Pratt & Whitney Co., Hartford, Conn. Assoc-Mem. A.S.M.E.
[2] See *Mechanical Engineering*, vol. 44 (1922), p. 813.

This discussion led to a proposal by the committee to furnish Professor Marx with gears of varying and measured degrees of accuracy with which to make tests similar to those in his previous experiments, described before the Society in 1912 and 1915.[1] The original plan was to make one gear of each pair identical, and to deliberately mismate the second gear of the pair on two sets, with a correct mating gear for the third set, thus giving pairs with errors of about 0.0006 in., 0.003 in. and 0.006 in. Professor Marx raised the point that it would be desirable to have one set of pairs as nearly like some of the previous ones tested as possible, so as to be able to compare the results of these tests with those of his previous ones. As a result, the three sets of pairs as finally furnished consisted of one set of Maag gears, one set of gears milled with the utmost care on Brown & Sharpe gear-cutting machines with Brown & Sharpe formed milling cutters, and one set of mismated gears cut with Maag cutters on a Maag gear-generating machine.

The first set when tested showed errors of from 0.0005 to 0.0010 in., the second set showed errors of about 0.002 in., while the third set showed errors of about 0.006 in. The mismating on the third set was accomplished by pairing a 15-deg. full involute gear with one of about 18½ deg. All gears had the same outside diameters and depth of teeth, and were made from castings poured in a single heat. The errors in these gears were profile errors, and not spacing errors as stated in the paper.

As the authors point out, the results obtained in the test on the second set of milled gears are remarkably consistent with the results of Professor Marx obtained in his previous tests with similar gears which were furnished him by the Brown & Sharpe Manufacturing Company. This consistency is even more remarkable because of the great difference in the strength of the materials used in the two sets of gears, that of the last gears tested being about 37 per cent stronger than the former ones. Table 5 is a comparison of the velocity coefficients given in Table 3 of Professor Marx's paper of 1915 with those shown in Fig. 11 of the present paper.

TABLE 5 COMPARISON OF VELOCITY COEFFICIENTS OF MARX (1915) AND FRANKLIN & SMITH (1924)

Pitch-line velocity, ft. per min.	Zero	1000	1200	1400	1600	1800
Velocity coefficient, Marx, 1915	1.000	0.484	0.455	0.435	0.420	0.410
Velocity coefficient, Franklin-Smith, 1924	1.000	0.516	0.500	0.482	0.468	0.461
Difference in velocity coefficients, per cent	0.000	+6.6	+9.9	+10.8	+11.4	+12.5

A study of Table 5 makes it evident that the milled gears used in these tests were fully equal in every respect to those used in the tests reported in Professor Marx's paper of 1915.

[1] Trans. A.S.M.E., vol. 34, p. 1323, and vol. 37, p. 503.

The results of these tests are significant. The differences were not as great as some of the members of the research committee expected, as the authors noted; but on the other hand, they were very much more than others expected. They give very strong evidence that the last thousandth of an inch in the accuracy of gear teeth has a very real value.

Table 6 gives the information shown in Fig. 11.

TABLE 6 VELOCITY COEFFICIENTS

Pitch-line velocity, ft. per min.	Zero	1000	1200	1400	1600	1800
Velocity coefficient (0.0006 errors)	1.000	0.696	0.682	0.667	0.636	0.576
Velocity coefficient (0.002 errors)	1.000	0.516	0.500	0.482	0.468	0.461
Velocity coefficient (0.006 errors)	1.000	0.375	0.344	0.320	0.302	0.290

The forms of the curve in Fig. 11 for the Maag gears shows a downward trend at the higher speeds, quite different from the two other curves. The probable cause of this is the fact that the testing machine was strained beyond its capacity in making these tests, as will be noted by the log of the tests, and the steel driving gears on the testing machine were practically destroyed in making them. With stronger and more rigid testing equipment, this curve would probably have a form very similar to the others.

These tests were not extensive enough to draw any final conclusions. It is interesting, however, to consider the nature of a formula which would introduce an accuracy factor and follow closely the curves as shown. Several equations could be found which would accomplish this. The following is one, in the form of the Barth equation:

$$\text{Velocity coefficient} = \frac{1000}{1000 + 700\sqrt{Ve}}$$

where V = pitch-line velocity, ft. per min.
e = error, in.

The values of this equation are shown in Fig. 12, in solid lines, for errors of 0.0006, 0.002, and 0.006. The curves in Fig. 11 of the paper under discussion are shown in dotted lines.

With the light that these tests throw upon the influence of accuracy, it is interesting to reconsider some of the data in Professor Marx's paper of 1915, as these tests are an extension of his previous ones. There seems to be no logical reason why the velocity coefficients in these previous tests should vary as regards the Brown & Sharpe gears and the Fellows stub-tooth gears except because of differences in accuracy.

Fig. 12 also shows in a solid line the curve representing the values of the foregoing equation when the error equals 0.0015 in., and a dotted line plotted from Table 7 of Professor Marx's paper of 1915 gives the velocity coefficients of Fellows 20-deg. involute stub-tooth gears.

Professor Marx states in his paper " that the actual test value of W sometimes comes out larger in the case of duplicate experi-

ments (all conditions the same) for the gear whose material subsequently showed the lower modulus of rupture in the flexure tests." The probabilities are that the conditions of accuracy are not the same and that the gears made of weaker material might have been more accurate.

These tests furnish additional evidence that the projected series

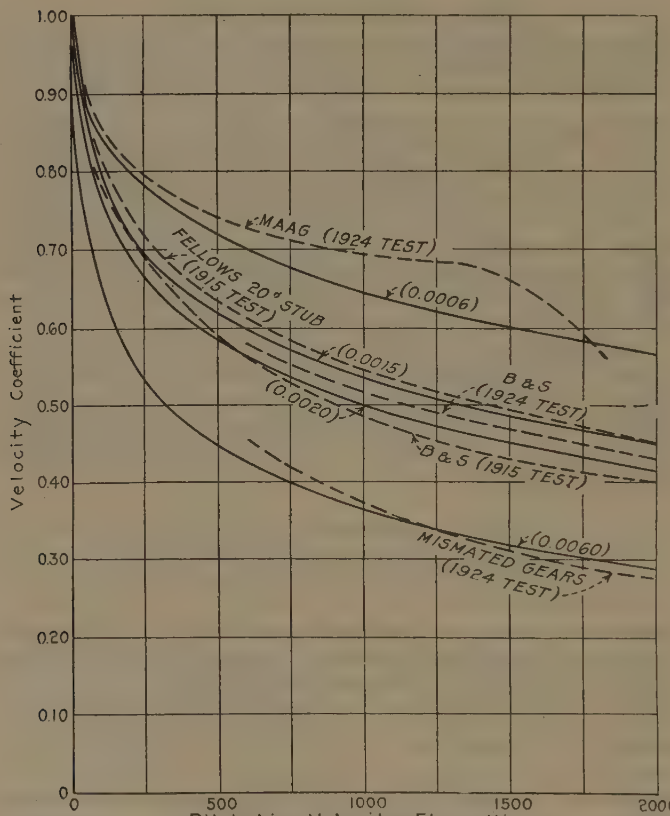


Fig. 12 Values of Equation for Velocity Coefficient

of tests to determine the effects of the inaccuracies in gear teeth which are to be made at the Massachusetts Institute of Technology on the Lewis testing machine will be of direct and practical advantage to all who are struggling with the problem of making better gears suited to the increasingly severe conditions which gears today must meet.

Ralph E. Flanders.[1] The authors do not state just what they mean by an inaccuracy of plus or minus 0.002 or plus or minus 0.006 in. At one point in the paper the inaccuracy is described as being one of indexing and at another it is described as being off center. The writer believes that possibly it may be neither of these two, but that it might be the inaccuracy as shown by the Saurer measuring machine, which would be in terms of the angular movement at the pitch line; that is, if one gear has uniform velocity the Saurer instrument would indicate that the other gear would sometimes move 0.002 in. ahead and at other times 0.002 in. behind the gear of uniform velocity. It probably would be helpful in using the authors' curves for the designing of gears or in the testing of gears, to know just what is meant by an inaccuracy of plus or minus some fraction of an inch. If it does mean, for instance, that the Brown & Sharpe cutter is set off center one or two thousandths of an inch on a gear of fair diameter, we must be very accurate in gear cutting. It would, in fact, indicate that serious weakness would result from errors so small as to be measured, probably, in ten-thousandths of an inch.

The curves shown in Figs. 7, 8, 10, and to some extent the resulting curves of Fig. 11, are interesting on account of the shape of the Maag gear. A tendency is shown to drop down at a velocity of about 1000 ft. and to come to some higher point again at about 1300 ft. velocity and then to drop more suddenly. This tendency is seen in all of the Maag gear tests to a considerable extent and to a somewhat lesser extent in the testing of the more accurately milled gears. It shows practically not at all in the most inaccurate set of gears. The shape of the gears raises a question as to whether or not the structure of the testing machine had a period of vibration, which reached its maximum at about 1300 r.p.m. and caused an increanse of stress for this number of revolutions. This calls attention to a matter well known to gear designers and manufacturers, namely, that many of the troubles that occur in the application of gearing, with relation to both strength and noise, do not necessarily originate in the gears themselves but appear in the mounting.

Important information that has been omitted is the length of time required for a complete test to point of rupture. The writer would also raise the question as to whether an endeavor was made to ascertain the shape of the teeth after prolonged running at heavy pressure and before rupture; that is, can we be sure that the tooth forms have not been so abraded just prior to rupture that they are no longer of the accuracy and shape necessary to carry the test out logically.

[1] Mgr., Jones & Lamson Mch. Co., Springfield, Vt. Mem. A.S.M.E.

A. T. Kasley.[1] The flywheel effect of the motor driving the gears is largely nullified by the Morse chain, and the flywheel effect on the brake drum is rather small. Would the curves not be considerably modified if flywheel effects on both sides of the gears being tested were increased to correspond more nearly to commercial conditions?

If additional tests should be conducted, it is suggested that the authors determine the result of increasing the flywheel effects, say, five times. There should not be too great a length of shaft between flywheel and gear.

Daniel Adamson.[2] The authors in Par. 21, state "it is to be particularly noted that the falling off of strength does not follow the law Mr. Wilfred Lewis thought he had deduced from Lasche's

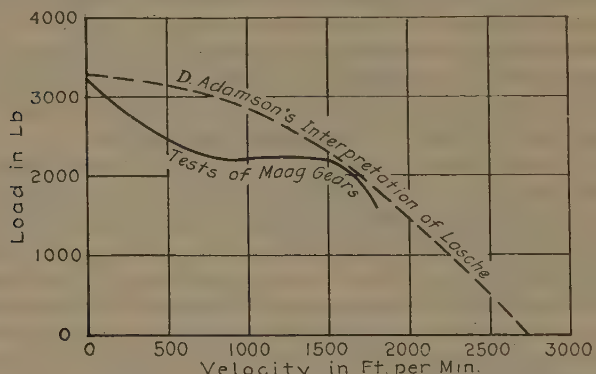


Fig. 13 Comparison of Authors' Tests on Maag Gears with Lasche's Formula

discussion and which led him to the obvious error that the allowable load becomes zero at speeds of 2600 or 2700 ft. per min."

The diagrams accompanying the paper present nothing to justify the reflection cast by the authors upon Mr. Lewis. As the authors say, they were unfortunately unable to carry their experiments over 2000 ft. per min., when the truth, or otherwise, of Lasche's ideas could have been more surely proved.

Fig. 13 is a reproduction of Fig. 7, upon which the writer has constructed in dotted lines a curve embodying his interpretation of Lasche's ideas, according to the formula showing the *loss* of strength to vary as the square of the velocity. Naturally such a curve must start at zero velocity with the result of the static tests (about 3250 lb.). For another point in the curve the breaking load at 1500 ft. per min. (about 2200 lb.) was taken. Continuing

[1] Research Department, Westinghouse Electric & Mfg. Company, Lester, Pa. Mem. A.S.M.E.

[2] Sr. Partner, Joseph Adamson & Co., Hyde, Cheshire, England. Mem. A.S.M.E.

the curve to zero load the writer was agreeably surprised to find that it passed through the base of his diagram at about 2700 ft. per min., the very speed held up to derision by the authors in the above-quoted paragraph.

The great difference between the dotted curve and the test curve (in the neighborhood of 700 ft. per min.), is noticeable. When the conditions of the test are more fully understood, it doubtless will be found that there is some explanation for this difference, altogether apart from the loss of strength due to velocity. It is only reasonable to believe, notwithstanding the tests quoted by the authors, that the reduction in strength will not be very serious at the lower velocities.

Loss of strength due to increased velocity = $V^2 \times 0.00045$

V	500	1000	1500	2000	2500	2700
Loss	112	450	1012	1800	2800	3280
3300 — Loss	3188	2850	2288	1500	500	20

Wilfred Lewis.[1] In the latter part of Par. 21 the authors state that " the falling off of strength does not follow the law Mr. Wilfred Lewis thought he had deduced from Lasche's discussion, and which led him to the obvious error that the allowable load became zero at speeds of 2600 to 2700 ft. per minute."

This statement shows a misunderstanding or misrepresentation of the writer's position. Lasche's discussion indicated that the limit might be 2600 or 2700 ft. per minute for rigid solids having the extreme values given by Walker and Marx, but it was also noted in the writer's paper that the limiting speed might be affected by the elasticity of the material, and Lasche's conclusions were simply cited as differing so radically from those of Walker and Marx as to suggest the need of further investigation. The writer's position was brought out more clearly at the Cleveland meeting of the American Gear Manufacturers Association a few months later and published in the *American Machinist,* December 13, 1923.

It should be noted, however, that the Maag lines in the upper curves of Figs. 7 and 10 dip very strongly toward the base line as though determined to cross it at or near 2600 to 2700 ft. per minute and thus sustain the " obvious error " of which the writer has been adjudged guilty.

What really happens is now in process of investigation by the A.S.M.E. Special Research Committee on Gears. A gear-testing machine has been built for the purpose and experiments will be conducted upon it at the Massachusetts Institute of Technology in the near future.

The Authors. There are a number of important points brought out in the discussion, upon which the authors have obtained further information which should add distinctly to the value of the paper.

[1] Pres., Tabor Mfg. Co., Philadelphia, Pa. Mem. A.S.M.E.

Several members have expressed their interpretations of what the authors meant by inaccuracy of spacing. This point is definitely answered by Mr. Buckingham, who supervised the making of the gears used in the test. It was the original idea to have milled gears indexed erroneously, but the gears finally furnished had error in profile rather than in pitch spacing. It is worthy of note that the effect in either case is identical, as we have the same departure from uniform rotation, thus producing the acceleration or retardation with the corresponding stresses. Thus the objection raised that gears of one type could not be fairly compared with gears of another type does not hold. The only difference in the intermediate curve due to substitution of Maag gears of equal inaccuracy would be to raise it enough to give the small difference between the strength of a 14½-deg. involute and a 15-deg. involute profile. However, assuming that the comparison is unfair, we may still say that the accurately made gears have twice the strength of the mismated ones, for, as stated by Mr. Buckingham, the mismated gears were of the Maag type and not milled gears as understood by the authors originally. There is nothing in the results or in our statement of conclusions to indicate that a 14½-deg. milled gear might not show substantially the same strength as the generated gears if the inaccuracies were of the same magnitude.

In regard to the amount of error being 0.002 in. ± etc., the authors would make it clearer by saying "errors of the order of 0.002 in." The notation 0.002 in. ± is not to be considered as a departure of that amount in both positive and negative directions, but is, as stated, a total value.

In reply to Mr. Burlingame, attention may be called to the fact that the paper clearly states (Par. 23) that all of the gears were furnished by Pratt & Whitney Co., and (Par. 8) that "in one class the ordinary milled gears were cut with a Brown & Sharpe milling cutter." Reference to Mr. Buckingham's discussion also shows that a Brown & Sharpe gear-cutting machine was used. It is noteworthy that the performance of these gears as pointed out by Mr. Buckingham and shown in Fig. 12 (due allowance being made for difference in material shown by flexure tests) is remarkably consistent with that of the gears furnished by the Brown & Sharpe Company for the Marx and Cutter tests, thus making unfair any implication that these gears were below standard. The question as to whether the B. & S. gears were of the same diameter and tooth depth as the Maag gears has been investigated by the authors and the dimensions were found to check very closely. Sample measurements are as follows:

	Outside diameter, inches	Root diameter, inches
Maag gear No. 30	6.202	5.756
B & S No. 15	6.200	5.769
Maag mismated No. 40	6.203	5.757
Maag mismated No. 49	6.199	5.757

The authors realize that the apparatus described in the paper is somewhat open to criticism. It was built about ten years ago, and is the first gear-testing apparatus that was used of which we have any record. Trouble, due mainly to the brake, was detrimental to the test, but time and money were lacking for the construction of other apparatus.

Concerning the suggestion that the tests should have been made on steel gears, it may be replied that the apparatus was stressed to its capacity to break cast-iron Maag gears at a velocity of 2000 ft. per min. and would not withstand a test on steel gears. It might be said here that the authors doubt points 31 to 36 of the Maag curves rather than the points objected to for the milled gears. Our reason for this doubt is as stated above and it was with great difficulty that these gears were broken. There was encountered sudden seizure of the brake which made the load indefinitely greater than the scale beam indicated. We note that it is precisely these probable erroneous points that Mr. Adamson and Mr. Lewis consider as corroborating their view. It should also be noted that Mr. Adamson, in Fig. 13, has taken a curve before reduction to common modulus for his comparison. This is obviously not correct.

The authors are misunderstood by Mr. Adamson in his discussion. There was no reflection cast by us upon any one. We simply stated that the results of the test showed that the falling off of strength does not follow a law which Mr. Lewis thought it would follow. To point out where actual results differ from *a priori* judgment previously published is certainly not casting any reflection.

Mr. Eaton makes certain just criticisms upon the limitations of the experiments and some valuable suggestions for further experimentation. However, we think he goes too far in stating that the omission of the detailed dynamic factors "entirely invalidate" our general conclusion. It must be borne in mind that the material and masses concerned were similar throughout. At the same velocities, then, the difference in mass effects would be precisely those due to the differences in profile and spacing (grouped as spacing errors) which produce accelerations and retardations of pitch velocity. And it is precisely the relative ultimate strength under similar conditions of mass and velocities that these tests disclose. As Professor Marx says, the results show relative performance against the respective integrated dynamic factors.

It is to be hoped that Mr. Eaton will employ the facilities at his disposal to carry out and publish such tests as he outlines. As pointed out in the closing paragraph of our conclusions (Par. 22), further tests, particularly at higher speeds, are greatly to be desired.

No. 1940

MECHANICAL SPRINGS

By Joseph Kaye Wood,[1] New York, N. Y.
Assoc-Mem. of the Society

The paper treats the subject of mechanical springs collectively in the hope of clarifying theories of design and of assisting in the ultimate standardization of springs. After defining a mechanical spring and a mechanical spring material, it considers the general cases of a unit cube stretched by a tensile force, of replacing the cube by a bar, and of applying the load transversely. From these are established load-deflection-rate formulas for flexure and torsion. Formulas for safe maximum load, safe maximum deflection, and safe maximum work are then derived in general terms containing constants which may be determined for stress method, material, form, etc. Under spring requirements of mechanical design, load-deflection characteristics are first considered, followed by those for safe maximum load and deflection and safe maximum work of resiliency. The paper then discusses the adaptability of springs to the requirements of mechanical design and the constants of material, dimension, stress method, and form of section. In the conclusion it is stated that the general or collective method of treating mechanical springs should eliminate much of the complexity and diversity of the subject.

SPRINGS and their design for mechanical use have always been viewed from the standpoint of some particular type, such as helical, spiral, flat, flat leaf springs stressed flexurally, helical springs stressed torsionally, rather than from the collective or general standpoint. This may seem strange since at the beginning of the art of designing springs in 1676, Dr. Robert Hooke described to the King of England his "new discovery" of a general law applicable to mechanical springs, and at the same time demonstrated the practicability of his theory by exhibiting what is probably the first spring-driven watch recorded in history. The general law as literally stated by Hooke is that "the power of any spring is in the same proportion as the tension thereof," in

[1] Consulting Engineer, New York, N. Y.

Contributed by the Machine Shop Practice Division and presented at the Annual Meeting, New York, December 1 to 4, 1924, of The American Society of Mechanical Engineers.

which the words "power" and "tension" were undoubtedly intended to mean load and stretching respectively.

2 It is the purpose of this paper to treat the subject of mechanical springs collectively, as was the natural tendency at the beginning of their use, in the hope that such a method will give a clearer picture of the greatly diversified and almost impracticable state to which the design of these very useful mechanical elements has steadily grown since Hooke's day. It is also hoped that this general method of treatment will be of considerable use in the ultimate standardization of springs.

3 The definitions given in connection with mechanical springs are usually vague being based upon general elastic properties or limited to some particular geometrical form of body having elastic properties. For a general treatment of the kind proposed it naturally seems logical to base the entire structure upon a sound general definition, framed somewhat as follows:

a A mechanical spring is an elastic body whose load-deflection rate and elastic range of deflection are of values suitable for mechanical use

b A mechanical spring material is an elastic material of the kind which when made into bodies of a shape and size suitable for use in mechanical design will function repeatedly and permanently as a mechanical spring.

4 In proposing this definition, one is reminded that the word "spring" has a very broad meaning, usually being connected with three different types of forces, hydraulic, chemical and elastic. In the case of the elastic type probably such bodies as branches of trees, bows, thin sword blades, and the like were first called springs by early man, and with the advent of the mechanical era Hooke introduced the first mechanical spring.

5 The springiness connected with the deformation of a piece of material is due to an inherent "elastic" force which is generally understood to be the resultant of the electromagnetic attractions between atoms deranged from their normal space lattice positions. The ratio of the fundamental atomic force to the relative displacement of the atoms is probably constant for a given material up to the point where slippage or plastic flow takes place. Since the resultant force and deflection are obtained by the application of simple cosine factors to the individual atomic forces and the corresponding displacements, it follows that they also must obey the straight-line law. Thus if a unit cube of metal be stretched an amount F by the tensile force P the ratio $P \div F$ would be equal to a constant E for that material. The constant E is called Young's modulus of elasticity after the man who first noted this relation, and a similar constant G for a torsional force is called the shearing modulus of elasticity. These two constants E and G form

the basis of determining the suitability of a material for use in mechanical spring design.

6 Considering the unit cube again in which the tensile load-deflection rate

$$P/F = E \text{ (numerically)} \quad \ldots \ldots \quad [1]$$

if the cube length be increased to l and the area to A then

$$P/F = EA/l \quad \ldots \ldots \quad [2]$$

and if the load P is applied transversely

$$\frac{P}{F} = E\frac{A}{l}\left(\frac{d}{l}\right)^2 k \quad \ldots \ldots \quad [3]$$

where d is the depth of the section and k a constant for flexure. For convenience $A \div l$ might be termed the bar index, which determines the load-deflection rate under maximum efficiency, that is when all parts of the bar are being stressed equally. The ratio $d \div l$ might be termed the lever index, which for flexure determines the relative values of the arms of a lever system. In a similar way k might be called the flexure constant, which in flexure would depend solely on the form of bar section because this alone changes the character of the stress gradient.

7 Even when the bar of material is coiled helically or spirally this expression still holds, but the length of the bar l will not be equal to the main lever arm as shown in the lever index of Formula [3]. The more generalized formula will therefore be as follows:

$$\frac{P}{F} = E\frac{A}{l}\left(\frac{d}{D}\right)^2 k \quad \ldots \ldots \quad [4]$$

where D = main lever arm. Similar reasoning as that above will show

$$\frac{P}{F} = G\frac{A}{l}\left(\frac{d}{D}\right)^2 k \quad \ldots \ldots \quad [5]$$

Since torsional loading is only another mechanical method of displacing atoms we should expect a constant relation between G and E for all metals. This very nearly happens to be the case, the small difference being due to the difference in Poisson's ratio. Assuming an average relation (about $G = 0.6\ E$), Formula [4] will apply for any method of stressing.

8 Continued loading of a piece of crystalline material will produce slippage or plastic flow, first, in a few of the crystals, and then in a gradually increasing number until the entire material is flowing plastically. While the plastic flow is taking place locally the greater bulk of the material is yielding elastically, and this condition will be accentuated in an aggregate of mixed soft and hard crystals. It is this simultaneous occurrence of these two actions which explains the characteristic shape of the stress-strain

diagram beyond the initial plastic-flow point. In continuing the tensile loading on the unit cube, a load P_m is reached whereat the initial plastic flow occurs, which when measured with ordinary commercial accuracy is numerically equal to the proportional limit S_p. With special means for obtaining considerably greater accuracy P_m might or might not be very much smaller than S_p, but at the same time it would approach a value equal to the fatigue endurance limit.

TABLE 1 FABRICATION OF FORMULAS

Col. No.	(10)	(11)	(12)	(13)	(14)
			Dimensional		
Row No.	Quantity required	Elastic constant of material	Bar	Mechanical advantage in method of stressing as gaged by lever index	Constant covering method of stressing and form of section
1	Load-deflection rate $= P \div F$	Modulus of elasticity $= E$ (or G)	Bar index $= A \div l$	Reciprocal square of lever index $= (d \div D)^2$	k
2	Safe maximum load $= P_m$	Proportional limit $= S_p$	Area of bar $= A$	Reciprocal of lever index $= d \div D$	m
3	Safe maximum deflection $= F_m$	Material index $= S_p \div E$ (or G)	Length of bar $= l$	Lever index $= D \div d$	$n = m \div k$
4	Safe maximum amount of work $= W_m = \frac{P \cdot F}{2}$	Modulus of resiliency $= S_p \div E$ (or G)	Volume of bar $= V = Al$	————	$u = \frac{mn}{2}$

9 If the cube length be increased to l and the area to A

$$P_m = S_pA \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [6]$$

and if the load be applied transversely

$$P_m = S_pA\left(\frac{d}{l}\right)m \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [7]$$

while the general formula for all methods of stressing including torsional is

$$P_m = S_pA\left(\frac{d}{D}\right)m \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [8]$$

In this formula as in Formula [4] the ratio $d \div D$ is called the lever index and m the stress-method and form constant.

10 By substituting the safe maximum load P_m in the load-deflection rate $P \div F$, the safe maximum deflection F_m is obtained, as follows:

$$F_m = \frac{S}{E}\, l \left(\frac{D}{d}\right)\frac{m}{k} \quad . \; . \; . \; . \; . \; . \; . \; . \quad [9]$$

assigning $n = m - k$ we have

$$F_m = \frac{S}{E}\, l \left(\frac{D}{d}\right) n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [10]$$

The new elastic material constant S/E will be referred to as the material index.

11 Since the safe maximum amount of work a bar of material can give is equal to the average force multiplied by the distance moved through, we may write

$$W_m = \frac{P}{2} F = \frac{S^2}{E} Vu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [11]$$

in which V equals the volume Al of the bar and u a constant equal to $mn \div 2 = m^2 \div 2k$. $S^2 \div E$ is generally known as the modulus of resiliency.

12 The formulas covered so far may be classified as in Table 1 which shows at a glance the great advantage of this general method of considering the elastic behavior of a bar under different

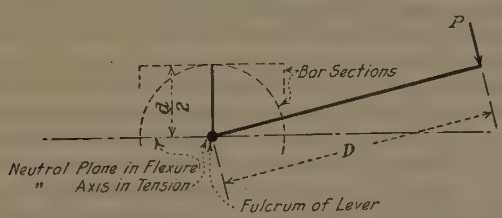


Fig. 1 Sketch Showing Significance of the Lever Index

conditions. Each row contains the factors in the formula giving the quantity shown in column 10. The terms in every column are interrelated in the same manner, that is in column 10, row 2 divided by row 3 equals row 1, while row 2 times row 3 divided by 2 equals row 4. This relation holds true for all the columns.

13 It should be noted that the result of multiplying row 2 by row 3 in column 13 is unity, which would be expected, since the amount of work done in lifting a mass a given height is independent of the mechanical method used, assuming 100 per cent efficiency. When stressing a bar entirely in tension the lever index is also unity because then each arm, i.e., d and D, is equal to zero. Fig. 1 shows more clearly the significance of the lever index. There are great numbers of these levers for a given section, probably one for each value of d varying by an amount equal to an atomic spacing from zero to maximum, but the constants given in column 14 take care of the integration of this varying lever arm over the entire section and also for the entire length.

14 Further observation will show that F_m requires a bar dimensional term of l, P_m a bar dimensional term of A, $P_m \div F_m$ of $A \div l$, and $PF/2$ of Al or V, all of which seems logical.

15 Inasmuch as the constant u in Formula [11] is unity when the bar is stressed entirely in tension and less when stressed otherwise, the following method of calculation in stress efficiency can be developed.

Pure tension in which every part of the bar is stretched an equal and maximum amount F_m is 100 per cent efficient. Flexure and torsion in which some parts of the bar are not stressed at all, and other parts are stressed the maximum amount while the remaining parts are stressed varying amounts between these two extremes, are $200u$ per cent efficient. Suppose the efficiency of stressing in a bar is 30 per cent, then to obtain the same amount of work in pure tension the volume of the bar would need to be only 30 per cent of the original volume. Symbolically,

$$\text{Per cent efficiency} = 100 \frac{W(\text{for flexure or torsion})}{W(\text{for tension})} = \frac{100u}{0.5}$$

In connection with work or resilience in elastic bars, a unit equal to one inch-pound might conveniently be called a "resilient," which however would be adding a new although related meaning to this word.

16 The principal quantities that determine the springiness of a bar of elastic material can vary over a considerable range since there is no limit to the values which the dimensional quantities in columns 12 and 13 of Table 1 can have, not to mention the considerable range the elastic constants in column 11 may have for a great variety of materials. It is therefore obvious that for the great variety of mechanical springs in use today some definite limitations must be set on the required quantities enumerated in column 10 of Table 1 for a spring as understood in its broadest sense. Before these limitations can be set however, the requirements for the mechanical apparatus needing spring action must be considered.

Spring Requirements of Mechanical Design

17 *Load-Deflection Characteristics.* In mechanical design it is not always necessary to have a straight-line load-deflection characteristic; in fact, in some special cases a decidedly curved characteristic with increasing load increments per equal deflection increments is desired. A flexible gear having a compound spring of this type was recently designed and invented by the author for electric street cars and locomotives. This is a desirable characteristic for automotive vehicles also and in some cases the excessive camber given to the leaf spring is good when positive due to the varying load arm D. Curved characteristics of the kind just described are good destroyers of oscillations, particularly when used in conjunction with frictional or liquid damping. In another instance a constant and variable deflection characteristic was

found to be desirable, the result being obtained by using a variable-radius cam to deflect the spring.

18 In the majority of cases, however, where mechanical springs are used the natural straight-line characteristic of elastic springs is found to be suitable, particularly from the standpoint of easy calculation and control in design. It is also very convenient when making calibration charts for commercial measuring instruments. There are some rare cases as in the design of scientific measuring instruments of high precision where so close an adherence to the proportionality law is required that even elastic springs will hardly satisfy. A spring for service of this kind must be carefully designed, manufactured, and heat-treated in order to insure that the spring material will be free from hysteresis and elastic after-effect which are the chief causes of unproportionality. Makers of commercial measuring instruments recognize this difficulty but of course they feel justified in claiming strict proportionality for their production of springs on the basis that the small difference is inappreciable for the service intended. Hysteresis and elastic after-effect, although too small to be measured in the ordinary commercial testing laboratory, are inevitable, since in an aggregate of crystalline grains oriented haphazardly as regards the direction of easy slip planes, interlocking of a unidirectional character is bound to occur. In an aggregate of soft and hard crystals under tension, it is obvious that the soft crystals, particularly those having the direction of their slip planes making 45 deg. with that of the external force, will slip first, while the hard crystals, particularly those having their slip planes coincident with the external load axis, will slip last. The merging of the plastic phase with the elastic phase which is more pronounced in some metals than in others, is the probable cause of hysteresis and elastic after-effect. This belief is supported by the fact that upon heating various kinds of metals to the proper temperatures, resulting in the elimination of elastic after-effect, the mobility of the plastic phase (considered as an uncooled liquid) is increased sufficiently to unlock the internally-strained crystals.

19 Since the load-deflection rate of elastic springs for all practical purposes is a straight-line relation and is more convenient as such in mechanical-design calculations, direct proportionality will be assumed throughout the remainder of the paper.

20 The load-deflection-rate requirement in mechanical design ranges from about 0.2 to 55,000 lb. per in. at the usual range of normal temperatures.

21 *Safe Maximum Load and Deflection.* These quantities unlike in the case of the load-deflection rate which for design purposes has to be within minimum and maximum limits, have only to meet a minimum requirement depending upon the service intended. As the safe-maximum-deflection requirement is never very severe, being less than a few inches, the demand is always for high safe loads, the largest springs carrying 85,000 lb.

22 *Safe Maximum Work or Resiliency.* Although this quantity contains both the load and deflection factors, it does not follow that it alone is sufficient to gage the requirements demanded in mechanical design, because it is possible to have a very high resilience figure with either the deflection factor too small or the load factor too small. The controlling quantities in mechanical-spring design are first, the load-deflection rate, and second, either the safe maximum load or deflection. The resilience has its function in spring design, however, and for that reason is considered sufficiently important for discussion. It ranges in value from 0.005 in-lb. (or "resilients") for small clock springs to 120,000 in-lb. for large railway springs.

Mechanical Springs

23 The adaptability of elastic springs to the requirements of mechanical design is the next logical step in determining the limitations which mark the mechanical-spring range. The quantities covered in Table 1 will be considered in the order given.

24 *Elastic Constants of Material.* The tensile modulus of elasticity, which for the fundamental unit cube is likewise the load-deflection rate, ranges from about 6,000,000 to 32,000,000 lb. per sq. in. for the mechanically useful metals at the usual range of normal temperatures. For any one metal the modulus varies about 10 to 20 per cent for moderately high temperatures. As stated previously, the effect of hysteresis on the load-deflection rate and consequently on the fundamental load-deflection rate, i.e., the modulus of elasticity, will not be considered.

25 The proportional limit as determined by the ordinary commercial testing machine ranges from about 25,000 to 270,000 lb. per sq. in. for the different spring metals at normal room temperatures. Such considerations as fatigue endurance, elevated temperatures, and unknown but severe conditions require the substitution of a lower figure in place of the proportional limit; for the first case the endurance limit, which may be considered as a "true proportional limit;" for the second case the proportional limit measured at the temperature desired; and for the third, the ordinary proportional limit modified by a factor of safety. Of course the selection of a spring material is influenced by the other physical properties, but since these do not apply directly to the calculation work they are used to influence the selection of a proper factor of safety.

26 The material index, or ratio of proportional limit to modulus of elasticity (using the tensile modulus), varies from about 0.001 to 0.009, while the modulus of resilience varies from about 25 to 2500 lb. per sq. in.

27 *Dimensional Indexes.* To obtain high flexibility, long lengths of material, in addition to reduced efficiency in stressing u, have to be used, and since long lengths of materials are inconvenient

in mechanical design, coiling helically and spirally is done, both in flexure and torsion. Thus lengths as great as 25 ft. and as low as 0.250 in. are used in mechanical springs. The cross-sectional area usually varies from 0.001 to 30.0 sq. in. The volume varies from 0.0001 to 3000 cu. in.

28 The bar index or ratio of area to length varies from 0.001 to 0.10 in., while the lever index may vary from 10 to 400.

29 Uniform-cross-section bars have been considered only in the previous discussion, but the introduction of a tapered bar, whether it be pointed or truncated, does not complicate the general method. In the case of a tapered spring, including the developed plan of a flat leaf spring, the bar index is very simply modified by a factor equal to the ratio of the minimum sectional area to the maximum area that is used in computing the bar index.

30 In the lever index the distance D is the maximum arm of the lever system of flexural or torsional stressing. In general it might be defined as that distance from the load axis to the fixed or stationary part of the spring; for helically coiled springs stressed torsionally it is the distance from the center of the helix to the axis of the bar; for similarly coiled springs stressed flexurally it is the distance from the load axis to the center of the helix; for spirally coiled springs it is from the load axis to the center of the spiral; while for flat springs stressed flexurally it is the distance from the clamped or supported end to the free or loaded end.

31 *Stress-Method and Form-of-Section Constant.* The constant k used in the determination of the load-deflection rate varies from 0.05 to 0.25, while the constant m used in the determination of the safe maximum load varies from 0.12 to 0.30. The constant $n = m \div k$, varies from 0.64 to 3.20, and $u = mn \div 2$ varies from 0.04 to 0.25; or another way of expressing the latter variation is that the efficiency of stressing used in mechanical springs varies from 8 to 50 per cent. This reduced efficiency, as stated previously, is due to the relatively low load-deflection rates and space limitations required in general mechanical design.

32 In nearly every case of stressing the intensity of the stress is symmetrical about either an axis or a plane, being zero in this vicinity and increasing to a maximum intensity on the exterior surface. The exceptions to this rule are in the case of torsionally stressed rectangular or elliptical bars; but this can be taken care of by modifying the stress-method and form-of-section constants with a factor equal to the ratio between the major and minor axes of the cross-section of the bar.

33 Since it is claimed by some that the stress does not increase uniformly from the neutral zone in any section when the order of flexibility is high, we might, if this is verified in time by experiment, apply another simple factor to the above constants which will depend upon the order of flexibility desired.

34 The values of the stress-method and sectional-form constants for some of the important types of springs are as follows:

CONSTANTS USED IN GENERAL SPRING DESIGN

Helical—stressed torsionally	k	m	n	$u \times 100$ per cent
Round (using modulus of elast = E)	0.05	0.16	3.20	25
" (" " " " = G)	0.12	0.25	2.00	25
Square (" " " " = E)	0.08	0.19	2.25	21
" (" " " " = G)	0.20	0.29	1.45	21
Helical—stressed flexurally				
Round and elliptical	0.06	0.12	2.0	12
Rectangular	0.08	0.16	2.0	16
Spiral—stressed flexurally				
Round and elliptical	0.06	0.12	2.0	12
Rectangular	0.08	0.16	2.0	16
Cantilever—stressed flexurally				
Round and elliptical	0.19	.12	.64	4
Rectangular	0.25	.16	.64	5

NOTE.—Assumed $G = 0.4E$ and S for torsion $= 0.63$ S for tension, which is only approximately true for materials in general.

35 These constants are interesting and of considerable value in selecting the right type of spring for a given service because they compare the important requirements for mechanical springs of different types when placed on an equal plane. Note that the per cent efficiency in flexural stressing of a rectangular cantilever is not as great as that in the torsional stressing of a round or rectangular bar.

36 As the study of fatigue of metals progresses, it is felt that the above constants will be found to bear some relation to the endurance limit.

SUMMARY

37 In the preceding discussion outlining a method of general mechanical-spring design and showing the limitations of elastic springs which are necessary to make them useful in mechanical design, four general formulas have been developed as follows:

Load-Deflection Rate

$$\frac{P}{F} = kE\frac{A}{l}\left(\frac{d}{D}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [12]$$

Safe Maximum Load

$$P_m = mSA\left(\frac{d}{D}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [13]$$

Safe Maximum Deflection

$$F_m = n\frac{S}{E}l\frac{D}{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [14]$$

$$= \frac{m}{k}\frac{S}{E}l\frac{D}{d}$$

Safe Maximum Work

$$W_m = u\frac{S^2}{E}V \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [15]$$

$$= \frac{m^2}{2k}\frac{S^2}{E}Al$$

38 As regards the elastic constants in these formulas, it might again be mentioned that in the case of torsional stressing the

modulus G can be used as in the usual practice, but in this case k must be multiplied by 2.5, and m by 1.5.

39 The material index can be conveniently labeled in accordance with the contemplated service as follows:

1 For severe fatigue service — kinetic material index

2 For elevated temperatures — temperature material index

3 For ordinary static conditions and normal temperatures — static material index.

40 In the first case, S should be the endurance limit, in the second case the proportional limit measured at the desired temperature, while in the third case it should be the ordinary proportional limit. In the second case E should also be slightly different, depending upon the service temperature.

41 In the design of a mechanical spring, Formula [12] is absolutely essential, with either Formula [13] or [14]. Formula [15], although very useful, is not sufficient for calculating a spring.

Conclusion

42 In the general or collective treatment of the subject of mechanical springs as covered above, the indications are that the complexity and diversity of this very important subject may be greatly eliminated, and if this end is ultimately attained through the instrument of this paper then the author will feel greatly repaid for his efforts. The layout of the component parts of each general formula and the very apparent relation of these parts to the fundamental laws laid down by Hooke and others should aid greatly in the standardization of mechanical springs. By further study and molding of this general method it will be possible to obtain a code of design which should give the mechanical designer a sense of masterly control over this part of his daily work.

DISCUSSION

Fritz K. Loeffler.[1] This paper represents a forward step on the subject of mechanical springs. It should prove to be of considerable value in paving the way to a suitable code of design for such springs. The writer believes, however, that it would have been advisable to have included in the paper a formula covering the oscillations of springs. It is to be hoped that the author will consider this important item in an extension of the general method of design which he proposes for springs.

J. W. Rockefeller, Jr.[2] The author has made a valuable contribution to the general subject of spring design in indicating a starting point from which it may be further expanded and developed. Springs generally have been viewed from the stand-

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[2] Engr., John Chatillon & Sons, New York, N. Y. Jun. A.S.M.E.

point of some particular type rather than in a general way. This fact may well be one of the chief contributing reasons for the dearth of information available on such subjects as hysteresis and temperature-modulus coefficient. While such subjects are of interest to individual users of springs, including the makers of commercial measuring instruments, it is not surprising that this class of individuals has not delved deeper into the cause of such phenomena.

The research program of the manufacturer is determined largely by practical necessity. For example, the case of direct-reading scales may be considered. Hysteresis was encountered; then by reducing the fiber stress in the steel it was found that hysteresis could be controlled and brought well within the limits of accuracy of the racks and pinions or the tapes and cams. As soon as this point was reached the attention of the manufacturer was redirected to problems of backlash, friction, and theoretical pivots in which the objectional third dimension will be eliminated. In fact, the springs themselves having been so perfected were looked to as a possible solution for grave problems in other parts of the mechanism, and one of the most recent developments in scale design in this country or abroad has been the substitution of a flat spring for the common pivot-type fulcrum.

The proper progress in spring research will be made only when the general importance of the subject is realized. The number of purposes for which springs are used warrants the study of springs for their own sake, if for no other reason. A fact that is given but little consideration, however, is the usefulness of a spring as a general test specimen. The laws of stress and strain that apply to springs are the general laws governing the behavior of materials. If hysteresis is a function of stress in springs, it must be present, although not easily measured, in many other forms of the same material, such as the heavier types of ordnance and steel cables. It is certainly a subject of general interest.

Hysteresis, a term borrowed from the field of electromagnetics, does not seem to mean precisely the same thing to all who use it. It may be used to describe the failure of a spring, loaded for some time, always to return upon unloading to the same extended length for a given load. If used in this sense we cannot make the positive assertion that the load-deflection rate has changed, since at least part of the discrepancy is due to the spring's possessing, after extending loading, a greater free length.

As to the cause of hysteresis (used in the above sense), it is quite likely to be something in the nature of internal friction. In subjecting a helical spring to a given load, there is an immediate deflection which is generally regarded as elastic. The secondary deflection taking place has usually been considered as being of a different nature from the first. The author has referred to the first as elastic and to the second as plastic. The writer prefers to

regard this secondary deflection as being of the same nature as the first, i.e., elastic, in fact merely a continuance of the initial deflection, retarded greatly by internal friction in the material. Perhaps this point may be clarified by analogy. It is common practice in scale design to prevent excessive vibration of the pointer in a direct-reading scale by employing a dashpot. This device will retard the action of the pointer without affecting materially the point at which it comes to rest. When such a device is used in a spring scale there will be a noticeable change in the velocity of the pointer as it approaches the point at which it will come to rest. This slowing down is brought about by the retarding effect of the dashpot, while the motion in its later, as well as its early, phases is produced by the elastic deformation of the springs.

The Author. The inclusion of the oscillation formula in a general method of spring design would be possible, since such a formula expresses a simple relation between the period or frequency and the load-deflection rate. The general formula would undoubtedly contain the constant $2\pi/\sqrt{g}$.

As to the question of hysteresis in mechanical springs, this is becoming more important every day in certain fields. The loads and deflections of springs used in sensitive length-measuring machines, aeronautical instruments, and precision weighing instruments are sufficiently small to make the errors due to hysteresis of appreciable magnitude. The hysteresis loop, formed somewhat symmetrically about the true load-deflection curve, due to the lagging of the deflection on loading and of the load on unloading, is large enough, even in steel, to offer difficulties in the adjustment of these springs. The usual remedy is to give such springs a careful heat treatment, which tends to eliminate or decrease the size of the hysteresis loop.

On the other hand, although it is necessary to consider ways and means of eliminating hysteresis in springs of the type just mentioned, it is necessary to induce this phenomenon artificially in other types of springs, such as in the equalization systems of electric locomotives and in the suspension systems of automobiles.

The artificial means employed are external surface friction, rubber absorbers, and liquid and gas dashpots which retard the deflections and loads, thus forming an artificial hysteresis loop.

The purpose of having springs with a large hysteresis is to damp or destroy undesirable oscillations. As mentioned in the paper the author has recently designed a flexible gear for use in electric locomotives and street cars in which the characteristics of the spring element are such as to decrease chattering slip in the drivers to a minimum.

Such questions in spring design as have been brought up during the discussion must be studied more thoroughly in order that the progress in the spring art may keep pace with that of the many other important mechanical elements.

No. 1941

THE STRENGTH AND PROPORTIONS OF WHEELS, WHEEL CENTERS AND HUBS

By R. Eksergian,[1] Philadelphia, Pa.
Member of the Society

The paper outlines an approximate analysis of the strength of wheel centers, particularly as applied to spoke wheels subjected to heavy lateral loads such as locomotive driving wheels. To generalize the work, the subject is extended to a study of other types of wheels with a view to approximating their strength characteristics. Inertia loadings and proportions of flywheels are considered briefly.

After a discussion of classification and loading, the author analyzes the characteristics of bending loadings and stresses, considering first the transmission of torque for wheels with heavy and with light rims, second, direct loading in the plane of the wheel, and third, lateral bending in wheel centers. The design of hubs subjected to a pressure fit is then analyzed, and the additional stresses in a locomotive driving wheel due to the offset load on the crank pin are considered. This is followed by a mathematical discussion of the effect of the shrinkage of tires on the compressive stresses in wheel centers. The counterbalance of locomotive wheel centers follows and the paper concludes with a consideration of inertia stresses and loadings of fly-wheels with special consideration of rotors for a.c. generators with rotating fields, and of the inertia proportions of flywheels.

THE object of this paper is to outline an approximate analysis of the strength of wheel centers, etc., more particularly as applied to spoked locomotive driving wheels subjected to large lateral loads. To generalize the work, the paper has been briefly extended to a study of other types of wheels. The inertia loadings and proportions of flywheels have also been briefly considered.

2 In any specialized phase of machine design, we may follow two distinct methods: (1) A statistical method, based on empirical data, or (2) a method based on approximate analysis. Either method alone would ultimately prove dangerous and the happy

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compromise is obviously a combination of these methods that is a rational development of the variation of the basic proportions so far as possible, combined with constants based on good practice. In analysis we are limited to simple elements only, and therefore for the more complicated parts we must approximate to equivalent proportions within the range of the mathematical analysis and the assumptions made are the basic criterion for the real value of the analysis.

3 The proportions of wheel centers, hubs, etc., are difficult to arrive at by analysis and these parts are usually designed from extended empirical data. The following analysis is offered by no means as a substitute for this procedure, but rather as an augmented guide in the proportioning of these elements for exceptional cases when practice is limited and as aiding in a more rational choice in our ordinary methods of proportioning.

Classification and Loading

4 A simple classification would be in the nature of the loading. With the exception of the simplest vehicle wheel, which is designed to sustain a vertical load, all wheels are subjected to the transmission of a torque in one form or other. This of course occurs in flywheels as well, since the primary object of a flywheel is, by the effect of its rim inertia, to regulate the speed by augmenting or decreasing the prime-mover torque. Moreover, the torque is often accompanied by a reaction or thrust which resolves into a concentrated or distributed loading along the periphery. Thus in a simple belted pulley, the belt reaction on the wheel resolves into a resisting torque balanced by the driving torque of the shaft at the hub and a distributed normal loading along the belt contact which is balanced by a reaction at the shaft. In a gear wheel the loading is concentrated approximately at a point along the periphery. So far the loading is coplanar, and the arms or spokes are subjected to bending in the plane of the wheel.

5 Next in order are wheels subjected to a large concentrated thrust which may or may not transmit torque, as is the case in vehicle wheels. Since the wheel in such cases must sustain a large vertical load, it must be designed as a supporting member between the roadway or rail and axle. There remains still bending in the vertical plane only, so the resultant loading on individual spokes is different from that transmitting a simple torque loading.

6 All vehicle wheels are subjected in a greater or less degree to side thrust, but to a far less extent in road vehicles than in railway work. This is because in a road vehicle the centrifugal force, distributed per axle, is the ordinary loading, the maximum not exceeding the total friction force of the vehicle, whereas in railway work the wheelbase and trucks are guided by a single concentrated side thrust which not only balances the centrifugal

force, but often overcomes the friction of a long, rigid wheelbase, this latter being by far the predominating loading. This introduces a new aspect, the transverse bending of the wheel plane.

7 In a locomotive driving wheel there is transverse bending both horizontally and vertically due to the offset piston loading from the plane of the wheel and the side thrust of the rail, sustaining larger vertical loads and transmitting a small torque to the rail.

8 A steam-locomotive driving wheel, however, is enormously reinforced by the wheel hub, crank, and counterbalance; moreover, possible lateral rail thrust is relieved on the main drivers by the end wheels, which suffer less rod thrusts. In the development of the electric locomotive, especially in the use of the quill drive, the spacing of wheel spokes is restricted and the reinforcement of the hubs is absent, while still sustaining large side thrusts at the rail. The analysis of such a wheel offers, therefore, a unique problem condensing, so to speak, a group of principles and analyses into one particular problem. It is therefore felt that a concentration of part of the analysis on this particular type is consistent with the purpose of the paper.

9 Finally, in wheels revolving at high speeds with heavy rims, as in flywheels, the centrifugal loading causes bending stresses in the rim. The flywheel problem has been attacked fairly rigorously by several authorities, but a brief survey of such analysis is not out of order.

Characteristic of Bending Loadings and Stresses

10 The nature of the stresses and loadings on the arms or spokes is dependent primarily on the relative rigidity of the rim. In general a heavy rim tends to distribute the loading on all the spokes, whereas a very light rim causes the entire loading to come on one or more individual spokes. An intermediate condition of the rim proportion offers an extremely complicated problem, but by various assumptions a simpler approximate analysis can be effected.

a Transmission of Torque

11 Pulley arms are made usually in one or two forms: straight arms and curved arms. The latter form, from a casting point of view, reduces the shrinkage stress in pulley castings. The tangential resisting force at the rim in the case of a belted pulley is distributed (not evenly) over approximately a semicircumference, whereas in a gear rim the load is concentrated at one or more adjacent tooth contacts.

12 With a heavy rim, the following assumptions are approximately true: The tangential shear loading per spoke is equally distributed, and the arm remains normal to the rim. That is, the arm is completely constrained at the rim and hub and the

slope of the elastic curve remains tangential to the radius at hub and rim. See Fig. 1.

Let α = angle between radii at hub and rim contacts a and b
T = torque transmitted in lb-in.
n = number of spokes
M_a = bending moment per arm at a in lb-in.
M_b = bending moment per arm at b in lb-in.
S = the common shear in one arm in lb.
$l = r_m - r_o$ = length of arm in in.

Then for the equilibrium of the heavy rim, (see Fig. 1)

$$\frac{T}{n} = Sr_m - M_b \text{ (approx.)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [1]$$

and for the equilibrium of the arm, (see Fig. 2)

$$Sl = M_a + M_b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [2]$$

while the equation for the elastic deflection (with the origin taken at the wheel center) is:

$$\int_{r_o}^{r_m} \frac{Mrdr}{EI} = r_m\alpha - r_m\alpha = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad [3]$$

where

$$M = M_a - S'(r - r_o)$$

From [2] and [3], assuming the moment of inertia constant and equal to the mean of its value along the arm,

$$M_a\left(\frac{l}{6} + \frac{r_o}{2}\right) = M_b\left(\frac{l}{3} + \frac{r_o}{2}\right)$$

hence

$$M_b = M_a\left[\frac{l+3r_o}{2l+3r_o}\right] = \frac{Sl}{3}\left[\frac{l+3r_o}{l+2r_o}\right] \text{ (lb-in.)}$$

$$= \frac{Sl}{3} \text{ (approx. when } r_o \text{ is small)}$$

while for the bending moment at a in terms of the shear,

$$M_a = \frac{Sl}{3}\left[\frac{2l+3r_o}{l+2r_o}\right] = \tfrac{2}{3}Sl \text{ (approx. when } r_o \text{ is small)}$$

The shear may be obtained from Eq. [1] and is

$$S = \frac{T}{n\left[r_m - \frac{l}{3}\left(\frac{l+3r_o}{l+2r_o}\right)\right]} = \frac{3T}{2nr_m} \text{ (approx.) (lb.)}$$

Approximately if we neglect the radius of the hub as small, the above reduces to

$$M_a = \frac{T}{nr_m}\, l = \frac{T}{n} \text{ (very approx.)}$$

which is exactly to be expected since the moment of the shear at the hub becomes negligible and the total torque is transmitted by bending.

13 In the other extreme with a very light rim, the flexibility prevents a bending constraint at the rim, and as a first approximation we may consider a concentrated reaction at the outer end of arm. If the torque loading is distributed as a uniform resistance around the periphery, then the load per arm is $\frac{T}{nr_m}$ which is the shear to which the arm is subjected, while the bending at a is

$$M_a = \frac{T}{nr_m} l$$

14 Thus the bending moment at the hub remains roughly the same while the bending moment at b is reduced to a very small value. The shear however is reduced over that for the case of a heavy rim.

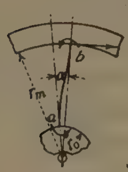


Fig. 1

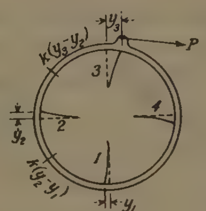


Fig. 2

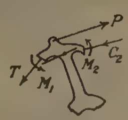


Fig. 3

15 On the other hand, the assumption has been made that the loadings per spoke are equal. This is by no means true, especially for a light rim, when the torque loading is unequally distributed along the periphery. Experiments noted by Professor Lanza on pulley wheels with comparatively light rims indicate that the arm adjacent to the point where the tight side of the belt leaves the pulley sustains a greater load than the other arms. A mathematical analysis of this point would, aside from being difficult, prove unprofitable. A rough picture of the variation per loading however can be shown as follows:

16 Consider a wheel with a flexible rim (Fig. 2). With an extremely flexible rim, the bending-moment resistance in the rim is small. Then for a crude approximation, assume a concentrated tangential loading at the rim at spoke (3) and a tangential tension and compression as existing in the rim. Then, for the angular deflection of a cantilever arm, at the rim,

$$\alpha = \frac{Fl^3}{3EIR} = \frac{FR^2}{3EI} \text{ (approx.)}$$

and for the elongation between two adjacent spokes, (very approx.)

$$\alpha_m - \alpha_n = \frac{T_{mn}}{AE} \cdot \frac{2\pi}{n} \quad \text{where } T_{mn} = \text{Tension (lb.)}$$
$$n = \text{no. of spokes}$$

17 Thus for the simple four-spoke wheel, with light rim, and a load applied at spoke (3), assuming only a hoop tension and compression to exist,

$$2(\alpha_2 - \alpha_1)k = \alpha_1 K$$
$$[(\alpha_3 - \alpha_2) - (\alpha_2 - \alpha_1)]k = \alpha_2 K$$
$$P - 2(\alpha_3 - \alpha_2)k = \alpha_3 K$$

where $k = 2AE$, $K = \dfrac{3EI}{R^2}$, P = Applied load on spoke (3).

18 From the equations we readily see that with a flimsy rim, the major loading is brought on arm 3. As the rim proportions

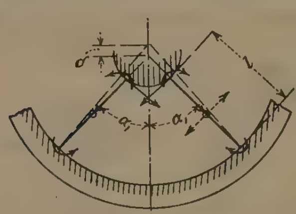


Fig. 4.

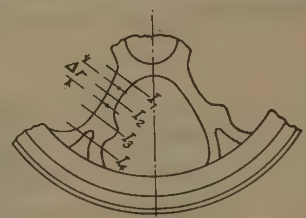


Fig. 5.

are increased, bending resistance is offered as shown on Fig. 3. Then:

$$T = PR = M_1 + M_2 + T_1 + C_2 + F \cdot R$$

where F = reaction of the arm at load P.

$$\therefore FR = PR - (T_1 + C_2) - (M_1 + M_2)$$

19 Thus the bending in the rim materially decreases the loading on the arm at the point of application of the tangential load; and thus transmits the loading to the other arms in a more even distribution.

20 For a preliminary design with a light rim, that is, when the section modulus of the rim is comparable with that of an individual spoke, the loading on a spoke may be considered double that of a distributed loading and the individual spoke as loaded as a simple cantilever for bending at sections adjacent to the hub, while the bending moment at sections adjacent the rim may be considered small, say, half that with a heavy rim.

21 With a heavy rim, that is, when the rim section modulus is several times that of an individual spoke, the loading may be assumed as equally distributed per spoke, and the preceding formulas are immediately applicable.

b Direct Loading in the Plane of the Wheel

22 Where the direct loading is most important, the loading is concentrated radially at the rim with a balancing reaction at the hub as in vehicle and railway wheels supporting heavy loads. The unbalanced thrust due to belt pulls, concentrated gear reactions, etc., when the rim is heavy, may be resolved into a radial axial loading combined with a torque on the rim. Therefore, as a first approximation the loading on the spokes will be considered as due to a simple radial axle loading. See Fig. 4.

Let $R =$ direct thrust along spoke, lb.
$S =$ shear reaction on spoke, lb.
$l =$ length of spoke, in.
$A =$ mean area of cross-section of spoke, sq. in.
$I =$ mean moment of inertia of spoke section
$\alpha =$ angle of spoke from radial direction of load
$P =$ applied load, lb.

23 The reaction of the spokes on the hub must balance the reaction of the applied load at the hub, that is,

$$P = \Sigma R \cos \alpha + \Sigma S \sin \alpha$$

24 Since under the radial loading, the hub is displaced with respect to the center of the rim a distance δ, then the displacement along a spoke is $\delta \cos \alpha$, while the total displacement tangentially is $\sin \alpha$. The radial strain is therefore $\frac{\delta \cos \alpha}{l}$ and the tangential deflection of the point of contraflexure evidently midway between the hub and rim, from either hub or rim, is, $\frac{\delta \sin \alpha}{2}$

The radial compression along the spoke is

$$\frac{R}{A} = E \frac{\delta \cos \alpha}{l} \qquad \therefore R = \frac{AE\delta \cos \alpha}{l}$$

and for the cantilever deflection of the point of contraflexure,

$$\frac{Sl^3}{24EI} = \frac{\delta \sin \alpha}{2} \qquad \therefore S = \frac{12EI\delta \sin \alpha}{l^3}$$

Therefore, for equilibrium of the hub or rim,

$$P = \left[\frac{AE}{l}(1+2\cos^2 \alpha \ldots .2\cos^2 \alpha_n) + \frac{12EI}{l^3}(2\sin^2 \alpha \ldots .2\sin^2 \alpha_n)\right]\delta$$

$$\delta = \frac{Pl}{AE\left[(1+2\cos^2 \alpha_1 \ldots .2\cos^2 \alpha_n) + 12\left(\frac{e}{l}\right)^2 (2\sin^2 \alpha_1 \ldots .2\sin^2 \alpha_n)\right]}$$

where

$$A\rho^2 = I$$

from which we may compute the values of R and S in the above equations. Then the maximum direct stress is

$$f_{max.} = \frac{E\delta}{l}$$

and the combined bending and direct stress for a lower inclined spoke is

$$f_d = \frac{E\delta \cos \alpha_1}{l} \text{ (for direct stress) (lb. per sq. in.)}$$

$$f_b = \frac{Sle}{2I} = \frac{6Ee\delta \sin \alpha_1}{l^2} \text{ (for bending stress) (lb. per sq. in.)}$$

and $f = f_d + f_b$

where e = distance from neutral axis to extreme fiber.

25 With a light rim the preceding distribution no longer exists. In this case a certain percentage of the total loading may be taken arbitrarily as causing direct compression in a single spoke, or a given percentage increase may be allowed for in computing the maximum stress by the above analysis.

26 In many cases the variation of cross-section is considerable along the spoke, and moreover, as in locomotive driving wheels, the spoke lengths vary; for such a case an integration method is necessary. See Fig. 5.

27 To account for the varying sections and lengths of a spoke, let

M_a = bending moment at hub, lb-in.
M_b = bending moment at rim, lb-in.
r_o = hub radius of spoke, in.
r_m = rim radius of spoke, in.
l = effective length of spoke, in.
A = cross-section of spoke at radius R, sq. in.
I = moment of inertia of cross-section in plane of bending, in.[4]
R = radial thrust in spoke, lb.
S = common shear reaction, lb.

Then

$$-E\delta \sin \phi = M_a \int_{r_o}^{r_m} \frac{r}{I}\, dr - \frac{M_b + M_a}{l} \int_{r_o}^{r_m} \frac{r^2 \cdot dr}{I} \quad . \ . \ [4]$$

$$M_a \int_{r_o}^{r_m} \frac{dr}{I} = \frac{M_b + M_a}{l} \int_{r_o}^{r_m} \frac{r \cdot dr}{I} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ [5]$$

$$S = \frac{M_b + M_a}{l} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ [6]$$

$$R = \frac{E\delta \cos \alpha}{\int_{r_o}^{r_m} \frac{dr}{A}} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ [7]$$

The unknowns are M_a, M_b, S and R, and we have the above four equations for solution. Combining and simplifying,

$$-E\delta \sin \phi = S\left[\frac{B^2 - AC}{A}\right] \quad \text{. . . . . [8]}$$

$$R = \frac{E\delta \cos \alpha}{D} \quad \text{. . . . . [9]}$$

where

$$A = \int \frac{dr}{I} = \Sigma\left(\frac{\Delta r}{I}\right); \qquad B = \int \frac{rdr}{I} = \Sigma \frac{r\Delta r}{I}$$

$$C = \int \frac{r^2 \cdot dr}{I} = \Sigma \frac{r^2 \Delta r}{I}; \qquad D = \int \frac{dr}{A} = \Sigma \frac{\Delta r}{A}$$

Then as before,

$$V = \Sigma R \cos \alpha + \Sigma S \sin \alpha$$

$$\therefore V = \left[\left(\frac{1}{D_o} + \frac{2 \cos^2 \alpha_1}{D_1} \ldots + 2\,\frac{\cos^2 \alpha_n}{D_n}\right) - \left(\frac{2 \sin^2 \alpha_1}{R_1} \ldots + \frac{2 \sin^2 \alpha_n}{K_n}\right)\right] E\delta$$

where

$$K = \frac{B^2 - AC}{A}$$

28 From the last equation we may compute the common deflection δ and, substituting in Equations [4] to [9], obtain the bending moments, shear, and direct thrust for any individual spoke at inclination α. Obviously the above analysis assumes the bending in the rim to be small as compared with that of an individual spoke.

c Lateral Bending in Wheel Centers

29 Many wheels are subjected to lateral bending, as gear wheels with helical gearing, vehicle wheels, especially railway wheels, etc. The lateral loading is usually concentrated at a point along the periphery. Here again the proportions of the rim are all important in the proportional loading coming on an individual spoke. With a light rim the loading is brought on one or more adjacent spokes, while with a heavy rim all the spokes are brought into action, some suffering only bending and others only torsion, while intermediate ones experience combined bending and torsion. The stresses resulting augment those due to direct loading and bending in the plane of the wheel.

For helical gears, with B = angle of helix, the side thrust may be taken approximately as:

$$F = P \tan B = \frac{T}{R} \tan B = 0.7\,\frac{T}{R} \text{ (approx.)}$$

where T = the torque transmitted, R = pitch-line radius. For simple vehicle wheels, assuming 0.6 as maximum coefficient of friction,

$F = 0.6\ W$ (min.) for skidding of one wheel
$= (1 \text{ to } 1.2)\ W$ (max.) for skidding per axle with applied side thrust
W = total wheel load.

For railway or locomotive wheels, the derailment side thrust

$$F = W\left(\frac{\tan\phi - f}{1 + f\tan\phi}\right)$$

where $\phi = 60$ deg., corresponding to the M.C.B. Standard (approx.).

Values of $\frac{F}{W}$ are as follows:

f	0	0.1	0.2	0.3
F/W	1.732	1.302	1.145	0.944

In service conditions, however, the maximum lateral thrust is not likely to exceed 0.7 W, while with automobile wheels the maximum side thrust will probably not exceed 0.4 W.

30 In no case, even with a heavy rim, are the spokes front and back of the hub equally loaded to a bending moment, a thrust along the axle; that is, the rim suffers an axial displacement in addition to its transverse bending. Therefore, unlike the case of a torque loading with a heavy rim, the spokes which suffer the greatest transverse bending are always adjacent to the applied lateral load. With a flexible rim, the bending of the rim throws greater load on spokes adjacent to the point of application of the side thrust.

d Lateral Stresses

31 Consider origin of axis at hub center. See Fig. 6.

Let M_a = bending moment at hub, lb-in.
M_b = bending moment at rim, lb-in.
S = common shear reaction, lb.
l = length of spoke, in.
R_o and R_m = radius of hub and rim, respectively, in.
I = moment of inertia of spoke section for transverse bending, in.^{4}
w = width of rim, in.

Slope along OR at wheel rim,

$$\phi' = \frac{\phi w \cos\alpha}{w} = \phi\cos\alpha$$

Then for any spoke

$$E(R_m\phi'-y) = M_a\int_{R_o}^{R_m}\frac{r}{I}\,dr - \frac{M_b+M_a}{l}\int_{R_o}^{R_m}\frac{r^2\cdot dr}{I} \quad . \quad [10]$$

$$E\phi' = M_a\int_{R_o}^{R_m}\frac{dr}{I} - \frac{M_b+M_a}{l}\int_{R_o}^{R_m}\frac{rdr}{I} \quad . \ . \ . \quad [11]$$

$$S = \frac{M_b+M_a}{l} \qquad \therefore M_a = Sl - M_b \quad . \ . \ . \ . \quad [12]$$

Now $y = \delta + R\cos\alpha\cdot\phi$

$$\therefore R\phi' - y = R\cos\alpha\cdot\phi - \delta - R\cos\alpha\cdot\phi = -\delta$$

Hence

$$AS - BM_b = -E\delta \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad [13]$$

$$CS - DM_b = E\phi\cos\alpha \quad . \ . \ . \ . \ . \ . \ . \ . \quad [14]$$

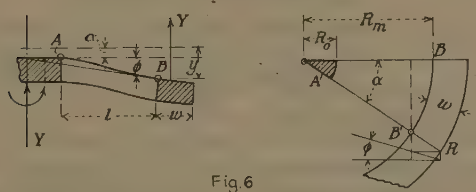


Fig. 6

where

$$A = l\Sigma\frac{r}{I}\Delta r - \Sigma\frac{r^2\Delta r}{I} = \frac{l}{6I}(l^2 - 2r_o^2) \text{ when } I \text{ is assumed constant}$$

$$B = \Sigma\frac{r\Delta r}{I} = \frac{l}{2I}(l+2r_o) \text{ when } I \text{ is assumed constant}$$

$$C = l\Sigma\frac{\Delta r}{I} - \Sigma\frac{r\Delta r}{I} = \frac{l}{2I}(l-2r_o) \text{ when } I \text{ is assumed constant}$$

$$D = \Sigma\frac{\Delta r}{I} = \frac{l}{I} \text{ when } I \text{ is assumed constant.}$$

For the torsion of a spoke, we note the slope of the rim along the rim at any spoke is

$$\text{Slope} = \frac{dy}{Rd\alpha} = \frac{d(\phi\cdot R\cos\alpha)}{Rd\alpha} = -\phi\sin\alpha$$

Therefore, the torsional moment reaction on the wheel rim for any spoke is

$$T = \frac{G\phi\sin\alpha}{\int\frac{dl}{J}} = K\phi\sin\alpha \quad . \ . \ . \ . \ . \ . \ . \quad [15]$$

where

$$J = \frac{\pi}{16} \frac{a^3 b^3}{a^2+b^2}$$

for an elliptic section, a and b being major and minor axes, and

$$J = k \frac{a^3 b^3}{a^2+b^2}$$

for a rectangular section, a and b being the long and short sides, respectively, where

$$k = 3.65-0.6 \frac{a}{b}$$

Then for the equilibrium of the wheel rim, if Y = lateral force,

$$YR_m = M_o+2M_1 \cos \alpha_1 \ldots .2M_n \cos \alpha_n + 2T_1 \sin \alpha_1 \ldots .2T_n \sin \alpha_n \quad . \; . \; . \quad [16]$$

$$Y = S_0+2S_1 \ldots .2S_n \quad . \; . \; . \; . \; . \; . \; . \; . \quad [17]$$

where

$$M = SR_m - M_b \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [18]$$

From Equations [13] and [14]

$$S = -\frac{D\delta + B\phi \cdot \cos \alpha}{AD-BC} \cdot E = C_3\delta + C_4\phi \cos \alpha$$

and

$$M_b = -\frac{C\delta + A\phi \cos \alpha}{AD-BC} \cdot E$$

where

$$C_3 = -\frac{DE}{AD-BC}$$

$$C_4 = -\frac{BE}{AD-BC}$$

Therefore

$$M = C_1\delta + C_2\phi \cos \alpha \text{ (lb-in.)}$$

where

$$C_1 = \left(\frac{C-DR_m}{AD-BC}\right)E; \qquad C_2 = \left(\frac{A-BR_m}{AD-BC}\right)E$$

Hence Equation [16], becomes

$$YR_m = nC_1\delta + (1+2 \cos^2 \alpha_1 \ldots . +2 \cos^2 \alpha_n)C_2\phi + 2K(\sin^2 \alpha_1 \ldots . + \sin^2 \alpha_n)\phi \quad . \; . \; . \quad [19]$$

and Equation [17], becomes

$$Y = nC_3\delta + C_4\phi(1+2 \cos \alpha_1 \ldots . +2 \cos \alpha_n) \quad . \; . \quad [20]$$

from which we may solve for δ and ϕ, and thus the bending moment and shear are readily obtained from the previous equations.

32 Other types of wheels that should be checked for lateral strength are disk-wheel centers as used in automobiles, etc. Assuming a maximum lateral load F and with R (in.) equal to radius of wheel, I equal to moment of inertia across a diametric section for lateral bending, and e equal to distance to outer fiber of section, then, for the maximum fiber stress,

$$f = \frac{6}{5} \frac{FR \cdot e}{I} \text{ (lb. per sq. in.)}$$

where $f = 10{,}000$ lb. per sq. in. for a rolled-steel disk wheel.

33 The stress along the outer fiber even for equal values of e is not uniform and the factor 6/5 as used by Bach, for cylinder heads, pistons, etc., takes account very approximately for the more or less concentration of the stress at different parts of the section.

Hubs and Crank Arms

34 In wheels subjected to pure torque with solid hubs, the hubs transmit the turning effort partly by the tangential friction along the periphery of the fit and partly as a concentrated tangential load on the key, the proportion of the loading depending on the nature of the press or shrinkage fit. It would seem desirable to relieve any concentration of loading at the keyway as far as possible, depending on the key rather as a safety device. Then the press fit must be sufficient to give a tonnage comparable with the maximum friction force at journal radius.

35 If F = tonnage, T = maximum torque, lb-ft., d = diameter of journal, in., and l = length of fit, in., then

$$F = 0.012 \frac{T}{d} \text{ tons,}$$

and assuming a coefficient of friction η, the intensity of normal pressure is

$$p_\eta = \frac{24}{\pi} \frac{T}{d^2 l \eta} \text{ (lb. per sq. in.)}$$

36 Since p is directly proportional to the allowance used, it is evident that the length of fit is primarily dependent upon the allowance to be used and the coefficient of friction at the fit.

37 In built-up flywheels, armatures, etc., the nave or hub is separated and usually clamped together by bolts or rings shrunk on around the hub. For such a construction, the torque loading is mainly transmitted by the keyway. For this reason the keys, preferably flat keys bearing on flat surfaces on the shaft, should be distributed around the shaft, say, with four for large shafts and two for small shafts. The bolts holding the hub portions together can be proportioned on the unbalanced centrifugal loading of a separated part of the flywheel, but with a shrunk ring

the tensile loading on the ring due to shrinkage fit is the all-important loading on the ring.

38 When a wheel fit resists lateral loads as well, discounting the lateral friction at the key, we have as in locomotive wheels large longitudinal loads along the axle to be resisted by the pressure fit. The tangential component resisting the torque in the plane of the wheel is then augmented in some such ratio as $\sqrt{F_t^2+F_l^2}:F_t$, where F_t = total tangential component (lb.) and F_l = total longitudinal force. Thus either the allowance or the length of fit must be increased or both. In a locomotive, however, the torque loadings are always small compared with the side-thrust and vertical loadings.

39 Finally, lateral bending of the plane of the wheel, combined usually with some direct loading, causes a change of pressure distribution at the fit surface. At one part the radial pressure is augmented, while at a diagonal point the radial pressure of the

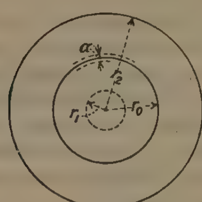
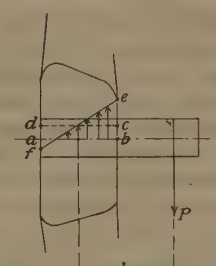


Fig. 7



Reaction of Hub on Pin
Fig. 8.

fit is relieved. It is not likely that these stresses can be added algebraically since the bending loading places the hub under a different elastic deformation. Moreover, with large allowances, this additional loading stresses concentric layers adjacent the fit beyond the elastic limit. This particular phase offers a channel for a very interesting research.

40 In the design of a hub subjected to a pressure fit we are concerned with two aspects: The strength of the hub (1) to resist the hoop tension imposed by the allowance used, and (2) to provide sufficient radial pressure at the fit to give sufficient frictional resistance to slippage under the maximum loadings. The former is very little affected by the outer diameter of the hub, since the maximum hoop tension is in major part affected only by the allowance used and is a local phenomenon at the differential annuli at the fit. The latter, however, does depend especially for ratios of outer diameter to inner diameter of hub below 2 on the particular ratio used. The radial pressure falls off very rapidly when the ratio of outer diameter to inner diameter falls below 1.6,

and more so with steel hubs than with cast-iron hubs. The necessary length of hub is obviously dependent upon the amount of frictional resistance that is needed, once the allowance is fixed. In Fig. 7, let

r_o = radius at fit surface = normal diameter of shaft, in.
r_1 = inner radius of shaft as with bored axle, in.
r_2 = outer radius of hub, in.
δ = allowance used, in.
p_r = Radial pressure, p_t = tangential tension, lb. per sq. in.
E = modulus of elasticity for shaft = 30,000,000, in.[4]
E' = modulus of elasticity for hub, in.[4]

From the fundamental equations for radial pressure and hoop tension,

$$p_r = \frac{B}{r^2} - A, \text{ and } p_t = \frac{B}{r^2} + A$$

We have immediately Lamé's formula, for the maximum hoop tension for a ring subjected to an internal pressure p, that is

$$p_t(\text{max.}) = p\left[\frac{r_2^2 + r_1^2}{r_2^2 - r_1^2}\right]$$

which is a form convenient to remember.

41 Tracing the analysis from the fundamental formula, it is evident that this applies to the adjacent concentric ring at the applied pressure. By the same reasoning the maximum hoop compression is, for a ring subjected to an outer pressure at outer radius R_2,

$$p_r(\text{max.}) = p\left[\frac{r_2^2 + r_1^2}{r_2^2 - r_1^2}\right]$$

42 In a compound cylinder, the allowance is the difference between the radii under no stress. When put under stress during the press fit, the outer ring is stretched and the inner compressed at the common radius of contact; that is,

Increase in diameter at common radius for

$$\text{outer cylinder} = \frac{p_{ro}}{E}\left[\frac{r_2^2 + r_0^2}{r_2^2 - r_0^2}\right] \cdot 2r_0$$

Decrease in diameter at common radius for

$$\text{inner cylinder} = \frac{p_{ro}}{E}\left[\frac{r_0^2 + r_1^2}{r_0^2 - r_1^2}\right] \cdot 2r_0$$

Therefore the allowance, that is, the total change in diameters, is

$$\delta = \frac{p_{ro}}{E}\left[\frac{r_2^2 + r_0^2}{r_2^2 - r_0^2} + \frac{r_0^2 + r_1^2}{r_0^2 - r_1^2}\right] \cdot 2r_0$$

With a solid shaft at the center it is evident that $p_r = -p_t$; hence from the fundamental equations, $B = 0$, and subtracting, $A =$

$p_t = -p_r$, and the shaft is therefore under a common compression loading throughout. Calling this common pressure in the shaft p_1, then $p_1 = p_r = -p_t$ for shaft.

Increase in diameter at shaft radius for hub $= \left(\frac{p_t}{E'} + \frac{p_t}{mE'}\right) d$

Decrease in diameter at shaft radius for shaft $= \left(\frac{p_1}{E} - \frac{p_1}{mE}\right) d$

Therefore the allowance, that is, the total change of diameters, is,

$$\frac{\delta}{d} = p_1 \left\{ \frac{1}{E} (1-1/m) + \frac{1}{m'E'} \right\} + \frac{p_t}{E'}$$

But from Lamé's equation

$$p_t = p_t \left(\frac{r_2^2 + r_0^2}{r_2^2 - r_0^2} \right)$$

so the fundamental equation for a pressure fit on a solid shaft is

$$\frac{\delta}{d} = p_1 \left\{ \frac{1}{E} (1-1/m) + \frac{1}{E'} \left(\frac{r_2^2 + r_0^2}{r_2^2 - r_0^2} + \frac{1}{m'} \right) \right\}$$

Assuming $m' = m$ and with a steel hub $E = E'$, then

$$p_1 = \left(\frac{k^2 - 1}{2k^2} \right) E \frac{\delta}{d}$$

where

$$k = \frac{\text{Outer diameter of hub}}{\text{Shaft diameter}}$$

With a cast-iron hub, $E' = \frac{E}{2}$ approx., and, assuming $m' = m = 0.3$,

$$p_1 = \frac{k^2 - 1}{3.3k^2 + 0.7} E \frac{\delta}{d}$$

which for the hoop tension of steel hub,

$$f = p_t = \frac{k^2 + 1}{2k^2} E \frac{\delta}{d}$$

and for the hoop tension of a cast-iron hub,

$$f = p_t = \frac{k^2 + 1}{3.3k^2 + 0.7} E \frac{\delta}{d}$$

43 From these equations Table 1 and Curve 1 have been constructed for various values of k with a solid shaft.

44 When hubs are pressed on hollow steel axles, as frequently occurs in locomotive practice, etc., assuming steel on steel, the radial pressure

$$p_r = \frac{1}{\left[\frac{k^2 + 1}{k^2 - 1} + \frac{1 + k^2}{1 - k'^2} \right]} \cdot \frac{E\delta}{d} \text{ (lb. per sq. in.)}$$

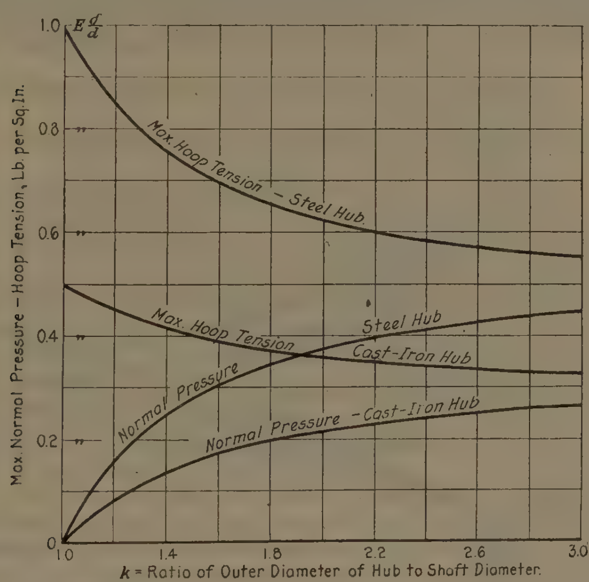
and the hoop tension in the hub

$$p_t = p_r \left[\frac{k^2+1}{k^2-1} \right]$$

where

$$k = \frac{\text{Outer diameter of hub}}{\text{Shaft diameter}} \; ; \; k' = \frac{\text{Shaft diameter of axle}}{\text{Hollow bore diameter of axle}}$$

45 The inner bore of a hollow-bore axle ranges from 37.5 per cent to 50 per cent of the outer diameter. Taking $k' = 0.375$ and



Curve 1 Pressures and Stresses at Fit

(E = modulus of elasticity, steel; δ = allowance; d = diameter of shaft.)

TABLE 1 COEFFICIENTS OF $E\frac{\delta}{d}$, SOLID SHAFT

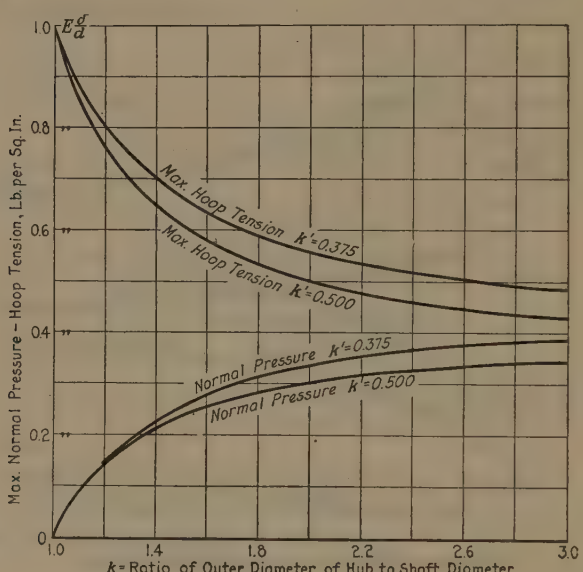
k	Normal pressure, steel on steel	Normal pressure, cast-iron on steel	Max. hoop tension, steel on steel	Max. hoop tension, cast-iron on steel
3.0	0.445	0.263	0.556	0.329
2.5	0.420	0.246	0.580	0.340
2.0	0.375	0.216	0.625	0.360
1.9	0.362	0.207	0.639	0.366
1.8	0.346	0.197	0.654	0.371
1.7	0.327	0.185	0.673	0.380
1.6	0.305	0.171	0.696	0.390
1.5	0.278	0.154	0.722	0.400
1.4	0.245	0.134	0.755	0.413
1.3	0.204	0.110	0.796	0.428
1.2	0.153	0.081	0.848	0.448

For cast iron $E' = \frac{E}{2}$ assumed.

0.500, respectively, Table 2 and Curve 2 have been constructed for various values of k.

46 With hollow-bore shafts, when the inner bore of shaft is $\frac{3}{8}$ of outer diameter, when $k = 2$ the pressure fit is reduced in holding power by 11 per cent, and when the hole is increased to $\frac{1}{2}$, the holding power is reduced by 20 per cent. Thus the allowance should be slightly increased accordingly as the diameter of the bore is increased.

47 With solid shafts, frequently no consideration is given to the flexibility of the shaft material, and the hoop tension is estimated merely on the allowance. By such a calculation with



Curve 2 Pressures and Stresses at Fit

(E = modulus of elasticity, steel; δ = allowance; d = diameter of shaft.)

TABLE 2 COEFFICIENTS OF $E\,\frac{\delta}{d}$, HOLLOW SHAFT

k	$k' = 0.375$ p_r	$k' = 0.375$ p_t	$k' = 0.500$ p_r	$k' = 0.500$ p_t
3.0	0.388	0.485	0.343	0.429
2.5	0.369	0.510	0.328	0.453
2.0	0.333	0.533	0.300	0.498
1.9	0.323	0.572	0.291	0.516
1.8	0.315	0.582	0.284	0.526
1.7	0.295	0.608	0.268	0.552
1.6	0.277	0.632	0.253	0.577
1.5	0.255	0.663	0.234	0.608
1.4	0.227	0.700	0.210	0.648
1.3	0.191	0.746	0.180	0.702
1.2	0.146	0.808	0.139	0.756

nominal hub values, the hoop tension is exaggerated by 60 per cent. Therefore it is felt that no simplification can be given to the above analysis since the elasticity of the shaft must be considered.

48 When the wheel center is subjected to lateral bending, there must be change in the distribution of normal pressure at the fit. Thus the uniform pressure on the lower half is increased directly by the load P, such that *abcd* (see Fig. 8) represents the direct increase in bearing load at the fit and the direct pressure is $P/(ab)D$ lb. per sq. in. In addition the fit must resist the bending moment Pl represented by a uniform pressure *dcef* when the intensity of pressure *ec* or *df* is $\dfrac{6Pl}{(ab)^2D}$ lb. per sq. in. Therefore the maximum pressure at b due to the loading P is

$\dfrac{P}{(ab)D}\left(1+\dfrac{6l}{ab}\right)$ lb. per sq. in., ab being the length of fit, which increases the pressure at the lower half.

49 A simple addition of the bending-load pressures to the normal pressure of the press fit is questionable, since the change in distribution of pressure may cause the normal pressure component due to the fit to change. Very often the maximum bending stress when added to the already existing hoop tension will result in the total stress at b exceeding the elastic limit, and for this case obviously no simple addition is possible. When the elastic limit is exceeded there is a weakening of the fit, but this can be remedied when due to large lateral bending loads by lengthening the fit, decreasing the allowance, or both.

50 From these curves, it is evident that the hoop tension, which is a maximum at the fit, is not materially affected even by large variations of k, i.e., the ratio of outer diameter of hub to shaft diameter, but depends primarily on the allowance used. On the other hand, the normal or radial pressure is found to decrease rapidly when k becomes less than 2. When the value of k falls below 1.7 the normal pressure falls off very rapidly and lower ratios should not be used. To recapitulate, the allowance is fixed by the allowable hoop tension of the material, whereas the normal pressure can be further modified considerably by a proper ratio of outer diameter to inner diameter of hub.

51 Finally the required tonnage of the press fit is dependent also on the peripheral area of the fit and the coefficient of friction. From experiments made by Sanford A. Moss, the estimated coefficient of friction for a tallow-grease fit was given as 0.038 and therefore the tonnage would be

$$T = 6\times 10^{-5} dlp_1 \text{ tons}$$

where d = diameter shaft (in.) and l = length of fit (in.).

52 Since the allowance is fixed by the allowable hoop tension, which for the normal pressure is fixed by the ratio k and the allow-

ance for a given tonnage, it is at once evident there must be a fairly definite value for l.

53 However, it is important to note that the coefficient of friction may vary considerably, depending upon the nature of the surfaces at the fit, the grease used, etc. Thus the tonnage required to press off wheels from axles may exceed 3 to 4 times the original maximum press fit.

54 The keyway prevents a symmetrical distribution of hoop tension around the periphery, resulting in a weakening of the fit.

55 It is to be noted that when k approaches unity, i.e., for a thin band, the hoop tension approaches $E\frac{\delta}{d}$, which amounts to neglecting the compression of the shaft. This applies to shrink bands on flywheel hubs.

56 Fortunately with a given allowance with ordinary hub proportions, the hoop tension of cast-iron hubs is roughly half that of steel hubs. Since the allowable tensions can also be taken in the ratio of 2 to 1 this accounts for finding frequently the same allowance with either steel or cast iron.

57 Allowances used in practice vary from 0.001 in. per inch of diameter to 0.0025 in. per inch. This gives hoop tensions with steel hubs from 18,700 lb. per sq. in. to as high as 47,000 lb. per sq. in., the latter being practically the elastic limit. In ordinary engine cranks and hubs, allowances of 0.0015 to 0.002 in. per inch of diameter are used, and in the locomotive wheel centers the allowance may reach 0.0025 in. per inch diameter. The allowance for cast-iron hubs should never exceed 0.001 in. per inch of diameter.

58 With tapered fits, the allowance is taken up by drawing in on the taper. If t is the taper on the diameter, then $t \times \text{draw} =$ allowance. Thus for a given allowance and taper, the draw or pull up on the taper can be simply determined.

59 On shrinkage fits the minimum temperature, if the coefficient of expansion is taken at 0.0000065 in. per deg. fahr., is

$$t = 1.54 \times 10^5 \times \frac{\delta}{d} \text{ deg. fahr.}$$

Flywheel Hubs

60 Split hubs are used with large flywheels, even with the remainder of the wheel cast solid, in order to reduce shrinkage stresses. Built-up wheels are cast in segments and joined both at hub and rim by bolts and shrink rings. Assuming a wheel split on a diametric section or a solid-rim wheel, neglecting the reinforcement of the rim, then approximately the radial load at the split diametric section of the hub is,

$$P = \tfrac{1}{2}\frac{W}{g} w^2 \frac{2}{\pi} R = 0.000107 n^2 R W \text{ lb.}$$

where W = total weight of flywheel, lb.
R = rim radius, ft., and
n = r.p.m.

61 This loading is distributed on the bolts and shrink rings. If no allowance is given for the rings, the bolts can be stressed to 3000 to 4000 lb. per sq. in. In estimating the normal or gripping pressure of the nave or hub on the shaft, let D = diameter of shaft, l = length of hub, a = cross-section of shrink ring, n = number of shrink rings, and D_1 = outer diameter of hub.

62 Then, if k is a fraction to allow for the compressibility of the hub and the decrease in shrinkage tension due to the centrifugal force of the flywheel segment, the gripping pressure on the shaft is approximately

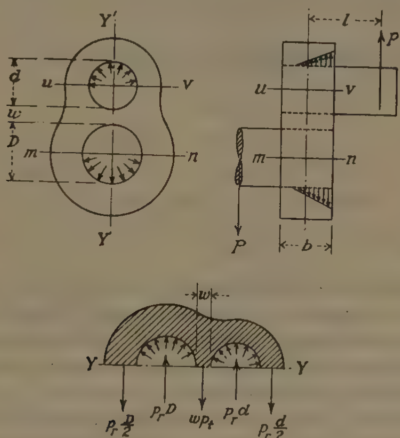


Fig. 9

$$p = \frac{nkE\delta a}{DD_1 l} \text{ lb. per sq. in.}$$

when assumed uniformly distributed and neglecting the radial pressure on the keys.

Proportion of Cranks

63 In a locomotive wheel the crank arm is cast as an integral part of the wheel center; while in general crank-arm proportions are basically determined from the principles used in hub proportioning. In a locomotive wheel center, spokes, rim, etc., as will be noted from the curves of hoop tension and radial pressure, have little effect on the proportions of the hub itself and may therefore be neglected. Additional bending stresses, however, are introduced due to lateral bending of the crank arm and hub caused by the offset load on the pin. Thus the lateral bending moment

on the crank is Pl combined with a direct tension P between the pin and axle. See Fig. 9.

64 At the dead-center position of crank the load on the upper half of pin hub and lower half of axle hub consists of a varying distributed pressure, the resultant of which is (reduced to center line)

$$\text{(1) A direct thrust } R = \frac{P}{2}\left[1 + \left(\frac{b^2 + 36l^2}{12bl}\right)\right] \text{ (lb.)}$$

$$\text{(2) A bending moment} = R\,\frac{b}{3}\left(1 - \frac{b}{12l}\right) \text{ (lb-in.)}$$

65 These are the bending moment and direct thrust that must be resisted at diametric sections uv and mn of pin and axle hubs, respectively,

66 Therefore, the maximum fiber stresses are

$$f = \frac{R}{(d_1 - d)b} + \frac{2Rb\left(1 - \dfrac{b}{12l}\right)}{(d_1 - d)b^2} \text{ lb. per sq. in. for section } uv$$

$$f = \frac{R}{(D_1 - D)b} + \frac{2Rb\left(1 - \dfrac{b}{12l}\right)}{(D_1 - D)b^2} \text{ lb. per. sq. in. for section } mn.$$

67 These stresses augment those due to the pressure or shrinkage fit of the axle and pin in the hubs. A section midway between the pins is subjected to a direct tension P and a bending moment Pl. In general it is possible to reduce the thickness b and the width across the section for the mid-section.

68 In locomotives, the stroke is frequently fixed by the diameters of axle and pin, and minimum width of metal between pin and axle; that is,

$$S = \frac{D + d}{2} + w$$

69 To obtain the minimum width w it is desirable to maintain the maximum hoop tension constant over the width w. The total tension on the section w is obviously half the sum of the projected radial pressures across the diameters D and d of the axle and pin, respectively, that is,

$$wp_t = p_r\,\frac{(D + d)}{2}$$

assuming same allowance per inch of diameter for both axle and pin. In the ordinary 2-to-1 ratio of hub diameter to axle diameter

$$p_t = \frac{5}{3}\,p_r$$

therefore

$$w = \frac{3}{5}\,\frac{D + d}{2} = 0.3(D + d) \text{ (in.)}$$

70 Decreasing w below this amount raises the hoop tension at w and causes an unequal distribution of radial pressure. Moreover, since the allowances used very often cause stresses nearly reaching the elastic limit, a marked decrease in w over the above value may result in this part being stressed over the elastic limit with a consequent loosening of the pin and axle.

Shrinkage of Tires and Compressive Stresses in Wheel Centers

71 In a shrinkage fit of a tire on a wheel, the internal diameter of the tire is made a small increment δ less than the diameter of the outer face of wheel center. The wheel center is therefore compressed an amount δ_2 on its external diameter while the tire is stretched an amount δ_1. The total difference δ between the original diameters of tire and wheel center at their fit, becomes,

$$\delta = \delta_1 + \delta_2 = \text{total shrinkage fit}$$

Assuming at first a symmetrical wheel center, with very small internal shrinkage stresses in the spokes, then, in Fig. 10, let

p_r = radial compression along rim, lb. per sq. in.
p_t = tangential hoop stress in tire rim, lb. per sq. in.
C = compressive load on spoke, lb.
C_r = compressive load in wheel rim, lb.
T = total tension in tire rim, lb.
p_c = spoke compression, lb. per sq. in.
p_{cr} = compression in wheel rim, lb. per sq. in.
n = number of spokes in wheel center
l = effective length of spoke, in.
A_t, A_s and A_r = cross-section of tire, spoke and rim, respectively, sq. in.
D = outer diameter of wheel center, in.

For the total load on one spoke,

$$C = 2(T - C_r) \sin\left(\frac{180}{n}\right) \text{ (lb.)}$$

For the compression of the wheel center, neglecting the compression of the hub as small,

$$\delta_2 = \frac{2p_c l}{E} = \frac{4(T - C_r)l}{A_s E} \sin \frac{180}{n} \text{ (approx.), (in.)}$$

For the compression of wheel rim,

$$\delta_2 = \frac{p_{cr}}{E} D$$

and for the stretch in the wheel tire,

$$\delta_1 = \frac{p_t}{E} D$$

Since $T = p_t A_t$, $C = p_c A_s$ and $C_r = p_{cr} A_r$, then,

$$p_c A_s + 2p_{cr} A_r \sin\frac{180}{n} = 2p_t A_t \sin\frac{180}{n} \quad . \quad . \quad . \quad . \quad [21]$$

$$\delta_2 = \frac{2p_c l}{E} = \frac{p_{cr}}{E} \cdot D \quad . \quad . \quad . \quad [22]$$

$$\delta_1 = \frac{p_t}{E} D \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [23]$$

$$\delta_1 + \delta_2 = \delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [24]$$

From these equations

$$p_c = \frac{2p_t A_t \sin\frac{180}{n}}{A_s + \frac{2A_r l}{D} \sin\frac{180}{n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [25]$$

And

$$p_t = \left[\frac{4A_t \sin\frac{180}{n}\, l}{A_s + \frac{2A_r l}{D} \cdot \sin\frac{180}{n}} + D\right] = \delta E \quad . \quad . \quad . \quad [26]$$

$$p_r = 2\frac{p_t A_t}{wD}, \text{ where } w = \text{width of rim} \quad . \quad . \quad . \quad . \quad [27]$$

72 As a numerical illustration, consider the following proportions of a 55-in. wheel center.

$\delta = 0.069$ in. $\quad D = 55$ in. $\quad l = 19$ in.
$A_s = 7.98$ (mean area) sq. in. $\quad A_r = 10.85$ sq. in.
$A_t = 21.5$ sq. in. $\quad w = 4.375$ in. $\quad n = 14$

$$\sin\frac{180}{14} = 0.222$$

Then

$$p_t = \left[\frac{4 \times 21.5 \times 19 \times 0.222}{7.98 + \frac{2 \times 10.85 \times 19}{55}\, 0.222} + .55\right] = 0.69 \times 30 \times 10^6$$

$$\therefore p_t = \frac{0.069 \times 30 \times 10^6}{92.6} = 22{,}300 \text{ lb. per sq. in.}$$

$$p_c = \frac{2 \times 22{,}300 \times 21.5 \times 0.222}{9.645} = 22{,}100 \text{ lb. per sq. in.}$$

$$p_r = \frac{2 \times 22{,}300 \times 21.5}{4.375 \times 55} = 4{,}000 \text{ lb. per sq. in.}$$

The compression of a spoke is $\dfrac{22{,}100}{30 \times 10^6} \times 19 = 0.014$ in.

73 If the flexibility of the rim and spokes is neglected, then the tension in the rim would be $\dfrac{0.069\times 30\times 10^6}{55} = 37{,}500$ lb. per sq. in.

74 This shows the necessity of considering the flexibility of the spokes and rim.

75 From Equation [25] it will be noted that the normal pressure at the rim increases with the area of the tire rim thus accounting for the loosening of tires after wear.

76 *Resultant Stress in Spokes.* This is dependent upon the initial shrinkage stress, which causes an initial tension in the spokes. The compression load due to shrinking on tires relieves this tension.

Counterbalance of Locomotive Wheel Centers

77 The design of a locomotive wheel center is closely related with the counterbalance problem, in fact the proportions of the

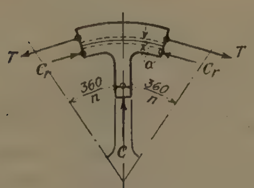


Fig.10

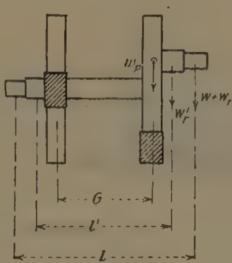


Fig.11

wheel center are determined to a considerable extent by the counterbalance required.

78 Two methods of counterbalancing are used: (1) Static balance, wherein the required weights are placed diametrically opposite the cranks in the wheel planes; and (2) Cross or dynamic balance, wherein any revolving unbalanced weight is balanced by two balancing component weights in the planes of the two wheels. In the balance of the revolving parts the former introduces unbalanced moments or couples which cause varying loadings at the rail and reactions at the pedestals, while in the latter these moments are completely balanced.

79 In a locomotive the reciprocating masses cause an unbalanced couple in a horizontal plane together with an unbalanced longitudinal force. Since a reciprocating weight may be considered as a component of a revolving weight, this component may thus be balanced by an equivalent revolving weight opposite in phase, either completely or partially at the expense of an unbalanced vertical component in a plane at right angles to the plane

of reciprocation. Thus to limit this vertical component, the reciprocating parts in the horizontal plane are only partially balanced.

80 In Fig. 11 let

W = weight of reciprocating balance
W_r = weight of revolving parts in plane of cylinder centers
W_r' = weight of revolving parts on main wheel at side-rod centers
W_r'' = weight of revolving parts on coupling driver at side-rod centers
W_p = weight of hub and pin in main driver
W_p' = weight of hub and pin in coupling driver
l = distance between cylinder centers
l' = distance between side-rod centers
G = distance between counterbalance centers

where the above weights are all reduced to crank radius.

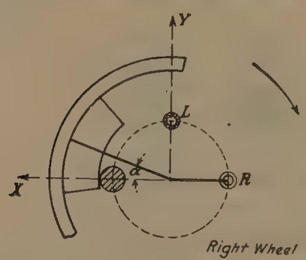


Fig. 12

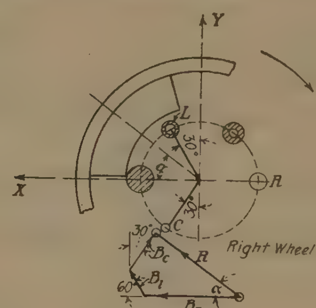


Fig. 13

81 *Two-Cylinder Locomotive,* cranks at right angles, assuming right crank leading left (See Fig. 12):

A — Counterbalance Components: Main Driver

1 Static balance at crank radius.
The balancing weight in wheel center reduced to crankpin radius diametrically opposite the crankpin, becomes,

$$R = W + W_r' + W_r + W_p$$

2 Cross or dynamic balance (right wheel) at crank radius.

$B_r = (W + W_r)\,\dfrac{l+G}{2G} + W_r' \cdot \dfrac{l'+G}{2G} + W_p$ component *opposite* right crank

$B_l = B_r - (W + W_r + W_r' + W_p)$ component *in phase* left crank

$\Sigma X = B_r \qquad \Sigma Y = B_l$

$R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2} \qquad \alpha = \tan^{-1} \dfrac{\Sigma Y}{\Sigma X}$ = angle of balance.

B — Counterbalance Components: Coupling Driver

1 Static balance at crank radius.

The balancing weight in this case is

$$R = W + W_r'' + W_p'$$

2 Cross or dynamic balance (right wheel) at crank radius.

$B_r' = W \dfrac{l+G}{2G} + W_r'' \dfrac{l'+G}{2G} + W_p'$ component *opposite* right crank

$B_r' = B_r' - (W + W_r'' + W_p')$ component *in phase* left crank

$\Sigma X = B_r' \qquad \Sigma Y = B_l'$

$R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2} \qquad \alpha = \tan^{-1} \dfrac{\Sigma Y}{\Sigma X} =$ angle of balance.

For the reciprocating weight W, we take

$$W = \frac{k \times \text{wt. of reciprocating parts}}{n}$$

where n = number of drivers

$k = 0.4$ to 0.66

82 In the balancing of a three-cylinder locomotive, with cranks phased at 120 deg. apart, the longitudinal force is balanced while the nosing is due to the unbalanced couple of the outside reciprocating masses in a horizontal plane, the center crank reciprocating mass obviously having no moment about a vertical axis through the center of gravity of the locomotive.

83 In the partial balance of the reciprocating masses, it is important to include the component for the center cylinder reciprocating mass as well since balancing the outside cylinder reciprocating masses alone would introduce an unbalanced longitudinal force. The unbalanced couple remains the same and is not affected by the center cylinder balance components.

84 Since the longitudinal vibration is eliminated on a three-cylinder locomotive, and the nosing due to the somewhat lighter reciprocating weight is slightly reduced over a two-cylinder locomotive, a smaller percentage of reciprocating balance can be used as compared with a two-cylinder locomotive.

85 *Three-Cylinder Counterbalance*, with cranks at 120 deg. and right leading left and left leading center crank: (See Fig. 13)

A — Counterbalance Components: Main Driver

$B_r = (W + W_r) \dfrac{l+G}{2G} + W_r' \cdot \dfrac{l'+G}{2G} + W_p$ component *opposite* right crank

$B_l = B_r - (W + W_r + W_r' + W_p)$ component *in phase* left crank

$B_c = \dfrac{W + W_{rc}}{2}$ component *opposite* center crank

where W_{rc} = unbalanced revolving weight of center crank

$$\Sigma X = B_r + B_l \cos 60° - B_c \cos 60° = B_r + (B_l - B_c) \cos 60°$$

$$\Sigma Y = (B_l + B_c) \sin 60°$$

$$R = \sqrt{\Sigma X^2 + \Sigma Y^2} \qquad \alpha = \tan^{-1} \frac{\Sigma Y}{\Sigma X}$$

B — Counterbalance Components: Coupling Driver

$B_r = W \dfrac{l+G}{2G} + W_r'' \dfrac{l'+G}{2G} + W_p'$ component *opposite* right crank

$B_c = \dfrac{W}{2}$ (see footnote)

$B_l = B_r - (W + W_r'' + W_p')$ component *in phase* left crank

$$X = B_r + (B_l - B_c) \cos 60°$$

$$Y = B_l \cdot \sin 60°$$

$$R = \sqrt{\Sigma X^2 + \Sigma Y^2} \qquad \alpha = \tan^{-1} \frac{\Sigma Y}{\Sigma X}$$

NOTE: Reciprocating component may be taken care of entirely on main driver.

86 Two types of counterbalance weight are ordinarily used in driving-wheel centers, (1) the segmental form and (2) the sector type. For the segmental type it may be readily shown that the chord AB of the balance is given very approximately by

$$AB = \sqrt[3]{\frac{12R \cdot r}{tw}}$$

where R = required counterbalance at crankpin radius, lb.
r = crank radius, in.
t = mean thickness of balance
w = weight per cu. in. of metal in balance

while for the sector type, the inner radius to the balance is given by

$$r_l = \sqrt[3]{r_m^3 - \frac{3Rr}{2tw \sin \theta}}$$

where θ = angular span of balance between spokes
r_m = rim radius of balance.

87 The above expressions are only very approximate guides, since with a more accurate calculation consideration must be given to the negative moments of the spokes where they intercept the balance and crank hub.

Inertia Loadings and Stresses — Flywheels

88 All wheels of course are subjected to inertia loadings, but the stresses resulting in certain types are negligible compared with that due to the loads which they transmit. Thus a locomotive driving wheel often reaches 250 r.p.m. and is an effective flywheel

but the journal and lateral loads cause very much larger stresses than those due to inertia loading. Wheels with heavy rims rotating at high linear velocities as in flywheels set up large inertia stresses both in the rim and arms. Such wheels have large polar moments of inertia and thus may be subjected to large inertia torque reactions, causing bending stresses in the plane of the wheel. Finally in exceptional cases we find in very high-speed wheels a gyroscopic action which causes lateral bending of the wheel center.

89 The centrifugal loading in a simple rim, neglecting the effect of the arms, causes a simple hoop tension in the rim.

Let w = angular velocity of the rim, radians per sec.
T = tension in rim, lb.
v = peripheral velocity, ft. per sec.
A = cross section of the rim, sq. in.
δ = density of the material, lb. per cu. in.
R = radius to center gravity of rim, ft.

Then

$$\frac{12\delta AR \cdot d\phi}{g} \omega^2 R = Td\phi$$

therefore

$$T = \frac{12\delta Av^2}{g} \text{ (lb.) and } f_t = \frac{12\delta v^2}{g} \text{ lb. per sq. in.}$$

90 Rules using this formula for the computation of limiting speeds of flywheels are erroneous, since in any actual flywheel the arms set up large bending stresses in the rim, while the tensile loading in the arms reduce the direct tension in the rim.

91 The angular acceleration or retardation of the rim causes bending stresses in the arms.

Let T = maximum torque acting on flywheel, lb-ft.
I = polar moment of inertia of entire flywheel
θ = angular displacement at any instant
ω = angular velocity, radians per sec.
R = mean radius of rim, in.
l = length of arm, in.

Then

$$T = I\frac{d^2\theta}{dt^2} = I\omega\frac{d\omega}{d\theta} \qquad \text{where } \omega = \frac{d\theta}{dt}$$

showing that the maximum acceleration can be obtained from the subnormal of the velocity-displacement curve. Then with n arms, the bending moment for any arm at the hub is

$$M = \frac{T}{n} l \text{ (lb-in.) in the plane of the wheel}$$

92 Finally the gyroscopic reaction of a high-speed wheel causes lateral bending. If the flywheel axis is suspended in a movable frame, any angular motion of this frame other than about the

flywheel axis, sets up a transverse torque on the flywheel equal to the rate of *change of the angular momentum.*

In Fig. 14 let M_g = gyroscopic torque = P (bearing reaction) $\times l$ Then if the axis (or frame suffers an angular displacement $d\phi$, the angular momentum vector changes to

$$I\omega' = I\omega(\text{approx.}) \text{ in direction } X'X'$$

The vectorial change in angular momentum is, $I\omega \sin d\phi = I\omega d\phi$. Hence the rate of change of angular momentum, equal to the gyroscopic torque, is,

$$Pl = M_g = \frac{d(I\omega d\phi)}{dt} = I\omega \frac{d\phi}{dt} = I\omega\psi \text{ (lb-ft.)}$$

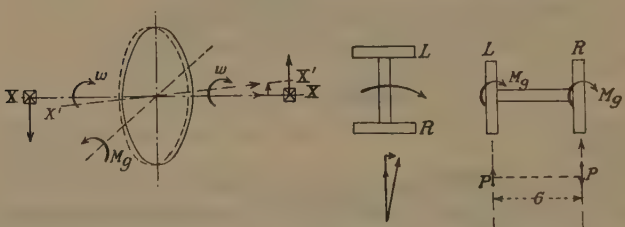


Fig. 14

Assuming the frame subjected to a simple harmonic oscillation, then

$$\phi = \phi_0 \sin \frac{2\pi}{T} \cdot t$$

And

$$\psi = \frac{d\phi}{dt} = \phi_0 \frac{2\pi}{T} \cos \frac{2\pi}{T} t$$

therefore

$$\psi \; max. = \phi_0 \frac{2\pi}{T}$$

93 Again in a vehicle wheel, the gyroscopic torque causes a change in the vertical loading at the rail or ground, that is,

$$PG = 2M_g = 2I\omega \frac{d\phi}{dt} = 2I\omega \frac{V}{R} = \frac{2IV^2}{rR} \text{ (lb-ft.)}$$

where G = track gage, ft., R = radius of curvature, ft., r = radius of driver, ft., and V = linear velocity of locomotive, ft. per sec.

94 The gyroscopic torque very slightly reduces the bending due to lateral flange reaction. It will be found only in exceptional cases where the gyroscopic reaction is appreciable, and therefore can be ordinarily entirely neglected.

Stresses in a Spoke Flywheel under Centrifugal Loading

95 Consider that portion of rim and arms between two adjacent arms. Due to symmetry the bending deflection is symmetrical with respect to either arm, and moreover the shears of the rim at the arms are equal and equal to half the tensile load in the arm.

In Fig. 15 let S_0 = initial shear at arm, lb.
S = shear at any section ϕ from arm, lb.
T_0 = initial tangential tension of rim at arm, lb.
T = tension in rim at any section ϕ from arm, lb.
M_0 = initial bending moment in rim at arm, lb-in.
M = bending moment in rim at section ϕ, lb-in.
A = cross-section of rim, sq. in.
δ = weight of rim per cu. in.
R = radius of rim, in.
v = peripheral velocity of rim, ft. per sec.
A_1 = mean cross-section of arm, sq. in.
E = modulus of elasticity
2α = total angle between two adjacent spokes.

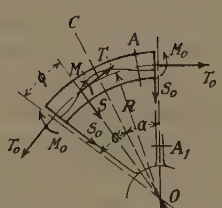


Fig. 15

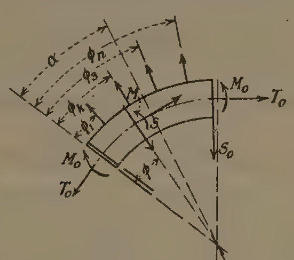


Fig. 16

Then the resultant centrifugal loading on the rim between two spokes is

$$2\int_0^\alpha \frac{12\delta A v^2}{g} \cdot \cos\phi d\phi = 24\,\frac{\delta A v^2}{g}\sin\alpha$$

and for the equilibrium of the rim, between two arms,

$$2S_0\cos\alpha + 2T_0\sin\alpha - 24\,\frac{\delta A v^2}{g}\sin\alpha = 0$$

therefore

$$T_0 = \frac{12\delta A v^2}{g} - S_0\cot\alpha$$

96 To determine the bending moment, consider the equation for the elastic deflection of the rim, noting that the slope due to symmetry is necessarily normal to the radius at the arms as well as midway between the arms, then

$$\int_0^\alpha \frac{Mds}{EI} = c\int_0^\alpha Md\phi = 0$$

Now

$$M = -2S_0R \cot\alpha \sin^2\frac{\phi}{2} + S_0R\sin\phi - M_0$$

$$= -2S_0R \left. \cot\alpha\left(\frac{1-\cos\phi}{2}\right) - \frac{\sin\phi}{2}\right] - M_0$$

$$= -S_0R\left[\cot\alpha - \frac{\cos(\alpha-\phi)}{\sin\alpha}\right] - M_0$$

therefore

$$\int_0^\alpha Md\phi = -S_0R[\alpha\cot\alpha - 1] - M_0\alpha = 0$$

Hence

$$M_0 = S_0R\left[\frac{1}{\alpha} - \cot\alpha\right]$$

From the equations for the equilibrium of a portion of the rim,

$$(S_0+S)\cos\frac{\phi}{2}\ (T_0+T)\sin\frac{\phi}{2} = \frac{24\delta Av^2}{g}\cdot\sin\frac{\phi}{2}$$

And

$$(T-T_0)\cos\frac{\phi}{2} - (S-S_0)\sin\frac{\phi}{2} = 0$$

The tension and shear in the rim are

$$T = \frac{12Av^2}{g} - S_0\frac{\cos(\alpha-\phi)}{\sin\alpha}$$

and

$$S = -S_0\frac{\sin(\alpha-\phi)}{\sin\alpha}$$

97 To calculate the initial shear S_0, consider the tension in the arm equal to $2S_0$ plus the centrifugal loading of the arm itself above the section under consideration. Then for the total tension at any section of the arm,

$$T_1 = 2S_0 + \frac{12\delta A_1v^2}{gR^2}\int_r^{R_m} rdr$$

$$= 2S_0 + \frac{12\delta A_1v^2}{gR^2}\left[\frac{R_m{}^2-r^2}{2}\right]$$

and for the total stretch or elongation of the arm at the rim,

$$\Delta R = 2S_0\int_{R_0}^{R_m}\frac{dr}{A_1E} + \frac{12\delta A_1v^2}{gR^2}\int_{R_0}^{R_m}\left(\frac{R^2-r^2}{2}\right)\frac{dr}{A_1E}$$

Neglecting the slight change of curvature of the rim, the total stretch of the rim between two arms is

$$\Delta(R\cdot 2\alpha) = 2\alpha\Delta R = \int_0^{2\alpha}\frac{TR}{AE}\,d\phi$$

Eliminating R from the preceding equations, and assuming the outer radius of arm R_m and rim radius R the same (approx.), then

$$S_0 = \frac{2\delta v^2}{g} \frac{\left[4R+3R_0\left(1-\frac{R_0^2}{3R^2}\right)\right]}{\left[\frac{2l}{A_1}+\frac{R}{A_a}\right]} \text{ (lb.)}$$

from which M_1S and T may be computed with the additional equations

$$M = -S_0R\left[\frac{1}{\alpha}-\frac{\cos(\alpha-\phi)}{\sin\alpha}\right] \text{ (lb-in.)}$$

$$S = -S_0\frac{\sin(\alpha-\phi)}{\sin\alpha} \text{ (lb.)}$$

$$T = \frac{12\delta Av^2}{g} - S_0\frac{\cos(\alpha-\phi)}{\sin\alpha} \text{ (lb.)}$$

98 During the acceleration of a flywheel with a heavy rim the arms are subjected to a torque in addition to the centrifugal loading. Ordinarily, however, the maximum acceleration occurs at low speed when the centrifugal loading is small, so the maximum loadings would not be superimposed.

Rotor for Rotating Field System in A-C. Generator

99 The equations of the flywheel-inertia loading must now be modified for the approximate concentrated loadings along the rim of the poles. The arms must be strengthened to transmit over full-load torque in addition to the centrifugal loadings that they sustain. Finally a direct loading is brought on the spokes due to possible unbalanced magnetic reactions along the air gap due to slight eccentricity of the rotor and armature. Since the sections of the rim, proportioned for magnetic conditions, are ordinarily much in excess of what is required for strength, we are primarily concerned only with the effect of the rim and pole inertia loadings on the arms. Assume the spokes symmetrically spaced between the arms.

In Fig. 16 let P = centrifugal pull of a field pole at angle ϕ_n with arm; then, using previous symbols, the tension in the rim at the arm is

$$T_0 = \frac{12\delta Av^2}{g} - S_0\cot\alpha + \tfrac{1}{2}P\Sigma\frac{\cos(\phi_n-\alpha)}{\sin\alpha}$$

and the bending moment at any section of the rim is

$$M = -S_0R\left[\cot\alpha - \frac{\cos(\alpha-\phi)}{\sin\alpha}\right] - M_0 - \Sigma P_nR\sin(\phi-\phi_n)$$

The equation of elastic deflection of the rim is complicated by the centrifugal reactions of the poles; thus

$$\int_0^\alpha Md\phi = \int_0^{\phi_1} Md\phi + \int_{\phi_1}^{\phi_2} Md\phi \ldots \int_{\phi_n}^{\alpha} Md\phi$$

since due to symmetry and the assumed symmetrical distribution of the poles between arms the integration need extend only between half the arc between the poles, and ϕ_k = angle of pole adjacent the mid-section. Hence

$$\int_0^\alpha Md\phi = -S_0R[(\alpha \cot \alpha - 1)] - M_0\alpha -$$

$$\int_{\phi_1}^{\alpha} PR \sin(\phi - \phi_1)d\phi \ldots - \int_{\phi_n}^{\alpha} PR \sin(\phi - \phi_k)d\phi = 0$$

Therefore

$$M_0 = S_0R\left[\frac{1}{\alpha} - \cot \alpha\right] + \frac{PR}{\alpha}[(\cos \alpha - \phi_1) \ldots \cos(\alpha - \phi_k) - k]$$

where $k = \frac{1}{2}$ number of poles between arms. Considering the equilibrium for a portion of the rim,

$$(S_0 + S)\cos\frac{\phi}{2} + (T_0 + T)\sin\frac{\phi}{2} = 24\frac{\delta A v^2}{g}\sin\frac{\phi}{2} + P\Sigma\cos\left(\phi_n - \frac{\phi}{2}\right)$$

$$(S - S_0)\sin\frac{\phi}{2} - (T - T_0)\cos\frac{\phi}{2} = P\Sigma\sin\left(\phi_n - \frac{\phi}{2}\right)$$

The tension and shear in the rim, in terms of the tension of the arm $2S_0$, are

$$T = \frac{12\delta A v^2}{g} - S_0\frac{\cos(\alpha - \phi)}{\sin \alpha} + P\Sigma\sin(\phi - \phi_n) + \frac{1}{2}P\frac{\cos\phi}{\sin\alpha}\Sigma\cos(\phi_n - \alpha)$$

$$S = -S_0\frac{\sin(\alpha - \phi)}{\sin\alpha} + P\cot\phi\Sigma\sin(\phi - \phi_n) - \frac{1}{2}P\frac{\sin\phi}{\sin\alpha}\Sigma\cos(\phi_n - \alpha) + \frac{P}{\sin\alpha}\Sigma\sin\phi_n$$

To obtain the shear in the rim at the arm, we proceed as before, neglecting the slight deformation of the heavy rim due to bending, then,

$$\Delta R = -\frac{1}{2\alpha}\int_0^{2\alpha}\frac{TR}{AE}d\phi$$

Therefore

$$\Delta R = \frac{R}{E}\left[\frac{12\delta v^2}{g} - \frac{S_0}{A\alpha}\right] - \frac{PR}{2\alpha AE}\left(\sum_0^n \cos(2\alpha - \phi_n) - \cos\alpha\Sigma\cos(\phi_n - \alpha)\right)$$

Equating this to the stretch of the arm obtained in the previous work gives the shear reaction S_0, and therefore the tension in the arm $T_1 = 2S_0$.

100 In addition we have the torque loading on the arms taken at twice full-load torque. The stresses due to bending under this torque loading must be added to the tension on the arm due to the rim.

101 The centrifugal pull of a field pole may be taken as

$$P = \frac{\pi^2 n^2}{900} \frac{w_p}{g} R_p (\text{lb.})$$

where

n = r.p.m.
w_p = weight of pole, lb.
R_p = radius to center of gravity, ft.

Inertia Proportions of Flywheels

102 This phase of the subject is extensive, and considerable literature has been devoted to it. However, a brief discussion of the essentials is consistent with the purpose of this paper.

103 In considering the effective moment of inertia, it is to be noted that the actual moment of inertia of the flywheel is augmented by that of all the other revolving parts. In general $I = Mk^2$, and the actual mass of the flywheel may be reduced to an equivalent mass at rim radius; that is, $I = Mk^2 = M_2R^2$. For the rim, the energy of the flywheel is

$$E_r = \frac{W_r}{2g} \omega^2 R^2 = \frac{W_r}{2g} V^2$$

for the arms,

$$E_a = \frac{W_a}{2gR} \omega^2 \int_0^R r^2 dr = \frac{W_a}{2g} \frac{V^2}{3} \text{ (approx.)}$$

hence $E = \left(W_r + \frac{W_a}{3}\right)\frac{V^2}{2g} = \left(W_r + \frac{W_a}{3}\right)\frac{R^2}{g}\omega^2$

where V = peripheral velocity of rim, (ft. per sec.)

$$M_e = \frac{1}{g}\left(W_r + \frac{W_a}{3}\right) \qquad I_e = \frac{R^2}{g}\left(W_r + \frac{W_a}{3}\right)$$

Plotting the variation of the engine torque; if E = *maximum* increased area in foot-pounds over the mean resistance, *assuming this resistance constant,* we have:

$$\Delta E = M_e \delta \cdot V^2 = I_e \delta w^2$$

where δ = total allowable speed variation $= \frac{V_2 - V_1}{V} = \frac{\omega_2 - \omega_1}{\omega}$

When the torque curve is reduced as by Fourier's series to the principal harmonics, we have, for the driving torque expressed in primary harmonics:

With single cranks

$$T = A + B \sin 2\theta - C \cos 2\theta$$

with two cranks

$$T = A' + B' \sin 4\theta - C' \cos 4\theta$$

and with multiple cranks

$$T = A'' + B'' \sin n\theta - C'' \cos n\theta$$

where $n =$ twice the number of cranks equally spaced (other than 180 deg.)

$A =$ resisting torque assumed constant.

For the equation of motion,

$$T - A = I \frac{d^2\theta}{dt^2} = I \frac{d\omega}{dt} = I\omega \frac{d\omega}{d\theta}$$

or in a simple case,

$$B \sin 2\theta - C \cos 2\theta = I\omega \frac{d\omega}{d\theta}$$

Since $I\omega \dfrac{d\omega}{d\theta} = \dfrac{dE}{d\theta}$, the minimum and maximum values of the kinetic energy E occur when $\dfrac{dE}{d\theta} = 0$, that is, when

$$B \sin 2\theta - C \cos 2\theta = 0$$

From this equation the limiting values of θ are obtained for the period of the unbalanced torque on the system; then

$$\int_{\theta_1}^{\theta_2} (T - A)\, d\theta = I\delta\omega^2$$

104 In many cases the resisting torque is not constant but is a definite function of the angle θ. If it is a periodic function of the angle θ, the resistance torque can be expressed in a harmonic series and then combined algebraically with the driving torque. Thus, if

$$T = A + B \cdot \sin 2\theta \ldots.$$
$$R = A' + B' \cdot \sin 2(\theta + \varepsilon) \ldots.$$

then $A = A'$ and $T - R = B \sin 2\theta \ldots . B' \sin 2(\theta + \varepsilon) \ldots .$

105 When part of the resistance torque is proportional to the difference between the angular displacement and that of the mean displacement, we have an elastic reaction introduced and therefore a possibility of large oscillations taking place when the angular velocity of rotation approaches the natural frequency of the system. In such a case as occurs in the driving of alternating-current generators, etc. the speed-variation limit based on the driving-

torque curve and the inertia of the flywheel is meaningless. What we are more concerned with is to avoid any possibility of an approach toward resonance, and next to maintain the amplitude of displacement rather than velocity as small as possible.

In Fig. 17 let

$\omega_s =$ synchronous angular velocity

$\omega = \dfrac{d\theta}{dt}$ = angular velocity of flywheel and rotor at any instant

$\theta_s =$ synchronous displacement, i.e., say, angular displacement of the rotating field produced by the stator of a-c. generators

$\theta =$ angular displacement of flywheel and rotor

$T_s =$ synchronous or elastic torque per unit angular displacement

$T_d =$ damping torque per unit angular velocity

$T =$ engine or driving torque impressed upon the system

$R =$ mean-load torque.

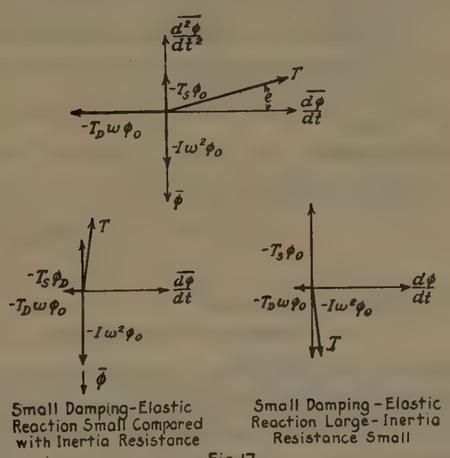


Fig.17

Then for the acceleration of the flywheel and other rotating parts,

$$T - T_s(\theta - \theta_s) - T_d\left(\frac{d\theta}{dt} - \omega_s\right) - R - = I\frac{d^2\theta}{dt^2}$$

where T is in the form,

$$T = T_0 + T_1 \sin(\omega t + e_1) + T_2 \sin(2\omega t + e_2) \ldots.$$
$$+ T_1' \cos(\omega t + e_1) + T_2' \sin(2\omega t + e_2) \ldots.$$

and obviously also

$$T_0 - R = 0$$

That is, the mean driving load equals the mean resistance. Let $\phi = \theta - \theta_s$ = relative angular displacement from the synchronous or mean position; then

$$\frac{d\phi}{dt} = \frac{d\theta}{dt} - \omega_s$$

and

$$I\frac{d^2\theta}{dt^2} = I\frac{d^2(\theta-\theta_s)}{dt^2} = I\frac{d^2\phi}{dt^2}$$

The equation of oscillation in terms of the relative displacement ϕ becomes

$$I\frac{d^2\phi}{dt^2} + T_d\frac{d\phi}{dt} + T_s\phi = T_n \sin(n\omega t + e) \ldots \text{(approx.)}$$

where $T_n \sin(n\omega + te) \ldots$ is the primary harmonic of the impulse of the torque curve.

106 The physical significance and interpretation of this equation can be shown by the use of a time-vector diagram. Considering for a moment the primary harmonic, its solution as in the form of

$$\phi = A \sin wt + B \cos wt$$

w = periodicity of engine torque harmonic.

That is the projection of a time vector $\phi = \sqrt{A^2+B^2}$ making an angle $wt + \tan^{-1}\frac{B}{A}$ with the initial reference line.

Differentiating twice,

$$\frac{d\phi}{dt} = Aw \cos wt - Bw \sin wt$$
$$= Aw(90° + wt) + Bw \cos(90° + wt)$$

and

$$\frac{d^2\phi}{dt^2} = Aw^2 \sin wt - Bw^2 \cos wt$$
$$= Aw(180° + wt) + Bw^2 \cos(180° + wt)$$

Thus the vector $\phi_0 = \sqrt{A^2+B^2}$ for the first and second derivative becomes rotated by 90 deg. and 180 deg., respectively, while its magnitude is increased to $w\phi_0$ and $w^2\phi_0$ respectively.

107 The elastic, damping, and inertia reactions, $-T_s\phi$, $-T_0\frac{d\phi}{dt}$ and $-I\frac{d^2\phi}{dt^2}$, are evidently reversed vectors proportional to $\overline{\phi}$, $\overline{\frac{d\phi}{dt}}$ and $\overline{\frac{d^2\phi}{dt^2}}$, respectively, and displaced 90 deg. from each other.

For the equilibrium of the time-vector diagram, evidently (Fig. 17)

$$T = \phi_0\sqrt{(Iw^2 - T_s)^2 + (T_d w)^2}$$

that is

$$\phi_0 = \frac{T}{\sqrt{(Iw^2 - T_s)^2 + (T_d w)^2}}$$

which gives the amplitude of oscillation. When Iw^2-T_s is large the amplitude is small, but when Iw^2-T_s approaches zero the amplitude becomes excessive, especially for small damping; that is, a resonance condition is approached.

108 Since the harmonic oscillations are independent, the preceding analysis immediately applies to the nth harmonic, and summing up the amplitude,

$$\phi = \frac{\Sigma\sqrt{T_n{}^2+T_n{}^{12}}\sin(nwt+e)}{\Sigma(I_n{}^2w^2-T_s)^2+(T_dnw)^2}$$

where T_n = amplitude of the primary harmonic torque.

109 The maximum amplitude would occur at a frequency when the elastic reaction balances the inertia force; that is,

$$In^2w_r{}^2\phi_n = T_s\phi_n \text{ for primary harmonic}$$

$$w_r = \sqrt{\frac{T_s}{In^2}} = \text{periodicity of fundamental at resonance.}$$

By the ordinary method, the elastic reaction is assumed nil, and with a simple harmonic impulse

$$\phi = \frac{T_n}{In^2w^2} = \text{initial amplitude}$$

Neglecting the damping as small and substituting the value T_n and T_s given above in the general equation, then

$$\phi_n = \frac{\phi}{1-\left(\frac{w_r}{w}\right)^2} = \frac{\phi}{1-\left(\frac{f_r}{r}\right)^2}$$

$$f = \text{frequency of primary harmonic}$$

where w_r and f_r = resonance periodicity and frequency.

110 This expression gives the amplification for a given frequency over the initial amplitude, i.e., as that occurring in a simple inertia system. The amplification may also be expressed in terms of the ratio of synchronizing torque or elastic reaction corresponding to the initial amplitude ϕ_a to the amplitude of the driving harmonic torque curve; that is,

$$\phi_n = \frac{T_n}{\frac{T_n}{\phi_a}-T_s} = \frac{T_n\phi_a}{T_n-T_s\phi_a} = \frac{\phi_a}{1-S}$$

where

$$S = \frac{T_s\phi_a}{T_n}$$

111 In a practical application, if the speed of rotation is given, and if we make the inertia of the flywheel such that

$$w = 1.41 w_r = 1.41 \sqrt{\frac{T_s}{In^2}} \text{ (radians per sec.)}$$

then the amplification becomes

$$\phi_n = \frac{\phi_a}{1-0.5} = 2\phi_a$$

where $\phi_a = \frac{T_n}{In^2w^2}$, T_n being the amplitude of the primary harmonic oscillations of the driving-torque curve.

112 In general the amplification of the displacement due to a simple inertia oscillation is given by the constant $\frac{1}{1-S}$, where S equals the ratio of the elastic reaction at the initial displacement, i.e., the displacement of a simple inertia system, to the amplitude of the impressed periodic force.

113 With steam and hydraulic turbines, multiple-crank engines, etc. the variation in driving torque is due primarily to variation in the steam supply set up by governor oscillations. The problem is therefore complicated by the dynamic oscillation of the governor. The oscillation of the rotor flywheel is

$$T - T_0 = I \frac{d^2\phi}{dt^2} + T_d \cdot \frac{d\phi}{dt} + T_s\phi$$

where T = driving torque and T_0 = load torque. Now T is a function of the steam-supply throttling, that is of the configuration of the governor mechanism, which is measured by the coordinate α. That is,

$$T = f(\alpha)$$

hence

$$T - T_0 = \Delta T = -\frac{\partial T}{\partial \alpha} \Delta\alpha = -\frac{\partial f(\alpha)}{\partial \alpha} \cdot \Delta\alpha$$

since T decreases as α increases. Therefore the dynamic equation of oscillation is

$$I \frac{d^2\phi}{dt^2} + T_d \frac{d\phi}{dt} + T_s\phi + \frac{\partial f(\alpha)}{\partial \alpha} \cdot \Delta\alpha = 0$$

From the kinetic reactions on the governor, we have, Fig. 18, (r being the governor gear ratio),

$$\frac{w}{g}(r\omega)^2 l^2 \frac{\sin 2\alpha}{2} - wl \sin\alpha - Sl \sin 2\alpha = \frac{w}{g} l^2 \frac{d^2\alpha}{dt^2}$$

where $w = \frac{d\theta}{dt}$ = angular velocity of rotor and flywheel and

$$2S \cos\alpha = Q + \frac{Q}{g}\frac{d^2y}{dt^2} + k \cdot \frac{dy}{dt}$$

where

$$\frac{dy}{dt} = 2l \sin \alpha \frac{d\alpha}{dt}$$

$$\frac{d^2y}{dt^2} = 2l \cos \alpha \left(\frac{d\alpha}{dt}\right)^2 + 2l \sin \alpha \frac{d^2\alpha}{dt^2}$$

Substituting for S and neglecting the term containing $\left(\frac{d\alpha}{dt}\right)^2$ as small, then

$$\left[\frac{w}{g} l + 2 \frac{Q}{g} l \sin^2 \alpha\right] \frac{d^2\alpha}{dt^2} + 2kl \sin^2 \alpha \frac{d\alpha}{dt} + (Q+w) \sin \alpha$$
$$- \frac{w}{g} r^2 l \frac{\sin 2\alpha}{2} \cdot \omega^2 = 0$$

For the variation of α and ω from their mean values, we have

$$\alpha = \alpha_0 + \Delta\alpha \text{ and } \omega = \omega_0 + \frac{d\phi}{dt}$$

hence

$$\frac{d\alpha}{dt} = \frac{d(\Delta\alpha)}{dt}, \quad \frac{d^2\alpha}{dt^2} = \frac{d^2(\Delta\alpha)}{dt^2}$$

and

$$\omega^2 = w_0^2 + 2\omega_0 \frac{d\phi}{dt}$$

$$\sin \alpha = \sin \alpha_0 + \Delta\alpha \cdot \cos \alpha_0$$

$$\frac{\sin 2\alpha}{2} = \frac{\sin 2\alpha_0}{2} + \cos 2\alpha_0 \cdot \Delta\alpha$$

Substituting in the above equation and noting that in the neutral position for equilibrium

$$Q + w = \frac{w}{g} r^2 w^2 l \cos \alpha_0$$

then

$$A \frac{d^2(\Delta\alpha)}{dt^2} + B \frac{d(\Delta\alpha)}{dt} + C\Delta\alpha + D \frac{d\phi}{dt} = 0$$

where

$$A = \frac{w}{g} l + \frac{2Q}{g} l \sin^2 \alpha_0$$

$$B = 2kl \sin^2 \alpha_0$$

$$C = (Q+W) \cos \alpha_0 - \frac{w}{g} r^2 l w_0^2 \cos 2\alpha_0 = \frac{w}{g} r^2 l w_0^2 \sin^2 \alpha_0$$

$$D = \frac{w}{g} r^2 l \sin 2\alpha_0 \cdot \omega_0$$

We have therefore the solution of two simultaneous differential equations

$$I\frac{d^2\phi}{dt^2}+T_d\frac{d\phi}{dt}+T_s\phi+\frac{\partial T}{\partial\alpha}\Delta\alpha=0\quad . \; . \; . \; . \quad [28]$$

$$A\frac{d^2(\Delta\alpha)}{dt^2}+B\frac{d(\Delta\alpha)}{dt}+C\Delta\alpha+D\frac{d\phi}{dt}=0\quad . \; . \quad [29]$$

Differentiating [28] we have

$$I\frac{d^3\phi}{dt^3}+T_d\frac{d^2\phi}{dt^2}+T_s\frac{d\phi}{dt}+\frac{\partial T}{\partial\alpha}\frac{d(\Delta\alpha)}{dt}=0$$

where $\frac{d\phi}{dt}$ is obtained from [29] and $\frac{d^2\phi}{dt^2}$ and $\frac{d^3\phi}{dt^3}$ by differentiating [29] successively. Hence the final equation reduces to

$$k_4\frac{d^4(\Delta\alpha)}{dt^4}+k_3\frac{d^3(\Delta\alpha)}{dt^3}+k_2\frac{d^2(\Delta\alpha)}{dt^2}+k_1\frac{d(\Delta\alpha)}{dt}+k_0\Delta\alpha=0$$

where

$$k_4=-I\frac{A}{D}$$

$$k_3=\frac{1}{D}(IB+T_0A)$$

$$k_2=-\frac{1}{D}(IC+T_dB+T_sA)$$

$$k_1=\frac{1}{D}\left(T_dC+T_sB-\frac{\partial T}{\partial\alpha}\right)$$

$$k_0=-\frac{T_sC}{D}$$

114 When two alternators operate in parallel, oscillations about their mean or synchronous position set up a mutual electromagnetic reaction between the two, and proportional to their relative displacement. (See Fig. 19.)

115 Neglecting the thermic resistance of the machines as small compared with their reactance Z_a, since $E_2=E_1=E_2'=$ the equal induced e.m.f. of the machines, then approximately,

$$I_s=\frac{E_r}{2Z_a}=\frac{E_1\sin\frac{\theta_s}{2}}{Z_a}$$

Since I_s bisects the angle θ_s,

$$\text{Hp.}=\frac{T_s\theta_s w_0}{550}=\frac{mEI_s\cos\frac{\theta_s}{2}}{746}$$

therefore

$$T_s\theta_s=0.37\frac{MEI_0\sin\theta_s}{w_0}$$

where m = number of phases and $I_0 = \dfrac{E}{Z_a}$ = short-circuit current. Therefore, as $\sin \theta_s = \theta_s$ for small angles, and $\theta_s = \theta_2 - \theta_1$, then

$$C(\theta_1 - \theta_2) = T_s\theta_s = 0.37\,\frac{MEI_0}{\omega_0}\cdot\theta_s \text{ (lb-ft.)}$$

116 That is, the electromagnetic mutual torque reaction between the machines is proportional to their relative displacement.

117 For just a first approximation neglect the damping reactions as small; then the dynamic equations become

$$I\frac{d^2\theta_1}{dt^2} = T_1 - T_0 + T_s(\theta_2 - \theta_1)$$

$$I\frac{d^2\theta_2}{dt^2} = T_2 - T_0 + T_s(\theta_1 - \theta_2)$$

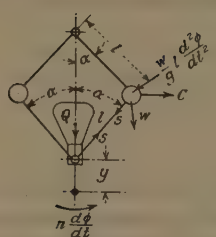


Fig. 18

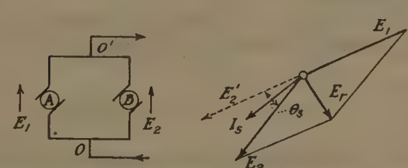


Fig. 19

Substituting the relative displacements $\phi_2 - \phi_1 = \theta_2 - \theta_1$ in place of the absolute and noting the primary harmonic torque as $T_{n1} = T_1 - T_0$ and $T_{n2} = T_2 - T_0$, then

$$I\frac{d^2(\phi_2 - \phi_1)}{dt^2} + 2T_s(\phi_2 - \phi_1) = T_{n2} - T_{n1} = T_{nr}$$

Thus we must combine the torque curves of both machines and plot the resultant primary harmonic of this combination. Then the maximum amplitude would occur when

$$\omega = \omega_r = \sqrt{\frac{2T_s}{In^2}}$$

where n = harmonic of the combined torque curve.

118 When the alternators are driven by a turbine, the oscillations are set up by the governor control. Then the equations of oscillations for the machines become

$$I\frac{d^2\phi_1}{dt^2} = \frac{\partial T}{\partial \alpha}\cdot\Delta\alpha_1 + T_s(\phi_2 - \phi_1)$$

$$I\frac{d^2\phi_2}{dt^2} = \frac{\partial T}{\partial \alpha}\cdot\Delta\alpha_2 + T_s(\phi_1 - \phi_2)$$

Then, as before,

$$I \frac{d^2(\phi_2-\phi_1)}{dt^2} = \frac{\partial T}{\partial \alpha} (\Delta\alpha_2 - \Delta\alpha_1) - 2T_s(\phi_2-\phi_1) \quad . \quad . \quad [30]$$

while the equations of the governor oscillations are

$$A \frac{d^2(\Delta\alpha_1)}{dt^2} + B \frac{d(\Delta\alpha_1)}{dt} + C\Delta\alpha_1 + D \frac{d\phi_1}{dt} = 0 \quad . \quad . \quad [31]$$

$$A \frac{d^2(\Delta\alpha_2)}{dt^2} + B \frac{d(\Delta\alpha_2)}{dt} + C\Delta\alpha_2 + D \frac{d\phi_2}{dt} = 0 \quad . \quad . \quad [32]$$

Differentiating [30] and substituting the values of $\frac{d\phi}{dr}$, $\frac{d^2\phi}{dt^2}$ and $\frac{d^3\phi}{dt^3}$ by successive differentiation of [31] and [32], respectively, we have a differential equation of the fourth degree as in the previous form

$$k_4 \frac{d^4v}{dt^4} + k_3 \frac{d^3v}{dt^3} + k_2 \frac{d^2v}{dt^2} + k_1 \frac{dv}{dt} + k_0 v = 0$$

where

$$v = \Delta\alpha_2 - \Delta\alpha_1$$

119 In the inertia proportioning of flywheels, the flywheel of moment of inertia I_1 may be connected with a rotor armature I_2 or other revolving mass with an elastic shaft between the two. If T_d = the driving torque, T_r the resisting torque, and T the neutral elastic torque of the shaft, then

$$T_d - T = I_1 \frac{d^2\theta_1}{dt^2}$$

$$T - T_r = I_2 \frac{d^2\theta_2}{dt^2}$$

and if $\phi = \theta_1 - \theta_2$ the relative displacement then

$$T = c\phi$$

Solving and substituting for ϕ,

$$T = -\frac{I_1 I_2}{I_1+I_2} \cdot \frac{d^2\phi}{dt^2} + \frac{I_2}{I_1+I_2} \cdot T_d + \frac{I_1}{I_1+I_2} T_r$$

If $T_d = T_r$ (approx.) then,

$$T = -\frac{I_1 I_2}{I_1+I_2} \cdot \frac{d^2\phi}{dt^2} + T_d = c\phi$$

Therefore, the equation of oscillation is

$$I_e \frac{d^2\phi}{dt^2} + c\phi = T_d$$

120 Dangerous oscillations would occur at the natural frequency of the system, that is, when

$$f = \frac{1}{2\pi}\sqrt{\frac{C}{I_e}} \qquad \text{where } I_e = \frac{I_1 \cdot I_2}{I_1 + I_2}$$

121 The same procedure can be used for three masses, though the simplest derivation is by considering the natural oscillations about the nodes of the shaft. Thus the angular displacement along the shaft is proportional to the distance from the node, and for one mode of oscillation in one direction and the middle mass in the opposite. The nodes will be taken at x and y from the end masses, respectively, along the shaft connecting the middle mass. The elastic reactions on the end masses are respectively $\frac{GJ_1}{x}\theta_1$ and $\frac{GJ_2}{y}\theta_2$, while that on the middle mass is the sum of the reactions of these two, where G = torsional modulus, J = polar moment of inertia of shaft, and θ is the angular displacement. The equations of motion are therefore

$$I_1\frac{d^2\theta_1}{dt^2} = -\frac{GJ_1}{x}\theta_1 = C_1(\theta_0 - \theta_1)$$

$$I_2\frac{d^2\theta_2}{dt^2} = -\frac{GJ_2}{y}\theta_2 = C_2(\theta_0 - \theta_2)$$

$$I_0\frac{d^2\theta_0}{dt^2} = -G\left(\frac{J_1}{l_1 - x} + \frac{J_2}{l_2 - y}\right)\theta_0 = C_1(\theta_1 - \theta_0) + C_2(\theta_2 - \theta_0)$$

Since

$$\frac{GJ_1}{x}\theta_1 = \frac{GJ_1\theta_0}{l_1 - x} \text{ and } \frac{GJ_2}{y}\theta_2 = \frac{GJ_2\theta_0}{l_2 - y}$$

A mode of oscillation is

$$\theta_1 = A_1 \sin wt$$
$$\theta_2 = A_2 \sin wt$$

and

$$\theta_0 = -A_0 \sin wt$$

Then

$$\omega = \sqrt{\frac{GJ_1}{I_1 x}} = \sqrt{\frac{GJ_2}{I_1 y}} = \sqrt{\frac{G}{I_0}\left(\frac{J_1}{l_1 - x} + \frac{J_2}{l_2 - y}\right)}$$

122 From which the nodal points may be computed and then the periodicity of the oscillation determined. Substituting directly for $\theta_1\theta_0$ and θ_2, then

$$I_0 A_0 = I_1 A_1 + I_2 A_2$$

and

$$A_1 = \frac{C_1 A_0}{I_1 w^2 - C_1} \qquad A_2 = \frac{C_2 A_0}{I_2 w^2 - C_2}$$

Therefore

$$I_1 I_2 I_0 w^4 - [C_1 I_2 (I_1 + I_0) + C_2 I_1 (I_2 + I_0)] w^2 + C_1 C_2 (I_1 + I_0 + I_2) = 0$$

or

$$Mw^4 - Nw^2 + P = 0$$

This is a quadratic equation with roots

$$w_1^2 = \frac{N + \sqrt{N^2 - 4MP}}{2M}$$

$$w_2^2 = \frac{N - \sqrt{N^2 - 4MP}}{2M}$$

where

$$M = I_1 I_2 I_0$$
$$N = C_1 I_2 (I_1 + I_0) + C_2 \cdot I_1 (I_2 + I_0)$$
$$P = C_1 \cdot C_2 (I_1 + I_2 + I_0).$$

which gives two modes of oscillation of natural frequencies.

$$f_1 = \frac{w_1}{2\pi} \text{ and } f_2 = \frac{w_2}{2\pi}$$

123 The inertia of the shaft may be taken care of approximately by adding one-third of its moment of inertia measured from a nodal point to the flywheel. Thus in two flywheel masses, the nodal point is located by

$$\frac{I_1}{I_2} = \frac{w_2}{w_1} = \frac{\theta_2}{\theta_1} = \frac{l_2}{l_1}$$

where $l_1 + l_2 = 1 =$ length of shaft. Hence

$$I_1' = \tfrac{1}{3}\left[\frac{I_2}{I_1 + I_2}\right] I_{shaft};\ I_2' = \tfrac{1}{3}\left[\frac{I_1}{I_1 + I_2}\right] I_{shaft}$$

for the equivalent inertia of the shaft located at either end.

APPENDIX

BENDING STRESSES IN THE RIM

124 Calculation of the bending stresses in the rim become extremely complicated for unsymmetrical loadings along the rim. With symmetrical loadings along the periphery the following analysis is of interest.

125 Assuming the symmetrical loading along the rim as a distributed tangential inertia loading along the rim due to the acceleration or retardation of the rim itself, we may closely approximate the bending in the rim and arms.

Let A = cross-section of rim, sq. in.
A' = cross-section of arm, sq. in.
$2a$ = total angle between adjacent rims
S = shear at any section in rim, lb.
M = bending moment at any section in rim, in-lb.
S_0 = shear at mid-section between arms, lb.
M_0 = bending moment at mid-section between arms, in-lb.
T = total tangential stress for any section in rim, lb.
T_0 = total tangential stress at mid-section of rim, lb.
M_b = bending-moment reaction between arm and rim, in-lb.
S_b = common shear in arm, lb.
t = tangential or circumferential loading per inch along the circumference, lb. per sq. in.
w = angular velocity of wheel, radians per sec.
R = radius of rim, in.
δ = density of metal per cubic inch = 0.283 lb. per cu. in.
ϕ = angle to any section of rim measured from mid-section between arms
M_a = bending moment in arm at hub, in-lb.
R_0 = outer radius of hub, in.
r = radius to any section in arm, in.
I = moment of inertia of cross-section in rim, in.^{4}
I' = moment of inertia of cross-section in arm, in.^{4}

126 For the inertia loading along the rim,

$$t = \frac{\delta}{g} AR \frac{dw}{dt} \text{ lb. per in.}$$

Then for the equilibrium of the rim CD (Fig. 20),

$$2S_0 \sin a + S_b = 2tR \sin a \quad \ldots\ldots\ldots [33]$$

$$M_b - S_bR + 2tR^2a = 0 \quad \ldots\ldots\ldots [34]$$

For the bending moment at any section of the rim,

From C to B, $M = M_0 - S_0R \sin \phi$, approximately . . . . [35′]

From B to D, $M = M_0 + S_0R \sin (2a - \phi)$, approximately . [36′]

Due to the symmetry of loading, evidently T_0 is the mean tension. Then,

$$\Delta R = \frac{T_0R}{nEA} = \frac{2T_0 \sin a}{EA'} \text{ ; } n = \text{no. of arms.}$$

Hence,

$$T_0\left(\frac{R}{nEA} + \frac{2 \sin a}{EA'}\right) = 0$$

Therefore, $T_0 = 0$ for the assumed symmetrical loading. Substituting S_0 from Eq. [33] in Eq. [35′] and Eq. [36′] respectively,

$$\text{From } C \text{ to } B, M = M_0 - tR^2 \sin \phi + \frac{S_b R \sin \phi}{2 \sin \alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [35]$$

$$\text{From } B \text{ to } D, M = M_0 + tR^2 \sin (2\alpha - \phi) - \frac{S_b R \sin (2\alpha - \phi)}{2 \sin \phi} \quad . \quad [36]$$

127 Since the spacing of the arms or spokes is ordinarily not less than 6, then the maximum value of $\alpha = 30$ deg. If we substitute for $\sin \phi$ and $\sin (2\alpha - \phi)$, the angles of ϕ and $2\alpha - \phi$, the maximum error would only be in the order of 4 per cent. Therefore [35] and [36] may be simplified to,

$$\text{From } C \text{ to } B, M = M_0 - tR^2\phi + \frac{S_b R \sin \phi}{2 \sin \alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [37]$$

$$\text{From } B \text{ to } D, M = M_0 + tR^2(2\alpha - \phi) - \frac{S_b R \sin (2\alpha - \phi)}{2 \sin \alpha} \quad . \quad . \quad [38]$$

which are the fundamental equations for bending in the rim.

128 From a consideration of the nature of the deformation of the rim it would be expected that due to symmetry points of contraflexure

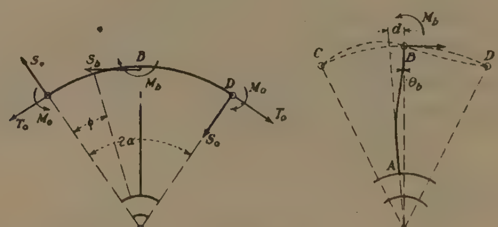


Fig. 20 Fig 21

along the rim would occur at mid-sections between adjacent arms, that is at those sections $M_0 = 0$. This may be readily proved from the deformation equation for the change of slope between symmetrical sections as follows:

$$\int_C^D M\,dS = \int_C^B M\,dS + \int_B^D M\,dS = 0$$

Hence,

$$\int_C^B M\,d\phi = -\int_B^D M\,d\phi$$

where

$$\int_C^B M\,d\phi = M_0\alpha - \frac{tR^2\phi^2}{2} + S_b R \frac{1 - \cos \alpha}{2 \sin \alpha}$$

$$\int_B^D M\,d\phi = M_0\alpha + \frac{tR^2\phi^2}{2} - S_b R \frac{1 - \cos \alpha}{2 \sin \alpha}$$

Hence $M_0 = 0$.

The equations of bending along the rim, therefore, simplify to,

$$\text{From } C \text{ to } B, M = -tR^2\phi + \frac{S_b R \sin \phi}{2 \sin \alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [39]$$

$$\text{From } B \text{ to } D, M = tR^2(2\alpha - \phi) - \frac{S_b R \sin (2\alpha - \phi)}{2 \sin \alpha} \quad . \quad . \quad . \quad [40]$$

129 These equations are evidently of the same form when measured from a section of the rim at the arm and the curves of slope and deflection relative to a tangent along the rim are symmetrical, but reversed from C to B and D to B respectively. Therefore, the deflection is zero at C and B and D, respectively.

130 Let n = normal displacement of the elastic curve from the center line of the rim and S the circumferential displacement along the rim. Then for the slope and deflection of the elastic deformation of the rim, Fig. 21,

$$\frac{dn}{dS} = \int \frac{MdS}{EI} = \frac{R}{EI}\int M d\phi$$

$$\therefore \frac{dn}{dS} = \frac{R}{EI}\left[\frac{-tR^2\phi^2}{2} - \frac{S_bR\cos\phi}{2\sin\alpha} + C_1\right]$$

$$n = \frac{R}{EI}\left[\frac{-tR^2\phi^3}{6} - \frac{S_bR\sin\phi}{2\sin\alpha} + C_1\phi + C_2\right]$$

When $\qquad \phi = 0, \quad n = 0, \quad C_2 = 0$

When $\qquad \phi = 0, \quad n = 0, \quad C_1 = \dfrac{tR^2\alpha^2}{6} + \dfrac{S_bR}{2\alpha}$

The slope at B, $\left(\dfrac{dn}{dS}\right)_B$ is,

$$\left(\frac{dn}{dS}\right)_B = \frac{R^2}{EI}\left[\frac{-tR\alpha^2}{3} + \frac{S_b}{2}\left(\frac{1}{\alpha} - \cot\alpha\right)\right] \quad . \; . \; . \; . \; [41]$$

131 Now consider the deformation of the arm. Evidently the angular displacement of section B relative to section A of the arm is,

$$\theta = \frac{d}{R} + \left(\frac{dn}{dS}\right)_B$$

The equations of elastic deformation for the arm are,

$$\theta = \frac{1}{EI'}\int_{R_0}^{R} M'dr$$

$$d = \frac{1}{EI'}\int_{R_0}^{R} M'(R-r)\,dr = R\theta - \frac{1}{EI'}\int_{R_0}^{R} M'r\,dr$$

Where $M' = S_b(R-r) - M_b + \dfrac{\delta}{g} A' \dfrac{dw}{dt}\displaystyle\int_r^R r'(r'-r)\,dr'$

$$= 2tR^2\alpha - S_br + \frac{\delta}{g}A'\frac{dw}{dt}\left[\frac{R^3}{3} + \frac{r^3}{6} - \frac{R^2r}{2}\right]$$

Therefore,

$$\left(\frac{dn}{dS}\right) = \frac{1}{EI'R}\int_{R_0}^{R} M'r\,dr$$

$$= \frac{1}{EI'R}\left[tR^2\alpha(R^2-R_0^2) - \frac{S_b}{3}(R^3-R_0^3) + \frac{\delta}{g}A'\frac{dw}{dt}\left(\frac{R^2R_0^2}{6}(R_0-R) + \frac{R^5-R_0^5}{30}\right)\right]$$

Equating this expression to Eq. [41] and solving for S_b,

$$S_b = \frac{\dfrac{t}{AR}\left[A\alpha R^3(R^2-R_0^2) + A'\left\{\dfrac{R^2R_0^2}{6}(R_0-R) + \left(\dfrac{R^5-R_0^5}{30}\right)\right\} + \dfrac{\alpha^3I'R^5A}{3I}\right]}{\dfrac{I'}{I}\dfrac{R^3}{2}\left(\dfrac{1}{\alpha} - \cot\alpha\right) + \left(\dfrac{R^3-R_0^3}{3}\right)}$$

Then, $M_b = 2tR^2a + S_bR$ in-lb., and for the maximum bending moment in the rim,

$$M_{\max} = -tR^2a + \frac{S_bR}{2},$$

where $$t = \frac{\delta}{g} AR \frac{dw}{dt} \text{ in-lb.}$$

132 These expressions are equally applicable to a uniform distributed loading along the periphery of the rim, where t is the loading per unit run along the rim. In this case, however, assuming uniform motion, the inertia loading along the arms is zero and the shear at B in the arm then reduces to,

$$S_b = \frac{\dfrac{t}{AR}\left[AaR^3(R^2-R_0^2) + \dfrac{a^2I'R^5A}{3I}\right]}{\dfrac{I'}{I}\dfrac{R^3}{2}\left(\dfrac{1}{a} - \cot a\right) + \dfrac{R^3-R_0^3}{3}} \text{ lb.}$$

133 In the case of a concentrated load we may arbitrarily increase the equivalent uniform loading by 2; that is, if T is the torque transmitted in ft-lb.

$$t = k \frac{6}{\pi} \frac{T}{R}$$

where $k = 1$ for uniform loading,
and $k = 2$ for concentrated loading.

DISCUSSION

S. Timoshenko.[1] The author discusses a very important problem on the proportioning of the locomotive wheel elements.

Because the external forces acting on the wheel under dynamic conditions are not accurately known, and because (due to the form of elements themselves) it is impossible by use of elementary formulas, to analyze the stresses accurately, it is difficult to arrive at suitable proportions for wheel centers, hubs, etc., by analysis alone. We agree with the author that analysis may be considered as an additional guide only in the proportioning of these elements for exceptional cases where our experience is limited.

In studying a complicated statically indeterminate system such as a wheel, the accuracy of analysis depends in a large degree upon the assumptions made upon which the analysis is based. In many cases it is logical to assume that the deflections due to extension or compression can be neglected in comparison with those depending on bending. On this basis it can be expected that the calculations of the author, made on the assumption that the rim can be considered as absolutely rigid, are accurate enough in the case of transmission of torque, and his conclusion on the more even distribution of the tangential load between the arms of the wheel is correct. Any non-uniformity in the distribution

[1] Engr., Research Dept., Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa. Mem. A.S.M.E.

of the tangential load depends principally on the extension of the rim, and this can be neglected in comparison with the deflections of spokes.

In the case of direct loading in the plane of the wheel we have entirely different conditions. Assuming again that the rim can be considered as absolutely rigid, the author arrives at the conclusion that the compression in the spokes is proportional to the cosine of the angle α between the spokes and the direction of the load, which is very far from the truth. This would mean that for $\alpha = 0$ and for $\alpha = \pi$ the same forces are acting in the spokes. But in actual conditions these forces are very far from being equal. The bending of the rim in this case cannot be neglected in comparison with extension and compression of the spokes. By taking into consideration this bending it may be found[1] that the spoke with $\alpha = 0$ is submitted to much greater stresses than the spoke with $\alpha = \pi$.

In the case of lateral bending of wheel centers, the accuracy of the assumption made depends principally on the ratio of flexural rigidity of the rim to that of the spokes. In cases when the rim has a large flexural rigidity, the results obtained on the assumption of an absolutely rigid rim may be considered as accurate enough.

In studying stresses in hubs and crank arms, the known formulas of Lamé are used. In applying these formulas for calculating increase in diameter at common radius for the outer cylinder and the decrease in diameter for the inner cylinder, the effect of lateral contraction is omitted. The correct formulas will be: Outer cylinder,

$$2r_0 \frac{pr_0}{E} \left\{ \frac{r_2^2 + r_0^2}{r_2^2 - r_0^2} + \frac{1}{m} \right\}$$

Inner cylinder,

$$2r_0 \frac{pr_0}{E} \left\{ \frac{r_0^2 + r_1^2}{r_0^2 - r_1^2} - \frac{1}{m} \right\}$$

Beginning with Par. 63, the question on minimum width of metal between pin and axle is discussed. Assuming[2] that the total tension on the section w is half the sum of the projected radial pressures across the diameters D and d, the author obtains as a minimum width,

$$w = 0.3(D+d)$$

But the total tension on the section w depends on the magnitude of w. In actual conditions it will be much less than that assumed

[1] General theory of bending of a circular ring in an elastic medium must be used for this purpose.

[2] Notations as in Fig. 9.

by the author. Taking, for instance, this tension equal to one-third the sum of the projected radial pressures, we obtain

$$w = 0.2(D+d)$$

which is in better agreement with the usual proportions.

Although the writer does not agree with all the results given in the paper, he nevertheless appreciates that it is the first attempt made to arrive at satisfactory proportions of wheels by analysis, and as such is a valuable piece of work.

The Author. In regard to direct loading in the plane of the wheel, the author agrees that the bending in the rim in this case would be of importance even with the assumption of a fairly rigid rim. This problem offers a unique solution by the application of Castigliano's principle. As for lateral bending of the wheel, he was unable to obtain a solution considering the flexibility of the rim, and therefore the assumptions were based on a fairly rigid rim as stated.

Formulas [**1**] and [**2**], in Par. 42, should of course include Poisson's ratio, but in the subsequent addition the terms containing Poisson's ratio cancel out; therefore the remaining formulas are not affected and are correct.

The formula for width between pin and axle is about double that used in extreme conditions found in practice. Of course the pressure falls off when the pressure fit of axle or pin acts *alone*, but with both acting together it is believed that the stress is more or less uniform, and if due to symmetry alone the total tension would be one-half the total radial pressures of axle and pin. This condition, however, increases the flexibility of the center portion, throwing a greater total stress on the outer hubs, which in turn reduces the total load on the center section until the condition as stated in Par. 70 is approached. This justifies the assumption recommended by Dr. Timoshenko and better accounts for conditions found in extreme practice.

No. 1942

AN EXPERIMENTAL INVESTIGATION OF NOZZLE EFFICIENCY

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Non-Member

The paper describes air-testing methods developed by the General Electric Co. for testing elements of turbines by simulating the conditions in the turbine with models and determining their relative performance, and from these results predicting the effect of similar changes on the turbine. The streamline and eddy flow through the passageway tested is clearly indicated by flow casts that show both the flow lines and the shape that caused them. The issuing stream of air is explored with impact tubes, static tubes and angle-measuring devices. A combined traversing, plotting and enlarging mechanism makes it possible to take a series of accurate impact traverses covering the whole of the jet. These traverses are mechanically evaluated and from them the efficiency of the nozzle is found by a method of graphical triple integration.

The paper gives curves from tests of two types of nozzles showing how improvement has been made and illustrating the value of the method in designing turbine elements. Illustrations of casts of other types of nozzles, right-angle bends and turbine exhaust hoods further emphasize the value and generality of the method.

THIS paper describes briefly methods developed by the General Electric Co. to accelerate advancement in the art of turbine design and to insure correct design of all the elements of a turbine that control and guide the flow of steam from the control valve through the nozzles, buckets, and exhaust hood to the condenser. The rapid improvement of the airplane is based on an infinite amount of painstaking wind-tunnel research, and in much the same way improvements in the turbine have been pointed out by the "turbine air test," which in reality is the turbine designer's wind tunnel.

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2 The air test was started over four years ago on top of a two-inch pipe and has grown so that there are now two test stands supplied through six-inch pipes with air at 90 lb. per sq. in. pressure. The air test is run continuously — that is days, nights and overtime. Over eleven hundred different models have been tested ranging in variety from those representing turbine problems to elements of turbo-air-compressors and ventilation of electrical machinery. Air-test methods similar to those developed could be adapted to ventilating and aerodynamical problems, in fact they are universally applicable to any problem that concerns the flow of a fluid, be it compressible or incompressible. The methods can be used to design streamline-valve passageways for water, air, or steam or to indicate the flow around the nose of a projectile, airplane, propeller, or ship model.

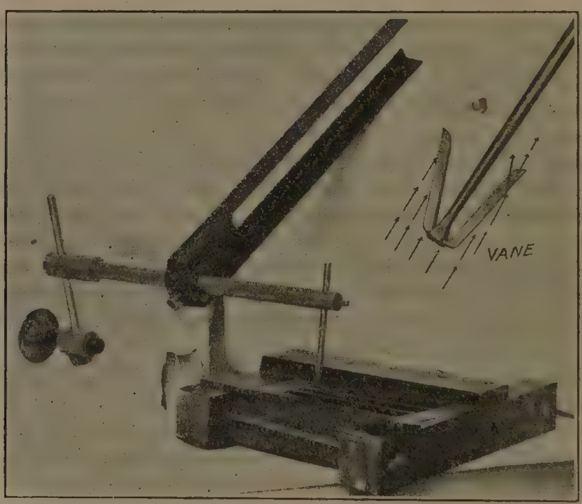

Fig. 1 Double-Vane Angleometer with Telescope and Protractor

3 The streamline and eddy flow through the passageway is clearly and beautifully indicated by flow casts that show both the flow lines and the shape that caused them. Also the issuing stream of air is explored with impact tubes, static tubes, and angle-measuring devices. In particular a combined traversing, plotting and enlarging mechanism has been devised that makes it possible to take quickly a series of accurate impact traverses covering the whole of the jet. With curve-drawing templets, these traverses are mechanically evaluated and from them the efficiency of the nozzle is found by a method of graphical triple integration. The final value of efficiency of a nozzle is the result of approximately nine hundred readings of impact pressure. Exhaust hoods, inter-

stage passageways, valves and the like are tested by measuring the volume flow through them for any initial pressure ahead of them and plotting pressure-flow curves.

4 The following condensed outline gives an idea of how thoroughly the turbine elements have been tested by means of models.

Throttle valves
Controlling valves
Intercepting valves

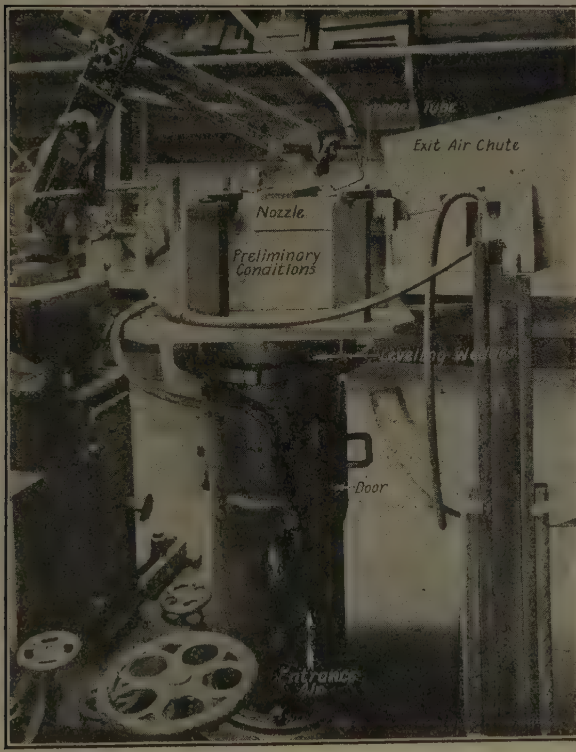

Fig. 2 Test Table Showing Part of Traversing Machine

Strainers
Elbows
High-pressure heads
High-pressure exhaust hoods
Crossovers from high-pressure turbine to low-pressure turbine
Low-pressure heads
Nozzles — for all stages and especially last stages
Buckets — for all stages and especially last stages
Exhaust hoods.

Description of Air-Test Method

5 For example: It is desired to find the relative efficiency of two proposed types of construction. A model nozzle will be made of each as shown on Fig. 1. The sides and ends are made of mahogany and the two plates of steel are let into the sides about

A B C

Fig. 3 Cast of Typical Preliminary Conditions for Testing Last-Stage Nozzles

A — Left-side view, 45 deg. approach; *B* — Back-view, discharge into paper and up; *C* — Right-side view, straight approach

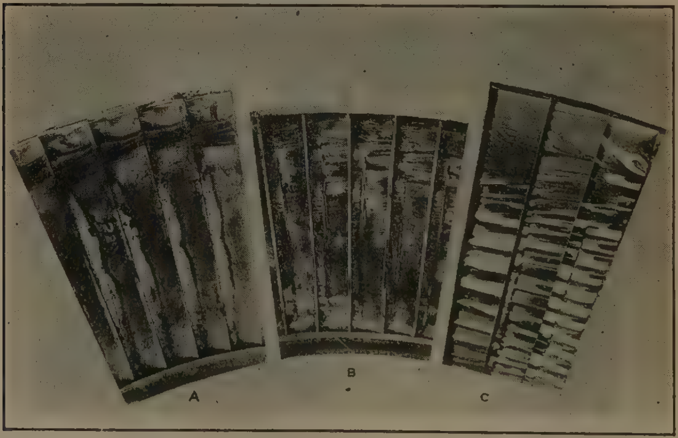

Fig. 4 Flow Lines on Exit Side of Nozzles Proving that Turbine Flow Conditions Are Duplicated in Air-Test Models

A — Turbine diaphragm; *B* — Turbine diaphragm; *C* — Air-test model

$\frac{3}{16}$ of an inch. Three passageways only are constructed. The discharge from the first and third passageways serves to support properly the center jet. The sides and ends of the model are held together with dowels and through bolts in order that it may be taken apart readily. Model nozzles for the center stages of a turbine are made full size. Those for the first stages are enlarged

two or three times while last-stage nozzles are generally made ½ to ⅓ full size.

6 Test results are worthless unless the nozzle being tested is fed by air in the same way that the corresponding nozzle in the turbine is fed by steam. Proper feeding requires that the nozzle be preceded with a suitable guiding passageway called the "preliminary conditions" (see Fig. 2) that simulate the discharge from

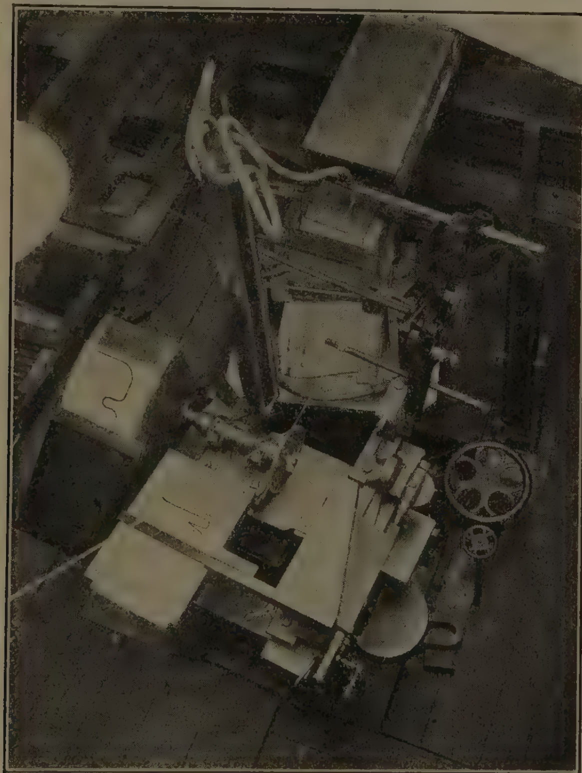

Fig. 5 Top View of Combined Traversing, Plotting and Enlarging Machine Improvised from a Scrapped Grinder

the previous bucket. The radial height of the preliminary conditions is made equal to the length of the active discharge portion of the preceding buckets.

7 The angles of discharge from the preliminary conditions into the nozzle entrance at the big and small diameter are equal to the absolute angles of discharge from the tip and root of the bucket. Angle measurements made of the steam discharge after wheels of turbines in operation indicate that the measured angle of dis-

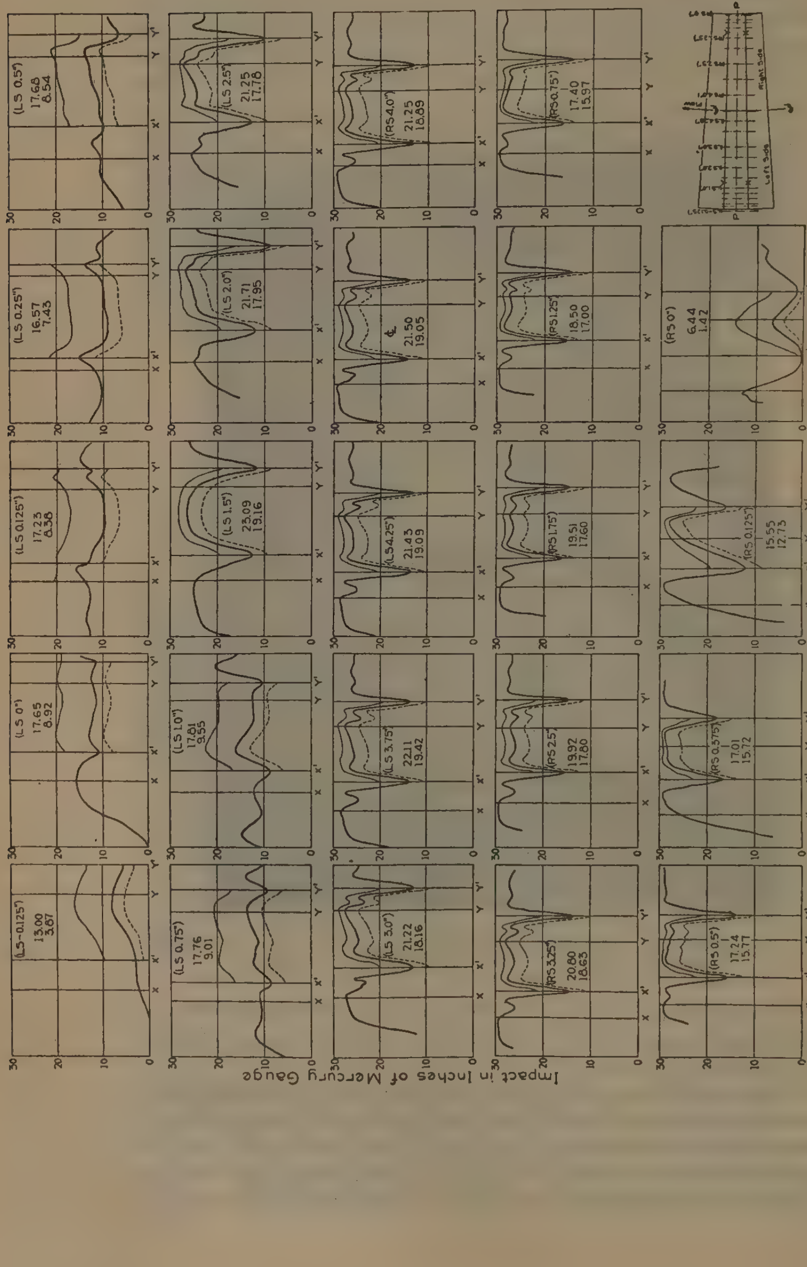

FIG. 6 SET OF IMPACT TRAVERSES ON AN INEFFICIENT TYPE OF NOZZLE

The heavy or middle curves of impact pressure as obtained on the plotting machine. The dotted or lower curves represent the actual energy and the lower number gives its value; the upper curves represent 100 per cent energy and the upper number gives its value. The ratio of these values gives the efficiency of any particular traverse. The dimensions in parentheses give the distance of the traverses from the nearest side of the nozzle as indicated in the key at the bottom right.

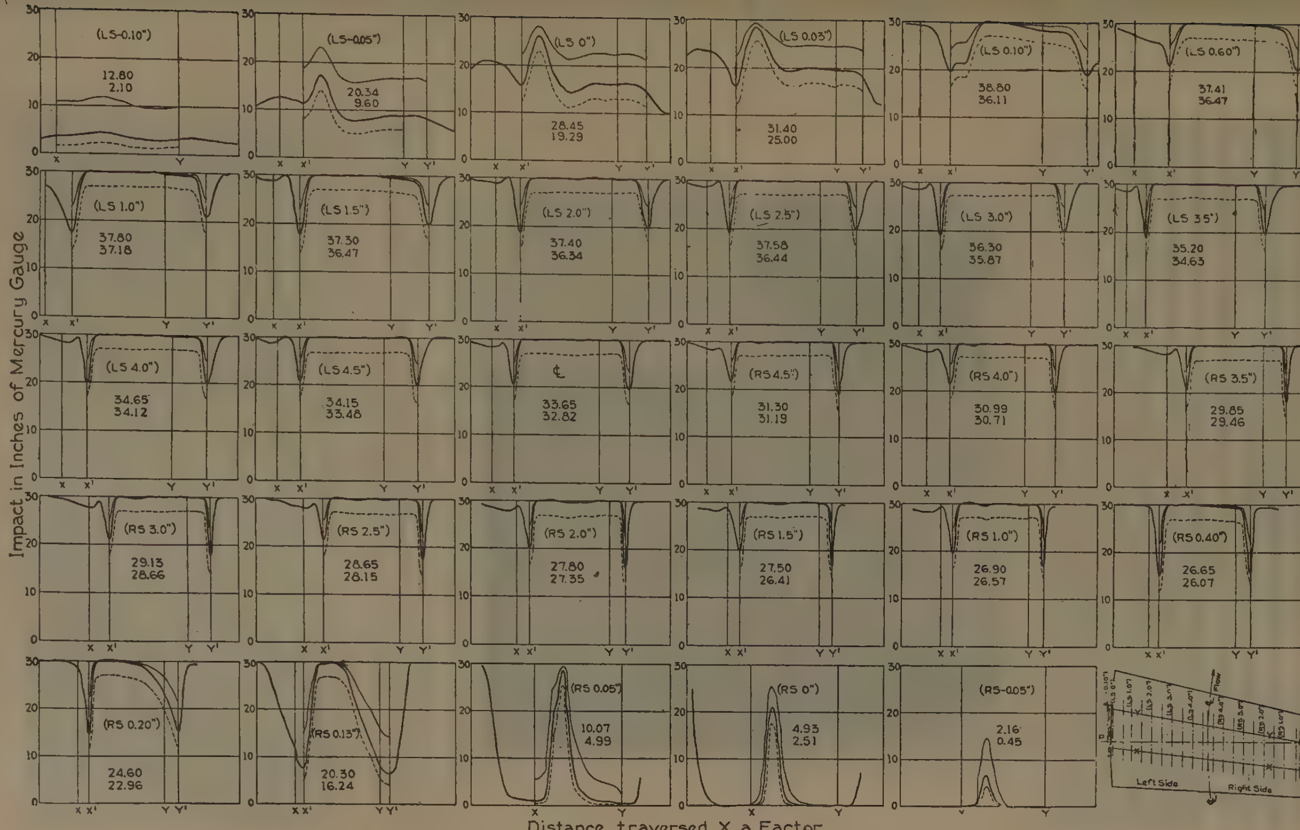

Fig. 7 Set of Impact Traverses on an Efficient Type of Nozzle

The heavy or middle curves of impact pressure as obtained on the plotting machine. The dotted or lower curves represent the actual energy and the lower number gives its value; the upper curves represent 100 per cent and the upper number gives its value. The ratio of these values gives the efficiency of any particular traverse. The dimensions in parentheses give the distance of the traverses from the nearest side of the nozzle as indicated in the key at the bottom right.

charge is practically equal to the angle obtained from a velocity diagram and consequently the latter angles are used to determine the angle for the preliminary conditions.

8 Fig. 3 shows a cast of the interior of such a nozzle and preliminary conditions. It will be seen that at the left side the approach to the nozzle is at 45 deg. while at the right side it is straight, due to the different absolute angle of discharge from the tip and root of the preceding bucket. This picture also shows clearly the two plates of the nozzle as well as the general shape of the nozzle in the radial-flow direction.

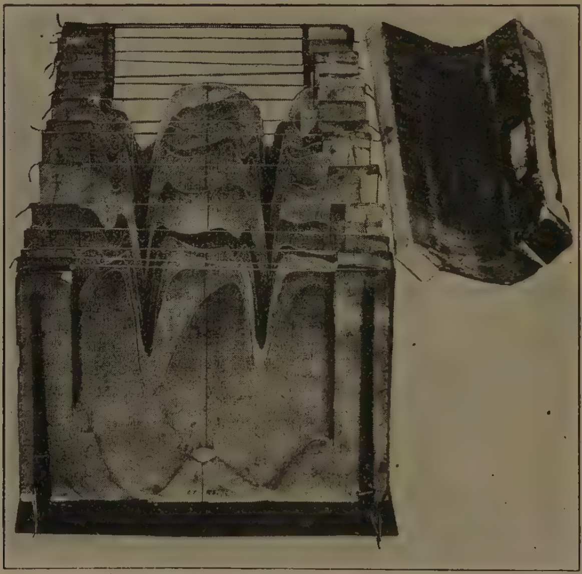

Fig. 8 Inefficient Type of Nozzle. Set of Traverses Cut Out and Lined Up with the Streamline Cast. Threads Show 100-Per Cent Values of Impact Pressure

9 Fig. 4 shows that a model nozzle fed in this way truly simulates turbine conditions. Figs. 4 *A* and *B* are views of the discharge surfaces of diaphragm plates that have been in operation for two or three years and show the markings caused by boiler compound and other impurities in the steam. Fig. 4 *C* shows similar markings obtained on the air-test model by placing dots of liquid paint on the nozzle surfaces and blowing at the proper velocity. In about a minute the paint took the form shown and the flow was stopped instantly with a clapper valve. The comparison proves several things. First, that the markings on the actual diaphragms are not due to water in the last stages of the turbine but are due to eddy flow of the steam itself, since in the

air-test model there was no water. Second, the air-test markings duplicate remarkably well the turbine markings. At the side at the big diameter there is the same eddy in both cases and there is also a throat line parallel to the exit edge of each plate. The close similarity of flow indicates that the model nozzle is being fed in the correct way and that therefore turbine conditions are being duplicated. Consequently gains shown by the air test should be realized in the turbine.

10 The nozzle and its preliminary conditions are bolted down on the top of the test stand shown in Fig. 2. The air is controlled by the valves at the bottom so that the pressure is constant in

Fig. 9 Set of Efficient Type of Nozzle Traverses Cut Out and Assembled and Lined Up with the Stream-Line Cast

the front of the nozzle at a value that will give a nozzle velocity corresponding to that in the turbine.

11 The initial pressure is measured by an impact tube projecting into the center line of the preliminary conditions and pointing in the direction of the approaching air. Thus it reads the total initial energy by adding the effect of velocity of approach to the static pressure. The discharge from the nozzle escapes upward to the right (see Fig. 2) through the exit air chute.

12 The center jet is explored in testing, overlapping somewhat into the first and third jets, by means of an impact tube held in the traversing mechanism shown in Fig. 5. This device is a combined traversing, plotting and enlarging machine improvised from a scrapped grinding machine. The three feeds are used to move the impact tube in three rectilinear directions. The location along

the path of the traverse of the point of the tube is known at all times by the position of the edge of the ruler on a flat table or by the edge of the tool rest which rubs on the revolving drum. By means of a steel wire and ratio wheel this drum can be made to revolve at any enlarged ratio up to twenty times full size. The impact tube is connected to both a simple U-tube and differential U-tube that are immediately in front of the operator. These tubes are selected so as to have uniform bore in order that it may be sufficient to read one side of the U-tube only and plot this value to the proper double scale.

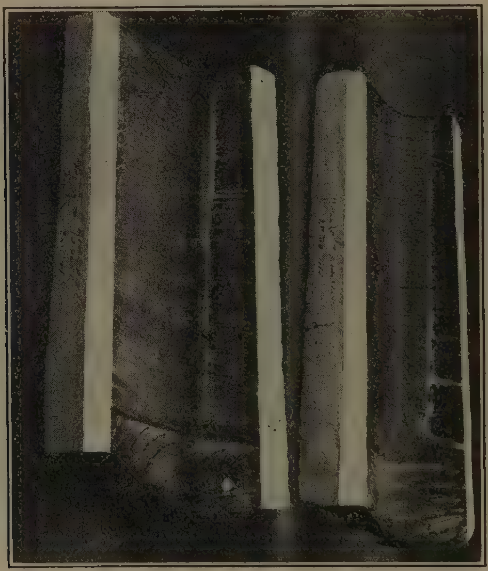

Fig. 10 Flow Casts of Inefficient Nozzles

13 In using the traversing machine the operator with his right hand moves the impact tube in small increments and with his left plots the value of impact pressure on the revolving drum. The points when connected give a curve like any one of the heavy curves on Figs. 6 or 7. It is clear that as the point of the impact tube moves along on a circumferential traverse from a region of high velocity in the center line of the first nozzle to a point in line with the partition exit edge that it will move into a region of lower velocity due to the loss caused by the plate and the eddies following it. This will produce a dip in the impact curve, the two dips corresponding to the two plate edges. Fig. 6 shows most of the traverses of the set for the first type of nozzle. The traverses are spaced further apart in the center of the nozzle

where there is little difference from one traverse to another while at the sides where the difference between traverses is great the spacing is reduced to $\frac{1}{8}$ in. or less.

14 The process of taking data is so simple and automatic that very accurate curves can be taken quickly and points on them checked if necessary. In testing models of last-stage diaphragms it is customary to run about 30 traverses and as each traverse contains about thirty-five points, some thousand values of impact pressure are read and plotted in order to obtain the efficiency of each nozzle tested. The model for a 16th-stage nozzle discharges 7000 cu. ft. of free air per min. which is expending energy at the rate of 358 hp., so it will be seen why such a model nozzle is made half size and limited to only three active passageways

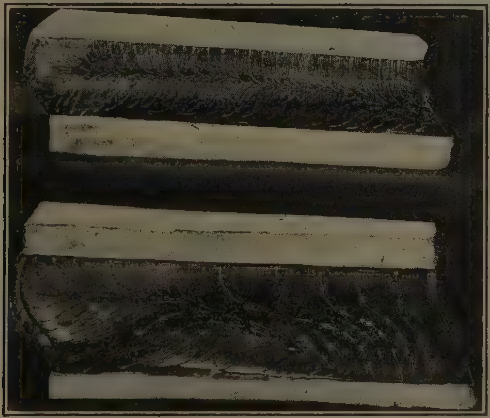

Fig. 11 Flow Casts of Inefficient Nozzles

and why the method of taking data must be accurate and quick. The whole set of impact traverses on one nozzle can be run off in one night. Tests requiring large flows of air are run at night because more air is then available. In fact some very large models have been tested on Sunday when exclusive use could be made of 14,000 cu. ft. of free air per min. at 90 lb. per sq. in. pressure, the entire air-compressor capacity of the Schenectady Works.

15 Fig. 7 shows a set of traverses on the second type of nozzle. The better performance is self evident since the dips are small and the impact pressure comes up to maximum between them. Before describing how to evaluate impact traverses it will be well to discuss paint casts and how they are made.

16 The best way that has been found to indicate streamline flow through a model is to paint the interior surfaces with a mixture of lamp black and neatsfoot oil. The model is then

clamped on to the test stand and blown at the proper velocity, generally about 1200 ft. per sec. The air blows away most of the paint and streaks the remainder out into fine lines in the direction of flow with clear places in between. In about a minute when the desired consistency of lines is obtained, the flow is

Fig. 12 Flow Cast of Efficient Nozzle

stopped instantly with a clapper valve. The model is removed and the interior filled with plaster of paris which, when it hardens, absorbs the lines. By taking the model apart the cast can be removed and lacquered thereby obtaining a permanent record of the streamline flow through the model as well as the shape

Fig. 13 Flow Cast of Efficient Nozzle

that produced that flow. Ever since turbines were first designed, streamline flow and eddies have been discussed; accordingly, this method is of immense value in showing what and where the eddies actually are, thereby eliminating guessing and much loose talking, and enabling real improvement in design to be made.

17 The flow casts of the first type of nozzle show that the flow is actually backward in places (see Figs. 10 and 11) and that in general the side of the nozzle toward the larger diameter is filled with a large spiral eddy. The impact traverses over the

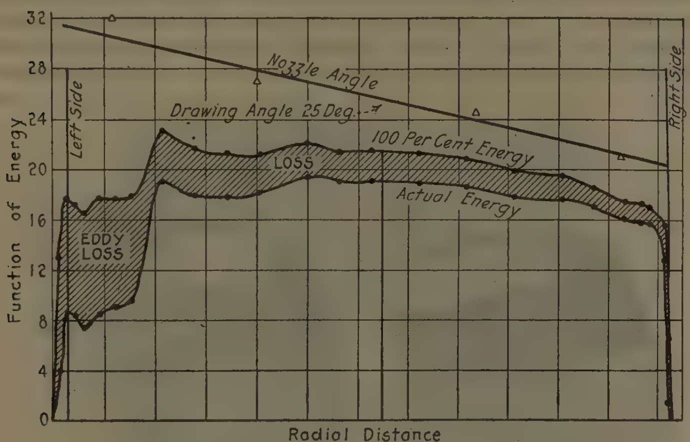

FIG. 14 LOSS DIAGRAM OF INEFFICIENT NOZZLE
(Total efficiency 82.9 per cent)

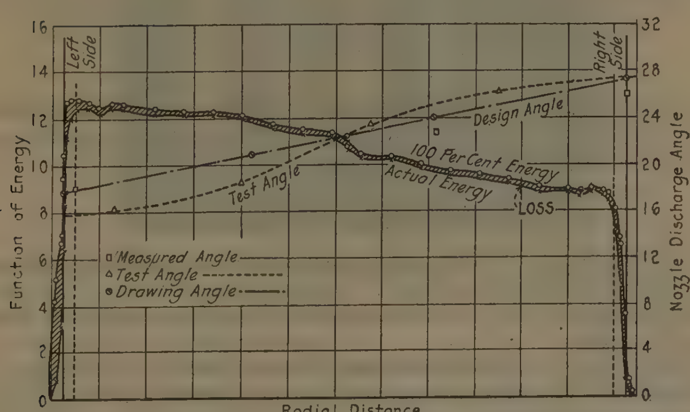

FIG. 15 LOSS DIAGRAM OF EFFICIENT NOZZLE
(Total efficiency 96.6 per cent; gain 13.7 per cent)

portion of discharge fed by this eddy come up to less than 50 per cent of their full value. This is significant since an impact traverse is run throughout the interior of the jet while the flow cast shows only the flow on the surface of the plates and walls.

Therefore the traverses prove that when this flow as indicated by lines on the flow cast is properly analyzed it can be interpreted as indicating the flow conditions throughout the center of the jet.

18 Fig. 8 is a photograph of the impact traverses on the first nozzle cut out of cardboard and mounted in their proper relative position and lined up with their flow cast. Threads indicate the 100 per cent value of impact pressure. In general, it will be seen that the traverses do not come up to maximum, that the partition dips are wide and deep, and that there are secondary dips in the

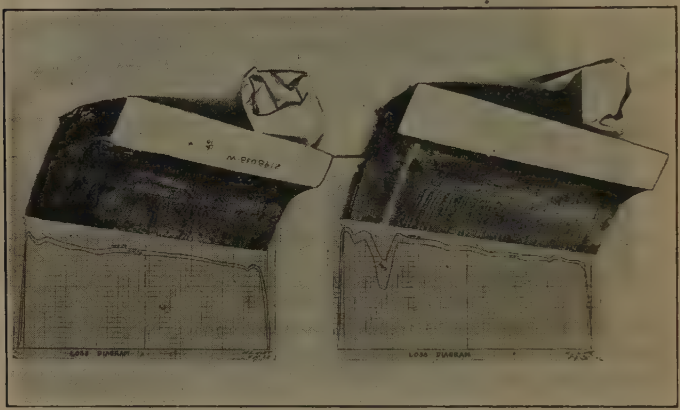

Fig. 16 Flow Casts and Loss Diagrams without and with Side Eddy

maximum portion of the traverses. Also at the back of the picture in the region occupied by the big eddy the traverses make no pretense of coming up to maximum.

Comparison of Two Nozzles

19 Contrasted with this first nozzle is the second nozzle whose traverses are shown on Fig. 7. The flow casts on Figs. 12 and 13 show that the good traverses are due to perfect streamline flow in the nozzle.

20 Fig. 9 shows the flow cast lined up with its set of impact traverses. The dips are narrow and no threads are needed to show the maximums.

21 From the foregoing it is evident that the second nozzle is a considerable improvement over the first nozzle but the question to be answered is: How much is this improvement in per cent? The answer is obtained by applying a method of graphical triple integration to the impact traverses. Briefly, the initial temperature and pressure are known and the final static pressure is atmospheric. If the impact pressure equals the initial pressure or

in other words comes up to maximum, the usual equations for the discharge of an elastic fluid from an orifice can be used to compute the mass and energy discharged per unit area. If the impact pressure is less than maximum a similar solution for mass and energy is possible if it is assumed that all of the initial energy that fails to show up as observed energy has been used to reheat the discharging air at atmospheric pressure. Therefore, in a region of low impact pressure there will be less mass and less energy

Fig. 17 Flow Casts of Buckets Showing View of Entrance Flats

per unit area per second for two reasons: The velocity of flow is less and the density is less.

22 Celluloid templets with impact pressure as an argument are used to draw in curves of mass and energy that are roughly parallel to the curve of impact pressure. Those curves can be seen faintly in Figs. 6 and 7. The area of the mass curve when multiplied by scale factors gives the total mass discharged per unit width. If this mass of air had expanded from its original pressure and temperature to the final atmospheric pressure with an efficiency of 100 per cent it would have set free an amount of energy that is called the 100-per cent energy. Consequently, the mass is multiplied by the 100-per cent energy per unit mass to obtain the

100-per cent energy. The area of the energy curve when multiplied by scale factors is the actual energy. Then the ratio of the two, the actual energy divided by the 100-per cent energy, is the efficiency of the traverse.

23 The efficiency of the nozzle is found by plotting the values of actual energy and 100-per cent energy found from each traverse as ordinates and the traverse spacing as abscissas thereby making a loss diagram as shown on Fig. 14. The area under the lower

Fig. 18 Flow Casts of Buckets Showing View of Concave Surface or Convex Cast Surface

curve is the actual energy and the area under the top curve is the 100-per cent energy and the ratio of the two is the efficiency of the nozzle. The area between the two curves, therefore, is the loss. A comparison of the loss diagram on Fig. 14 with the right-hand flow cast on Fig. 10 shows that the excessive loss near the left side is due to the eddy. The loss diagram for the second nozzle (see Fig. 15) has no excessive loss at this place and the cast also shows no eddy. A comparison of the efficiencies of the two nozzles shows that the first nozzle had a total efficiency of 82.9 per cent and the second nozzle a total efficiency of 96.6 per cent, a gain of 13.7 per cent.

Other Typical Tests

24 Fig. 16 shows flow casts of two additional nozzles lined up with their loss diagrams and again proves that the side eddy causes loss and that lines on the surface of the cast can be interpreted as showing loss throughout the jet.

25 Figs. 17 and 18 show flow casts of long buckets and indicate the disturbed character of flow through a bucket. The flow is

Fig. 19 Plaster Casts of 90-Deg. Turns. Blade Corner, Square Corner, Round Corner. Comparative Losses are Approximately as 1: 5: 6

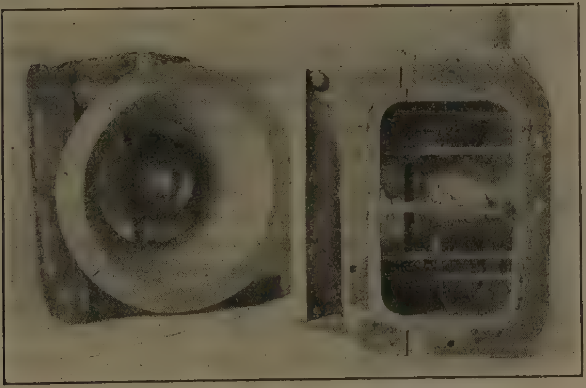

Fig. 20 Model of 30,000-kw. Exhaust Hood

backward in many places and the ends of the bucket are filled with large eddies.

26 Fig. 1 shows the set-up for measuring the angle of discharge. The design of the double-vane angleometer is the result of much experimenting with various shapes of vane and the vane shown has been the only type that has been found to work properly at high velocity. Flat vanes shaped like a flag split a high-velocity jet and read the angle of one current of flow or the other current

of flow. A double-vane angleometer has its active edge inclined to the impinging jet so that each increment of edge has an undisturbed stream striking it. Sufficient torque is developed along the line of the active edge to turn the vane and so indicate the angle. The angle is read by means of a telescope with cross hairs.

27 Valves, right-angle turns, exhaust hoods, and the like are tested by measuring the flow through them for any pressure ahead of them and comparing their flow curves with the flow curve of an orifice of equal area. Fig. 19 shows flow casts of three types of right-angle turns. The square corner is slightly better than the round corner while the addition of blades reduces the loss in the turn to one-sixth of its previous value. The improvement in streamline flow caused by the blades explains this tremendous reduction in loss.

28 Fig. 20 shows two views of a model 30,000-kw. exhaust hood, the view at the left directly into the preliminary conditions or annular opening being obtained with a mirror. The struts are removable so that their effect on exhaust-hood loss can be determined.

Advantages of the Air-Test Method

29 From the foregoing it will be seen that these methods of investigation have many advantages which previous methods have lacked: namely,

a The performance of each part of the nozzle is observed in detail and so cause and effect can be isolated in a way that is impossible in any nozzle-testing machine which gives only the final result and from the nature of its operation cannot differentiate between the good and bad features that contribute to that result.

b Actual diaphragms of a turbine under construction can be tested and their relative performance in the turbine predicted. The diaphragms are tested by blowing three or five nozzle passageways preceded by their proper preliminary conditions. Diaphragms set up in this way have had changes made in the nozzles while on the test stand and efficiency tests made after each change. The whole diaphragm was then altered to the most efficient condition and the gains indicated confirmed by water rates on the turbine.

c Nozzles, buckets, etc., are tested by measuring the loss directly, that is, the distance between 95 and 100 per cent. Other methods measure the effect from 0 to 95 per cent and take the result from 100 per cent to find the loss. Consequently the air test gives highly accurate differentials of values of efficiency that are near 100 per cent.

d The models for test can be made cheaply and quickly from easily worked materials like wood, plaster, and white metal and slight alterations made with solder or plaster. With a new feature of design, it is no longer necessary to follow a " hunch," but instead a model can be constructed and tested within a week and in almost all cases a definite answer can be given as to how much gain or loss will be caused by the change. This feature is of immense value in developing an art like turbine design where considerable time separates conceived ideas and test results and where in most cases there could be no comparable test results because of the many new features in every succeeding design.

e The performance of large parts of a turbine, for instance exhaust hoods, may be investigated in small-sized models where it is both impracticable and expensive to experiment with the actual turbine.

f This method gives an independent check on results obtained by other methods of testing, such as steam reaction nozzles tests and water-rate tests on turbines.

30 When properly controlled the three methods have been found to give remarkably interchangeable results. Thus a gain indicated by the turbine air test can be confirmed by a steam reaction nozzle test and definitely proved by a turbine water-rate test. Thus, if the air test shows a gain of, say, 3 per cent, a gain of about 3 per cent will be shown by the other two independent methods of testing.

31 In conclusion, all of the foregoing sounds so very simple and straightforward that it would be well to point out that a few isolated experiments are misleading and dangerous. For instance, nozzle and bucket action are so interrelated that tests of either one alone may give wrong conclusions. Therefore a background of all the previous work done, the eleven hundred models tested, has to be freely drawn upon in order properly to design models for test and in order correctly to interpret test results.

DISCUSSION

F. O. Ellenwood.[1] Accurate data relating to turbine nozzles and buckets have been needed for many years and it now seems that this new method of obtaining information is reliable and far superior to many older ones. The ingenious methods outlined in this paper may be expected to assist the turbine designer very materially, and the author is therefore to be congratulated for the work he has done.

Certain points, however, are not entirely clear in the paper. These are outlined in the following questions:

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When using model nozzles in the tests for efficiency, are substantially the same results obtained whether the models are full-size, one-half size, or one-third size? If not, how is correction made for the various ratios used as mentioned in Par. 5?

Have duplicate tests been run on the same models or nozzles, and if so how closely do they check? The answer to this question may be given in Figs. 6 and 7, but these reproductions have been so reduced that it has not been possible to obtain much information from them.

Have any tests been made with air carrying a spray of water with it? If so, how do the results compare with relatively dry air?

From Fig. 19 and Par. 27, it appears that the square corner is slightly superior to the round corner. Should we then not expect less pressure drop in air or steam lines having sharp turns instead of rounded ones? Is not this contrary to all data previously obtained on pipe-line losses?

Can the apparatus probably be adapted to make tests on moving blades?

G. B. Warren.[1] The work described by the author is probably without precedent and different from anything that has ever been done. It is a part of the research program on turbines that the General Electric Company has been attempting to carry out for the past few years. Although the work described by the author is only part of the program it is a very important part. Nevertheless, it has been found necessary to carry along with this method, two or three other methods of research on the elements of steam turbines. The reason for this is that we have never been willing to trust any one method of finding out how turbines should be made. In addition to the work described in the paper, extensive nozzle reaction tests are being conducted, these tests being made with steam rather than with air. Also, there has been under way for some time an extensive series of steam-turbine tests in which is used a comparatively large steam turbine, into which can be put different kinds of nozzles and buckets, similar to those shown in the paper.

In this research work, the new ideas of things that are different are usually tested first with air, first because it is by far cheaper than any other method of testing and it permits a larger variety of things to be tested. After results have been obtained by this method, the other methods are used to check up the results and determine whether the air test results were really accurate and could be transferred to the turbine itself. The underlying principle that has governed us in all of this work has been the knowledge that turbine inefficiency is due to certain losses that occur in the

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steam flow through the turbine. The whole idea has been to analyze and locate these losses and discover what causes them. This done, we endeavor to ascertain the means that will eliminate them or minimize them as far as possible. It would be interesting to know how the results of these researches have been reflected in turbine efficiency. However, since so many varieties of turbines are involved, this question cannot be answered. The principal accomplishment has been the elimination of the uncertainty of getting a good or a bad design; that is, it has been possible to predict with reasonable accuracy whether the design will be good or bad, and by eliminating the bad points of design we have been able, with considerable certainty, to arrive at a turbine design that will represent the best results obtainable at that time. We are endeavoring to bring every possible facility of science and engineering to bear on the subject of turbine efficiency in an effort to produce a turbine that has the minimum possible loss.

A. G. CHRISTIE.[1] The writer would call attention to the great importance apparently attached to what the author calls the "preliminary conditions"; that is, the form of the passageway and the direction of the jets entering the nozzle. This is of particular interest for the reason that in a review of the work of the steam-nozzle research committee of the Institution of Mechanical Engineers at London, it is apparent that they encountered the same problems. Their values and efficiencies seem to depend on what might be called preliminary conditions. This may account for some of the differences in results of some of the earlier nozzle research work.

It would be interesting to know if the same methods as described in the paper have been applied to blading tests. Many of us have tried to draw conclusions from streamlines as they appeared on blades in turbines. In some of the low-pressure blading there is an appearance that water is being thrown off at the periphery. In England an attempt has been made to draw off water at the blading point and some improvement in efficiency has been claimed on that account. We do not know definitely whether this water is being withdrawn. If there is a possibility of draining the water it might be worth considering in connection with some of our American machines.

The illustrations showing the flow of air around right-angle bends are of considerable interest. If there is less velocity in the fluid passing around a right-angle bend it would seem inadvisable to put in a long-radius bend at that point. It would be of considerable interest if the author would give some information as to the relative velocity in this case.

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H. N. Davis.[1] The very interesting methods described by the author have a possible application to fields quite remote from steam turbines. For instance, these methods might be applied to studying the flow of steam through superheaters, throttle valves, and exhaust passages. In certain studies that have been made on locomotives it has been found that the loss through the superheaters and throttle valves amounted to from 5 to 10 per cent of the available energy, and in locomotive boosters the losses are sometimes as high as 10 per cent in exhaust passages. All these losses might conceivably be materially reduced by a proper application of the method of this paper.

The Author. The question of the superiority of the square corner has been brought up in the discussion. It is quite true that the loss in any given turn is reduced by substituting a square corner for the larger circumference of the turn, keeping the smaller circumference the same. The advantage of a square corner over a round corner decreases as the radius of the center line of the turn is increased, and when this radius is three or four times the diameter of the pipe, the superiority of the square corner practically disappears. Increasing the radius of the center line of a turn reduces the loss in the turn, consequently the turn just described has only slightly more loss than turn number 165, Fig. 19. It is probable that if corner 165 were substituted for corner 165 X, the gain that would accrue would probably more than repay the expense of putting in the blades. Of course, if there were dust and oil or other foreign matter in the passageway that might clog the blades, it might be impracticable to use them.

In regard to the velocity in the bends, the flow through the models was measured for various initial pressures and therefore the data cover the entire range of velocity from 0 to 1000 ft. per sec. A comparison of the flow curves shows that with different velocities of flow about the same part of the velocity head is lost in the turn.

In regard to the scale size of the model, it is necessary to change the scale somewhat. If the model of a first-stage nozzle were made actual size it would be too small to give accurate results, and if the last-stage models were made full size they would require for testing more air than is available. Hence the first-stage nozzles are enlarged and the last-stage nozzles reduced two or three times. However, in all this work it is customary to make a model of the nozzle of the present construction, as well as of the new construction. These, of course, will be made to the same scale and results can be compared accurately and any effect of scale eliminated. In the case of exhaust hoods that are only one-sixteenth or one-twentieth full size, data are taken over a wide

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range of velocity so that a velocity for the model may be selected that is equivalent to the conditions on the turbine by allowing for scale and the difference in the gas constant R and the temperature T between air and steam.

Duplicate tests on the same nozzle can be made to check within one-fifth of one per cent for the following reasons: If an impact traverse is repeated, it will superimpose on the first traverse. The traverses are enlarged as much as twenty times to increase their accuracy. The efficiency of a nozzle is the result of approximately 1000 readings of impact pressure. The loss is measured directly as explained in Par. 29-*c*.

Some work is now being done with moisture in the air. The big gain to be obtained at first was to eliminate eddies in the nozzle. This has been done fairly well and no eddies are permitted in nozzle diaphragms of present design. At present much work is being done with moisture in the steam, and attempts are being made to ascertain how moisture causes loss in a nozzle and how to eliminate such loss.

The condition of motion in bucket blades is simulated in the air test by making the angle of approach to the bucket in the model the same as the angle of approach of the steam to the moving bucket in the turbine. Preliminary conditions are important when testing a nozzle, but they are three or four times as important when testing a bucket. Nozzle and bucket action are tied together so closely that it is impossible to isolate the latter. Tests on buckets alone are misleading.

No. 1943

LARGE OIL ENGINES, WITH SPECIAL REFERENCE TO THE DOUBLE-ACTING TWO-CYCLE TYPE

By Charles Edward Lucke,[1] New York, N. Y.
Member of the Society

Efforts to build large oil engines have been growing in number and in variety of means selected to meet marine requirements, stimulated by the success of motorships and the desire to increase their size or utility in prevailing sizes. Possible stationary applications of larger oil engines have also contributed additional motives and other solutions. Much of this work has not been published and is therefore not as well known to the engineering profession as it deserves by reason of its high quality and the foundation it lays for the future. This paper is concerned with a review of some of this work, with special reference to double-acting engines.

One of the problems considered is that of the possibility of raising the mean pressure in the cylinder to get high horsepower per cubic foot of displacement and without rise of maximum pressure to thereby reduce the weight per horsepower. High injection-air pressures, extra injection air, and supercharging of cylinders are included here in addition to studies of shape of combustion chamber, position of sprayer, and design of spray valve. A second one is that of determining the maximum diameter of cylinder that can be operated at very high or at moderate mean pressures without injury to the metal of cylinder, piston, or cylinder head, with particular reference to the effect of special designs of these parts for resisting heat damage at a given heat-generation rate or the corresponding relation of diameter and mean indicated pressure.

A third problem is that of securing results in two-cycle cylinders as nearly as possible equal to those of the earlier developed and more widely standard four-cycle, to determine how close the former can be brought to half the weight per horsepower of the latter on the one hand, and on the other, for the same size of cylinder and similar engine structures, how close to twice the maximum horsepower of a four-cycle engine it may be possible to build the two-

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cycle. Still another, from the standpoint of double action of the piston, assuming a given diameter and mean pressure or heat-generation rate established in four- or two-cycle single-acting cylinders, and a construction of cylinder, piston and cylinder head not injured for these values, is that of determining the practicability of making both ends develop the same power in both four-cycle and two-cycle engines. This includes the determination of how close to half the weight per horsepower of the single-acting it may be possible to build the double-acting engine or how close to twice the maximum horsepower of the single-acting it may be possible to make the maximum horsepower of the double-acting engine. Some of the double-acting designs published are reviewed, including the rodless piston and the double opposed pistons.

Introduction

DEVELOPMENTS in prime movers always have been and always will be matters of major interest to engineers, and any progress in the direction of enlarging the scope of usefulness of the internal-combustion engine with its inherently high thermal efficiency is of special importance. The success of the small units in the form of the high-speed gasoline engines, especially in the field of transportation, has been so remarkable as to have repeatedly raised the question as to the future of the large units in the form of the Diesel oil engines burning fuel oil. This uncertainty about the large engines would of itself justify an engineering survey, but when, in addition, it is known that much work has been done, most of which is as yet unpublished, and that the results of this work indicate new possibilities for large engines, a review of the situation is doubly important.

2 There never has been any doubt about the high thermal efficiencies of Diesel oil engines, and such doubt as once existed as to their reliability or maintenance costs has been completely dissipated by the success of the motorship, as great in its way as that of the automobile on land and of aircraft above. However, even in spite of the proved stability of the motorship engine when properly designed, with due recognition of that body of engineering data which is steadily accumulating through experience and research, there have been, and in some respects there still are, many questions to be answered about the future of the large oil engine. This is but an indication of zones of unexplored possibilities which year by year will be narrowed by increasing knowledge, with definite answers to questions of how progress may be made and in what direction there is no hope of gain.

3 Of the big-engine questions of broader significance, perhaps the two that stand out or are most often heard are: How may the zone of economic use be widened for such Diesel engines as have become more or less standardized as to design or size? and, second, how large, how much larger than these standards in horsepower

capacity, may good oil engines be built? The answers to these questions involve all the facts, questions, and experience so far accumulated and in addition some degree of speculation, but every day some speculation may be replaced by facts and accomplishments.

4 Widening of the scope of economic utility of large oil engines in existing sizes whether in marine or in stationary fields of competition with other prime movers is mainly a question of reduction of weight, and through it a reduction of first cost on the assumption of a more or less constant ratio of labor to material in production. Reduction of first cost reduces fixed charges in the cost of power, and this is the biggest single item against oil engines in competition with steam. Reduction of first cost of large oil engines toward the cost of steam plants would result in a wholly different picture of adoption and use than has heretofore prevailed. It is not at all necessary to improve efficiency or fuel consumption, nor is it necessary to reduce maintenance or attendance below existing figures, though, naturally, any gains here will help the oil engine, provided they do not prevent reduction of first cost. Reduction of cost is primarily a matter of weight reduction in pounds per horsepower, except when it is accomplished by the use of special high-priced materials, or when it requires excessively expensive shop operations or higher ratios of labor to material in building the engine. It is true, however, that there are some applications where weight reduction would result in new adoptions of oil engines even without cost reductions on other grounds than power costs compared with steam, as, for example, in naval vessels.

5 One set of engineering problems of primary interest in connection with large oil engines is concerned with the means whereby weight reductions may be made with more or less proportionate reductions in cost and without loss of reliability in operation or life of parts. When, as is the case here, more than one possible course of procedure is open, there arises the additional problem of weighing one against the other: the problem of choice.

6 The second broad question, that of maximum possible size, with or without weight reduction but, of course, preferably at lesser weight per horsepower than prevails at present, is perhaps of equal importance but really of greater engineering interest. What applications will be made of oil engines of larger sizes than have prevailed in ships, commercial or naval, and in stationary installations will of course depend on just what may be their characteristics as to weight, cost, and reliability, with efficiency maintained. It is, however, quite certain that such availability must result in some adoptions and changes in engineering practice, and anything in this direction is sure to be important.

7 A second set of engineering problems in connection with large engines is therefore concerned with the determination of

maximum possible horsepower per cylinder, and of the means whereby horsepower per cylinder may be made greater without loss of other good qualities, and preferably also with reduction of weight and first cost. As in the problem of reduction of weight per horsepower, so also in this problem of increase of horsepower per cylinder, all of the several possibilities must be weighed and a selection must be made, and in the selection there must necessarily be some elements of opinion and judgment.

PART I

Sizes and Arrangements of Large Engines

8 Among the larger sizes of Diesel engines, those in which the arrangement of parts was first brought to a state of acceptably satisfactory perfection on a considerable scale were of the four-cycle air-injection type, single-acting, some with trunk pistons, but all of the larger, more reliable designs with the crosshead style of main running parts. Efforts to reduce the weight per horsepower of these single-acting four-cycle engines have extended over many years and still continue. Here two quite independent lines of attack are recognized in addition to the perfectly obvious one of increased speed with its limitations of life above recognized values for piston speed. The first of these is concerned with securing higher mean effective cylinder pressures by suitable treatment of air and fuel supply and their proper combination through controlled cylinder combustion. The second is directed toward the metal and its weight in pounds per cubic inch of cylinder, measured by the product of area, stroke, and number. There is obviously no relation between these two lines of attack since maximum pressures may be assumed equal in all Diesel engines, though they do together determine the weight per horsepower.

9 Increase of indicated mean pressure is being secured in various ways, among the most notable of which are the following: Higher spray-air pressures to carry fuel to the more remote points of the air charge and by agitation of the whole charge to effect the combustion of more fuel per pound of air than otherwise, reducing free oxygen in the exhaust to the lowest possible value with no carbon monoxide; extra spray air injected at points other than the spray valve to increase turbulence and to produce similar effects; two spray valves instead of one to reduce the distance the fuel must travel to find its air, and to reduce the necessity for extra turbulence; supercharging to increase the weight of air for a given piston displacement and so to raise the mean pressure for the same completeness of oxygen utilization by fuel as shown by free oxygen in the exhaust, it having been employed with and without the former means of raising mean pressure.

10 By research along such lines it has been found possible to secure indicated mean pressures of 150 lb. per sq. in. without supercharging, and 176 lb. per sq. in. with supercharging. With such high values established as attainable it is a fact that the mean pressures in actual use for engines at regular work are very much lower, especially in cylinders of considerable size. The values for these commercial engines seem to have been established by Burmeister & Wain's experience in their motorship engines, the first to secure a position of real success on a considerable scale at sea, and these values for mean indicated pressures are 100 lb. per sq. in. for stationary, and 88 lb. per sq. in. for marine engines under constant full load in cylinders of 30 in. diameter, more or less. Some of these four-cycle engines have been rated a little higher than this but many are found with lower values, and it is fair to say that the usual values for indicated mean pressures are 60 per cent — more or less — of those proved by research to be obtainable.

11 Not very much has been accomplished in the direction of decrease of metal weights of the engine structure for a given cylinder size for these four-cycle single-acting engines, nor does it seem possible to go very far here without increase of cost per pound as much as or more than pounds per cubic inch may be reduced. Experience has proved the necessity for the closed box type of frame in the interest of pump lubrication and of maximum frame stiffness for maintenance of bearing alignment, and with steel tie rods to main bearing to take direct tension loads of gas pressures acting on cylinder heads. Such an engine structure designed with maximum allowances for safety and reliability for such lårge four-cycle, crosshead, six-cylinder, long-stroke marine engines of about 30 in. bore, will weigh about 4.20 lb. per cu. in. of stroke volume. This may be regarded as a maximum. Neglecting other frame arrangements, especially open classes, as not suitable, the minimum for the enclosed box type of frame may be taken as the value for the German trunk-piston submarine engine of the late war, where special material was used and pound costs further raised by much increase in labor of construction. Here the largest cylinder bore was 21 in., with stroke equal to bore, or 7273 cu. in. of stroke volume, and the weight was 16,600 lb. per cylinder, or 2.28 lb. per cu. in. There is, therefore, $4.20-2.28 = 1.92$ lb. per cu. in. range between a design so high in pound cost as not to reduce cost per horsepower but increase it — and with trunk piston, and the other design which, while satisfactory from the pound-cost standpoint and safe in service, may yet be reduced somewhat.

12 These difficulties in reduction of weight per horsepower for the most widely used type arrangement, the four-cycle single-acting engine, which increase as horsepower per cylinder becomes larger, have led other investigators to approach the problem from

different angles. The first of these is the substitution of the two-cycle arrangement for the four-cycle. Here there is nothing to be saved in pounds per cubic inch of stroke volume — rather an increase may be expected, because the scavenging pump and its connections will weigh more than the four-cycle valve gear that it displaces. Nor is there any prospect of using higher mean pressures — quite the contrary, because even with the identical air and fuel conditions the mean indicated pressure for the effective part of the stroke will be equal to that of the four-cycle cylinder, and for the whole stroke something less by an amount dependent on the length of the exhaust port, which measures the ineffective part of the stroke. Such exhaust ports are 20 per cent of the whole stroke, more or less, so that the effective stroke is 80 per cent ± of the whole, and the indicated mean pressures 80 per cent ± of the values for four-cycle cylinders, provided combustion conditions are equal.

13 On the other hand, however, doubling the number of working strokes for speeds equal to the four-cycle results in the two-cycle engine, otherwise similar, approaching toward 75 per cent the weight per horsepower. How close this approach may be depends on the length of the exhaust port, the relation between the indicated mean pressures for the effective stroke of the two-cycle and the full stroke of the four-cycle, and the amount of excess weight per cubic inch of the scavenging pump of the former over the valve gear of the latter. There is, of course, a higher rate of heat generation per square foot of cylinder approaching twice the value of the four-cycle for equal bores, which would seem at first glance to limit the two-cycle to smaller cylinders and perhaps in the attainment of lower weight per horsepower would result in a lower maximum horsepower per cylinder, so that both ideals would not be attainable. At one period these matters were much more speculative than they are now, and they have been the basis of some rather extended debates, not always kept within the bounds of professional discussion; but experience has been accumulating rapidly through a great variety of two-cycle research investigations and engineering developments by practically all of the engine-building nations.

14 It has been definitely established that combustion conditions can be secured in two-cycle cylinders which are quite as good as in four-cycle, so that attainable indicated mean pressures for the former for the effective part of the stroke may be accepted as equal to values for the latter, full stroke. It has also been proved that air charging of the two-cycle cylinders by ports with no head valves whatever, can give air charges necessary for such equalization of mean indicated pressures[1] if the scavenging is

[1] The Diesel Engine of Today, by A. Naegel. *Zeit. Ver. Deut. Ing.*, June, 1923.

properly done. Therefore heads may be so simplified, compared with multi-ported four-cycle heads, as to be able to withstand uninjured the higher heat rates typical of the two-cycle engine of bore equal to the four-cycle. This is a most important accomplishment because it contributes directly to the attainment of both ideals at once: the reduction of weight per horsepower and larger horsepower per cylinder; but whether the amount of these gains is sufficient is yet to be seen, as is also the question of still other means of approach.

15 For such single-acting two-cycle engines, indicated mean pressures for the full stroke have reached the value of 122 lb. per sq. in.[1] with an effective stroke of 77 per cent, making equivalent mean pressure 158 lb., which checks well against the four-cycle maximum values. As in the case of the four-cycle, however, the rating values are lower, Sulzer using the value of 93 lb. per. sq. in., equivalent for same port lengths to 120 lb. per sq. in. for cylinders of about the same bore — between 20 and 30 in. These ratings correspond to substantially twice the heat-generation rate per square foot of the four-cycle engine, and are indicative of the ability of the simpler structure of the two-cycle cylinder, especially the head, to stand higher heating rates than four-cycle forms with more complicated parts when made in similar ways of casting. It seems, therefore, that there is a real gain in horsepower per cylinder for two-cycle cylinders of the same bore compared with four-cycle, but which, as in the case of four-cycle, is less than the maximum attainable by combustion — about 50 per cent, more or less. Indirectly this points to the practicability of higher heat-generation rates per square foot in two-cycle cylinders of equal thickness and diameter, and to the probability of more favorable metal conditions for high over low periodicity of heat charges with equal hourly rates. While there is some increase in weight per cubic inch (about 0.5 lb.), there is a substantial reduction made in weight per horsepower by doubling the impulses.

16 Finally, there is to be noted in this series of efforts to reduce weight still another and the most recent: doubling the number of impulses by the double-acting method, with its parallel of double opposed pistons. A double-acting piston should give nearly twice the output with piston-rod allowance obtainable from the single-acting, either two-cycle or four-cycle, with not much increase in weight over the standard single-acting crosshead designs, provided combustion conditions are equal and permit of equal indicated mean pressures. This mode of decreasing weight per horsepower does not at all increase the heat-generation rate over the single-acting because each end acts independently. Therefore it not only contributes something to the first ideal of cost reduction

[1] The Development of the Sulzer Engine, by L. S. LeMesurier Inst. Mech. Engrs., January, 1923.

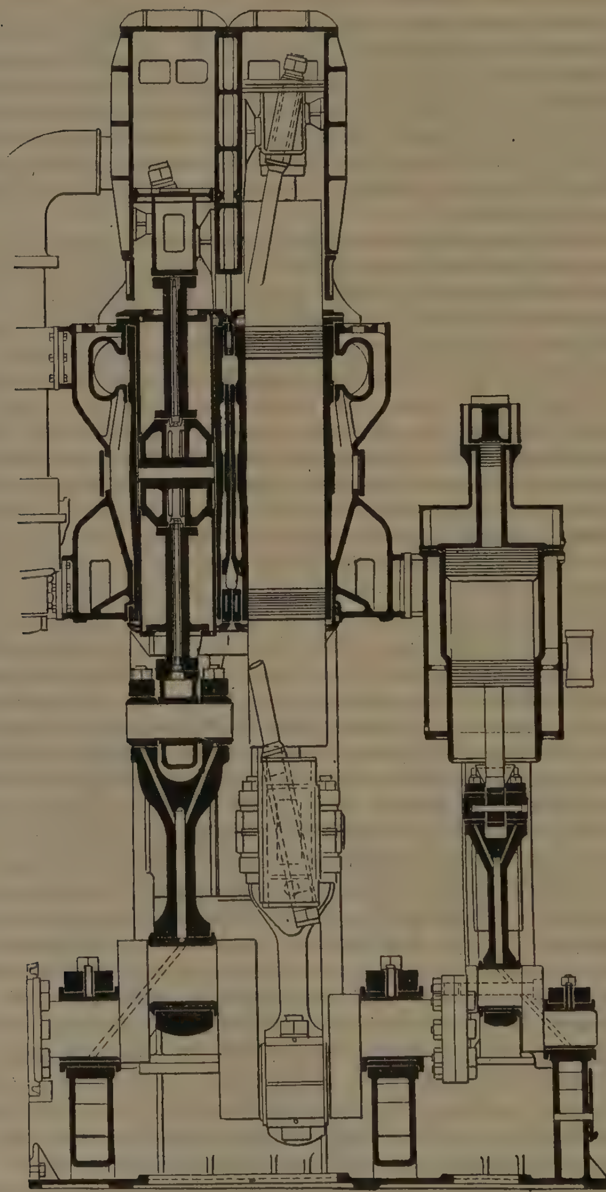

FIG. 1 CAMMELLAIRD-FULLAGAR OPPOSED-PISTON TWO-CYCLE ENGINE, SIDE ELEVATION

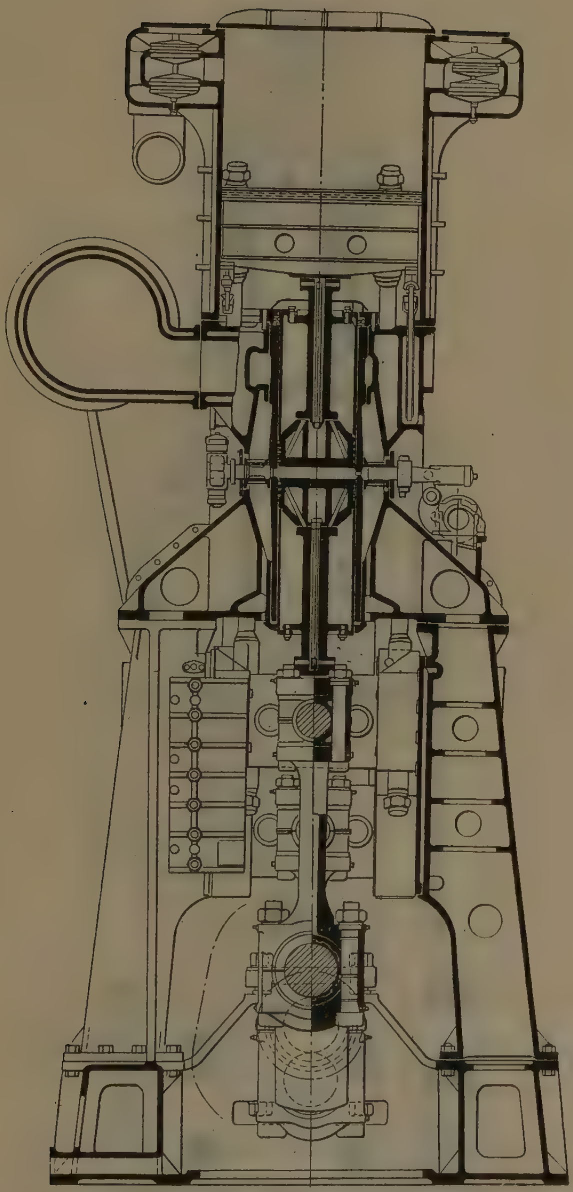

FIG. 1 (Continued) END ELEVATION

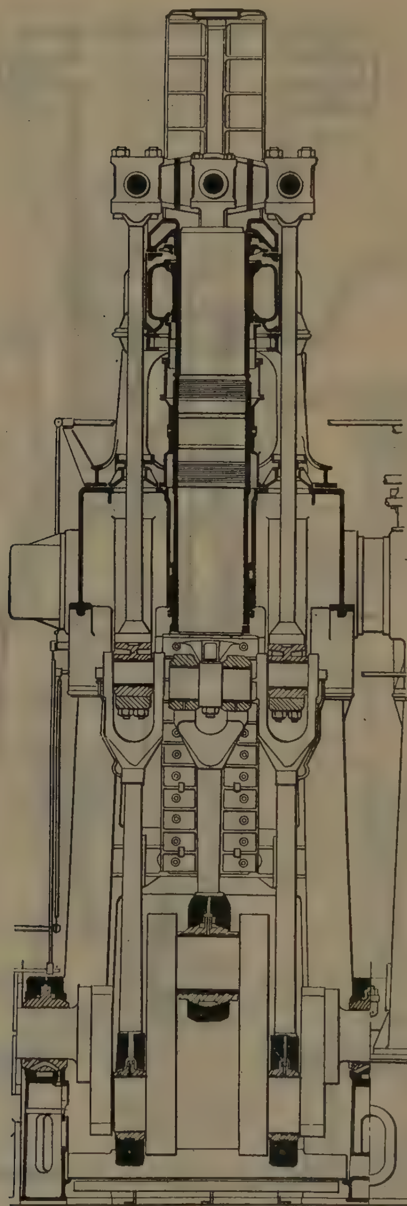

FIG. 2 DOXFORD JUNKERS OPPOSED-PISTON TWO-CYCLE ENGINE, SIDE ELEVATION

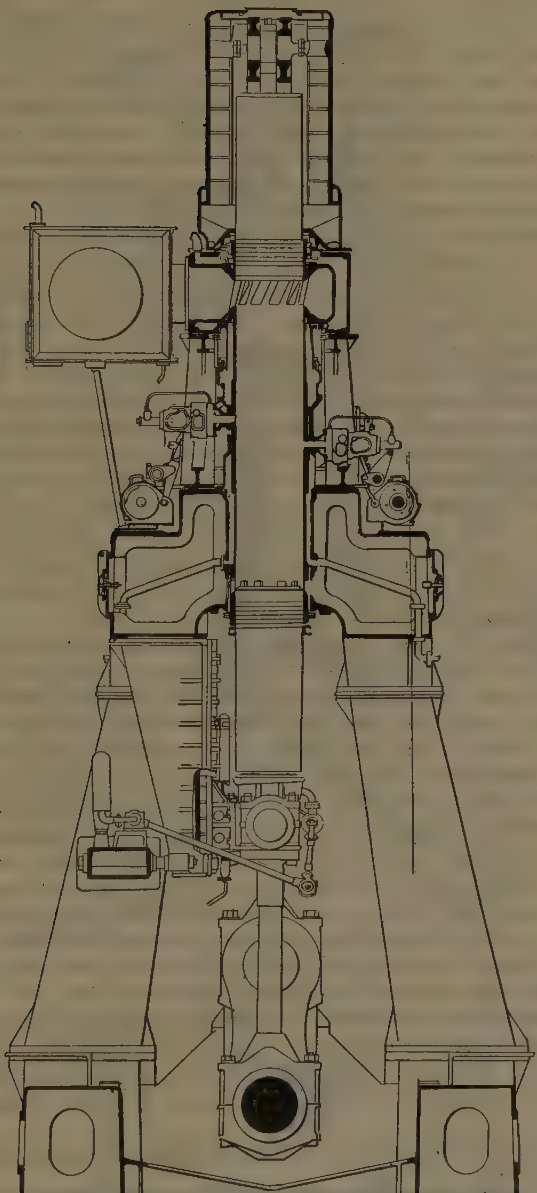

FIG. 2 (Continued) END ELEVATION

through weight, but, unlike other directions of effort, it contributes quite as much to increasing the horsepower per cylinder and the maximum practicable horsepower of the engine. So obvious does this seem that it excites some wonder at the deficiency of commercial, or even naval, double-acting engines in service, either two- or four-cycle. An examination of this question indicates a very interesting situation but one not at all unnatural, considering the pioneer nature of the project and the great cost of all experimental development typical of big-oil-engine work.

17 The first to be noted are the opposed-piston engines of Junkers and Fullagar, which are arrangements for doubling the output of a cylinder of given bore by a single-acting working piston in each end, and differ from each other mainly in the running gear connecting the two pistons to the crankshaft as in Fig. 1, the Cammellaird-Fullagar,[1] and Fig. 2, the Doxford Junkers.[2] Both involve directly or indirectly the structural ideas that cylinder heads and piston rods are unsafe elements in an oil engine, and they are accordingly eliminated. They also involve the functional idea that when the air enters one end of the cylinder-driving hot products out at the opposite end, scavenging is better than when otherwise arranged. The mechanism arrangements as to effects on frame loads, turning effort, bearing pressures and other similar matters are also features of some interest, but are secondary to the opposed-piston plan of doubling the output of the cylinder.

18 Next in order comes the true double-acting piston but without the piston rod, which in relation to the two previous opposed-piston plans accepts part of the ideas for which they stand and rejects the rest. As worked out in the North British design[3] of Fig. 3, the piston rod is still avoided as dangerous, but the scavenging-air admission at one end and exhaust escape at the other end are retained as desirable, so very desirable indeed as to justify a moving cylinder acting as a valve member with a novel mechanism both for the cylinder sleeves and for the piston with wristpin passing through slots. However, fixed cylinder heads are retained, and herein is one difference, though they do look like pistons as the cylinder slides past, and are fitted with rings.

19 Piston rods have not been avoided by all designers seeking to double the output of a cylinder, nor have they ever proved dangerous or even troublesome where used, beginning with the well-standardized double-acting gas engines. It is therefore of special interest to follow the piston-rod type or normal double-acting-engine development, especially as to ideas involved. Toward the

[1] The Largest Cammellaird-Fullagar Machine, *Motorship* (British), June, 1921.

[2] 3000 I. Hp. Single-Screw Motorship "Eknaren," *Engineering,* Nov. 10, 17, 1922.

[3] The North British Two-Stroke Double-Acting Diesel Engine, *Engineering,* June 29, 1923.

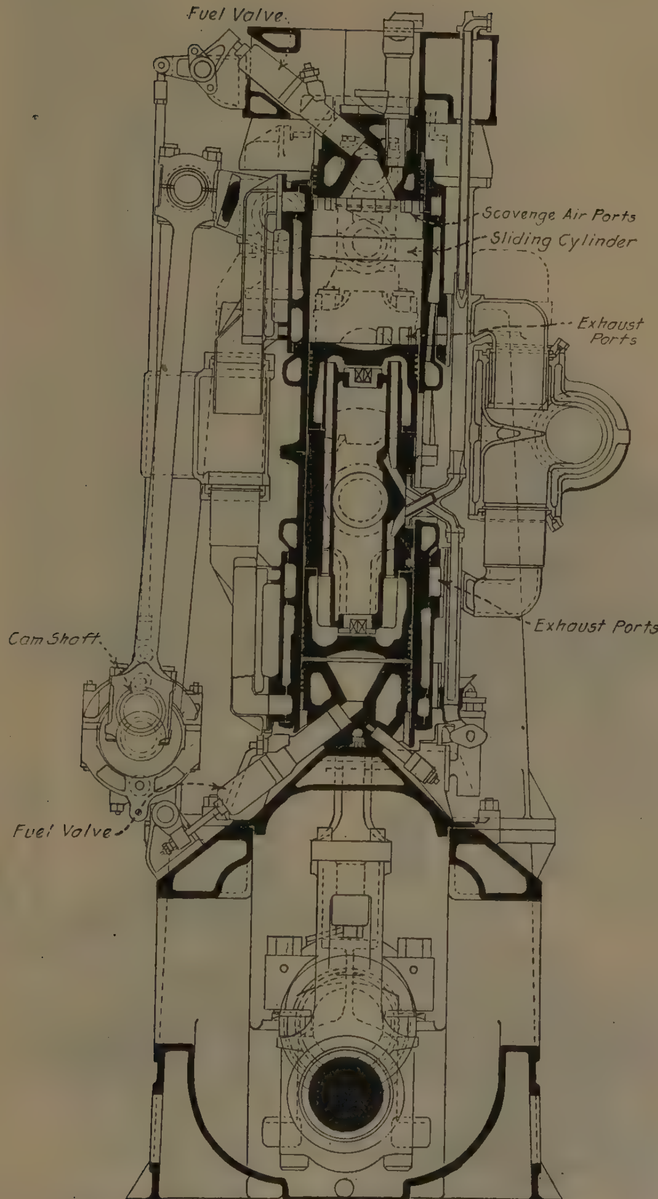

FIG. 3 NORTH BRITISH RODLESS DOUBLE-ACTING TWO-CYCLE ENGINE

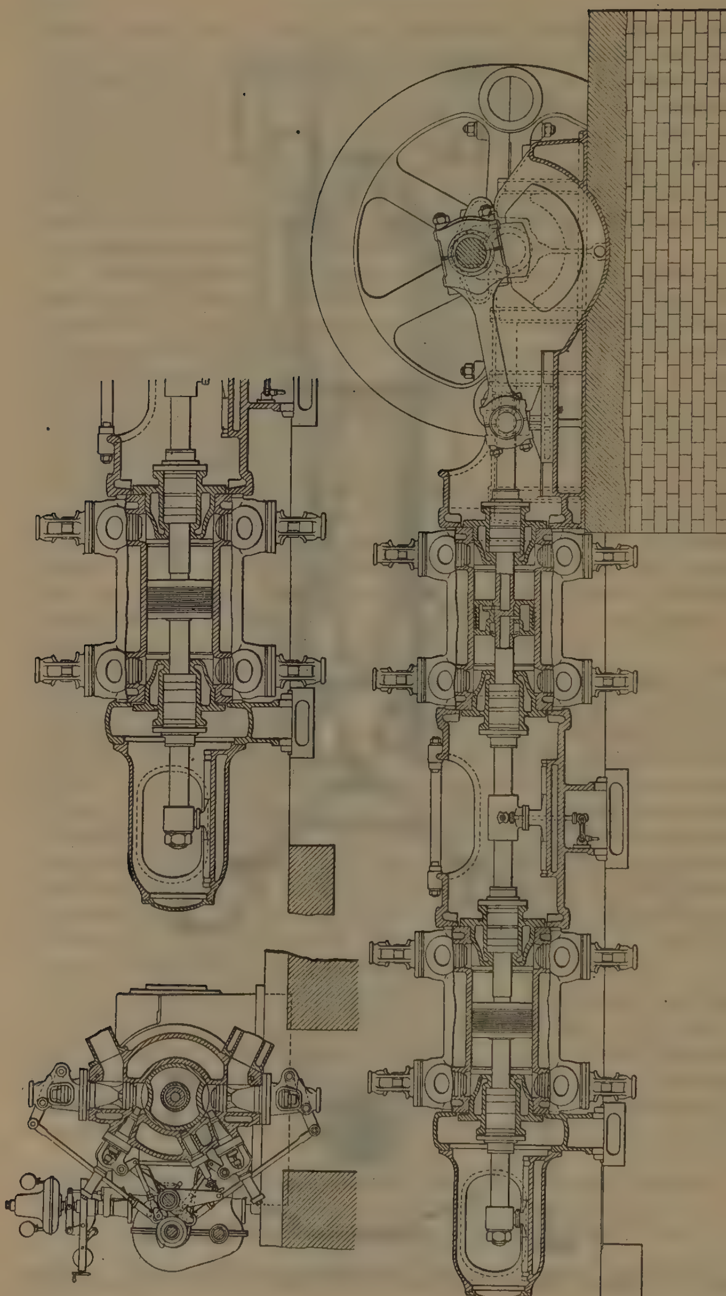

Fig. 4 Augsburg Horizontal Tandem Double-Acting Four-Cycle Engine

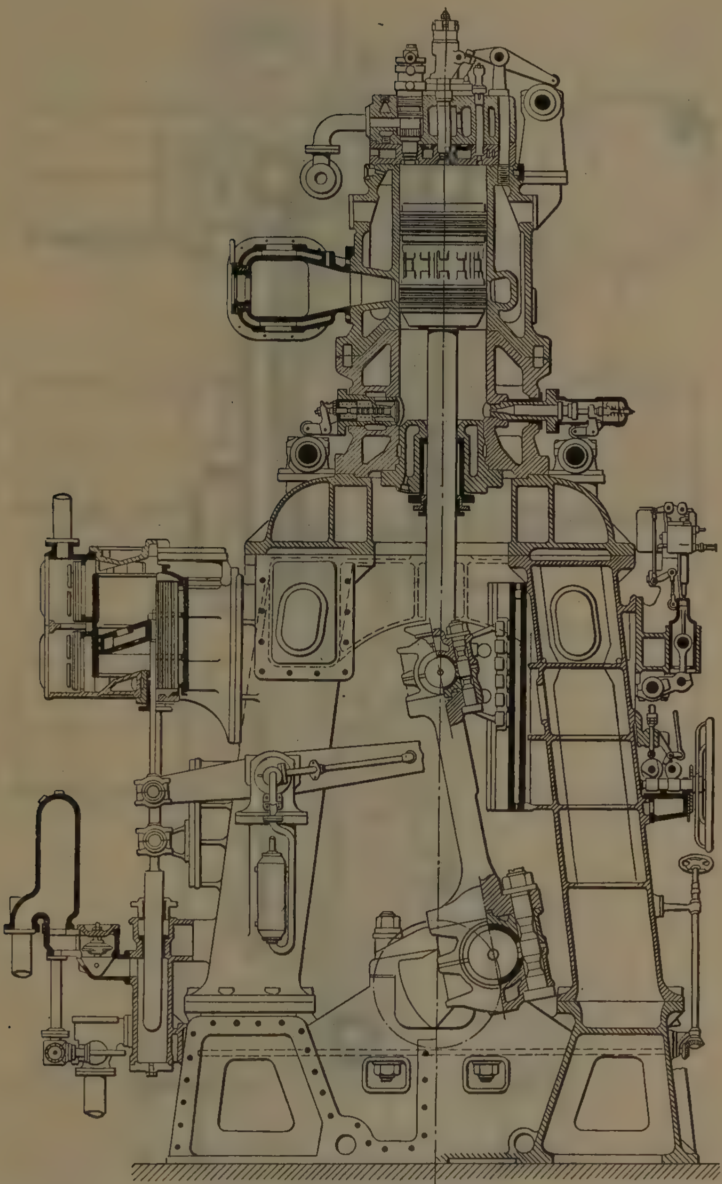

Fig. 5 Blohm & Voss Vertical Double-Acting Two-Cycle Engine, M. S. "Fritz"

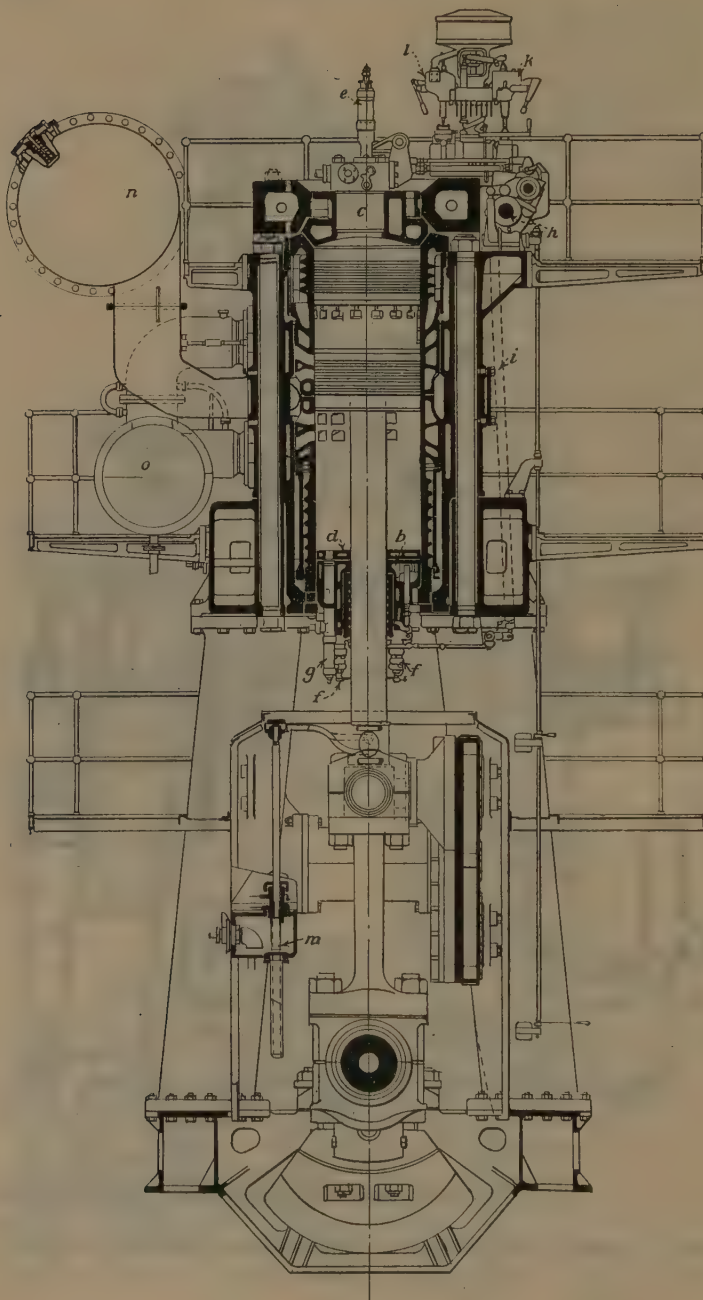

Fig. 6 M. A. N. Vertical Double-Acting Two-Cycle Engine

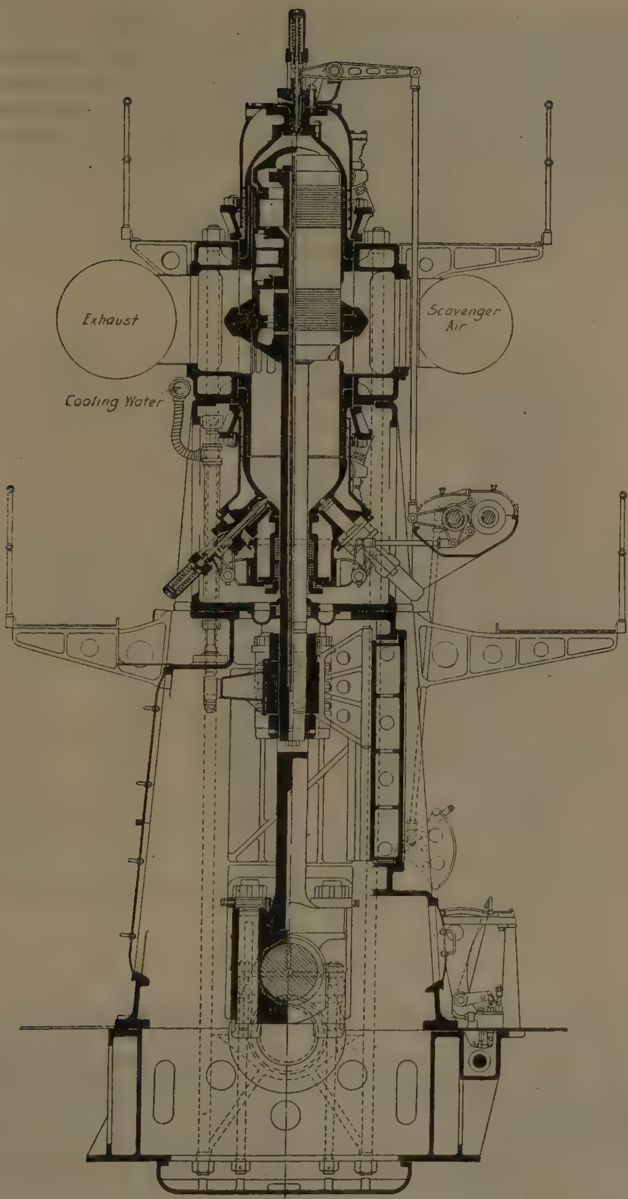

Fig. 7 Worthington Vertical Double-Acting Two-Cycle Engine

end of the period of perfecting the large double-acting gas engine, practice settled down to the adoption of the four-cycle horizontal tandem arrangement, the two-cycle having been abandoned because of the losses of gas at exhaust ports during scavenging and in spite of its greater simplicity of mechanism even with valve scavenging. Design had become pretty well standardized and service reliability well established.

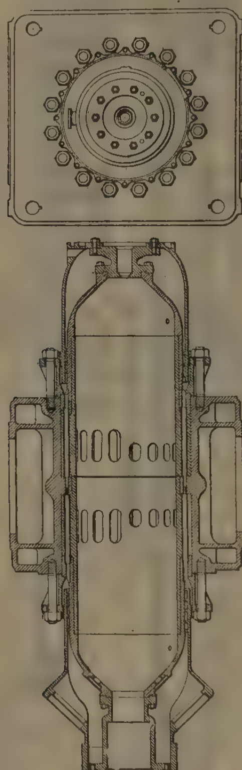

FIG. 8 WORTHINGTON STEEL JOINTLESS CYLINDER AND HEAD

20 Reduction of clearance between piston and cylinder head to get high compression made a clearance pocket at each valve, one under the inlet on top and the other above the exhaust valve under the cylinder, as in Fig. 4, illustrating an Augsburg[1] design.

21 This design is obviously unsuited to marine work, and has other limitations, more or less apparent, which limit its interest; but the vertical designs, inherently lighter, are of special interest,

[1] Double-Acting Diesel Engines, Cosmos, *Motorship*, July, 1920.

especially those that are operating two-cycle, in view of their greater simplicity and their attainment of maximum possible number of impulses with consequent promise of least weight per horsepower, as well as maximum horsepower per cylinder, subject to proof that piston rods will give no trouble, that cylinder heads are safe, and that cylinders working in both ends can be held so as to permit complete freedom of expansion of heated parts. To this must be added, also subject to proof, that combustion conditions can be as good in double- as in single-acting, it already having been established that they can be as good in two-cycle as

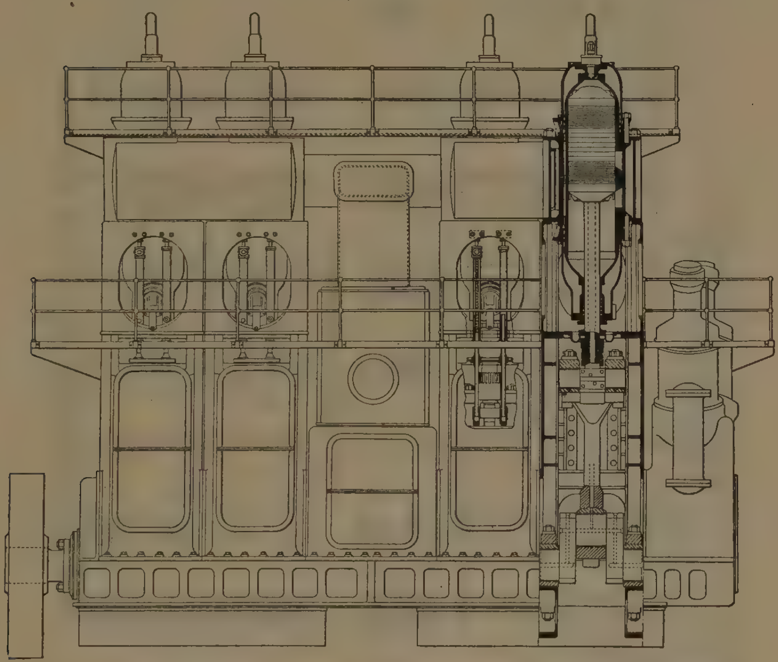

FIG. 9 WORTHINGTON ENGINE, LONGITUDINAL SECTIONAL ASSEMBLY

in four-cycle, and that side-port scavenging gives as good an air charge as a four-cycle, so that nothing can be gained by the end-to-end scavenging of the opposed-piston engines.

22 Without attempting to review the whole of the work along these lines, there are three engines of special interest, all two-cycle. The first one invites attention from the fact that it was the first double-acting oil engine[1] to be installed in a ship, the German

[1] First Motorship with Double-Acting Two-Stroke-Cycle Engines, R. Dreves. MECHANICAL ENGINEERING, July, 1921, and *Zeit. Ver. Deut. Ing.*, April, 1921.

TABLE 1 LARGE OIL ENGINES

A Four-Cycle, Single-Acting

No.	Engine name	B.hp.	I.hp.	No. cyl.	R.p.m.	Bore, in.	Stroke, in.	Hp. per cyl. B.	Hp. per cyl. I.	Mean press. B.	Mean press. I.	Refer.*
1	Burmeister & Wain	1250	....	6	125	24.8	37.8	208	....	72	....	4
2	Burmeister & Wain (Barclay-Curle Co)	....	2500	8	....	20.88	28.75	....	313	....	....	10
3	Burmeister & Wain	3000	....	8	100	31.49	47.24	375	....	80.5	....	34
4	Glenapp (Burmeister & Wain)	....	3200	8	120	29.53	43.31	....	400	....	89.2	....
5	Burmeister & Wain	1750	....	6	85	29.12	50.12	....	....	63	....	24
6	Werkspoor	1250	....	6	125	23.0	41.0	208	....	77	....	4
7	Werkspoor	1560	....	6	110	26.38	47.24	260	....	72.5	....	34
8	Werkspoor, M. S. *Ashbee*	1500	2000	6	110	27.0	47.0	250	333	66.6	....	36
9	Werkspoor	1650	....	6	110	26.5	47.25	275	....	69.5	....	24
10	Vickers, solid injection	1250	....	6	118	24.5	39	208	....	76	....	33
11	Vickers, Scottish Mardin, etc.	1250	1560	6	118	24.5	39.0	208	260	76	....	....
12	Augsburg	....	....	....	....	27.5	47.0	....	....	....	....	30
13	Tosi-Beardmore, M. S. *Pinzon*	1250	....	6	120	24.37	33.37	208	....	88	....	10
14	Krupp	1400	1867	6	125	....	....	233	....	....	....	37
15	Motorenwerke-Mannheim	1600	....	6	120	27.16	39.37	267	317	77.5	....	2
16	McIntosh-Seymour	1700	2250	6	115	28.0	48.0	283	375	61.3	....	36
17	Cramps (B. & W.), Wm. Penn	....	2250	6	115	29.2	45.25	....	375	....	85.5	35

B Two-Cycle, Single-Acting

No.	Engine name	B.hp.	I.hp.	No. cyl.	R.p.m.	Bore, in.	Stroke, in.	Hp. per cyl. B.	Hp. per cyl. I.	Mean press. B.	Mean press. I.	Refer.*
1	Sulzer	1250	....	4	102	23.6	37	310	....	76.3	....	4
2	Sulzer	1800	....	4	90	26.77	47.24	450	....	71.5	....	32
3	Sulzer stationary, 1915	4500	....	6	132	30	40	750	....	....	94	3
4	Sulzer experimental, 1914	2000	....	1	....	39.375	43.3125	2000	....	....	122	3
		2058	3297	1	149.6	39.375	43.3125	....	....	....	....	....
5	Sulzer	4000	....	6	100	32.0	48.0	666	....	68.5	....	34
6	Ansaldo, San Giorgio	1250	....	4	110	24.8	35.4	310	....	66	....	33
7	Ansaldo	1100	....	4	100	25.8	35.4	275	....	64.0	....	34
8	Germania Werft, Kiel, M. S. *Zoppot*	1675	2360	6	106	22.64	39.37	280	....	70	....	2
9	Carels (1911)	....	....	....	150	38.375	38.375	1250	....	....	....	....
10	Nobel, Swedish	1600	....	4	106	26.58	36.22	400	....	....	....	....
		1635	2022	....	107	....	....	410	....	....	....	2
		1958	2401	....	108.4	....	....	490	....	....	....	....
11	Bethlehem	2900	4000	6	116	25.5	48	480	....	67.5	....	....

* These reference numbers apply to items in the Bibliography.

33

TABLE 1 LARGE OIL ENGINES—CONTINUED

No.	Engine name	B.hp.	I.hp.	No. cyl.	R.p.m.	Bore, in.	Stroke, in.	Hp. per cyl. B.	Hp. per cyl. I.	Mean press. B.	Mean press. I.	Refer.*
	C Two-Cycle, Opposed-Piston and Rodless, Double-Acting											
1	Palmer-Fullagar	3000	....	6	90	23	36	500	....	73.5	....	32
2	Cammellaird-Fullagar	1250	....	4	125	18.5	25	310	....	71	....	4
3	Doxford	2610	3050	4	76.7	22.375	45.5	650	....	90	....	33
4	Doxford	1850	....	3	85	21.25	42.5	615	....	90	....	32
	D Four-Cycle, Double-Acting											
1	M. A. N.	1600	....	2	....	23.33	31.50	800	....	....	....	5
2	N. E. Werkspoor	600	....	1	95	31.5	55.0	600	....	58.0	....	32
3	Burmeister & Wain	6750	8150	6	....	33.0	59.12	1130	....	70	....	32
	E Two-Cycle, Double-Acting											
1	North British	2000	....	3	100	24.5	44	667	....	63	....	38
2	Durenberg, 1910	12000	....	6	160	33.46	41.34	2000	....	....	....	2
	Valve Scavenging	3220	....	1	145	33.46	....	....	3573	....	144	....
3	Blohm & Voss, M. S. *Fritz*	850	....	3	110	18.9	27.95	283	....	75	....	39
4	M. A. N.	2000	....	3	75	27.5	46.25	667	....	63	....	32
5	Worthington	2900	....	4	95	28.0	40.0	2900	....	64.8	....	....

* These reference numbers apply to items in the Bibliography.

motorship *Fritz*, Fig. 5. The working cylinder used valve scavenging, thus retaining the idea that proper scavenging requires admission and outlet to be at opposite ends. Put into service in 1915, it represents the first practical outcome of a series of large-scale developments of double-acting-engine ideas conducted by M.A.N.,[1] begun in 1910 in an effort to develop Diesel ships for the German Navy and extending up to the present time. The last published outcome of this series is the double-acting, two-cycle, port-scavenging design[1] of Fig. 6, the research work having demonstrated the complete effectiveness of port scavenging for the double-acting two-cycle as had previously been demonstrated for the single-acting.

[1] The Diesel Engine of Today, A. Naegel. *Zeit. Ver. Deut. Ing.*, June, 1923.

23 Finally, in this series of developments of double-acting engines of the simplest arrangement of major running parts, with the plain piston rod and with the simplest of cylinder charging arrangements — that of the two-cycle port-scavenging — there is to be noted the Worthington design, which carries simplification of the cylinder and head structure farther than before. This is a forged-steel shell, including head and most of one half of the cylinder, Figs. 7, 8, 9 and 10, shaped somewhat like a hollow pro-

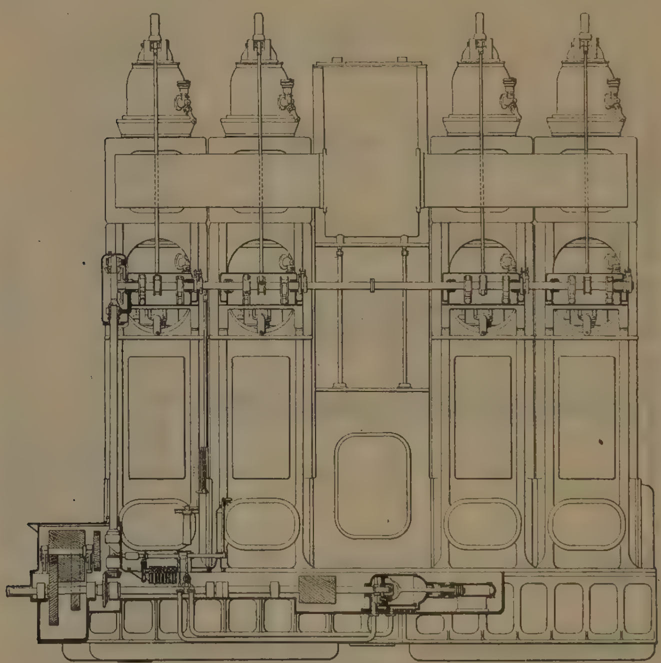

Fig. 10 Worthington Engine, Side View Showing Valve Gear

jectile at each end, with the parts held together and against a base plate carrying ports and frame connections near the middle. A thin cast-iron liner provides a piston rubbing face, and a cast-iron outer shell forms the water jacket in such a way as to permit complete freedom of expansion of the heated member, while the long bore of the central base or frame member assures axial alignment for free working of the piston.

24 This cylinder construction by the use of forged steel is thinner than one of cast iron, in inverse ratio to the allowable working stresses, roughly one-third, and is wholly free of joints or webs which involve localization of expansion stresses, though its ductility

compared with cast iron makes it capable of greater resistance even if such were present. It is therefore capable of safely carrying very much higher rates of heat generation than one of cast iron for a given bore, and as a consequence may safely carry such rates as are characteristic of larger bores than are possible for cast iron.

25 Some items of interest, including the relation of fuel consumption to size and arrangement of large oil engines, are now to be examined in a more analytical way after reviewing some of the notable large engines that have been built. The main data of some of these engines are collected for easy reference in Table 1. Of these large engines there are two that stand out as of maximum interest in connection with size, both being experimental machines and each the biggest of its kind.

26 The first of those is the experimental Sulzer engine of 2000 b.hp. in one cylinder, tested by Dr. A. Stodola[1] in 1914, a single-acting, port-scavenging engine. It developed 2058 b.hp. or 3297 i.hp. for 6 hours at 149.6 r.p.m. with a mean indicated pressure of 122 lb. per sq. in. or 159 lb. per sq. in. on effective stroke with a high scavenging air pressure of 7 lb. per sq. in., and at 4.55 lb. per sq. in. the mean pressure was 110 lb. per sq. in. or 143 lb. per sq. in. on effective stroke.

27 The other is of the Nuremberg engine, also experimental, of 12,000 b.hp. in 6 cylinders, or 2000 b.hp. per cylinder at 160 r.p.m., reported by Prof. A. Naegel[2] to have operated at loads of 10,800 to 12,000 b.hp. for five days continuously during a period of three months of trials. Other tests of one cylinder in 1917 carried 3573 i.hp. at 145 r.p.m. with a mean indicated pressure of 144 lb. per sq. in., and a friction loss of 10 per cent or 14.4 lb. per sq. in., exclusive of scavenging and compression, which were separate. This, like the former engine, was a two-cycle one, but unlike it was double-acting and valve-scavenged, and 33.46 in. by 41.34 in.

28 Neither of these high-capacity engines ever came into commercial use, but from them much information of great value has been obtained.

[1] The Development of the Sulzer Engine, by L. S. LeMesurier. Inst. Mech. Engrs., January, 1923.

[2] The Diesel Engine of Today, A. Naegel. *Zeit. Ver. Deut. Ing.*, June, 1923.

PART II

Forces and Weights

29 As the maximum pressures in cylinders of all classes of Diesel engines may be taken as equal in any comparative study, it follows that, for a given bore and stroke, the maximum load in pounds due to gas pressure, acting to stress the major fixed parts, is identical for the two- and four-cycle classes. With negligible or no difference in inertia and centrifugal forces of the major running parts, the same is true for them. Therefore, whatever difference in metal weights there may be in two-cycle as compared with four-cycle engines of equal bore, stroke, and number of cylinders, must be in the minor parts, particularly the scavenging pump weight with connections as compared with the weight of the four-cycle valve gear. Direct comparison of some standard designs of large single-acting two- and four-cycle marine engines of similar framing, of the closed box type, and having crossheads indicates that the weight for each class is about constant when measured in pounds per cubic inch of stroke volume (cylinder area times stroke) for equal numbers of cylinders.

30 The difference is a measure of the excess weight of the scavenging equipment. Such figures for pounds per cubic inch of stroke volume are independent of cylinder diameter in the range of sizes of interest, but they decrease somewhat with increase in number of cylinders from three to six.

31 A double-acting engine of the same bore and stroke as a single-acting receives the same head-end loading due to gas pressures, putting the frame in tension, and a loading at the crank end about 10 per cent less because of the rod, putting the frame in compression. Thus, the double-acting cylinder requires no more frame metal than is required for the single-acting to resist these direct loads. The same metal carries its load a large part of the time, just as in a single-acting engine, comparing two-cycle with four-cycle. More metal is, however, necessary to convert a single- to a double-acting cylinder: in the cylinder with its heads and a stuffing box, in the piston, and in various other items, including connections. The total is about one and a quarter times the weight per cubic inch for single-acting engines. For similar construction details and materials, that is, similar machine design, the double-acting engines will therefore weigh a little more per cubic inch of stroke volume than the single-acting, and the excess may be greater for two- than for four-cycle, because the cylinder must be longer.

32 It is desirable, in order to compare the four types, to set down some values for these weights per cubic inch as a basis for judging comparative weights per horsepower for similar machine

design, avoiding as far as possible individual differences in machine design.

Let w = weight of a six-cylinder four-cycle single-acting engine

wx_1 = added weight of a four- over a six-cylinder four-cycle single-acting engine

wx_2 = added weight of a three- over a four-cylinder four-cycle single-acting engine

wS_s = weight of a single-acting two-cycle engine scavenger over four-cycle valve gear

wS_d = weight of a double-acting two-cycle engine scavenger over four-cycle valve gear

wD_4 and wD_2 = weight of four- and two-cycle double-acting engines, respectively, over the four-cycle single-acting engine

V = cubic inches of stroke-bore volume.

33 On account of starting torque and turning effort, a marine engine must have a minimum of six cylinders when four-cycle single-acting, four when two-cycle single-acting, two when two-cycle double-acting and three when four-cycle double-acting.

34 The weight per horsepower for each style will then be as follows in terms of the weight per cubic inch for the four-cycle single-acting engine:

Weight per Horsepower, Lb.

Style	Six cylinders	Four cylinders
4-cycle single-acting	$\frac{792000}{PN}\left(\frac{W}{V}\right)$	$\frac{792000}{PN}\left(\frac{W}{V}\right)(1+x_1)$
2-cycle single-acting	$\frac{792000}{PN}\left(\frac{W}{V}\right)\left[\frac{1+S_s}{2}\right]$	$\frac{792000}{PN}\left(\frac{W}{V}\right)\left[\frac{1+x_1S_s}{2}\right]$
4-cycle double-acting	$\frac{792000}{PN}\left(\frac{W}{V}\right)\left[\frac{1+D_4}{2}\right]$	$\frac{792000}{PN}\left(\frac{W}{V}\right)\left[\frac{1+x_1+D_4}{2}\right]$
2-cycle double-acting	$\frac{792000}{PN}\left(\frac{W}{V}\right)\left[\frac{1+S_d+D_2}{4}\right]$	$\frac{792000}{PN}\left(\frac{W}{V}\right)\left[\frac{1+x_1+S_d+D_2}{4}\right]$

Style	Three cylinders
4-cycle single-acting	$\frac{792000}{PN}\left(\frac{W}{V}\right)(1+x_1+x_2)$
2-cycle single-acting	$\frac{792000}{PN}\left(\frac{W}{V}\right)\left[\frac{1+x_1+x_2+S_s}{2}\right]$
4-cycle double-acting	$\frac{792000}{PN}\left(\frac{W}{V}\right)\left[\frac{1+x_1+x_2+D_4}{2}\right]$
2-cycle double-acting	$\frac{792000}{PN}\left(\frac{W}{V}\right)\left[\frac{1+x_1+x_2+S_d+D_2}{4}\right]$

35 It is evident by inspection that for equal values of r.p.m. = N, and mean effective pressure = P, the weights per horsepower are in proportion to the numbers in brackets, with that for the six-cylinder, four-cycle, single-acting engine as unity. The numerators in the brackets are the relative weights per cubic inch of stroke volume.

36 To make a true comparison would require evaluation of the various quantities — which cannot be done precisely because there

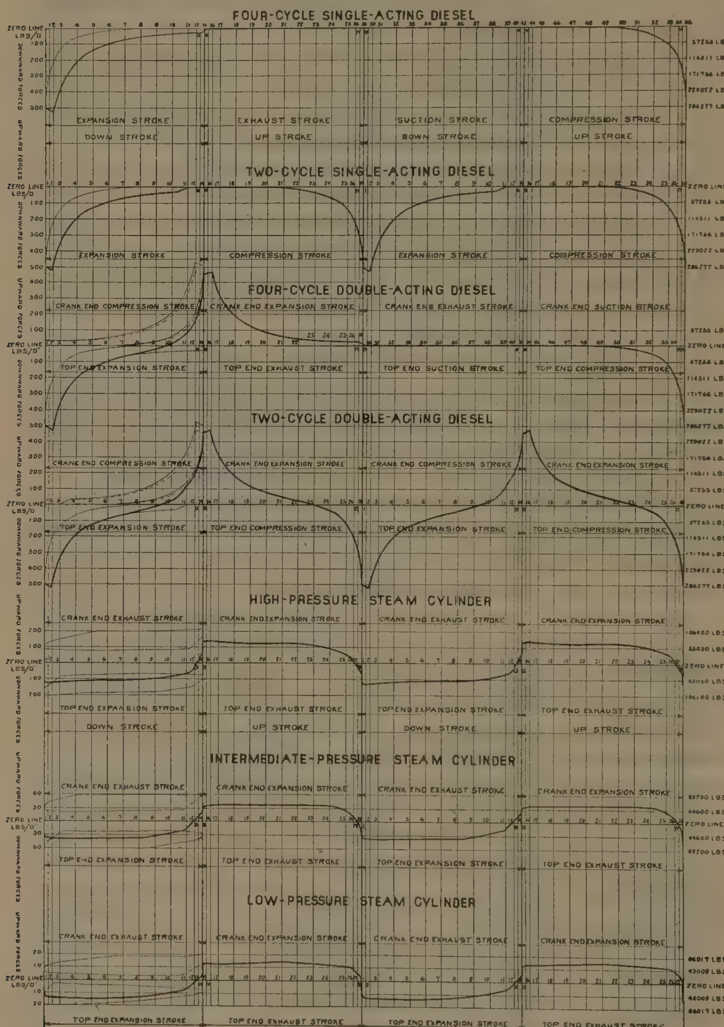

FIG. 11 UPWARD AND DOWNWARD VERTICAL FORCES—FRAME LOADS
(Upward forces show above, downward forces below, the zero line; forces as plotted apply to piston; frame forces are in the opposite direction.)

are no really homologous designs of engines in all classes and styles, nor do various builders adopt the same values of speed and mean pressure for either combustion conditions or wall-temperature

limits. It does, however, seem desirable to make some reasonable approximation to see type relations as nearly correctly as possible. These may then be compared with known or published figures,

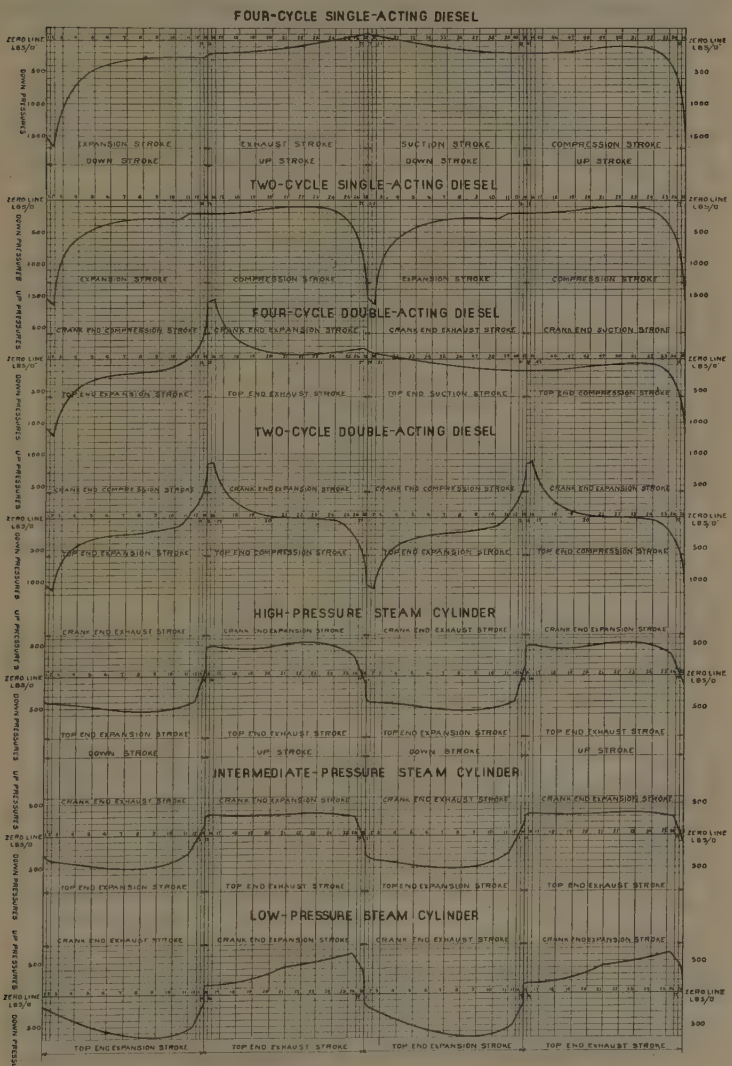

FIG. 12 WRISTPIN-BEARING PRESSURES

(Pressures below the zero line show downward pressures; above the zero line, upward pressures.)

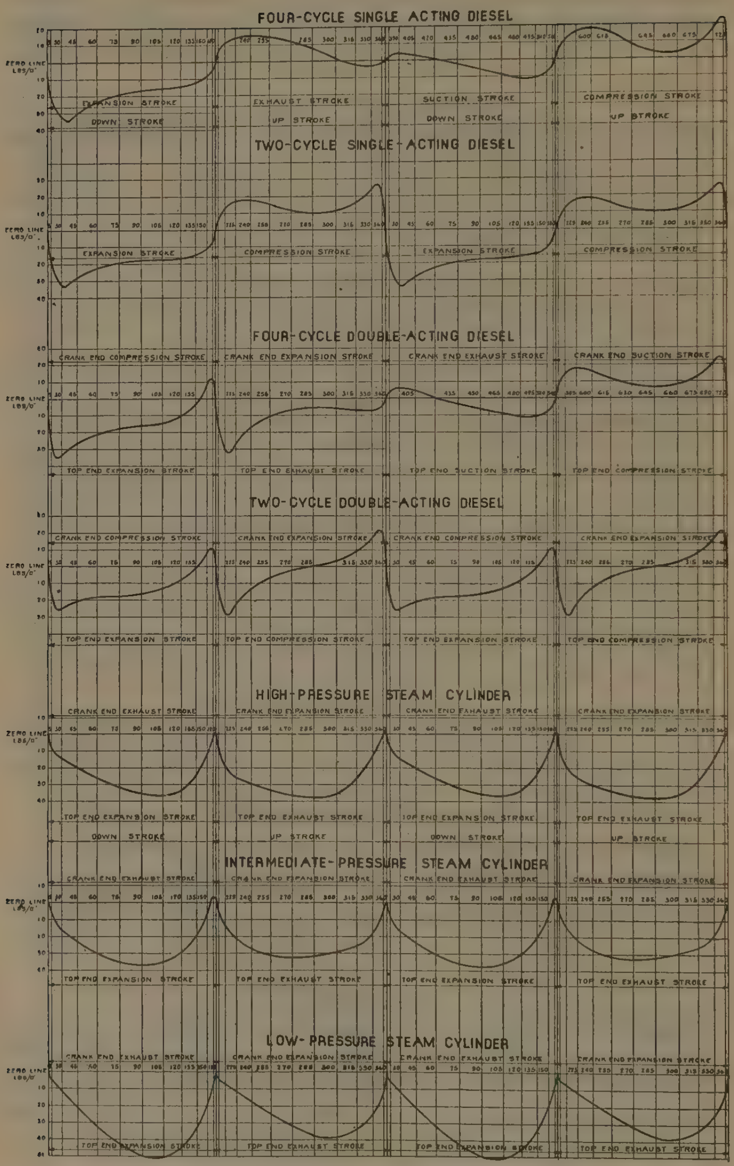

FIG. 13 CROSSHEAD-GUIDE PRESSURES

(Pressures below the zero line show pressures on guides; those above the zero line, pressures on gibs for ahead running.)

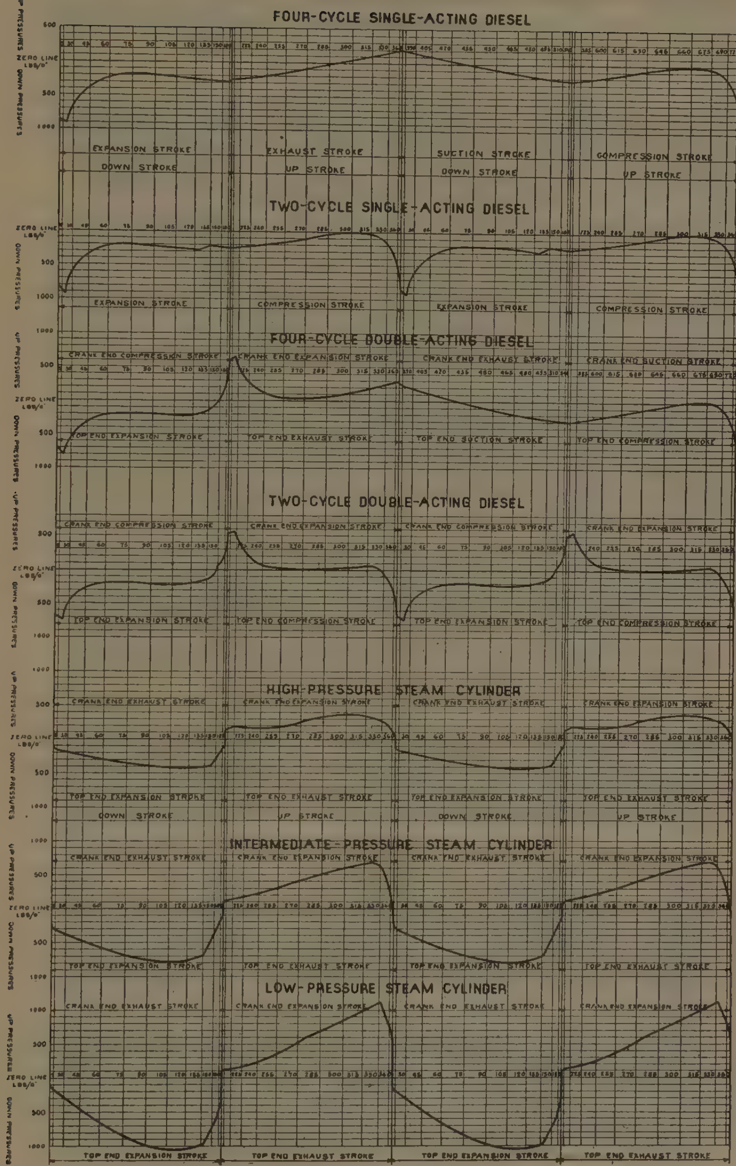

FIG. 14 CRANKPIN-BEARING PRESSURES
(Upward pressures show above, downward pressures below, the zero line.)

and finally compared with ratios derived from a force analysis, with determinations of average metal loading compared with maxi-

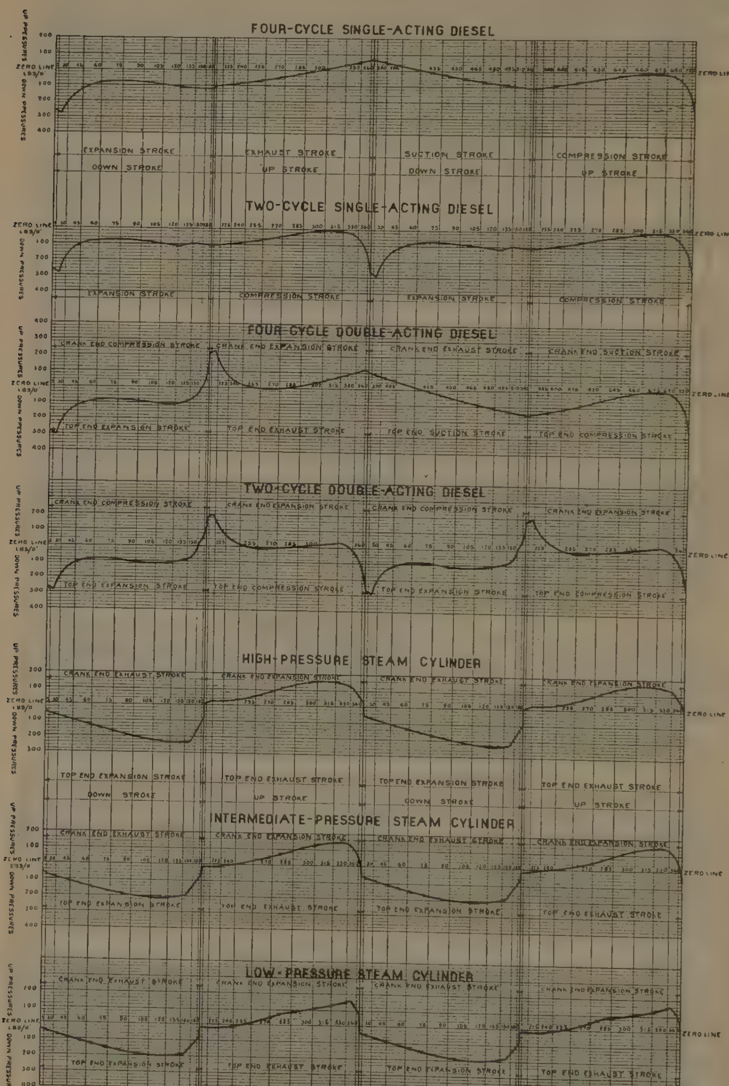

FIG. 15 MAIN-BEARING PRESSURES
(Upward pressures show above, downward pressures below, the zero line.)

mum. No one basis is in itself conclusive as a guide to the most promising directions of future effort.

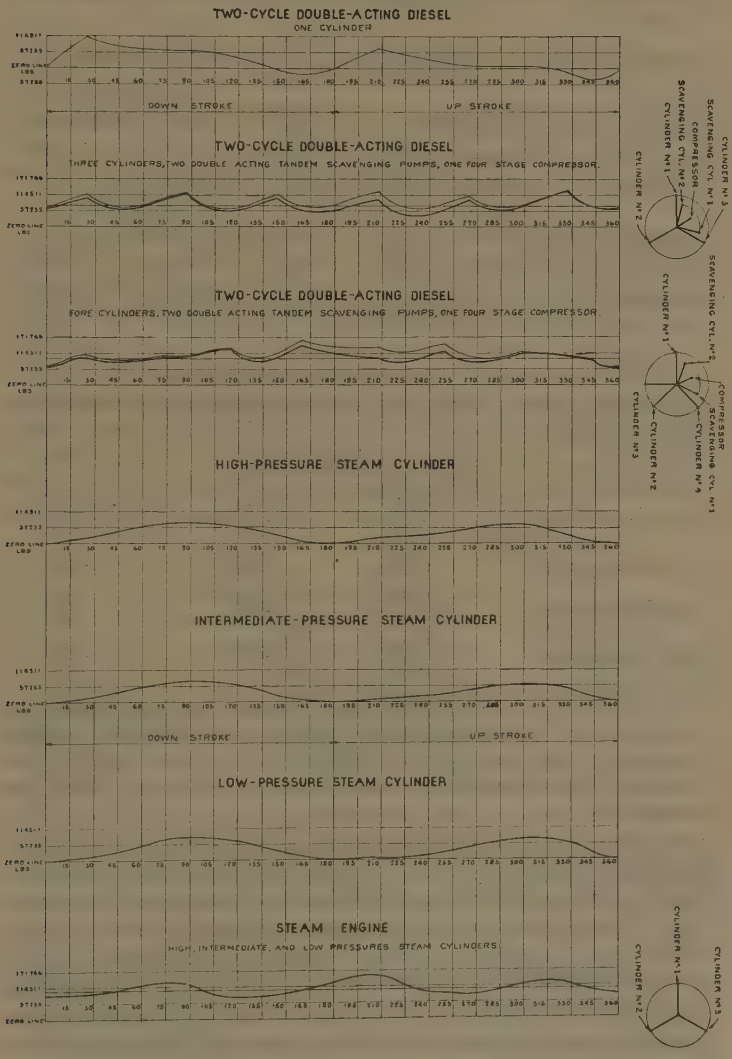

FIG. 16 TURNING EFFORTS

37 From some comparisons of weights of a number of three-, four- and six-cylinder engines, it seems that the following are

reasonable values, and close enough to the truth for type comparison:

$x_1 = 0.11$; $x_2 = 0.08$; $S_s = 0.12$; $S_d = 0.09$; $D_4 = 0.22$; $D_2 = 0.30$.

$$1+x_1 = 1.11;\quad 1+x_1+x_2 = 1.19;\quad \frac{1+S_s}{2} = \frac{1.12}{2} = 0.56;$$

$$\frac{1+x_1+S_s}{2} = \frac{1.23}{2} = 0.615;\quad \frac{1+x_1+x_2+S_s}{2} = \frac{1.31}{2} = 0.655;$$

$$\frac{1+D_4}{2} = \frac{1.22}{2} = 0.61.\quad \frac{1+x_1+D_4}{2} = \frac{1.33}{2} = 0.665;$$

$$\frac{1+x_1+x_2+D_4}{2} = \frac{1.41}{2} = 0.705;\quad \frac{1+S_d+D_2}{4} = \frac{1.39}{4} = 0.35;$$

$$\frac{1+x_1+S_d+D_2}{4} = \frac{1.50}{4} = 0.375;\quad \frac{1+x_1+x_2+S_d+D_2}{4} = \frac{1.58}{4} = 0.395.$$

Substituting these values, the weight relations per horsepower are as follows:

Style	Weight per Horsepower, Lb. Six cylinders	Four cylinders	Three cylinders
4-cycle single-acting	$\frac{1.0}{PN}$	$\frac{1.11}{PN}$	$\frac{1.19}{PN}$
2-cycle single-acting	$\frac{0.56}{PN}$	$\frac{0.615}{PN}$	$\frac{0.655}{PN}$
4-cycle double-acting	$\frac{0.56}{PN}$	$\frac{0.665}{PN}$	$\frac{0.705}{PN}$
2-cycle double-acting	$\frac{0.35}{PN}$	$\frac{0.375}{PN}$	$\frac{0.395}{PN}$

38 Reference to this tabulation indicates that in general the double-acting, four-cycle and the single-acting, two-cycle engines are more or less similar as to weight per horsepower even with full allowance for differences in mean pressure and r.p.m. Both are lighter than the four-cycle, single-acting engine for equal values of these quantities. On the other hand, however, the double-acting, two-cycle engine is as much lower than they are in lb. per hp. as they in turn are lighter than the four-cycle, single-acting engine.

39 Referring to published figures for engines that are in service and as nearly similar in machine design as circumstances permit, a large four-cylinder, two-cycle, single-acting engine with attached scavenging pump weighed 340 lb. per hp. at $P = 57$, against a six-cylinder, single-acting, four-cylinder value of 575 lb. per hp. at $P = 62$, both at 90 r.p.m. This is in the ratio $\frac{575}{340} = 1.69$. Using the value of P in the table, the ratio is $\frac{162}{108} = 1.51$, the difference indicating the uncertainty of the coefficients and range in machine

design. A two-cycle, double-acting engine of four cylinders and comparable as to speed gives a weight ratio in comparison with the six-cylinder, four-cycle, single-acting of $\frac{575}{209} = 2.75$ with $P = 64$. The table makes the ratio $\frac{162}{59} = 2.57$, about the same degree of difference as above.

40 While, therefore, the constants that have been estimated cannot be accepted as having the values assigned, they do seem to be close enough for a general type comparison, though not for any detail work.

41 As the numerators in the brackets are relative weights per cubic inch of stroke volume, it becomes possible with any specific data to make check calculations by a direct valuation of the several sums of the coefficients over all, and then, by difference, each one in turn.

42 There are given below some weight figures for a six-cylinder, single-acting, four-cycle engine, 26 in. by 42 in., of crosshead design, with corresponding figures for a Worthington 28-in. by 40-in. double-acting, two-cycle engine, and as an interesting check, figures for a triple-expansion steam engine of comparable size, 26 in. by 43 in. by 74 in., and 48 in. stroke.

Classification	Weights of engines, pounds			Pounds per cu. in. of stroke volume		
	Triple steam, excluding boilers, etc.	4-cycle 6-cyl., single-acting	2-cycle 4-cyl., double-acting	Steam	4-cycle single-acting	2-cycle double-acting
Major fixed parts	172,252	258,200	305,540			
Major running parts	58,073	118,330	123,500			
Other parts	55,959	186,350	186,960			
Total	286,274	562,880	616,000	0.95	4.20	6.25

43 The three designs have stroke volumes of 301,632; 133,812; and 98,560, respectively, which makes the weight per cubic inch 0.95 for steam, 4.20 for four-cycle, single-acting, and 6.25 lb., for two-cycle, double-acting engines, including scavenger. The ratio for the last two is $\frac{6.25}{4.20} = 1.48 = 1+x_1+S_d+D_2$. By addition of the tentatively used coefficients this becomes $1.00+0.11+0.09+0.03 = 1.50$. No one check of this kind is of any particular value, but many cross-checks will in time pretty well establish all these coefficients and their ranges, removing one of the many complexities in the relative characteristics of engines of different types.

44 One prime variable must always be the unit weights in pounds per cubic inch of stroke volume, and while class totals are

given above, it is of considerable interest to be able to compare the principal items in the main parts as follows:

	Parts weights (pounds)		
	Triple steam	Four-cycle six-cylinder single-acting	Two-cycle four-cylinder double-acting
Major fixed parts			
Bedplate	45,797	64,000	53,230
Frame and housings	39,648	108,000	135,376
Cylinders and liners	66,399	50,000	45,120
Cylinder heads	8,724	31,000	46,000
Main bearings	11,684	15,200	25,820
Total	172,252	258,200	305,546
Major running parts			
Crankshaft	33,088	61,100	44,000
Connecting rods	10,880	30,680	36,360
Crossheads	4,402	9,450	10,960
Piston rods	2,579	9,000	14,840
Pistons	7,124	8,100	17,600
Total	58,073	118,330	123,500

45 It is only through figures like these that it becomes possible to see what happens when the engine type is changed, especially from single- to double-acting; and even passing from four-cycle to two-cycle with attached scavenging pump, the weight per cubic inch is increased by only 50 per cent while the horsepower is increased to something less than four times for equal cylinders, depending on the other factors discussed, as shown in the tables.

46 This increase in weight is not the result of overstressing but rather the consequence of the new arrangement, part being due to the scavenging pump. The loads on the main parts are shown by the force diagrams and are consistently related, as is clear from the values determined from these diagrams.

47 Force and pressure diagrams have been prepared over four full strokes for one cylinder of each of the four classes of oil engines, assumed equal in bore and stroke, and for each of the three cylinders of the triple-expansion steam engine, high, intermediate, and low. The first of these diagrams is shown in Fig. 11, and represents to a stroke base the upward and downward forces acting to load the cylinder head, and hence the frame down to the main bearings. The regular periodic use of the metal every stroke, typical of the steam engine, is clearly approximated by the double-acting, two-cycle oil engine, but by no other arrangement. In all cases the oil engines have the same pressures over the effective stroke and a low mean value is chosen within the range of common use, 94.2 lb. per sq. in. for full-stroke four-cycle and effective-stroke, two-cycle engines, which gives 80 lb. for full-stroke, two-cycle engines. For the steam engine the mean pressures are 98.3 high, 34.5 intermediate, and 11.8 low. These general relations are shown in Table 2. From the diagrams the following ratios of maxi-

mum to mean loads are found as measures of extent of use of metal.

	Maximum and mean frame loads, lb. per sq. in.					
	Upward forces			Downward forces		
	Maximum	Mean	Ratio	Maximum	Mean	Ratio
4-cycle, single-acting	....	....	....	520	42.4	12.26
2-cycle, single-acting	....	....	....	520	82.6	6.3
4-cycle, double-acting	470	35.8	13.1	520	40.2	12.9
2-cycle, double-acting	470	64.6	7.3	520	74.2	7.0
Steam, high	136	55.2	2.47	126	51.3	2.46
Steam, intermediate	37.4	17.1	2.19	38.8	15.7	2.47
Steam, low	15.0	6.5	2.31	14.0	5.6	2.5

48 The next step in the force analysis is to examine the running parts loading and the bearing pressures. The same cylinder pressures act on the piston and give the piston rod and all connected parts forces equal to those acting on the frame except for the small effect of a piston rod. There are to be evaluated, however, the inertia forces of reciprocating parts and the corresponding centrifugal forces of rotating parts at appropriate points to get correct resultants. The values of these items are given in Table 3.

49 Considering the wristpin, the resultant is the combined effect of cylinder pressures, reciprocating inertia, and gravity weight of the moving parts. The reciprocating inertia acts favorably in oil engines to reduce the high initial cylinder pressures and adds to the low terminal or release pressures at the end of the stroke. It is, of course, of no help on idle strokes and is harmful rather than beneficial in the steam engine. It is of greatest value in equalizing the resultant forces in the double-acting two-cycle engine, and higher values for the inertia here used would give better results. This is a direct indication that such an engine may advantageously be run much faster than the speeds here considered. The wristpin bearing-pressure relations in Fig. 12 show clearly the greater regularity of bearing pressures for the two-cycle oil engines and the maximum and mean values compare favorably with the steam engine as tabulated below.

	Wristpin-bearing pressures, lb. per sq. in.					
	Upward			Downward		
	Maximum	Mean	Ratio	Maximum	Mean	Ratio
4-cycle, single-acting	110	8.9	12.4	1650	236	7.0
2-cycle, single-acting	....	....	....	1640	321	5.11
4-cycle, double-acting	940	77.7	12.1	1200	148	8.1
2-cycle, double-acting	860	86.8	9.9	1140	170	6.7
Steam, high	540	219	2.46	540	222	2.43
Steam, intermediate..	360	131	2.75	500	201	2.48
Steam, low	630	177	3.57	690	265	2.6

50 Introducing the angularity of the rod and the areas of the crosshead slides, ahead and astern sides, Fig. 13 results. These pressures are low in comparison with the steam engine, indicating

conservative design and perhaps a possibility of later reduction of dimensions, as shown by the tabulation:

	Crosshead-guide pressures, lb. per sq. in.					
	Right			Left		
	Maximum	Mean	Ratio	Maximum	Mean	Ratio
4-cycle, single-acting	35	5.87	5.97	24	4.95	4.85
2-cycle, single-acting	34	9.6	3.54	27	7.38	3.66
4-cycle, double-acting	35	8.0	4.37	24	3.43	7.0
2-cycle, double-acting	29	9.8	2.96	22	2.2	10.0
Steam, high	38	26.9	1.41	..	....	....
Steam, intermediate	37	25.7	1.44	..	....	....
Steam, low	51	30	1.7	..	....	....

51 Passing to the crankpin, there is a centrifugal force due to the rod end to be considered in finding the resultant, in addition to the other items, thus making the vertical forces at the crankpin a little different from those at the wristpin. This is shown in Fig. 14 in comparison with Fig. 12, as further modified by relative bearing areas.

52 Here the peaks due to low reciprocating inertia are still present, but the pressures are low compared with those in the steam engine and would be even better with higher inertia through higher speed. The same situation exists as to main-bearing pressures as shown in Fig. 15 and the general similarity of the two-cycle, double-acting oil engine to the steam engine is very striking. These relations are clearly set forth in the tabulation following:

	Crankpin and main-bearing pressures (vertical) lb. per sq. in.											
	Crankpin						Main bearings					
	Up			Down			Up			Down		
	Max.	Mean	Ratio	Max.	Mean	Ratio	Max.	Mean	Ratio	Max.	Mean	Ratio
4-cycle, S. A.	200	17.7	11.3	900	153	5.9	65	5.5	11.8	280	60	4.67
2-cycle, D. A.	....	....	....	900	211	4.27	...	....	....	285	80	3.56
4-cycle, D. A.	650	54.8	11.85	800	118	6.78	225	18.8	11.96	300	53.2	5.69
2-cycle, D. A.	560	50.5	11.1	740	119	6.22	200	13.5	14.85	280	55	5.08
Steam, high	370	129	2.88	430	168	2.62	155	45.7	3.39	235	86	2.73
Steam, intermed.	670	197	3.41	800	293	2.73	150	33.7	4.46	210	78.7	2.67
Steam, low	1100	281	3.91	1060	394	2.69	160	36.2	4.42	200	72.7	2.75

53 The favorable values of bearing pressures so clearly set forth for the oil engines compared with the steam engine of good standard design, is very striking, especially for the two-cycle, double-acting oil engine.

54 Combining the tangential forces at the crankpin for one crank with the same forces for three and four cranks of the double-acting, two-cycle oil engine gives the turning effort for a three- and a four-cylinder engine when, in finding the resultant, there is also included the negative effort of the scavenging pump and spray-air compressor. This is done in Fig. 16 and also for the three cylinders of the triple-expansion steam engine. Inspection of these turning efforts shows the double-acting oil engine to have a more favorable ratio of positive and negative fluctuations above and below the mean in relation to the steam engine. Evaluation of the areas shows

that the maximum single energy increment compared to the mean, which fixes the coefficient of irregularity, has the values given below:

Turning Effort: Coefficients of Irregularity

Three-cylinder, double-acting, two-cycle........	$= \frac{0.51}{11.9} = 0.043$
Four-cylinder, double-acting, two-cycle..........	$= \frac{0.55}{14.9} = 0.037$
Triple-expansion steam engine..................	$= \frac{0.86}{16.4} = 0.053$

55 Summarizing this part of the study, it is established that the double-acting, two-cycle arrangement offers the greatest prospect for both weight reduction per horsepower and increase of horsepower per cylinder, unless it should appear to be necessary to reduce mean pressures below values established for single-acting engines. It is also established that as small a number of cylinders as two will give satisfactory turning efforts with crank-shaft loads, and hence shaft sizes, that compare favorably with those on steam engines when the oil engine is two-cycle, double-acting, so that this arrangement gives a very short length of engine, other things being equal, even with allowance for scavenging pump and compressor at one end.

PART III

Performance

56 While the performance figures of various engines as published are often contradictory and confusing, there are some reports of tests of more than usual accuracy which may be made the basis of at least a tentative effort to clarify this situation. It is of considerable importance at this stage of progress in the development of large oil engines to establish such absolute quantities and such relations as may aid in selecting lines and which may be followed as most promising for development, if it should appear that any one were measurably better than another. The most important quantities to be examined in relation to each of the four possible styles of engine arrangement are indicated mean pressure with the mechanical losses in the engine leading to brake mean pressure, and indicated thermal efficiency with corresponding brake thermal efficiency and fuel consumption.

57 It has been pointed out that the really important thing in widening the scope of the economic utility of the oil engine is reduction of weight, and through it reduction of cost per horsepower on the one hand, and an increase in the horsepower per cylinder, preferably by the same means, if possible. Thermal efficiency is of secondary importance and may well be sacrificed

TABLE 2 FORCE-DIAGRAM DATA

Number of line	Type of engine	Diameter of piston, in.	Area of piston, sq. in.	Stroke, in.	Volume cu. in. cu. ft.	M.i.p. lb. per sq. in.	I.hp. cyl. i.hp.	Ratio i.hp.	Crosshead pin Diameter, in.	Crosshead pin Length, in.	Crosshead pin Area, sq. in.	Piston area Pin-area ratio
1	4-cycle, single-acting, Diesel engine	27	572.50	40	22900 13.25	94.2	245	100	10	8.5x2	170	3.37
2	2-cycle, single-acting, Diesel engine	27	572.56	40	22900 13.25	80	416	170	10	8.5x2	170	3.37
3	4-cycle, double-acting, Diesel engine	27 27—8.75	572.56 —60.13 512.43	40	22900 13.25 20500 11.85	94.2	245 +219 464	190	10.5	11.375x2	239	2.395
4	2-cycle, double-acting, Diesel engine	27 27—8.75	572.56 —60.13 512.43	40	22900 13.25 20500 11.86	80	416 +372 788	322	10.5	11.375x2	239	2.395
5	High-pressure steam cylinder	26 26—6	530.93 —28.27 502.66	48	25500 14.75 24100 13.92	98.3	569 541 1110	453	7	7.75x2	120.2	4.42
6	Intermediate-pressure steam cylinder	43.5 43.5—6	1486.17 —28.27 1457.90	48	71300 41.2 69800 40.3	34.5	560 550 1110	453	7	7.75x2	120.2	12.34
7	Low-pressure steam cylinder	74 74—6	4300.84 —28.17 4272.57	48	206200 119.5 205000 118.7	11.8	553 551 1104	425	7	7.75x2	120.2	35.8

Number of line	Crankpin				Main bearing				Crosshead guide			Crosshead gibs		
	Diameter, in.	Length, in.	Area, sq. in.	Ratio	Diameter, in.	Length, in.	Area, sq. in.	Ratio	Width, in.	Length, in.	Area, sq. in.	Width, in.	Length, in.	Area, sq. in.
1	18	15	270	2.122	17.5	24x2	840	0.68	22	33.5	737	10.75x2	33.5	452
2	18	15	270	2.122	17.5	24x2	840	0.68	22	33.5	737	10.75x2	33.5	452
3	18	16.75	302	1.9	17.5	21.25x2	744	0.768	21	37	777	6.125x2	37	454
4	18	16.75	302	1.9	17.5	21.25x2	744	0.768	21	37	777	6.125x2	37	454
5	14.5	12.75	185	2.865	14.5 14.5	14.5 14.5	210 210	1.263 1.263	14	26	364	14	26	364
6	14.5	12.75	185	8.02	14.5 14.5	14.5 17.5	210 254	3.54 2.93	14	26	364	14	26	364
7	14.5	12.75	185	23.25	14.5 14.5	17.5 19.5	254 283	8.47 7.60	14	26	364	14	26	364

to a reasonable degree, if by so doing something may be gained in the desired and necessary direction, but it would be especially satisfactory should it appear that the main ideals may be attained without loss of fuel consumption, and doubly so if some gain, however small, might also result. This really appears to be the case for the double-acting, two-cycle arrangement, and is one of the things that makes this direction of effort particularly interesting, even without utilization of the possibilities of raising mean pressures to high values, more or less equally available to all four of the typical arrangements — two- and four-cycle, single- and double-acting.

58 Indicated mean pressure of the thermodynamic cycle representative of the Diesel process of transforming heat of hot gases into work, is a function of the cut-off as in the uniflow steam engine, the diagram of which is identical except for the slope of the compression and expansion lines due to differences in fluids. It may be assumed that the maximum pressure is the same for all Diesel engines and with it the compression ratio, and for purposes of comparison this pressure may be set at 500 lb. per sq. in. above atmosphere. On this basis, assuming adiabatic compression and expansion of gases having

TABLE 3 INERTIA AND CENTRIFUGAL FORCES

Number of line	Wristpin: Weight of piston and water, lb.	Wristpin: Weight of piston rod, lb.	Crosshead, lb.	Total lb.	Lb. per sq. in. piston area	Crankpin: Inertia coefficient, lb. per sq. in.	Crankpin: ⅔ connecting-rod wt., lb.	Crankpin: Total lb.	Lb. per sq. in. piston area	Inertia coefficient, lb. per sq. in. piston area
1	1700	2100	2100	5900	10.3	47.3	3200	9100	15.9	73.2
2	1900	2300	2100	6300	11.0	50.6	3200	9500	16.6	76.3
3	2100	3100	2800	8000	14.0	64.3	4700	12700	22.2	102.3
4	4600	3100	2800	10500	18.35	84.4	4700	15200	26.5	122.
5	1021	860	1467	3348	6.32	8.58	2202	5550	10.46	57.7
6	1751	860	1467	4078	1.746	9.63	2202	6280	4.23	23.4
7	4352	860	1467	6679	1.553	8.58	2202	8881	2.06	11.4

Number of line	Connecting rod, weight, lb.	Total lb.	Lb. per sq. in. piston area	Rotating forces: ½ connecting-rod weight, lb.	Rotating forces: Lb. per sq. in. piston area	Rotating forces: Centrifugal forces, lb. per sq. in.	Crank-pin and webs, lb.	Total on main bearings, lb.	Lb. per sq. in. piston area	Centrifugal forces, lb. per sq. in.	Crankshaft: Wt., lb.	Crankshaft: Lb. per sq. in. piston area
1	7100	13000	22.7	3600	6.3	29	3400	7700	13.45	62	10000	17.5
2	7100	13400	23.4	3600	6.3	29	3400	7700	13.45	62	10000	17.5
3	9500	17500	30.6	4800	8.4	38.7	3400	8700	15.2	70	10000	17.5
4	9500	20000	35.0	4800	8.4	38.7	3400	8700	15.2	70	10000	17.5
5	3302	6650	13.25	1700	3.21	17.7	3300	5000	9.43	52.1	9000	16.96
6	3302	7380	4.97	1700	1.15	6.35	3900	5600	3.77	20.8	11400	7.67
7	3302	9981	2.32	1700	0.4	2.21	4500	6200	1.44	7.95	13100	3.05

the physical constants of air, the indicated mean pressures are found by calculation to be as follows:

Diesel Air-Cycle Mean Pressure

Cut-off, per cent displacement	3	6	9	12	15	18
Mean pressure, lb. per sq. in.	32.54	63.68	91.72	119.16	144.70	163.24
Cut-off, per cent displacement	21	24	27	30	33	
Mean pressure, lb. per sq. in.	188.00	212.50	232.60	252.30	269.10	

59 The cut-off is a function of the oil burned in the air charge in general, but actually is determined by several variables acting together and is not determinable exactly by any process of calculation without many assumptions, the validity of which is so much in doubt as to make the results of very doubtful value. In the engine cylinder still other variables enter, including relations be-

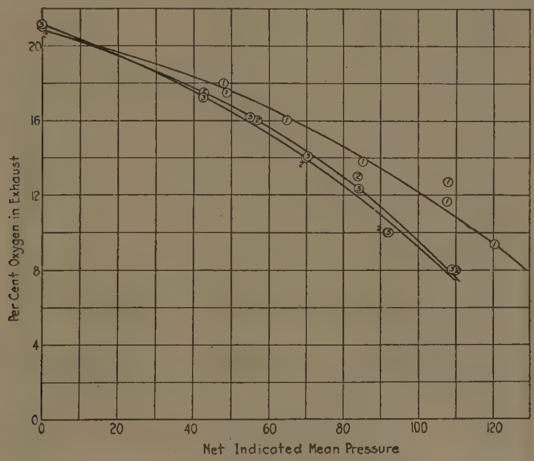

Fig. 17 Indicated Mean Pressure in Relation to Oxygen in Exhaust

tween variables assumed in thermodynamic calculations to be independent, such, for example, as temperature before compression or oxygen density in relation to indicated mean pressure, and appearance of carbon monoxide before the oxygen in the exhaust has been brought to zero, to cite only two.

60 It is of some interest, however, to set down limits to cut-off for assumed complete combustion of a fuel oil in air as one basis of comparison of actual cylinder performance, and to note its relation to incomplete utilization of air in terms of excess air or of free oxygen in the exhaust.

61 A fuel oil may be taken as more or less of the composition $C = 0.85$, $H_2 = 0.12$, and non-combustible $= 0.03$, ultimate analysis by weight. Its combustion requires 13.95 lb. of air per pound of oil, and its calculated heat of combustion, high value, is 18,660

TABLE 4 INDICATED MEAN PRESSURE IN RELATION TO OXYGEN IN EXHAUST AND EXCESS AIR

Engine	Bore, in.	Stroke, in.	R.p.m.	I.m.p.	Oil		Air per lb. oil	Exhaust		Reference No.*
					C	H		O_2	Air, excess	
Fraser and Chalmers, four-cycle, air-injection	21.5 in.	22.0 in.	300	108.0 85.4 64.8 48.3 49.0 107.7 120.5	82.96	11.87	13.75	12.7 13.8 16.0 18.0 17.5 11.65 9.39	1.42 1.82 3.06 5.79 4.92 1.18 0.77	11
Royal Tech. College, Mellanby and Gray	....	....	....	0 43 57 70 84 92 110	....	....	....	21.0 17.5 16.0 14.0 12.5 10.0 8.0	....	10
Four-cycle, air-injection	....	....	....	....	....	....	....	....	....	Mellanby
M. A. N. four-cycle, air injection	400 (mm.)	600 (mm.)	160	117.0 97.7 77.5 58.8 38.5	....	....	....	....	1.57 1.89 2.5 3.63 5.0	1

* These reference numbers refer to items in the Bibliography.

B.t.u. per lb., when 14,544 and 60,626 are taken as the heat of formation of the products per pound of carbon and hydrogen, respectively, and the heat of formation of the fuel-oil compounds as 5 per cent of the heat of formation of products. On this basis there would be liberated 1250 B.t.u. per pound of products for complete combustion and full oxygen utilization.

62 Just how much this will affect the cut-off can be calculated provided two assumptions are made for the adiabatic Diesel cycle: first, temperature before compression, and second, heat suppression or the fraction not producing temperature rise during combustion because of dissociation, or because of radiation to walls at that

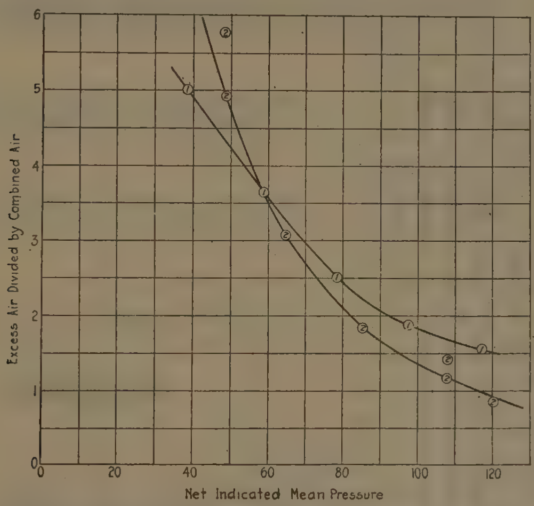

FIG. 18 INDICATED MEAN PRESSURE IN RELATION TO EXCESS AIR

time. It is sufficient for present purposes to note, however, that for cold air (60 deg. fahr.) and no heat suppression, the cut-off would be 33 per cent. For any value of heat suppression it would be smaller and in proportion to the effective heat, e.g., 22 per cent cut-off for one-third heat suppression. For higher initial temperature and for the same completeness of combustion, the cut-off would be shorter and inversely proportional to the absolute temperatures at beginning of compression. The oxygen in the exhaust products or the excess air resulting from non-attainment of as long a cut-off as might be possible will be zero for the above limiting cut-off of 33 per cent with 60 deg. fahr. initial temperature and no heat suppression; and for higher temperature and some heat suppression, zero oxygen in products will correspond to shorter cut-offs. For any given initial temperature and heat

TABLE 5 I.M.P. AND B.M.P.—FOUR-CYCLE OIL ENGINES WITHOUT COMPRESSOR—GAS AND GASOLINE ENGINES

OIL ENGINES

No.	Engine	R.p.m.	B.hp.	I.hp.	Cyl.	Bore, in.	Stroke, in.	B.m.p.	Net I.m.p.	F.m.p.	Refer.*
1	Vickers, solid-injection	381.3	12.15	....	12	14.5	15.0	84.9	106.2	21.3	13, 14,
		119.8	....	....	6	24.5	39.0	80.1	102.0	21.9	15, 16
2	Still, solid-injection	....	....	....	....	22	36	92.4	109.8	17.4	
								84.3	98.5	14.2	
3	Experimental, horizontal, single-cylinder	77.9	7.07					10.5	20.6	8.9	
		101.0	19.1					21.85	34.6	9.3	
		112.2	26.2					27.0	36.3	10.0	
		142.1	53.8					43.7	53.9	10.8	
		184.0	103.0					64.7	78.6	12.6	
		216.0	124.7					66.6	78.6	12.0	
		119.1	32.3	....	1	19	24	31.2	38.7	10.3	
		141.4	53.6					43.7	48.0	11.4	
		157.8	73.2					53.5	62.5	11.8	
		180.2	100.9					64.7	77.8	12.7	
		195.7	111.8					65.9	77.2	12.7	
		223.5	129.1					66.6	80.5	11.8	
		225.0	130.0					66.6	75.3	11.8	
		247.0	142.7					66.6	77.7	11.2	
4	British Admiralty exptl. submarine engine, air-injection without compressor	309.0	268.0	310				109.0	126.0	17.0	
		282.0	212.0	254				95.0	114.0	19.0	
		262.0	173.0	207				83.6	100.0	16.4	
		245.0	138.0	174				71.5	90.0	18.5	
		222.0	106.0	135				60.0	76.0	16.0	
		....	314.0	373	....	20.0	20.0	126.0	150.0	24.0	16
		....	313.0	358				126.0	144.0	18.0	
		....	288.0	344				127.0	150.0	23.0	
		....	284.0	322				126.0	143.0	17.0	
		....	283.0	334				126.0	149.0	23.0	

					GAS ENGINES					
A	Crossley	170.0	....	....	....	21.0	30.0	76.4	88.7	12.3	17
		240.0	....	....	....	12.0	18.0	75.2	86.5	11.3	17
B	Tangye	190.0	....	....	....	16.5	23.0	68.7	79.5	10.8	17
C	Körting	162.0	....	....	....	16.92	27.55	61.6	72.5	10.9	17
D	Ehrhardt-Sehmer	150.0	....	....	....	24.4	29.52	57.3	69.0	11.7	18
E	Westinghouse, D A	149.0	....	....	4	23.5	33.0	79.0	99.0	20.0	19
F	Allis-Chalmers	83.0	....	....	4	42.5	54.0	38.6	53.0	14.4	20
			GASOLINE ENGINES AT PEAK OF B.M.P.—R.P.M. CURVE								
a	Liberty	1400	....	....	12	5.0	7.0	119.5	131.3	11.8	
	Liberty	1400	....	....	12	5.0	7.0	132.0	141.8	9.8	
	Liberty	1600	....	....	12	5.0	7.0	132.0	144.2	12.2	
b	Packard	1200	....	....	8	4.75	5.25	132.0	145.8	13.8	21
		1400	....	....	12	4.75	5.25	124.5	139.1	14.6	
c	Hispano-Suiza	1200	....	....	8	4.72	5.11	118.5	131.1	14.6	
		1400	....	....	8	5.51	5.91	128.0	147.0	19.0	

*These reference numbers apply to items in Bibliography.

suppression, oxygen will be inversely proportional to cut-off, and so also will be excess air.

63 There is as a result a definite relation between the excess or unused air or free oxygen in exhaust products, and the indicated mean pressure, both for the hypothetical thermodynamic cycle and for the working cylinder, similar qualitatively but different quantitatively. This fact makes a study of actual mean indicated pressures as related to free oxygen and excess air a matter of considerable importance, as showing not only how close to the limit an engine may be working, but at any value of indicated mean pressure showing also by the values for oxygen and excess air something about the initial temperature of the charge and heat suppression combined. It is difficult to separate these, but in many cases compared it is possible to assume heat suppression to be the same, and then initial charge temperatures may be judged relatively.

64 One of the most interesting reports of direct experimental results of the relation between indicated mean pressure (i.m.p.) and free oxygen in exhaust products is

that of Prof. A. L. Mellanby[1] on a four-cycle air-injection engine, reproduced in Fig. 17 with results of some other tests as collected in Table 4. He concludes that the slope of the line is such as to suggest that i.m.p. = 140 is the limit for zero oxygen, a figure of great interest in comparison with a common working limit of 125 lb. per sq. in. for overload and the usual marine trial figure of 100 lb. per sq. in. A second curve, Fig. 18, completes the picture. The other test figures plotted are those reported by Prof. W. H. Watkinson[2] for a four-cycle, air-injection Fraser & Chal-

[1] Clyde Marine Oil Engines, A. L. Mellanby, Inst. Mech. Engrs., June, 1923.

[2] The Fraser & Chalmers Heavy-Oil Engines, *Engineering*, June 6, 1924.

TABLE 6 I.M.P. AND B.M.P.—FOUR-CYCLE OIL ENGINES WITH ATTACHED COMPRESSORS

No.	Engine	R.p.m.	B.hp.	I.hp.	Cyl.	Bore, in.	Stroke, in.	B.m.p.	Net I.m.p.	C.m.p.	B.m.p. + C.m.p.	Refer.*
1	Beardmore	124	1267	....	6	24.4	38.4	75.5	98.0	....	....	22, 14
	Tosi	129.6	1332	....	6	24.4	38.4	76.0	100.5	....	....	
2	M. A. N.	190	....	....	....	11.3	18.1	88.2	106.0	....	....	17
								73.2	93.0			
								43.3	66.0			
3	M. A. N.	100	....	....	6	27.5	47.3	52.8	65.7	....	....	23
								53.6	67.0			
1	Beardmore-and Wain	123	....	....	8	23.2	31.6	70.5	90.3	....	....	17
								69.8	89.5			
5	Werkspoor	125	....	....	6	22.1	39.4	77.0	98.7	....	77.0	17
6	Mannheim	120	1600	....	6	23.6	39.4	77.0	90.5	....	....	2
7	M. A. N.	158	79.6	103.4	....	15.75	23.6	87.1	113.5	5.66	92.8	1
		159	67.4	89.4				73.5	97.5	5.46	79.0	
		161.5	50.3	72.0				54.2	77.5	5.06	59.3	
		163.0	33.0	55.0				35.2	58.8	4.92	40.1	
		164.5	15.6	36.3				16.6	38.5	5.07	21.7	
8	Mirrlees	250	....	....	....	12.0	18.25	80.7	109	....	....	17
		250						70.0	97			
		250						46.7	73			
		250						23.3	48.6			
		250						7.8	33.8			

9 Mirrlees	170	....	....	4	18.9	26.8	79.1 75.6 59.3 58.7 40.8 79.8 77.2 58.8 58.1 41.1	104.0 101.0 82.0 82.0 63.4 110.0 108.0 86.0 86.0 68.0	....	....	17
10 Fraser-Chalmers	300 299.8 301.6 301.3 302.5 300.5 301.1	1000 749.7 502.7 251.0 .... 1001.7 1154.4	1306.6 1033.1 1788.1 586.3 299.0 1305.1 1463.6	....	21.5	22	82.6 62.0 41.3 20.7 .... 82.6 95.0	108.0 85.4 64.8 48.3 49.0 107.7 120.5	8.1 7.8 7.5 7.0 .... 8.1 8.2	90.7 69.8 48.8 27.7 .... 90.7 103.2	11

* Reference numbers apply to items of the Bibliography

mers engine of new design, and by Prof. W. Riehm for a M.A.N. engine of the same class. These show differences in cylinder conditions clearly by the relation of one series of test values to another for equal values of i.m.p. Confirming the initial density effect on oxygen for a given i.m.p. value are the very unusual figures of Dr. Riehm for an engine tested with supercharges, 17.72 in. by 16.54 in. at 300 r.p.m. giving for values of i.m.p., 115.0, 146.5, and 176.0, with excess air of 1.86, 1.44, and 1.17, respectively.

65 All of the previous figures are on four-cycle engines with air injection, which with sufficient spray air at a high enough pressure can agitate the air charge and drive the fuel charge through it sufficiently far to get good utilization in combustion chambers of the usual small range of shapes typical of single-acting engines. There might be a question as to the correspondingly effective utilization of the air in the rod end of a double-acting engine, but in view of the practicability of using two or more spray valves in such large cylinders, there is every reason to believe that equally good utilization of air can be secured, and this is confirmed by such reports as are available as to mean pressure attained. At the present time this is not so pertinent a matter as that of finding a construction that will not be injured by the high heat-generation rates

typical of large cylinders with even the moderate mean pressures in common use corresponding to about half the air utilization, more or less.

66 Considering now the two-cycle cylinder and the possibility of developing indicated mean pressures as high as the four-cycle, it must first be noted that with the low air-utilization characteristic of four-cycle cylinders at normal rating values of indicated mean pressures, the primary problem in the two-cycle is not one of securing an equal air charge in view of low air-utilization limits imposed by

TABLE 7 I.M.P. AND B.M.P.—TWO-CYCLE OIL ENGINES WITH ATTACHED SCAVENGING PUMP AND COMPRESSOR

No. Engine	R.p.m.	B.hp.	I.hp.	Cyl.	Bore, in.	Stroke, in.	B.m.p.	I.hp.	S.m.p.	C.m.p.	S.m.p. + C.m.p.	B.m.p. + S.m.p. + C.m.p.	Reference*
1 Sulzer	149.9	1791	....	....	39.4	43.5	89.4	104.0	....	....	....	....	14, 24
Sulzer	148.8	1986	....	....	39.4	43.5	99.75	111.5	....	....	....	....	
2 Sulzer	101.6	1264.5	....	....	23.62	37.0	75.0	92.0	....	....	....	....	14, 25
3 Sulzer	101.2	1262	....	....	23.62	37.0	75.4	103.5	....	....	....	....	14, 25
4 Sulzer	290.0	550	....	4	340(m.)	540(m.)	85.0	124.0	....	....	....	....	2
		410	....	4									
5 Vickers	141.2	1042	....	....	30	36	114.7	124.1	....	....	....	....	14, 15
6 Cammellaird-Fullagar	117.3	1037	....	4	18.5	25	65.0	78.1	....	....	....	....	14, 26
	117.3	....	1420				64.97	89.0	4.9	5.71	10.61	75.58	
	118.3	....	1648				76.8	102.4	5.11	5.46	10.57	87.37	
7 Cammellaird-	58.5	....	473				36.29	59.5	1.23	6.45	7.68	43.97	
Fullagar	90.5	....	818	4	18.5	25	46.62	66.6	....	6.72	....	....	
	110.6	....	1220				57.86	81.5	5.4	6.42	11.82	69.68	
	121.1	....	1526				68.48	92.8	6.96	7.71	14.67	83.15	
	120						62.0	90.8					
	120						60.5	87.5					
	119						64.8	91.9					
	120						68.0	95.5					
8 Cammellaird-	110						57.5	81.5					
Fullagar	100	....	....	4	18.5	25	51.5	73.5	....	....	....	....	23
	90						46.0	67.0					
	80						42.5	68.0					
	70						39.0	63.0					
	60						39.0	60.5					
							36.5	59.5					

9 Cammellaird-Fullagar	105	1000	1400	4	18.5	25.0	....	....	....	....	....	....	
10 Bethlehem	116	2900	4000	6	25.5	48.0	67.5	93.0	....	....	....	....	
	87.3	810	1137				45.1	63.1	2.91	5.05	7.96	53.06	
	84.7	970	1290				55.5	74.9	2.90	5.16	8.06	63.56	
	85.9	1232	1582				70.5	90.5	2.96	5.40	8.36	78.86	
	87.4	1456	1797				81.1	100.1	3.17	5.50	8.67	79.77	
	64.3	930	1142				70.2	86.2	1.91	5.05	6.96	77.16	
	65.2	746	970				55.5	72.1	1.90	5.05	6.95	62.45	
11 Nobel	65.9	578	770	....	26.58	36.22	42.7	57.0	2.05	4.76	6.81	49.51	12
	45.2	375	496				40.3	53.3	1.26	4.50	5.76	46.06	
	106.6	948	1342				43.3	61.4	3.36	5.05	8.41	51.71	
	106.0	1237	1623				56.6	74.3	3.36	5.30	8.66	65.26	
	107.0	1635	2022				74.1	92.0	3.50	5.53	9.03	83.13	
	105.1	1752	2128				81.1	98.5	3.48	5.65	9.13	90.41	
	108.4	1958	2401				88.0	108.0	3.66	5.75	9.41	97.41	
12 M. A. N. Exp'l D. A.	145	....	....	4	33.44	41.38	135.0	150.0	....	....	....	....	1

* References apply to items in the Bibliography.

the ordinary cast-iron metal walls, but merely one, as Dr. Naegel[1] has pointed out, of conserving scavenging-pump work while supplying a sufficient air charge. This question, however, takes on a new interest in connection with the substitution of thin steel walls in place of cast-iron, as such walls can stand higher heat-generation rates per square foot, corresponding to higher indicated pressures. This fact makes the question of adequacy of air charge by two-cycle scavenging methods of greater importance for the future than it has been in the past, or than it is at present.

67 It has long been known by those investigating the two-cycle method of charging a cylinder with air that many of the untested ideas held by others concerning the problem were wrong. Nothing is easier than arranging inlet and exhaust openings and drawing arrows on paper to illustrate air flow and air displacement of products. The difficult thing is to make the air follow the arrows. Yet it is not so difficult to get a good air charge after all, and even with only a moderate expenditure of scavenging-air work, not much in excess of the so-called pumping loss of four-cycle engines equipped with valves of one-third of the cylinder diameter, more or less. Conversion of scavenging-air pressure to air velocity, forming enter-

[1] Loc. cit.

ing streams of known velocity and direction, and the changing of direction of streams by wall faces, more or less equivalent to turbine vanes, so as to have the least short-circuiting of air from inlet to outlet, with maximum spreading of stream and displacement of burnt gases, constitutes the scavenging problem. This is not at all different in kind from that of room ventilation by incoming air streams or from the control of gases over heating and cooling surfaces, for best distribution. Lack of space forbids detailed discussion of this interesting subject here, but satisfactory conclusions are rapidly accumulating.

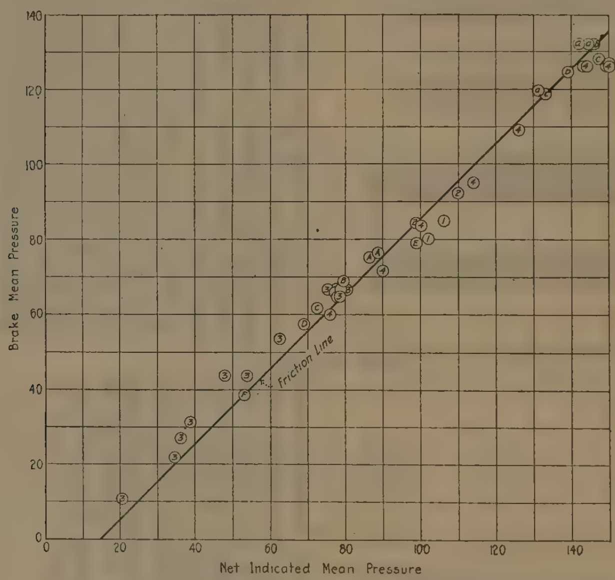

FIG. 19 RELATION OF I.M.P. TO B.M.P. FOR FOUR-CYCLE ENGINES WITHOUT COMPRESSORS

68 One such conclusion is to the effect that an excess volume of entering air is necessary and that the increase of scavenging ratio — pump to working-cylinder displacement — adds to the oxygen content remaining in the cylinder after exhaust ports close, up to a maximum of 1.6, more or less, with smaller and smaller gains as this limit is approached. High scavenging pressures are not necessary and, of course, are undesirable. In fact, the scavenging pressure may be so low as to make valve resistance a matter of quite as great importance as compression work, thereby leading to the abandonment of automatic valves in favor of mechanical ones and large ports. With large air volumes at low scavenging pressures it is entirely possible that the air-charge temperature in a two-cycle cylinder may be lower than in a four-cycle with its fixed

clearance of 7 to 8 per cent full of burnt gases at exhaust temperature. This means greater density, and with oxygen content substantially or actually equal to that of pure air, a possibility of indicated mean pressures higher in two-cycle than in four-cycle engines over the effective stroke of the former or higher excess air and free oxygen for a given mean pressure less than the maximum, if for metal or other reasons the maximum is not to be used. Proof of the purity of the air charge in port-scavenged engines is being obtained by cylinder sampling apparatus, and the most complete reports on this subject are those of Dr. Naegel,[1] of Dresden. He finds for an Augsburg double-acting two-cycle

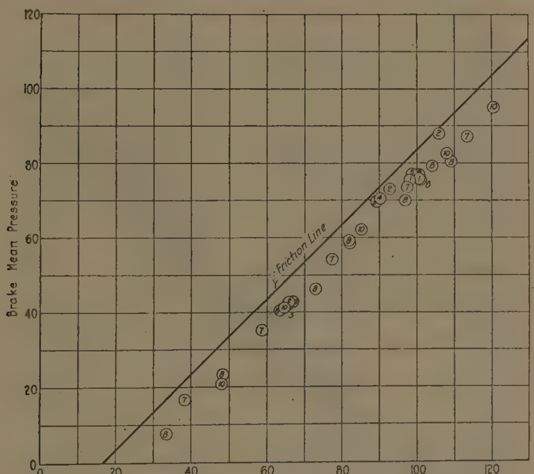

Fig. 20 Relation of I.m.p. to B.m.p. for Four-Cycle Engines with Attached Compressor

engine an oxygen content in the cylinder at the time of exhaust-port closure of 19 to 20 per cent at two points near the head and near the exhaust port, respectively, and for a Sulzer design of single-acting engine, 20.2 and 19.7 per cent, respectively. This is confirmed by Professor Nicholson's report of 19.3 per cent oxygen in the cylinder of a Scott-Still engine after scavenging.

69 This is substantially pure air, and with a proper scavenging ratio it may be cooler than is possible in a four-cycle cylinder.

70 It may therefore be concluded that for the effective stroke of the two-cycle cylinders, indicated mean pressures equal to those for four-cycle may be obtained, and with the same or greater unused air, if the value for i.m.p. used is less than the maximum.

[1] Loc. cit.

71 Of the gross indicated horsepower, the part that becomes available as shaft or brake horsepower can be determined only by subtracting the losses in the engine. These losses are of three classes, representing the power absorbed, first, in the mechanism itself; second, in the compression of spray air; and third, in the pumping of the working air. For each of these there is an equivalent mean pressure, f.m.p., c.m.p. and s.m.p. or p.m.p., so that the net brake mean pressure is given for the various classes of engines by the following:

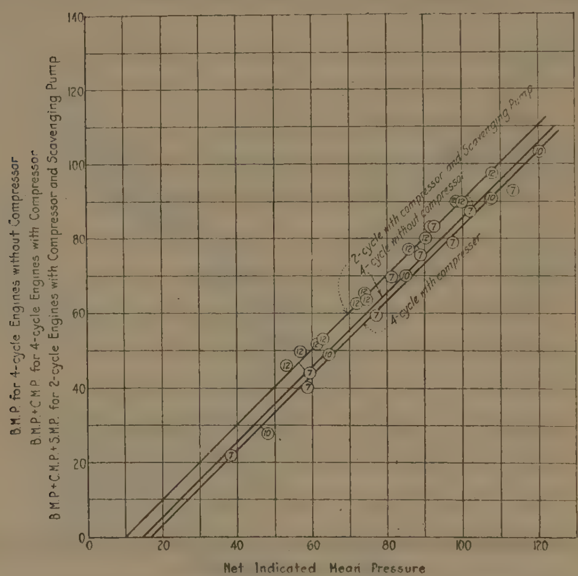

FIG. 21 FRICTION OF SINGLE-ACTING OIL ENGINES

For two-cycle engines with attached air compressors and scavenging pumps,

$$\text{b.m.p.} = \text{i.m.p.} - (\text{f.m.p.} + \text{s.m.p.} + \text{c.m.p.}) \quad . \; . \; . \quad [1]$$

For two-cycle engines with attached scavenging pumps but no compressors,

$$\text{b.m.p.} = \text{i.m.p.} - (\text{f.m.p.} + \text{s.m.p.}) \quad . \; . \; . \; . \; . \quad [2]$$

For four-cycle engines with attached air compressors,

$$\text{b.m.p.} = (\text{i.m.p.})_G - (\text{f.m.p.} + \text{p.m.p.} + \text{c.m.p.}) \quad . \; . \; . \quad [3]$$

For four-cycle engines without attached compressors,

$$\text{b.m.p.} = (\text{i.m.p.})_G - (\text{f.m.p.} + \text{p.m.p.}) \quad . \; . \; . \; . \quad [4]$$

72 The values for these various quantities are to be determined only from tests because no rational analysis can lead to reliable results, though such analysis is of value in examining experimental data. Ordinarily the indicated mean pressure as found experimentally is the net value, the four-cycle pumping loss being automatically subtracted in planimetering the indicator card, and this, the only value available as the pumping loss, is normally not reported. On this basis the four-cycle relations are changed by introducing i.m.p. = $(\text{i.m.p.})_G$ − p.m.p., so that

For four-cycle engines with attached air compressors,

$$\text{b.m.p.} = \text{i.m.p.} - (\text{f.m.p.} + \text{c.m.p.}) \quad . \ . \ . \ . \ . \ . \quad [5]$$

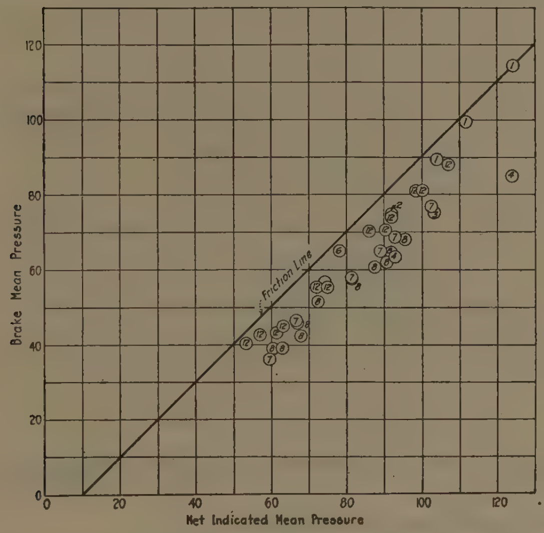

FIG. 22 RELATION OF I.M.P. TO B.M.P. FOR TWO-CYCLE ENGINES WITH ATTACHED SCAVENGING PUMP AND COMPRESSOR

For four-cycle engines without attached compressors,

$$\text{b.m.p.} = \text{i.m.p.} - (\text{f.m.p.}) \quad . \ . \ . \ . \ . \ . \ . \quad [6]$$

73 Test data are available for some engines of these different classes which are sufficiently complete for an evaluation of all the above variables, though, of course, such tests are not yet so numerous as to fix these values with the conclusiveness that is desirable. Some of these test figures have been collected in Tables 5, 6, and 7 for three of the four classes of engines, and these are used as the basis of the determination of mechanical losses in each type of engine.

74 As scavenging pumps of two-cycle engines vary in ratio of displacement of pump to that of working pistons, in scavenging

pressure, and in valve resistance, the first step in the analysis of two-cycle engines with attached scavenging pumps is to eliminate s.m.p. as a variable. This can be done when the test data include separate determinations of scavenging indicated horsepower from which to determine s.m.p. and (b.m.p.+s.m.p.). The next step is similar in kind but concerned with air compression and determination of (b.m.p.+c.m.p.) for four-cycle engines, or with the above (b.m.p.+s.m.p.+c.m.p.) for two-cycle engines.

75 When such steps have been taken and the results plotted, i.m.p. against (b.m.p.) or (b.m.p.+s.m.p.) or (b.m.p.+c.m.p.) or (b.m.p.+s.m.p.+c.m.p.) as the case may be, the points always lie in a straight line.

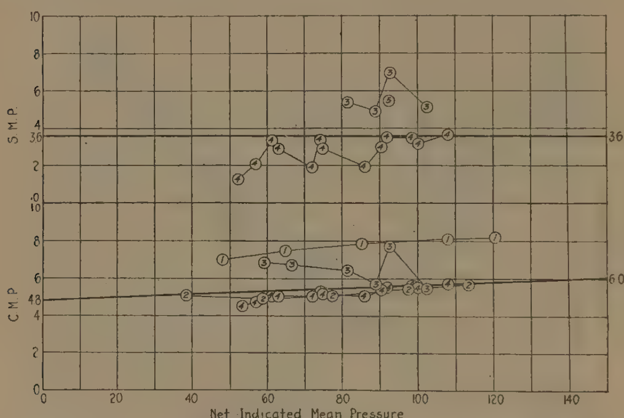

Fig. 23 Scavenging and Compressor Mean Pressures

This line has the general equation

$$\text{b.m.p.} = b \times \text{i.m.p.} - a \quad . \; . \; . \; . \; . \; . \; . \; . \quad [7]$$

This is equivalent to a friction mean pressure of

$$\text{f.m.p.} = (b-1) \times \text{i.m.p.} + a \quad . \; . \; . \; . \; . \; . \; . \quad [8]$$

76 In this expression the constant a represents a part of the friction that is independent of the cylinder pressures, while the first term represents the rest of it, an amount varying with the cylinder pressure determined by the constant $(b-1)$, which is the slope of the line.

77 Considering the second term it can be shown that if A is its value for the four-cycle mechanism with compressor, then $\frac{A}{2}$ will be the equivalent for the two-cycle mechanism, neglecting an excess d of scavenger friction over that of a four-cycle valve

gear. A corresponding addition e for the compressor mechanism will give the result for air-injection engines. Similarly, for double-acting engines the value $\frac{A}{2}$ applies to the four-cycle and $\frac{A}{4}$ to the two-cycle. Expressing by $B = (b-1)$ the slope of the line, which careful plotting and fairing of lines indicates is the same for all, the friction of the several types may be expressed by the following, introducing a term r for the friction of a second set of piston rings.

For single-acting, four-cycle engines without compressor,

$$\text{f.m.p.} = A+B\times \text{i.m.p.} \qquad [9]$$

For single-acting, four-cycle engines with compressor,

$$\text{f.m.p.} = A+e+B\times \text{i.m.p.} \qquad [10]$$

For single-acting, two-cycle engines with scavenging pump but no compressor,

$$\text{f.m.p.} = \frac{A}{2}+d+B\times \text{i.m.p.} \qquad [11]$$

For single-acting, two-cycle engines with scavenging pump and compressor,

$$\text{f.m.p.} = \frac{A}{2}+d+e+B\times \text{i.m.p.} \qquad [12]$$

For double-acting, four-cycle engines with compressor,

$$\text{f.m.p.} = r\left(\frac{A}{2}\right)+e+B\times \text{i.m.p.} \qquad [13]$$

For double-acting, two-cycle engines with scavenging pump and compressor,

$$\text{f.m.p.} = r\times\left(\frac{A}{4}\right)+d+e+B\times \text{i.m.p.} \qquad [14]$$

78 The experimental results reported in Table 5 for the four-cycle engines without compressors are plotted in Fig. 19. The fair line for vertical oil engines at constant speed is located, and from it the values of the constants are found to be $A = 14.8$, $B = 1.0$.

79 Here the British Admiralty tests by Hawkes are especially valuable, because, in addition to more than usual accuracy, the spray air was supplied by a separate compressor and the mean cylinder pressures run high but include a considerable range. Each test point is indicated and its relation to the fair straight line is clear. On this sheet are plotted points from the table for other engines — a horizontal oil engine at variable speed, horizontal gas engines, and vertical aircraft gasoline engines, all single-acting, with two cases of large double-acting tandem twin gas engines.

It is most remarkable how closely the results agree, considering the truly enormous range of cylinder sizes and speeds, in addition to fuels and cylinder pressure, and judged in relation to possible errors in measuring i.m.p. For the aircraft engine the points were selected at the peak of the mean-pressure-speed curve.

80 The results for the four-cycle engines with attached compressors are next plotted in Fig. 20, but because of variations in values of spray-air pressures and c.m.p. no attempt is made at locating a fair line. Compressor-work data are not available for but two, the M.A.N. and the Fraser & Chalmers, and is different for these. To locate the friction line, the line for four-cycle engines without compressors from Fig. 19 is reproduced in Fig. 21, on which are located the points for these two tests, plotting vertically

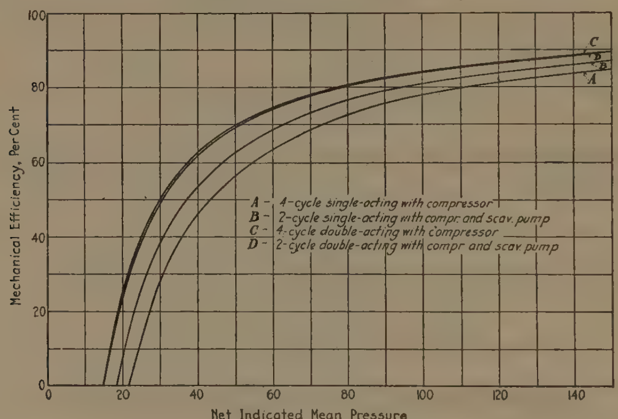

Fig. 24 Mechanical Efficiencies

the sum b.m.p. and c.m.p., and locating the fair line representing the friction of the mechanism with attached compressor.

81 On the axes of i.m.p. the difference between the intercepts of these lines gives the value e, the constant mean pressure equivalent to friction of the compressor mechanism, the slopes of the two lines being unity. In this way the value is found $e = 1.9$.

82 This friction line for four-cycle engines with attached compressors is located on Fig. 20 for comparison.

83 Passing to the two-cycle engines there is a shortage of data for compressorless engines, but there are some good data for present purposes on engines with attached compressors and scavenging pumps. These results are plotted in Fig. 22. For these tests where both scavenging pump and compressor mean pressures are given, the values b.m.p.+c.m.p.+s.m.p. are plotted on Fig. 21 besides the value of b.m.p. for four-cycle engines without compressors, and

the values of b.m.p.+c.m.p. for four-cycle engines with compressors. This checks the slope of the lines as unity for all styles of engines and determines the value of friction of scavenging-pump mechanism over four-cycle valve gears to be $d = 0.7$. This friction line is drawn in on Fig. 22 for comparison.

84 With these determinations of the constants the friction of single-acting engines may be regarded as at least tentatively established in a proper relation, one to another. It is furthermore possible, in the absence of test data, to approximate the value for double-acting engines with reasonable accuracy.

85 All of these constants have been evaluated except r. From all available information on four-cycle, single-acting trunk pistons it seems as if the pistons and rings accounted for some 70 per cent of the total constant mechanical friction, and that a set of rings represent about 15 per cent of the 70 per cent, or 10 per cent of the total in round numbers. Accepting this value $r = 1.1$ in the absence of any direct measurements on large engines, and adding up the various terms for the four classes of engines of major interest, the friction for each is as follows:

Four-cycle single-acting,

$$\text{f.m.p.} = A+e = 14.8+1.9 = 16.7$$

Two-cycle single-acting with attached scavenging pump,

$$\text{f.m.p.} = \frac{A}{2}+d+e = \frac{14.8}{2}+0.7+1.9 = 10.0$$

Four-cycle double-acting,

$$\text{f.m.p.} = r\times\left(\frac{A}{2}\right)+e = 1.1\times\left(\frac{14.8}{2}\right)+1.9 = 10.04$$

Two-cycle double-acting,

$$\text{f.m.p.} = r\times\left(\frac{A}{4}\right)+d+e = 1.1\times\left(\frac{14.8}{4}\right)+0.7+1.9 = 6.67$$

86 For purposes of fair comparison of the four typical arrangements it is necessary to reduce values of scavenging and compressor mean pressures to a common fair value for all, which may be compared with such actual values as are available.

87 From the available test reports some figures of spray-air mean pressures are collected in Table 8, with indicated mean pressures in the working cylinders and the spray pressures used in atmospheres. Similarly, some corresponding values for scavenging mean pressures are collected in Table 9, with scavenging pressures and the corresponding working mean pressures. Both sets of results, plotted in Fig. 23, show considerable irregularities, with, however, a tendency toward constancy or independence of

both c.m.p. and s.m.p. with reference to i.m.p. at values dependent on whatever spray-air and scavenging supply pressures are used.

88 This is a natural result of absence of regulation of delivery pressure by means that reduce mean pressures appropriately, as may be done but not yet generally practiced.

89 It is of course possible to calculate the mean pressures pretty closely for any assumed condition of valve resistance and delivery pressure for scavenging pumps, and get its equivalent s.m.p. referred to the working pistons through the cylinder ratios (displacement), and to do the same thing for three- or four-stage

TABLE 8 SPRAY-AIR MEAN PRESSURES

Engine	I.m.p.	Spray pressure, atmos.	C.m.p.
Fraser & Chalmers, Four-cycle	108.0	68	8.1
	85.4	61	7.85
	64.8	60	7.5
	48.3	60	7.0
	107.7	71	8.1
	120.5	75	8.2
M. A. N., four-cycle	113.5	63	5.66
	97.5	60	5.46
	57.5	50	5.06
	58.8	46	4.92
	38.5	46	5.07
Fullagar, two-cycle	89.0		5.71
	102.4		5.46
	59.2		6.45
	66.6		6.72
	81.5		6.42
	92.8		7.71
Nobel, two-cycle	61.1	48	5.05
	74.9	50	5.16
	90.5	55	5.40
	100.1	57	5.50
	86.2	48	5.05
	72.1	48	5.05
	57.0	42.5	4.76
	53.3	38	4.50
	61.4	48	5.05
	74.3	53	5.30
	92.0	59	5.53
	98.5	63	5.65
	108.0	67	5.75

air compressors, but for purposes of type comparison of the four engine classes a flat assumption is made consistent with facts and indicated by the heavy lines on Fig. 23.

90 For scavenging air the value is taken as the high value reported by Professor A. Rosborg[1] for the Nobel engine tested by him on the assumption of no regulation and s.m.p. = 3.6 lb. per sq. in.

91 For spray-air compression the value taken is just above the points for the M.A.N. engine and assumed to be a straight line It is represented by c.m.p. = 4.8+0.008 i.m.p., lb. per sq. in.

[1] 1600 B.HP. Nobel-Diesel Marine Engine, George J. Steinheil, *The Engineer*, January 27 and February 3, 1922.

92 Collecting all of these losses, it is possible to set down values for the difference between indicated and brake mean pressures and for mechanical efficiencies E_m that, while not necessarily applicable exactly to any one engine, yet may be accepted tentatively as giving their relative positions one to another, or the type characteristics. Accordingly for the four styles, all with attached compressors —

Four-cycle single-acting engines:

$$\begin{aligned} \text{i.m.p.} - \text{b.m.p} &= \text{f.m.p} + \text{c.m.p.} = 16.7 + 4.8 + 0.008\,\text{i.m.p.} \\ &= 21.5 + 0.008\ \text{i.m.p.} \end{aligned}$$

$$E_m = 1 - 0.008 - \frac{21.5}{\text{i.m.p.}} = 0.992 - \frac{21.5}{\text{i.m.p.}}$$

Two-cycle single-acting engines with attached scavenger:

$$\begin{aligned} \text{i.m.p.} - \text{b.m.p.} &= \text{f.m.p.} + \text{s.m.p.} + \text{c.m.p.} \\ &= 10 + 3.6 + 4.8 + 0.008\,\text{i.m.p.} \\ &= 18.4 + 0.008\ \text{i.m.p.} \end{aligned}$$

$$E_m = 0.992 - \frac{18.4}{\text{i.m.p.}}$$

Four-cycle double-acting engines:

$$\begin{aligned} \text{i.m.p.} - \text{b.m.p.} &= \text{f.m.p.} + \text{c.m.p.} = 10.04 + 4.8 + 0.008\,\text{i.m.p.} \\ &= 14.84 + 0.008\ \text{i.m.p.} \end{aligned}$$

$$E_m = 0.992 - \frac{14.84}{\text{i.m.p.}}$$

Two-cycle double-acting engines:

$$\begin{aligned} \text{i.m.p.} - \text{b.m.p.} &= \text{f.m.p.} + \text{s.m.p.} + \text{c.m.p.} \\ &= 6.67 + 3.6 + 4.8 + 0.008\ \text{i.m.p.} \\ &= 15.07 + 0.008\,\text{i.m.p.} \end{aligned}$$

$$E_m = 0.992 - \frac{15.07}{\text{i.m.p.}}$$

93 These values are plotted in Fig. 24, and show clearly the relative positions of the four classes, the most interesting conclusion being the superiority in mechanical efficiency of double-acting over single-acting engines.

94 Indicated thermal efficiency is independent of the mechanism of the engine, being wholly a question of fuel combustion in relation to compression and expansion of the working gases, so that for similar combustion-chamber conditions, engines of all four classes would have the same indicated efficiency and fuel consumption.

95 Whatever actual differences there may be in fact must be traced to gas sources or gas-temperature relations to combustion chamber walls. It is therefore a matter of considerable importance to check up and determine to what extent it may be true

that actual differences in fuel consumption of Diesel engines are due substantially or actually to their differences in mechanical losses just discussed.

96 On thermodynamic grounds the adiabatic Diesel air cycle yields results for thermal efficiency as a function of cut-off as was the case for indicated mean pressure, so that indicated efficiency becomes a function of indicated mean pressures. The result of such a calculation are given below.

Indicated Thermal Efficiency (Air Card)

Cut-off	Mean pressure	Efficiency	Cut-off	Mean pressure	Efficiency
3	32.54	0.537	21	188.00	0.462
6	63.68	0.519	24	212.50	0.447
9	91.72	0.513	27	232.60	0.431
12	119.16	0.498	30	252.30	0.418
15	144.70	0.489	33	269.10	0.404
18	163.24	0.474			

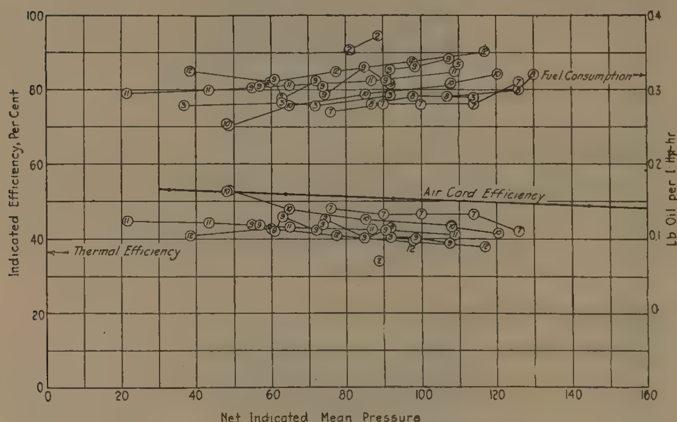

FIG. 25 INDICATED THERMAL EFFICIENCY AND FUEL CONSUMPTION

97 The cut-off corresponding to a given amount of heat liberated is, as was the case for indicated mean pressure, a function of the initial temperature of the charge and of the heat suppression, the increase of specific heat at high temperatures playing its part. Combustion in a cylinder is, moreover, never at a constant pressure and probably never completed at the time expansion begins, as practically all indicator cards by the slope of their expansion lines show evidence of some afterburning, but the amount is or can be the same in all cylinders working with air injection.

98 For reasons of this sort actual thermal efficiencies must be lower and are lower than the air-card values, and the relation is still the subject of study and speculation as to the individual effect of the several causes.

99 From various sources test values of indicated fuel consumption have been collected and converted into equivalent thermal efficiencies based on the high calorific value of the fuel in Table 10. In some cases the higher calorific power was not reported and for these it was estimated at the value 1.06 times the low value, which is probably as close to the truth as the accuracy of determination of indicated horsepower, a quantity always justly suspected. The fuel consumptions, which are always given, are plotted on the same sheet (Fig. 25) with the thermal efficiencies for graphical comparison.

100 Reference to the curves shows fair agreement between the trend of the lines for actual indicated thermal efficiency and at values represented roughly at 0.8, excluding some abnormally high

TABLE 9 SCAVENGING MEAN PRESSURES

Engine	I.m.p.	Scavenging pressure	S.m.p.
Fullagar	89.0		4.9
	102.4		5.11
	81.5		5.4
	92.8		6.96
Sulzer	82.5		5.46
Nobel	63.1	1.18	2.91
	74.9	1.17	2.90
	90.5	1.21	2.96
	100.1	1.35	3.17
	86.2	.72	1.91
	72.1	.71	1.90
	57.0	.77	2.05
	53.3	.45	1.26
	61.4	1.50	3.56
	74.3	1.50	3.36
	92.0	1.62	3.50
	98.5	1.59	3.48
	108.0	1.82	3.66

values, a fact that gives additional weight to thermodynamic calculations and invites further study of such comparisons.

101 Variations in these results for any one engine, due to fluctuation of operating conditions in spite of care to hold them steady, are of the same order of magnitude as differences between two engines, and differences between two-cycle and four-cycle thermal efficiencies are of the same magnitude or less. It must be concluded, therefore, that all Diesel engines of the air-injection class have the same indicated thermal efficiency and all are subject to fluctuations of about the same degree, due to adjustments or other operating conditions.

102 From this it follows that the fuel consumption per net or brake horsepower-hour will be determined by the mechanical losses, and as these are less in the double-acting than in the single-acting engines, it must follow that double-acting engines are more efficient than single-acting.

103 It appears, therefore, that by developing the double-acting cylinder for large oil engines there is not only a material reduction in weight or cost for more or less identical machine

TABLE 10 INDICATED THERMAL EFFICIENCY AND FUEL CONSUMPTION

No.	Engine	Bore, in.	Stroke, in.	R.p.m.	I.m.p.	Lb. oil per I.hp.	B.t.u. per lb.	B.t.u. per I.hp.m.	Indicated thermal efficiency	Reference*
1	Diesel Engine Users' Assn.	General				0.416	18,000	7490	0.34	28
2	Still	22	36		88.9	0.372			0.34	
	two-cycle				81.2	0.353				
3	Sulzer	25.2	41.73	100	114	0.289				29
	two-cycle				63	0.282				
					37	0.278				
					92	0.292				
					72	0.278				
4	Burmeister & Wain									
	1418 Hp.					0.326				30
	2299 Hp.					0.302				
	3178 Hp.					0.306				
5	Beardmore-Tosi					0.326				30
6	Camellaird-Fullagar	18.5	25.0	115		0.28	19,200	5370	0.468	
7	British Admiralty	14.5	15	380	126	0.31	19,400	6000	0.42	31
	experimental				114	0.28		5420	0.465	
	four-cycle				100	0.28		5420	0.465	
					90	0.28		5420	0.465	
					76	0.27		5220	0.48	
8	Doxford air-	20	20	300	130	0.32				
	injection, sub-				126	0.30				31
	marine, two-				107	0.29				
	cycle				98	0.29				
					87	0.28				
9	Nobel,	26.58	38.98	106	63.1	0.288	19,100	5500	0.458	12
	two-cycle				74.0	0.304	19,100	5790	0.436	
					90.7	0.312	19,100	5950	0.423	
					85.1	0.33	19,100	6290	0.401	
					72.1	0.312	19,100	5950	0.423	
					57.0	0.304	19,100	5790	0.436	
					54.7	0.302	19,100	5750	0.437	
					61.0	0.314	19,100	6000	0.421	
					74.5	0.293	19,100	5580	0.451	
					92.0	0.307	19,100	5850	0.430	
					92.0	0.327	19,100	6120	0.403	11
					98.5	0.33	19,100	6290	0.401	
					107.5	0.342	19,120	6539	0.387	

10	Fraser & Chalmers, four-cycle	21.5	22	300	108.0	0.308	19,120	5900	0.431	
					85.4	0.294	19,120	5610	0.451	
					64.8	0.278	19,120	5300	0.478	
					48.3	0.523	19,120	4840	0.526	
					49.0	0.251	19,120	4800	0.530	
					107.7	0.306	19,120	5850	0.435	
					120.5	0.321	19,120	6150	0.414	
11	Sulzer, two-cycle	26.77	47.24	95	21.7	0.296	19,100	5150	0.447	2
					43.5	0.299	19,100	5700	0.442	
					65	0.306	19,100	5820	0.432	
					87	0.312	19,100	5930	0.425	
					109	0.323	19,100	6160	0.410	
12	M. A. N., four-cycle	15.75	23.62	160	117.0	0.35		6660	0.378	1
					97.6	0.338		6420	0.392	
					77.5	0.324		6190	0.408	
					59.8	0.310		5900	0.427	
					38.5	0.324		6190	0.468	

* These reference numbers apply to items of the Bibliography

design, especially frame types and materials of construction, a lesser number of cranks needed with a subsequent shortening of engine or reduction of floor space, and a very considerable increase in power for a cylinder of same bore, but that in addition there is a gain in fuel economy. It also appears that the two-cycle, double-acting engine exceeds in aggregate advantages the double-acting, four-cycle engine, though in the matter of fuel economy they are equal.

104 It remains now to examine the question of wall heating and to compare the limits of heat-generation rates of these four classes for a given bore or horsepower capacity per cylinder with the corresponding maxima for bores, or power per cylinder.

PART IV

Heat-Generation Rates and Metal Protection

105 As a result of heat generation in the combustion chamber, the walls of the cylinder liner or cylinder proper, the cylinder head and piston head, all of which in large engines have circulating water or oil on their backs, will become hotter on the gas side due to heat reception, in spite of conduction to the liquid. The body of the metal will have a mean temperature higher than the liquid, and there will be a straight-line temperature differential between the liquid side of the metal and the gas side up to the face metal on the gas side. Experimental measurement by metal temperatures by properly made and properly applied thermocouples proves this beyond question. There is, however, a rational certainty of another temperature condition at the inner face for which satisfactory experimental data are lacking. This condition is due to the fact that the heat is received by the metal at a fluctuating rate, which must cause a periodic rise and fall of the temperature of the inner skin in much the same manner qualitatively as in the brick of furnace regenerators. In the latter, temperature measurements are comparatively easy because the period of fluctuation is half an hour, more or less. These measurements prove conclusively the existence of a temperature wave of maximum amplitude at the face, decreasing as the heat flows through the brick, and finally being damped to constancy at some distance from the face. In the regenerator brick the face is alternately heated by hot waste gases and cooled by incoming air, thus making the face temperature rise regularly above and below the mean steady value at the interior. The conditions are very similar in the metal of the combustion chamber walls in spite of the fact that here the skin temperature fluctuates always above the steady temperature in the body metal beyond the skin. It must fluctuate at the face and a steady value only a short distance from the face is experimentally established.

106 As the rate of heat generation rises by increase of speed or by increase of mean pressure in a given cylinder, there arises in every such cylinder a tendency to injure the metal. For every detail of construction there is a limiting diameter for a given mean pressure and speed, or for a given diameter a limiting mean pressure beyond which it is not safe to go because of cracks or burns which at that time appear. This is the main reason for the recognized fact that in large cylinders of cast iron the mean pressure for rating is never as high as it is possible to secure by combustion. It is also the reason for much intensive study of design of these parts, from which many new details have developed that have added much to reliability of these parts, especially those constructions in which one-piece castings have been re-

placed by multiple-unit members providing specifically for protection against injury. Each of these has contributed something to the satisfactory operation of large oil engines at higher ratings in larger cylinders than once was possible. Each, however, has its own limits and there still exists the need of further extension of the limits, even to the point, if possible, where metal injury ceases to limit the mean pressure to be carried, or to limit the diameter. Not until then will it be possible to build oil engines of any horsepower per cylinder that may be needed, subject only to adequacy of control of combustion conditions. Modern designs of water-tube boilers can generate steam at very high rates without injury to the metal, and it has been established that the only limit to such rates is in the furnace capacity to generate heat. This is the ideal for the oil engine: to devise structural arrangements, to select materials and to design the structural details of the heated parts, cylinder, piston, and head, with rod, so that they will be uninjured by heat at whatever rate it may be produced. The forged-steel cylinder and piston construction peculiar to the Worthington double-acting engine is a new development and marks a near approach to this ideal.

107 This being a fact, it becomes a matter of primary importance in large engines to consider this matter of metal injury with a view to devising improved structures or finding new materials, or both, and with the understanding that every little gain is worth while, even though the ideal of no limit to heating rate may still be far off.

108 It is not possible to analyze this phase of the problem by rational means alone. No formula can be set up that will give accurately the stresses or predict the heating-rate limit beyond which cracks or burns will occur for any part of an oil engine. This is due to the very great complexity of the stress-producing causes and their interrelation, which is so great as to defy formulations without assumptions of unfavorable validity. Real progress must be based on experience with each new fact analyzed as well as may be in the light of rational possibilities, and then checked and rechecked. In time such a procedure, starting with empirical formulations, will ultimately lead to rational expressions that give results not contradicted by the facts of experience but in accord with them.

109 This is a slow process, but in the meantime it becomes possible from time to time to discover remedies from qualitative analyses of facts that make for real progress. Many steps of this sort have been taken, as previously noted, and others will follow in proportion as the attention of the engineering profession becomes focused on the problem of permanence of metal walls enclosing high-pressure air supporting the combustion of oil at high average rates in B.t.u. per hr. per sq. ft., and intermittently at rates expressed in terms of B.t.u. per injection per sq. ft. Some

of the best suggestions may well come from engineers who have had no contact whatever with Diesel engines, but who have had experience in one of the hundreds of lines of activity in which protection of heated walls against injury is a matter of regular concern.

110 The two heat-rate units noted at once call attention to the fact that the metal-protection problem has two parts, which, while related, have a considerable degree of independence. The first is steady heat flow, and the second, heat-storage frequency. The first recognizes expansion stresses varying with the mean temperature of the metal, and with the temperature drop across the metal section that varies with the gradient in degrees per inch, and with the thickness. The gradient varies with the heat-generation rate, B.t.u. per hr. per sq. ft. The second recognizes that heat is stored in a metal layer at actual moments of generation, being received in part by hot gas contact and in part by direct radiation of the gaseous mass and of solid particles of dust, ash, or carbon, probably mainly the latter. This produces momentarily high interior skin temperatures lasting but very short periods of time, but regularly recurring more or less, as in the filament of a flashing incandescent lamp.

111 In direct contact with this high frequency and high temperature of skin there is a hot air charge in which oxygen weight per cubic inch is higher than in pure oxygen at atmospheric conditions because of the pressure of 35 atmospheres, more or less, that prevails. This may well be the direct cause of burning of metal in a manner somewhat like that of the oxygen cutting torch working through a cold bar but acting more slowly because of the very short periods during which the temperature necessary for the reaction exists and the promptness with which it is damped out by the cooler metal mass behind the skin. Confirmation of this is found in fact that metal in the direct path of sprays is more rapidly burned than elsewhere, as should be the case due to greater intimacy of contact of the gases and the metal, the so-called scrubbing effect, added to local temperature excesses.

112 Independent of such burning there is necessarily a high degree of expansion effort of the skin metal, regularly followed by contraction according to frequency of injection. This produces an intensification of the thick-cylinder effect, which tends to make the inner metal fail before the outer main mass has taken its load. The appearance of the spider webs of fine cracks first noticed in large gas engines must be more or less closely related to this condition.

113 Remedies for both of these effects of high momentary skin temperatures may be found in more than one direction. The first is in the use of ductile instead of brittle metal, forged, rolled, or pressed metal instead of castings, and metal selected for appro-

priate yielding properties under load, without lowered, and preferably with higher, elastic limit. The second is in the reduction of the temperature of the main metal mass just back of the skin, the temperature above which the skin temperature fluctuates and which, being the base temperature, must fix the maximum when other things are equal. This is a matter of temperature difference through the wall and a function of heat-generation rate per hour per square foot and of thickness. The third is frequency reduction. For equal hourly rates per square foot the B.t.u. per injection per square foot will vary inversely with the number of injections per hour and the skin temperature likewise. Therefore, high injection frequency is most desirable, and it is for this reason that high-speed engines give less trouble from metal injury than low-speed engines at equal heat-generation rates. It is also the reason for the fact established by experience that two-cycle engines give less trouble than four-cycle for equal hourly generation rates, or, what is equivalent, may carry higher hourly generation rates with the same metal reliability in large cylinders than four-cycle engines.

114 The use of forged steel for cylinders and pistons instead of the traditional cast iron seems, therefore, by its ductility to contribute directly to the reduction of the inner-skin injuries of local stress failure and burning, and its adaptability to the two-cycle port-scavenged cylinder, requiring no complicated ports and valve housings that would make adoption difficult or impossible, also contributes to further reduction by increasing the injection frequency and reducing the skin-temperature peak to one-half.

115 There is, however, a further contribution of the steel wall toward prevention of injury of the two surface types through its direct effect on reduction of the temperature of the main metal mass. Such steel walls may be made thinner than cast iron just about in inverse proportion to the allowable working stresses which are three or more to one, depending mainly on the type of steel, of which there is now available a good range of choice. Other things being equal, the temperature drop would be one-third or less in steel walls than in cast iron for the same hourly rate of heat generation, or with the same temperatures that now prevail, three or more times the hourly rate would be permissible with a greater margin of metal stability, due to the ductility of the metal. Freedom from webs, ribs, or thickened sections common to castings, and especially when removable cylinder heads are used, adds still more margin.

116 With no doubt as to the inherent possibility of thin steel as a wall material for oil-engine combustion chambers, of reduction of wall-injury hazard at equal heating rates, or what is equivalent, an increase in horsepower per cylinder over cast-iron construction, it becomes a matter of considerable interest to estimate how much gain might be expected. As already pointed out, there is no really satisfactory way of doing this by any rational

formulation, but it is possible to make an estimate based on certain assumptions that seem reasonable.

117 One such basis of estimating the increase in horsepower per cylinder that might be expected by the use of steel cylinders for double-acting, two-cycle engines, is that of assuming equal temperature differences through the wall to those now in successful use. That this is conservative is supported by the fact that no allowance is made for absence of localized stresses due to thick spots or irregular sections, nor for the ductility of the material. To carry through such an estimate, however, other assumptions must be made for several other factors that appear in the formulation. It may therefore be concluded that, however interesting such estimates may be, they will be too speculative to form a basis of any sound prediction.

118 For the present it may justly be said that the steel cylinder does make it possible to increase the horsepower per cylinder just as does two-cycle adoption compared with four and double-acting compared with single, so that the combination of steel cylinder, two cycles and double acting seems to offer the widest prospect at present in this direction. It has also been shown by this review that it is in about the same position as to number of cranks, shaft length, and weight per horsepower, and with higher overall efficiency than obtain for single-acting engines.

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DISCUSSION

Elmer A. Sperry.[1] The author has brought into his paper a large number of most useful collateral data. He has placed before us much that has been done in the effort to improve the power and lower the weight of combustion engines, starting with the four-cycle single-acting, bringing it down to the two-cycle double-acting with the various contributions, both piston-rod and rodless, eliminating cast iron and using steel to get the last word in lightness of construction.

The question has been raised, "Is this two-cycle double-acting the last word?" the writer believes that it must not be allowed

[1] President, Sperry Gyroscope Co., Brooklyn, N. Y.. Mem. A.S.M.E.

to be, and that it is by no means the last word. The combustion engine gives us possibilities so rich in promise, deriving mechanical energy so directly from heated gases, that it would be a sad day for the art if the constructions dealt with in the paper were to be considered as final. Take the Worthington engine, to which the author has himself made a number of contributions. Here is an engine whose weight is still of the order of 220 lb. per hp. The paper states that it gives only 64.8 lb. net mean effective pressure

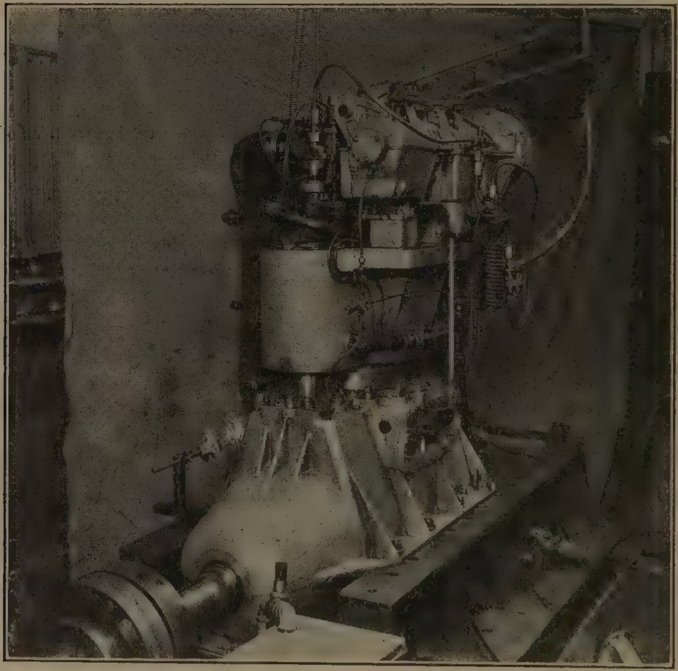

Fig. 26 Sperry Compound Oil Engine

to the crank. This low figure, together with the presence of CO in the exhaust — which seems to be characteristic, probably comes from the very great exposure in the combustion chamber to chilled walls at the lower end where not only the piston and cylinder heads are chilled, but also the cylinder walls. Furthermore, there is a large, quite intensely cooled piston rod occupying the center of the cylinder for the entire length of the combustion space. This piston rod necessarily conducts all of the cooling medium, both in and out, required for both ends of the piston.

The weight of the engine is still far too great. The cost of all of these engines, in the last analysis, bears a more or less direct

relation to the weight and cannot be very greatly separated therefrom. Hence, if this construction is the last word, the day of universal adoption of these engines is remote, although they are almost ideally direct from the heated gases to the crankshaft. For one, the writer does not believe that this form of engine is anywhere near the final word, but that it is only a passing phase. If this be true, what is to be the next step in lowering the weight and cheapening the cost of production of these engines?

The march of progress from the four-cycle single-acting engine has been in perfectly logical steps. The next equally logical step in the progression must be compounding. It is from the combustion engine that we should expect logically the great result from compounding, due to one simple fact. That is, we are dealing with much larger pressure ranges than we have at present with steam. Furthermore, there is the entire absence of condensation problems. We have pure gases throughout, inasmuch as our exhaust temperatures are still above 212 deg. fahr.

If there are any authorities on the compounding of combustion engines, the writer believes that he should be entitled to no little consideration from this standpoint. He has to date caused six compound engines to be built in ratios of 6, 8, and 10 to 1. He has also caused the investment of probably ten times as much capital as any other person or group in this particular line of research. It will not be out of place to state here that practically all of the findings presented in the writer's paper on this subject [1] have been very completely substantiated in the course of the subsequent development. These constructions have included a very light engine that has passed departmental acceptance tests as to performance, indicating that progress is being made along the lines of this important development. Engines weighing from 11 to 40 lb. per hp. are included in the list mentioned.

One very important achievement in this department which has the most fundamental bearing on matters gone into very fully by the author, especially in the last paragraph of the paper, cannot be overlooked. This lies along the line of advance work indicating that limitations heretofore thought to be fundamental as to heat transference and heat gradients, and limitations as to thermal capacity of cylinder walls, break down entirely in the light of what we now consider as the most ordinary performance in the combustion chamber and cylinder of the compound engine. The limitations that have been considered so vital, and about which so much has been written, do not seem to hold, and may be considered as yielding to very much larger values.

What we want in these engines is from 30 to 50 lb. per hp. with cheap construction, instead of 350 or 220 lb., and even then depending on steel for lightness — and this must come. As a basic

[1] Trans. A.S.M.E. vol. 43, p. 677.

proposition, however, if we are ever going to make these engines lighter and cheaper we must make each cylinder do more work. It has been found not at all difficult to do this. Many papers and books have been written on the heat limitations of cylinder walls, etc., but these so-called limitations do not hold. Combustion cylinders must give between 200 and 250 lb. instead of the 64.8 lb. mean effective pressure referred to in this paper. It may be of interest to know that the compound engine passing the acceptance tests gave over 240 lb. net mean effective pressure that reached the brake, i.e., the brake mean effective pressure was 240 lb.

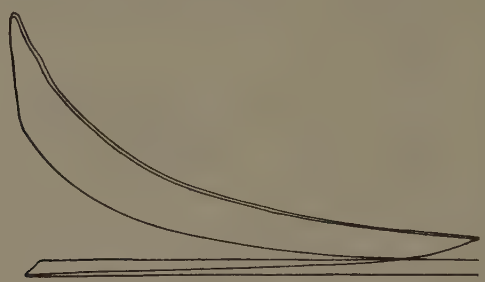

Fig. 27 High-Pressure Card of Sperry Compound Oil Engine

Fig. 28 Low-Pressure Card of Sperry Compound Oil Engine

throughout the test for each combustion cylinder, with solid-fuel injection and without supercompressed air being furnished to the engine. The weight of the engine in this case was of the order of 11 lb. per b.hp.

This method of appraising values treats the combined low-pressure and scavenging pump in exactly the same manner as the same elements are treated in the two-cycle engine. A combustion cylinder is a combustion cylinder and always presents about the same line of problems as to induction, eduction, and fuel valves, injection, tightness, and lubrication. The question is, what mean effective pressure can we realize on the crankshaft and at the brake for each combustion cylinder present? The above high mean

effective pressures to the brake explain why we should expect lighter and cheaper engines by taking this next logical step in combustion-engine development.

In starting the analysis years ago, the only indication that the combustion-chamber limitations, so-called, were unsound was that those same limitations as to size of cylinder came at about the same diameter in both the two- and four-cycle engine, where the heats present are easily at the rates of two to one, and that the heats of the compound engine lay between the two. The compound engine has taught the following fact of great usefulness to this art with regard to greater heat capacities of the combustion chamber walls: The heated gases in contact with the cylinder

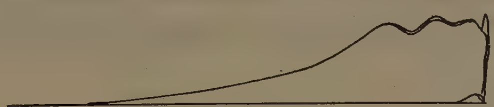

FIG. 29 AIR-PUMP CARD OF SPERRY COMPOUND OIL ENGINE

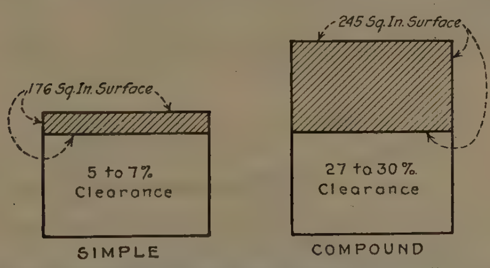

FIG. 30 RELATIVE CYLINDER AREAS EXPOSED TO COMBUSTION IN SIMPLE AND COMPOUND ENGINES

walls should affect them in two ways: First, the factor of the heat intensity, second, the number of molecules in contact. First their activity and then their number. Any consideration that omits either of their proper derivatives is in error. From one very definite point of view, the values for the heat gradients and general activity that the walls must face, which are expressed as the product of these qualities, are the correct ones. However this item is taken, both quantities are indispensable to the phenomenon in hand. It remains now to examine the quantities as they have been taken, and not in the new light thrown on the subject by our experience with the compound engine.

The performance of these combustion cylinders has been practically perfect, and is not different from that of the ordinary Diesel four-cycle experience. The cylinders are working normally in every way; the surfaces are splendid; no particular care is

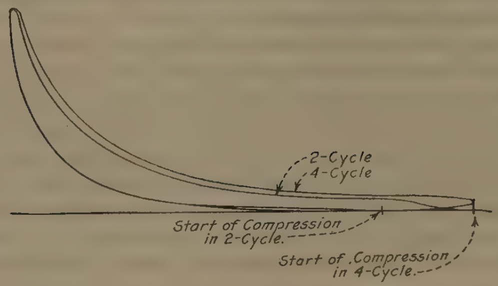

Fig. 31 Two- and Four-Cycle Cards of 68 and 73 M.e.p., Respectively

(Mean Effective Pressure, Lb. per Sq. In.: Net, 2-cycle, 63; 4-cycle, 73. Under expansion, 2-cycle, 108; 4-cycle, 122. Under compression, 2-cycle, 35; 4-cycle, 38.)

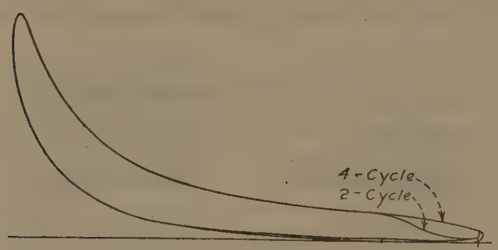

Fig. 32 Two- and Four-Cycle Cards of Higher M.e.p. than Shown in Fig. 31

(Cards more like those obtained in regular full performance. Mean Effective Pressure, Lb. per Sq. In.: Net, 2-cycle, 98; 4-cycle, 101. Under expansion, 2-cycle, 150; 4-cycle, 154. Under compression, 2-cycle, 55; 4-cycle, 57.)

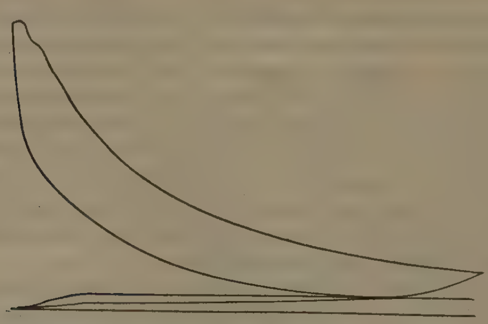

Fig. 33 Card of Sept. 10, 1924, 245 B.hp. at 360 R.p.m.

(Mean Effective Pressure, Lb. per Sq. In.: Net, 245; Under expansion, 435; Under low pressure part, 55; Under compression, 178; Under induction, 58.)

taken in lubrication; the rings never stick, and for the last four years have given us no trouble at all. We have simply forgotten that we have any such thing as a combustion cylinder. The wall

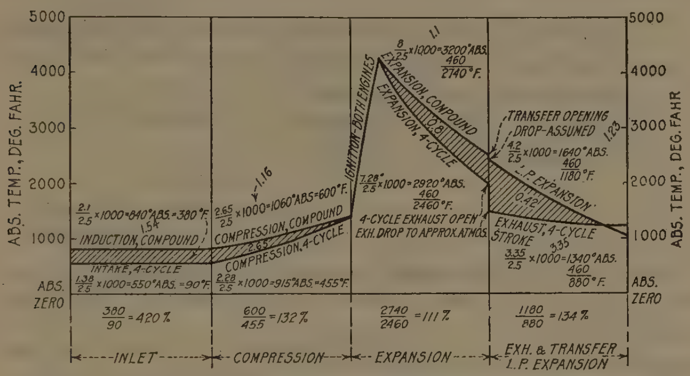

Fig. 34 Diagram of Cyclic Temperatures of 4-Cycle and Compound Engines

(4-cycle simple engine, 1429 deg. abs. = 969 deg. fahr., average for whole cycle or 720 deg. of crankpin path.; for compound engine, 1695 deg. abs. = 1235 deg. fahr., average for whole cycle or 720 deg. of crankpin path.)

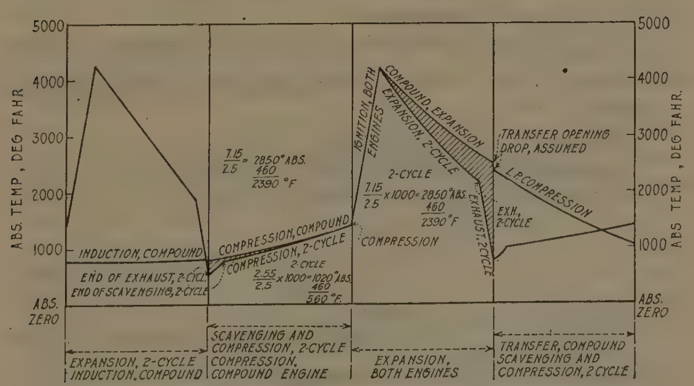

Fig. 35 Diagram of Cyclic Temperatures of 2-Cycle and Compound Engines

(2-cycle engine, 1880 deg. abs. = 1450 deg. fahr., average for 720 deg. crankpin path; compound engine, 1695 deg. abs. = 1235 deg. fahr., average for 720 deg. crankpin path.)

limitations that are supposed to exist do not exist. Here is metal taking four or five times the heat it was thought to be capable of taking. The writer believes that we can go far beyond what there now is any indication of in the literature on the subject.

An expression for the specific intensity of the heat attack upon the walls of the combustion-space heads and cylinder walls in various engines follows:

I — Diesel Single-Acting Combustion Cylinder for 720 deg. of Crank. Values on the order of the author's paper (see Fig. 31).

a Four-cycle (net m.e.p. to brake = 73 lb.).

Indicated m.e.p. under expansion	122 lb.
Indicated m.e.p. under compression	38 lb.
Indicated m.e.p., say, inhalation and exhaust strokes	0
	160 lb. for the 720 deg. of crank travel.

Average molecules in contact, expressed in atmospheres (14.7 lb. per. sq. in.), of the mean effective pressures existing = 160 lb. per sq. in. $160 \div 14.7 = 10.9$.

10.9×1429 deg. fahr. absolute (see Appendix) = intensity of heat attack = 15,500.

b Two-cycle (net m.e.p. to brake = 63 lb.).

Indicated m.e.p. expansion curve	108 lb.	180 deg.	crankshaft.
Indicated m.e.p. compression curve	35 lb.	180 deg.	
Indicated m.e.p. expansion curve	108 lb.	180 deg.	
Indicated m.e.p. compression curve	35 lb.	180 deg.	
	286 lb.	720 deg.	

$286 \div 14.7 = 19.5$ atmospheres.

19.5×1880 deg. absolute (see Appendix) = intensity of heat attack = 36,200.

II — Highest Full-Performance Values Attained in Diesel Engines (see Fig. 32).

a Four-cycle (net or brake m.e.p. = 101 lb.).

Indicated m.e.p., expansion part of curve	154 lb.
Indicated m.e.p., compression part of curve	57 lb.
Indicated m.e.p., say, inhalation and exhaust strokes	0
	211 lb. for 720 deg. of crank.

$211 \div 14.7 = 14.4$ atmospheres.

14.4×1429 deg. absolute (see Appendix) = intensity of heat attack = 20,060.

b Two-cycle (net or brake m.e.p. 98 lb.).

Indicated m.e.p. expansion stroke	150 lb.
Indicated m.e.p. compression stroke	55 lb.
Indicated m.e.p. expansion stroke	150 lb.
Indicated m.e.p. compression stroke	55 lb.
	410 lb. for 720 deg. of crank.

$410 \div 14.7 = 27.85$ atmospheres.

27.85×1880 deg. absolute (see Appendix) = intensity of heat attack = 52,000.

III — Compound, Single-Acting Combustion Cylinder. These values are rather high though they are the same as maintained throughout Government acceptance tests (see Fig. 33).

Four-cycle (net or brake m.e.p. = 245 lb. for each combustion cylinder).

Indicated m.e.p., expansion curve........435 lb.
Indicated m.e.p., low-expansion extension of expansion curve.................. 55 lb.
Indicated m.e.p., compression stroke......178 lb.
Indicated m.e.p., induction stroke........ 58 lb.

Total for the 720 deg. of crankshaft...... 726 lb. per sq. in.

$726 \div 14.7 = 49.5$ atmospheres.

49.5×1695 (see Appendix) = intensity of heat attack on walls = 83,750. (About 70 per cent greater than the highest, about twice the average of the two two-cycle, or nearly five times the average of the two four-cycle cylinders.)

Appendix

The following figures give the direct comparison between the heat units to which the high-pressure pistons and other walls of the Sperry compound are exposed, as compared with the Worthington double-acting and the Burmeister & Wain four-cycle. Figures on engine sizes and pressures are taken from the author's paper, and also the accepted method of comparison used by him.

	Brake hp.	Indicated m.e.p.	r.p.m.	B.t.u. per sq. ft. of piston per hp.
Sperry compound	245	260	350	2,245,000
Worthington double-acting two-cycle..	615	79	90	645,000
Worthington double-acting two-cycle..	900	85	125	944,000
Burmeister & Wain, four-cycle........	315	89.2	120	478,000

A point that stands out prominently in this connection is that although with the compound there is about five times the air present, burning five times the fuel per injection, the cooled surface exposed is only about 1.4 times as great. For example, in a 10-in. cylinder with 10-in. stroke, for a 6 per cent clearance the height of the clearance volume will be about ⅝ in. Again for a compound clearance of 27 per cent the height of the clearance of cylindrical shape will be $4\frac{1}{2} \times \frac{5}{8}$ or about 2 13/16 in. high. The wall area will be the circumference of a 10-in. cylinder × height of wall or 19.6 sq. in. for a simple, and 88 sq. in. for a compound engine. Adding this to the piston and head area, which is equal in both cases, the total area exposed to combustion will be 176 sq. in. for the simple engine and 245 sq. in. for the compound engine. (See Fig. 30.) The compound has five times the clearance volume and therefore contains five times the amount of air. It can be assumed that five times the amount of fuel per charge can be burned in the combustion chamber, which has only about 1.4 times the surface exposed to heat. This works a definite gain for compounding.

Figs. 31 to 35 show the method pursued in obtaining the values of the various pressure and heat components used in the latter part of the discussion. These were compiled by Mr. Herman Scharnagel, engineer in charge of the compound-oil-engine development for many years.

O. E. Jorgensen.[1] The Worthington double-acting two-cycle Diesel engine was briefly mentioned by the author, but as it is the original American engine of its type, it is fitting that some facts pertaining to its history should here be recorded.

The European motorship has developed and now prospers under conditions materially different from those under which American ships operate. The principal trade is between Europe and the Far East, or South America, or other distant ports, and the competing steamers which, on account of their greater number, determine the freight rates, are coal burners. Coal takes up valuable cargo space and is expensive at the outlying coaling stations, but fuel oil can be carried for the round trip from the place of cheapest oil. Besides a considerable saving in fuel cost, these motorships derive further advantages from an increased carrying capacity and a reduction in crew wages.

The majority of privately owned American ships trade in the coast-to-coast service and compete against oil-burning steamers which have already saved the coal bunker space, the coal passers, and most of the stokers. Fuel oil can be had at about the same price at both ends of the run so that no great advantage is obtained by reduced bunker weight, as the heavier machinery weight of the standard motorship to a great extent absorbs it.

Remunerative business is more or less assured for the operator of a long-distance motorship if the machinery can be depended upon to do its work, notwithstanding the excess weight and price of the machinery compared with steam machinery. It is natural for such an operator to be conservative; hence the preponderance of the single-acting four-cycle motorship, which was first in the field. The American motorship operator works on a narrow margin of profit. If the American motorship is to become a factor in shipping, a motor of lighter weight and lower cost must be developed and produced in this country.

This was the writer's appraisal of the situation in 1920 when the Worthington Pump and Machinery Corporation instituted a research into the causes of the slow advance of the motorship in American shipping and the possibility of its improvement. As chief engineer of the Burmeister & Wain Company in Denmark, he had had considerable experience in the construction of 4-cycle Diesel engines and was responsible for the design of the engines of the first motorships made by that company, including the pioneer motorship *Selandia*, which has demonstrated its good qualities in thirteen years' operation. Despite the writer's consequent preference for the 4-cycle type, the outcome of his endeavor to solve the problem of finding a type of lighter weight and lower cost, combined with the economy and reliability of the present

[1] Consulting Marine Engineer, Worthington Pump and Machinery Corpn., New York, N. Y. Mem. A.S.M.E.

standard motorship engine, was an entirely new type; namely, the Worthington double-acting, 2-cycle Diesel engine, which is based on the writer's conception, patents, and design.

An intensive utilization of the running gear is the natural and rational way of economizing in engine weight and cost per horsepower; the double-acting two-cycle engine is on a par with the steam engine in this respect; no system can go further and this system seemed therefore the logical one to adopt. Low first cost demands simplicity in design. As the running gear on which the marine steam engine has finally standardized is the simplest yet devised, this general type was adopted with the expectation that possible difficulties arising from a divided combustion space in the bottom end could be overcome.

In the choice between valve and port scavenging, preference was given to the latter for several reasons. In port scavenging the piston is used as a slide valve for the exhaust as well as for the scavenging air. The valve motion is simplified, the cylinder itself can be shaped with more freedom, and larger areas are available for the entrance of scavenging air, resulting in a lower air pressure, more efficient scavenging, and higher mechanical efficiency.

At the time, it was known in a general way that the difficulty in making a reliable two-cycle engine of large power was the intensive heat stresses set up in the walls of the combustion chamber. Such stresses will crack these parts and cause delay and expense in the operation of the engine, and may more than offset any saving in first cost. The problem, which is purely economic, would not be properly solved unless a design of the cylinder was found which would raise its margin of safety to a level with the general engine parts, or at least to a par with the corresponding parts of a four-cycle engine. It was not a question of merely improving existing designs: The heat-stress evil had to be eradicated, even at the expense of other valuable features of the engine. This sacrifice, fortunately, proved unnecessary.

Detailed information is now available regarding the experiments made in the M. A. N. works with the famous 33 by 41½-in. double-acting two-cycle engine, beginning in 1910 and continuing up to 1917. The history of this engine is given in *Zeitschrift des Vereines deutscher Ingenieure,* vol. 67, (1923), pp. 1093 and 1134. The heat stresses in some cases would crack a cylinder after a few hours run, often in the place where the valve cages penetrate the cylinder wall. In an article by Dr. A. Naegel, in the same journal, vol. 67 (1923), p. 725, are given some temperature measurements indicating a flow of heat through the walls of this engine of 57,000 B.t.u., corresponding to an evaporation of 60 lb. of water per sq. ft. per hr. This proves what was previously only surmised, that the flow of heat through the walls of the combustion space of a Diesel-engine cylinder is of the same order as the heat flow through the plates in a boiler furnace; the pressure in the Diesel

cylinder of 500-550 lb. per sq. in. is considerably higher than is common in boiler practice. Furthermore, it is constantly changing between this high value and nothing. But while the boiler furnace is constructed of forged-steel plates, in a self-contained shape, of a single thickness, generally free from stays and bracings, and cooled by boiling water, the Diesel-engine cylinder in its conventional design is made of cast iron with flat surfaces in the cylinder head. It has complicated double-wall castings in which the outside cool wall is braced against the inner hot wall and the surfaces are cooled by cold water. Judging from general experience with steam boilers, it is small wonder that Diesel-engine cylinders should give trouble.

This parallel with boiler design led to the design of the Worthington double-acting two-cycle-engine cylinder. The combustion takes place in a steel shell of single thickness, made of high-grade chrome-vanadium steel of great ductility. The thickness of this shell is reduced to that used in boiler furnaces. The shape

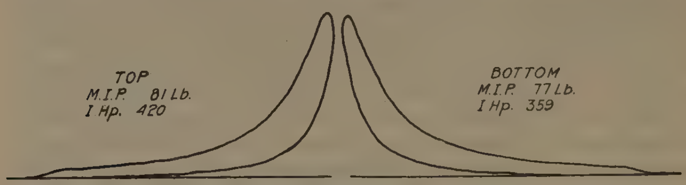

Fig. 36 Typical Indicator Cards from Test of a 27-in. by 40-in. One-Cylinder Double-Acting Two-Cycle Diesel Engine

(R.p.m., 90; b.hp. for both cards, 617; injection pressure, 820 lb. per sq. in.)

makes it self-contained, and it is free from any bracing or staying to other parts. The fastening of the cylinder at the cold end is such as to permit this steel shell to expand freely, unhindered by any other parts. The surface presented to the fire is smooth and free from all projecting parts or corners on which the flames would otherwise concentrate their heat. The combustion space is thus designed on lines which have proved reliable in steam-boiler design, but that part of the cylinder which is swept by the piston and where the expansion takes place, is lined with cast iron and follows conventional ideas of steam engines and Diesel engines.

In order to avoid any added heat problem due to the double action of the engine, this cylinder, as well as the piston, has been designed as two single-acting cylinders supported on the same frame and two single-acting pistons placed on the same rod. This idea has been carried out to the fullest extent.

The first step to carry this design out was taken in 1921 when a 14- by 19-in. horizontal experimental engine was built. The cylinder was of exactly the same design, using a steel shell, as the later large engine, even though it was realized that this precaution was probably unnecessary in an engine of these dimensions. It was

done in order to investigate as many of the novel features of the design as possible on a small scale before the building of the large engine was decided upon. Experiments were carried out over a wide range of speeds and loads; the behavior of the cast-iron liner pressed into the steel shell was carefully observed; the horizontal fuel-spray valve was developed; the scavenging and combustion space shapes were studied; the piston cooling, the cylinder cooling, and the piston-rod packing were developed, and during this work the final large engine took more and more definite shape. In October, 1923, it was decided to begin the actual work on the 27- by 40-in. vertical engine. It was started in the following June; a ten-day full-power test was run in August, and a thirty-day endurance trial was successfully carried out without a stop under continuous supervision day and night by the U. S. Shipping Board engineers from September 26 to October 26. After the run the engine was opened up for inspection and its condition was found as perfect as when the run was started. The main data from the test are given in Table 11.

TABLE 11 DATA ON TEST OF A ONE-CYLINDER 27-IN. BY 40-IN. DOUBLE-ACTING TWO-CYCLE DIESEL ENGINE

Beginning of test	10:05 p. m., September 25, 1924
Termination of test	10:05 p. m., October 25, 1924
Duration of test	720 hr.
Total number of revolutions during test	3,879,921
Average speed	89.8 r.p.m.
Average brake load	2400 lb.
Length of brake arm	15 ft.
Average brake hp	615
Average indicated m.e.p., top cylinder	81 lb. per sq. in.
Average indicated m.e.p., bottom end	77 lb. per sq. in.
Average indicated hp	778
Fuel-oil consumption, average of four 2-hour tests, per b.hp-hr.	0.428 lb.
Gravity of fuel used	28 to 32 deg. B.
Fuel-oil consumption per i.hp. per hr	0.339 lb.
Mechanical efficiency measured during the four tests	79.5 per cent
Average lubricating oil consumed in the cylinder and piston rod stuffing box per 24 hr	4.03 gal.
Average exhaust temperature	510 deg. fahr.

Typical indicator cards taken during the test are shown in Fig. 36.

Fig. 7 gives a general idea of the design of the large engine and it is interesting to compare it with Fig. 6 showing the latest M. A. N. design of an engine of a similar type but having a cylinder of cast iron. The simplicity of the Worthington engine, which is obtained principally by the use of the strong, ductile, self-contained steel cylinder taking up the longitudinal as well as the bursting stresses, stands out very clearly by this comparison.

Louis Illmer.[1] It is pleasing to note that the author, together with many other engineers, has now become a strong advocate of the two-stroke engine. This is a development with which the

[1] Development Engineer, Patent Attorney, Brewer-Titchener Corpn., Cortland, N. Y. Mem. A.S.M.E.

writer has long been associated, and during this time has repeatedly had occasion to defend the two-stroke principle. The paper appears to favor the double-acting two-stroke engine rather too much, without giving sufficient credit to competing types. For instance, the weight data given for the double- and single-acting types are not thought to be quite as simple as would appear from the given presentation of these relations.

In the first place, the working parts in a single-acting engine are only subjected to a repeated or one-way load, while in the double-acting engine they are subjected to a reversing load. This difference in the character of loading calls for approximately 50 per cent greater cross-sectional strength in the baseplate, shaft, frame, and all the moving parts in order to maintain a corresponding factor of safety.

Objection might also be raised as to the method of comparing the weights of different engine types and sizes on the unit-displacement basis. Such parts as the baseplate, shaft, rods, etc., for any given cylinder dimension remain more or less independent of the stroke ratio. It is thought that any particular line of such engines can be more closely analyzed by taking the engine weights on the basis of $D^2\sqrt{L}$, where these notations refer to bore and stroke, respectively. As applied to single-acting Diesel engines, this relation shows that submarine engines usually weigh about one-half as much as large cargo engines, and that the two-stroke engines weigh from 10 to 20 per cent more than do four-stroke engines of corresponding cylinder dimensions.

The weight per horsepower, assuming a fixed maximum pressure in all cases, is dependent upon the mean effective pressure that can be realized at the engine shaft. The two-stroke engine suffers from a considerable loss of effective stroke due to the use of piston-controlled exhaust ports, and in single-acting engines this loss generally amounts to more than 15 per cent of the gross stroke. If on the basis of the effective stroke the mean effective pressure in two-stroke engines is taken as equal to that attained in a four-stroke engine, then under favorable conditions a delivery of about $1\frac{2}{3}$ more power may be expected than from the four-stroke Diesel engine of equal cylinder dimensions.

In the case of the double-acting two-stroke engine there are further deductions in power capacity involved. Allowance must be made for the loss of displacement resulting from the use of a piston rod and since the rod diameter in high-pressure oil engines assumes a size of about five-sixteenths to one-third that of the bore dimension, this in itself will reduce the crank-end piston displacement by about 10 per cent. Furthermore, owing to the peculiar angular relations resulting from the short connecting rod, the crank-end exhaust ports are usually overrun by about three-halves more in port length than is the case at the head end. The effect of this extra loss in effective piston stroke is to further reduce

the charge-holding capacity as compared to that of the head-end cylinder. With these deductions, the total power capacity of a double-acting two-stroke cylinder would hardly be expected to exceed three times that of a single-acting four-stroke cylinder having equal bore dimensions and speed.

The situation is further complicated by the fact that the crank end of a two-stroke cylinder is difficult to charge properly with fresh air, especially when the charging is effected through intake ports overrun by the piston. The principles underlying porting and charging of two-stroke oil engines have been discussed by the writer in a recent paper presented before the Society,[1] from which it would appear that the piston rod of the double-acting engine might easily cause the incoming charge to be improperly deflected and thus interfere with trapping the desired volume of air charge behind the piston prior to exhaust-port closure.

No doubt these difficulties have been solved in the new Worthington oil engine. In an article in *The Iron Age* of September 4, 1924, it was stated that the present paper contemplated giving test results as to economy and other performances attained with the first Worthington engine. It would be of considerable interest to learn how the actual results compared with those expected.

In this connection it might be pointed out that the fuel-valve injection into the head can readily be made to impinge against the crown of the piston and thus attain the most effective distribution of fuel throughout the available air. This condition does not, however, apply to the crank end because of rod interference, and the indicated test results would further disclose just how effective the plural nozzle arrangement is in making the crank end of the cylinder deliver its proper share of the power.

The author lays considerable stress on the use of a steel cylinder liner, and it is contended that this allows of a greatly increased rate of heat offtake. The reasons for this deduction are not apparent, since steel cylinder liners, preferably with thin cast-iron bushings, have long been used without showing any marked advantages over high-grade cast-iron cylinders. While it is true that steel cylinders may be made somewhat thinner, the statement made in Par. 115 that this reduction in thickness is of the order of 3 to 1 is not borne out by the showing made in Fig. 8 of the Worthington cylinder. As scaled, the liner thickness appears to be about one-sixteenth of the cylinder bore. It is not clear how it could be much thinner and still provide for proper strength, stiffness in machining, and allowance for rebore.

The subject of proper thickness for cast-iron cylinder walls and liners was treated at some length by the writer in a paper on Heat Flow through Cylinder Walls,[2] and a number of liners for double-

[1] Trans. A.S.M.E., vol. 43, 1921, p. 649.
[2] Trans. Society of Automotive Engineers, vol. 13, part 1, 1918.

acting two-stroke engine cylinders were designed and tested in practice. As applied to Diesel work such liners when made of air-furnace iron need not be thicker than one-twelfth the cylinder bore, that is, about four-thirds of the thickness indicated for the steel liner.

The reason why a close-grained iron liner is almost as effective as a steel liner has to do with the difference in their moduli of elasticity, which vary approximately with the relative tensile strengths of the respective materials in question. The coefficient of thermal conductivity is about the same in both cases. The resulting heat flow sets up tension stresses in proportion to the temperature gradient. While the strength of liner steel is about twice that of air-furnace iron, the temperature stresses resulting from heat flow will likewise be about twice as high in the case of steel due to its higher modulus E. Hence for equal thickness for liners, the factor of safety will remain substantially the same in either an iron or a steel liner if the direct tension due to bursting pressure is neglected; but a relatively small increase in thickness of the iron liner serves to put its factor of safety on a par with the steel liner. It is therefore not clear why the use of a steel liner should allow for so much greater heat flow without destructive effects. To meet the situation to better advantage, it is thought that bronze liners provided with iron bushings point the way to further progress in this direction.

It has usually been deemed advisable to line steel cylinders with cast-iron bushings for the purpose of facilitating lubrication and to increase the wearing qualities of the rubbing surface. In the latter respect it is generally acknowledged that hard cast iron on cast iron provides for better bearing surfaces, especially under the severe conditions obtaining in an engine cylinder.

Lastly, the matter of heat flow through the cylinder walls assumes especial importance in the case of double-acting two-stroke engines. The S. A. E. paper previously referred to shows that for the same size and speed, the heat flow through the four-stroke oil or gas engine is about five-eighths of that through a two-stroke engine under otherwise similar conditions. Since the maximum size and speed of engines as used for marine work are about the same in both two- and four-stroke cargo engines, it will be apparent that the problem of disposing of the augmented rate of heat flow in the case of the two-stroke engines must be most carefully handled.

A further point to be kept in mind is that the maximum bore temperature of the cylinder liner must be kept within rather closely prescribed limits, otherwise lubrication troubles are likely to arise. Analysis of quite a large number of sizes and types of engines as reported in the S. A. E. paper shows that a bore temperature of from 400 to 450 deg. fahr. should not be exceeded.

In Par. 106 of the paper under discussion, a reference made to the heat flow through modern boiler shells might lead one to infer that this rate is carried beyond that used in engine practice. This clearly is not the case, for, taking an evaporation of 10 lb. per hour per square foot of boiler surface, this would correspond to only about 700 B.t.u. heat flow per square inch per hour, while in small high-speed engines the heat flow is frequently carried up to from 10 to 20 times that stipulated for boilers.

In view of the already high rate of heat flow that must be taken care of in large engines, it would seem that not much further advantage in the way of power capacity per unit piston displacement is to be gained by supercharging the cylinder. The rate of flow in the large conventional internal-combustion engine is already so high that it imposes a limitation upon allowable piston speeds.

J. C. Shaw.[1] While the paper is interesting, the writer considers it misleading in giving a true picture of the large oil engine, particularly as applied to the marine field. One is led to infer that the four-cycle engine is destined to be replaced by the two-cycle, double-acting type, and which finds its ideal in the Worthington single-cylinder experimental engine.

The two-cycle engine is as old as the four-cycle, and has not lacked either in money or engineering ability for its development. The present status, however, in the marine field is that over three-quarters of the deep-sea tonnage is driven by four-cycle engines, and new tonnage now being laid down is preponderantly four-cycle. Shipowners must therefore have reasons other than those that can be expressed by academic formulas. Chief of these are the agreed reliability and freedom from trouble which has been experienced with the four-cycle engine.

The author has laid much stress on the cylinder-head design of the engine under discussion, but very little has been said in regard to the piston. From the sections through the cylinder it will be noted that it travels a considerable portion of its time over the heated double exhaust ports.

Tables of data have been given for four- and two-cycle engines, with and without air injection, and with and without attached air compressors, and from these the author has derived formulas for comparative weights, bearing pressures, etc., and has arrived at a set of mechanical-efficiency curves, shown in Fig. 24 for the various types. The writer does not consider the four-cycle engines given as representative of this class, particularly those shown in Tables 5 and 6, from which the mechanical efficiencies are derived. These, it will be noted, are mostly either of the high-speed type,

[1] Asst. to Chief Engineer, Wm. Cramp & Sons Ship and Engine Bldg. Co,. Philadelphia, Pa. Mem. A.S.M.E.

or of comparatively small powers. In Table 1 the engines designated as No. 1 and No. 2 were constructed by Burmeister & Wain over ten years ago, and the third engine credited to the same company is of a cylinder size that never has been built by them or their licensees.

The efficiency curve for the single-acting, two-cycle engine it will be noted is higher than that for the four-cycle engine, which is contrary to usual opinion.

The author has not taken into consideration the more recent developments with the four-cycle engine which put it on a very favorable basis with the two-cycle engine in regard to horsepower output per unit weight. This is in the employment of very high stroke-bore ratio, as high as 2.5, with correspondingly increased piston speeds, and the use of supercharging, which permits of about 20 per cent higher mean indicated pressures to be carried. The two-cycle engine is limited in its stroke-bore ratio on account of the difficulty of scavenging and the length of exhaust ports. High piston speed also adversely affects the mechanical efficiency, by reason of the increased scavenging pressure required. Supercharging with the two-cycle engine is part of the scavenging, and to this in no small part has been due the present success of this engine, and allowed mean pressure in the cylinders approaching that of the four-cycle engine.

Supercharging in the four-cycle engine consists in supplying air at about ½ lb. pressure to the intake manifold by means of a small independently driven blower. This sweeps the clearance space at the end of the exhaust stroke, when the inlet and exhaust valves are open simultaneously, free of burned gases, and also increases the volumetric efficiency to unity, or slightly above unity. An additional advantage is the partial cooling of the hottest part of the combustion space by air, similarly as in the two-cycle engine. The power required for the blower is about one per cent of that of the main-engine output, and is more than compensated for by the increase in mechanical efficiency.

In Par. 39 the author states that the estimated weight of the four-cylinder, double-acting Worthington engine, designed for developing 3000 shaft hp. at 90 r.p.m., is 209 lb. per shaft horsepower, and has compared it with a single-acting four-cycle engine which is given as 575 lb. per shaft horsepower. This latter figure is difficult to reconcile with present-day practice, as four-cycle engines of the same power and revolutions as stated, and using moderate supercharging, as previously mentioned, are being constructed which weigh 275 lb. per shaft horsepower. The comparison of the Worthington engine should have been made with the double-acting, four-cycle engine listed under heading D of Table 1. This engine is being installed in a twin-screw motorship, and weighs 197 lb. per shaft horsepower without the supercharger.

H. SCHRECK.[1] The argument has been opened regarding the reason for ship owners' leaning more toward four-cycle than two-cycle Diesel engines. That is, why are there at present more ships with four-cycle engines in operation than with two-cycle engines? During the years 1910 to 1915 the interest as to large marine Diesel engines was mostly centered in two-cycle engines, and during those years this type made great progress in Germany, England, Italy, France, and Belgium; its development, however, ceased during the war. While in this country the leaning is to a great extent toward the four-cycle engine, the manufacturers of large engines in Germany, England, and Italy devote their efforts again today to the two-cycle engine.

The writer was a co-worker in the development of the engine shown in Fig. 5, and will state that the results obtained from it are proof enough of the possibility of a reliable double-acting two-cycle Diesel engine. The difficulties with which the two-cycle Diesel-engine cylinder head had to contend are the same as we have in the design of a four-cycle engine, that is, the reduced section of material between the fuel valve, intake, and exhaust valve on the four-cycle engine, and the fuel valve and scavenging valves on the two-cycle engine. Today practically every two-cycle engine has scavenging ports instead of scavenging valves in the head, which eliminates the difficulties on the cylinder head of the two-cycle engine. Other parts of the cylinder, which are stressed equally as hard on the two-cycle engine, have been very well laid out on the engine which the author has described, and justify us in assuming that the earlier difficulties of the two-cycle engine have been well taken care of.

The writer would call attention to the early success of the Carels two-cycle engine, built by Carels in Belgium during the above-mentioned period. This engine was built in on a number of ships. That further progress on this engine was stopped is due wholly to the war, at which time the factory was destroyed.

Economic conditions during the past years have delayed the development of the two-cycle engine, but recent development and an ever-growing demand for larger sizes of Diesel engines will bring the two-cycle engine very quickly to its proper place in the engineering world.

R. D. GATEWOOD.[2] There are a number of conclusions that can be drawn from the data compiled by the author, and probably a number of different opinions as to the values of some of these conclusions. Many reasons can be given for the failure of the Diesel engine to advance further than it has either in the stationary

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[2] Dept. of Maintenance and Repair, U. S. Shipping Board, New York, N. Y.

or the marine field. Some of these are plausible, but from the Shipping Board point of view the principal one is the cost of the engine. If a type of engine can be evolved that can be put into ships at a reasonable cost, it will be bought in large numbers. The Shipping Board could, to advantage, begin at once to Dieselize 200 vessels, with a total of half a million horsepower. The Shipping Board problem, too, is only a small portion of the problem of the marine world. As far as the cost of a single-acting engine is concerned, there is not now, nor is there likely to be, a great differentiation between two-cycle and four-cycle single-acting engines. Such differentiation as does exist is, for all practical purposes, hardly appreciable. This is not the case with double-acting engines. Just why the double-acting engine has not advanced further, either here or abroad, is not clear, but it may be safely stated that for our present fleet the single-acting engine has no future whatever if double-acting engines of equal reliability can be developed. For the larger powers, such as will unquestionably be required in the development of a Shipping Board fleet or a Government fleet, the writer can see only the double-acting type or some type other than the single-acting engine, for the reason that the single-acting type cannot be put into a ship at a reasonable cost.

It seems futile and useless for so much engineering talent to take up this problem, without radiating from some one point. Would it not be much better if a start could be made, all workers converging toward a given point? The Shipping Board certainly does not want 15 or 20 different kinds of engines. It would prefer to have relatively few. This applies equally to the general marine world. It would be advisable if all who are interested in this problem would concentrate on one or two types of double-acting engines and spend their time and money developing these types rather than developing a heterogeneous lot of engines, each designer traveling his own road. If that could be done, progress would be much more rapid in the marine field.

The Author. The practical situation with regard to the position of the double-acting engine in relation to the single acting is fully summed up in the discussion of Captain Gatewood in his statement that "the single-acting type cannot be put into a ship at a reasonable cost." This fact makes it unnecessary to review the technical details involved in comparisons any further for the present.

Whether the double-acting engine of the future motorship is to be two-cycle or four-cycle, or both, will be determined by experience in the near future, but that there are clear advantages for the two-cycle, all things considered, no one can doubt from the present state of knowledge, as set forth in the paper.

Just what is the position of the compound engine, or what it may become, is difficult for most people to judge now, because there is so little experience available as a basis for such judgment. This makes the discussion of Mr. Sperry especially interesting, even though it does not carry conviction. The engineering world must be grateful for his contribution of facts and data on compounding and must admire his courage in pursuing development along such radical lines. From what has so far been presented it is impossible to carry out any analysis to fundamental units that will give a clear picture of the real relation of the compound to the single-expansion engine divested of accidental peculiarities of a single design. It does seem clear, however, that the very high mean pressures reported are not truly comparable with existing standards, because they are not referred to the same low-pressure displacement, and are somewhat misleading. The very low weights per horsepower reported are not proved to be inherent consequences of compounding rather than of overspeeding, special machine design with too low weights per cubic foot of displacement, or the effect of a small cylinder bore. Further data will be awaited with much interest on the actual weights, costs, and fuel consumptions of compound engines equal in horsepower and speed to existing engines on which data have been established. Not until then can any one judge the compound engine with even a fair approach to accuracy.

In the meantime it does seem pretty well established that for large oil engines the double-acting type has arrived and that future plans for oil-engine power will be very much influenced by it.

[illegible] few. This applies equally to the general motor [illegible]. It would be advisable if all who are interested in this problem would concentrate on one or two types of double-acting [illegible] and record their lives and [illegible] these types [illegible]

No. 1944

GAS TURBINES

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Member of the Society

and

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The paper presents a statement of the brake thermal efficiencies that may be obtained from gas turbines of various types, and discusses the possibilities and limitations of these types of heat engines. Calculated performances, with tables of computed values, are given for the following types: (1) *Explosion turbine with regeneration,* (2) *Explosion turbine with regeneration and air cooling,* (3) *Explosion turbine with regeneration and water injection,* (4) *Explosion turbine combined with steam turbine operated from exhaust-heat boiler,* (5) *Explosion turbine combined with steam turbine operated from exhaust-heat boiler, with charge precompression and reduced back pressure,* (6) *Constant-pressure-combustion turbine with regeneration,* (7) *Constant-pressure-combustion turbine with regeneration and with cooling of the buckets by steam jets, and* (8) *Constant-pressure-combustion turbine with regeneration and with steam injection into the combustion space.*

In the conclusion it is stated that a review of the possibilities of the gas turbine does not give much hope of realization of efficiencies such as would encourage attempts to overcome the many difficulties with which the gas turbine is surrounded.

In two appendices are considered the entropy chart for gases and the representation of the proposed cycles on this chart, as well as the fundamental formulas and methods used in the computation of the data included in the tables.

THE object of this paper is to present a fairly comprehensive statement of the brake thermal efficiencies that may be obtained from gas turbines of various types. Such efficiencies are functions of the efficiency of the compressor and of the efficiency

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ratio of the turbine (turbine efficiency). No attempt is here made to estimate the compressor and turbine efficiencies which may be realized now or in the future, but calculations of brake thermal efficiencies have been made for a range of compressor and turbine efficiencies which, it is thought, includes both the present and the future possibilities.

2 The only published values of the attainable efficiencies of gas turbines, calculated on the basis of the true (variable) specific heats of the gases,[1] are contained in the fifth edition of Stodola's Dampf- und Gas-Turbinen, which is not as yet available in the English language. These values are exclusively for turbines of the explosion type and for a certain specified range of conditions. The purpose of this paper is to extend that range and to include also constant-pressure-combustion turbines. In addition there are given the efficiencies for the more promising modifications of the simple cycle, such as (1) regeneration, (2) water injection into the combustion space, (3) steam generation by the exhaust gases for direct action on the gas turbine, (4) steam generation by the exhaust gases for use in a separate steam turbine, and (5) the use of an exhauster for extending the operating range of the gas turbine.

Expectation of High Efficiencies not Justifiable

3 The common expectation of improved efficiency for a gas turbine as compared with a reciprocating engine rests on an analogy with the steam turbine. This expectation may be dismissed as baseless. For the same pressure and temperature range the non-condensing steam turbine is less efficient than the non-condensing reciprocating engine. The higher efficiencies obtained with steam turbines result primarily from the use of higher superheats and lower condenser pressures than are practicable with reciprocating engines. If the gas turbine were able to utilize a more extended temperature and pressure range than is possible with a reciprocating engine a similar result might be expected, but instead, as will be shown later, the gas turbine is subject to limitations of pressure and temperature which do not exist for the reciprocating engine, and consequently offers no possibility of increased efficiency. Furthermore, since the gas turbine operates with exhaust at atmospheric pressure, or, if an exhauster is used, with only moderate vacuum, the fluid-friction losses will be greater than in a steam turbine with high vacuum. The gas turbine will be found to have a lower brake thermal efficiency than a reciprocating engine operating throughout the same pressure and temperature range.

[1] For efficiencies calculated on the basis of constant specific heat of the gases the reader is referred to the paper by Dr. H. N. Davis, "A P Q Plane for Thermodynamic Cyclic Analysis", Proc. Am. Acad. of Arts and Sciences, vol. xl, pp. 629-654.

The Field of the Gas Turbine

4 It is not easy to predict exactly the field which a practical gas turbine might occupy. For smaller sizes it cannot compete with the reciprocating engine which is available in compact form, of moderate weight, and of considerably higher efficiency. For 1000 hp. or over, the advantage in diminished size and weight might in certain cases cause its choice in preference to the more efficient Diesel engine. As compared with a steam plant, the absence of boiler and condenser equipment might in some cases be decisive, even though the brake thermal efficiency is not likely to exceed that of a first-class steam-turbine plant and the fuel is likely to be more costly.

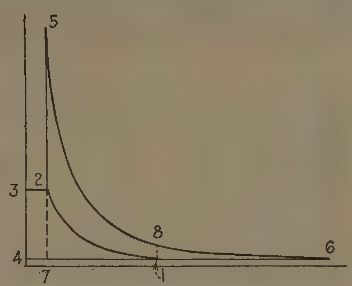

Fig. 1 Explosion-Turbine Cycle

Possible Gain in Efficiency from the More Complete Expansion of the Gas Turbine

5 The gas-turbine cycle has the advantage over the reciprocating engine of permitting complete expansion to the exhaust pressure.

6 In the *explosion-turbine* cycle, Fig. 1, the compressor work is 1234; the turbine work is 567 if the fresh charge is not admitted

TABLE 1 COMPARISON BETWEEN EXPLOSION CYCLES WITH ADIABATIC COMPRESSION AND (1) WITH EXPANSION TO THE INITIAL CHARGE VOLUME (1258, FIG. 1) AND (2) WITH COMPLETE EXPANSION (1256, FIG. 1)

Assumptions: Gas fuel; excess air, 20 per cent; initial temperature, t_1, 200 deg. fahr. (366 deg. cent. abs.); initial pressure, 14.7 lb. per sq. in. abs. (1.033 kg. per sq. cm. abs.)

Adiabatic Compression (Numerical suffixes apply to Fig. 1)					
Ratio of compression, p_2/p_1		4	6	8	10
Compression volume ratio, v_1/v_2		2.72	3.65	4.52	5.33
Compression temperature, T_2	deg. cent. abs.	538	602	648	688
	deg. fahr.	509	624	707	779
Explosion temperature, T_5	deg. cent. abs.	2309	2351	2383	2408
	deg. fahr.	3696	3772	3830	3875
Explosion pressure, p_5	kg. per sq. cm. abs.	17.7	24.2	30.4	36.2
	lb. per sq. in.	237	329	418	500
Expansion to Original Volume					
Temperature after expansion, T_5	deg. cent. abs.	1801	1685	1606	1563
	deg. fahr.	2782	2574	2431	2354
Pressure after expansion, p_8	kg. per sq. cm. abs.	5.08	4.75	4.53	4.41
	lb. per sq. in.	58	53	50	48
Indicated thermal efficiency, per cent, E_o		18.5	24.0	27.5	29.1
Complete Expansion					
Temperature after expansion, T_6	deg. cent. abs.	1277	1208	1161	1133
	deg. fahr.	1840	1714	1630	1580
Expansion ratio, v_6/v_5		9.50	12.03	14.34	16.49
Indicated thermal efficiency, per cent, E_c		34.6	37.6	39.7	40.7
Increase in efficiency resulting from complete expansion, per cent = $100\ (E_c-E_o)/E_o$		87.0	56.7	44.0	39.8

to the combustion chamber till after expansion is complete, but is 567+3274 if the fresh charge is admitted at the instant when the expansion has fallen to the pressure p_2. With expansion to the initial volume only, as in a reciprocating engine, the turbine work would be 7581 and 7581+3274 for the same two cases, respectively. The gain in cycle efficiency from complete expansion, V_6, compared with expansion to the original volume, V_1, is shown in Tables 1 and 2. Table 1 is calculated for adiabatic compression, which is the condition approximated to in a reciprocating engine. Table 2 is for isothermal compression, which is the rational ideal in a multi-stage centrifugal compressor such as would presumably be used in a gas-turbine plant. The conditions assumed are: $p_1 = 14.7$ lb. per sq. in. abs.; $t_1 = 200$ deg. fahr. (366 deg. cent. abs.); exhaust pressure, 14.7 lb. per sq. in. abs.; gas fuel; excess air, 20 per cent; no heat losses; no utilization of heat of exhaust gases; compressor efficiency, 100 per cent; turbine efficiency ratio, 100 per cent. It will be seen that the gain in cycle efficiency is greatest with low ratios of compression. With a ratio of compression of 4 the increase in cycle efficiency with isothermal compression is from 27.8 to 39.3 per cent, or 11.5 per cent, which is an increase of 41.4 per cent.

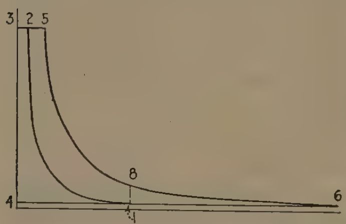

FIG. 2 CONSTANT-PRESSURE-COMBUSTION-TURBINE CYCLE

TABLE 2 COMPARISON BETWEEN EXPLOSION CYCLES WITH ISOTHERMAL COMPRESSION AND (1) WITH EXPANSION TO THE INITIAL CHARGE-VOLUME (1258, FIG. 1), AND (2) WITH COMPLETE EXPANSION (1256, FIG. 1)

Assumptions: Gas fuel; excess air, 20 per cent; initial temperature, t_1, 200 deg. fahr. (366 deg. cent. abs.); initial pressure, 14.7 lb. per sq. in. abs. (1.033 kg. per sq. cm. abs.)

Isothermal Compression

(Numerical suffixes apply to Fig. 1)

Ratio of compression, p_2/p_1	4	6	8	10
Explosion pressure p_5 { kg. per sq. cm. abs.	24.9	37.3	49.7	62.1
Explosion pressure p_5 { lb. per sq. in.	340	515	695	870
Explosion temperature T_5 { deg. cent. abs.	2200	2200	2200	2200
Explosion temperature T_5 { deg. fahr.	3500	3500	3500	3500
Expansion to Original Volume				
Temperature after expansion, T_8 { deg. cent. abs.	1543	1381	1271	1185
Temperature after expansion, T_8 { deg. fahr.	2318	2026	1828	1674
Pressure after expansion, p_8 { kg. per sq. cm. abs.	4.35	3.90	3.59	3.34
Pressure after expansion, p_8 { lb. per sq. in.	47.5	41	36	33
Indicated thermal efficiency, per cent, E_o	27.8	33.9	37.9	41.0
Complete Expansion				
Temperature after expansion, T_6 { deg. cent. abs.	1117	1019	951	900
Temperature after expansion, T_6 { deg. fahr.	1551	1375	1252	1160
Expansion ratio, V_6/V_5	12.22	16.68	20.77	24.62
Indicated thermal efficiency, per cent, E_c	39.3	42.9	45.4	47.2
Increase in efficiency resulting from complete expansion, per cent $= 100\ (E_c - E_o)/E_o$	41.4	26.5	19.8	15.1

With adiabatic compression the gain is still greater, viz., from 18.5 to 34.6 per cent, or an increase of 87 per cent, but the actual efficiencies are lower.

TABLE 3 COMPARISON BETWEEN CONSTANT-PRESSURE-COMBUSTION CYCLES WITH ADIABATIC COMPRESSION AND (1) WITH EXPANSION TO THE INITIAL CHARGE VOLUME (1258, FIG. 2), AND (2) WITH COMPLETE EXPANSION (1256, FIG. 2)

Assumptions: Excess air, 100 per cent; initial temperature, t_1, 200 deg. fahr. (366 deg. cent. abs.); initial pressure, p_1, 14.7 lb. per sq. in. abs. (1.033 kg. per sq. cm. abs.); oil fuel.

Adiabatic Compression

(Numerical suffixes refer to Fig. 2)

Ratio of compression, p_2/p_1		10	12	14	16
Compression volume ratio, v_2/v_1		0.1854	0.1620	0.1455	0.1332
Combustion volume ratio, v_5/v_2		2.726	2.635	2.552	2.479
Compression temperature, t_2	deg. cent. abs.	679	712	745	778
	deg. fahr.	763	822	882	941
Combustion temperature, t_5	deg. cent. abs.	1851	1877	1903	1929
	deg. fahr.	2872	2919	2966	3013
Expansion to Original Volume					
Pressure after adiabatic expansion, p_8	kg. per sq. cm. abs.	4.306	4.130	4.012	3.952
	lb. per sq. in.	47	44	42.5	41.5
Temperature after adiabatic expansion, t_8	deg. cent. abs.	1525	1464	1421	1400
	deg. fahr.	2286	2176	2098	2060
Indicated thermal efficiency, per cent, E_o		30.9	35.0	38.1	39.6
Complete Expansion					
Temperature after adiabatic expansion, t_6	deg. cent. abs.	1092	1054	1032	1015
	deg. fahr.	1506	1438	1398	1368
Ratio of expansion, v_6/v_5		5.90	6.74	7.59	8.41
Indicated thermal efficiency, per cent, E_c		44.5	47.7	49.6	51.1
Increase in efficiency resulting from complete expansion, per cent = $100\,(E_c-E_o)/E_o$		44.0	36.3	30.2	29.0

TABLE 4 COMPARISON BETWEEN CONSTANT-PRESSURE-COMBUSTION CYCLES WITH ISOTHERMAL COMPRESSION AND (1) WITH EXPANSION TO THE INITIAL CHARGE VOLUME (1258, FIG. 2), AND (2) WITH COMPLETE EXPANSION (1256, FIG. 2)

Assumptions: Excess air, 100 per cent; initial temperature, t_1, 200 deg. fahr. (366 deg. cent. abs.); initial pressure, p_1, 14.7 lb. per sq. in. abs. (1.033 kg. per sq. cm. abs.); oil fuel.

Isothermal Compression

(Numerical suffixes apply to Fig. 2)

Combustion temperature, t_5, 1604 deg. cent. abs. (2919 deg. fahr.)

Ratio of compression, p_2/p_1		10	12	14	16
Expansion to Original Volume					
Temperature after adiabatic expansion, t_8	deg. cent. abs.	1252	1180	1128	1080
	deg. fahr.	1794	1664	1571	1484
Pressure after adiabatic expansion, p_8	kg. per sq. cm. abs.	3.532	3.330	3.188	3.049
	lb. per sq. in.	35	33	31	29.5
Indicated thermal efficiency, per cent, E_o		31.4	34.7	36.7	38.7
Complete Expansion					
Temperature after adiabatic expansion, t_6	deg. cent. abs.	928	885	849	821
	deg. fahr.	1211	1134	1069	1018
Ratio of expansion, v_6/v_5		5.78	6.62	7.42	8.20
Indicated thermal efficiency, per cent, E_c		40.3	42.3	44.0	45.3
Increase in efficiency resulting from complete expansion, per cent = $100\,(E_c-E_o)/E_o$		28.3	21.9	19.9	17.1

7 In the *constant-pressure-combustion* cycle, Fig. 2, the increase in work from complete expansion is shown by the area 861. The gain in cycle efficiency is shown in Table 3, which is for adiabatic compression, and in Table 4, which is for isothermal compression.

The conditions assumed are the same as for the explosion cycle except that the fuel is oil and the excess air is 100 per cent. The cycle efficiencies are naturally higher, owing largely to the much higher compression ratios proper to this cycle, and the gain from complete expansion is correspondingly less. For a ratio of compression of 10, with isothermal compression, the increase in efficiency is from 31.4 to 40.3 per cent, or an increase of 28.3 per cent; with adiabatic compression it is from 30.9 to 44.5 per cent, or an increase of 44 per cent.

8 Tables 1 to 4 are calculated using variable specific heats. Attention is called to the high temperature at the end of expansion. The constant-pressure-combustion cycle is the more satisfactory in this respect with values from 928 to 821 deg. cent. abs. (1211 to 1018 deg. fahr.), which are not much beyond practicable temperatures of the present day. As will be shown later, however, reheating from turbine inefficiency would raise these temperatures very considerably in any actual turbine.

Practical Limitations of Gas Turbine

9 The actual construction of a gas turbine offers various practical difficulties resulting from the high temperatures to which the combustion chamber, nozzles, turbine disk, buckets, and casing are subjected. Of these members the turbine disk and buckets alone are necessarily subjected to high stress and, since they cannot be water-jacketed, a definite limit has to be set to the temperature to which they can be subjected. The temperature of the disk and buckets will be approximately the same as that of the exhaust gases, or, as it will be called here, the "casing temperature." In a simple gas turbine the casing temperature will be greater than the temperature of the gas discharged from the orifice as a result of reheating through bucket friction, windage, and residual kinetic energy. This reheating can be determined for any assumed turbine efficiency, and in the cases considered in this paper is found to range from about 100 to 400 deg. fahr. By the use of special cooling devices, such as steam jets or cooling air, the casing temperature may be reduced even below the spouting temperature of the gases.

10 The present practical limit of casing temperature appears to be about 450 deg. cent. (850 deg. fahr.). Holzwarth is able to operate at this temperature with buckets of soft low-carbon steel. Rateau[1] reports a temperature of 1200 deg. fahr. in his exhaust-gas turbine with rotor of tungsten tool steel. As practically all steels have lost half their tensile strength at 1000 deg. fahr. there does not appear much chance at present of operating at temperatures in excess of that figure.

[1] *The Engineer*, Nov. 3, 1922.

Selection of Type of Turbine

11 The limitation in casing temperature results immediately in another important limitation of the gas turbine, namely, the type of turbine which may be employed. Neither the reaction type nor pressure staging can be used, because, even with the maximum possible pressure drop, it is difficult to get the gases down to a temperature which is low enough to be practicable. With a single pressure stage, velocity staging is necessary in order to keep down the centrifugal stresses if the rotor is to run with a satisfactory peripheral speed. A practicable design can be made with two velocity stages and a corresponding bucket speed of about 22 per cent of the spouting velocity; the turbine efficiency estimated from steam-turbine experience will be less than 70 per cent, exclusive of windage losses. With three velocity stages and a corresponding bucket speed of about 15 per cent of the spouting velocity, the similar turbine efficiency will be less than 60 per cent. It appears, then, that a single-pressure-stage, two-velocity-stage impulse turbine is the only practical turbine arrangement at the present time; it has been accepted as such by all recent designers. The turbine arrangement being fixed, the maximum spouting velocity which can be efficiently utilized is determined by the maximum permissible peripheral speed, which, in turn, is limited by the strength of the rotor material at the casing temperature.

Limitation of Combustion Temperature

12 The limitations of casing temperature and of spouting velocity determine the maximum permissible temperature of combustion and thereby set definite limits to the attainable efficiency of a turbine operating without special cooling device. The control of combustion temperature is through the control of the amount of fuel burned per unit weight of air compressed. With Diesel engines at full load it is usual to operate with 100 per cent excess air; with explosion engines the excess air is not usually greater than 30 per cent. It will be found that in a gas turbine, without special cooling device, the excess air must be increased very largely, with the unfortunate result that the ratio of the positive work done on the turbine, W_t, to the negative work of precompressing the air before combustion, W_c, is much less than in the usual reciprocating engine.

Overall Efficiency

13 This ratio $R = \frac{W_t}{W_c}$ is most important in determining the efficiency of a gas turbine. The compression work has to be done in a separate machine, which would presumably always be a centrifugal compressor. The net work of the turbine is given by

$$W = W_t \times E_t - \frac{W_c}{E_c}$$

where E_t and E_c are the efficiencies of turbine and compressor respectively, or

$$W = W_t\left(E_t - \frac{1}{RE_c}\right)$$

14 The value of E_t, as indicated above, will be less than 0.7; the efficiency of a centrifugal compressor E_c, for high ratios of compression, on the isothermal basis, is probably not greater than 0.7. Examining the factor $\left(E_t - \frac{1}{RE_c}\right)$, it will be seen that each of the three quantities in it should be as high as possible. With E_t and E_c both equal to 0.7, this factor becomes negative for $R = 2$; that is, no work can be obtained from the turbine under these conditions. Similarly for E_t and E_c both equal to 0.6, R must be greater than 2.77 in order that the turbine may be able to operate at all.

15 The quantity $\frac{W}{W_t - W_c}$ may be called the "overall efficiency" of the turbine; this may be written,

$$\frac{W_t\left(E_t - \frac{1}{RE_c}\right)}{W_t - W_c} = \frac{W_t\left(E_t - \frac{1}{RE_c}\right)}{W_t\left(1 - \frac{1}{R}\right)} = \frac{E_t R - \frac{1}{E_c}}{R - 1}$$

Values of this quantity are given in Table 5.

TABLE 5 VALUES OF $\frac{E_t R - \frac{1}{E_c}}{R - 1}$

	Turbine efficiency, per cent								
	0.55			0.65			0.75		
Compressor efficiency, per cent	0.55	0.65	0.75	0.55	0.65	0.75	0.55	0.65	0.75
Values of R: 2	−0.7200	−0.4380	−0.2330	−0.5200	−0.2380	−0.0330	−0.3200	0.0380	0.1670
2.5	−0.2966	−0.1086	0.0280	−0.1300	0.0580	0.1946	0.0366	0.2246	0.3613
3.0	−0.0850	0.0560	0.1585	0.0650	0.2060	0.3085	0.2150	0.3560	0.4545
3.5	0.0420	0.1548	0.2368	0.1820	0.2920	0.3768	0.3220	0.4348	0.5168
4.0	0.1266	0.2207	0.2890	0.2600	0.3540	0.4223	0.3933	0.4873	0.5557
4.5	0.1857	0.2677	0.3262	0.3142	0.3942	0.4548	0.4443	0.5248	0.5834
5.0	0.2325	0.3030	0.3542	0.3575	0.4280	0.4792	0.4825	0.5530	0.6042
5.5	0.2677	0.3304	0.3760	0.3888	0.4526	0.4977	0.5122	0.5748	0.6204
6.0	0.2960	0.3524	0.3934	0.4160	0.4724	0.5134	0.5360	0.5924	0.6334

Utilization of the Heat of the Exhaust Gases

15 In any gas turbine it will be found that a considerable fraction of the heat of combustion escapes with the exhaust gases. A good turbine performance is not possible unless this heat is utilized. Various procedures have been used or seem of sufficient promise to justify analysis:

1 The exhaust gases may pass through a regenerator for heating the fresh air. With a counterflow arrangement the entering

air may be heated, after compression, to the temperature of the exhaust gases

2 The exhaust gases may generate steam which
- *a* May be sent into the combustion space in a constant-pressure turbine, or
- *b* May expand through steam nozzles and do work on the buckets (thereby cooling them) or on a special turbine on the same shaft, or
- *c* May drive a special condensing steam turbine and supply part or all of the power for operating the compressor; or
- *d* Various combinations of the above may be used.

Pressure Limits in Gas Turbines

16 There is no special condition limiting the compression pressure used in a gas turbine, but the increase in efficiency from high compression pressures is less than in reciprocating engines because the efficiency of centrifugal compressors falls off at high pressures in consequence of the high density of the air in the later stages and the resulting increase in frictional losses. The exhaust pressure will be in excess of atmospheric pressure when the heat of the exhaust gases is utilized in a regenerator or boiler, unless a gas exhauster is employed. As an exhauster acts on the gases after their volume has been diminished by cooling, its use may show an increase in thermal efficiency.

17 Many modifications of the gas turbine other than those mentioned above have been suggested and employed. The injection of water into the combustion space in conjunction with preheating the compressed air by the exhaust gases is analyzed in this paper. There have been several so-called gas-turbine projects employing an oscillating water column (as in the Humphrey pump) and a hydraulic turbine. These are really water-piston engines and are not considered here. The most ingenious is that of Stauber.[1]

Explosion Turbines

18 Gas turbines may be classified in the same two groups as reciprocating engines, constant-volume-combustion or explosion turbines, and constant-pressure-combustion turbines. These are strongly differentiated.

19 In the constant-volume-combustion or explosion turbine the action is intermittent. The compressed air and fuel are introduced into the combustion chamber, the admission valve is closed, and the mixture is ignited and exploded; the nozzle valve is then opened and the products of explosion pass through the nozzle to the turbine buckets. After the pressure in the combustion chamber has fallen sufficiently, cooling air may be sent through

[1] Stodola, loc. cit., p. 1015.

the combustion-chamber nozzle and on to the turbine buckets, as in the Holzwarth turbine. The nozzle valve is then closed and the cycle begins again. Attention may be drawn here to certain features of this process. In order to operate, it is essential that the mixture in the combustion chamber should be explosive at the moment of ignition. This requirement limits the permissible amount of excess air, but to an extent which is not determinable from existing data on explosive mixtures. With air preheating, the temperature of the charge will be high — a condition favorable to ignition. If the fuel is injected very rapidly into air preheated to the ignition temperature and with the great turbulence which would exist under those conditions, the combustion may take place, as in a Diesel engine, with any quantity of fuel, however small; but the combustion, though very rapid, will not be a true explosion.

20 Another special condition which is inherent in the explosion turbine is that the spouting velocity of the gases during the expansion period will fall from a maximum at the instant of opening the nozzle valve to zero when the pressure in the combustion chamber is equal to that in the turbine casing. This is not a condition favorable to high turbine efficiency. Stodola[1] shows for a particular case, with initial spouting velocity of 4526 ft. per sec., that the velocity will have fallen to 3280 ft. per sec. when 66 per cent of the charge has passed, and to 1640 ft. per sec. after 96 per cent has passed. He finds a mean turbine efficiency of 63.5 per cent in this case as compared with an efficiency of 70 per cent for the most favorable gas velocity. These turbine efficiencies do not take windage losses into account.

Constant-Pressure-Combustion Turbine

21 In the constant-pressure-combustion turbine there is a steady flow of air and fuel to the combustion chamber and a steady discharge of products of combustion through the nozzles. The temperature conditions offer much more difficulty than with the explosion type. Temperatures must be kept down either by using a very large excess of air, by injecting water or steam into the combustion space, by separate air or steam jets acting on the buckets, or by other device. This type, which has given highest efficiencies in reciprocating (Diesel) engines, does not promise similar efficiencies in the gas turbine in consequence of the low ratio of positive to negative work, R, and the low value of compressor efficiency, E_c, which results from the high compression pressures. On the other hand, turbine efficiencies, E_t, are higher than in the explosion turbine, as indicated in the preceding paragraph.

[1] Loc. cit., p. 1007.

Fuels

22 Either gaseous or liquid fuels may be used in a gas turbine. Gaseous fuels must be precompressed and may add seriously to the negative work of the cycle, especially if the gases are of low heating value and the excess air is low. Liquid fuels require injection devices similar to those used in Diesel or semi-Diesel engines. The form of the combustion space can be made more favorable than is possible in reciprocating engines.

Conditions for High Efficiency

23 The conditions assumed for the calculations are in some respects not practicable at present — this is especially true as regards casing temperatures. The efficient gas turbine must wait for metallurgical developments which will yield a metal better able to retain high strength at high temperatures than any now available.

24 No attempt is made to calculate probable turbine efficiencies, E_t, but it is evident that certain conditions, such as the reduction of windage losses, will favor higher efficiencies. The constant-pressure-combustion turbine, exerting a constant torque, will have a smaller percentage windage loss than the explosion turbine with intermittent applications of a lower average torque. A turbine with back pressure reduced by an exhauster will also have a smaller windage loss. On the other hand, an explosion turbine with short expansion period followed by prolonged cooling by low-pressure air, will have a low turbine efficiency if the conditions are such that the turbine acts like a blower and speeds up the cooling air as it passes through the buckets. Stodola [1] estimates the use of over 16 per cent of the turbine power to carry out the cooling by low-pressure air in this manner. With high-pressure air it is probably not less.

Efficiencies of Various Cycles

25 The calculation of the efficiency of a gas turbine, using variable specific heats, can be carried out most readily by the use of a temperature-entropy chart such as is given by Stodola.[2] The process of calculation and the values of the specific heats used are given in Appendix No. 1. In most of the cases considered these calculations have been made for some assumed casing temperature and for a series of compressor and turbine efficiencies. The turbine efficiencies cover all the losses, including nozzle friction, bucket friction, windage, residual kinetic energy, and machine losses, that is, they are the ratio of brake work to the adiabatic total-heat drop from combustion-chamber conditions to exhaust pressure. All

[1] Loc. cit., p. 1006.

[2] Loc. cit.

the energy lost in the turbine is assumed to be used in heating the gases at constant pressure from the temperature at the end of adiabatic expansion to the casing temperature. With a fixed casing temperature and a fixed combustion pressure this requires a different combustion temperature for each assumed turbine efficiency.

26 Certain conditions have been assumed for all the cases considered — with a few exceptions as noted:

a When gaseous fuel is used, it is assumed to have the volumetric composition; $H_2 = 11$; $CO = 22$; $CO_2 = 10$; $N_2 = 57$ per cent, and a low heating value of 102 B.t.u. per cu. ft. at 14.7 lb. per sq. in. abs. and 60 deg. fahr.

b When oil fuel is used, it is assumed to have the weight composition: $C = 86.5$; $H_2 = 11.2$; $N_2 = 2.3$ per cent, and a low heating value of 18,610 B.t.u. per lb.

c The effect on volume of molecular shrinkage due to combustion is neglected; the resulting error in the calculated efficiency is less than 1 per cent. The change in chemical composition is in other respects taken into account as shown in Appendix No. 1.

d The heat of combustion is assumed to be the same at constant pressure and at constant volume. The actual difference is less than 0.5 per cent. Similarly the variation of heat of combustion with temperature is neglected.

e The pressure drop between the air compressor and the combustion chamber is neglected.

f Compression efficiencies are all on the isothermal basis. The table in Appendix No. 3 gives the corresponding efficiencies on the adiabatic basis. The compressed air is supposed to have the room temperature when it reaches the regenerator or combustion chamber.

g The radiation loss to the walls of the combustion chamber is assumed to be 10 per cent of the heat of combustion. In most cases this heat is regarded as lost, but in certain specified cases it is utilized in generating steam.

h The pressure of the exhaust gases before passing through the regenerator or steam boiler is generally assumed to be 1.2 kg. per sq. cm. (17.06 lb. per sq. in. abs.) when exhausting to the atmosphere.

i Most of the calculations assume an initial air pressure of 1 kg. per sq. cm. abs. (14.22 lb. per sq. in. abs.), which is 3 per cent less than the standard atmosphere. The difference between this pressure and the standard atmosphere may be considered as the pressure drop of the air entering the compressor. Its effect on efficiency is quite negligible.

j Spouting velocities are always theoretical quantities assuming no nozzle loss.

CALCULATED PERFORMANCE OF GAS TURBINES

A—Explosion Turbine with Regeneration

27 The highest turbine-casing temperature which seems likely to be practicable, according to present indications, is about 500 deg. cent. (932 deg. fahr.). With exhaust gases at that temperature, and with an efficient regenerator, a temperature of 450 deg. cent. (842 deg. fahr.) for the charge entering the combustion chamber should be obtainable. The efficiencies attainable under these conditions and with compression-pressure ratios 5, 10, 15 and 20 are given in Table 6. Blank spaces indicate negative values of the brake thermal efficiency, i.e., the turbine work is not sufficient to drive the compressor. The temperatures after adiabatic expansion are the temperatures at which the gases would leave a frictionless expansion nozzle. The reheating due to nozzle and bucket friction, windage losses, and residual kinetic energy is the difference between the casing temperature and the temperatures after adiabatic expansion; in this case it ranges from 146 to 470 deg. fahr. The amount of excess air is so great (ranging from 533 to 1796 per cent) that no explosive mixture is possible; the cycle would presumably have to be carried out by injecting the fuel rapidly as in a semi-Diesel engine. The spouting velocities are the maximum values (disregarding nozzle friction) and diminish from the given values to zero. They are moderate and can be readily taken care of without necessitating excessive peripheral speed for the turbine buckets. The ratio of bucket speed to spouting velocity should be lower for gas turbines than for steam turbines because of their greater windage loss.

B—Explosion Turbine with Regeneration and Cooling

28 The conditions of Table 6 have been selected as apparently those giving maximum efficiency for an explosion gas turbine with regenerator but without any special cooling device. If the gases reach a temperature in excess of 500 deg. cent. (932 deg. fahr.) in the turbine casing, it becomes necessary to add some device to keep down the temperature of the buckets and turbine disk. If the cooling device is a jet of high-pressure air expanding on to the buckets, it will consume power for compressing the air, and all of this power will not be recovered at the buckets. If it is low-pressure air the buckets will do work on it. The magnitude of the power required for this purpose depends upon the details of the particular scheme used.

29 Stodola[1] gives charts showing the efficiencies of explosion turbines with regenerative heating of the compressed charge to 550 deg. cent. (1022 deg. fahr.) and with casing temperatures of 600, 800 and 1000 deg. cent. (1112, 1472 and 1832 deg. fahr.). Table 7 has been compiled from these charts. The tabulated

[1] Loc. cit., Figs. 1048, 1049 and 1050.

TABLE 6 CALCULATED PERFORMANCE OF EXPLOSION GAS TURBINES WITH REGENERATIVE HEATING OF THE COMPRESSED CHARGE TO 450 DEG. CENT. (842 DEG. FAHR.) AND TURBINE-CASING TEMPERATURE 500 DEG. CENT. (932 DEG. FAHR.)

Assumptions: Back pressure at gas turbine 1.2 kg. per sq. cm. abs. (17.06 lb. per sq. in. abs.); gas fuel; heat loss to walls of combustion chamber 10 per cent of heat of combustion; compression efficiencies on isothermal basis.

Compression pressure, kg. per sq. cm. abs. (lb. per sq. in. abs.)	5 (71.1)					10 (142.2)					15 (213.4)					20 (284.5)				
Turbine efficiency, per cent	55	60	65	70	75	55	60	65	70	75	55	60	65	70	75	55	60	65	70	75
Explosion pressure kg. per sq. cm. abs.	6.80	7.00	7.17	7.35	7.54	15.05	15.55	16.10	16.70	17.30	23.6	24.6	25.6	26.6	27.7	32.7	34.1	35.6	37.2	38.8
Explosion pressure lb. per sq. in. gage	82	85	88	90	93.5	197.5	200	214.5	223.5	231.5	321.5	335.5	349	364	369	450	470	492	515	538
Explosion temperature deg. cent. abs.	988	1012	1038	1064	1091	1089	1125	1162	1207	1247	1145	1188	1235	1283	1338	1177	1232	1285	1343	1407
Explosion temperature deg. fahr.	1319	1362	1409	1456	1504	1497	1566	1632	1713	1785	1602	1679	1764	1850	1947	1660	1758	1854	1958	2073
Temperature after adiabatic expansion deg. cent. abs.	641	653	665	678	693	575	592	609	627	648	539	558	577	597	620	512	532	552	574	600
Temperature after adiabatic expansion deg. fahr.	694	716	738	761	788	576	606	637	669	707	511	545	579	615	657	462	498	534	574	621
Excess air, per cent	1796	1622	1475	1342	1220	1235	1097	973	867	775	1032	908	800	706	622	935	815	708	615	533
Initial spouting velocity, ft. per sec.	2930	2980	3032	3085	3143	3535	3610	3690	3776	3862	3845	3942	4040	4143	4232	4068	4170	4280	4390	4490
	Brake thermal efficiency, per cent																			
Compressor efficiency per cent 55					2.3				0.7	8.46				2.6	10.5				4.2	12.6
60				1.9	8.3				7.2	14.40			0.5	9.1	16.5			1.9	10.6	18.5
65			0.6	7.3	13.3			5.50	12.8	19.50			6.5	14.6	21.5			7.9	16.1	23.3
70			5.8	12.0	17.6		3.14	10.7	17.5	23.80		3.2	11.7	19.2	25.7		4.36	13.2	20.7	27.5
75		3.4	10.2	16.1	21.4		7.96	15.1	21.5	27.50		8.1	16.3	23.3	29.5		9.42	17.7	24.9	31.2

Blank spaces under brake thermal efficiencies indicate negative values, i. e.;—the turbine work is not sufficient to operate the compressor.

TABLE 7 BRAKE THERMAL EFFICIENCIES, PER CENT, OF EXPLOSION GAS TURBINES WITH REGENERATOR

Data from Figs. 1048, 1049 and 1050 of Stodola's "Dampf- und Gas-Turbinen," 5th edition

Assumptions: Back pressure at gas turbine 1.2 kg. per sq. cm. abs. (17.06 lb. per sq. in. abs.); fuel, producer gas with low heating value 1100 kg. cal. per cu. m. (123 B.t.u. per cu. ft.) at 1 atm. pressure and 60 deg. fahr.; regenerative heating of compressed air to 550 deg. cent. (1022 deg. fahr.); heat loss to combustion chamber 10 per cent of heat of combustion; compression efficiencies on isothermal basis.

	Turbine efficiency, per cent																								
	55					60					65					70					75				
Compressor efficiency, per cent	50	60	65	70	75	55	60	65	70	75	55	60	65	70	75	55	60	65	70	75	55	60	65	70	75
Compression pressure, kg. per sq. cm. abs.	Casing temperature 600 deg. cent. (1112 deg. fahr.)																								
5	-19.0	-12.7	-6.0	0.0	4.2	-10.3	-3.6	2.2	7.5	11.0	-2.7	3.7	9.4	14.0	17.2	4.1	10.1	15.2	19.4	22.8	10.4	15.9	20.5	24.5	28.0
10	-14.5	-7.0	-1.0	4.9	9.1	-4.7	2.1	7.9	13.2	10.2	4.1	10.0	15.4	20.3	23.7	11.5	17.0	22.0	26.3	29.9	18.8	24.8	28.5	32.3	36.1
15	-13.7	-6.0	0.5	6.0	10.5	-3.6	3.4	9.0	14.4	18.2	5.5	11.6	17.0	21.9	25.2	13.5	19.0	23.7	28.2	31.7	20.8	26.0	30.5	34.2	37.5
20	-14.0	-6.3	0.5	6.0	10.5	-3.6	3.4	9.2	14.5	18.7	5.8	12.0	17.3	22.3	26.0	14.3	19.4	24.5	29.0	32.8	21.5	26.8	31.0	35.0	38.0
	Casing temperature 800 deg. cent. (1472 deg. fahr.)																								
5	0.4	3.7	6.7	9.2	11.3	5.0	8.2	11.3	13.5	15.5	9.5	12.4	15.2	17.6	19.5	13.5	16.3	19.1	21.0	23.0	17.2	20.0	22.7	24.7	26.2
10	2.9	6.9	10.3	13.3	15.5	9.7	13.4	15.8	18.7	21.0	14.5	17.8	20.7	23.3	25.6	19.5	22.6	25.4	27.9	30.0	24.3	27.3	29.8	32.3	34.0
15	3.2	7.9	11.5	14.5	17.3	10.0	14.0	17.3	20.5	23.0	16.0	19.5	22.6	25.3	27.8	21.5	25.0	27.8	30.2	32.6	27.2	30.0	32.7	35.0	37.0
20	3.0	7.9	11.5	14.8	17.3	10.0	14.4	17.3	20.5	23.0	16.4	20.5	23.2	25.8	27.8	22.5	26.1	28.5	31.4	33.7	28.5	31.3	34.2	36.5	38.7
	Casing temperature 1000 deg. cent. (1832 deg. fahr.)																								
5	7.0	9.1	11.1	12.6	14.2	10.5	12.5	14.5	16.0	17.5	14.0	16.0	17.8	19.2	20.5	17.2	19.2	20.7	22.2	23.8	20.4	22.3	23.7	25.1	26.2
10	10.5	13.1	15.1	17.3	18.7	15.0	17.3	19.5	21.5	22.8	19.3	21.6	23.7	25.4	26.7	23.5	25.4	27.4	29.0	30.5	27.3	29.3	31.0	32.7	33.8
15	11.4	14.2	16.4	18.5	20.5	16.3	19.0	21.2	23.4	25.0	21.0	23.5	25.7	27.6	29.2	25.4	27.8	29.8	31.7	33.0	29.7	32.0	33.8	35.4	36.7
20	11.0	14.2	16.5	19.0	20.8	16.3	19.5	21.6	24.0	25.6	21.5	24.2	26.4	28.5	30.2	26.3	28.8	30.9	32.9	34.5	30.8	33.7	35.1	37.0	38.3

values cannot be regarded as attainable unless there should be developed turbine-rotor materials capable of withstanding these high casing temperatures. The additional work required for cooling the rotor may be taken into account by assuming lower values of the compressor or turbine efficiency. With prolonged cooling by low-pressure air, alternating with the discharge of the hot explosion gases, the windage loss of the turbine will be much increased and the turbine efficiency correspondingly reduced. The higher the casing temperature (calculated on the assumption of no air cooling) the greater will be the negative cooling work.

C. Explosion Turbine with Regeneration and Water Injection

30 Another possible method of lowering the casing temperature is by injecting water into the combustion space. This case has

TABLE 8 CALCULATED PERFORMANCE OF EXPLOSION GAS TURBINES WITH REGENERATIVE HEATING OF THE COMPRESSED CHARGE TO 450 DEG. CENT. (842 DEG. FAHR.), WITH WATER INJECTION INTO THE COMBUSTION CHAMBER, AND WITH TURBINE-CASING TEMPERATURE 500 DEG. CENT. (932 DEG. FAHR.)

Assumptions: Compression pressure 15 kg. per sq. cm. abs. (213.4 lb. per sq. in. abs.); back pressure at gas turbine 1.2 kg. per sq. cm. abs. (17.06 lb. per sq. in. abs.); fuel, oil; excess air 100 per cent.

Turbine efficiency, per cent		55	60	65	70	75
Ratio of weight of water injected to weight of air and fuel		0.415	0.387	0.358	0.328	0.297
Explosion pressure	kg. per sq. cm. abs.	23.4	24.3	25.3	26.4	27.6
	lb. per sq. in., gage	318	331	345	358	378
Explosion temperature	deg. cent. abs.	1134	1174	1220	1270	1320
	deg. fahr.	1582	1654	1737	1827	1935
Temperature after adiabatic expansion	deg. cent. abs.	543	561	580	600	620
	deg. fahr.	518	550	585	621	657
Initial spouting velocity, ft. per sec.		4190	4270	4360	4460	4560
		Brake thermal efficiency, per cent				
Compressor efficiency per cent	55	7.7	11.3	14.8	18.5	22.3
	60	9.8	13.4	17.0	20.7	24.5
	65	11.8	15.3	19.0	22.6	26.5
	70	13.5	17.0	20.6	24.2	28.0
	75	15.0	18.6	22.3	28.0	29.7

been calculated for combustion with 100 per cent excess air, a casing temperature of 500 deg. cent. (932 deg. fahr.), regenerative heating of the compressed air to 450 deg. cent. (842 deg. fahr.), and a compression-pressure ratio of 15. For each assumed turbine efficiency there is a definite weight of water which must be injected to give the desired casing temperature; this weight is found by graphic methods and is given in Table 8. The explosion pressures and temperatures and the spouting velocity are seen to be moderate; but the possible brake thermal efficiencies are not very promising — not more than 20 per cent could probably be realized.

D — Explosion Turbine Combined with Steam Turbine Operated from Exhaust-Heat Boiler

31 Regenerative heating of the compressed air increases the explosion and casing temperatures for a given amount of fuel

burned, or, if the casing temperature is fixed, increases the amount of excess air necessary to keep down the temperatures. It consequently increases the negative work of the compressor and diminishes thereby the overall efficiency. A better method of utilizing the heat of the exhaust gases is to generate medium- or low-pressure steam with it and to use this steam in a steam turbine. With a high exhaust temperature, a combined efficiency of boiler and economizer of 80 per cent may be assumed as a possibility, and a steam-turbine efficiency of 20 per cent. The steam-plant efficiency is then 16 per cent, and, in most cases, the steam-turbine work will be more than is required for driving the compressor.

32 The gas turbines built by Holzwarth are of this kind, with the steam turbine driving the compressor and with all the gas-turbine work available. As the casing temperatures are high, air cooling is also employed. This case has been calculated by Stodola[1] for casing temperatures of 600, 800 and 1000 deg. cent. (1112, 1472 and 1852 deg. fahr.) and for a steam-plant efficiency of 16 per cent. Any excess of steam-turbine work over that required to drive the compressor is added to the gas-turbine work. The efficiencies attainable are given in Table 9 and are seen to be higher than the corresponding efficiencies in Table 7, especially for the lower compressor efficiencies. It should again be remembered that the additional negative work imposed by the air cooling is not here taken into account, unless lower turbine efficiencies are assumed for this purpose. When air cooling is employed the exhaust-gas temperature falls, so that the attainable steam-plant efficiencies would be lowered and the brake thermal efficiencies correspondingly reduced below the values of Table 9. The plant becomes very complicated with air compressor, gas turbine, boiler, steam turbine, condenser, feed pump, etc.

33 The conditions of Table 9 demand a large excess of air. A further analysis of this cycle is given in Table 10, in which the excess air is kept at 50 per cent, which would presumably insure an explosive mixture. The casing temperature will then vary both with the amount of reheating (turbine efficiency) and with the compression ratio. This case is investigated for no compression and for compression-pressure ratios of 2 and 3. The casing temperatures are very high, ranging from 1039 to 1325 deg. cent. (1902 to 2417 deg. fahr.) so that cooling, with its attendant losses, is necessary; no account of these losses is taken in Table 10.

34 Three different efficiencies are tabulated. Column 4 gives the brake thermal efficiency in the case where the compression work is done by the steam turbine and no excess work is available from that turbine. The last three columns show the steam-plant efficiencies that are necessary if the steam turbine develops exactly

[1] Loc. cit., Figs. 1051, 1052 and 1053.

TABLE 9 BRAKE THERMAL EFFICIENCIES, PER CENT, OF EXPLOSION GAS TURBINES IN WHICH THE HEAT OF THE EXHAUST GASES IS USED TO GENERATE STEAM AND DRIVE A STEAM TURBINE

Data from Figs. 1051, 1052 and 1053 of Stodola's "Dampf- und Gas-Turbinen" 5th edition

Assumptions: Back pressure at gas turbine 1.2 kg. per sq. cm. abs. (17.06 lb. per sq. in. abs.); fuel, producer gas with low heating value 1100 kg. cal. per cu. m. (123 B.t.u. per cu. ft.) at 1 atm. press. and 60 deg. fahr.; heat loss to combustion chamber 10 per cent of heat of combustion; efficiency of steam boiler 80 per cent, of steam turbine 20 per cent, of steam plant 16 per cent; compression efficiencies on isothermal basis.

Turbine efficiency, per cent	55					60					65					70					75				
Compressor efficiency, per cent	55	60	65	70	75	55	60	65	70	75	55	60	65	70	75	55	60	65	70	75	55	60	65	70	75
Compression pressure, kg. per sq. cm. abs.	Casing temperature 600 deg. cent. (1112 deg. fahr.)																								
2	15.2	16.3	17.5	18.4	18.8	17.0	18.0	19.0	19.8	20.4	18.5	19.5	20.5	21.3	22.4	20.2	21.2	22.2	22.8	23.2	21.7	22.7	23.5	24.4	24.8
4	14.7	16.4	17.9	19.1	20.3	17.2	19.0	20.5	21.5	22.9	19.7	21.5	22.8	24.1	25.1	22.3	24.0	25.3	26.5	27.5	24.7	26.2	27.0	28.5	29.7
6	12.8	15.0	16.5	18.2	19.5	15.7	17.8	19.5	21.0	22.4	18.8	20.8	22.5	23.9	25.2	22.0	24.0	25.4	26.7	28.0	25.1	27.0	28.5	29.7	31.0
8	11.6	13.8	16.0	17.8	19.2	15.1	17.3	19.2	21.0	22.5	18.8	21.0	22.8	24.3	25.8	22.2	24.2	26.0	27.5	28.7	25.5	27.6	29.3	30.5	32.0
10	11.3	14.0	16.0	18.0	19.6	15.0	17.5	19.8	21.5	23.0	19.0	21.3	23.5	25.0	26.5	22.8	25.0	27.0	28.5	29.9	26.4	28.5	30.3	31.8	33.2
	Casing temperature 800 deg. cent. (1472 deg. fahr.)																								
2	19.6	20.4	21.1	21.6	22.2	21.0	22.0	22.5	23.0	23.5	22.6	23.3	24.0	24.5	25.0	24.2	25.0	25.5	26.0	26.4	25.8	26.5	27.1	27.6	28.0
4	20.1	21.1	22.5	23.4	24.1	22.3	23.5	24.5	25.5	26.3	24.6	25.8	26.7	27.7	28.5	27.0	28.0	29.0	30.0	30.7	29.4	30.5	31.5	32.4	33.2
6	19.5	20.8	22.3	23.2	24.1	22.0	23.5	24.8	25.7	26.7	24.8	26.2	27.3	28.3	29.3	27.5	29.0	30.1	31.0	32.0	30.5	31.7	33.0	33.8	34.5
8	19.0	20.7	22.2	23.2	24.3	22.0	23.7	25.0	26.0	27.2	25.0	26.5	27.8	28.8	29.8	28.0	29.5	30.7	31.7	32.7	31.2	32.5	33.9	34.9	35.8
10	18.8	20.7	22.2	23.5	24.6	22.0	23.9	25.3	26.5	27.7	25.3	27.2	28.5	29.7	30.8	28.6	30.3	31.7	32.7	33.8	32.0	33.5	34.8	35.9	37.0
	Casing temperature 1000 deg. cent. (1832 deg. fahr.)																								
2	22.0	22.5	23.0	23.6	24.0	23.5	24.0	24.5	25.0	25.7	25.2	25.7	26.8	26.7	27.2	26.8	27.3	27.9	28.2	28.7	28.5	29.0	29.5	30.0	30.2
4	23.2	24.0	25.0	25.7	26.5	25.6	26.5	27.3	28.0	28.5	27.8	28.8	29.5	30.2	30.8	30.1	31.0	31.7	32.5	33.0	32.4	33.3	33.9	34.6	35.2
6	22.8	24.0	25.1	26.0	26.8	25.5	26.8	27.8	28.7	29.5	28.2	29.5	30.4	31.2	32.0	30.8	31.8	32.8	33.6	34.5	33.5	34.5	35.4	36.2	37.0
8	23.0	24.5	25.5	26.5	27.5	25.8	27.2	28.2	29.2	29.9	28.5	29.8	31.0	32.0	32.7	31.5	32.6	33.6	34.6	35.4	34.3	35.4	36.4	37.3	38.2
10	23.1	24.5	25.6	27.0	27.8	26.1	27.5	28.6	29.7	30.7	29.3	30.5	31.7	32.8	33.7	32.5	33.7	34.8	35.7	36.5	35.3	36.5	37.6	38.5	39.5

TABLE 10 CALCULATED PERFORMANCE OF EXPLOSION GAS TURBINES WITHOUT REGENERATION BUT WITH EXHAUST HEAT USED TO GENERATE STEAM AND DRIVE A STEAM TURBINE, WITH CHARGE PRECOMPRESSION AND EXHAUST TO ATMOSPHERE

Assumptions: Gas fuel; excess air 50 per cent; back pressure at turbine 3 in. of water above exhaust pressure; temperature of charge entering combustion chamber 288 deg. cent. abs. (59 deg. fahr); heat lost to walls of combustion chamber = 10 per cent of heat of combustion

Turbine efficiency, per cent	Casing temperature, deg. cent. abs.	Casing temperature, deg. fahr.	Brake thermal efficiency when steam turbine work = compressor work, per cent	Brake thermal efficiency, per cent, including steam-turbine work (steam-plant efficiency 15 per cent) — Compressor efficiency, per cent: 55	60	65	70	75	Steam-plant efficiency when steam-turbine work = compressor work, per cent — Compressor efficiency, per cent: 55	65	75
				Ratio of compression = 3							
50	1448	2147	21.9	23.2	23.9	24.6	25.1	25.6	13.2	11.2	9.6
55	1414	2086	24.1	24.9	25.6	24.3	24.8	25.3	13.6	11.5	10.0
60	1380	2024	26.4	26.9	27.7	28.2	28.8	29.4	14.1	11.9	10.2
65	1346	1964	28.6	28.8	29.6	30.2	30.7	31.1	14.6	12.4	10.7
70	1312	1902	30.8	30.7	31.5	32.1	32.6	33.2	15.1	12.8	11.1
				Ratio of compression = 2							
50	1501	2242	18.6	23.6	24.1	24.5	24.8	25.1	7.9	6.7	5.8
55	1472	2190	20.4	25.3	25.8	26.2	26.5	26.8	8.1	6.9	5.9
60	1443	2138	22.3	26.9	27.3	27.7	28.1	28.4	8.3	7.0	6.1
65	1414	2086	24.1	28.4	28.9	29.3	29.6	29.9	8.6	7.3	6.3
70	1385	2034	26.0	30.0	30.4	30.8	31.2	31.5	8.8	7.5	6.5
				No compression							
50	1598	2417	11.97			23.7					
55	1580	2384	13.17			24.7					
60	1562	2352	14.35			25.7					
65	1544	2320	15.55			26.7					
70	1526	2288	16.75			27.8					

the power required by the compressor; as this ranges from 5.8 to 15.1 per cent it may be regarded as a possibility. Cols. 5 to 9 give the brake thermal efficiency of combined gas and steam turbines when the steam plant has an efficiency of 15 per cent. It may be noted that, under these conditions of operation, precompression of the charge is not particularly valuable; for example, if compressor efficiency is 70 per cent and turbine efficiency is 60 per cent, the brake thermal efficiency with a compression-pressure ratio of 3 is 28.8 per cent as compared with 25.7 per cent when no compression is used.

35 Supplementary data for this case are given in Table 10*a*, which shows that the spouting velocities are high and that, in this respect, the condition with no precompression of the charge

TABLE 10*a* SUPPLEMENTARY DATA FOR TABLE 10

Ratio of compression		1	2	3
Explosion pressure	kg. per sq. cm. abs.	7.47	14.94	22.41
	lb. per sq. in.	92	198	317
Explosion temperature	deg. cent. abs.	2151	2151	2151
	deg. fahr.	3412	3412	3412
Temperature after expansion	deg. cent. abs.	1418	1212	1103
	deg. fahr.	2093	1722	1526
Spouting velocity, ft. per sec.		4570	5130	5400

offers least difficulty. On the other hand, it is the condition with highest casing temperature and consequently of maximum cooling work losses.

E — Explosion Turbine Combined with Steam Turbine Operated from Exhaust-Heat Boiler with Charge Precompression and Reduced Back Pressure

36 By the addition of an exhauster to a gas turbine, the density of the medium in which the turbine rotates can be reduced and the windage losses consequently diminished. Spouting velocities will increase if the other conditions are unchanged, and this may necessitate a speeding up of the rotor and an increase in windage losses from this cause. The additional work of increased expansion of the explosion gases is offset by the work required to be done on the exhauster. Since the exhauster acts on the exhaust gases after they have been cooled (desirably to atmospheric temperature), the negative work of the exhauster may be less than the increase in work done on the turbine, and an actual increase in brake thermal efficiency may be possible. Table 11, in which the performance of a turbine plant of this type is given, shows but little change in brake thermal efficiency as compared with Table 10, which is the generally similar case without exhauster; any advantage is to be looked for principally in reduction of windage loss. The complexity of the plant is even greater than in the preceding case in consequence of the addition of the exhauster. With fuels containing sulphur there will be diluted sulphuric acid in the cooled exhaust gases, and corrosion of the economizer and

TABLE 11 CALCULATED PERFORMANCE OF EXPLOSION GAS TURBINES WITH CHARGE PRECOMPRESSION AND WITH AN EXHAUSTER FOR REDUCING THE BACK PRESSURE AT THE GAS TURBINE. THE HEAT OF THE EXHAUST GASES IS UTILIZED IN GENERATING STEAM TO DRIVE A STEAM TURBINE

Assumptions: Oil fuel; excess air 50 per cent; temperature of charge entering combustion chamber 288 deg. cent. abs. (59 deg. fahr.); back pressure at gas turbine 0.2 kg. per sq. cm. (2.84 lb. per sq. in.) greater than exhauster pressure; ratio of compression = ratio of exhaustion; temperature of water fed to boiler 15 deg. cent. (59 deg. fahr.); heat lost to walls of combustion chamber = 10 per cent of heat of combustion.

Turbine efficiency, per cent	Casing temperature		Brake thermal efficiency when steam-turbine work = compressor work, per cent	Brake thermal efficiency, per cent, including steam-turbine work (steam-plant efficiency, 15 per cent)					Steam plant efficiency if steam-turbine work = compressor + exhauster work, per cent		
				Compressor efficiency, per cent					Compressor efficiency, per cent		
	deg. cent. abs.	deg. fahr.		55	60	65	70	75	55	65	75
Ratio of compression = ratio of exhaustion = 1.5											
50	1496	2234	17.4	22.2	22.7	23.1	23.5	23.8	8.4	7.1	6.2
55	1472	2190	19.1	23.7	24.2	24.7	25.0	25.3	8.6	7.3	6.3
60	1447	2145	20.9	25.2	25.7	26.2	26.5	26.8	8.8	7.5	6.5
65	1422	2100	22.6	26.6	27.1	27.5	27.9	28.2	9.0	7.6	6.6
70	1395	2052	24.3	28.1	28.6	29.0	29.4	29.7	9.3	7.9	6.8
Ratio of compression = ratio of exhaustion = 2											
50	1435	2124	21.6	21.5	22.4	23.1	23.7	24.3	15.0	12.9	11.2
55	1402	2064	23.7	23.3	24.2	24.9	25.5	26.1	15.6	13.4	11.5
60	1370	2006	25.9	25.1	26.0	26.7	27.4	27.9	16.3	13.8	11.9
65	1335	1944	28.1	27.0	27.9	28.6	29.3	29.8	16.8	14.2	12.4
70	1305	1890	30.3	28.9	29.7	30.5	31.1	31.5	17.4	14.8	12.8
Ratio of compression = ratio of exhaustion = 3											
50	1360	1988	26.6	19.6	21.0	22.1	23.1	24.0	26.0	22.0	19.1
55	1317	1912	29.3	21.9	23.2	24.4	25.4	26.3	27.2	23.0	20.0
60	1275	1832	31.9	24.1	25.5	26.7	27.7	28.5	28.4	24.1	20.8
65	1235	1764	34.6	26.4	27.8	28.9	29.8	30.8	29.8	25.2	21.8
70	1195	1692	37.3	28.6	30.0	31.2	32.2	33.0	31.3	26.4	22.9
Ratio of compression = ratio of exhaustion = 1. (No compression or exhaustion)											
50	1600	2421	10.4			22.4					
55	1585	2396	11.5			23.3					
60	1570	2369	12.5			24.1					
65	1555	2342	13.5			25.0					
70	1540	2315	14.5			25.8					

the exhauster would have to be guarded against. From the last three columns of Table 11 it is apparent that the steam turbine would not be able to drive both the compressor and the exhauster for ratios of compression (= ratio of exhaustion) greater than 2.

37 Table 11*a* contains supplementary data for the conditions of Table 11.

TABLE 11*a* SUPPLEMENTARY DATA FOR TABLE 11

Ratio of compression = ratio of exhaustion		1	1.5	2	3
Explosion pressure	kg. per sq. cm. abs.	7.35	11.05	14.70	22.04
	lb. per sq. in.	90	143	194	299
Explosion temperature	deg. cent. abs.	2117	2117	2117	2117
	deg. fahr.	3352	3352	3352	3352
Temperature after expansion	deg. cent. abs.	1447	1233	1100	938
	deg. fahr.	2150	1760	1520	1229
Spouting velocity, ft. per sec.		4480	5110	5450	5810

38 Further calculations for this cycle are given in Table 12, with the following differences in conditions from Table 11: Gas fuel instead of oil fuel; back pressure at turbine 3 in. of water above exhauster pressure instead of 2.84 lb. per sq. in.; calculation made only for ratio of compression pressure = ratio of exhauster = 1.5. The resulting brake thermal efficiencies are improved by the reduction in back pressure; the windage losses would also be reduced.

F — Constant-Pressure-Combustion Turbine with Regeneration

39 The general advantages of the constant-pressure-combustion have been touched on in Par. 7. If no cooling device is used the maximum permissible casing temperature may be taken as 500 deg. cent. (932 deg. fahr.) and the corresponding regenerative heating to 450 deg. cent. (842 deg. fahr.). These are the same as the conditions assumed for the explosion cycle of Table 6. The calculated performance of this cycle is given in Table 13 for the same compression ratios as in Table 6; the efficiencies obtained are not very different in the two cases and are higher for the explosion cycle except at low compression-pressure ratios. The casing temperature limits the maximum combustion temperature and thereby limits the amount of fuel that may be burned. The excess air consequently is very high, ranging from 485 to 1580 per cent. This results in a high ratio of the negative work of compression to the positive work of the turbine and thereby results in a low brake thermal efficiency.

G — Constant-Pressure-Combustion Turbine with Regeneration and with Cooling of the Buckets by Steam Jets

40 The constant-pressure-combustion turbine without cooling (Table 13) is necessarily of low efficiency. Air cooling entails con-

TABLE 12 CALCULATED PERFORMANCE OF EXPLOSION GAS TURBINES WITHOUT REGENERATION BUT WITH EXHAUST HEAT USED TO GENERATE STEAM AND DRIVE A STEAM TURBINE, AND WITH EXHAUSTER TO REDUCE BACK PRESSURE ON GAS TURBINE

Assumptions: Ratio of compression = ratio of exhaustion = 1.5; temperature of charge entering combustion chamber 288 deg. cent. abs. (59 deg. fahr.); fuel, gas; back pressure at turbine 3 in. of water above exhauster pressure; excess air 50 per cent; heat loss to walls of combustion chamber = 10 per cent of heat of combustion; temperature of water fed to boiler, 15 deg. cent. (59 deg. fahr.)

Turbine efficiency, per cent	Casing temperature, deg. cent. abs.	Casing temperature, deg. fahr.	Brake thermal efficiency when steam-turbine work = compressor + exhauster work, per cent	Brake thermal efficiency, per cent, including excess steam-turbine work (steam-plant efficiency, 15 per cent): Compression efficiency, per cent 55	60	65	70	75	Steam-plant efficiency when steam-turbine work = compressor + exhauster work, per cent: Compression efficiency, per cent 55	65	75
50	1487	2218	19.3	23.7	24.2	24.6	25.0	25.3	9.02	7.64	6.62
55	1457	2164	21.3	25.3	25.8	26.3	26.6	27.0	9.28	7.86	6.80
60	1427	2110	23.3	26.9	27.4	27.9	28.3	28.6	9.56	8.09	7.00
65	1397	2056	25.2	28.6	29.1	29.5	29.9	30.2	9.84	8.34	7.22
70	1367	2002	27.2	30.2	30.7	31.2	31.6	31.9	10.15	8.60	7.44

TABLE 13 CALCULATED PERFORMANCE OF CONSTANT-PRESSURE-COMBUSTION GAS TURBINES WITH REGENERATIVE HEATING OF THE COMPRESSED CHARGE TO 450 DEG. CENT. (842 DEG. FAHR.) AND TURBINE-CASING TEMPERATURE 500 DEG. CENT. (932 DEG. FAHR.)

Assumptions: Back pressure at gas turbine 1.2 kg. per sq. cm. abs. (17.06 lb. per sq. in.); gas fuel; heat loss to walls of combustion chamber 10 per cent of heat of combustion; compression efficiencies on isothermal basis

Compression pressure, kg. per sq. cm. abs. (lb. per sq. in.)	5 (71.12)					10 (142.24)				
Turbine efficiency, per cent	60	65	70	75	80	60	65	70	75	80
Maximum combustion temperature, deg. cent. abs.	944	962	982	1002	1022	1023	1054	1085	1117	1151
Maximum combustion temperature, deg. fahr.	1240	1272	1308	1344	1380	1382	1438	1494	1551	1612
Temperature after adiabatic expansion, deg. cent. abs.	651	664	678	692	706	593	612	631	651	672
Temperature after adiabatic expansion, deg. fahr.	713	736	761	786	811	608	642	676	712	750
Excess air, per cent	1580	1442	1325	1222	1127	1113	994	890	799	717
Spouting velocity, ft. per sec.	2662	2702	2739	2770	2800	3237	3290	3340	3387	3438
Brake thermal efficiency, per cent — Compressor efficiency, per cent 55				3.1	9.2				6.1	12.5
60			2.8	9.0	14.7			5.5	12.2	18.1
65		1.7	8.2	14.1	19.4		4.0	11.1	17.4	22.9
70	.2	6.8	12.9	18.5	23.5	1.5	9.3	16.0	21.8	26.9
75	4.8	11.1	16.9	22.2	27.0	6.4	13.7	20.1	25.6	30.4

Compression pressure, kg. per sq. cm. abs. (lb. per sq. in.)	15 (213.36)					20 (284.48)				
Turbine efficiency, per cent	60	65	70	75	80	60	65	70	75	80
Maximum combustion temperature, deg. cent. abs.	1073	1106	1142	1181	1223	1106	1146	1190	1236	1283
Maximum combustion temperature, deg. fahr.	1472	1531	1596	1666	1742	1531	1603	1683	1765	1850
Temperature after adiabatic expansion, deg. cent. abs.	561	580	601	624	649	536	559	583	608	634
Temperature after adiabatic expansion, deg. fahr.	550	585	622	664	709	505	547	590	635	682
Excess air, per cent	940	830	732	646	572	830	724	634	556	485
Spouting velocity, ft. per sec.	3535	3595	3653	3715	3780	3726	3790	3870	3950	4032
Brake thermal efficiency, per cent — Compressor efficiency, per cent 55			.4	8.0	15.3			1.4	9.92	18.0
60			7.1	14.2	20.8			8.0	15.9	23.5
65		5.5	12.8	19.3	25.5		5.5	13.6	20.9	28.1
70	2.6	10.7	17.5	23.7	29.5	2.4	10.9	18.4	25.3	32.0
75	7.7	15.4	21.7	27.4	33.0	7.5	15.5	22.6	29.2	35.6

siderable losses. Another possibility in cooling is to generate steam by the exhaust gases and to expand this steam through nozzles on to the gas turbine, which thus serves both as a gas and a steam turbine. The steam will be used inefficiently in this case as it will be operating as a non-condensing turbine, but, as it discharges as wet steam at 212 deg. fahr., it will serve as an excellent cooling medium and will reduce the casing temperature.

41 Two cases of this cycle have been analyzed. In both of them oil fuel is used with 100 per cent excess air, a bucket peripheral velocity of 800 ft. per sec. is assumed, the back pressure at the turbine is taken as 1.2 kg. per sq. cm. abs. (17.06 lb. per sq. in. abs.) and the exhaust gases are supposed to be cooled to 100 deg. cent. (180 deg. fahr.) above the steam temperature in the exhaust-heat boiler and then to go through a regenerator. It is also assumed in both cases that the 10 per cent of the heat of combustion which goes to the combustion-chamber walls is used in evaporating water in the jackets around it. The compression pressure is 7.10 kg. per sq. cm. abs. (101 lb. per sq. in. abs.). The steam-nozzle velocity coefficient is taken as 0.95. The combustion temperature is 1601 deg. cent. abs. (2370 deg. fahr.).

42 I — In the first case it is assumed that steam is generated at such pressure that it has the same spouting velocity as the gases (3600 ft. per sec.) so that it can be efficiently utilized by the turbine. The gas and steam turbines are assumed to have 70 per cent efficiency, the compressor 60 per cent.

43 Computations on this basis yield the following results:

Weight of steam generated per lb. of exhaust gas, lb.		0.214
Pressure of steam,	kg. per sq. cm. abs.	75
	lb. per sq. in. abs.	1070
Gas temperature after adiabatic expansion,	deg. cent.	783
	deg. fahr.	1441
Casing temperature of gas alone,	deg. cent.	952
	deg. fahr.	1745
Quality of steam after adiabatic expansion		0.751
Quality of steam after reheating in turbine		0.845
Casing temperature with combined gas and steam,	deg. cent.	687
	deg. fahr.	1269
Brake thermal efficiency, per cent		20.5

44 II — The steam pressure necessary to give the required spouting velocity in case I is very high (1070 lb. per sq. in. abs.). It seemed desirable to make computations also for the case in which the steam acts only on the second row of buckets and consequently has a spouting velocity of 1800 ft. per sec. It was assumed for this case that the steam-turbine efficiency was 70 per cent, gas-turbine efficiency 75 per cent, compressor efficiency 65 per cent. The computed results follow.

Weight of steam generated per lb. of exhaust gas, lb.		0.314
Pressure of steam,	kg. per sq. cm. abs.	3.2
	lb. per sq. in. abs.	45.5
Gas temperature after adiabatic expansion,	deg. cent.	783
	deg. fahr.	1441
Gas temperature of gas alone,	deg. cent.	925
	deg. fahr.	1697
Quality of steam after adiabatic expansion		0.946
Quality of steam after reheating in turbine		0.97
Casing temperature with combined gas and steam,	deg. cent.	640
	deg. fahr.	1184
Brake thermal efficiency, per cent		19.9

45 It will be seen that the cases calculated above yield casing temperatures of 687 and 640 deg. cent., respectively, which are too high to be practicable. With larger excess air this condition could be remedied but at the cost of reduced brake thermal efficiencies. These efficiencies are already quite low. This case does not appear to offer appreciably better prospects than case F (Par. 39) and it adds the complication of a waste-heat boiler.

H—Constant-Pressure-Combustion Turbine with Regeneration and with Steam Injection into the Combustion Space

46 Another method of using the steam generated in an exhaust-gas boiler is to discharge it directly into the combustion space. With this process the pressure at which the steam is generated does not have to be so high as in case G-I (Par. 42), where the steam acts independently on the turbine buckets. It is assumed to be generated at 10 per cent above the compression pressure. As no additional cooling is used, the casing temperature is limited to 500 deg. cent. (932 deg. fahr.). The exhaust gases are assumed cooled to 300 deg. cent. (572 deg. fahr.) in the boiler, the compressed air is heated by the regenerator to 200 deg. cent. (392 deg.

TABLE 14 CALCULATED PERFORMANCE OF CONSTANT-PRESSURE-COMBUSTION TURBINE WITH STEAM INJECTION INTO THE COMBUSTION SPACE, WITH REGENERATION TO 200 DEG. CENT. (392 DEG. FAHR.) AND WITH TURBINE-CASING TEMPERATURE OF 500 DEG. CENT. (932 DEG. FAHR.)

Assumptions: Oil fuel; excess air, 200 per cent; steam pressure, 10 per cent greater than compression pressure; back pressure, 1.2 kg. per sq. cm. abs. (17.06 lb. per sq. in. abs.); steam generated by cooling exhaust gases to 300 deg. cent. (572 deg. fahr.) and by 10 per cent of heat of combustion going through combustion-chamber walls; feedwater heated to 50 deg. cent. (122 deg. fahr.) by exhaust gases.

Compression-pressure ratio		40	17.6	10
Combustion temperature after steam admixture	deg. cent. abs.	1128	1120	1116
	deg. fahr.	1571	1557	1550
Temperature after adiabatic expansion	deg. cent. abs.	460	569	661
	deg. fahr.	369	565	730
Turbine efficiency ratio, per cent.		54.9	64.2	76.1
Spouting velocity, ft. per sec.		4210	3840	3500
		Brake thermal efficiency, per cent		
Compressor efficiency, per cent	55		5.8	13.8
	60		9.5	16.8
	65	3.3	12.6	19.2
	70	6.7	15.2	21.4
	75	9.7	17.5	23.2

fahr.), and the feedwater is heated, by the exhaust gases, to 50 deg. cent. (122 deg. fahr.). In addition to the heat from the exhaust gases, the 10 per cent of the heat of combustion which goes to the jacketed walls of the combustion chamber is assumed to be used in generating steam. The back pressure at the turbine is 1.2 kg. per sq cm. abs. (17.06 lb. per sq. in. abs.).

47 This case was calculated for an assumed excess of air. This gives a definite combustion temperature and also, for each compression-pressure ratio, a definite temperature at the end of adiabatic expansion. With a casing temperature limited to 500 deg. cent. this fixes the amount of reheating in the turbine and therefore the turbine efficiency. In Table 14 the turbine efficiencies are those which must be reached if the casing temperature is not to exceed the stated value.

48 When calculated for 100 per cent excess air, the turbine efficiencies must be over 85 per cent if the casing temperature is to be kept down to 500 deg. cent. (932 deg. fahr.), even for very high compression pressures.

49 With 200 per cent excess air, the ratio of weight of steam to weight of products of combustion is 0.144 and the temperature of combustion before the injection of the steam is 1273 deg. cent. abs. (1832 deg. fahr.). The calculated efficiencies and other data are given in Table 14.

50 An examination of this table shows that with turbine efficiencies of 54.9, 64.2, and 76.1 per cent, the necessary compression-pressure ratios, if the casing temperature is to be kept down to 500 deg. cent., are 40, 17.6, and 10, respectively. With turbine efficiency of 76.1 per cent and compressor efficiency of 75 per cent (both values beyond existing possibilities) the brake thermal efficiency is only 23.2 per cent.

Conclusions

51 The advantages which the steam turbine has over the reciprocating engine are chiefly high rotative speed, absence of cylinder oil, large power per unit, low weight, low attendance cost, compactness, simplicity, small weight, and higher efficiency. The gas turbine also has the first three items as probable advantages but the other items cannot be definitely claimed, at any rate in so great a degree as in the steam turbine. A simple gas turbine without use of the exhaust heat would have too low an efficiency to be practical. If a regenerator is added, but no provision for cooling is made (cases A and F, Pars. 27 and 39), the operating conditions have to be such that the efficiencies obtainable are very low. If cooling is provided the weight and complication of the plant increase. With the explosion turbine (which appears to be the most practical in view of the lower compression pressures required and the possibilities of cooling) the use of valves in the

combustion chamber introduces a type of complication from which the steam turbine is free and which may be expected to be the source of much trouble.

52 A review of the possibilities of the gas turbine does not give much hope for the realization of efficiencies such as would encourage attempts to overcome the many difficulties with which this machine is surrounded. Even with the increase in compressor and turbine efficiencies which may be expected to result from further developments it seems highly improbable that brake thermal efficiencies as high as 25 per cent could be obtained. The maximum that has been claimed up to the present time is about one-half this quantity, or 13 per cent. This figure should be compared with brake thermal efficiencies of about 34 per cent obtained with Diesel engines, and about 38 per cent with the Still engine.

53 The exhaust-gas turbine, as used for supercharging aircraft engines, is not considered here. It is the only commercially successful gas turbine at the present time, but it is merely an attachment to an internal-combustion engine permitting the utilization of the work of expansion down to the exhaust pressure. Tables 1 to 4, inclusive, show the improvement in cycle efficiency theoretically possible by the addition of an exhaust-gas turbine.

APPENDIX NO. 1

The Entropy Chart

54 The computations involved in the paper were carried out with the aid of Stodola's temperature-entropy diagram (loc. cit.). The chart, of which Fig. 3 is a partial outline, is drawn for one kg-mol of gas, i.e., for a weight of gas in kg. equal to its molecular weight. The equation of state for one kg-mol of gas is

$$pv = RT$$

where p is the pressure in kg. per sq. cm., v the volume in cu. m. per kg-mol, R a constant equal to 848 for all gases, and T the absolute temperature centigrade. The horizontal lines running across the diagram are isothermals and are labeled in deg. cent. abs., the solid lines p, p_0, p_1, p_{11} are isobaric, corresponding to different values of pressure expressed in kg. per sq. cm. The dotted lines v_1, v_{11}, v_{111}, v_{1111} are lines of constant volume expressed in cu. m. per kg-mol.

55 A linear variation of specific heats with temperatures was assumed in most cases in the paper, so that:

$$C_p = a_p + bT \text{ kg-cal. per kg-mol}$$
$$C_v = a_v + bT \text{ kg-cal. per kg-mol}$$

where

$$a_p = 6.660 \text{ kg-cal. per kg-mol}$$
$$a_v = 4.670 \text{ kg-cal. per kg-mol}$$

and

$$C_p - C_v = a_p - a_v = AR = 1.985 \text{ kg-cal. per kg-mol}$$

where $A = \frac{1}{427}$ is the thermal equivalent of mechanical energy. The coefficient b was assigned the following values:

Substance	Values of b
Diatomic gases (H_2, O_2, N_2, CO)	1.00×10^{-3}.
Water vapor H_2O	2.90×10^{-3}.
Carbon dioxide CO_2	4.40×10^{-3}.

56 The expressions for the total heat H and the intrinsic energy U are as follows:

$$H = \int_0^T C_p dT = a_p T + \frac{b}{2} T^2 + \frac{c}{3} T^3$$

$$U = \int_0^T C_v dT = a_v T + \frac{b}{2} T^2 + \frac{c}{3} T^3$$

57 The entropy at the point 0 ($T_0 = 300$ deg. cent. abs. $p_0 = 1$ kg. per sq. cm., $v_0 = 25.44$ cu. m. per kg-mol) is taken as zero. An adiabatic is a straight line parallel to the line connecting the point O with the proper value of b on the scale B. The adiabatic drawn through point O is a line of zero entropy and is the axis from which entropies are

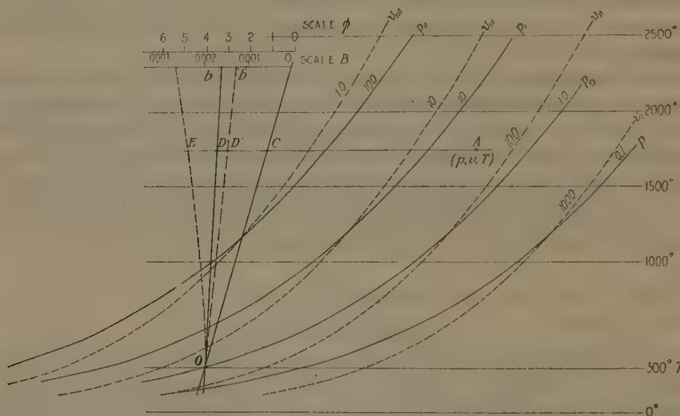

Fig. 3 Temperature-Entropy Diagram

measured, so that the entropy of any point A defined by a system of coördinates p, T, and v is represented on the diagram by the distance AD (Fig. 3) measured on the scale of entropies ϕ. Analytically the value of the entropy ϕ is given by:

$$\phi = a_v \log_e \frac{T}{T_0} + b(T - T_0) + AR \log_e\left(\frac{v}{v_0}\right)$$

also by:

$$\phi = a_p \log_e \frac{T}{T_0} + b(T - T_0) - AR \log_e (p/p_0)$$

In both equations the sum of the first and third terms is represented by the distance AC in Fig. 3, and the second term by CD.

58 The accuracy obtained with the assumption of a linear variation of specific heats is in general sufficient for all practical purposes; in cases where a more accurate theory is desired, the expressions for the specific heats of CO_2 and H_2O were taken as follows:

$$C_p = a_p T + b'T + cT^2 \text{ kg-cal. per kg-mol}$$
$$C_v = a_v T + b'T + cT^2 \text{ kg-cal. per kg-mol}$$

a_v and a_p retaining their former values, while the remaining coefficients were taken as shown below:

Substances	Coefficients	
	b'	c
Carbon dioxide, CO_2	8.004×10^{-3}	-2.337×10^{-6}
Water vapor, H_2O	1.263×10^{-3}	1.180×10^{-6}

59 The entropy of any point A is now represented by the distance AE (Fig. 3) and is given by the expressions

$$\phi = a_v \log_e \frac{T}{T_0} + b'(T-T_0) + \frac{c}{2}(T-T_0^2) + AR \log_e (v/v_0);$$

and

$$\phi = a_p \log_e \frac{T}{T_0} + b'(T-T_0) + \frac{c}{2}(T^2-T_0^2) - AR \log_e (p/p_0)$$

In both equations the sum of the first and last terms is represented by AC in Fig. 3, the second term by CD' and the third by $D'E$. Also

$$H = \int_0^T C_p dT = a_p T + \frac{b'}{2} T^2 + \frac{c}{3} T^3 \text{ kg-cal per kg-mol}$$

$$U = \int_0^T C_v dT = a_v T + \frac{b'}{2} T^2 + \frac{c}{3} T^3 \text{ kg-cal per kg-mol}$$

Adiabatics are now parabolas, OE, obtained by the addition of the abscissas $ED' = \frac{c}{2}(T^2-T_0^2)$ to the value of the entropy AD'. A cardboard templet was found both convenient and sufficiently accurate for drawing such adiabatics.

60 If $m_1, m_2 \ldots m_k$ be the molecular weights of the constituents of a gaseous mixture and $w_1, w_2 \ldots w_k$ their total weights, the number of mols $n_1, n_2 \ldots n_k$ of each gas present is: $n_1 = w_1/m_1$, $n_2 = w_2/m_2 \ldots n_k = w_k/m_k$. The molecular weight m of the mixture is:

$$m = \frac{\Sigma mn}{\Sigma n} = \frac{\Sigma w}{\Sigma n}$$

The coefficients b and c in the expression for the specific heats are given by

$$b = \frac{\Sigma bn}{\Sigma n}$$

$$c = \frac{\Sigma cn}{\Sigma n}.$$

In the mixture. a_p and a_v retain the constant values which they have for the constituents.

APPENDIX NO. 2

The Cycle

61 The fresh charge is compressed along 12 (Fig. 4) in case of adiabatic compression (where 12 is drawn parallel to OA connecting the

point of zero entropy with the value of b for the fresh charge on scale B), or along $12'$ in the case of isothermal compression.

62 In cases with regeneration the charge is heated at constant pressure from 2 or $2'$ up to 3 at which point the combustion starts. If no regeneration is provided, point 3 coincides with either point 2 or $2'$ according to whether the compression was carried out adiabatically or isothermally.

63 The combustion is thermodynamically equivalent to heat addition from an external source, along 35 in the constant-pressure turbine, or $35'$ in the constant-volume turbine. The gases then expand adiabatically along 56 (or $5'6$) to the back pressure 6. The expansion adiabatic is drawn parallel to OC, where C is the value of b for the products of combustion under the assumption of a linear variation of specific heats. In cases when the parabolic law of variation of specific heats is used, the expansion adiabatic is a parabola $5_1'6_1$ or 5_16_1 whose horizontal distance from the parabolic axis OD is constant. The expansion adiabatics cut the line of original volume at the points 8 and 8_1, respectively. After reheating at constant pressure by the turbine losses, the gases reach the condition shown at $6'$.

64 The condition of the gases after leaving the regenerator or steam boiler is represented by 10.

General Equations

65 In addition to the symbols already explained in the preceding paragraphs, the following will be used:

- E_t Turbine efficiency ratio
- E_c Compressor efficiency (isothermal basis)
- W_{tp} Theoretical turbine work with constant-pressure combustion, kg-cal. per kg-mol.
- W_{tv} Theoretical turbine work with constant-volume combustion, kg-cal. per kg-mol.
- W_c Work of precompression of the charge, kg-cal. per kg-mol.
- W Net turbine work, kg-cal.
- Q Heat of combustion of the fuel
- $E = W/Q$ Brake thermal efficiency
- m Molecular weight of fresh charge
- m_1 Molecular weight of products of combustion
- m_2 Molecular weight of water
- n Number of mols before combustion
- n_1 Number of mols after combustion
- n_a Number of mols of excess air
- n_2 Number of mols of water
- w Weight of working substance, kg.
- w_2 Weight of water injected, kg.
- i_0 Total heat of water entering the cycle, kg-cal. per kg.
- i' Heat of liquid water at saturation temperature, kg-cal. per kg.
- l Internal latent heat of water, kg-cal. per cal.
- h_1 Total heat of steam at nozzle bowl, kg-cal. per kg.
- h_s Total heat of steam at saturation, kg-cal. per kg.
- ϕ Entropy per kg-mol of gas
- λ Ratio of the weight of water used in the cycle to the weight of the products of combustion
- V_0 Spouting velocity from a frictionless nozzle (in constant volume turbines, the initial spouting velocity)
- x Fraction of the heat supplied lost through the walls of the combustion chamber.

66 All computations are based on one kg-mol of gaseous fuel or on one kg. of oil fuel.

67 The effect of combustion is to change the number of molecules

$$w = mn = m_1 n_1$$

For a gas of the composition considered,

$$m_1 = 28.84 + 2.66/n_1$$

For all turbines cycles, the available total-heat drop is

$$\Delta H = H_5 - H_6$$

and

$$V_0 = 91.54\sqrt{\Delta H/m_1}, \text{ m. per sec.}$$

For isothermal compression,

$$W_C = ART_1 \log_e (p_2/p_1)$$

For adiabatic compression,

$$W_C = H_2 - H_1$$

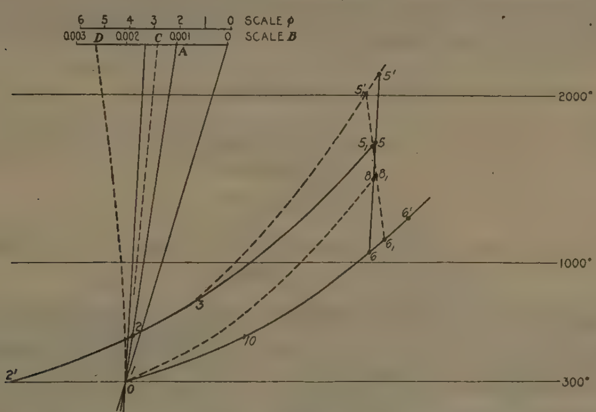

FIG. 4 GAS-TURBINE CYCLES REPRESENTED ON TEMPERATURE-ENTROPY DIAGRAM

68 In a constant-pressure cycle:

$$W = \left(n_1 W_{tp} E_t - n \frac{W_C}{E_C}\right) \quad . \quad . \quad . \quad . \quad . \quad [1a]$$

$$W_{tp} = \Delta H \quad . \quad . \quad . \quad . \quad . \quad [2a]$$

$$H'_6 = H_6 + (1 - E_t) W_{tp} \quad . \quad . \quad . \quad . \quad . \quad [3a]$$

$$H_5 = H_3 + (1 - x)\frac{Q}{n_1} \quad . \quad . \quad . \quad . \quad . \quad [4a]$$

69 The corresponding expressions for a constant-volume cycle are:

$$W = \left(n_1 W_{tv} E_t - n \frac{W_C}{E_C}\right) \quad . \quad . \quad . \quad . \quad . \quad [1b]$$

$$W_{tv} = H_5' - H_6 - Av_5'(p_5' - p_2) \quad . \quad . \quad . \quad . \quad . \quad [2b]$$

$$U_5' = U_3 + (1 - x)\frac{Q}{n_1} \quad . \quad . \quad . \quad . \quad . \quad [3b]$$

$$H_6' = H_6 + (1 - E_t) W_{tv} \quad . \quad . \quad . \quad . \quad . \quad [4b]$$

70 Temperatures at different points of the cycle were determined from the corresponding values of total heats or intrinsic energies.

71 In cases where the casing temperature, T_6', was assigned a definite value, the temperature at the beginning of the adiabatic expansion, T_5 or T_5', corresponding to any given precompression pressure, p_2, and turbine efficiency ratio, E_t, was determined by a series of systematized trials. The temperature T_3 being given, n_1 can be evaluated from Equation [3a] (or [3b]). The corresponding amount of excess air n_a was determined as follows:

72 One kg-mol (26.7 kg.) of gas requires 0.796 kg-mols of air for complete combustion, yielding 1.630 kg-mols of burned gases. The excess air, n_a, is given by

$$n_a = n_1 - 1.630$$

and the per cent excess air

$$e = \frac{n_1 - 1.630}{0.796} = 1.256\, n_1 - 2.049$$

73 In the explosion turbine with water injection into the combustion chamber before the opening of the nozzle valve, the temperature, T_5'', after water injection was found from the corresponding value of the intrinsic energy, U_5'',

$$U_5'' = \frac{1}{n_1'}\,(n_1 U_5' + i_0 w_2 - u_s w_2)$$

where

$$\begin{aligned} n_1' &= n' + n_2 \text{ mols} \\ n_2 &= w_2/m_2 \text{ mols} \\ w_2 &= \lambda w \\ u_s &= i' + l \end{aligned}$$

The value of u_s is nearly independent of pressure and is taken as

1115 B.t.u. per lb., or, 620 kg-cal. per kg.

74 The case of the constant-pressure turbine with a separate steam jet impinging on the bucket wheel was treated as follows: The temperature, T_3, and the percentage of excess air being given, the combustion temperature T_5 was determined. The drop, ΔH, of total heat was found from the assigned value of the spouting velocity; this together with the assumed value of turbine efficiency ratio, E_t, gives the points 6 and 6′. The requisite precompression pressure of the gas was then determined by solving, for p_2, the equation

$$\phi_5 = \phi_6$$

The corresponding values for the steam cycle were similarly determined. Assuming an arbitrary value of λ to start with, the value of n_2 was found from

$$n_2 = n_1 \frac{\lambda m_1}{m_2} \text{ mols per mol of gases}$$

Calling p_s and p_g the partial pressures of steam and gas, respectively,

$$p_s + p_g = p_6'$$

On the other hand,

$$p_g \cdot V = n_1 R T_6' \qquad p_s \cdot V = n_2 R T_6'$$

from which

$$\frac{p_s}{p_g} = \frac{n_2}{n_1} \text{ and } p_s = \frac{n_2}{n_1 + n_2}\, p_6'$$

Knowing p_s and p_6', the corresponding saturation temperature T_s, and total heat, h_s, for steam can be found. The total heat of steam, h_6',

leaving the bucket wheel having been previously determined (= total heat at nozzle exit+steam-turbine losses), the temperature, T_9, resulting from the mixing in the casing of the gas and steam, can be computed by equating the amount of heat given up by the gases in cooling down to the casing temperature to the amount of heat required to raise the temperature of the steam to the same temperature

$$n_2\left[m_2h_S+a_p(T_9-T_S)+\tfrac{1}{2}b_2(T_9^2-T_S^2)+\frac{c_2}{3}(T_9^3-T_S^3)-m_2h_8'\right]$$
$$=[a_p(T_6'-T_9)+\frac{b'}{2}(T_6'^2-T_9^2)]n_1$$

where b_2 and c_2 are the values of the coefficients for steam, while b' is the corresponding value for the products of combustion. The value of T_9 being found by trial from the above equation, the amount of heat given up by the casing mixture in cooling at constant pressure to the steam-boiler stack temperature, T_{10}, (which in this case is taken 100 deg. cent. above the steam temperature in the boiler) was computed and this, together with the heat lost through the walls of the combustion chamber, gave the total amount of heat, Q_S, available for the generation of steam:

$$Q_S = (n_1+n_2)\,[a_p(T_9-T_{10})+\tfrac{1}{2}b(T_9^2-T_{10}^2)+\tfrac{1}{3}c(T_9^3-T_{10}^3)]+xQ$$

kg-cal., where b and c are coefficients for the mixture corresponding to the assumed value of λ.

75 If h_1 be the total heat of the steam at the steam nozzle, the weight of water evaporated is

$$w_2 = \frac{Q_S}{h_1-i_0}$$

and the value of λ' actually obtained

$$\lambda' = \frac{w_2}{m_2}$$

The computations were repeated until the assumed and the computed values of λ became identical.

76 In the constant-pressure turbine with steam injection into the combustion chamber, the heat Q'_S evolved by the casing mixture in passing through the waste heat boiler is given by

$$Q'_S = n_1[a_p(T_6'-T_{10})+\tfrac{1}{2}b(T_6'^2-T_{10}^2)+\tfrac{1}{3}c(T_6'^3-T_{10}^3)]$$

Since T'_6 and T_{10} are in this case assigned definite values, Q_S' is a function of b and c only which in turn depend on the ratio λ. The heat lost through the walls of the combustion chamber is:

$$Q_S'' = xQ$$

The sum $Q_S'+Q_S''$ being plotted against λ, the intersection of this curve with a curve representing the amount of heat required to evaporate the water gives the value of λ to be used. The temperature T_5'' after water injection was found by solving for T_5'' the equation:

$$n'[a_p(T_5-T_5'')+\frac{b'}{2}(T_5^2-T_5''^2)+\frac{c'}{3}(T_5^3-T_5''^3)]$$
$$= n_2[a_p(T_5''-T_S)+\frac{b_2}{2}(T_5''^2-T_S^2)+\frac{c_2}{3}(T_5''^3-T_S^3)]$$

where T_S is saturation temperature of steam; b', c' are coefficients for the products of combustion; b_2, c_2 are coefficients for water vapor;

$$n_2 = w_2/m_2, \qquad w_2 = \lambda w$$

77 In dealing with explosion-turbine cycles, with or without exhaustion, the following relations were employed:

$$W_C = ART_1\log_e r, \text{ kg-cal. per kg-mol}$$

where r is the ratio of compression = ratio of exhaustion.

The total amount of heat contained in the exhaust gases is

$$Q_8 = n'(H_6'-H_1)$$

The minimum efficiency E_8 of the steam-turbine plant necessary to perform the work of precompression and exhaustion is

$$E_8 = E_C \frac{Q_8}{W_C(n_1+n_0)}$$

The brake thermal efficiency E_1 with the work of precompression and exhaustion done entirely by the waste-heat steam plant, is

$$E_1 = \frac{n_1 W_{tv}}{Q}$$

where n_1 and n_0 are the number of mols of products of combustion and air, respectively. Assuming $E_8 = 0.15$, the corresponding brake thermal efficiency E_2 would be given by

$$E_2 = \frac{W + 0.15Q_8}{Q}$$

APPENDIX NO. 3

COMPRESSOR EFFICIENCIES ON ADIABATIC BASIS CORRESPONDING TO STATED EFFICIENCIES ON THE ISOTHERMAL BASIS

(Ratio of specific heats = 1.4)

Ratio of compression, P_2/P_1	Isothermal efficiencies, per cent					Ratio of compression, P_2/P_1	Isothermal efficiencies, per cent				
	55	60	65	70	75		55	60	65	70	75
1.5	58.3	63.5	68.9	74.2	79.4	9	76.5	83.5	90.4	97.5	104.4
2.0	60.8	66.4	71.9	77.4	82.9	10	77.9	85.0	92.0	99.2	106.2
2.5	62.8	68.5	74.3	79.9	85.7	11	79.1	86.4	93.5	100.8	107.9
3.0	64.5	70.5	76.4	82.2	88.2	12	80.3	87.6	94.9	102.3	109.5
3.5	66.1	72.2	78.3	84.3	90.3	13	81.3	88.8	96.2	103.6	111.0
4.0	67.6	73.8	80.0	86.1	92.2	14	82.3	89.8	97.2	104.8	112.3
4.5	68.8	75.2	81.5	87.7	93.9	15	83.1	90.8	98.3	105.9	113.5
5.0	69.9	76.2	82.6	88.9	95.2	16	84.0	91.7	99.3	107.0	114.7
5.5	70.8	77.2	83.6	90.1	96.4	17	84.8	92.6	100.3	108.1	115.8
6.0	71.7	78.3	84.7	91.3	97.7	18	85.7	93.5	101.2	109.1	116.9
7.0	73.4	80.1	86.8	93.6	100.1	19	86.5	94.4	102.2	110.1	117.9
8.0	75.0	81.9	88.6	95.6	102.4	20	87.2	95.2	103.0	111.0	119.0

DISCUSSION

Louis C. Loewenstein.[1] The authors, in their excellent paper, present the theoretical side in a very admirable manner and cover the field reasonably well, but the writer hopes that their prophecy as to the future of the gas turbine is not correct. He has been interested in gas-turbine development for the past twelve years and does not agree with their prophecy. For instance, a combination of Diesel engine and gas turbine looks promising. The Diesel engine receives precompressed air and compresses it to the ignition pressure. The gases on expanding in the Diesel cylinder drop in pressure and temperature and exhaust into a gas turbine which

[1] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

further utilizes the energy of these gases. The authors limited their calculations to centrifugal compressors, although there is no reason why they could not get the advantage of higher efficiencies by using reciprocating compressors. A combination of Diesel engine and gas turbine gives a rather simple plant of low cost and of easy manipulation. The difficulty of using ferrous metal for the buckets of the gas turbine because of the high temperature of the hot gases could be overcome, for instance, if the buckets were made of quartz. The addition of water to the hot gases, sometimes resulting in producing sulphuric acid, may work well with quartz buckets because they will not deteriorate as will ferrous buckets.

H. H. Suplee.[1] The author has referred to the injurious effect of heat on the buckets. There is one other fact that will have to be taken into account, namely, the oxidation of the buckets. The discharge of the jet upon the turbine constitutes an oxidizing flame like that of an oxidizing blowpipe. Experience has shown us that under ordinary circumstances the surface of a steel bucket will first be covered with a film of oxide, which is then blown off by the sand-blast action of the jet. The buckets will be washed away by this continued oxidation and polishing long before they are worn out. It will be necessary to construct the buckets of some material such as quartz or some modification of steel, or some other material that will not be readily oxidized.

The Author. The authors have made no prophecy with reference to the exhaust-gas turbine, referred to by Doctor Loewenstein. On the contrary, they have expressly stated that the exhaust-gas turbine is practical and is of value in utilizing, without much difficulty, power which would otherwise be thrown away.

In regard to the use of reciprocating compressors, there is little point in building gas turbines if it is necessary to add large reciprocating compressors; it is assumed that the main use of a gas turbine will be in the large sizes.

The authors do not discuss in their paper the practical side of the gas turbine. The paper gives the results of certain calculations, which are so presented that the designer, having assumed a compressor efficiency, a turbine efficiency ratio, and the maximum temperature of the buckets, can ascertain the overall thermal efficiency which may be obtained under these conditions.

[1] New York, N. Y. Mem. A.S.M.E.

No. 1945

INTAKES FOR POWER PLANTS

INCLUDING A DESCRIPTION OF EXPERIMENTS AND RESULTS ON A LARGE-SCALE MODEL

By Robert W. Angus,[1] Toronto, Canada

Member of the Society

The purpose of this paper is to set forth the desirable features of intakes placed in rivers or other bodies of water for supplying water for hydraulic turbines and other equipment. The paper makes special reference to protection against ice and other large floating substances. It includes a description and discussion and conclusions with reference to an extensive set of experiments made by the author at Niagara Falls, Canada, on a large model of the Niagara River, their immediate object being the design of a suitable intake for the Queenston hydro-electric plant.

THE construction of a water intake for a power plant or water works is frequently simple, where the volume of water taken in is relatively small and free from objectionable floating material, but in water and steam power plants and in many water-works installations the problem is a very serious one. The difficulty of the problem depends on the volume of water to be taken in, the depth below the surface at which the intake may be placed, the velocity of the water in the source of supply, and on the amount of floating material present. Where there is much ice the design must be very carefully worked out to be effective.

2 In the northern parts of the United States and in Canada, ice produces one of the most serious troubles and has been the cause of more or less frequent shutdowns in plants, and has also caused serious damage to the turbines and machinery. Many illustrations of this might be given, but the great power developments on the Niagara River are typical, these being mainly located where the water is shallow, the current moderate, and the volume of water drawn in very large. This river carries vast quantities of ice at certain times of the year, since in addition to what forms in the river itself, there is the accumulation from Lake Erie, the ice from which, with a strong wind, may be blown

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over against one shore or the other, and thus forced into the power plants on that side of the river. In this instance the plants are closed down at times for short periods due to ice blocking the intakes.

3 The photographs shown herewith give some idea of what has happened in these cases, Fig. 1 indicating an ice condition on the Niagara River at Hog Island.

4 This paper has in view primarily the design of an intake to avoid drawing in ice, and usually if ice can be avoided other floating substances not in suspension are likely to be avoided, although the plans investigated would not serve in case of sand or small stones.

Desirable Properties of an Intake

5 An intake must be simple in form and easily constructed. This is self-evident when it is remembered that they are used under water and frequently must be constructed without unwatering. Further, the danger of blocking up increases as the intake becomes more intricate in form, and the difficulty of cleaning is also increased.

6 The intake must be thoroughly reliable and positive, even to the extent of sacrificing some efficiency for certainty of operation. It is very necessary that an intake should never fail to supply the desired volume of water, and it is therefore better to have a small amount of ice and debris enter it, if necessary, a good part of the time under part loads, than to make the design so complicated as to render it liable to choke up when the draft is heavy. On the whole, an intake that functions properly under full load will have little chance of drawing in debris at other times.

7 The intake should have a large factor of safety or operation — greater, indeed, than other parts of the plant, partly because of the lack of knowledge of effective design, as demonstrated by the failure of so many intakes under critical conditions. On the other hand, money may be needlessly wasted by making large and ineffective designs. The loss of head in the intake should be small, as it produces a decrease in the effective head on the plant.

8 A large factor of safety naturally suggests an intake covering a considerable area, but it must be kept in mind that the larger the area occupied, the greater the field of ice from which the intake may draw. It is also well known that, while the larger area would tend to decrease the volume of water drawn in from each square foot of surface, yet it may follow that an imperfect design may be little benefited by an increase in size, since the draft may not be equally distributed over the area covered by it.

9 This statement is evident to any one observing the action of intakes, for there are many cases where the intensity of draft varies enormously over the area covered by the intake, and where the distribution is far from uniform. In fact, it is one of the most

difficult things in hydraulics to arrange for a uniform draft of water over a prescribed inlet area.

10 Where water in proportionately large quantities is drawn from a flowing stream, such as a river, there will be a natural decrease in velocity below the intake, and this must be compensated for in some manner or else the intensity of the floating ice or debris will increase just over the intake, thus making the danger more serious. Manifestly if any change in the velocity of the water in the river takes place at the intake, it should be an increase.

11 The most obvious way of keeping up the velocity would be to make the river shallower in passing the intake, but if this is done to any great extent, the tendency is to divert the water

Fig. 1 Mouth of Welland River, Showing Ice Field Extending from Niagara River; Lighthouse on Hog Island

away from the intake over toward the center of the stream, and thus defeat the very object in view. If compensation is attempted by narrowing the river with a diverting wall or dam, there is danger of concentrating shore ice above it and of forming a pool of dead water with floating debris above the diverter and close to the intake.

12 It is clearly impossible to avoid drawing in some ice or debris, since at times these may be found at all depths of the river or other source, but the minimum of foreign matter will be taken in where the water is drawn from the bottom of the source. If the intake is designed to function in this way, then some scheme must be devised to deal with logs, etc., rolling along the bottom, and provision must be made for taking them through the intake, or for removing them from it, if it is not possible to make them pass over it.

13 If the intake openings are not large, then foreign substances will stick in them and will have to be removed by divers or some other means, often a very difficult matter.

14 The distance from which a floating mass will be drawn into an intake depends very largely upon the velocity of the water entering the latter, hence, unless the intake is in very deep water, the velocity of the entering water must be low. Here again attention is called to the fact that this condition is not necessarily met when the quotient of the volume of the water to be taken in divided by the area of the intake is small, since the velocity on at least half of the intake may be much higher than this quotient.

15 The foregoing considerations have been set out at some length so that a clear conception of the problem may be established. A number of the principles stated have been fairly well understood for years, and yet the fact is that many intakes are unable to meet a severe test such as comes at an unfavorable time. Examples of inadequate designs are not wanting, and one well-known case may be cited where the designer did not realize the value of low velocities, as his design shows a mean velocity of entry of the water of 4.5 ft. per sec. and it fails to handle the ice with any satisfaction, while the plant suffers considerably from shutdows.

Experimental and Research Work on Intakes

16 Unfortunately very little experimental work has been done on intakes, or at least little is available, and consequently former mistakes are being repeated. The experiments about to be described may therefore prove of interest, not because they are particularly exhaustive, but because they indicate some of the causes of failure and may prove helpful to those facing this important problem.

17 The experiments referred to were made by the author in connection with the large development recently built at Niagara Falls by the Hydro-Electric Power Commission of Ontario, Canada. As is well known, the intake for this development is at the mouth of the Welland River, which is some two miles above the Horseshoe Falls; the river surface at this point is only a slight distance below the level of Lake Erie, and vessels may navigate from Buffalo down the river to the village of Chippawa, which is situated on the Welland River.

18 The level of the Niagara River at this point varies from 558 ft. to 561 ft. above mean sea level, and the width of the river is slightly over one mile. At the intake location the depth of the river at 600 ft. from shore was only 15 ft. with the water at El. 561 ft. and 12 ft. at El. 558 ft., and the depth rather decreases further out. At a point opposite the mouth of the Welland River the measured velocity was 4.2 ft. per sec. and the

volume of water passing down the Niagara River within a distance of 600 ft. from shore varies from 25,000 cu. ft. per sec. when the river is at El. 558 ft. to 32,000 cu. ft. per sec. when the river is at El. 561 ft. The corresponding quantities for 1200 ft. are 45,000 and 59,000 cu. ft. per sec., respectively.

19 At the time the investigation was in progress it was the intention to draw 15,000 cu. ft. per sec. from the Niagara River for the plant at Queenston, but in view of the fact that a safety factor was desired, the experimental work was planned with the possibility of larger drafts of water than the above, and, indeed, later developments have shown that the draft will in time exceed the 15,000 cu. ft. per sec. It was not desired to have the intake extend further out from shore than was necessary, partly on account of difficulties of construction in the river.

20 If the intake was not to extend out more than 600 ft. from shore, the problem confronted was that of drawing from a river 12 to 15 ft. deep, having a velocity of 4.2 ft. per sec., heavily burdened with ice at certain seasons, approximately one-half the available water in this width without causing serious ice trouble in the plant; and even if the intake extended twice as far, the draft would be about one-third of the supply. Extra care was also necessary because the water was to be taken down an open canal 13.5 miles long, without any means of disposing of ice along the route.

21 In view of the importance of the problem and the lack of information, a model of the river and canal was decided on, and fortunately a space was available at the Dufferin Islands, Niagara Falls, Ontario, where it was possible to build to a scale of one-twentieth full size, a model of the Niagara River out to about 2000 ft. from shore, and for a length of 4000 ft., there was also room to include 2800 ft. of the Welland River. The model was so laid out that the intake site would be at about the center. The supply of water for the experiments was introduced at a distance of 170 ft. above the beginning of the model of the bed.

22 In order to make it true in every way, the bed of the model was constructed to agree with what was known of the contours of the beds of the Niagara and Welland rivers, and very great care was taken in the grading, stakes being set up every 10 ft. in each direction. Gages were also set in the model to represent the river stage, and it will be noted that since the experiments were made for El. 558 ft. to 561 ft. of the river, the depth of water in the model was only from 7.2 to 9 in., and this small depth necessitated the very greatest care in the whole scheme.

Laws Followed in Using the Model

23 In using the model it was necessary to determine the scale relations for the discharge and velocity from the physical dimensions. While there is no absolute information on the point, the

laws of hydraulic similarity have nevertheless been fairly well established, the experience shows that models will represent the

Fig. 2 General View of Model with Improved Welland River in Foreground and Niagara River in Background

Fig. 3 Supply Pond on Right with 18-ft. Weir Feeding the Model on the Left

actual results with great accuracy,[1] those obtained with model turbines, etc., being remarkable in their correspondence with the

[1] See paper by B. F. Groat, Trans. A. S. C. E., vol. lxxxii, p. 1138.

full-size machines. In this case the physical dimensions were reduced to the adopted scale of one-twentieth, which made the slope of the water surface and bed in the model the same as that in the river. The cross-sectional area of the water was therefore reduced in the ratio of $1:(20)^2$ or $1:400$.

24 Since the bottom resembled the actual river bed in contour, and since the roughness of the bottom was reduced in about the same proportion, it was thought that the coefficient in the Chézy formula, $v = c\sqrt{rs}$, would be the same in the model as in the actual river; in other words, the decrease in Kutter's n would about compensate for the decrease in r, so that the velocity v would vary as the square root of the hydraulic radius r. On this

FIG. 4 METERING BRIDGE AND STRUCTURE TO ALLOW MEASUREMENTS TO BE MADE ON INTAKE WHICH IS IN PLACE

basis the velocity in the model should be reduced in the ratio $1:\sqrt{20}$, and this was the general ratio adopted, although the model experiments were made at several rates of discharge of the river and hence were not vitally affected by inaccuracy in this rule. The discharge in the model should thus be reduced in the ratio $1:400\sqrt{20}$ from that in the corresponding part of the river, giving for El. 561 a discharge of 52.56 cu. ft. per sec. and for El. 558 ft. a volume of 39.60 cu. ft. per sec. to be passed down the model river.

25 The desired volume of water was admitted to the model over an 18-ft. weir, Figs. 2 and 3, which was 280 ft. above where the intake was placed, and by suitable arrangement of stones and obstructions the water was forced to spread over the entire model in the same proportion as in the river. While adjusting and dis-

tributing the volume, the river stage was controlled by a dam below the intake, and after some experimenting the adjustment was made so as to represent actual conditions.

26 In Fig. 2 will also be seen the model of the bed of the reconstructed Welland River with the model intake at its mouth where the men stand, and a 9-ft. weir to measure the quantity drawn off in the intake, the control of this quantity being effected by the slats shown just above the weir.

27 The arrangements thus made it possible to send any desired volume of water down the river, to adjust its gage height independently, and to draw off on the intake any desired volume.

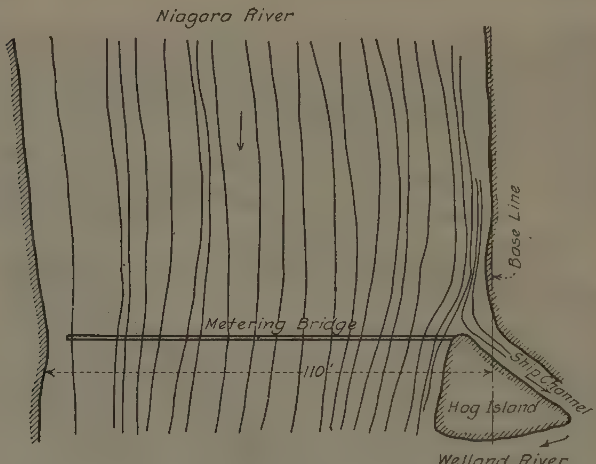

Fig. 5 Paths of Deep Floats in Model River Channel as it Originally Existed Before Any Intake Was Constructed

Leakage of Water from the Model

28 Leakage was a matter of great concern and no doubt some did occur owing to the nature of the materials, although precautions were taken to prevent it. In order to have a check on this, as well as to check the desired distribution of the velocity in the bed, a current-metering station was arranged just below the intake to correspond to a section of the river at which discharges had been measured. This station was made in the form of a bridge, Fig. 4, across the main channel to the left of the men shown on Fig. 2 (see also Fig. 5).

29 It will thus be seen that the discharge by the 18-ft. weir should equal that by the current meter and the 9-ft. weir combined, and a number of comparisons were made from this standpoint in the course of the experimental work. The following are some of the results obtained:

Measurements by			Total of last two columns
18-ft. weir	9-ft. weir	Current meter	
48.25	0	49.1	49.1
48.25	0	49.2	49.2
49.0	5.65	45.2	50.85

Many of the results thus agreed quite well, showing little leakage, although others were not so accurate.

Method of Carrying Out the Experiments

30 The first procedure was to get a distribution of water over the model similar to that existing in the Niagara River before the intake was placed in it, and the model was therefore first made without any intake, but with the original river with Hog Island, the ship channel, and all details as shown in Fig. 5. By the use of the current meter on the bridge and also by the use of floats, the proper distribution was effected through the placing of stones and obstructions just below the 18-ft. weir and also by adjustment of the dam below the intake, the final results being as shown in Fig. 11, the actual conditions agreeing with the model fairly well.

31 The floats in this case were bottles sunk to a depth of about 2.5 in., and the path and velocity of each float were taken by stretching wires across the model at 25-ft. intervals, each wire having graduations on it 5 ft. apart. A float was then placed in the channel at the desired distance from shore and 25 ft. above the first wire, and as the bottle passed under the first wire, one observer signaled and a second observer on the bridge noted the time on a stop watch, also noting the distance out from shore and plotting the point on cross-section paper, containing a plan of the river, at the same time; the same procedure was followed with all of the wires. In this way five points were plotted on the path of each float, also the approximate intermediate path between wires, the field plotting much decreasing the chance of error.

32 Having thus adjusted the conditions to correspond to those in the Niagara River, the channel was unwatered and the intakes built into place. During the two seasons thirty-five models were experimented on, covering the different types with their various modifications. In placing these intakes, the area to be covered by each was staked off and great care was taken not to disturb the remainder of the river bed. It is not practicable to give full information here on all the intakes used, nor complete details of the results obtained, but the method was the same in all of them and the first one will be described more fully than the others.

33 The primary object in the experiments was to get an intake which would avoid taking in a troublesome amount of ice and debris; and hence it was desired to know what effect would be produced on the stream lines of the river, how much tendency there was for the intake to disturb the conditions at a greater

distance from shore than it extended, and what tendency there was to draw in floating ice and debris as well as substances floating below the surface. The loss of power due to friction, etc., involved in the design is also important, and it is, of course, desirable to keep the cost down to the lowest figure consistent with good design.

34 To study the effect of the intake on cakes of ice, floats made of oak blocks of different sizes were used, these being heavy enough to float about 90 per cent submerged. Bottle floats were used to arrive at subsurface effects and these were sunk to different depths to suit the case. The paths of the floats were plotted

FIG. 6 VIEW OF INTAKE *A* LOOKING FROM POWER CANAL TOWARD NIAGARA RIVER; MODEL UNWATERED

in the way already described, but in the vicinity of the intake extra wires were used and more detailed information on the paths of the floats was obtained. Distribution of the flow was also determined by the use of the current meter, this being also used to find the velocity at the different points around the intake. In addition to these data, the actual directions of the water particles were determined in many cases by means of a light string attached to the lower end of a stick, to the upper end of which a protractor was attached.

TYPES OF INTAKES TRIED

35 In general there were three types of intakes experimented on, and for want of a better nomenclature these may be briefly referred to as (*a*) open intakes, (*b*) closed intakes, and (*c*) combined intakes. In this paper open intakes are those having no

physical structure to separate the surface water in the river from that in the power canal; that is to say, boats not drawing too much water could pass freely from the source of supply into the power canal right over the intake. By closed intakes is meant those where the water from the source of supply passes through a closed tube of some length on its way to the power canal; such intakes have a series of tubes opening at one end into the main river or source and into the power canal at the other, and all water entering the latter must pass through the tubes. Combined intakes are referred to as those with tubes where the water enters along a considerable length of the tube, but like the closed type

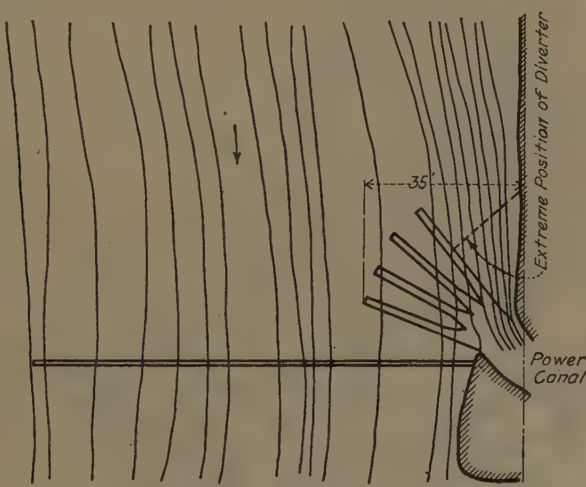

Fig. 7 Paths of Deep Floats in River with Intake *A* Drawing 8.5 Cu. Ft. per Sec.; River Gage, 560.5 Ft.

a curtain wall of some kind cuts off the surface river water from the power canal.

36 The first intake was entirely an open one of a type invented by B. F. Groat, and is shown in Figs. 6 and 7 in place. Mr. Groat has stated[1] in a paper his ideas on an intake, and as it was worked out in this investigation it consisted of four open slots or fingers sunk in the bed of the river and discharging into a single channel at the entrance to the power canal. The sides of the fingers were made of plank and were vertical, and the bottom of the river was built with a rising gradient between these fingers and thus compensated for the water drawn off by the intake; the bed of the model just below the last slot was therefore higher than just above the first one.

[1] Trans. A. S. C. E., vol. lxxxii, p. 1138.

37 The set of observations taken on the first intake operating under one condition — not, however, that for which this was best suited — is given in Fig. 8, the figures denoting the velocities in feet per second by the current meter and the arrows giving both the direction and sense of the particles. The direction indicator proved much more sensitive than the current meter. In each case, of course, the water drawn off by the intake was measured, as well as the flow in the river and its height.

38 With this particular intake, experiments were made with some variations in the slots, and with diverters or walls running

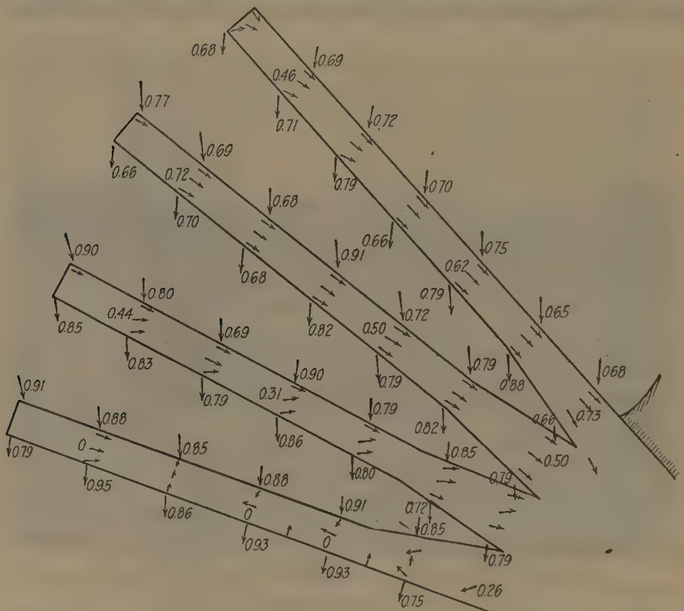

Fig. 8 Intake *A* in Place. Diagram Showing Method of Studying the Action of an Intake. Figures Denote Velocities and Lines Show Direction of Currents

out from the shore above the intake to various distances; but both shallow and deep floats went into the canal in large numbers, particularly those nearest the shore, and the intake in this form did not give satisfaction. The author does not wish to be misunderstood in this matter, for Mr. Groat states that he was able to keep out floating materials in experiments he tried. Probably some other proportions or dimensions than those used were necessary, but it did not seem advisable at the time to continue work on it.

39 Modifications of this were made, these broadly taking the form of a curtain wall reaching down below the water surface, so that floating matter had to pass under a submerged wall before

entering the canal, the bottom of the wall corresponding to El. 548.5 ft. This was along the line adopted in many earlier intakes, but proved of little value, as block and bottle floats coming close

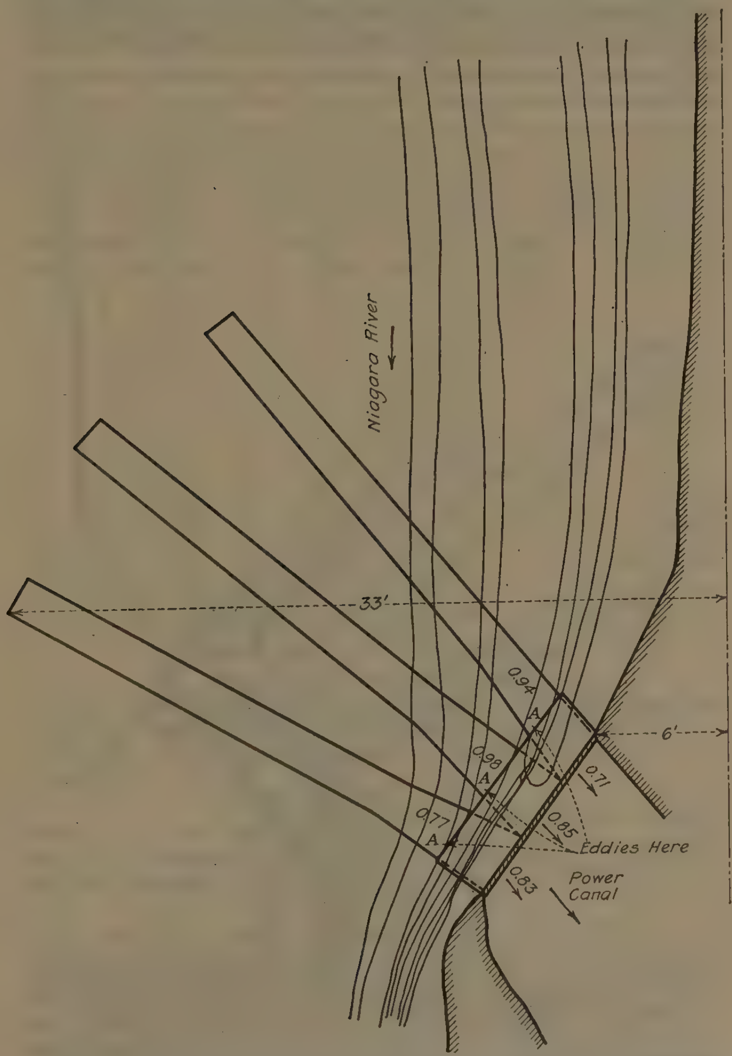

Fig. 9 Intake *B* Drawing 8.05 Cu. Ft. per Sec.; River Gage, 560.6 Ft. Figures Give Velocities. Loss, 0.56 In.

to the wall invariably went down under its face into the canal. This intake was then modified by the addition of a submerged horizontal platform 28 in. wide in front of the wall, as shown in

Fig. 9, but again, while this proved helpful, it had the objection that eddies formed at A, and there was also relatively dead water near the vertical wall. The block floats had a tendency to bunch over the intake, but there was clear evidence of a distinct improvement over the vertical wall alone, and the volume was fairly

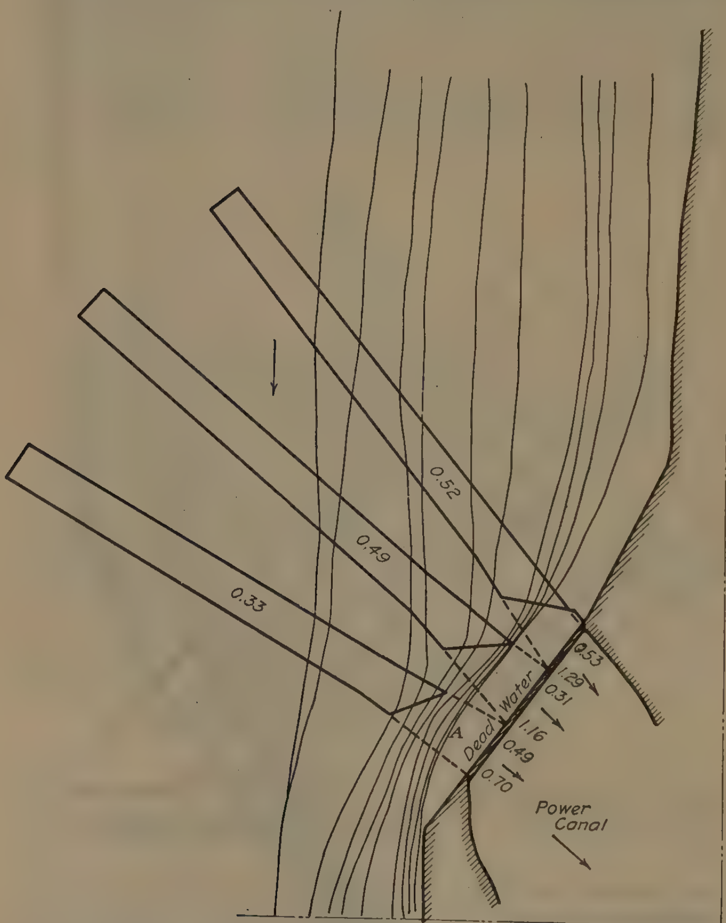

FIG. 10 PATHS OF DEEP FLOATS WITH INTAKE C DRAWING 8.1 CU. FT. PER SEC.; RIVER GAGE, 560.5 FT. FIGURES GIVE VELOCITIES

uniformly distributed over the area of the opening. The intake loss in this case was increased and amounted to 0.56 in. when drawing 8.05 cu. ft. per sec., which would correspond to 14,400 cu. ft. per sec. in the actual case.

40 This suggested that a proper design of cover might produce the desired results, provided that eddies could be avoided and the

loss was not too large. Naturally the experimental season was short and it was not possible to digest the results at leisure, so that the changes to be made had to be decided upon quite rapidly. In the next intake, therefore, the cover was extended out further, with the result that the floats decreased in speed as they went down over the intake and a number of them entered the lower slot, the reason being that the amount of water taken out by each slot caused a decrease in the river velocity, which finally became critical and could not carry the floating matter past.

41 Triangular corners were next added, forming intake *C* shown in Fig. 10. A number of experiments were made with this arrangement with different lengths of slots and different drafts, studies being made by current meter and floats. In this type

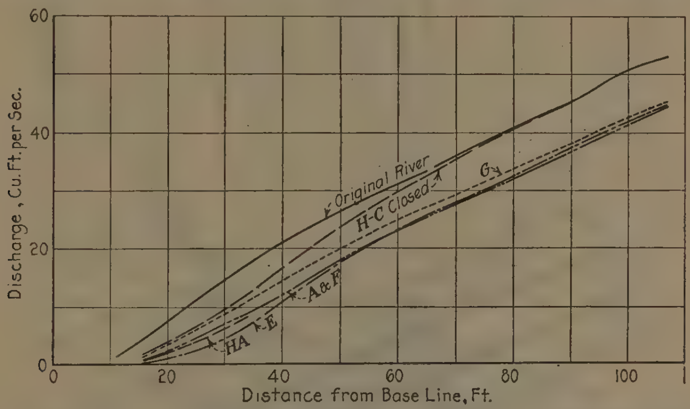

Fig. 11 Total Discharge Down the River at the Distances from the Base Line. Discharge Measured at the Bridge below the Intakes. The Letters on the Curve Denote the Intake Referred to

floats 4.5 in. deep failed to go in, but the disadvantage was that a large eddy formed near the vertical curtain wall at *A*, which held the floats for some time but finally released them to go down the river.

42 That this intake drew water largely from near shore was proved when the slots were shortened without producing any material effects on the intake. The current meter invariably showed that where the water passed from the intake into the canal the velocity at the downstream side of the slot was very much more than at the upstream side, a feature that was undesirable and unexpected.

43 Some curves to show the effect of the intakes on the distribution of water in the river are given in Fig. 11, where the upper one corresponds to the undisturbed river and the others to the intakes, none of which has much effect on the river beyond

its own length, the two lines remaining a constant distance apart beyond the end of the intake.

44 The next type, *D*, consisted of a trench having a uniform width of 6.8 ft. and length of 18.8 ft., the depth below the river bed averaging about 1 ft. This trench was divided into three slots

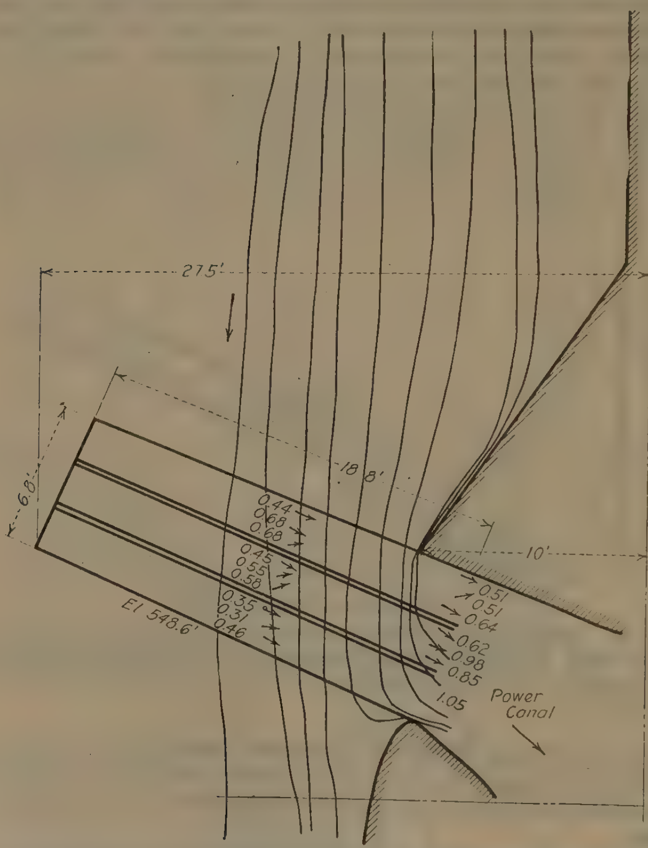

Fig. 12 Intake *D* Drawing 8.0 Cu. Ft. per Sec.; River Gage, 560.4 Ft. Paths of Deep Floats. Figures Denote Velocities in Slots

by vertical planks as shown in Fig. 12, the depth of the river being decreased on passing down over the intake for reasons already stated. With this intake left entirely open many floats went in, as the discharge velocity into the canal was quite high, but inasmuch as the loss of head due to this high velocity could largely be recovered with a proper diffuser, it was thought worthy of trial. In the earlier trials with this type the lower slot proved

to be of little value, but by raising the river bed below this slot it was made to take its full load.

45 The open form of this intake having proved ineffective, a vertical submerged wall was next used, and afterward a combination with this of a horizontal table 20 in. wide and with various

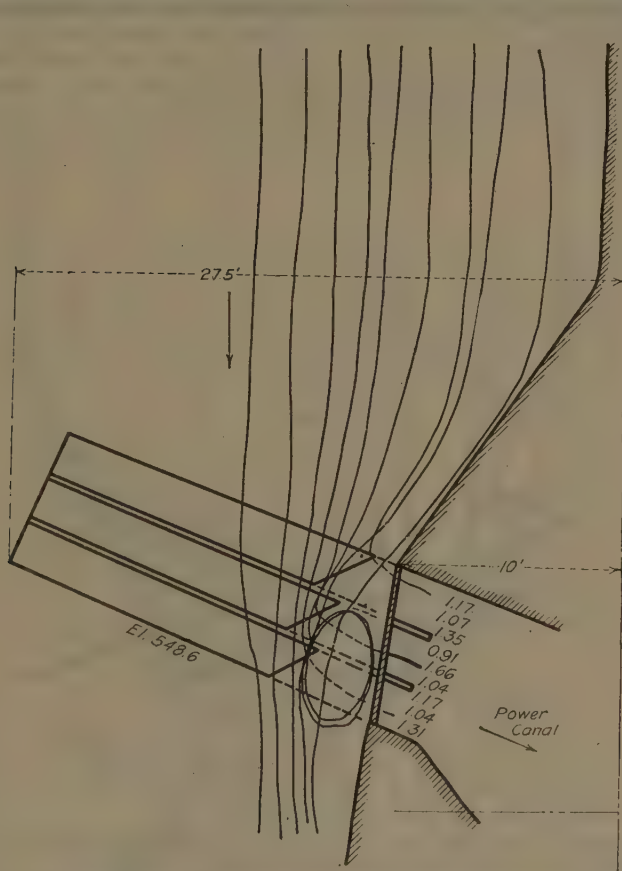

Fig. 13 Paths of Deep Floats with Intake *E* Drawing 7.72 Cu. Ft. per Sec. River Gage, 560.3 Ft. Figures Denote Velocities in Slots

positions of the diverter, but very little difference appeared to exist between this and the first type, and neither could be considered a solution to the problem. Triangular covers were also added, but the large eddy still remained and it held the floats close to the intake and passed them back over it, although few of them entered. Different forms of diverters were tried for the

purpose of reducing this sweep, and one scheme, intake *E*, is shown in Fig. 13 where the draft is fairly evenly distributed on the three tubes.

46 This finished the work on the open intakes, none of which gave promise of meeting the conditions demanded. They did, however suggest certain lines along which the later experiments were made.

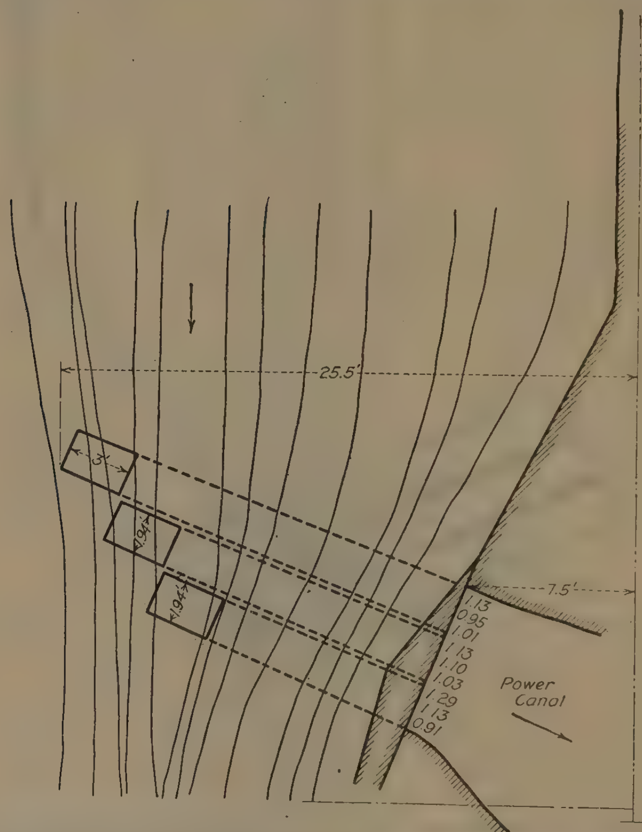

Fig. 14 Paths of Deep Floats with Intake *F* Drawing 8.1 Cu. Ft. per Sec.; River Gage, 560.5 Ft. Figures Denote Velocities

Closed and Combined Intake Models

47 The first of these was made from the open intake just described by covering over the slots with boards level with the river bed and leaving one opening 3 ft. by 1.94 ft. in each, in the position shown in Fig. 14. This design showed a marked improvement over those already tried in every respect except that there was an increased loss of head, but the paths of the floats shown

in Fig. 14 and effect of the draft shown in Fig. 11 indicate a fairly good type of intake. The current meter also showed very good distribution of the load over the various slots. Two features were evident: (*a*) there was a large friction loss, and (*b*) there was a marked tendency to draw water from a distance further out than the intake tubes extended. The velocity was also lowered in the river below the intake.

48 Slots were next cut in the covers along the tops of the channels, these being narrowed down as they approached the shore

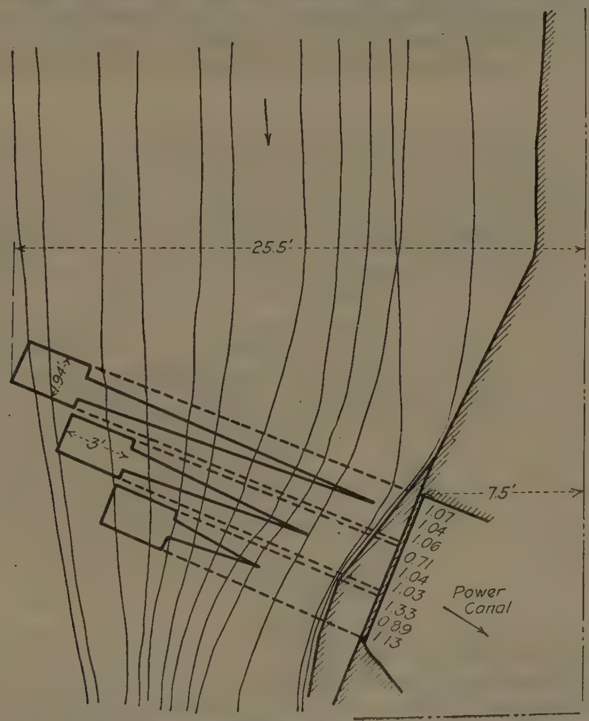

FIG. 15 INTAKE *G* DRAWING 8.2 CU. FT. PER SEC. WITH RIVER GAGE, 560.1 FT. FIGURES DENOTE VELOCITIES

for the purpose of distributing the draft along the length and to make the intake more effective over the entire area occupied by it. The results of this modification proved very encouraging, the friction loss decreased, and the stream lines were straighter than on any other of the models. Practically none of the floats entered the intake, not even those 3 in. deep and exposed to an upstream wind of 10 miles an hour. This type of intake formed the basis on which one of the second season's plans was based, but lateness of the season prevented a full investigation being made at this juncture. This intake is shown in Fig. 15 and the results obtained in Fig. 11.

49 During the first season two other forms of intake were suggested and it was thought well to try them in a general way. From the original opening 6.8 ft. wide used in the last set of experiments the two intermediate planks were removed and replaced by a single one in the center, thus making two channels 3.3 ft. wide and lengthened to 25.75 ft. Each channel was then covered over in such a way that water could enter along the entire length of one side of each channel, the other side being closed: that is, the cover was nailed down solid to one edge of the channel and the other edge of the cover raised 0.30 ft. to let water in. There were thus two vertical openings 0.30 ft. high and 24 ft. long through which all the water entering the canal had

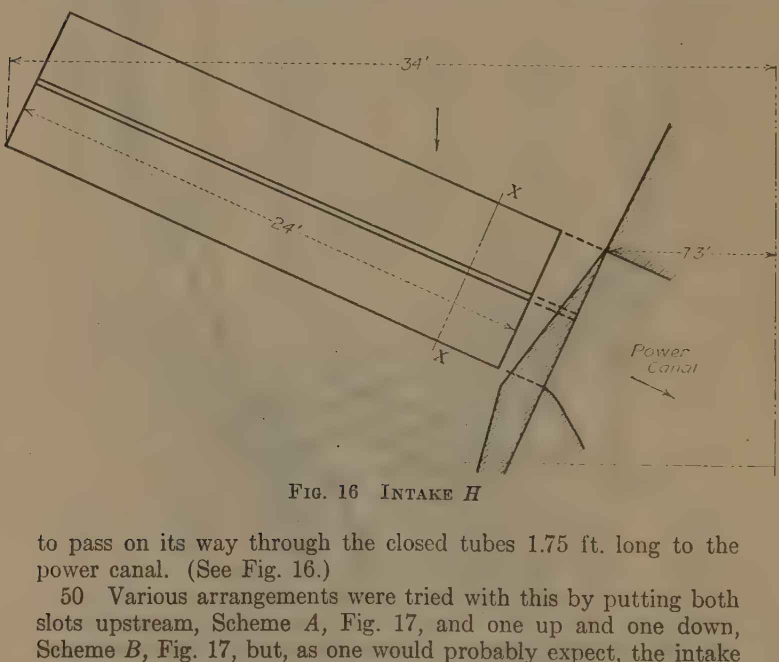

Fig. 16 Intake *H*

to pass on its way through the closed tubes 1.75 ft. long to the power canal. (See Fig. 16.)

50 Various arrangements were tried with this by putting both slots upstream, Scheme *A*, Fig. 17, and one up and one down, Scheme *B*, Fig. 17, but, as one would probably expect, the intake was not a success, the floats entering the shore end of both openings, and the lower opening seemed to catch a large proportion of what came near it. Part of the trouble was undoubtedly due to the small depth of water on top of the cover, and possibly with more time for experiment a satisfactory device could have been made. There are, however, inherent defects, as uniformity of draft is impossible with constant width of opening in this type of intake, and also the change of depth that occurs at the intake due to the covers' forming a type of obstruction which pushed out the water, as a study of Fig. 11 shows. This means a marked decrease in the water available at the intake and a very decided dead-

ening of the water below it, causing ice to jam, as was indicated by the floats. There was also a standing wave above the opening.

51 A modification of this intake was next built by making the cover out of separate slats, somewhat in the form of a louver, and laid flush with the river bottom in the manner shown in Scheme *C*, Fig. 17. This type would evidently be troublesome and costly in construction and therefore objectionable from that standpoint, but when the slats were so adjusted that the openings for the water varied from nothing at the shore end to a desired amount at the outer end, the result was very good. Uniform width of opening for the entire length caused relatively heavy draft at the shore end and a bad eddy below it.

Results of Experiments

52 When the experiments above described were completed, construction work on the new pipe line for the Ontario Power

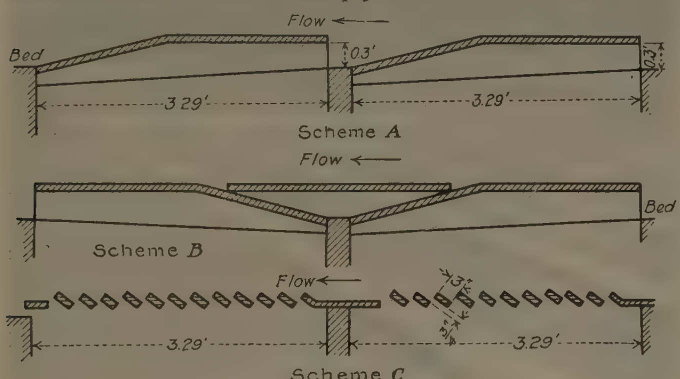

Fig. 17 Sections through Intake Shown in Fig. 16 along Line *X-X*

Company compelled the work to be discontinued, and this gave time for a careful study of the results. The work covered a careful investigation of a number of proposed intakes and further enabled certain conclusions to be drawn as to the desirable and undesirable features. These may be briefly summarized as follows, it being remembered that part of these conclusions apply only to a location similar to the one proposed:

a It appears that an open type of intake, i. e., one where there is no physical structure to cut off the surface water, is not desirable. Such an intake failed to keep out surface debris, more especially when there was an unfavorable wind. Its losses are very small and cost of construction comparatively low.

b A vertical curtain wall extending well above and below the water surface, forming submerged openings for water, is of little value, as debris is drawn down its surface and into the intake.

Making the wall wide at the bottom, so as to produce the effect of a tube instead of an orifice, improves the conditions.

c Draft of the water must extend over a considerable area, as any localizing of the draft produces excessive velocities and causes the entry of debris. There is also a stagnant area just below this point, in which an eddy forms, holding suspended material and passing it back over the intake.

d Unusual care is required to prevent localized draft. For long tubes with uniform openings along their length there is a drop in gradient inside the tube toward the power canal, and this causes concentration of draft at the inner end of such tubes, making the outer end of little value and causing high velocities at the shore end.

Fig. 18 View of Johnson and Wahlman Model, Taken from Bed of the Niagara River just below the Model. Welland River and Power Canal on the Right

e The maximum velocity of water entering the intake must be low at all points.

53 The experiments indicated that at least two methods, after intakes *F* and *G*, might be used to meet the desired conditions and consequently two forms of intake were devised, and the second season's work was done on these two only. The first of these intakes was designed by Messrs R. D. Johnson and P. Wahlman, and consisted of a series of six tubes drawing in water for the greater part of their length; while the second intake was designed by the author, and consisted of six large openings on the bed of the river, each connected with the power canal by a separate tube.

The Johnson Intake Model

54 The Johnson intake model was made up of six parallel tubes, each having a total length of 69 ft. 6 in. and being placed

at 5 ft. centers. Of the total length of the tube, the inner 4 ft. 6 in. served as a diffuser with expanding section; the next 55 ft. was of uniform size 10 in. square inside, while the outer 10 ft. tapered from the 10-in. by 10-in. section to a 3-in.-square section at the outer end. Along the face of the outer 40 ft. of each tube there

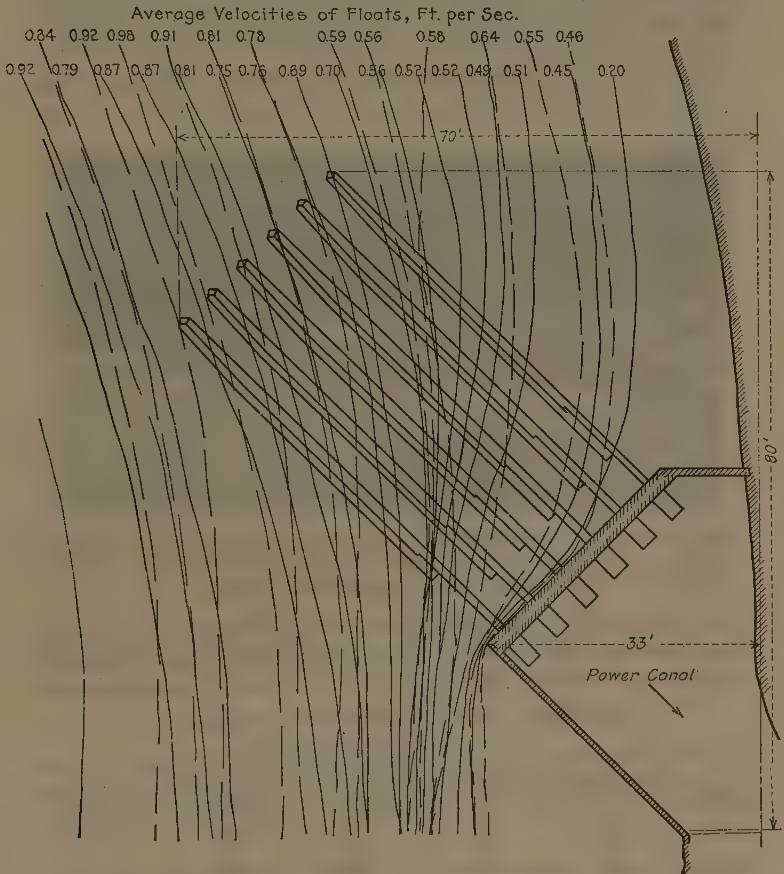

FIG. 19 JOHNSON AND WAHLMAN INTAKE MODEL—PATHS OF DEEP FLOATS. BROKEN LINES CORRESPOND TO INTAKE CLOSED AND DRAWING NO WATER. FULL LINES TO A DRAFT OF 8.72 CU. FT. PER SEC.; RIVER GAGE, 559.8 FT.

was a vertical opening in which guide vanes were set so as to form ports which would direct the water inward along the tube axis. These are shown in place in Fig. 18.

55 The tubes had been carefully worked out theoretically and designed with sufficient shock loss to try to produce uniform draft

along the entire outer 40 ft. Naturally the port areas have a relatively smaller area at the inner end than at the outer, and the areas and angles of the ports were very carefully graded to suit the desired conditions. If the objects of the design were realized, the intake would draw uniformly over an area 40 ft. by 25 ft. at the river bed, or in the full-sized intake over an area of 400,000 sq. ft., and hence the vertical component of the velocity would be very small.

56 A general view of this intake in position is shown on Fig. 19, together with the stream lines for a draft of 8.72 cu. ft. per

Fig. 20 The Author's Model, Taken from the Bed of the Niagara River just above the Intake (Grading Unfinished). Power Canal at Upper Left, Metering Bridge in Center

sec., corresponding to 15,600 cu. ft. per sec. in the actual intake, with the river at El. 559.8 ft. For reference the stream lines with the intake closed are shown dotted, and it is seen that the draft

TABLE 1 PITOT MEASUREMENTS IN TUBES

Tube	Discharge in Tube—Cu. Ft. per Sec. One-quarter way in	Half-way in	Three-quarters in	Full discharge of tube	Measurement by weir
A	0.27	0.73	0.88	1.12	....
B	0.26	0.70	0.76	1.18	....
C	0.33	0.64	1.05	1.23	....
F	0.31	0.68	0.94	1.24	....
A	0.70	1.26	1.85	2.36	....
C	0.74	1.20	1.73	2.48	....
A	....	....	....	2.11	
B	....	....	....	2.10	
C	....	....	....	2.09	
D	....	....	....	2.14	12.90
E	....	....	....	2.15	
F	....	....	....	2.10	
			Total	12.69	

is very uniform over the area. Pitot-tube studies were also made on this intake to find how effectively each section of the tube

operated, and measurement of the discharge was made for each quarter of the length of the part of the tube containing the ports.

57 These measurements showed a very uniform division of the discharge among the several tubes, the discharge for any one tube not differing more than a small percentage from the average for the six, and it was also interesting to find that the sum of the discharges measured by pitot tube was within two to five per cent of that taken by weir measurement at the lower end of the canal. In view of the fact that the velocities were usually between one

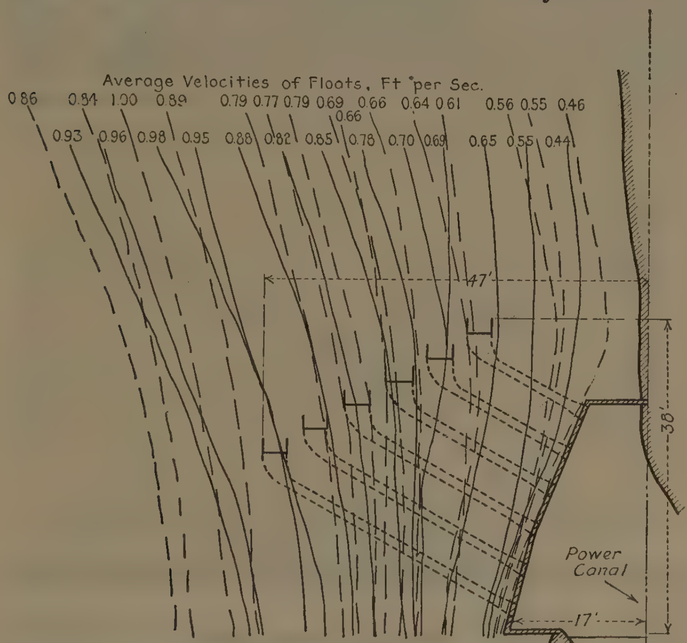

Fig. 21 The Author's Intake Model—Deep Floats. Broken Lines show Paths of Floats with the Intake Closed and Drawing No Water. Full Lines are for a Draft of 9.56 Cu. Ft. per Sec.; River Gage, 559.6 Ft.

and two feet per second and that the observations had to be taken with moderate speed in the field, these results are fairly satisfactory.

58 The distribution of draft in the tubes is shown in Table 1, which gives the discharges taken from an average of six velocity measurements at each section.

59 The loss of head in this intake was measured carefully and amounted to 1.45 in. with a draft of 15,000 cu. ft. per sec. (8.72 cu. ft. per sec. on the model) and was 2.15 in. with a draft of 20,000 cu. ft. per sec. In the first model the diffuser at the inner

end was not well proportioned and a second set of experiments was made with an improved diffuser and also with the inner end of the tube shortened 10 ft. so as to bring the inner port 10 ft. closer to the shore. This modified intake shown in Fig. 19, worked quite as well as the former one, indicating that the tubes could be reduced by 200 ft. in length, making them extend 1100 ft. beyond the diverter, without impairing their efficiency, and that possibly still greater reduction might be made.

60 The modified intake showed a reduction in friction loss of head to 0.70 in. at a draft of 15,000 cu. ft. per sec. and 1.20 in. at 20,000 cu. ft. per sec., which are only about one-half the former results. Lack of time prevented farther experimenting on this intake.

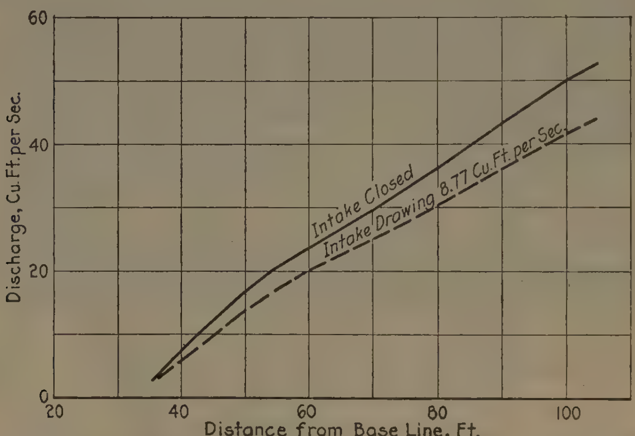

FIG. 22 TOTAL DISCHARGE UNDER BRIDGE WITH JOHNSON AND WAHLMAN INTAKE

THE AUTHOR'S INTAKE MODEL

61 This intake was designed on a different line from the former one and its general appearance is shown in Figs. 20 and 21. There were six tubes with a diffuser on the inner end of each, and the outer end was a large rectangular opening 24 in. by 36 in.; the tubes were sunk below the bed of the river, with the opening lying on the river bed. The tubes all had a uniform depth of 10 in. and a width proportioned to produce uniform draft in each tube as well as possible. An examination of Figs. 19 and 21 gives the relative sizes of this and the former intake, and it is also evident that this intake draws from a narrow strip of river reaching out diagonally from shore, rather than from the large rectangular area used by the Johnson scheme.

62 The results of float experiments on this intake at River El. 559.6 ft. when drawing 9.56 cu. ft. per sec., corresponding to

17,100 cu. ft. per sec. on the full-sized intake, are shown in Fig. 21, and the conditions in the river with the intake shut off are shown by broken lines, from which it will be seen that there would be a slight tendency for thickening of ice over the openings. The floats were 6 in. deep in many cases and no float entered the intake even at reduced river flow. Floats passing over the intake showed a slight hesitation on passing the downstream edge of the opening, but by comparing the mean velocities of two floats, one of which passed over the opening and the other passed between two openings, no difference was observable.

63 Pitot-tube studies on this intake were also made and the results of some of them are given in Table 2; they show that the draft was quite evenly divided, and that slightly different

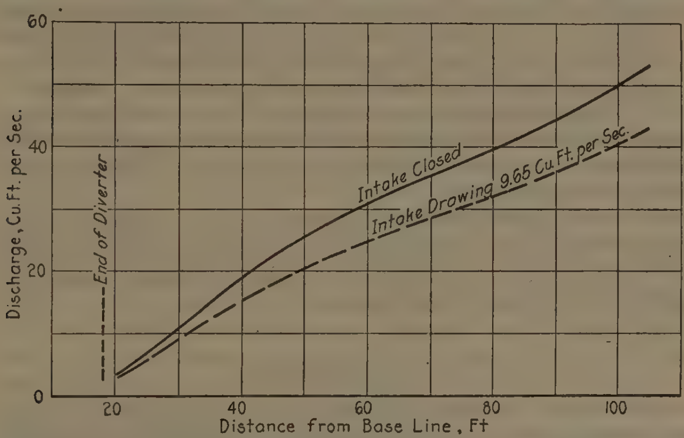

FIG. 23 TOTAL DISCHARGE UNDER BRIDGE WITH THE AUTHOR'S INTAKE

proportions of tubes would make this more uniform still. The loss of head with a draft of 9.56 cu. ft. per sec. on the model was 0.59 in. and at 11.95 cu. ft. per sec. was 0.964 in., the former corresponding to the full-sized discharge of 17,100 cu. ft. per sec. and the latter to a discharge of 21,400 cu. ft. per sec., and the loss computed to the discharge of 15,000 cu. ft. per sec. is 0.46 in. in the first case and 0.47 in. in the second.

TABLE 2 PITOT-TUBE MEASUREMENT IN TUBES—DISCHARGE IN CU. FT. PER SEC.

Tube						
A	*B*	*C*	*D*	*E*	*F*	Total
1.69	1.64	1.63	1.59	1.53	1.57	9.65 (Weir gave 9.56)
2.08	2.02	2.09	1.95	1.89	1.96	11.99 (Weir gave 11.95)

64 It will be noticed that there is very good agreement between the weir and the pitot-tube measurements and similar agreement was found in all the experiments.

65 Both the Johnson and Wahlman intake and the author's intake were tried with reduced river discharges and also with drafts much in excess of the 15,000 cu. ft. per sec., as the above results indicate. Neither scheme was satisfactory if over 50 per cent of the river flow was cut off, even when drawing its normal amount of water.

66 Figs. 22 and 23 show that there is a marked similarity in the general effect of these two intakes as far as the river flow is concerned, and that both intakes affected the entire width of the river, as is evidenced by the fact that the two lines on each figure continually diverge as they go out from shore. In this connection it will be well to reëxamine Fig. 11, in which only intake *G* had a similar tendency, the others having very little effect on the river beyond the point of the intake, being, no doubt, too small.

67 That the intakes must be made to draw from lines normal to the shore rather than parallel with it is evident, and an effective design may be made to distribute its suction over a much greater distance than it extends.

68 Only a part of the experimental work done is shown in the paper owing to lack of space. The author expresses his thanks to the Hydro-Electric Power Commission of Ontario, Canada, for permission to publish these results.

DISCUSSION

B. F. Groat.[1] The author refers to a paper presented before the American Society of Civil Engineers, entitled Ice Diversion, Hydraulic Models and Hydraulic Similarity. The paper is confined principally to experiments on models. Doubtless limited space prevented reference to a subsequent illustrated article by the writer on Ice Diversion for St. Lawrence River Power Company.[2] The ice-diversion works referred to in this article were designed and installed by the writer for the St. Lawrence River Power Company at Massena, N. Y. These works have been in use since December, 1918.

The Massena plant has been operated for many years. As much power has always been drawn during the winter as ice conditions would permit, excepting in 1907 and 1908 when stagnant industry compelled a general shutdown. The power demand for the manufacture of aluminum at this plant requires maximum capacity load at all times, and the winter load curve prior to the installation of ice diversion may be adopted as an accurate indication of the

[1] Consulting Engineer, Philadelphia, Pa. Mem. A.S.M.E.
[2] *The Canadian Engineer,* vol. 39, Nov. 25, 1920, p. 545.

limitation of power output by ice conditions. Improvements were made from time to time during the decade preceding the installation of ice-diversion works, but there was little or no increase in output during the winter months owing to ice troubles, and frequently the plant was almost shut down for considerable periods.

The following is a comparison of the power output during the first quarter of 1919, the ice-diversion works being in operation, with the average power output during the first quarters of the preceding ten years when no ice diversion was in operation, and also with the average of the three monthly power-output maxima for January, February, and March during the same ten years:

	Kilowatts
Ten-year average without ice diversion	11,600
Maximum (1915-1916) without ice diversion	21,500
Actual, 1919, with ice diversion	41,700
Increase, 1919 over 10-year average	30,100
Increase, 1919 over previous maximum	20,200

After making additions for some average increase during December and April, in which months there are sometimes heavy losses, it may be shown that there was an average annual increase of about 30,000 kw. for four months, or about 10,000 kw-years per average ice season. There was no serious trouble with ice entering the canal during the two extreme seasons, 1918-1919, and 1919-1920. Nor has there been any serious trouble due to ice entering the canal since the installation of the ice-diversion works. Practically all this increase is due to ice diversion, since, otherwise, ice would fill the canal and cut down the output as formerly.

The seriousness of ice trouble is not confined to losses of power resulting from shutdowns. Frequently costly damage results to hydraulic works and power plant which, in a single case, may run into many thousands of dollars. Damages of such magnitude are sometimes inflicted within a few seconds of time.

The author, in Par. 38, refers to experiments with a model, ascribed to the writer, an "open" intake, which did not give satisfaction. But he further states that "probably some other proportions or dimensions than those used were necessary, but it did not seem advisable at the time to continue work on it." But he then indicates, in Par. 39, that modifications of this model were made which brought forth further experiments. In view of these statements it has been deemed advisable to call attention to the successful works at Massena, based upon the invention of the writer.

In connection with this subject reference may be made to a valuable paper on hydraulic models, by Francesco Marzolo,[1] containing a number of references to other literature upon the subject.

[1] *Giornale del Genio Civile,* vol. 55, Apr. 30, 1917, p. 187.

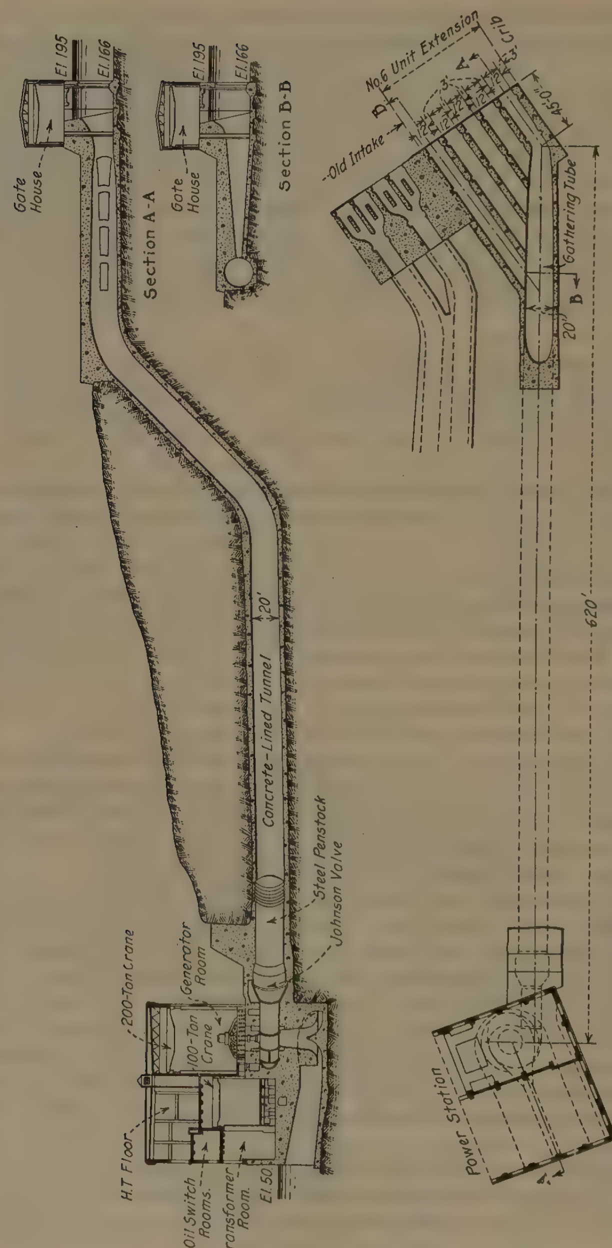

FIG. 24. GENERAL ARRANGEMENT OF INTAKE AT SHAWINIGAN FALLS

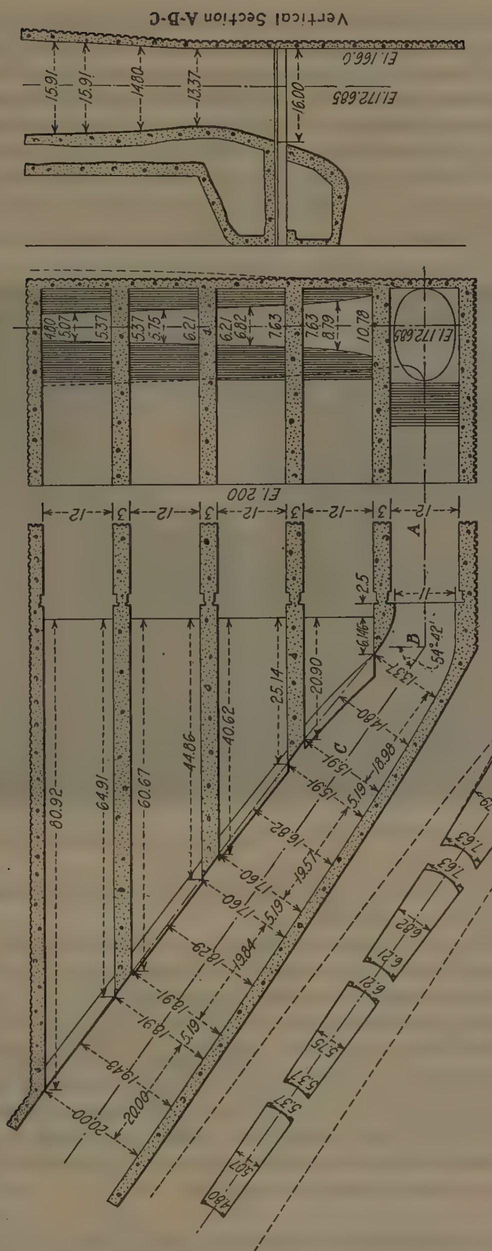

FIG. 25 DISTRIBUTOR INTAKE AT SHAWINIGAN FALLS FOR 2940 CU. FT. PER SEC.

R. D. Johnson.[1] The author does not seem to have made clear the net result of the comprehensive experiments which were instituted and directed by the engineers of the Hydro-Electric Power Commission, and which found the Wahlman intake to be the most satisfactory solution of their problem. This intake, which has been partly constructed, is the evolution of an idea conceived and used by Mr. Wahlman in a South American water-power plant where it was necessary to remove sand along the entire width at the bottom of a forebay. It became desirable to provide suckers

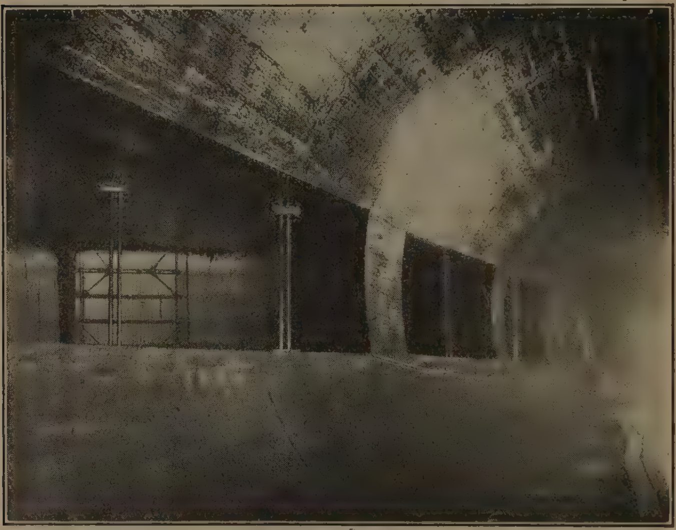

Fig. 26 Interior of Gathering Tube at Shawinigan Falls

which would take in water evenly throughout their length, although discharging from only one end.

This idea lends itself quite naturally to a water intake where it is desirable to so distribute the flow over a large area of the river that there is no appreciable vertical velocity at any point which might tend to cause obstruction by means of floating ice.

In Par. 55, the author says, "If the objects of the design were realized the intake would draw uniformly over an area 40 ft. by 25 ft. at the river bed, or in the full-sized intake over an area of 400,000 sq. ft." The writer thinks that attention should be called to the fact that the objects were realized to the fullest extent which could have possibly been expected.

[1] Hydraulic Engineer, Johnson & Wahlman, New York, N. Y. Mem. A.S.M.E.

In Par. 54, the author speaks of guide vanes set in a manner properly to direct the water. While it is perfectly true that these guide vanes were placed for that purpose, yet it was later discovered that they were quite unnecessary and in the finished design for the full-size structure the guides were omitted as far as possible; in fact, they were only inserted occasionally as mechanical spacers for structural reasons.

This idea of drawing water uniformly has a useful application rather different from that described in the paper. Figs. 24 to 26 show a design which was built by the Shawinigan Water and Power Company at Shawinigan Falls, Quebec. The problem here was to enter water into a 20-ft. tunnel out of a forebay in such manner that the flow would be uniformly distributed through a considerable width of trash racks.

Fig. 24 shows the general arrangement with the water entering at a substantially uniform rate through five passages. This is made possible by the proper dimensioning of the slot where the ends of these passages enter the gathering tube. Fig. 25 shows a larger plan of the intake proper and probably requires no further description. Fig. 26 is a photograph taken inside of the gathering tube and looking outward through one of the passages toward the trash racks. The vertical bars which appear to be obstructions in the passage are merely temporary supports for electrodes placed to carry out certain water measurements by the salt velocity method. The behavior of this intake was highly satisfactory to its owners.

The theory on which these slots are proportioned has been proven to be accurate and sufficiently flexible to enable a preconceived rate of flow to be realized in any arbitrary manner which may be selected. That is to say, it is quite feasible to arrange for unequal drafts from one end of the slot to the other with any prescribed variation, which may conform more perfectly to certain requirements than strict uniformity might do.

The Author. The author appreciates the additional information supplied by Mr. Groat, and is glad to see this added to the paper. He regrets that he neglected to mention the article in *The Canadian Engineer* referred to by Mr. Groat.

Regarding Mr. Groat's criticism of Pars. 38 and 39, the modifications referred to were begun by building on the original framework previously described, and were such as could be made without digging out the bed; only very slight variations in the dimensions and positions of the slots would have been possible under these circumstances — too slight, in the author's opinion, to be worth while.

After agreeing on the site at Dufferin Islands, the Hydro engineers left the work entirely in the author's hands, with the under-

standing that intake *A* was first to be installed; and, with the exception of intake *H*, the remainder of the first season's work, involving very much more than the paper describes, was devised and carried out by the author, his only assistant being Mr. J. J. Traill, B. A. Sc., at that time a member of the University Staff. In the second season's work the one intake was suggested, designed, and developed by the author alone.

Intake *G*, tried in September, 1918, was worked out quickly and in a rough way in the field, being based on the same ideas as submitted by Mr. Johnson to the Commission in a report dated Jan. 31, 1919, dealing with his intake, full information on the author's first season's work being available to him.

The meaning of Par. 55 would be more clear if the last sentence read "The objects of the design were to make the intake draw uniformly over an area, etc." The author never understood the objects of the guide vanes in the model, Fig. 18, and is glad they are to be left out.

The results of the experiments do not differentiate appreciably between the intakes shown in Figs. 18 and 20 and both worked well, but the marked difference in construction leaves room for a difference of opinion as to which to adopt. The author believes there are other equally good solutions, and hopes the work described may prove of help to those trying to solve this difficult problem.

Information on the working and efficiency of the design shown in Figs. 24 and 25 is contained in an article by W. R. Way in *The Engineering Journal,* the journal of the Engineering Institute of Canada, for October, 1924.

No. 1946

A METHOD FOR THE ECONOMIC DESIGN OF PENSTOCKS

By H. L. Doolittle,[1] Los Angeles, Cal.
Member of the Society

The author presents an original, simple graphic method for the economic design of penstocks. Given a certain flow of water, length of piping, and profile of penstock, curves can be rapidly drawn for pipes of varying diameter which indicate the frictional loss and its value in dollars. Other curves indicate the cost of a certain diameter of piping, which includes such items as the cost of pipe in place, interest, and depreciation. The costs being plotted as ordinates, and the lengths of pipe as abscissas, the total area under a curve representing the summation of these curves indicates the total cost for any diameter of piping selected. It is therefore possible to determine by inspection the most economical system. In view of the large number of variables entering into the problem, it is believed that the method presented should be of considerable help to the designer, and should eliminate guesswork in the selection of suitable pipe diameters.

When laying out a penstock for the first time the designer is impressed by the large number of variables entering into the problem and may be at a loss to know how to arrive at a definite solution. In the end he will probably resort to some trial-and-error method for determining the most economic size, and even after a great amount of tedious calculation may not be certain that he has arrived at the best design. It was in an effort to obtain a more rational way of working that the following graphic method was evolved. This is not claimed to be an exact solution of an intricate problem, but it is believed to be correct enough for all practical purposes and very simple in its application.

2 This discussion will apply only to a pipe line used in connection with a hydroelectric plant. Also the simplest kind of pipe will be assumed as it is desired to present only the manner of

[1] Asst. Constr. Engr., So. Cal. Edison Co.

Contributed by the Power Division and presented at the Annual Meeting of The American Society of Mechanical Engineers, New York, Dec. 1 to 4, 1924.

applying the method instead of working out a number of different designs.

General Description

3 In the method presented it is taken as a working premise that "the most economical penstock will be that in which the sum of the annual value of the power lost in friction and the annual charges for interest, depreciation, taxes, etc., is a minimum."

4 The fundamental principle of the method is as follows: If a curve be plotted with lengths of pipe as abscissas and *total* annual costs *per foot* as ordinates, the area under the curve will be proportional to the total annual cost of the entire line. The ordinate at any point represents the cost of the pipe per foot at that paricular position in the line. If, now, several curves be drawn on the same chart for different diameters of pipe, the most economical pipe will be the one made up of the various diameters giving the minimum area between the pipe curves and the x-axis. This can be made clear most easily by means of an illustrative problem.

5 Assume the simple case of a hydroelectric plant to utilize a maximum of 540 sec-ft. of water in two units, there being one penstock for each unit. Assume further that this is a stream-flow plant using all the water in the river up to the maximum capacity of the units.

Variables Entering into the Problem

6 In determining the economic size of penstock to be used for a water-power plant, the following items must be given consideration:

a Daily variation of flow through the pipe
b Estimated load factor of the plant over a term of years
c Profile of the line
d Number of pipes
e Kind of pipe to be used
f Diameter and thickness of pipe
g Value of power lost in friction
h Cost of pipe installed
i Cost of piers and anchors
j Annual cost of pipe in place
k Increased thickness to provide for water hammer or corrosion
l Maximum permissible velocity.

These items will now be discussed in order to point out their bearing on the problem.

7 *Daily Variation of Flow; Estimated Plant Load Factor.* These items being closely related will be discussed together.

When it is remembered that the friction in the pipe varies roughly as the square of the velocity of flow, it is seen that the

value of the power lost in friction cannot be calculated unless the variations in velocity be taken into account. These variations in velocity may result from the load demand on the plant, as in the case of a plant operating on water from a storage reservoir, or, in the case of a plant without storage, from the variable flow of the river. The second case would obtain in a large system where stream-flow plants generally operate at maximum possible capacity and the regulation is taken on the plants having storage.

8 The case assumed is that of a stream-flow plant, so it is first necessary to obtain an average hydrograph of the river flow. If records of flow for several years are available, these can be combined to give an average yearly hydrograph. Such an average

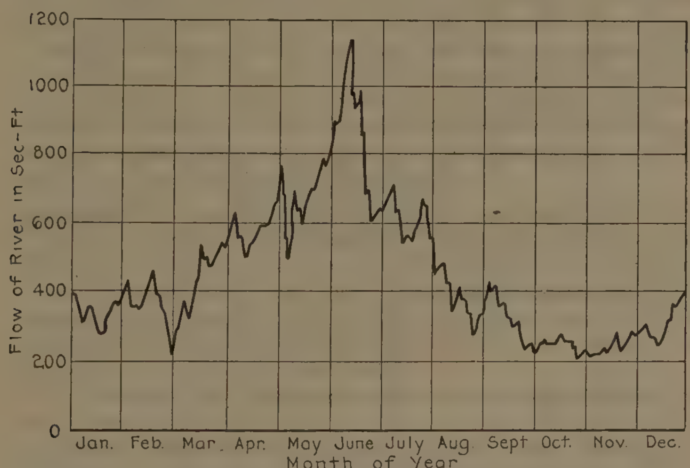

Fig. 1 Average Yearly Hydrograph of River Flow

curve of flow is shown in Fig. 1. On account of the variation of flow with the square of velocity it would be incorrect to take the mean flow of the river below 540-sec-ft. as the capacity for which the pipes are to be designed. The friction could be calculated for various points and then averaged, but this would be a very lengthy operation. This calculation can be performed by the following simple method.

9 Since the plant is assumed to operate at the maximum load corresponding to the water in the river, it is immaterial when the variation in load occurs, but essential that the percentages of time be known during which the plant operates at the various loads. It is therefore necessary to plot a curve with the flow of the river in second-feet as ordinates and percentages of time as abscissas. This curve (Fig. 2) can be made from the average hydrograph, Fig. 1, or direct from the records of river flow over a term of years.

10 For simplicity let it be assumed that the friction varies as the square of the velocity, although any other relation could be

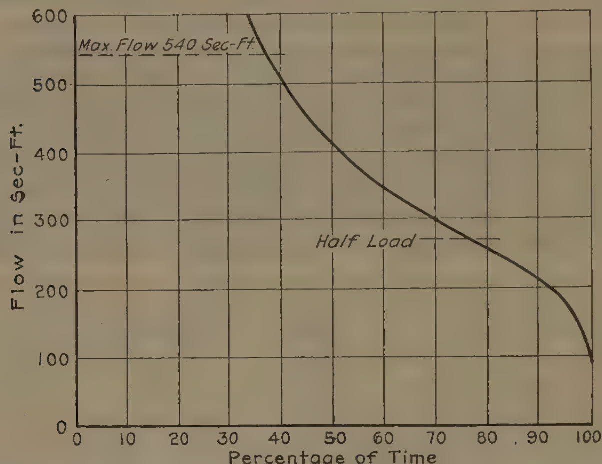

FIG. 2 CURVE SHOWING RELATION BETWEEN FLOW AND PERCENTAGE OF TIME

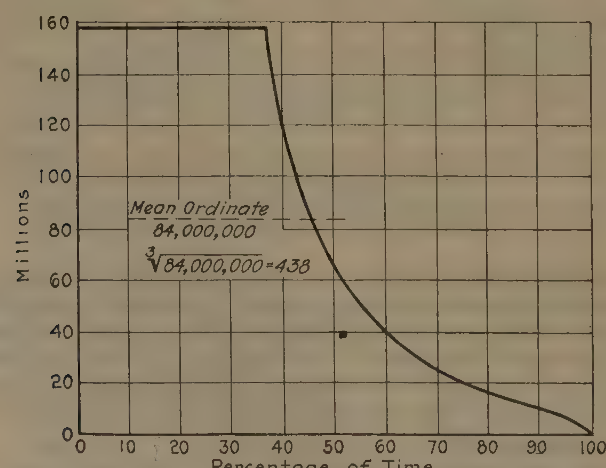

FIG. 3 CURVE WHOSE ORDINATES ARE THE CUBES OF THE ORDINATES OF FIG. 2

used if desired. The power lost by friction is proportional to the product of the flow and the friction head, and, since the flow is proportional to the velocity, it follows that the power loss due to friction varies as the cube of the velocity. It is now possible to

find the volume of water which, flowing at a constant rate throughout the year, will produce the same total frictional power loss as the variable flow. This is done by taking the cube root of the mean of the cubes of the ordinates to the curve of Fig. 2. Fig. 3 shows the cube of Fig. 2 curve, and also the cube root of its mean. This mean of the cube curve is 438 sec-ft. All calculations will be greatly simplified by designing the two pipes for a constant flow of 438 sec-ft., or 219 sec-ft., each.

11 It might be pointed out that in the case of a plant operating on storage water the curve of Fig. 2 should be calculated from the predicted output of the plant rather than from the flow of the river.

12 *Profile of the Pipe.* The nature of the profile on which the pipe is to be laid will have an effect on the economic size. Assume in the present case, that the profile of the pipe is as shown in Fig. 4. On this chart is also shown a line representing the maximum water level in the forebay to which the pipe connects. It is now necessary to construct Table 1, giving the actual calculated lengths of pipe from the top of the line for equal increments of head. In this case increments of 5 ft. are assumed.

13 *Number of Pipes.* In some cases it is considered good practice to install a separate pipe for each unit, while in others there may be one pipe at the top of the profile branching into two or more pipes as the power house is approached. This method will indicate at what point on the profile it is economical to increase the number of pipes. In this example it has been assumed that there will be one pipe for each unit.

14 *Kinds of Pipe.* There are many kinds of pipe available for use in penstock design. If there is a long, flat grade at the top of the profile it may be economical to install wood-stave pipe or light riveted steel pipe. As the head increases it may be more economical to use welded pipe. The most economical kind of pipe to use at any location can be readily determined.

15 *Diameter and Thickness of Pipe.* There are an indefinite number of pipes that can be used, varying from those having a constant diameter throughout to those decreasing in size toward the bottom of the line. It is the determination of the points at which to change diameter and thickness that is the most difficult in any trial-and-error calculation. This is a very simple operation with the method herein described. Pipe diameters should vary by 6-in. increments as this will permit nesting them for shipment, and smaller increments would have very little effect on the economics of the design.

16 *Value of Power Lost in Friction.* The value placed on the power lost due to friction in the pipe has a large effect on the determination of the size of pipe to adopt. Obviously a smaller pipe can be used if the value of power is low, as this would justify a greater frictional loss. The proper price to place on this power

might be taken as the selling price of the power developed, or the cost to produce the power at the power house. This can be settled as the designer wishes, although it is believed that the cost to produce at the power house is more nearly correct.

17 *Cost of Pipe Installed.* The application of this method is much simplified if a price per pound or per foot can be adopted for the cost of the pipe in place. This does not have to be a constant price but can vary for different diameters and thicknesses, or even for different locations on the profile. The method permits the widest latitude in the variation in costs. In applying a cost per foot this cost must include a proportionate amount for the cost of girth joints and other special work. As we are concerned largely with the relative cost of pipes of different sizes and kinds,

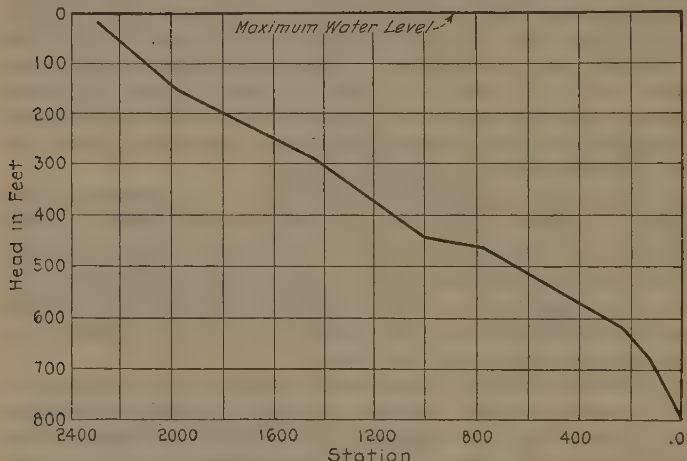

Fig. 4 Actual Profile of Penstock

it is proper to omit items of cost that are common to all pipes, as cost of tramways, construction plants, etc.

18 *Anchors and Piers.* The cost of anchors and supporting piers affects the problem to some extent and, if desired, an amount may be added to the cost of pipe per foot to cover this item. It is probable that in most cases the cost of these is nearly constant for all pipes, so this item may generally be disregarded.

19 *Annual Cost of Pipe.* This item should cover interest, depreciation, taxes, etc., also any expense for upkeep that varies with the size or kind of pipe. In most cases a percentage of the cost of the pipe in place will be sufficiently correct.

20 *Increased Thickness.* This method permits making allowance for any increase in thickness desired to provide for corrosion or water hammer. The increase for corrosion is made by adding a constant quantity to the calculated thickness for a given head.

The increase for water hammer can best be made by so plotting the line in Fig. 4, representing maximum water level, that it will give the maximum static head plus water hammer at every point in the line. This will also change the figures in Table 1, which are now shown for the static head only.

21 *Maximum Velocity.* Sometimes it may be considered desirable to limit the maximum velocity in the pipe. In these cases the diameter corresponding to the maximum flow is calculated and this figure used as the minimum diameter on the charts. It is probable that this method of determining economic size will give higher velocities at the bottom of the line than would be ordinarily expected.

TABLE 1 PROFILE OF PENSTOCK

(Actual lengths of pipe in feet from forebay for 5-ft. increments of head)

Head	Length	Head	Length	Head	Length	Head	Length
150	300	315	905	480	1640	645	2155
155	310	320	920	485	1660	650	2170
160	325	325	935	490	1680	655	2185
165	335	330	955	495	1700	660	2200
170	350	335	970	500	1720	665	2215
175	365	340	985	505	1740	670	2230
180	380	345	1000	510	1760	675	2240
185	400	350	1015	515	1770	680	2250
190	420	355	1035	520	1780	685	2260
195	440	360	1055	525	1790	690	2275
200	460	365	1065	530	1805	695	2285
205	480	370	1080	535	1815	700	2295
210	500	375	1095	540	1830	705	2305
215	515	380	1110	545	1840	710	2315
220	535	385	1125	550	1855	715	2325
225	555	390	1145	555	1865	720	2335
230	575	395	1160	560	1880	725	2345
235	595	400	1175	565	1895	730	2355
240	615	405	1190	570	1915	735	2365
245	630	410	1205	575	1930	740	2375
250	650	415	1220	580	1945	745	2385
255	670	420	1240	585	1960	750	2395
260	690	425	1255	590	1975	755	2405
265	710	430	1270	595	1990	760	2415
270	730	435	1285	600	2010	765	2425
275	750	440	1300	605	2025	770	2435
280	770	445	1315	610	2045	775	2445
285	790	450	1330	615	2060	780	2455
290	810	455	1345	620	2075	785	2465
295	830	460	1360	625	2090	790	2475
300	850	465	1375	630	2105	795	2480
305	870	470	1390	635	2120	800	2500
310	890	475	1500	640	2140		

Numerical Example

22 An actual case will now be worked out on the basis of the data previously given and the following assumptions.

23 The average hydrograph over a term of years is as shown in Fig. 1. In Fig. 2 is shown the curve of flow for two pipes based on percentage of time. Fig. 3 gives the cube of the curve in Fig. 2 with its mean of 438 sec-ft., which is to be used for design as previously discussed in Par. 7, et seq. It is assumed that the two pipes carry an equal amount of water at all times. If this is not the case, a separate determination of the mean flow must be made for each pipe. It is also assumed that the profile is as given in Fig.

4 and that the actual length of pipe for 5-ft. increments of head is as given in Table 1 (no allowance will be made for corrosion or water hammer). In addition to the foregoing the following data will be used:

Cost of riveted pipe in place, cents per lb.	9.5
Coefficient of friction, *n*	0.015
Unit stress in steel, lb. per sq. in.	11,000
Total annual cost for interest, etc., per cent	9
Value of power per year for 1 ft. friction	$1000

24 The first step is to prepare a table such as Table 2, giving data for different diameters of pipe and various thicknesses of plate. A rough calculation based on average velocities will give a

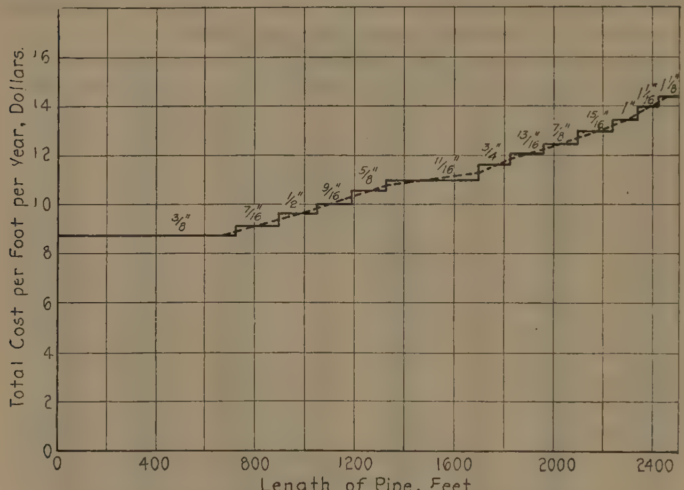

Fig. 5 Curve for Riveted Pipe 5 ft. in Diameter

fair idea of the diameters to assume in making up the table. The diameters and thicknesses are first assumed and entered in columns 1 and 2. The maximum head for which a given diameter and thickness can be used is next calculated, using any stresses, factor of safety, etc., that the designer desires; these heads are entered in column 3. From Table 1 should be obtained the lengths of pipe from the top of the line for the heads just calculated. These are shown in column 4.

25 Columns 5, 6, 7, and 8 give the weights *per foot of pipe* for the pipe shell, girth joints, longitudinal joints, and the complete pipe. The weights of pipe per foot are now multiplied by 9 per cent of the price per pound for the pipe installed, thus giving the values for interest, depreciation, etc., as shown in column 9.

26 The friction per foot of pipe is next calculated, taking account, if desired, of increasing friction due to greater thicknesses

of plate. In this case, for simplicity, the friction has been assumed to be constant for a given diameter. This friction head is next multiplied by the assumed value per year for 1 ft. friction (in this case $1000), which gives the costs entered in column 10. The addition of columns 9 and 10 gives the total annual cost per foot of pipe for all diameters and thicknesses.

TABLE 2 RIVETED-PIPE COSTS

(Weights and costs are per foot of pipe. Data assumed: Cost of pipe in place, 9½ cents per lb.; interest, depreciation, etc., 9 per cent; cost of 1 ft. friction, $1000 per year; friction coefficient $n = 0.015$)

(1) Diam.	(2) Thickness, in.	(3) Head, ft.	(4) Length, ft.	(5) Net wt. pipe, lb.	(6) Wt. girth joint, lb.	(7) Wt. long. joint, lb.	(8) Total wt. lb.	(9) Int. and depn.	(10) Cost of friction	(11) Total annual cost
	3/8	150	300	433	43	30	506	$4 32	$0.27	$4.59
	7/16	175	365	506	51	37	594	5.07	0.27	5.34
	1/2	200	460	578	58	45	681	5.82	0.27	6.09
	9/16	225	555	651	65	53	769	6.57	0.27	6.84
	5/8	250	650	724	72	66	862	7.37	0.27	7.64
	11/16	275	750	796	80	73	949	8.10	0.27	8.37
	3/4	300	850	869	122	80	1071	9.15	0.27	9.42
	13/16	325	935	942	132	86	1160	9.91	0.27	10:18
	7/8	350	1015	1016	142	93	1251	10.70	0.27	10.97
9 ft. 0 in.	15/16	375	1095	1089	152	99	1340	11.45	0.27	11.72
	1	400	1175	1162	163	106	1431	12.23	0.27	12.50
	1 1/16	425	1255	1237	173	113	1523	13.01	0.27	13.28
	1 1/8	450	1330	1310	182	119	1612	13.79	0.27	14.06
	1 3/16	475	1500	1384	194	126	1704	14.57	0.27	14.84
	1 1/4	500	1720	1458	204	133	1795	15.35	0.27	15.62
	1 5/16	525	1790	1532	214	139	1885	16.10	0.27	16.37
	1 3/8	550	1855	1605	224	146	1975	16.89	0.27	17.16
	1 7/16	575	1930	1679	235	152	2066	17.65	0.27	17.92
	1 1/2	600	2010	1751	245	159	2155	18.41	0.27	18.68
	3/8	160	325	409	41	30	480	4.10	0.36	4.46
	7/16	185	400	478	48	37	563	4.81	0.36	5.17
	1/2	210	500	546	55	45	646	5 52	0.36	5.88
	9/16	235	595	615	62	53	730	6.24	0.36	6.60
	5/8	265	710	684	68	66	818	6.99	0.36	7.35
	11/16	290	810	753	75	73	901	7.70	0.36	8.06
	3/4	315	905	822	115	80	1017	8.69	0.36	9 05
8 ft. 6 in.	13/16	345	1000	891	125	86	1102	9.43	0.36	9.79
	7/8	370	1055	960	134	93	1187	10.14	0.36	10.50
	15/16	395	1160	1029	144	99	1272	10.89	0.36	11.25
	1	425	1255	1099	154	106	1359	11.60	0.36	11.96
	1 1/16	450	1330	1170	164	113	1447	12 37	0.36	12.73
	1 1/8	475	1500	1239	174	119	1532	13.10	0.36	13.46
	1 3/16	505	1740	1309	183	126	1618	13.81	0.36	14.17
	3/8	170	350	386	39	30	455	3.89	0.50	4.39
	7/16	195	440	450	45	37	532	4.55	0.50	5 05
	1/2	225	555	515	52	45	612	5.23	0.50	5.73
	9/16	250	650	580	58	53	691	5.91	0.50	6.41
	5/8	280	770	645	65	66	776	6.64	0.50	7.14
	11/16	310	890	709	71	73	853	7.29	0 50	7.79
	3/4	340	970	775	109	80	964	8 24	0.50	8.74
	13/16	365	1065	839	117	86	1042	8.91	0.50	9.41
	7/8	395	1160	905	127	93	1125	9.60	0.50	10.10
8 ft. 0 in.	15/16	420	1240	969	136	99	1204	10.30	0.50	10.80
	1	450	1330	1036	145	106	1287	11.00	0.50	11.50
	1 1/16	480	1640	1100	154	113	1367	11.69	0.50	12.19
	1 1/8	505	1740	1167	163	119	1449	12.39	0.50	12.89
	1 3/16	535	1815	1230	172	126	1528	13.06	0.50	13.56
	1 1/4	560	1880	1298	182	133	1613	13.79	0.50	14.29
	1 5/16	590	1975	1361	191	139	1691	14.45	0.50	14.95
	1 3/8	620	2075	1430	200	146	1776	15 19	0.50	15.69
	1 7/16	645	2155	1492	209	152	1853	15.83	0.50	16.33
	1 1/2	675	2240	1562	219	159	1940	16.59	0.50	17.09

(Continued on following page)

TABLE 2 RIVETED-PIPE COSTS (*Continued*)

(1) Diam.	(2) Thickness, in.	(3) Head, ft.	(4) Length, ft.	(5) Net wt. pipe, lb.	(6) Wt. girth joint, lb.	(7) Wt. long. joint, lb.	(8) Total wt. lb.	(9) Int. and depn.	(10) Cost of friction	(11) Total annual cost
7 ft. 6 in.	3/8	180	380	362	36	30	428	3.66	0.69	4.35
	7/16	210	500	422	42	37	501	4.28	0.69	4.97
	1/2	240	615	483	48	45	576	4.92	0.69	5.61
	9/16	270	730	543	54	53	650	5.55	0.69	6.24
	5/8	300	850	605	61	66	732	6.25	0.69	6.94
	11/16	330	955	665	67	73	805	6.88	0.69	7.57
	3/4	360	1055	727	102	80	909	7.76	0.69	8.45
	13/16	390	1145	787	110	86	983	8.40	0.69	9.09
	7/8	420	1240	849	119	93	1061	9.07	0.69	9.76
	15/16	450	1330	909	127	99	1135	9.70	0.69	10.39
	1	480	1640	972	136	106	1214	10.39	0.69	11.08
	1 1/16	510	1760	1031	144	113	1288	11.00	0.69	11.69
	1 1/8	540	1830	1095	154	119	1368	11.69	0.69	12.38
	1 3/16	570	1915	1153	162	126	1441	12.31	0.69	13.00
	1 1/4	600	2010	1218	171	133	1522	13.01	0.69	13.70
	1 5/16	630	2105	1278	179	139	1596	13.64	0.69	14.33
	1 3/8	660	2200	1342	188	146	1676	14.32	0.69	15.01
	1 7/16	690	2275	1401	196	152	1749	14.94	0.69	15.63
	1 1/2	720	2335	1466	205	159	1830	15.63	0.69	16.32
	1 9/16	750	2395	1527	214	166	1907	16.30	0.69	16.99
7 ft. 0 in.	3/8	195	440	338	34	30	402	3.44	0.98	4.42
	7/16	225	555	394	39	37	470	4.02	0.98	5.00
	1/2	255	670	451	45	45	541	4.62	0.98	5.60
	9/16	290	810	508	51	53	621	5.24	0.98	6.22
	5/8	320	920	565	56	66	687	5.87	0.98	6.85
	11/16	355	1035	621	62	73	756	6.46	0.98	7.44
	3/4	385	1125	679	95	80	854	7.30	0.98	8.28
	13/16	415	1220	735	103	86	924	7.90	0.98	8.88
	7/8	450	1330	793	111	93	997	8.52	0.98	9.50
	15/16	480	1640	849	119	99	1067	9.12	0.98	10.10
	1	515	1770	908	127	106	1141	9.76	0.98	10.74
	1 1/16	545	1840	965	135	113	1213	10.37	0.98	11.35
	1 1/8	580	1945	1023	143	119	1285	11.00	0.98	11.98
	1 3/16	610	2045	1080	151	126	1357	11.60	0.98	12.58
	1 1/4	640	2140	1138	159	133	1430	12.22	0.98	13.20
	1 5/16	675	2240	1193	167	139	1499	12.81	0.98	13.79
	1 3/8	705	2305	1254	176	146	1576	13.47	0.98	14.45
	1 7/16	740	2375	1309	183	152	1644	14.06	0.98	15.04
	1 1/2	770	2405	1370	192	159	1721	14.72	0.98	15.70
6 ft. 6 in.	3/8	210	500	314	31	30	375	3.21	1.43	4.64
	7/16	240	615	366	37	37	440	3.76	1.43	5.19
	1/2	275	750	419	42	45	506	4.32	1.43	5.75
	9/16	310	890	472	47	53	572	4.89	1.43	6.32
	5/8	345	1000	525	52	66	643	5.50	1.43	6.93
	11/16	380	1110	577	58	73	708	6.05	1.43	7.48
	3/4	415	1220	631	88	80	799	6.83	1.43	8.26
	13/16	450	1330	683	96	86	865	7.39	1.43	8.82
	7/8	485	1660	737	103	93	933	7.97	1.43	9.40
	15/16	520	1780	789	110	99	998	8.53	1.43	9.96
	1	555	1865	844	118	106	1068	9.12	1.43	10.55
	1 1/16	590	1975	896	125	113	1134	9.70	1.43	11.13
	1 1/8	625	2090	961	133	119	1203	10.30	1.43	11.73
	1 3/16	660	2200	1003	141	126	1270	10.86	1.43	12.29
	1 1/4	690	2275	1058	148	133	1339	11.44	1.43	12.87
	1 5/16	725	2345	1110	155	139	1404	12.00	1.43	13.43
	1 3/8	760	2415	1166	163	146	1475	12.61	1.43	14.04
	1 7/16	795	2480	1216	170	152	1538	13.15	1.43	14.58
	1 1/2	830	2500+	1274	178	159	1611	13.77	1.43	15.20

(Continued on following page)

TABLE 2 RIVETED-PIPE COSTS (*Continued*)

(1) Diam.	(2) Thickness, in.	(3) Head, ft.	(4) Length, ft.	(5) Net wt. pipe, lb.	(6) Wt. girth joint, lb.	(7) Wt. long. joint, lb.	(8) Total wt. lb.	(9) Int. and depn.	(10) Cost of friction	(11) Total annual cost
6 ft. 0 in.	3/8	225	555	290	29	30	349	2.98	2.20	5.18
	7/16	260	690	338	34	37	409	3.49	2.20	5.69
	1/2	300	850	387	39	45	471	4.02	2.20	6.22
	9/16	340	985	435	44	53	532	4.55	2.20	6.75
	5/8	375	1095	485	49	66	600	5.12	2.20	7.32
	11/16	415	1220	533	53	73	659	5.63	2.20	7.83
	3/4	450	1330	583	82	80	745	6.36	2.20	8.56
	13/16	490	1680	631	88	86	805	6.88	2.20	9.08
	7/8	525	1790	681	95	93	869	7.42	2.20	9.62
	15/16	560	1880	729	102	99	930	7.95	2.20	10.15
	1	600	2010	780	109	106	995	8.50	2.20	10.70
	1 1/16	635	2120	828	116	113	1057	9.04	2.20	11.24
	1 1/8	675	2240	879	123	119	1121	9.59	2.20	11.79
	1 3/16	710	2315	927	130	126	1183	10.10	2.20	12.30
	1 1/4	750	2395	978	137	133	1248	10.66	2.20	12.86
	1 5/16	785	2465	1026	144	139	1309	11.19	2.20	13.39
	1 3/8	825	2500+	1077	151	146	1374	11.74	2.20	13.94
5 ft. 6 in.	3/8	245	630	266	27	30	323	2.76	3.51	6.27
	7/16	285	790	310	31	37	378	3.23	3.51	6.74
	1/2	325	935	355	36	45	436	3.72	3.51	7.23
	9/16	370	1080	400	40	53	493	4.21	3.51	7.72
	5/8	410	1205	445	45	66	556	4.75	3.51	8.26
	11/16	450	1330	489	49	73	611	5.21	3.51	8.72
	3/4	490	1680	535	75	80	690	5.90	3.51	9.41
	13/16	530	1805	579	81	86	746	6.37	3.51	9.88
	7/8	570	1915	625	88	93	806	6.89	3.51	10.40
	15/16	615	2060	669	94	99	862	7.36	3.51	10.87
	1	655	2185	716	100	106	922	7.88	3.51	11.39
	1 1/16	695	2285	760	106	113	979	8.36	3.51	11.87
	1 1/8	735	2365	807	113	119	1039	8.87	3.51	12.38
	1 3/16	775	2445	852	119	126	1097	9.37	3.51	12.88
	1 1/4	815	2500+	898	126	133	1157	9.89	3.51	13.40
5 ft. 0 in.	3/8	270	730	241	24	30	295	2.52	6.23	8.75
	7/16	315	905	282	28	37	347	2.96	6.23	9.19
	1/2	360	1055	323	32	45	400	3.42	6.23	9.65
	9/16	405	1190	363	36	53	452	3.86	6.23	10.09
	5/8	450	1330	404	40	66	510	4.36	6.23	10.59
	11/16	495	1700	445	45	73	563	4.81	6.23	11.04
	3/4	540	1830	486	68	80	634	5.41	6.23	11.64
	13/16	585	1960	527	74	86	687	5.87	6.23	12.10
	7/8	630	2105	568	80	93	741	6.33	6.23	12.56
	15/16	675	2240	609	85	99	793	6.77	6.23	13.00
	1	720	2335	650	91	106	847	7.24	6.23	13.47
	1 1/16	765	2425	692	97	113	902	7.71	6.23	13.94
	1 1/8	810	2500+	734	103	119	956	8.17	6.23	14.40
4 ft. 6 in.	3/8	300	850	218	22	30	270	2.30	11.04	13.34
	7/16	350	1015	254	25	37	316	2.70	11.04	13.74
	1/2	400	1175	291	29	45	365	3.12	11.04	14.16
	9/16	450	1330	328	33	53	414	3.54	11.04	14.58
	5/8	550	1720	364	36	66	466	3.98	11.04	15.02
	11/16	550	1855	401	40	73	514	4.39	11.04	15.43
	3/4	600	2010	438	61	80	579	4.95	11.04	15.99
	13/16	650	2170	475	67	86	628	5.36	11.04	16.40
	7/8	700	2295	512	72	93	677	5.79	11.04	16.83
	15/16	750	2395	549	77	99	725	6.19	11.04	17.23
	1	800	2500	587	82	106	775	6.62	11.04	17.66

27 From the description it may appear that these calculations are somewhat laborious, but with tables of friction and weights of pipe, etc., it is quite a simple matter. The figures of Table 2 should not be used for the purpose of actual design as the data assumed were solely for the purpose of illustration and would probably not apply to an actual case.

28 It is now possible to plot a curve for each diameter of pipe as shown in Fig. 5. This curve for a 5-ft.-diameter pipe is plotted by using the lengths given in column 4, of Table 2, as abscissas and the total annual costs per foot in column 11 as ordinates. Each step in the curve represents a change in thickness. Obviously the area under this curve represents the total annual cost

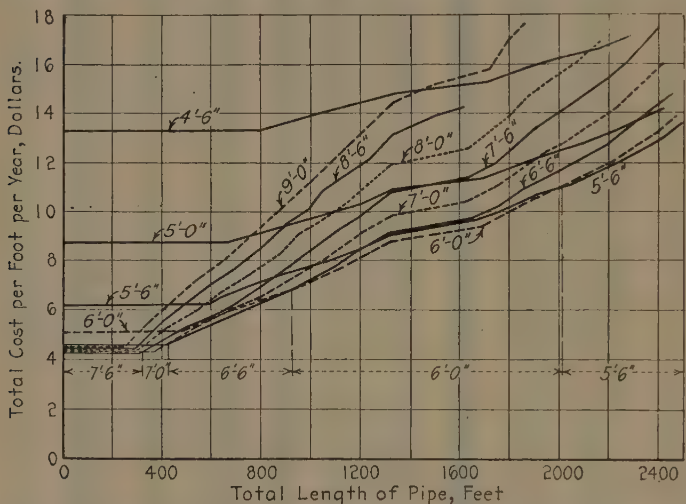

FIG. 6 CHART OF ANNUAL-COST CURVES FOR SEVERAL PIPES, FROM WHICH THE MOST ECONOMICAL PIPE CAN BE DETERMINED BY INSPECTION

of the entire 5-ft.-diameter pipe. The chart can be somewhat simplified if, instead of drawing a stepped curve, a line be drawn through the average costs of each pair of thicknesses.[1] This simplified curve is represented by the dotted line in Fig. 5.

29 If, now, a chart be prepared on which are plotted the annual-cost curves for several pipes it will furnish immediately the means of determining, by inspection, the most economical pipe. Such a chart is shown in Fig. 6, and it is apparent that the pipe of varying diameter represented by the combination of the bottom lines encloses the minimum area and consequently has the least annual cost.

[1] The author is indebted to R. M. Peabody for the suggestion of this simplification.

30 As it is necessary to vary thicknesses by increments of $\frac{1}{16}$ in. it may be found that the bottom line does not represent a pipe tapering regularly from the top but that the lines of different diameters cross and recross in an irregular manner. It is, however, a simple matter to choose from the curves a regularly tapering pipe which will have practically the same yearly cost as the theoretical minimum. Fig. 7 shows the final design as selected from

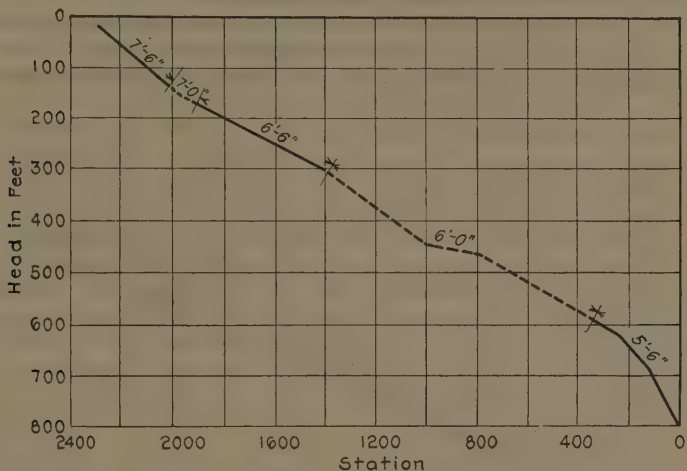

FIG. 7 MOST ECONOMICAL PIPE AS DETERMINED BY CURVES OF FIG. 6

the curves in Fig. 6 and these diameters, lengths, and thicknesses are given in Table 3.

TABLE 3 LENGTHS, THICKNESSES, AND DIAMETERS OF PIPE IN FIG. 7

Length, ft.	Thickness, in.	Diameter, ft.-in.	Length, ft.	Thickness, in.	Diameter, ft.-in.
320	$\frac{3}{8}$	7-6	350	$\frac{13}{16}$	6-0
105	$\frac{3}{8}$	7-0	110	$\frac{7}{8}$	6-0
75	$\frac{3}{8}$	6-6	90	$\frac{15}{16}$	6-0
115	$\frac{7}{16}$	6-6	135	1	6-0
135	$\frac{1}{2}$	6-6	45	$\frac{15}{16}$	5-6
140	$\frac{9}{16}$	6-6	125	1	5-6
40	$\frac{5}{8}$	6-6	100	$1\frac{1}{16}$	5-6
55	$\frac{9}{16}$	6-0	80	$1\frac{1}{8}$	5-6
110	$\frac{5}{8}$	6-0	80	$1\frac{3}{16}$	5-6
125	$\frac{11}{16}$	6-0	55	$1\frac{1}{4}$	5-6
110	$\frac{3}{4}$	6-0	...	...	...

31 Fig. 8 shows the final curves for riveted and welded pipes based on assumptions differing from those given in the previous example. Each curve represents the most economical welded or riveted pipe, but it is seen that a combination of welded and riveted pipe will be the most economical.

32 It will now be apparent that the method has a wide application and is susceptible of many variations. It has been shown in

the example that the economics of welded and riveted pipe can be compared. In the same manner curves can be plotted on the same chart for wood-stave or any other kind of pipe; also a curve showing the combined cost of two or more smaller pipes can be plotted in order to determine whether or not it is economical to install multiple pipes. This method would seem to be applicable to the solution of any pipe-line problem that will permit the calculation of the total yearly cost of the pipe per foot.

33 The assumption can be simplified or made more elaborate as the designer wishes. The illustrations given are for pipes in connection with a hydroelectric plant, but there is no apparent reason

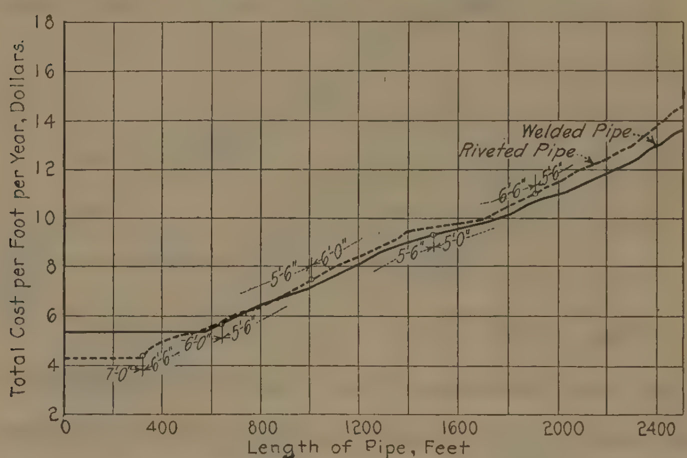

FIG. 8 CHART SHOWING COMPARISON BETWEEN RIVETED AND WELDED PIPE

why the method could not be adapted to the economic design of many other kinds of pipe lines or conduits.

DISCUSSION

ROBERT L. DAUGHERTY.[1] The graphical method presented by the author for the determination of the most economical design of a penstock, is simple, accurate, and practical. It is of especial value in the case of a high-head plant where the diameter of the pipe line and the thickness of metal both vary through a considerable range. While theoretically both diameter and thickness should vary continuously with the head, for practical reasons these dimensions change only by steps. This method shows beyond

[1] Professor of Mechanical and Hydraulic Engineering, California Institute of Technology, Pasadena, Cal. Mem. A.S.M.E.

question the location of the point of transition from one dimension to the next. It has the advantage of being also entirely general and independent of any assumptions save the fundamental one that the sum of the fixed charges plus the value of the power lost should be a minimum. That is, it is immaterial whether the cost of the pipe per pound be constant or not, whether the friction factor is independent of the diameter or not; or in other words, it is not necessary to have a series of constants.

The writer, however, desires to present an algebraic solution of this problem for the following reasons: (1) An algebraic equation shows the general nature of the case more clearly; (2) an approximately correct solution may be obtained in a few minutes; (3) with this approximate solution in hand one then knows roughly the diameters and thicknesses to use in the graphical method and so need compute only a few of the many values otherwise required. Thus, referring to the example presented by the author one finds tables constructed for diameters from 9 ft. 6 in. to 4 ft. 6 in. and for thicknesses from $\frac{3}{8}$ in. to $1\frac{1}{2}$ in. One has very little idea in the beginning as to what dimensions should be used in such tables, and so many of them are superfluous. On the other hand, if a few minutes are first spent in making an approximate solution by the algebraic method presented by the writer, it may be seen, as will be shown later, that one could omit all values for diameters of 9 ft., 8.5 ft., and 4.5 ft. at least. Also many of the thicknesses could be omitted from the tables for the diameters that are to be considered. Thus, in the table for 5.5 ft. diameter, all thicknesses below $\frac{7}{8}$ in. would be known to be of no interest. In a practical case the writer proposes that his method be used first to obtain a close approximation and then the graphical method be applied next to obtain greater precision.

The mathematical solution offered rests upon the following assumptions:

1 For a given type of pipe the cost per pound is constant
2 The weight per foot of length of a given type of pipe is equal to a constant times the square of the diameter times the head
3 The coefficient of friction in a given type of pipe is constant and independent of either diameter or velocity.

The first assumption is usually in accordance with the facts. It will be shown later that there is a variation for the riveted-steel pipe presented in the author's tables of about 14 per cent from the minimum to the maximum values for the factor involved in the second assumption, or about 7 per cent deviation from the mean, and about twice as much for the friction coefficient. Since these tables cover a wider range of diameters than are involved in the actual solution, the percentage error actually involved is less.

The following notation will be used:

h = head in feet of water
d = diameter of pipe in feet
t = thickness of pipe wall in inches
v = velocity in feet per second
M = value of 1 ft. head in dollars
i = interest and depreciation rate
b = cost of pipe per pound in dollars
q = flow in cubic feet per second
c = pipe coefficient in Chézy's formula
= $16.15/\sqrt{f}$ where f = friction factor
s = allowable stress in pounds per square inch
e = efficiency of riveted joint
w = weight of pipe per foot of length = adt.

The value of M would be determined by the flow in the pipe line and the value to be attached to a unit of power. The quantity designated by a is proportional to the density of the material in pounds per cubic feet multiplied by $\pi/12$. To obtain the value of a it is necessary to multiply in turn by a factor which represents the ratio of the complete weight to the net weight (without the riveted joints) or col. 8/col. 5 of the table in the author's paper.

The head lost in friction per foot length of pipe is

$$\frac{v^2}{c^2r} = \frac{q^2}{0.154c^2d^5}$$

and its value in dollars per year is obtained by multiplying by M. Thus the annual value of the power lost per foot of length is

$$\frac{B}{d^5} \quad \ldots\ldots\ldots\ldots \quad [1]$$

where $B = Mq^2/0.154c^2$.

The thickness of the pipe wall is given by

$$t = \frac{2.6hd}{se} \quad \ldots\ldots\ldots\ldots \quad [2]$$

and the weight per foot of length is

$$w = adt = \frac{2.6ahd^2}{se} \quad \ldots\ldots\ldots\ldots \quad [3]$$

The cost of the pipe is determined by multiplying [3] by b and the fixed charges by multiplying in turn by i. Thus the annual fixed charges are

$$Ahd^2 \quad \ldots\ldots\ldots\ldots \quad [4]$$

where $A = 2.6aib/se$.

Equation [2] leads to walls that are too thin for very low heads. In such an extreme case we may fix a constant value for t,

and this then makes the weight per foot of length $w = adT$, where T is the fixed value. The annual fixed charges are then

$$Cd \qquad [5]$$

where $C = aTib$.

The total annual cost of the pipe per foot of length may then be expressed as

$$y = Ahd^2 + \frac{B}{d^5} \qquad [6]$$

The most economical design is such that y is a minimum. Differentiating [6] and equating to zero,

$$d = \left(\frac{2.5B}{Ah}\right)^{1/7} \qquad [7]$$

is obtained as the proper diameter. The total minimum cost per foot of length is then found by substituting [7] in [6] so that

$$y = 1.82A^{5/7}B^{2/7}h^{5/7} \qquad [8]$$

In similar manner for a pipe of constant thickness T where the above economic solution is impractical, we obtain

$$y = Cd + \frac{B}{d^5} \qquad [9]$$

$$d = \left(\frac{5B}{C}\right)^{1/6} \qquad [10]$$

$$y = 1.57B^{1/6}C^{5/6} = 1625T^{5/6}A^{5/6}B^{1/6} \qquad [11]$$

as giving the most economical values under this limitation.

In order to apply these equations to the design of the riveted steel penstock considered by the author, it was necessary to compute values of the factors A, B, and C. The following values were selected more or less arbitrarily:

$$A = 0.000385 \qquad B = 16{,}500 \qquad C = 0.61.$$

By Equation [10] the upper portion of the penstock where the thickness was limited to $\frac{3}{8}$ in. as a minimum, was found to be 7.17 ft. in diameter. However, it is shown later on that the values of the factors that really apply under the low heads are appreciably different from those for the higher heads, or in other words, A, B, and C are not really constants but vary somewhat with the head and diameter. For the upper part of the penstock it will be found that better values are such as to make

$$B = 20{,}500 \qquad C = 0.57$$

Applying these values in Equation [10] we obtain a diameter of 7.48 ft. The value determined by the graphical method was 7.5 ft., as only 6-in. intervals were considered. Hence the two methods may here be said to be in exact agreement.

In Fig. 9 is shown the solution of Equation [7] giving the diameter as a function of the head. The dotted portion of this curve for heads below 180 ft. represents that portion of the pipe where the thickness would be less than the minimum allowable value. The steps shown in Fig. 9 are the values in 6-in. intervals as determined by the author. It may be seen that there is a very close agreement between these values and the curve. If one had only the curve as a guide he would select the same diameters as the author, but would be somewhat uncertain as to the proper transition point from one to the other.

In Fig. 9 is also shown a curve for the thickness of the pipe if its diameter were to vary continuously and the thickness could

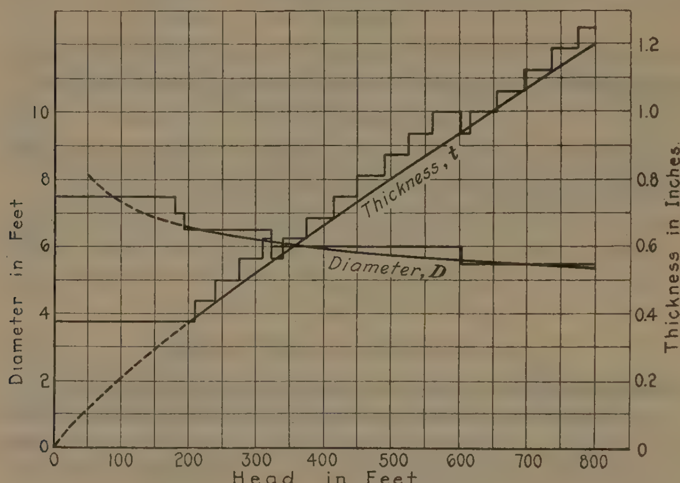

FIG. 9 SOLUTION OF EQUATION [7]

also vary continuously. The steps that are shown are the thicknesses in $\frac{1}{16}$-in. intervals, as determined by the author. The curve really represents the maximum value of the head for which a given thickness at the specified diameter may be used. Consequently we should expect the steps to be all to the left of the curve and that each horizontal line should end on the curve. This is seen to be the case where the curve for the diameter coincides with one of the 6-in. intervals used. The farther the actual diameter is from the value shown by the curve, the farther the steps showing the thickness are from its curve.

The cost of the pipe per foot of length as determined by Equation [8] is shown in Fig. 10. The steps again represent the actual values as found by the graphical method. Since both diameter and thickness of metal must vary by steps rather than continuously, the cost will vary in steps, and all costs should be

above the curve for reasons similar to those in the preceding paragraph. The cost curve is here seen to be very satisfactory in the neighborhood of 400 ft. head but is 5 per cent too low for 800 ft. head. By a proper selection of the factors A and B for 800 ft. head the point X is moved to Z and the dotted curve shown is obtained, but these values of A and B would be too high for the lower heads. This is another instance of the necessity of using different values of A and B for different portions of the curve to obtain the highest degree of accuracy.

It should be made clear, however, that this algebraic method need be applied only to find the diameters at different heads.

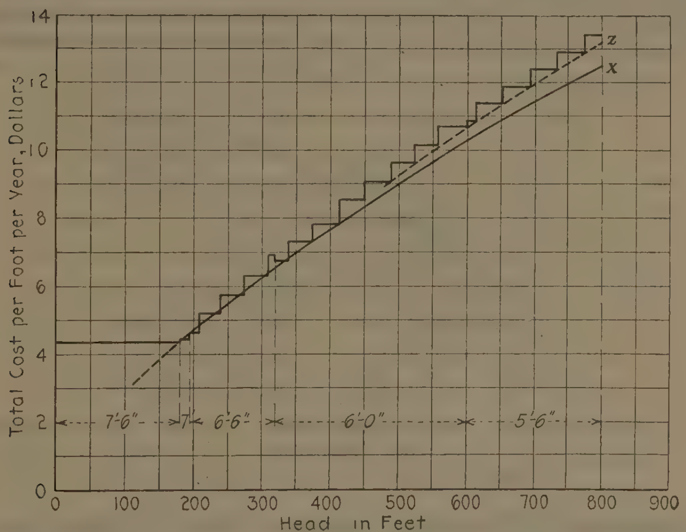

Fig. 10 Cost of Most Economical Pipe
(Smooth curve from formula; steps from graphical solution.)

With this information the thicknesses in steps may be computed directly by Equation [2], and the exact costs are then readily determined by direct calculation. By this procedure a fairly high degree of accuracy may be attained with a minimum of labor. The curves for thickness and cost from Equation [8] were added merely for general interest.

The results given by Equation [7] are not altered appreciably by reasonable changes in the factors A and B, especially since both of them are found to change in the *same* direction as functions of the head. And it would take quite a decided change in the curve of Fig. 9 to cause one to select different diameters, as long as 6-in. intervals were adhered to. But since A and B tend to change in the same direction, the error in the value for total cost

is more noticeable. But even at that it is not more than 5 or 6 per cent at the most, as shown in Fig. 10. Furthermore this curve is not necessary to determine the design of the penstock if Equations [7] and [2] are employed.

In order to determine the actual variation in the factors A and B, a brief study was made of the data presented in the author's tables. In Fig. 11 are shown some curves of K as a function of both head and diameter. According to the writer's assumption, this should be a constant. Actually it varies so that the maximum value is about 14 per cent more than the minimum. It may be seen to increase for a given diameter as the head increases, and for a given head it increases as the diameter decreases. This is because the riveted joint becomes a greater percentage of the net weight of the pipe either as the head increases or the diameter

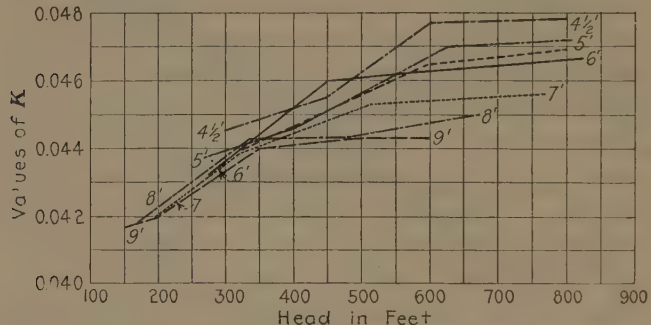

FIG. 11 VALUES OF K FOR RIVETED STEEL PIPE FOR VARIOUS HEADS AND DIAMETERS

(Weight per foot = Kd^2h; $K = 2.6a/11{,}000 \times e$.)

diminishes. It is believed that with a series of welded pipes this factor would be found to be much more nearly constant. It would appear that a riveted steel pipe would present the most unfavorable case of any so far as this mathematical solution is concerned.

The assumption was also made that the coefficient of friction was constant, so that the loss of head should vary inversely as the fifth power of the diameter only. In Fig. 12 is shown the variation of the factor B as a function of diameter. It is seen to vary about 28 per cent from the minimum to the maximum values. This factor B is directly proportional to the friction coefficient f in the expression $f(L/d)v^2/2g$, or inversely proportional to the square of c in the formula $v = c\sqrt{rs}$. According to the curve obtained in Fig. 11, the friction factor f decreases as the diameter increases. This is in accordance with experience. However, the loss of head does not usually vary as the square of the velocity but rather as some lower power, and to compensate for this f must diminish as

v increases. Hence the friction coefficient f may be expressed as a function of $d \times v$, and it will increase or diminish as this product changes. Now for a constant rate of discharge through a tapering pipe, the velocity diminishes faster than the diameter increases and hence this product is smaller for the larger pipe. This would indicate a higher value of f, or, in other words, the friction should vary as the dotted curve shown in Fig. 12.

On the other hand, if the pipe were assumed to be rough enough so that the loss of head really varied as the square of the velocity, then f should be independent of the velocity and would decrease with increasing diameter. However, the writer is unable to see how it could vary as much as 28 per cent, and believes that the truth is somewhere between the two curves shown in Fig. 12. If that is the case, the value of B is more nearly a constant than it appears in this discussion. Again, this would make the analytical

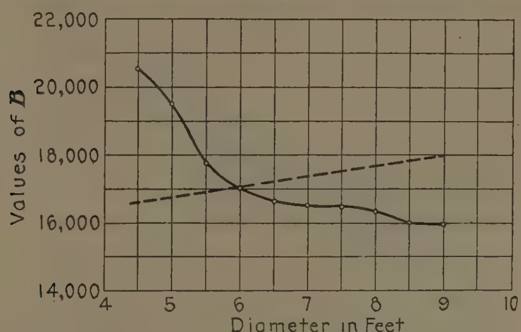

Fig. 12 Variation of Factor B as a Function of Diameter
(Annual value of power lost in friction = B/d^5.)

method agree more nearly with the graphical method without the necessity of changing values of the factors for different portions of the pipe.

The writer desires to emphasize that this discussion is in no sense to be construed as a criticism of the author's very excellent paper. His contribution is a presentation of a method of solution of a problem whatever the numerical values of the data may be and however much they may vary. But the convenience and accuracy of the writer's method depends upon the constancy of certain factors. This analysis of the author's data is purely for the purpose of finding out how much these factors vary in the case of the example presented, and to decide whether this variation is comparable to what one might find in any practical case. The writer believes that the variation of the weight with diameter and head is greater in the case of the riveted steel pipe here considered than in the case of a welded pipe, for instance. And it would appear that the variation in the coefficient of friction is here greater than one

would usually expect. Hence, applying the analytical method, using constant values of A and B in this particular instance, is a satisfactory test of its generality. It is seen that it gives very close agreement throughout the range. If one took the trouble to use different values of A and B for different portions of the pipe, by inspecting critically the results in Figs. 11 and 12, much greater precision would be obtained, but it would hardly be worth while, as the results are intended to be considered largely as a first approximation.

Another matter of interest is the comparison of two different types of pipe, such as riveted and welded. Assume for the purpose of the discussion that the welded pipe costs more per pound, but that the friction coefficient is less than for the riveted pipe. This means that the value of the factor A is greater and that of B is

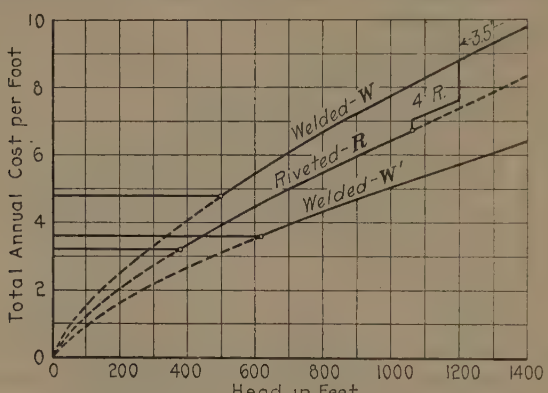

FIG. 13 CHANGING FROM RIVETED TO WELDED PIPE

smaller than for riveted pipe. From Equation [7] it may be seen that the diameter of the most economical welded pipe will then be less than that of the most economical riveted pipe. If we assign both maximum and minimum limits to the thickness of metal that may be employed, it may be seen from Equation [2] that the welded pipe may then be used for a higher head than the riveted pipe because of its smaller diameter. Also the minimum thickness is not reached with a riveted pipe until a lower head is attained.

If $A^{5/7}B^{2/7}$ from Equation [8] is greater for the welded pipe, then the riveted pipe would be cheaper at all heads. We should change from riveted to welded, however, if the head becomes high enough, because we have reached a maximum thickness of pipe wall and, though we might continue a short distance with a smaller-diameter riveted pipe, it would soon cease to be less economical than a welded pipe. This is shown in Fig. 13, where a certain riveted pipe reaches its maximum allowable thickness at a head of 1068 ft.

with a diameter of 4.5 ft. Then follows a section of 4-ft. riveted pipe up to 1200 ft. This is not the most economical size, but is required because of the limitation imposed. Beyond 1200 ft. head welded pipe is more economical than any size of riveted pipe that physical limitations will permit. When it reaches its limitation of thickness, it will be necessary to use either a smaller diameter than the economic solution would require or to divide it into two pipes.

If $A^{5/7}B^{2/7}$ from Equation [8] is less for the welded pipe than for the riveted pipe, then welded would be used as long as the economic solution could be applied. But when lower heads are reached so that the minimum allowable thickness becomes a determining factor, there are two possible cases. If the values of A and B are small enough so that for the welded pipe the value of $A^{5/6}B^{1/6}$ is less than for the riveted pipe, then the former would be used for the entire penstock. But if $A^{5/6}B^{1/6}$ is more for the welded pipe, we then have the case shown in Fig. 13 and labeled W'. It is seen that riveted pipe would be used below 445 ft. and welded pipe above that value.

It is possible to have such values of A and B that the costs for welded and riveted pipes are the same where the economic solution applies, but in this case the horizontal lines of Fig. 13 would not coincide unless the values of A and B were identical. It is believed that these cases are all there are, based on the assumption that these factors are constant.

The mathematical equations present the various quantities as functions of the head, hence the curves are plotted with head as a coördinate. The author uses the length of the pipe as a coördinate instead. Since in general the length along the pipe is an irregular function of the head, this tends to make the curves irregular. It has the possible advantage that the total area under the cost curve is then the total annual cost of the entire penstock. The use of either head or length as a coördinate, however, will enable one to obtain the correct solution.

R. T. Livingston.[1] The particular problem considered in the paper has been attacked by numerous authorities and one can find, with little searching, a large number of solutions, all differing somewhat but almost all giving results close to one another. The author seems to have considered nearly all the factors entering into the problem, and to have found a practical solution.

The problem, however, is not only a problem of economic design of penstocks alone, but is really a general one — the economic selection of any piece of energy-transforming or distributing apparatus. While it may not be apparent, it is nevertheless true, that a penstock or pipe line is in direct competition with

[1] Instructor, Mech. Eng. Dept., School of Engineering, Columbia University, New York, N. Y. Assoc.-Mem. A.S.M.E.

the electric transmission line for distributing electrical energy, and also with the coal or tank car for distributing energy in the form of coal or oil.

In the selection of competitive equipment there are two bases equally important but wholly different: the first is suitability and the second is cost. It is possible that two pieces of apparatus in competition may differ widely as to suitability and also as to cost. It is not unusual for the less suitable apparatus to have a lower annual cost than the more suitable one, and hence it may be selected. In *Mechanical Engineering,* June and July, 1924, in a paper entitled The Value of Efficiency in Transforming and

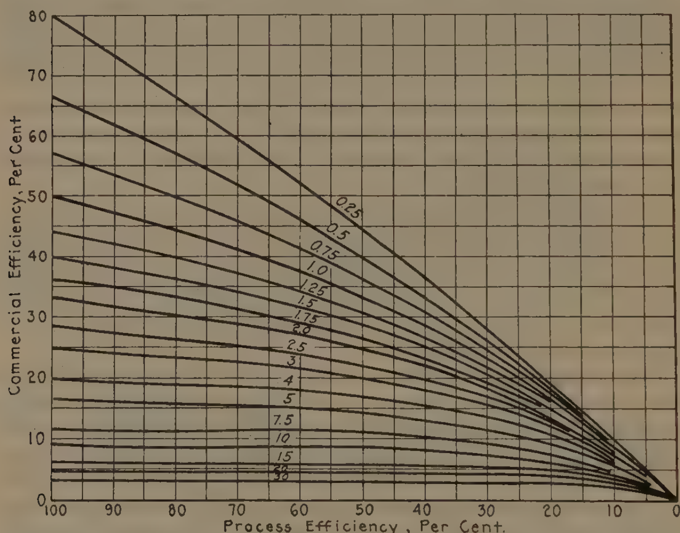

FIG. 14 SOLUTION OF LUCKE'S FORMULA FOR COMMERCIAL EFFICIENCY CURVES FOR DIFFERENT VALUES OF $(I + O)/A$.

Distributing Energy,[1] Dr. Charles E. Lucke devised a formula for a factor to compare such pieces of equipment which differ in both suitability and cost. This factor he terms "commercial efficiency." The formula is reproduced herewith together with a graphical solution thereof (Fig. 14).

Cost in cents per million B.t.u. at $B(B)$ = Cost at $A(A)$, plus inefficiency increment (xA), plus investment increment (I), plus operating disbursement increment (O), or

$$E_c = \frac{E}{1 + E\left\{\frac{1 + O}{A}\right\}},$$

where E = process efficiency and E_c = commercial efficiency.

[1] See abstract on p. 1245 of this volume.

While Dr. Lucke's paper related essentially to fuel-consuming apparatus, nevertheless it is possible to apply it to hydraulic apparatus. He has used "cents per million B.t.u." as a basic value. Hydraulic engineers will naturally use some other term, preferably cents per kw-hr. This equation is of vital interest because it introduces a term not always considered — the cost of energy supplied to the device — and connects this with the three other most important factors, viz., the inefficiency cost, the investment cost and the operating cost. All of these are naturally annual costs. The formula is not only applicable to the penstock as a whole but to any single part of the penstock, and also to any single part or the whole of the power-producing or power-transmitting plant. Its value lies in the fact that it is absolutely general and that it provides a simple method of judging between any number of similar devices. It is not the writer's idea that this formula can supersede the author's method, or in fact any similar thorough analysis. It may be used, however, in an original investigation to determine which is the best of a number of different types of apparatus and to approximately determine power sizes in relation to their costs. This latter is an important item when the financial outweigh the engineering considerations.

O. V. Kruse.[1] The author offers an interesting method for an economic penstock design. He has reduced the many variables to a definite comparable basis and the results so secured are unquestionably accurate.

It might, however, be appropriate to discuss the effect of water hammer on the assumed allowable stress or factor of safety which is definitely determined in using the author's method. The paper suggests under Item 11 that the water-hammer gradient be plotted for the entire penstock length, thus determining in advance the maximum pressure at all points and allowing the necessary additional thickness. Since the water-hammer gradient depends on the velocity, it would be necessary to assume definite lengths and diameters of penstock before such a gradient could be plotted. The average velocity for the entire penstock must be known before the water hammer can be determined.

It appears to the writer that such a procedure would defeat the primary object of the author's method, as it would introduce "cut and try" computations. Furthermore, an independent study of the effect of water hammer on a definite design would seem to be more logical.

The pressures in any penstock might be divided into two parts:

1. Those due to the regular operating conditions where the maximum is equal to the static level in the surge tank or forebay at the top of the penstock.

[1] Larner Engineering Company, Philadelphia, Pa.

2. Those due to the momentary pressure rise caused by a quick closure of the turbine gates, the maximum occurring when the full load flow is cut off in the minimum governor time.

Under the first condition the metal in the penstock is subjected to a dead load consisting of the static pressure. An additional live load is present, due to the pressure changes caused by ordinary small movements of the turbine gates, changes in draft-tube vacuum and similar causes. The proportionate magnitude of this live load varies considerably in different installations. The engineer generally throws this item into the factor of safety of the material, but he recognizes the existence of the live load and chooses a suitably low working stress for the material.

Under the second condition the penstock must be designed to withstand just as positive a force as exists under the first condition, although this force is momentary and infrequent. It is not the same as a live load which might produce metal fatigue, but is more like a shock occurring possibly only to or three times a year. The Allievi theory[1] offers a definite and accurate method of determining water hammer. There is no longer any reason to throw this item into the factor of safety. It is, however, entirely justifiable to stress the metal higher to meet this shock than would be allowable under the first condition. Here again the judgment of the engineer is called upon to choose the allowable stress for the material.

It is the writer's opinion that two definite allowable figures for working stress, to meet the two conditions above mentioned, should be determined before proceeding with the design. With these figures fixed, the procedure set forth by the authors will establish the design for the ordinary operating conditions.

The design so established should be tested throughout the entire length for water-hammer conditions. The length, area and plate thickness of the various sections are known, as is the maximum full load velocity. The water-hammer gradient is readily established by the use of the Allievi method and hence the actual maximum pressure due to this cause is known for the entire penstock. If the stress in the penstock at any point due to water hammer exceeds the previously established allowable limit, the design should be modified accordingly. This might mean still further alterations to reestablish an economic design, but such an adjustment is unavoidable when the penstock is designed to meet both conditions.

Using the example shown by the author, the water-hammer gradient shown in Fig. 15 has been computed. This is based on a maximum flow of 270 sec-ft. and a governor time of three seconds. The average speed of propagation of the pressure wave is approxi-

[1] See paper by N. R. Gibson, Trans. Am. Soc. C. E. vol. lxxxiii.

mately 3400 ft. per sec., while the average velocity of water is 9.07 ft. per sec. The ordinates between this gradient and the contour of the penstock will give the water-hammer pressures. It will be noted that this pressure amounts to 1200 ft. at the bottom. Therefore, with the penstock designed for 11,000 lb. stress under 800 ft., the stress under the water-hammer condition will be 16,500 lb. Similarly, these stresses can be determined for the entire length of the penstock. Obviously the use of the maximum flow or 270 sec-ft. is proper for the water-hammer conditions,

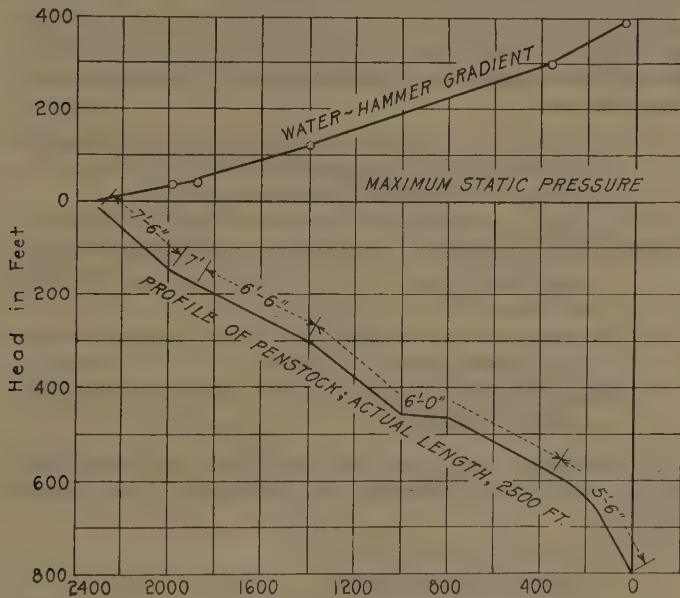

FIG. 15 WATER-HAMMER GRADIENT CURVE

while the average flow as indicated by the author is correct for the original design.

The water hammer, rather than the normal conditions, may in some instances establish the plate thickness. This is quite apt to be true where the penstock is long and the head comparatively low. On the other hand, the regulation of the plant generally prohibits the low-head penstocks, and a surge tank is installed. This serves the dual purpose of regulating the flow in the conduit and eliminating the long penstock with its high water-hammer pressures. In any event the water-hammer check of the penstock design might result in relocating the surge tank nearer the power house in order to hold to the penstock plate thicknesses as established for normal conditions.

The use of governor-actuated relief valves might in some instances warrant a higher allowable stress for the water-hammer pressures. The investigation, however, should be conducted without considering the relief valves, and the penstock made strong enough to withstand the forces in the event of failure of the relief valve.

R. M. Peabody.[1] In the design of a penstock, or other carrier of energy, in which the capacity increases with the cost, the usual assumption is that the most economical size is that for which the sum of the value of the energy lost in transmission plus the annual fixed charges on the carrier is a minimum. Several other assumptions for the economic size of penstocks might be considered, such as the following:

I The value of the power lost in friction plus the annual fixed charges on the penstock shall be a minimum. (This is the usual assumption above mentioned.)

II The cost per unit of power produced shall be a minimum.

III The rate of return on the total plant investment, over and above fixed charges and operating expenses, shall be a maximum.

IV The net revenues over and above fixed charges and operating expenses shall be a maximum.

V The total plant cost, per installed continuous horsepower, shall be a minimum.

It is interesting to express these conditions analytically, for a steel pipe penstock, and compare the differences, if any, between them.

The following notation will be used:

L = length of penstock, ft.
H = head on turbine, exclusive of friction loss in penstock, ft.
h_f = total friction loss in penstock, ft.
h = static head at any point in the penstock, ft.
q = mean annual flow, sec-ft.
c = coefficient in Chézy formula
d = diameter of penstock at any point, ft.
s = unit stress in penstock, lb. per sq. in.
e = efficiency of longitudinal joint
t = thickness of pipe, in.
w = weight per ft. of pipe, here assumed to be $140dt$
W = total weight of penstock, lb.
b = cost of pipe per lb.

[1] Chief Mechanical Designer, So. California Edison Co., Los Angeles, Cal. Mem. A.S.M.E.

i = rate of interest, depreciation, etc., on penstock
P = cost of plant, exclusive of penstock
r = rate of interest, depreciation, etc., on plant
O = annual operating expense; wages, supplies, etc., which will not vary appreciably with the small variation in output due to different assumptions for penstock design
p = selling price of power, per hp-yr., delivered to turbine casing.

Condition I. The power lost in friction in the penstock is $\frac{qh_f}{8.8}$, and the value of this power is $\frac{pqh_f}{8.8}$. The total cost of the penstock is bW, and the annual fixed charges on the penstock are ibW.

To satisfy Condition I, the function

$$y = \frac{pqh_f}{8.8} + ibW \quad \ldots\ldots\ldots \quad [12]$$

must be a minimum. This will be a minimum when the similar function

$$y' = \frac{pqh_f}{8.8L} + \frac{ibW}{L} \quad \ldots\ldots\ldots \quad [13]$$

is a minimum.

$$\frac{h_f}{L} = \frac{q^2}{0.154c^2d^5} \text{ and } \frac{W}{L} = w = 140dt = \frac{364hd^2}{es}$$

Therefore

$$y' = \frac{pq^3}{1.36c^2d^5} + \frac{364ibhd^2}{es}, \quad \ldots\ldots\ldots \quad [14]$$

or

$$y' = \frac{A}{d^5} + Bhd^2$$

where

$$A = \frac{pq^3}{1.36c^2} \text{ and } B = \frac{364ib}{es}$$

Differentiating [14], equating to zero, and solving for h,

$$h = \frac{5A}{2Bd^7} \quad \ldots\ldots\ldots \quad [15]$$

which is the relation between h and d necessary to satisfy Condition I.

Condition II. The annual cost of producing power is made up of fixed charges on plant and penstock plus operating expense, or

$$\text{Annual cost} = rP + O + ibW$$

The amount of power delivered to the turbine is $\frac{q(H-h_f)}{8.8}$. The cost per horsepower year is, therefore,

$$y = \frac{rP+O+ibW}{\frac{q(H-h_f)}{8.8}} \quad \ldots\ldots\ldots \quad [16]$$

which is to be a minimum. Equation [**16**] will be a minimum when the similar function

$$y' = \frac{\frac{rP+O}{L} + \frac{ibW}{L}}{\frac{H}{L} - \frac{h_f}{L}}$$

is a minimum. Substituting for $\frac{h_f}{L}$ and $\frac{W}{L}$,

$$y' = \frac{\frac{rP+O}{L} + \frac{364ibhd^2}{es}}{\frac{H}{L} - \frac{q^2}{0.154c^2d^5}}$$

or

$$y' = \frac{K+Bhd^2}{T-\frac{N}{d^5}} \quad \ldots\ldots\ldots\ldots \quad [17]$$

where

$$K = \frac{rP+O}{L}, \quad T = \frac{H}{L},$$

$$B = \frac{364ib}{es}, \quad N = \frac{q^2}{0.154c^2}.$$

Differentiating, equating to zero, and solving for h,

$$h = \frac{5KN}{2BTd^7 - 7BNd^2} \quad \ldots\ldots\ldots \quad [18]$$

which gives the relation between h and d necessary for Condition II.

Condition III. The rate of return on the plant investment is the gross revenues, less fixed charges and operating expenses, divided by the plant investment, or, expressed algebraically,

$$y = \frac{\frac{pq(H-h_f)}{8.8} - [rP+O+ibW]}{P+bW} \quad \ldots\ldots \quad [19]$$

Dividing both numerator and denominator of the right-hand member of the equation by L and substituting for $\frac{h_f}{L}$ and $\frac{W}{L}$ gives

$$y = \frac{\dfrac{\left[\dfrac{pqH}{8.8} - (rP+O)\right]}{L} - \dfrac{pq^3}{1.36c^2d^5} - \dfrac{364ibhd^2}{es}}{\dfrac{P}{L} + \dfrac{364bhd^2}{es}}$$

$$= \frac{E - \dfrac{A}{d^5} - Bhd^2}{F + Ghd^2} \quad \ldots \ldots \ldots \ldots \ldots [20]$$

where

$$E = \frac{\left[\dfrac{pqH}{8.8} - (rP+O)\right]}{L}, \quad B = \frac{364ib}{es},$$

$$A = \frac{pq^3}{1.36c^2}, \; F = \frac{P}{L}, \quad G = \frac{364b}{es}.$$

Differentiating, equating to zero, and solving for h,

$$h = \frac{5AF}{(2BF+2EG)d^7 - 7AGd^2} \quad \ldots \ldots \ldots [21]$$

which gives the relation between h and d to satisfy Condition III.

Condition IV. Total revenue, less fixed charges and operating expenses, can be expressed thus:

$$y = \frac{pq(H-h_f)}{8.8} - [rP+O+ibW] \quad \ldots \ldots [22]$$

which is to be a maximum. Equation [22] may be written

$$y = \frac{pqH}{8.8} - \frac{pqh_f}{8.8} - rP - O - ibW$$

Note that the first, third, and fourth terms are constants, so that y will be a maximum where

$$y' = -\frac{pqh_f}{8.8} - ibW$$

is a maximum, or changing sign, when

$$y'' = \frac{pqh_f}{8.8} + ibW$$

is a minimum.

This is the same as Equation [12] for Condition I, showing that Conditions I and IV are simply different ways of saying the same thing.

Condition V. The total plant cost is $P+bW$ and the cost per continuous horsepower developed is

$$y = \frac{P+bW}{\dfrac{pq(H-h_f)}{8.8}} \quad \ldots\ldots\ldots \quad [23]$$

This will be a minimum when the similar function

$$y' = \frac{\dfrac{P}{L}+b\dfrac{W}{L}}{\dfrac{H}{L}+\dfrac{h_f}{L}} \quad \ldots\ldots\ldots \quad [24]$$

is a minimum.

Substituting for $\frac{W}{L}$ and $\frac{h_f}{L}$,

$$y' = \frac{\dfrac{P}{L}+\dfrac{364bhd^2}{es}}{\dfrac{H}{L}-\dfrac{q^2}{0.154c^2d^5}} = \frac{F+Ghd^2}{T-\dfrac{N}{d^5}} \quad \ldots\ldots \quad [25]$$

Differentiating, equating to zero, and solving for h,

$$h = \frac{5FN}{2GTd^7-7GNd^2} \quad \ldots\ldots\ldots \quad [26]$$

which gives the relation between h and d to satisfy Condition V.

If, in Condition I, the "value" of the power lost in friction is taken to be the cost of power delivered to the turbine, the analysis is as follows:

$$\text{Cost per hp-yr.} = \frac{rP+O+ibW}{\dfrac{q(H-h_f)}{8.8}} \quad \ldots\ldots \quad [16]$$

$$\text{Power lost in friction} = \frac{qh_f}{8.8}$$

The "cost" of power lost in friction plus the annual charges on the penstock is

$$y = \frac{rP+O+ibW}{\dfrac{q(H-h_f)}{8.8}} \times \frac{qh_f}{8.8}+ibW$$

$$= \frac{rP+O+ibW}{H-h_f}\,h_f+ibW \quad \ldots\ldots \quad [27]$$

which is to be made a minimum.

Equation [27] will be a minimum when the similar function

$$y' = \frac{\dfrac{rP+O}{L} + ib\,\dfrac{W}{L}}{\dfrac{H}{L} - \dfrac{h_f}{L}} \times \frac{h_f}{L} + \frac{ibW}{L} \quad . \quad . \quad . \quad . \quad [28]$$

is a minimum.

Substituting values of $\dfrac{h_f}{L}$ and $\dfrac{W}{L}$,

$$y' = \frac{\dfrac{rP+O}{L} + \dfrac{364ibhd^2}{es}}{\dfrac{H}{L} - \dfrac{q^2}{0.154c^2d^5}} \times \frac{q^2}{0.154c^2d^5} + \frac{364ibhd^2}{es}$$

$$= \frac{K+Bd^2}{T-\dfrac{N}{d^5}} \times \frac{N}{d^5} + Bd^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [29]$$

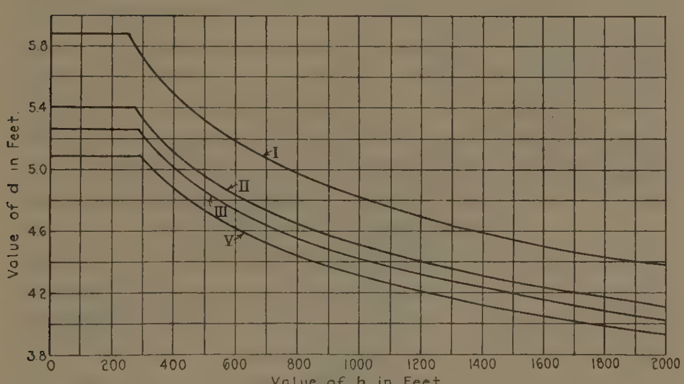

FIG. 16 RELATIONS OF h AND d CORRESPONDING TO EQUATIONS FOR CONDITIONS I, II, III, AND V, FOR A PLANT INVESTMENT OF \$5,000,000.

Differentiating, equating to zero, and solving for h,

$$h = \frac{5KN}{2BTd^7 - 7BNd^2}$$

which is the same as Equation [18] of Condition II, showing that Conditions I and II give the same result if the "value" of the power lost in friction is taken to be the cost of power delivered to the turbine casing.

In order to bring out the difference between the conditions just analyzed, sets of hypothetical values will be assumed and the various equations worked out.

For simplicity, the penstock is assumed to be of steel pipe, straight, and of uniform slope. The head at the top is zero, and at the bottom is H. The minimum thickness of metal is $\frac{3}{8}$ in. The values assumed are for a penstock the cost of which is a considerable portion of the total plant cost.

$L = 5000$ ft.
$H = 2000$ ft.
$q = 150$ sec-ft.
$c = 100$
$s = 10{,}000$ lb. per sq. in.
$e = 100$ per cent
$t = \dfrac{2.6hd}{s} = 0.00026hd$

$b = \$0.10$
$i = 10$ per cent
$P = \$5{,}000{,}000$
$r = 10$ per cent
$O = \$200{,}000$
$p = \$35$ per hp-yr.

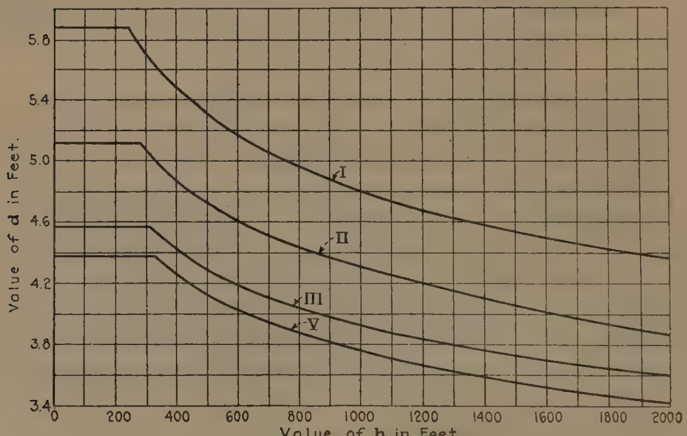

FIG. 17 RELATIONS OF h AND d CORRESPONDING TO EQUATIONS I, II, III, AND V FOR A PLANT INVESTMENT OF \$2,000,000

Substituting the above values in Equations [15], [18], [21] and [26]

$$\text{Condition I, } h = \frac{59{,}600{,}000}{d^7} \qquad [15a]$$

$$\text{Condition II, } h = \frac{35{,}160{,}000}{d^7 - 128d^2} \qquad [18a]$$

$$\text{Condition III, } h = \frac{29{,}950{,}000}{d^7 - 153d^2} \qquad [21a]$$

$$\text{Condition V, } h = \frac{25{,}080{,}000}{d^7 - 128d^2} \qquad [26a]$$

Fig. 16 shows the relation of h to d for penstocks corresponding to the above equations. Table 4 shows the cost, horsepower developed, revenue, etc., for each of the four cases.

TABLE 4 COST, HORSEPOWER DEVELOPED, REVENUE, ETC., FOR PLANT INVESTMENT OF $5,000,000

Condition	I	II	III	V
h_f	27.1	38.2	42.1	48.1
$H - h_f$	1972.9	1961.8	1957.9	1951.9
Net hp. developed	33,628	33,439	33,372	33,270
Hp. lost	462	651	718	820
Total revenue	$1,176,980	$1,170,265	$1,168,020	$1,164,450
Value of hp. lost	$16,170	$22,785	$25,130	$28,700
Cost of penstock	$415,310	$361,165	$349,160	$331,030
Interest, depr., etc., on penstock	$41,531	$36,116	$34,916	$33,103
Total plant cost	$5,415,310	$5,361,165	$5,349,160	$5,331,030
Total int., depr., etc.	$541,531	$536,116	$534,916	$533,103
Total int., depr., etc., plus operating cost	$741,531	$736,116	$734,916	$733,103
Net annual revenue	$435,449	$434,149	$433,104	$431,347
Annual value of power lost, plus annual charge on penstock	$57,701	$58,901	$60,046	$61,803
do., per cent of Condition V	93.4	95.30	97.10	100.00
Cost per hp-yr.	$22.05	$22.01	$22.02	$22.03
do., per cent of Condition V	100.09	99.91	99.95	100.00
Rate of return on capital invested, per cent	8.04	8.09	8.10	8.09
do., per cent of Condition V	99.38	100.00	100.12	100.00
Cost per installed hp.	$161.04	$160.32	$160.29	$160.23
do., per cent of Condition V per hp. developed	100.50	100.06	100.04	100.00
Cost of plant, per cent of Condition V	101.58	100.56	100.34	100 00
Cost of penstock, per cent of Condition V	125.40	109.07	105.44	100.00

If the plant cost P is taken as $2,000,000, instead of $5,000,000, the following equations result:

$$\text{Condition I, } h = \frac{59{,}600{,}000}{d^7} \quad \text{[15a]}$$

Note that P has no effect on Condition I.

$$\text{Condition II, } h = \frac{25{,}100{,}000}{d^7 - 128d^2} \quad \text{[18b]}$$

$$\text{Condition III, } h = \frac{12{,}000{,}000}{d^7 - 153d^2} \quad \text{[21b]}$$

$$\text{Condition V, } h = \frac{10{,}030{,}000}{d^7 - 128d^2} \quad \text{[26b]}$$

Fig. 17 shows the relation of h and d for these equations. Table 5 shows the cost, horsepower developed, etc., with P = $2,000,000 instead of $5,000,000.

Note that in both tables the cost per horsepower-year and the rate of return is practically the same for the four conditions, but that the cost of the penstock is, in Table 4, 25 per cent, and, in Table 5, 68 per cent greater for Condition I than for Condition V. The power developed is only one per cent more in Table 4 and only four per cent more in Table 5, for Condition I over Condition V.

It would seem, especially in cases where the amount of capital available for construction is limited, that the design should be based on Condition II, III or V, rather than Condition I.

TABLE 5 COST, HORSEPOWER DEVELOPED, REVENUE, ETC., FOR PLANT INVESTMENT OF $2,000,000

Condition	I	II	III	V
h_f	27.1	48.0	75.8	101.5
$H - h_f$	1972.9	1952 0	1924.2	1898.5
Net hp. developed	33,628	33,272	32,798	32,360
Hp. lost	462	818	1292	1730
Total revenue	$1,176,980	$1,164,520	$1,147,930	$1,132,583
Value of hp. lost	$16,170	$28,630	$45,220	$60,567
Cost of penstock	$415,310	$331,520	$279,265	$244,485
Int., depr., etc., on penstock	$41,531	$33,152	$27,926	$24,448
Total plant cost	$2,415,310	$2,331,520	$2,279,265	$2,244,485
Total int., depr., etc.	$241,531	$233,152	$227,926	$224,448
Total int., depr., etc., plus operating cost	$441,531	$433,152	$427,926	$424,448
Net annual revenue	$735,449	$731,368	$720,004	$708,135
Annual value of power lost, plus annual charge on penstock	$57,701	$61,782	$73,146	$85,009
Cost per hp-yr.	$13.13	$13.02	$13.05	$13.12
do., per cent of Condition V	100.08	99.20	99.40	100.00
Rate of return on capital invested, per cent	30.45	31.32	31.59	31.55
do., per cent of Condition V	96.8	99.27	100.13	100.00
Cost per hp. developed	$71.82	$70.08	$69.49	$69.36
do., per cent of Condition V	103.55	101.04	100.19	100.00
Cost of plant, per cent of Condition V	107.62	103.87	101.56	100.00
Cost of penstock, per cent of Condition V	168.7	135.5	114.2	100.00

The analytical methods outlined are not practical for actual design. They are used only in this case to compare the principles involved. The author's graphic method is the best for practical use and can be applied to any of the conditions discussed.

R. D. Johnson.[1] The use of the cube of the velocity by the author is ingenious and helpful in the solution of such problems as the one under consideration.

The author uses the word " penstock " in a sense rather different from the sense in which it is used in the East, where the penstock is generally known as the short part of the flume leading from the surge tank to the power house. The long line usually has some other name to distinguish it from this short pipe. The author's use of the word may be somewhat misleading in connection with the determination of the economic size.

While the author's methods are well adapted to the determination of the economical size of a long feeder from the forebay to the surge tank, perhaps several miles distant, other considerations enter from that point to the power house. Considering only the cost of lost power and the cost of the additional size of the long pipe line, it is comparatively easy to determine the most economical size; from the surge tank down to the power house, however, the question of friction loss is not the determining factor. The inertia

[1] Hydraulic Engr., New York, N. Y. Mem. A.S.M.E.

effects particularly determine the size of the penstock. The velocity of the water as it leaves the surge tank on the way down to the power house may even, at times, be reduced one-half, completely changing the law of economical sizes at that point. In many instances where regulation is of importance and where the penstock from the surge tank to the power house is comparatively short, the question of friction loss is almost never the determining factor so far as the size of the pipe is concerned. After problems of regulation have been solved and the sizes determined for proper pressure rises and proper inertia effects, as compared to the flywheel effect at the power house, the friction loss would be a secondary computation. If it proved by any chance to be too high from economical considerations, the penstock might be enlarged but in the writer's opinion that almost never would occur. The foregoing are the reasons for defining exactly what constitutes the penstock. The author's computations are far more generally applicable to the long reach of pipe line, running more or less on the level, than they are to the short reach of pipe from surge tank to power house.

If the penstock, as the author calls it, runs all the way to the power house and there is no surge tank, and if regulation is taken care of by other plants in parallel, then there is no reason why this strict method of economical determination should not be adopted throughout the whole length of the pipe.

Lewis F. Moody.[1] In this very interesting paper the author has given us an excellent basis for a method of selecting the most economic size of penstock or pipe line for a power plant, for any part of its length from forebay to power house. He has well stated the conditions surrounding the problem when he refers at the outset to "the large number of variables entering into the problem." By selecting those variables which have a major influence on the solution, neglecting the variables of minor effect, and by then showing how these can be systematically treated and their effects determined by a straightforward graphical process, he has given us valuable assistance in the obtaining of a simple rational method of solution. The writer hopes that the author will continue this work. He desires to offer as a constructive suggestion that the method be extended to include in a definite manner the effect of water hammer, since this cannot properly be considered a minor variable. In almost every practical case it will be one of the determining factors.

The paper recognizes that water hammer is a factor to be considered, and states that "the increase in water hammer can best

[1] Consulting Engineer, I. P. Morris Department, Wm. Cramp & Sons Ship & Engine Building Co., Philadelphia, Pa. Mem. A.S.M.E.

be made by so plotting the line in Fig. 4, representing maximum water level, that it will give the maximum static head plus water hammer at every point in the line." This of course is merely the initial step in considering this factor and to work through the remainder of the method with water hammer included is a real problem which will not only involve a considerable extension of the method, but will, in general, materially alter the solution. The line of maximum water level in Fig. 4 will not be a fixed horizontal line, but will be different in form and position for every different penstock size and every different manner of variation of size from end to end. Moreover there will be two such lines for each design of penstock; one for decreasing flows and the other for increasing flows. The line for decreasing flows will govern the pipe thickness and that for increasing flows will place a minimum limit upon the diameter. If the water-hammer gradient for increasing flows is neglected, it might be found when the design had been worked out according to the method detailed in the paper, that the pipe will collapse under a sudden increase in load on the power unit, so that the solution would hardly remain an "economical" one. This point has been given insufficient consideration in many plants.

The writer would note one other point which relates to Professor Daugherty's discussion of the paper rather than to the paper itself. Professor Daugherty makes the useful suggestion of substituting the analytical equivalent for the graphical solution. The following change is suggested in the analytical method:

The head lost in friction per foot of pipe is expressed by Professor Daugherty as $\frac{v^2}{c^2 r}$. It would be much more accurate to express the loss of head[1] as $\frac{v^2}{c^2 r^{\frac{4}{3}}}$, or[2] $\frac{v^2}{c^2 r^{1.4}}$.

Professor Daugherty mentions the slight inaccuracy in using the square of the velocity; but the unity exponent of the hydraulic radius is a much greater source of inaccuracy; it is just as easy to use a more accurate value. Using the Strickler value, the annual value of power lost per foot of pipe length (Professor Daugherty's Equation [**1**]) becomes $\frac{B}{d^{5\frac{1}{3}}}$; and Equation [**6**], the total annual cost of the pipe per foot of length, that is, the fixed charges plus the value of power lost, becomes:

$$y = Ahd^2 + \frac{B}{d^{5\frac{1}{3}}}$$

[1] A. Strickler, *Schweizerische Bauzeitung*, June 7, 1924 — "Beiträge zur Frage der Geschwindigkeitsformel" etc.).

[2] P. Forchheimer, "Hydraulik."

Differentiating for a minimum, we have, instead of Equation [7], for the value of the most economical diameter

$$d = \left(\frac{2.67B}{Ah}\right)^{\frac{1}{7.33}}.$$

Equation [8], the total cost of the most economical pipe per foot becomes:

$$y = 1.797\ (Ah)^{0.728}\ B^{0.273}.$$

If these revisions are applied the analytical solution will agree much more closely with the graphical results. The curve in Fig. 10, if computed by the revised formula [8], will fall very close to the dotted curve Z, agreeing with the graphical solution. This analytical solution, however, like the graphical method, is deficient in its lack of provision for water hammer.

W. F. Uhl.[1] The author could have more properly entitled his paper, A Method for the Partial Economic Design of Pipe Lines. The method outlined hardly applies to that portion of the conduit supplying hydraulic turbines which is generally known as penstock. The method, however, would apply to the pipe-line portion of it. The method is only a partial one for the reason that three things must be investigated before we come to the point of pipe-line design. One of these is the quantity of water available. Methods such as described for the pipe line are likely to give us solutions that are sometimes called more precise than accurate, for the reason that the fundamental information is not as nearly correct as the final solution. The question of the amount of water available is a question of predicting what will happen in the next 20 or 30 years, hence, in many instances where there are few if any records of run-off, our solution may be merely a guess.

Another question which might be troublesome is that of the value of water power. It is quite evident that the value of water power for the same plant may be very different at one time than another. Water power may be worth two or three cents a kilowatt-hour where it replaces peak load. On the other hand, where it can only take base load it may be worth only four to six mills.

[1] Hydraulic Engr., Chas. T. Main, Boston, Mass. Mem. A.S.M.E.

No. 1947

THE ZOELLY TURBINE-DRIVEN LOCOMOTIVE

By H. Zoelly,[1] Zürich, Switzerland

Non-Member

This paper, by the general manager of the Escher-Wyss Company and designer of the well-known locomotive turbine bearing his name, gives particulars regarding a 1000-hp. experimental condensing locomotive driven by a six-stage impulse turbine and employing a cooler for the condensing water in which air is brought into intimate contact with the water, heated and saturated with vapor, the heat necessary for evaporation being withdrawn from the water to be cooled. Mention is also made of a 2000-hp. locomotive of the same type now being built at the Krupp Works, Essen, which differs from the experimental machine only in a few sundry details; after which theoretical considerations regarding feedwater heating and recooling are discussed at some length. A table is included which presents calculations of the steam and coal consumptions of simple and compound condensing and non-condensing piston locomotives using superheated steam, and of the Krupp turbo-locomotive, all with and without feedwater heating, and shows the turbo-locomotive when employing feedwater heating and preheating of combustion air to have a coal consumption of but one-half that of a simple locomotive provided with a feedwater heater.

THE steam locomotive as we know it to-day is the result of a century of development, but in spite of present-day simplicity and reliability the attendant drawbacks are still lack of efficiency and insufficient capacity. For any one familiar with the efficiencies of modern steam plants it is positively pitiful to see hundreds of thousands of locomotives running about with maximum overall efficiencies of 7 to 9 per cent — very often as low as 5 to 6 per cent — whereas with up-to-date stationary steam plants efficiencies up to and in excess of 20 per cent are attained with ease. The inefficiency of the steam locomotive was recognized long ago, however, and in order to improve matters many railway lines were electrified. Large and powerful power stations were built and equipped with high-efficiency turbo-generators, and not-

[1] Chairman of Board of Directors, Escher Wyss & Co.

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withstanding the incidental transformation of power a very much higher degree of efficiency was obtained than with the ordinary locomotive. At the same time a second advantage was also realized, namely, that the capacity of the electric locomotive could be increased. Notwithstanding the great advantages offered by this solution, however, there are great drawbacks attached thereto, such as enormous costs, complete dependence on a few generating stations, and long transmission lines. The great war put a stop to electrification schemes practically everywhere, while at the same time the cost of coal rose to an extent previously unheard of. Under these circumstances the problem of improving the existing locomotive became imperative.

2 The direction in which improvement lay was clearly indicated, viz., it meant following up the development of stationary steam plants. In the average stationary steam plant the steam supplied by the boiler has a total heat of 1350 B.t.u. steam at 213 lb. and 662 deg. fahr. With a vacuum of 28.55 in., 427 B.t.u. are available for transmission into energy and 923 B.t.u. are lost in the condenser. In the case of an ordinary locomotive working with the same initial steam only 236 B.t.u. could be used, 1114 B.t.u. being lost owing to the locomotive's exhausting into the atmosphere. To improve the locomotive, then, it is necessary to improve the ratio of the number of heat units used to that of those lost in the process, which can evidently be done in two ways. One way would be to introduce condensation, thus giving similar conditions to those prevailing in the case of stationary steam plants. The other, which would of course represent an improvement for both locomotive and stationary plant, is to increase the steam pressures to values much higher than those obtaining to-day.

3 If it were possible to increase the boiler pressure to 1434 lb. per sq. in. and the total temperature to 750 deg. fahr., the total heat of the steam would be 1329 B.t.u., and if again we expanded down to 28.55 in. the total heat lost would only be 810 B.t.u. while 519 B.t.u could be utilized. The advantage thus gained would be enormous, for not only would the available drop be much larger, but at the same time the total heat of the initial steam would be less. For the present-day locomotive — exhausting into the atmosphere — the available heat drop would be 353 B.t.u., and 976 B.t.u would be lost. This would constitute a remarkable improvement on what actually takes place, but it would still be nothing compared with the improvement to be obtained by introducing condensation.

4 It scarcely needs to be pointed out that in the case of the steam locomotive high pressures should only be introduced after successful experience in connection with stationary plants, conditions on a locomotive being so extremely difficult and considerations of reliability and safety having to come before everything else. The greatest improvement to the existing locomotive consists, therefore, in the introduction of condensation. Without going into

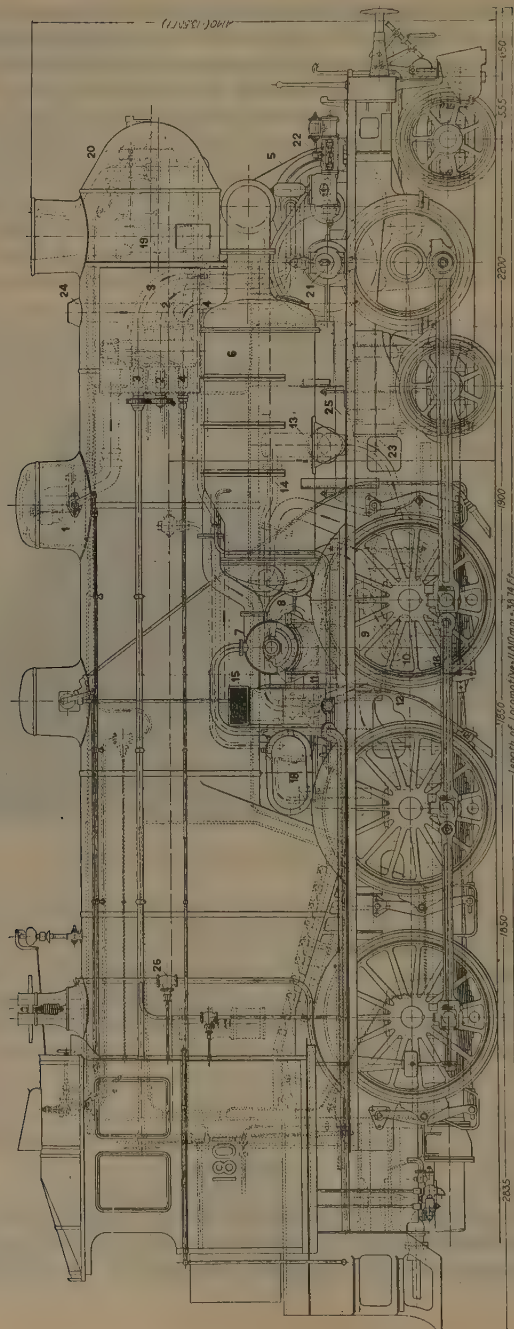
details, it may be said at once that for the low-pressure part of the condensing locomotive at least, it will be impossible to make use of cylinders, as the dimensions required for utilizing the relatively good vacua would be too great for them to be housed on the locomotive. For this reason it is only natural that the practice which has been followed in the case of modern power plants should be adopted, namely, the introduction of turbine drive combined with condensation. All engineers working on the subject of improving the steam locomotive as we know it to-day have come independently to the same conclusion. Among those that may be mentioned in this connection are Ramsay and Reid-MacLeod in Great

Fig. 1 Zoelly 1000-Hp. Turbo-Locomotive

Britain, and Ljungström in Sweden. The means employed in realizing the fundamental principle, however, were entirely different. The following gives a detailed account of the principles applied in the turbine-driven condensing locomotive built by Escher Wyss & Co., Zürich, in conjunction with the Swiss Locomotive Works, Winterthur, both in Switzerland, according to the Zoelly patents.

General Description of the Zoelly Turbine-Driven Locomotive

5 The locomotive in its present form is shown in Figs. 1 and 2. This experimental machine was constructed by altering a standard piston locomotive (type B 3/4) belonging to the Swiss Federal



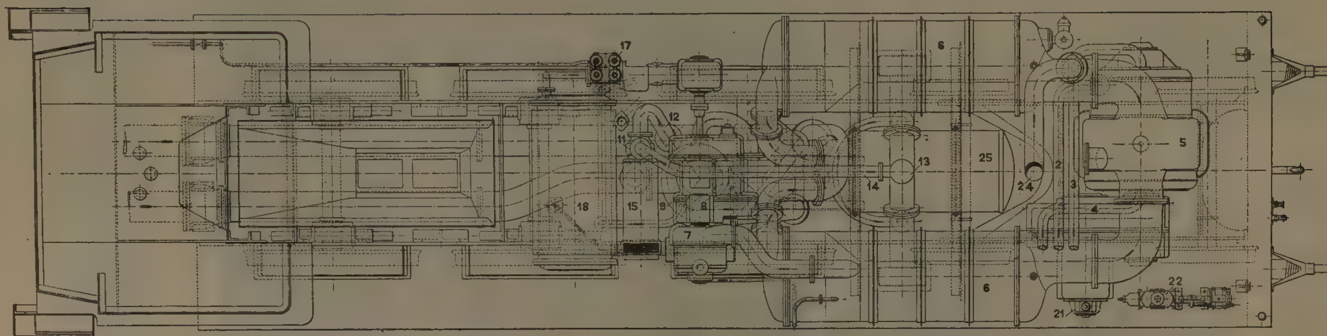


FIG. 2 ELEVATION AND PLAN OF ZOELLY 1000-HP. TURBO-LOCOMOTIVE SHOWN IN FIG. 1 (DIMENSIONS IN MM.)

1, Main governor valve
2, Steam valve for running ahead—normal
3, Steam valve for running ahead—overload
4, Steam valve for running astern
5, Main turbines for running astern and ahead.
6, Condensers
7, Turbine driving condenser auxiliaries
8, Reduction gear
9, Circulating water pump
10, Centrifugal pump—water for air ejector
11, Water-jet air ejector
12, Water delivery pipe from pump 13 to ejector
13, Air collector for air from condensers
14, Air suction pipe from collector to ejector
15, Air separator
16, Condensate pump
17, Feed pump
18, Feedwater heater
19, Furnace fan
20, Turbine driving the furnace fan
21, Oil pump for main turbines
22, Stand-by oil pump
23, Oil cooler
24, Atmospheric exhaust
25, Air storage for automatic air brake
26, Live steam for turbine on recooler

Railways and the type B 3/4 shown in Fig 3. Data of the transformed locomotive are as follows:

Maximum speed	47 mi. per hr.
Boiler pressure	200 lb.
Superheat	662 deg. fahr.
Heating surface of furnace	132.5 sq. ft.
Total heating surface	1060.0 sq. ft.
Surface of superheater	380 sq. ft.
Total heating surface inclusive of superheater	1450.0 sq. ft.
Grate surface	25 sq. ft.
Diameter of driving wheels	60 in.
Rigid wheelbase	6 ft.
Total wheelbase	25 ft.
Total wheelbase, including tender	58.4 ft.
Weight of locomotive, empty	60 tons
Weight in running order	65 tons
Adhesion weight	45.6 tons
Length over buffers, including tender	68 ft.
Extreme height	13.8 ft.

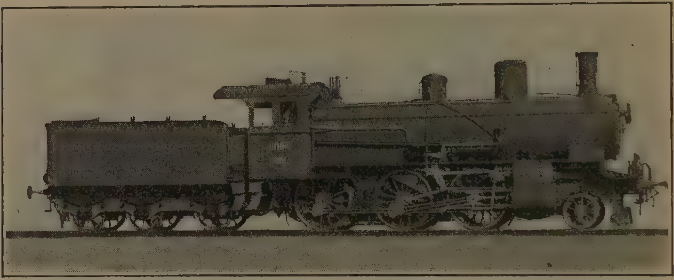


Fig. 3 Original Piston Locomotive Before Altering into a Turbo-Locomotive

The principal change in general appearance as compared with the original locomotive was the replacement of the cylinders by the turbine.

6 *Boiler.* The boiler in use is the unchanged one of the original locomotive, provided with a Schmidt superheater. A turbine-driven fan has been arranged in the front part of the smokebox as a substitute for the draft produced by the exhaust steam in the case of the ordinary locomotive.

7 *Main Turbines.* The new locomotive has been designed for the same performance as the old one, so that the turbine for running ahead is designed to give 1000 hp. at the crankpin. The turbine itself is a 6-stage impulse Zoelly turbine. The astern turbine consists of a simple compound wheel and is erected in the same casing as the ahead turbine (see Fig. 4). The turbine rotor, comprising both ahead and astern wheels, is made out of a solid block, the blades being inserted in slots in the wheel rims in accordance with approved practice. The turbine drives through double-

reduction gear (1st reduction, 1: 7; 2nd, 1: 4.1) a jackshaft carrying the crank and crankpins, the drive to the wheels being obtained in the usual way.

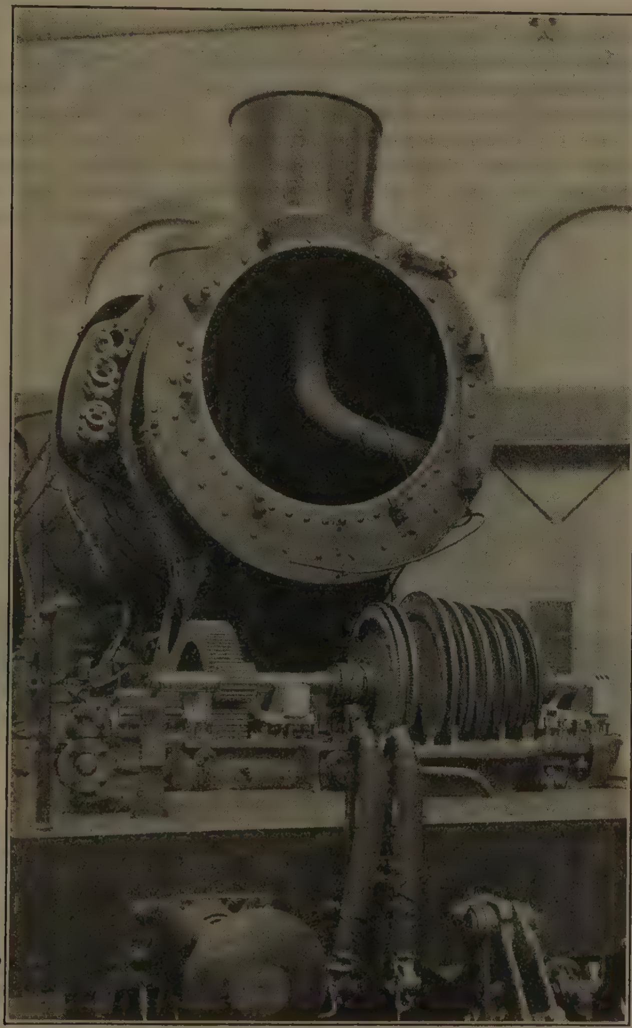


Fig. 4 Turbine Rotor and First Reduction Gear

8 The turbine casing with reduction gear, intermediate and dummy shafts, and all bearings are mounted on a one-piece steel casting (Fig. 5), which is rivèted to the locomotive frames. The turbine is placed in front of the boiler, its axis being parallel to the locomotive axles.

9 Referring to Fig. 2, steam admission to the ahead or astern turbines is controlled by means of valves 2 and 3, for ahead, and 4 for astern, which are operated by hand from the driver's cab. For running ahead, two groups of nozzles have been provided in the first guide wheel, one allowing the passage of about 11,000 lb. of steam when fully open, and the other 4400 lb. According to load, one or the other of the valves — or both — is fully open. Intermediate quantities are obtained by throttling with the main governing valve 1. For running astern only one valve has been provided, allowing a total of 15,500 lb. of steam to pass. Smaller quantities are likewise obtained by throttling down with valve 1. The efficiency of the astern turbine is lower than that of the ahead turbine,

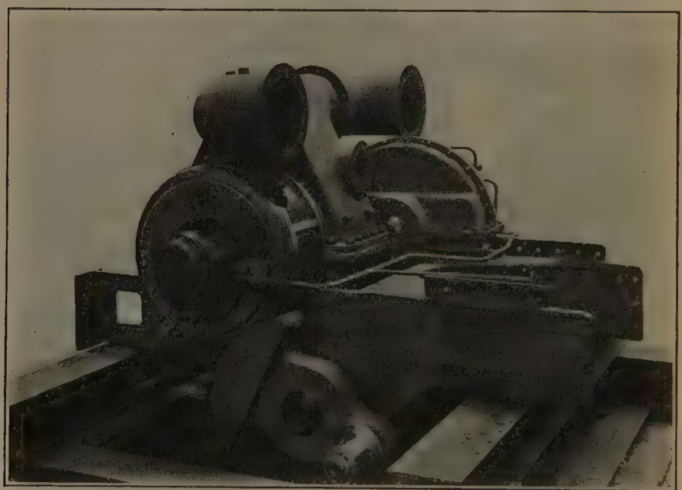


Fig. 5 Steel Casting with Turbine

but a locomotive must for other reasons run ahead as a rule, astern running only being provided for switching, maneuvering, or in case of emergency.

10 The maximum traveling speed of the locomotive is 47 miles per hour and is limited by the type of locomotive, the driving wheels having a diameter of only 60 in. At this speed the turbine makes 7500 r.p.m. The turbine speed is of course proportional to the traveling speed. When running ahead the astern wheel rotates in the vacuum as is generally the case in marine propulsion. The wheel friction for the simple astern turbine is small, and the losses are therefore insignificant.

11 *Condensers.* From the exhaust end of the turbine — common to both the ahead and the astern turbine — the steam passes in about equal quantities to condensers placed longitudinally on

each side of the boiler. These condensers are water cooled and are of the surface type.

12 *Auxiliaries.* The general arrangement of the principal apparatus is shown in Fig. 6. All auxiliaries of the condensing plant are driven by one small turbine 7 (see Fig. 2) revolving at 9000 r.p.m. This turbine, through a reduction gear which lowers its speed to 1200 r.p.m. and bevel gearing, drives a vertical shaft

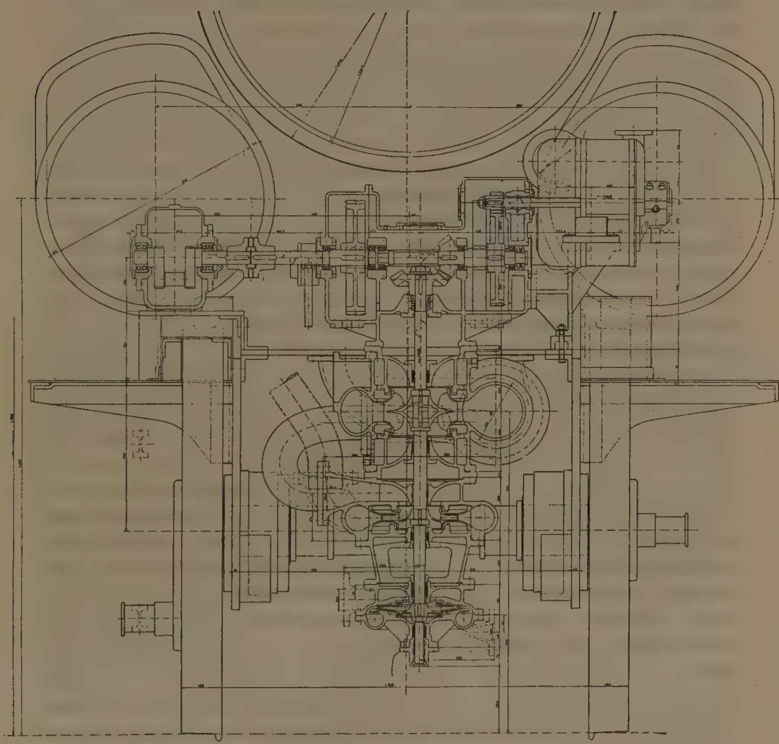


Fig. 6 Condensing Auxiliaries

carrying the circulating-water, air, and condensate pumps. The circulating pump takes water from the tender, forcing it through the condensers and back again to the recooler. The air pump discharges at a pressure of about 75 lb. to the water-jet air ejector 11, Fig. 2, which air ejector communicates with the two condensers. In the air separator 15 the air can escape out into the atmosphere, the water returning to the suction side of the circulating pump.

13 The condensate from the condensers is led to the condensate pump, which latter discharges at about atmospheric pressure into

the feed pump 17 placed on the side platform of the locomotive. This is a reciprocating pump running at 59 r.p.m. only, but driven through a second reduction from the auxiliary turbine. The feed pump discharges direct through a preheater into the boiler.

14 The turbine is a three-stage Zoelly impulse turbine and receives steam at 11 lb. gage pressure, being connected to the condenser. The steam used in the turbine is exhaust steam from a back-pressure turbine driving the ventilator of the recooler.

15 *Furnace Fan.* For producing the furnace draft special means have to be provided, exhaust steam being no longer available. In its earliest form the locomotive was equipped with a forced draft producing pressure under the grate. After a long series of tests, however, it was found necessary to change over to the suction principle and to adopt the solution shown in Fig. 2.

16 The fan is of the centrifugal type, provided with a spiral casing and is capable of producing a depression in the smokebox of 8.2 in. of water. A maximum of 280 cu. ft. of flue gases can be discharged per second while running at 1500 r.p.m. The fan is driven by a small turbine through a gear with a transmission ratio of 1: 6. The turbine receives live steam and exhausts with a back pressure of about 7 lb. gage to the feedwater heater. The admission of steam to the turbine is regulated by a valve operated from the driver's cab. The connection of the heater with the exhaust steam of the turbine driving the furnace fan is very useful, as there is a certain ratio between the quantity of feedwater and the exhaust steam from that turbine. If for some reason or other there should be no water in the heater and the steam could therefore not condense, a safety valve opens a connection from the heater to the condenser. The condensate of the heating steam always escapes directly to the condenser and, together with the condensate of the main circuit goes to the boiler.

17 This design adopted for the experimental machine lacks the advantage of the usual draft producer, i.e., the proportion in the draft to the quantity of steam required in the main turbine. This, however, can be realized on condensing locomotives by bleeding steam from the main turbine. With this type of turbine the quantity of steam thus bled is strictly in proportion to the total quantity of steam going through the turbine, thus giving the right proportion of steam admitted by the small turbine. In this case the turbine driving the fan will be of the low-pressure condensing type. The heater in this case must be heated in a different way to that described above and could be heated by bled steam or by exhaust steam from any other auxiliary turbine, flue gases or by a combination of the different means.

18 *Air Pump for Automatic Air Brake.* On the experimental machine the usual Westinghouse air pump — exhausting to atmosphere — is used. The design is not very appropriate, however, as the exhaust steam is lost, and in any case could not very well be

used on account of the oil it contains. In future designs the natural course will be to employ a rotary pump, which can be driven by the same auxiliary turbine that drives the condensing auxiliaries, thus returning all steam to the boiler.

19 *Boiler Feedwater.* In principle the condensing locomotive does not need any additional water for boiler feeding, as the water in the boiler is working on a closed circuit. Practically, however, it is imposible to eliminate leakage losses, loss through the steam whistle, and last but not least, loss through train heating. In order to obtain the full advantage from the condensing locomotive it is essential that none other than clean, soft water gets to the boiler. This can be done in two different ways. It is possible to have a special tank for boiler feedwater on the tender, the feeding being effected by injector as in the case of the ordinary locomotive, or water from the cooling tank may be used, which water has to be cleaned before being sent to the boiler. The Krupp Company, of Essen, which holds a Zoelly license, cleans the make-up water by sending it to a small evaporator. The feedwater evaporated in the evaporator escapes into the condenser, where it condenses and is sent to the boiler along with the condensate of the main circuit. Instead of leading the steam from the evaporator direct from the condenser, it is also possible to send it to a low-pressure turbine, or to a certain stage of the machine, thus doing useful work.

20 In the experimental machine neither of these two solutions has been resorted to. For boiler feeding the cooling water is used direct, being fed by the steam injector if necessary.

21 *Lubrication.* Each turbine has its own lubricating system, comprising an oil tank and a geared pump driven from a shaft of the reduction gear. Gears and bearings are under forced lubrication, the oil for the main turbines passing through an oil cooler connected with the cooling-water circuit of the condensers. Referring to Fig. 2, 22 is an auxiliary oil pump which does not really belong to the locomotive, but which has been installed for use in case of emergency. The lubrication of running gear is carried out in the usual manner.

22 *Recooler.* The most vital part of the condensing locomotive working with water as refrigerating medium in the condenser is the recooler. All the heat units taken from the steam in the condenser go to the cooling water, which of course has to be recooled in order to be able to be used in a cyclic process.

23 The recooler is a separate vehicle taking the place of the usual tender and providing room for coal and eventually make-up water for boiler feeding. It works on the vaporization principle, air being brought into very intimate contact with the water to be cooled and being thus heated and saturated with water vapor. The heat necessary for evaporation is withdrawn from the water to be cooled. The heat to be absorbed in such manner is enormous, amounting in the case of the 1000-hp. experimental machine to

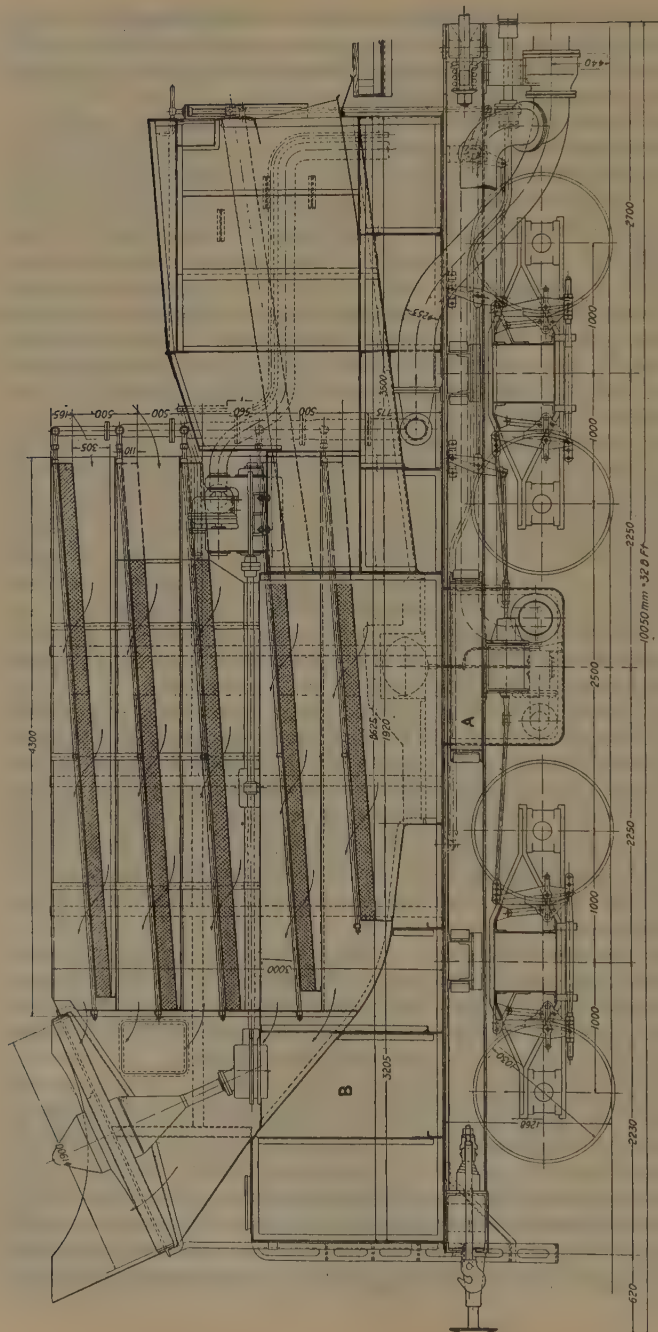


Fig. 7 Recooler for Zoelly 1000-Hp. Turbo-Locomotive (Dimensions in mm.)

about 5,760,000 B.t.u. per hour, and increasing in proportion for larger machines.

24 This cooler has been developed from the countercurrent type of cooling tower, and utilizes only the natural draft produced by the running of the train. Although excellent results were obtained when running at normal speed, the cooling was nevertheless insufficient when ascending steep gradients on low speeds or starting up the train, in which cases the heat transmitted to the condenser was considerable but the ventilation nil. This led to the introduction of a fan which made the cooling effect less dependent on the natural draft. In order to prevent large quantities of water from being extracted mechanically by the air, it was found necessary to abandon the practice of dividing up the water into small particles. This, however, reduced the amount of water surface that could come in contact with the air, and in order to obtain the required surface and prevent water losses it was found necessary to introduce some filling material to which the water could adhere. The recooler in its present form represents the result of conjoint work with the Krupps. Fig. 7 shows the recooler of the experimental machine.

25 The recooler comprises a certain number of elements working in parallel as regards water and air. Each element consists of a wrought-iron channel of practically rectangular cross-section. In the longitudinal direction this channel is divided diagonally into two halves by means of perforated trays containing Raschig rings, i.e., small tubes of about equal length and diameter. The water to be cooled is led to these trays by tubes acting as sprayers, the air passing in counterflow. In the experimental cooler shown in Fig. 4 the cooling elements are so disposed that the natural current of air produced by the traveling train can enter the cooling element direct. The fan provided produces a sufficient current of air when the train is stationary or traveling at a very slow speed, and also augments the normal current when traveling at ordinary speeds. It is driven through a gear by a small back-pressure turbine, as mentioned before. Admission of steam to the turbine is controlled by means of a valve from the driver's cab.

26 The cooled water flows from the cooling element back into a tank, whence it is again drawn into the circulating pump. As a certain amount of water is evaporated in the process, it becomes necessary to add a corresponding amount in order to keep the circulating quantity constant. For this purpose there is provided the large storage tank *B* on the tender, which communicates with the suction tank *A* by means of floaters.

27 The water in the tender need not necessarily be so clean and pure, as it does not enter the boiler nor come into contact with such parts as could seriously affect the working of the plant.

28 Room is provided in the tender for 11 tons of coal and 6 tons of water.

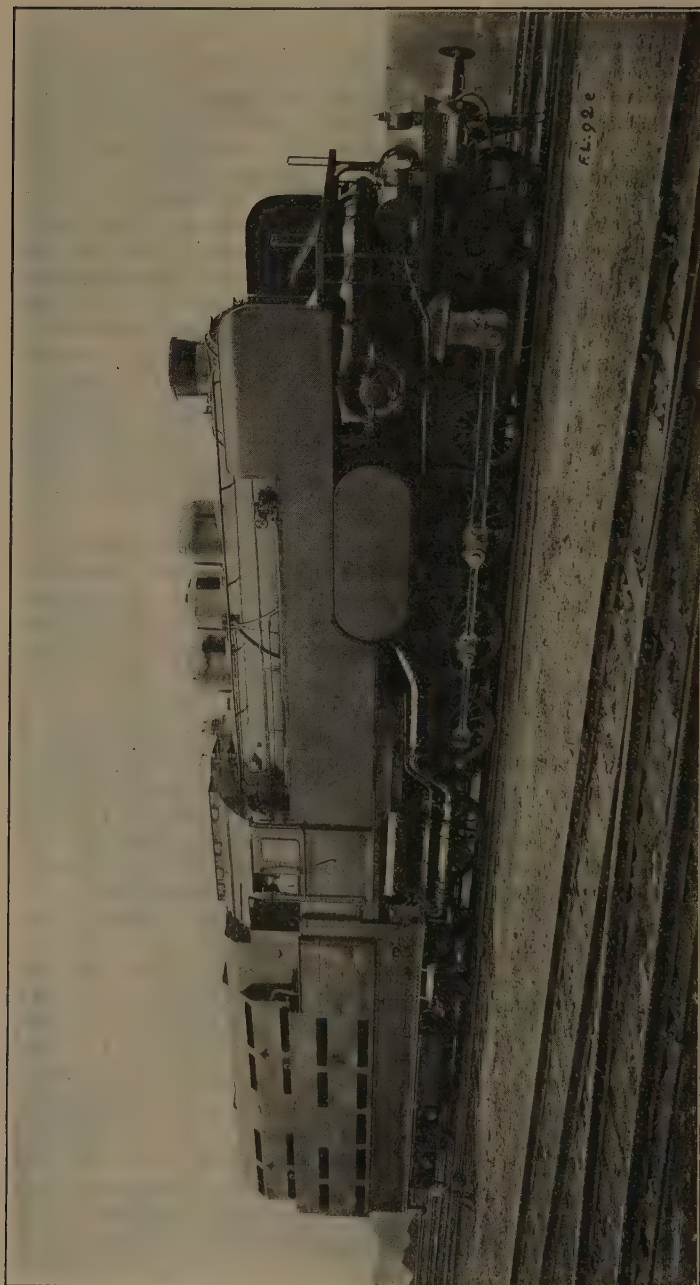


Fig. 8 2000-Hp. Zoelly Turbo-Locomotive Built by Krupp

29 The recooler car and locomotive are coupled in the usual way. The connections of both suction and discharge pipes for the cooling-water system are made by means of sliding ball-and-socket joints. On removing the coupling bolts these allow of the tender's being taken away from the locomotive. Connections for live steam and for exhaust steam from the turbine driving the fan of the recooler are of very small dimensions only and are made by flexible pipes.

30 Before dealing with the theory of the most important parts of the turbine-driven locomotive and its superiority over the ordi-

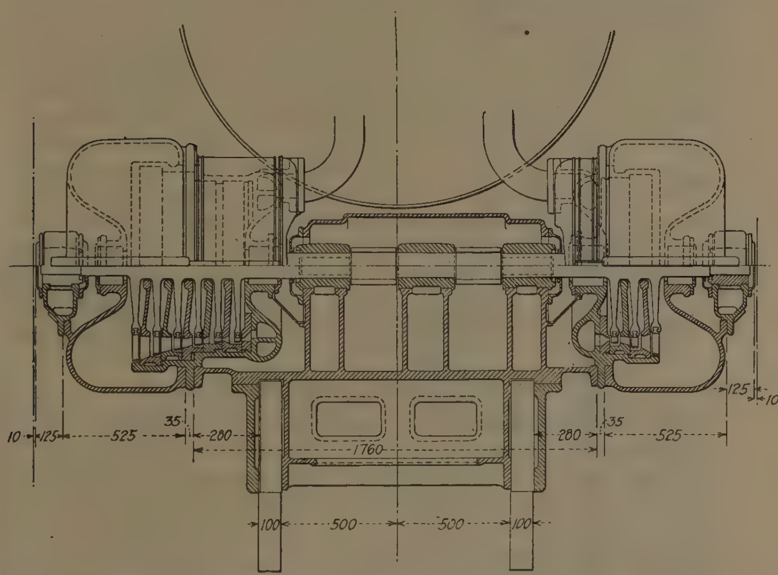


Fig. 9 Longitudinal Section Through Turbines

nary locomotive, a brief sketch will be given of the 2000-hp. locomotive built at the Krupp works under license to use the Zoelly patents.

2000-Hp. Zoelly Turbo-Locomotive Built by Krupp

31 Fig. 8 shows the Krupp locomotive, which is now completed and will shortly be tested. The general arrangement is absolutely identical with that of the Zoelly experimental machine, the only differences being in sundry details. Having twice the energy to transmit, it was found necessary to locate the reduction gear in about the middle of the driven axle. For this reason the turbine has been divided into separate ahead and astern turbines, placed symmetrically to the locomotive axle outside the longitudinal bars of the frame.

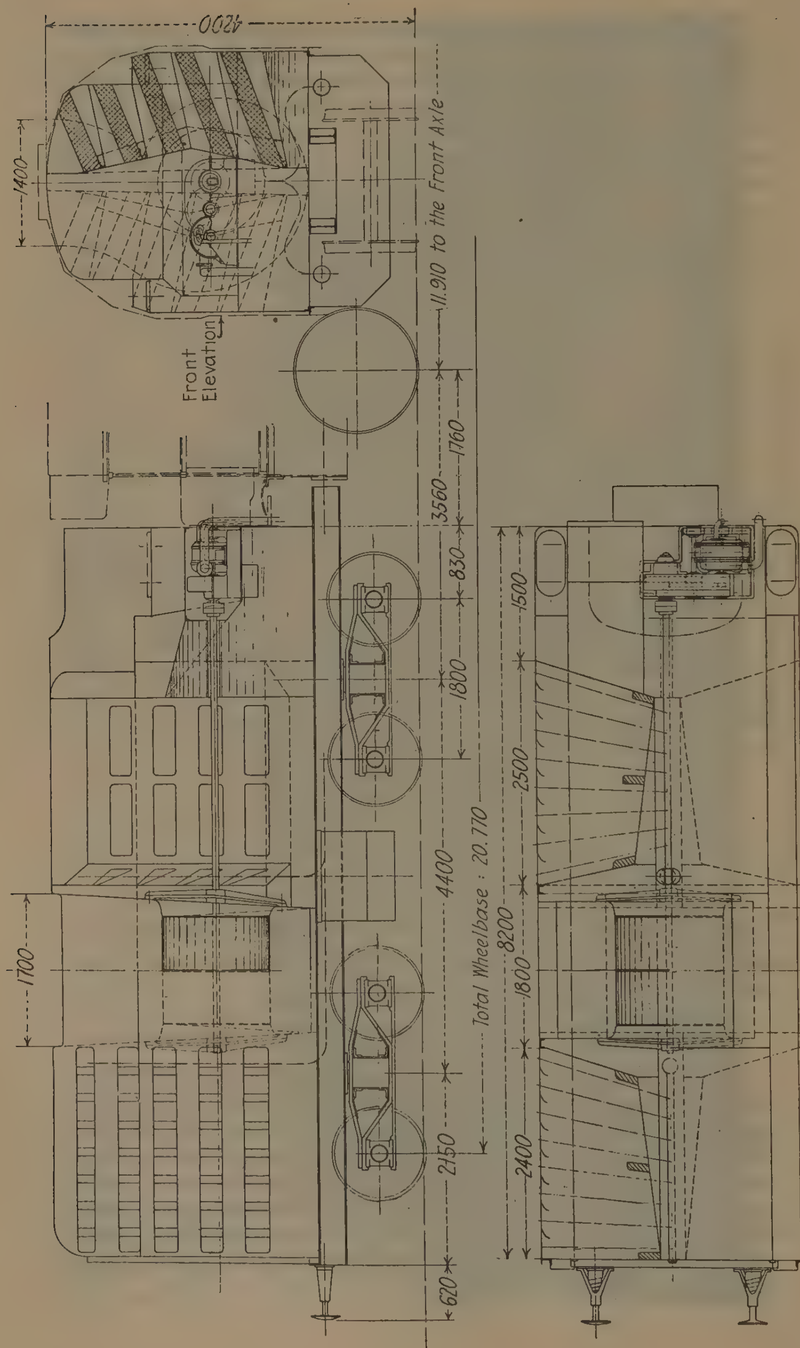


FIG. 10 RECOOLER OF 2000-HP. TURBO-LOCOMOTIVE

32 A sectional view of the turbines is given in Fig. 9. The turbines and gears, including intermediate and dummy shafts, are mounted on a one-piece steel casting, which can be slid as a whole onto the main bar frames of the locomotive. The latter extend through the casting and carry the buffers.

33 The condensers are led crosswise to the boiler, under the latter, and are connected in series. The condensing auxiliaries are in principle arranged as on the experimental machine. The furnace fan has been mounted just in front of the firebox under the boiler and is connected to the smokebox by a large wrought-iron channel. In this channel is arranged a feedwater heater. The fan exhausts again in a similar channel leading to the stack. The turbines driving the furnace fan as well as the turbine driving the condensing auxiliaries are condensing machines fed with live steam. The turbine driving the ventilator of the recooler is a back-pressure turbine exhausting into a feedwater heater arranged in series and ahead of the one already mentioned.

34 A diagrammatical view of the tender is given in Fig. 10. The cooling elements are arranged crosswise to the axis of the tender and symmetrically to it. They communicate on one side with the atmosphere, and on the other with a central channel leading to a centrifugal fan. As shown, a double fan may be used. For still greater capacities the arrangement can be doubled or trebled.

Theoretical Considerations

35 *Recooler.* The recooler will be considered first, as the vacuum obtainable determines the calculation of the turbines and auxiliaries. The better the vacuum, the greater the total heat which can be transferred to useful work. It is known from stationary plants that with surface condensers the vacuum directly depends on the cooling-water temperature. In the case of the condensing locomotive the vacuum therefore depends upon the temperature to which the cooling water can be lowered in the recooler.

36 The maximum temperature that the air going through the recooler can attain is that of the warm cooling water. If it be assumed that the air at this maximum temperature is fully saturated with water vapor, then the difference in total heat between the air entering and that leaving the recooler is identical with the amount of heat withdrawn from the water. When a turbine is working against different vacua it is possible, assuming definite initial steam conditions and constant turbine efficiency, to calculate the amount of heat which has to be extracted from the steam in the condenser, or, what is the same thing, from the cooling water in the recooler. If it be further assumed that the vacuum in the turbine exhaust corresponds to the water-vapor tension of water at a temperature which is 9 deg. fahr. higher than the cooling water leaving the condenser, we can calculate the amount of air

necessary to obtain a certain vacuum for a given quantity of steam. This calculation has been made and in Fig. 11 curve I shows the result for cooling air at 59 deg. fahr. and 70 per cent saturated. The air necessary for cooling is delivered by a fan. Assuming constant surface area for the cooler for all the different vacua, the resistance through the cooler would, of course, increase with the

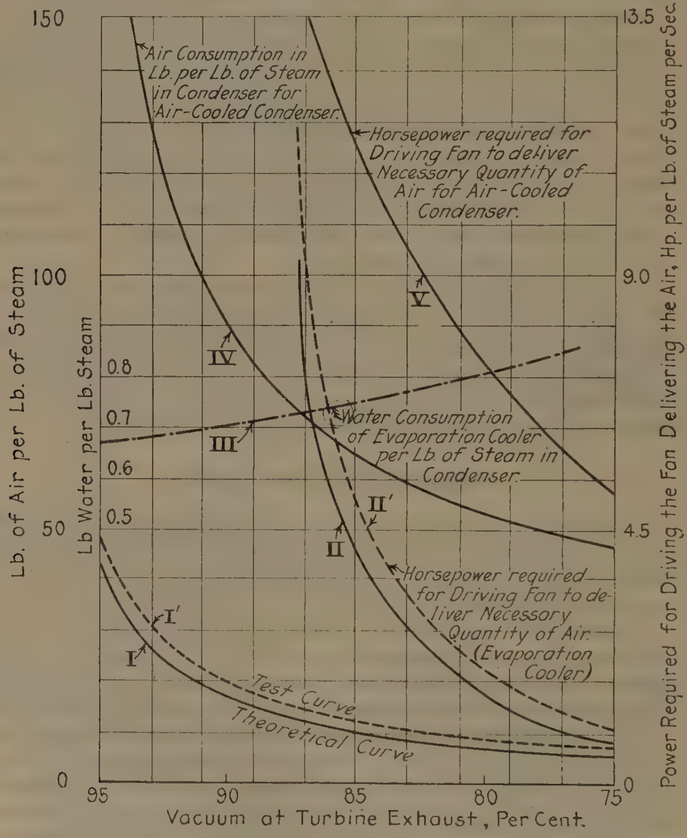


FIG. 11 AIR CONSUMPTION FOR EVAPORATION AND SURFACE COOLERS

quantity of air, and the power required for driving the fan would augment exceedingly rapidly with better vacua. The conditions are illustrated in curve II, the horsepower referring to the power necessary for each pound of steam per second. A long series of tests has shown that for full load the vacuum will be somewhere near 90 per cent which, of course, would only apply to maximum loads. On partial loads the vacuum would increase on the amount of steam sent to the condenser being less.

37 Curve III gives the water consumption of the recooler in pounds per pound of steam. Curves I′ and II′ give test results (air quantity and power absorbed by fan) which show that actual conditions do not differ materially from the theoretical.

38 To show the efficiency of the Zoelly cooling system as compared with that obtained by employing an air-cooled condenser

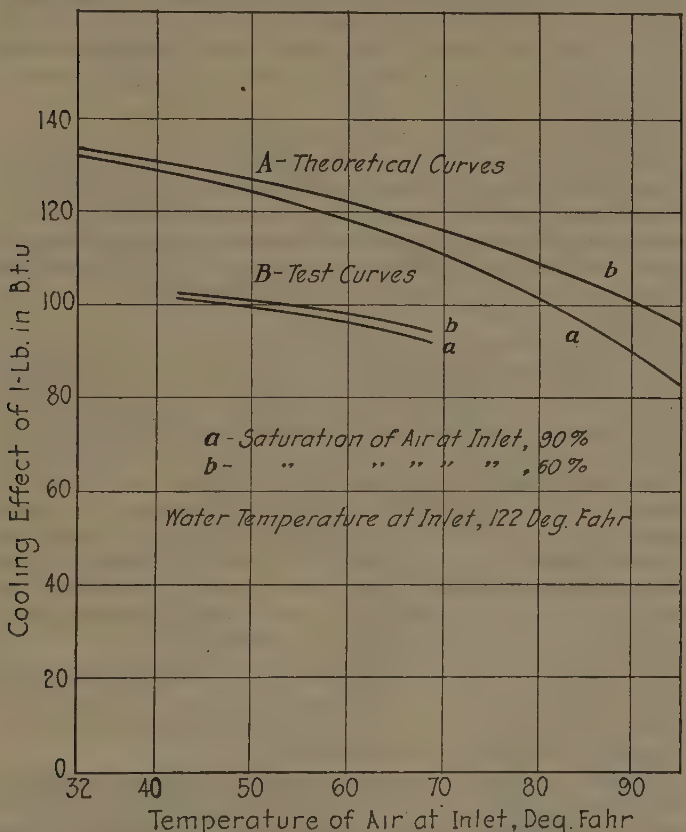


Fig. 12 Variation of Cooling Effect with Varying Air Conditions on Evaporation Coolers

curve IV in Fig. 11 gives the necessary quantity of air on the assumption that the air could warm up to 122 deg. fahr., and curve V the power required for the fan per pound of steam per second. The difference as compared with the evaporator cooler is enormous. The Zoelly system of cooling involves the necessity of having a condenser and recooler, but affords important advantages. The coefficient of heat transmission from steam to water in a sur-

face condenser is about 490 B.t.u. per deg. fahr. per sq. ft. per hr., whereas that for air-cooled condensers from steam to air is only a trifle over 8 B.t.u.[1] The surface of the air condenser must therefore be 60 times larger than that of a water-cooled condenser. This makes it possible to have on the Zoelly locomotive both condenser and turbines on the same carriage as the boiler, thus overcoming all difficulties involved in having vacuum connections between tender and locomotive. The relatively small vacuum space is easily kept airtight, while the system further allows of the locomotive's being the real driving part, thus adhering so far to the old and conventional design.

39 The curves in Fig. 11 have been drawn for average air conditions [15 deg. cent. (59 deg. fahr.) and 70 per cent saturated]. In Fig. 12 curves are given to show the variation of cooling capacity when air conditions are changing. Curves *A* are theoretical curves, while curves *B* embody the results of experiments, from which will be seen that the effective influence is less than the theoretical, owing to the fact that the effective air quantity is greater than the theoretical.

40 It was of course impossible to calculate the recooler, and very extensive tests were necessary to clear up the very complicated relationships and to get the data for actual design. The complicated nature of the task of finding the most economical design, i. e., to get the maximum cooling effect for a given surface with the minimum of weight and driving power, will be seen from the numerous variable factors, viz., material of filling bodies, height, quantity of water, velocity of air, distribution of water, water and air temperatures, etc. As a result of all tests we found that for average conditions — air 58 deg. fahr., cooling water 122 deg. fahr., corresponding to about 85 per cent vacuum, a cooling effect of 44,260 B.t.u. per sq. ft. an hour is obtained.

41 There still remain many interesting details in connection with this cooler, but their discussion would lead too far for the present paper, the scope of which is confined to giving a general idea only of the Zoelly turbo-locomotive.

42 *Main Turbine.* For turbine-driven locomotives superheated steam is used in any case, as this gives the highest efficiency. So long as it is necessary to use the conventional type of boiler it will scarcely be possible to assume other steam conditions than those obtaining for ordinary locomotives, say, 215 lb. pressure and 662 deg. fahr. temperature. The vacuum has been fixed in the preceding paragraphs and the output desired is given, so that as variable factors in the turbine calculation there are the turbine speed, the wheel diameters, and the number of stages. These elements have so to be chosen that the efficiency will be as high as possible. On

[1] R. P. Wagner, *Organ für die Fortschritte des Eisenbahnwesens,* Jan. 15, 1924, p. 3.

the locomotive we are limited in the matter of wheel diameters as well as in the number of stages. If these are determined the speed is dictated, any departure from which speed would mean a certain loss in efficiency, which in relatively wide limits is not of the utmost importance. The turbine speed determines the gear ratio and dimensions. For normal conditions the turbine speed is round about 6000 to 8000 r.p.m., but it also depends upon the maximum

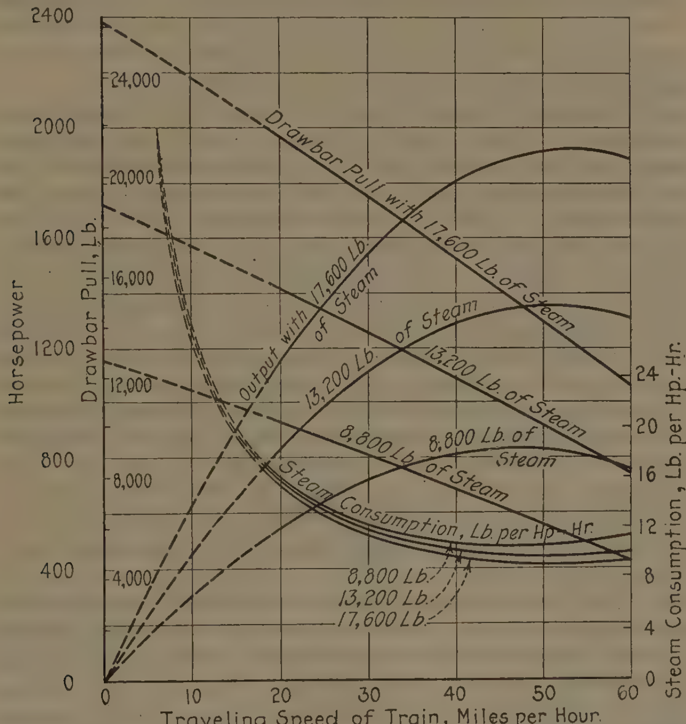


FIG. 13 OUTPUT, DRAWBAR PULL, AND STEAM CONSUMPTION OF 2000-HP. ZOELLY TURBO-LOCOMOTIVE FOR DIFFERENT SPEEDS AND STEAM QUANTITIES

speed the train can attain, this in turn being limited by the stresses allowed for.

43 A locomotive turbine has to be calculated not only for one particular speed and one load, as is usually done for stationary plants, but for different quantities of steam flow and for different speeds for each steam flow, in order to obtain full information as to the performance of the locomotive. Such curves as calculated for the Krupp locomotive, showing output and drawbar pull for different speeds and steam quantities, are given in Fig. 13. For

running astern a sacrifice in efficiency is admitted on account of the extreme limitation of space. The only point to consider is that sufficient initial torque must be available for accelerating the heaviest of trains.

44 *Auxiliary Turbines.* The limitation in size and weight is very important, but efficiency is also extremely important. Small wheel diameters call for multiple stages and the turbine would thus be too long. In order to overcome this difficulty several turbines are connected in series. This arrangement in conjunction with high running speeds (9000 to 10,000 r.p.m.) satisfies the demand for high efficiency combined with small dimensions.

45 *Feedwater Heating.* The heat contained in the stack gases has long since been utilized in stationary plants, the economizer being a very well-known device. Several feedwater heaters of this type have been tried for locomotives, but without success. The draft available on ordinary locomotives is not sufficient to overcome the additional resistance of such a heater. In turbine-driven locomotives, however, where the draft has to be produced artificially, this difficulty does not present itself and therefore waste-gas feedwater heaters can be employed to advantage. The gases leave the stack with a temperature of about 750 deg. fahr., and can be cooled down to about 350 deg. fahr., thus giving 95 B.t.u. per lb., corresponding to about 1 lb. of feedwater. If the condensate leaves the condenser at a temperature of 122 deg. fahr., it would therefore be possible to heat the feedwater to 217 deg. fahr. The total heat of the steam in the boiler being 1350 B.t.u., the saving effected by heating the feedwater thus amounts to 7.5 per cent. It is a matter of practice to see how much of this heat can actually be utilized.

46 There are other possibilities of heating the feedwater with steam, which latter can be exhaust steam from auxiliary turbines or steam bled from the main or an auxiliary turbine. Steam being exhausted at atmospheric pressure from a back-pressure turbine, as in the case of the turbine driving the furnace fan of the experimental turbine, has a total heat of about 1150 B.t.u. Were this steam used in a low-pressure turbine working to the condenser, probably 72 B.t.u. could be transferred to useful work, but the steam would enter into the condenser with 1080 B.t.u., of which 990 B.t.u. would go to the cooling water and thus be lost. Using the exhaust steam in the heater, all the 1150 B.t.u. can be utilized. Under normal steam conditions the feedwater can be heated up to about 302 deg. fahr. The condensate having a temperature of 122 deg. fahr., we can therefore use 180 B.t.u. per lb. of feedwater. The maximum quantity of exhaust steam which can be utilized is accordingly one-fifth of the quantity of condensate. To heat up to this high pressure, of course, steam at a pressure higher than that of the atmosphere has to be used.

47 The greatest locomotive economy would be effected by preheating the combustion air with stack gases and the feedwater with steam in a series of feedwater heaters, all heated by steam of different pressures bled from the main turbine or exhaust steam at suitable pressure. Steam bled from the main turbine or steam from the furnace fan will always be the most convenient, as the proportion of steam to feedwater is adjusted automatically.

Steam and Coal Consumption

48 The steam consumption of a turbine working condensing is very much better than that of a reciprocating engine working non-condensing. The condensing machines, however, require a number of auxiliaries also consuming steam and thus lowering the overall efficiency to a certain extent. In calculating the coal consumption it is necessary to consider carefully what preheating system shall be employed, as the coal consumption depends to a very great extent upon this factor.

49 Table 1 gives the results of such a calculation for the 2000-hp. Zoelly turbo-locomotive built by Krupp and described above, as well as for a piston locomotive.[1] The steam and coal economy of the turbo-locomotive is calculated for three different methods of preheating and also without preheating, and the beneficial effect of employing a suitable heating system is evident.

Test Results

50 The Krupp locomotive has only just been completed, so that test results will not be available before a certain lapse of time. Tests have been carried out with the experimental machine, but only over a very short line (35 miles) with very variable grades, so that there was no possibility of making coal-consumption tests. All that could be determined was the steam consumption, which may be compared with that of a corresponding piston locomotive making the same journey with the same load. Considering the preheating employed on the turbo-locomotive and the fact that for the auxiliaries only saturated steam was used, the consumption of heat in the turbo-locomotive for the round trip was only 4,230,000 B.t.u., whereas that of an ordinary locomotive — same train and identical conditions — was 5,904,000 B.t.u. (calculated from the difference in water level in the tender before and after the run and the initial steam conditions). The tests were repeated four times, the average being given. The water consumption of the recooler was 11,880 lb. Of this quite a large amount represented mechanical loss through leakage.

[1] From *Kruppsche Monatshefte,* Essen, Germany, 1924, p. 24.

TABLE 1 COMPARATIVE STEAM AND COAL CONSUMPTIONS OF DIFFERENT LOCOMOTIVE TYPES

	Piston-Locomotives, Non-Condensing				Piston-Locomotives, Condensing		Turbo-Locomotives			
	Superheated-Steam Simple 2-Cyl.		Superheated-Steam 4-Cyl. Compound		Superheated-Steam Simple Compound			With Feedwater Heating		
	Without preheating	With preheating	Without preheating	With preheating	Without preheating	With preheating	Without preheating	With steam	With smoke gases	Preheating feedwater and combustion air
Boiler pressure, lb. per sq. in. abs.	213	213	213	213	213	213	213	213	213	213
Pressure at steam chest, lb. per sq. in. abs.	199	199	199	199	199	199	199	199	199	199
Temperature in steam chest, deg. fahr.	572	572	572	572	572	572	662	662	662	662
Exhaust pressure, lb. per sq. in. abs.	14.2	14.2	14.2	14.2	2.13	2.13	2.13	2.13	2.13	2.13
Steam per i.hp-hr. (including all auxiliaries), lb.	16.97–16.28	16.97–16.28	16.5–15.61	16.5–15.61	13.83	11.55	10.59	11.08	11.08	11.08
Saving in steam consumption compared to simple locomotive working with superheated steam, non-condensing, lb.	0	0	0.47–0.67	0.47–0.67	3.14–2.45	5.42–4.73	6.38–5.69	5.89–5.20	5.89–5.20	5.89–5.20
Do., per cent	0	0	2.76–4.1	2.76–4.1	18.5–15.1	31.9–29.0	37.6–35.0	34.7–31.9	34.7–31.9	34.7–31.9
Total heat required in boiler per hp-hr., B.t.u.	44,860–41,680	33,830–32,470	36,850–34,880	32,910–31,140	30,410	25,510	20,120	19,510	17,980	16,410
Coal consumption (heating value of 11,700 B.t.u. assumed), lb. per hp-hr.	3.23–3.10	2.88–2.77	3.14–2.97	2.81–2.52	2.60	2.18	1.72	1.67	1.54	1.40
Saving in coal compared to a simple locomotive working with superheated steam and feedwater heating, lb.	—0.35 to —0.33	0	—0.26 to —0.20	0.07–0.25	0.28–0.17	0.70–0.59	1.16–1.05	1.21–1.10	1.34–1.23	1.48–1.37
Do., per cent	—12.4 to —12.1	0	—9.3 to —8.9	2.43–8.9	9.3–5.65	24.1–21.0	40.0–38.0	41.8–39.5	46.5–44.4	51.1–49.2

51 In spite of the various parts comprised in a turbo-locomotive, its operation is simple and less attention on the part of the personnel is called for than is the case with the ordinary locomotive. Before starting the driver speeds up the turbine driving the fan on the recooler, which operation consists solely in opening the

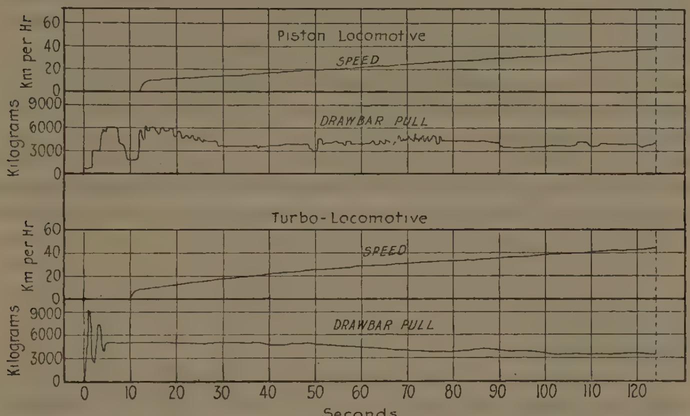


FIG. 14 STARTING DIAGRAMS FOR PISTON AND TURBO-LOCOMOTIVES

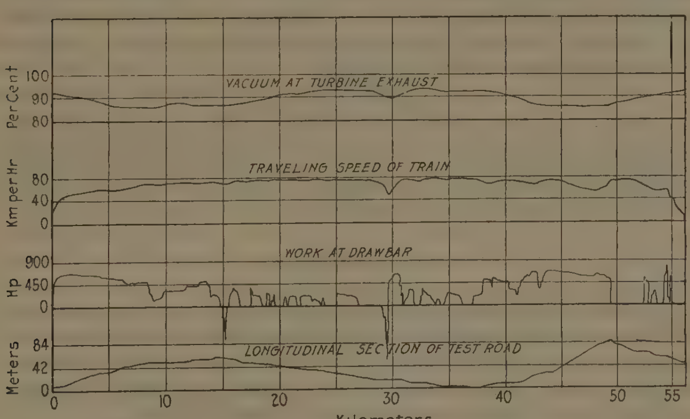


FIG. 15 DYNAMOMETER-CAR DIAGRAMS

corresponding valve. As this turbine is connected in series with the auxiliary turbine driving the condensing auxiliaries, the condensing plant is started automatically. After the working vacuum has been obtained, which takes several minutes, the steam admission valve to the main turbine is opened wide, and when it is desired to start the train it is only necessary to open the governor valve. On the experimental machine it is also necessary to have the furnace fan

started, which, however, only requires a valve to be opened. The opening of this valve is goverened in such a way that the boiler pressure is kept constant. Boiler feeding is no longer necessary. To keep the locomotive going at a certain speed it is no longer necessary to vary the degree of admission; all that is required is simply to throttle the steam admission to the required extent. If for any reason the output is insufficient with the governor valve wide open, then the overload valve allows an increase.

52 All that is required to stop the locomotive is to close the governing valve. The auxiliaries are not to be shut down unless for a prolonged stop.

53 To change from running ahead to running astern, after first having stopped the machine, the valve for running ahead is closed and that for running astern is opened, after which all that is necessary is to open the governing valve.

54 Starting up takes place very smoothly, as may be seen from Fig. 14, and the fears expressed by many engineers that a turbine-driven locomotive would not have sufficient initial torque are absolutely unfounded.

55 The running of the turbine-driven machine is also very smooth compared with that of a piston locomotive, this being on account of the absence of reciprocating masses. The drawbar pull also is extremely constant, as is shown by the smooth line in the diagram (Fig. 14), taken with the dynamometer of the Swiss Federal Railways. In the diagram for the piston engine in Fig. 14 every stroke of the piston is marked.

56 Fig. 15 gives the results of a test made with the dynamometer car of the Swiss Federal Railways. The meanings of the various curves are given in the diagram. Noteworthy is the excellent vacuum maintained throughout the whole of the trip, which averaged 90 per cent at an air temperature of 68 deg. fahr.

Future Prospects

57 The experimental locomotive described has a capacity of 1000 hp. and that built by Krupp one of 2000 hp., but this is by no means the limit of the possibilities. Studies and complete designs have been made for locomotives up to 5000 hp., neither locomotive nor recooler presenting any particular difficulty.

58 Today a further development of the turbine-driven locomotive is impending, namely, the employment of very high pressures, their practical utilization being only a matter of finding a suitable boiler. As far as the turbine is concerned, the designs are all ready.

Conclusion

59 Numerous trials made with the Zoelly turbine-driven locomotive described have proved that it is fit for the rail. Wagner, a

German authority on railroad operation, states that "as a whole the Zoelly locomotive is of astonishing simplicity for a condensing locomotive, and for a completely new design is remarkably reliable and well fit for railway service.[1]"

60 The advantages of the turbo-locomotive are manifold: High economy in fuel, amounting to very nearly 50 per cent; low water consumption, the water not necessarily being good water for the purpose; boiler kept clean; very smooth running on account of the absence of reciprocating masses, etc. There can be no doubt that the turbine-driven locomotive will very soon enter into successful competition not only with the old piston locomotive but also with schemes of electrification which are expected to result in greater efficiency in the utilization of coal. Its greater economy gives increased capacity with the same boiler, thus opening a new era of steam locomotive traction.

DISCUSSION

G. B. Warren.[2] Although electrification is the ideal toward which we should strive, the fact remains that even the most ardent exponents of railway electrification hesitate to say that more than 30 per cent of the railway mileage in this country could be profitably electrified now or in the near future, even if the enormous capital required for the investment were available. It therefore appears that we must have the mobile plant power with us for some time to come, and the condensing steam locomotive appears as one of the next logical developments.

A condensing locomotive immediately demands a turbine on the low-pressure end of the cycle, but not necessarily on the high-pressure end. The writer has made a careful study of the condensing-turbine-locomotive problem, and has become convinced that the best solution is to replace the tender of a reciprocating locomotive of the present type with a car carrying not only the fuel and water, but also a low-pressure turbine and condensing equipment, with either mechanical or electrical drive. The low-pressure turbine would be driven by the exhaust steam from the present type of locomotive. A preliminary design and study indicates that the operating characteristics, cost of development, etc., would be most favorable when using direct-current electric drive with an electric motor geared to the wheels and driven from a low-pressure turbo-generator. This idea is not new, but was suggested in Germany in 1908 by Prosper L'Orange.

Such a low-pressure-turbine locomotive would be about 50 to 53 ft. in length, and the combined wheel base of a 380,000-lb.

[1] *Organ für die Fortschritte des Eisenbahnwesens*, Jan. 15, 1924.

[2] Turbine Engineering Department, General Electric Company, Schenectady, N. Y. Jun. A.S.M.E.

Mikado locomotive and connected low-pressure-turbine locomotive would be less than 95 ft. The weight of the total combination would be about 100 tons more than that of the present locomotive and tender.

The cylinders of the reciprocating locomotive would exhaust into the pipe leading back to the low-pressure turbo-generator, and the draft would be supplied by an electrically-driven blower. If desirable, the auxiliaries could be electrically operated from motors driven by a low-pressure auxiliary turbo-generator whose turbine steam flow was either in series or parallel with the main low-pressure unit.

Such a low-pressure unit, at speeds of 10 miles per hour and above, when operating on the exhaust steam of the engine, would have a drawbar pull of from 75 per cent to 85 per cent of that of the present locomotive, whose pulling ability would be practically unaffected by the presence of the low-pressure locomotive. The starting drawbar pull could be made to equal or exceed by from 10 per cent to 25 per cent that of the locomotive by bleeding a comparatively small amount of live steam into the low-pressure turbine. The power of the combined unit would be around 5000 hp. when used with a Mikado engine of 2500 to 3000 hp. That is, the hauling capacity of a present type locomotive could be practically doubled without increasing the fuel consumed or the crew required for operation, and the train loadings or speed increased by a corresponding amount.

All the auxiliaries would be driven by the exhaust steam. They would start and stop and fluctuate in speed with the starting and stopping or change in load of the engine without the attention of the engineer. The admission of high-pressure steam could be handled in the same manner as the booster of the present locomotive. The electrical control, if used, would be relatively simple and automatic.

The condenser would return condensed water to the boiler, which should result in a considerable reduction in boiler maintenance. The maintenance of the low-pressure turbo-electric locomotive should be very low, and a considerable reduction in maintenance per locomotive should result.

The oil from the engine cylinders could be removed from the feedwater before being returned to the boiler and a periodic washing of the condenser tubes and turbine blades would prevent any trouble due to the lubricating oil.

The cost of such a low-pressure-turbine locomotive would probably not exceed very much that of the reciprocating steam locomotive with which it is used. It should be considerably less than that of a complete turbine locomotive whose power is equal to the power of the combined low-pressure turbine and the reciprocating locomotive. Since, if the electric drives used — driving motors, generator control equipment, turbine auxiliaries, etc. —

are such as to permit utilizing present standard equipment to a great extent, the development and cost would be low.

Such a combination has all of the advantages and economy of a complete turbine locomotive, and many more in addition. In the first place the present reciprocating locomotive is an exceedingly fine machine. It is the product of a hundred years of intensive evolution and development, and has a remarkably low initial cost per horsepower. Despite its comparatively low thermal efficiency it is holding its own in a highly competitive field with electrification, internal-combustion locomotives, turbine locomotives, etc. If now, instead of completely revolutionizing locomotive practice, as would the complete turbine locomotive, we just add to the present machine a low-pressure condensing element without impairing the machine in any way, we have greater assurance of obtaining the results expected and with very much less development, change in manufacture or operation, and at a greatly reduced capital investment. Hence railroads can, if desired, bring present equipment up to the highest possible efficiency by the addition of such a low-pressure condensing turbine locomotive.

The indicated Rankine-cycle efficiency of the modern superheated reciprocating steam locomotive cylinder is in the neighborhood of 75 per cent at 50 to 45 per cent cut-off, and most of the loss is in the pressure released when the exhaust valve opens. Part of this is recoverable by the low-pressure turbine system. The mechanical efficiency of transmitting the indicated power to the crankpin must be exceedingly high. No turbine system at present available can do more than equal this efficiency on the high-pressure to atmospheric part of the cycle in the capacities used on locomotives and with the losses in transmission allowed. It is probable that the efficiency would be even less, and certainly no high-pressure, non-condensing turbine system approaches the simplicity of the cylinders, valve gear, and side rods of the reciprocating locomotive. As boiler pressures are increased, the difficulties of using a high-pressure turbine will increase, while the reciprocating engine will be unimpaired.

With the low-pressure turbine system the difficulties of the transmission of power to the wheels is cut in half. This is an item of some importance when locomotives of 5000 hp. are considered. The starting characteristics of the combination are more favorable than with a steam locomotive, which would not be true of the geared or alternating-current electric-drive turbine locomotive.

The low-pressure turbine will have larger clearances, fewer stages, longer buckets, and higher efficiency, will be less liable to injury, and will have a much better sustained efficiency than a high-pressure turbine.

As regards the weight efficiency of the Zoelly turbine locomotive when compared to the standard reciprocating steam locomotive, unfortunately the weight of the turbine-locomotive tender is not

given in the paper. If we assume it to be 100,000 lb. when loaded, and the locomotive to be 130,000 lb., and the maximum power as 850 hp., then the weight, including the tender, would be 271 lb. per hp., and excluding the tender, 153 lb. per hp.

The weight of the American Locomotive Company H-10 Mikado locomotive is 535,000 lb. with loaded tender and 350,000 lb. without tender. The maximum power is approximately 2700 hp., making the weight 198 lb. per hp. with tender and 124 lb. per hp. without tender.

The Pennsylvania Railroad L-1s Mikado engine weighs 500,000 lb. with loaded tender and 315,000 lb. without tender, and develops approximately 2500 hp. maximum. This makes 200 lb. per hp. with tender and 126 lb. per hp. without tender.

The No. 5000, 3-cylinder Mountain type American Locomotive Company locomotive weighs 570,000 lb. with loaded tender and 390,000 lb. without tender, and develops approximately 2700 hp. This gives a weight of 211 lb. per hp. with tender and 136 lb. per hp. without tender.

It can thus be seen that the Zoelly locomotive with the assumed tender weight is only 35 per cent heavier per hp. than standard steam locomotives representing the best American practice, and without tender it is only 17 per cent heavier per hp. This is on the basis of the comparison involving American locomotives of three times the capacity of the turbine locomotive. This is not a fair comparison, inasmuch as the weight efficiency should be better as the capacity of the machine increases, other conditions being equal.

The proposed combination of a Mikado reciprocating locomotive and a low-pressure turbine-driven condensing locomotive would have a weight of approximately 820,000 lb. when loaded with water and coal, and would have a maximum power of about 4500 hp. This would make a weight of 183 lb. per hp., which compares very favorably with the best American practice including the tender. It does not follow, therefore, that a condensing turbine locomotive must be considerably heavier than a standard non-condensing locomotive. If properly designed, the reduction in boiler capacity, water and fuel storage per hp. should offset to a very great extent the weight of condensing equipment required.

C. A. Jacobson.[1] Fundamentally a locomotive is a machine to transform chemical energy into the work of pulling a train. The reciprocating steam locomotive releases heat energy in the boiler, transmits it in the form of fluid steam to the cylinder and there converts it into work, and by the driving machinery into drawbar pull. The turbine locomotive is apparently not a device which has in it the inherent characteristics that make it suitable for hauling

[1] New York Central Railroad, New York, N. Y.

trains without adding other complications and transmissions. Instead of that we should try to get back to the fundamental idea of a locomotive which will give us the necessary transformation of energy without any transmission mediums. The ideal in that respect would be a direct-driven Diesel locomotive, but unfortunately that has some bad characteristics for starting. The turbine locomotives that have been developed are complicated machines with many parts. It has been stated that the turbine and generator can be placed on the tender, driving the wheels by electric motors, and that the cost should not be any more than that of the present steam locomotives, but the proposition hardly seems reasonable. Furthermore, there is one fundamental difficulty with the steam locomotive. It is incapable of utilizing the maximum capacity unless the men operating it handle it correctly. This is true of the reciprocating steam locomotive, and the same difficulty will apply to the turbine locomotive.

The final criterion of a locomotive is the cost of hauling trains per thousand ton-miles, and in that we have many elements other than fuel cost. If we saved a large percentage of the fuel cost, but only by increasing the first cost and the fixed charges, we would perhaps be no better off than if we kept the same old steam locomotive.

Unquestionably, power machinery can be designed and built that will do almost anything that may be necessary, but in the final analysis the cost settles the question.

R. Eksergian.[1] In the consideration of the possible adoption of the turbine locomotive by American railroads, the problem should be viewed from two major aspects: (1) the physical characteristics of the turbine locomotive as compared with the reciprocating locomotive, and (2) the general economic advantages, if any.

Thermodynamically it is immediately admitted that decreasing the back pressure to a moderate vacuum by the use of a condenser results in an increased thermal efficiency; that for taking advantage of even a moderate vacuum a reciprocating engine is impossible, due to constructive limitations; hence the inevitable necessity of the turbine in conjunction with the condenser.

On the other hand, weight efficiency which ultimately represents the maximum hauling capacity of a locomotive limited by the fixed weight limitations of the roadway, bridges, etc., is a basic criterion of locomotive performance. Drastic efforts in increasing thermal efficiencies are not likely to be very effective in locomotive design, unless they react in increased weight efficiency.

Locomotive performance may be divided into two ranges: (1) the adhesive range, wherein the tractive force is dependent upon

[1] Engr., The Baldwin Locomotive Works, Philadelphia, Pa. Mem. A.S.M.E.

the adhesive weight on the drivers, and (2) the horsepower range which depends upon the performance of a locomotive as a power plant. We are thus concerned in mechanically designing a mechanism that can transmit large torque loadings at low speeds, and then proportioning a minimum-weight power plant to give the maximum tractive force at the higher speeds for a given axle, as well as total, weight of the locomotive. It is thus evident that high thermal economies do not necessarily imply satisfactory locomotive performance.

Assuming a satisfactory condenser, and considering the torque characteristics and steam consumption of the turbine as a variable-speed motor, let

T = torque developed by a turbine stage element, lb-ft.
C_1 = absolute velocity of jet at entrance to movable blade, ft. per sec.
ϕ = ratio of relative velocity at exit to that at entrance, ft. per sec.
U = peripheral velocity of blade, ft. per sec.
R = mean radius of blade, ft.
W = weight of steam per second, lb. per sec.
α_1 = angle of jet to entrance of movable blade.

Impulse Type	Reaction Type
$T = \frac{W}{g} RC_1(1+\phi)\left(\cos\alpha_1 - \frac{U}{C_1}\right)$	$T = \frac{W}{g} RC_1\left(2\cos\alpha_1 - \frac{U}{C_1}\right)$
For maximum efficiency,	
$\frac{U_m}{C_1} = \frac{\cos\alpha_1}{2}$	$\frac{U_m}{C_1} = \cos\alpha_1$
Then rated torque	
$T_m = \frac{W}{g} RC_1(1+\phi)\frac{\cos\alpha_1}{2}$	$T_m = \frac{W}{g} RC_1 \cos\alpha_1$
$T = 2T_m \frac{1+\phi}{1+\phi_m} \frac{\cos\alpha_1 - \frac{U}{C_1}}{\cos\alpha_1}$	$T = T_m \frac{\left(2\cos\alpha_1 - \frac{U}{C_1}\right)}{\cos\alpha_1}$
Hence when $U = 0$, $T = T_i$	
$T_i = 2T_m \frac{1+\phi}{1+\phi_m}$	$T_i = 2T_m$

For the variation of torque of either type,

$$T = T_i\left(1 - \frac{U}{C_1\cos\alpha_1}\right) = T_i - KU. \text{ (A straight-line variation.)}$$

Assuming $\phi_i = \phi_m$ approximately, then

$$T_i = 2T_m$$

That is, the initial torque for the same steam consumption corresponding to the torque at rated speed is approximately double the torque at rated speed.

In a reciprocating locomotive, maximum horsepower is obtained around 1000 ft. per minute piston speed in 200 r.p.m. of the drivers, and the tractive force at this speed may be taken approximately for a road engine as 0.43 of the initial tractive force. That is, the initial to the rated tractive force is in the ratio of 2.3 to 1.

Further investigation shows clearly that the torque characteristics of the turbine compare favorably with those of the reciprocating steam locomotive.

The steam consumption of a reciprocating locomotive, the draft and the consequent coal consumption all increase with the speed up to the limit of the adhesion range, say at 80 r.p.m. Above this the steam consumption decreases to its minimum value around 200 r.p.m. along the horsepower range.

With the turbine locomotive, full steam blast — i.e., the steam consumption per hour at rated horsepower — or more is required during the adhesion range, say, to 80 r.p.m. of the drivers. Thus at the very low speeds the turbine locomotive is not as economical as the reciprocating locomotive. In the case of the 50 per cent cut-off locomotive for slow drag service this economy in favor of the reciprocating locomotive is more emphatic. However, with a change in gear ratio for freight service and considering the very frequent small accelerating period for passenger locomotives, the loss in excessive steam consumption with the turbine at the low speeds is not so excessive as might at first be supposed. At the higher revolutions the turbine locomotive is of course superior to the reciprocating steam locomotive.

The transference of rotating motion to reciprocating motion by means of a jackshaft, etc. and side rods is not a simple problem. For a given adhesion weight, the rod loads, axles, etc. are considerably in excess of ordinary reciprocating-steam-locomotive proportions. It may be stated very emphatically that 180,000 lb. adhesive load is the extreme limit of a single jackshaft drive, whereas comfortable proportions may be limited to 150,000 lb. adhesive load per jackshaft. The adhesive loading on modern American locomotives ranges from 180,000 to 240,000 lb. for passenger service, and from 240,000 to 300,000 lb. and over for freight service, showing the necessity, in a possible adoption of the turbine locomotive for our large motive power, for the use of two separate units.

As a concrete example of the difficulties in the design of the machinery, the jackshaft driving only three pairs of drivers with axle loadings of 60,000 lb., that is 180,000 lb. adhesive weight, may be considered.

Assuming a driver 72 in. in diameter with a 16-in. crank throw, a limiting value of stroke to driver diameter for wheel-center

proportions may be taken at 0.45. Then, assuming ample torsional flexibility at the jackshaft in the form of a flexible gear or pinion, etc., the maximum side-rod load is,

$$K = \frac{0.33 \times 60{,}000 \times 3 \times 36 \times 1.41}{16} = 190{,}000 \text{ lb.}$$

and with poor alignment this may reach a value of as high as 270,000 lb. For the bearing value on the pin, the side-rod load will be taken at $\frac{190{,}000}{1.41} = 134{,}500$ lb.

Pin diameter $= \sqrt{\frac{134{,}500}{1{,}500}} = 9.5$ in. Use $9\frac{1}{2}$ by $9\frac{1}{2}$-in pin at the jackshaft, and a 10 by 9-in. pin at the driver. (See Fig. 16.)

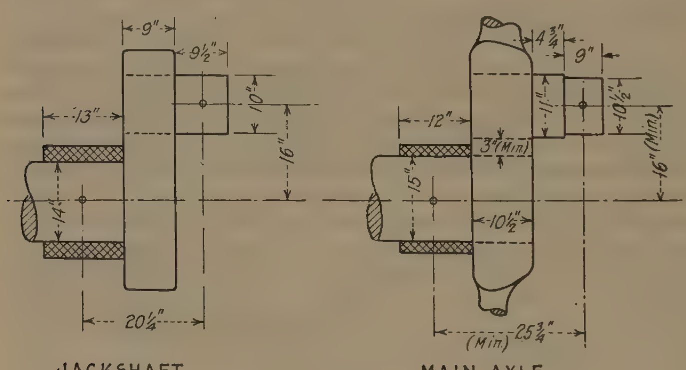


FIG. 16

It is to be noted that the pin-bearing value is somewhat high and therefore a larger pin might be necessary in a final design.

For the jackshaft and axle using possible peak loadings for strength proportions, and rod thrust at 90 deg. crank position for mean bearing loadings with due consideration for peak bearing loads, then (using a 14-in. jackshaft and 15-in. axle) the peak stresses will be,

$$\text{Jackshaft: } \frac{270{,}000 \times 20.25 \times 32}{\pi \times 14^3} = 20{,}300 \text{ lb. per sq. in.}$$

$$\text{Main Axle: } \frac{270{,}000 \times 25.75 \times 32}{\pi \times 15^3} = 21{,}000 \text{ lb. per sq. in.}$$

$$\text{Main Pin: } \frac{270{,}000 \times 9.75 \times 32}{\pi \times 11^3} = 19{,}000 \text{ lb. per sq. in.}$$

The bearing values will be

$$\text{Jackshaft: } \frac{134{,}000}{13 \times 14} = 740 \text{ lb. per sq. in.}$$

$$\text{Pin: } \frac{134{,}000}{95} = 1410 \text{ lb. per sq. in.}$$

In a corresponding reciprocating-locomotive design, the maximum main-rod load for the same weight, assuming a 30-in. stroke and a 72-in. driver, is

$$P = \frac{0.33 \times 180{,}000 \times 36}{1.41 \times 0.85 \times 15} = 120{,}000 \text{ lb. (max.)}$$

This permits less overhang, smaller axles, pins, etc., and is but 63 to 45 per cent of the loading in a turbine locomotive.

It is interesting to note that in reciprocating locomotives in no case do we find more than two pairs of drivers driven by one side rod, the front and rear coupling drivers in a Santa Fe type being driven by the middle or main driver.

With the larger torque loadings required at the jackshaft, the design of the reduction gearing becomes complicated by the very limited space available, necessitating possible overhanging of the turbines.

Moreover, the rod loadings previously stated are based on the assumption that sufficient torsional elasticity is obtained at the jackshaft. Some form of torsional spring system at the jackshaft is of primary importance, to reduce possible impact loadings on the rods and journal bearings. Finally, for sufficient gear reduction the driver diameter may be increased by the main-gear diameter for proper clearances. With the larger driver diameter and the need of limiting the throw on account of centrifugal forces and the balance, especially on the jackshaft, the rod loadings may become excessive.

Thus, it is clearly seen that a direct mechanical drive is very much limited in horsepower and adhesive-force capacity, especially for the large locomotives used on American railways, unless two sets of turbines, jackshafts, etc., are used. A combined system by the use of two units, one on the locomotive driven by a high-pressure reciprocating piston and the other under the tender driven by a low-pressure turbine, has certain merits. By this method, however, considerable throttling of high-pressure steam to the low-pressure turbine would be needed for the starting period, since the discharge steam from the reciprocating locomotive during this period would be insufficient to give full-load torque. By giving greater adhesion weight to the reciprocating unit a small reduction in the initial steam consumption might be effected.

The turbine locomotive is wrapped up, so to speak, in the condenser, and the justification of the turbine is of course based on the vacuum produced by the condenser. Simple calculations

will quickly show that even with poor vacuums, a reciprocating compound engine developing 3000 hp. with our gage limitations, and even with articulated running gear, is constructively impossible; hence the inevitable necessity of the turbine in conjunction with the condenser. Therefore, a major problem for development includes the design of a minimum-weight condenser giving a high vacuum under ordinary operating conditions and at reasonable cost.

In judging the ensemble design, a serious factor is the weight efficiency. This should not exceed 200 lb. per hp. and preferably should be around 150 to 170 lb. per hp.

Another consideration is the arrangement of wheelbase, etc., for proper tracking and reducing concentrated axle loading without undue overall length.

The introduction of any new type of motive power is ultimately dependent on the equivalent operating cost, where:

Equivalent operating cost = (1) Interest on the investment in initial cost + (2) depreciation + (3) fuel operating cost + (4) maintenance cost + (5) labor and accessories operating cost + (6) change-over cost distributed over a period of years − (7) the reduction in cost of train movements − (8) the reduction of maintenance-of-way costs.

Items (6), (7) and (8) are obviously difficult to arrive at, while the two latter are very often not even considered in conjunction with the cost of motive power, but they exist nevertheless and often completely justify the introduction of a new type of motive power. Thus electrification is often entirely justified in congested traffic zones by (7) alone, even when the other items may be abnormally high.

It is quite characteristic that engineers outside of the railroad field have laid undue stress on the thermal side, and although it is admitted that (3) is a large part of the total cost, the percentage saving on (3), even if large, may be more than offset by the other items. The real economy of the continually increasing thermal economies on reciprocating steam locomotives is actually the increased horsepower capacity of the locomotive for a given total weight, that is, the weight efficiency. Drastic efforts in increasing thermal efficiencies are not likely to be very effective in locomotive design unless they result in considerable weight efficiency. With the fixed limitation of the roadway, bridges, etc., it is evident that weight efficiency measures the tonnage-hauling capacity of the locomotive, with the corresponding reduction of train movements, maintenance cost per ton hauled, etc.

Assuming a reasonable weight efficiency, the reduction in fuel costs plus the advantage of a better counterbalance is offset by the increased cost of the locomotive, and probably due to somewhat higher maintenance caused by the crank drive and the change-over maintenance methods. In the writer's opinion, probably the

biggest economic obstacle is the high development cost of the turbine locomotive.

C. E. Chambers.[1] The writer wishes to propound two questions. Is this locomotive being developed for tonnage only or for speed as well as tonnage? Also, would it be possible to cut out the turbine, attached to the locomotive tender, or must it remain constantly in service?

Spencer Miller.[2] It would be interesting to learn what prompted the design of this turbine locomotive. Was it prompted by the high cost of fuel in Europe, or the high cost of labor?

Perhaps the experience with logging machinery on the Pacific Coast may be cited as an example of high wages affecting the problem of fuel. Logging machinery is extensively employed on the Pacific Coast to haul logs from the forests to logging railroads. In the past the waste wood that is scattered all about these logging machines has been employed for fuel, and Oregon fir is a splendid fuel because it contains a large amount of pitch. There is an unlimited quantity of this fuel, and there are no transportation charges to be added. But it has been found that the cost of cutting it up for the furnaces has become almost prohibitive. In consequence of this, to a large extent wood fuel has been abandoned and oil substituted. And yet this oil has to be transported from Southern California to Oregon. And in spite of the cost of the oil fuel itself and the transportation charges, nevertheless it is the more economical to use, and more dependable for maintaining the steam pressures at a point where the logging machinery will be most efficient. Labor today in Oregon costs $7 or $8 per day plus the cost of board and lodging. Another factor of the problem is the difficulty of controlling the labor. In spite of orders to the contrary, the men insist on sawing up the best of the logs because they are easier to split. In consequence of this, we witness the spectacle of the finest sort of fuel left to rot on the ground, or what is worse, being responsible for disastrous forest fires, and oil, which has to be transported 1000 miles, used for making steam.

Very few boilers are equipped with the customary appliances for saving fuel. Feedwater heaters are rare, and superheaters as well, because it has been found that whatever economies they may effect theoretically are largely offset by the cost of extra labor in keeping them in order.

C. C. Egbert.[3] One reason for the development of the locomotive described by the author is the high cost of coal, which is $10

[1] Supt., Motive Power and Equipment, Central R. R. Co. of N. J., Jersey City, N. J. Mem. A.S.M.E.
[2] Chief Engr., Lidgerwood Mfg. Co., New York, N. Y. Mem. A.S.M.E.
[3] Cons. Engr., Niagara Falls, N. Y. Mem. A.S.M.E.

per ton in Europe. On account of this high cost of coal there has been a marked development in the efficiency of the steam turbine during the past few years. In the smaller sizes, say, from 4000 hp. down, the saving in steam attained by the adoption of high-speed turbines coupled through reducing gears has amounted to about 30 per cent, making geared units of from 3000 hp. to 4000 hp. of about the same efficiency as slower speed, direct-connected units of from 12,000 to 15,000 hp. It is quite natural to suppose that the author had this fact in mind in applying the turbine to the locomotive.

E. N. Trump.[1] A locomotive recently built in Europe with the uniflow cylinder has been a success. It had combined with it, however, a device to take the place of the condenser. An examination of the tests that have been made on some Pennsylvania locomotives shows a back pressure in all cylinders of about 15 lb. per sq. in. That is a large percentage of the power, to be used only for making a draft for the fire. The nozzles are either so inefficient or so badly proportioned that they put a tremendous back pressure on the cylinders. The uniflow engine is most efficient with a vacuum. With back pressure the efficiency of the cycle is so much impaired that it is a question whether it will pay. With a good vacuum, such as is used in ordinary turbines, a tremendous increase of efficiency is obtained by the entire elimination of initial condensation in the uniflow cylinder.

Professor Stumpf, who invented the uniflow cycle, has also devised a system of induced exhaust to induce a draft in the other cylinders of a multiple-cylinder engine. He uses a nozzle so arranged that the exhaust in one cylinder induces a vacuum in the next one. By this system he has obtained as high as 80 per cent vacuum. This is as good as is necessary for the reciprocating engine and an increase of about 20 per cent in economy is expected. A locomotive using this system was built for the Russian Soviet by a concern in Norway. It has proved quite successful and fulfilled the competitive tests.

This induced exhaust is useful, not only in locomotives, but also in other places where exhaust steam is used for process work or heating. The toe of the card represents power which is used to reduce the pressure on the cylinder and at the same time to give the required back pressure which might be wanted for other purposes. Two or three cylinders with cranks at 90 or 120 deg. are necessary. The writer believes that the next locomotive will be a three-cylinder one with induced exhaust.

A. F. Johnson.[2] In connection with condenser troubles, which are admittedly present in turbine locomotives, the utilization of

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[2] Asst. Prof., Mech. Drawing, George Washington University, Washington, D. C. Mem. A.S.M.E.

the uniflow engine with attending simplified construction might well be considered for locomotives. This engine has excellent economical characteristics over a wide load range and tests have shown that a non-condensing uniflow engine may equal a turbine with condenser in efficiency. The uniflow locomotive could be run non-condensing, thus eliminating condenser troubles such as will be encountered in the turbine condensing locomotive.

W. E. Symons.[1] Last year in the United States the total fuel bill of the railways was $617,000,000, while the amount charged to locomotives was about $556,000,000. The average value of coal consumed by a locomotive is more than $10,000 a year and in some cases it is as high as $15,000 a year. The number of pounds of coal burned per thousand gross ton-miles of freight moved last year was about 136. The year before it was 176. The Pittsburgh and Lake Erie Railroad had a coal consumption per thousand ton-miles of freight of less than 70. Other railways have records somewhat approaching this low figure but any kind of locomotive or any improvement on our present steam locomotive that will materially reduce the amount of fuel consumed per passenger car-mile or per thousand freight ton-miles, or that will add to the efficiency of our railways as a transportation unit, will receive a very cordial welcome, and its use will be extended.

L. B. Jones.[2] From an operator's standpoint a turbine locomotive has two outstanding advantages. The first is the saving of fuel. The second is the fact that the turbine locomotive has no reciprocating parts. Railroad men will admit that the limit of power that can be transmitted through two crankpins has been very nearly reached. We are watching with interest the development of three crankpins and three-cylinder locomotives and the possibility of getting away from the enormous stresses that must be taken care of in the present steam locomotive.

Another point that deserves consideration is the behavior of the condenser locomotive in a tunnel. Many railroads have their heaviest grades approaching or in a tunnel. Any condition that will reduce the condensing power will naturally reduce the tractive energy of the locomotive and might cause trouble. This is one of the problems that will have to be met in a condensing locomotive.

The Author. The idea put forward of making a driving tender is one which the author thoroughly studied at the very beginning. It is certainly a problem to maintain the present piston locomotive and to increase its capacity and efficiency at the same time. But

[1] Cons. Engr., and Editorial Work, Angus Sinclair Pub. Co., New York, N. Y. Mem. A.S.M.E.

[2] Master Mechanic, Pennsylvania Railroad Company, Harrisburg, Pa. Mem. A.S.M.E.

the real turbo-locomotive has its advantages. The reciprocating masses of the piston locomotive form a very distinct drawback. Further, absolute cleanness of the feedwater can be obtained only with an exclusively turbine-driven locomotive, as only with such a locomotive can absolutely no trace of oil enter the boiler. The beneficial influence of clean feedwater has been clearly seen on the author's experimental machine, where, even after long periods of operation, the boiler was absolutely clean. Regarding weight, a real turbo-locomotive will be more advantageous than a piston locomotive with driving tender.

The driving tender nevertheless has its importance and the author believes it will be exceedingly useful during the transition period from the piston locomotive to the turbo-locomotive.

It may be of interest to note that under a Zoelly license Henschel & Son, of Cassel, Germany, are building for the German State Railways such a driving tender. This tender carries in one casing an ahead and an astern Zoelly turbine which work through a reducing gear to a dummy shaft with crank and crankpins, and with coupling rods to the driving wheels. The turbines operate on exhaust steam from the cylinders and exhaust directly into a rigidly coupled condenser. All the condensing auxiliaries are also arranged on the tender and are driven by an auxiliary turbine that receives live steam in order to maintain a constant vacuum. This is essential also for light load and no load, the main turbine running on high speed.

As mentioned in the discussion, the weight of the experimental locomotive with tender is 106 tons. At the rating of 1000 effective hp. at the crankpins this gives a weight of 240 lb. per hp. The Krupp locomotive with tender weighs 178 tons and its turbine develops 2000 effective hp. at the crankpins. The Krupp locomotive therefore weighs only 198 lb. per hp., and further reduction is certainly possible in new designs. It is absolutely certain that for still larger machines the weight per horsepower can be considerably reduced, and beyond doubt the Zoelly turbine-driven locomotive can very favorably compete with the piston locomotive on the count of weight.

On the Continent the interest taken in the turbo-locomotive, owing to its advantages regarding efficiency, weight, and cost of maintenance, is very great, and some of the largest engineering firms have themselves secured licenses under the Zoelly patents. Among these are Schneider in Le Creusot, France, and Krupp, Henschel & Son, and the Hannoversche Maschinenbauaktiengesellschaft in Germany.

The development of the turbine-driven locomotive is progressing rapidly, and a locomotive to work with a boiler pressure of 850 lb. per sq. in. has been designed and is in the first stages of construction.

No. 1948

THE VALUE OF EFFICIENCY IN TRANSFORMING AND DISTRIBUTING ENERGY

By Charles E. Lucke,[1] New York, N. Y.

Member of the Society

Realizing that cheaper heat and cheaper power are closely related national problems requiring engineering methods for their solution, the author in this paper gives valuable data to serve as a guide for the efficient transformation and distribution of energy. Costs of primary energy at the source, transportation or transmission costs, and costs at point of use are presented in tables and charts. Methods and costs for the preparation of a suitable working fluid for the generation of water and steam power are given, and the transformation of working-fluid energy into brake-horsepower energy is shown by means of numerous tables. Other tables give hydroelectric, steam-turbine, and Diesel oil-engine power costs. The paper concludes with an interesting summation of total costs and a comparison of economies of small and large stations.

IN SETTING UP measures of competitive value there are two steps necessary, or rather there are two typical but wholly different sorts of measure. The first is "suitability," the other is "cost." Two alternatives may not be equally suitable, and if the difference is large the more unsuitable one must be rejected regardless of cost. However, when there are two alternatives each equally suitable for a given purpose, the choice must rest on costs. That one which produces the result at least cost is the better, so that the cost of the service prevails when the suitabilities are equal. This is the case when in a steam plant one boiler-room equipment is to be compared with another, or oil engines, steam turbines, or hydraulic turbines are to drive electric generators. There is, however, a third case, and it is the most difficult of all. This is where suitability and costs are both unequal, and when the lesser suitability corresponds with lesser costs.

[1] Professor of Mechanical Engineering, Columbia University.

Presented at a meeting of the Metropolitan Section of The American Society of Mechanical Engineers, New York, April 22, 1924. Abridged.

2 Having selected any one of these sets of alternatives, there must next be established a cost basis by which to judge between two or more that are equally suitable — and which therefore are really competitive from the suitability standpoint — or where costs must be considered, with unequal suitability to reach a decision. It usually happens that suitability is easily judged and is often quite obvious.

3 There is, however, never anything obvious about relative costs of results by alternative or competitive means, especially, as

TABLE 1 CONVERSION EQUIVALENTS—CENTS PER KW-HR. TO CENTS PER MILLION B.T.U.

Cents per kw-hr.	Cents per 1,000,000 B.t.u.	Cents per kw-hr.	Cents per 1,000,000 B.t.u.	Cents per kw-hr.	Cents per 1,000,000 B.t.u.
0.1	29.282576	3.6	1054.172736	7.1	2079.062896
0.2	58.565152	3.7	1083.455312	7.2	2108.345472
0.3	87.847728	3.8	1112.737888	7.3	2137.628048
0.4	117.130304	3.9	1142.020464	7.4	2166.910624
0.5	146.412880	4.0	1171.303040	7.5	2196.193200
0.6	175.695456	4.1	1200.585616	7.6	2225.475776
0.7	204.978032	4.2	1229.868192	7.7	2254.758352
0.8	234.260608	4.3	1259.150768	7.8	2284.040928
0.9	263.543184	4.4	1288.433344	7.9	2313.323504
1.0	292.825760	4.5	1317.715920	8.0	2342.606080
1.1	322.108336	4.6	1346.998496	8.1	2371.888656
1.2	351.390912	4.7	1376.281072	8.2	2401.171232
1.3	380.673488	4.8	1405.563648	8.3	2430.453808
1.4	409.956064	4.9	1434.846224	8.4	2459.736384
1.5	439.238640	5.0	1464.128800	8.5	2489.018960
1.6	468.521216	5.1	1493.411376	8.6	2518.301536
1.7	497.803792	5.2	1522.693952	8.7	2547.584112
1.8	527.086368	5.3	1551.976528	8.8	2576.866688
1.9	556.368944	5.4	1581.259104	8.9	2606.149264
2.0	585.651520	5.5	1610.541680	9.0	2635.431840
2.1	614.934896	5.6	1639.824256	9.1	2664.714416
2.2	644.216672	5.7	1669.106832	9.2	2693.996992
2.3	673.499248	5.8	1698.389408	9.3	2723.279568
2.4	702.781824	5.9	1727.671984	9.4	2752.562144
2.5	732.064400	6.0	1756.954560	9.5	2781.844720
2.6	761.346976	6.1	1786.237136	9.6	2811.127296
2.7	790.629552	6.2	1815.519712	9.7	2840.409872
2.8	819.912128	6.3	1844.802288	9.8	2869.692448
2.9	849.194704	6.4	1874.084864	9.9	2898.975024
3.0	878.477280	6.5	1903.367440	10.0	2928.2576
3.1	907.759856	6.6	1932.650016		
3.2	937.042432	6.7	1961.932592		
3.3	966.325008	6.8	1991.215168		
3.4	995.607584	6.9	2020.497744		
3.5	1024.890160	7.0	2049.780320		

is usually the case, when one cost factor is least for one alternative and another least for some different alternative. One of the commonest cases of this sort is that in which the process involves an energy loss and for which, therefore, the efficiency is less than 100 per cent for all alternatives, but where the investment costs are inversely related to efficiencies, with operating costs miscellaneously distributed. This complex case is especially aggravated, assuming power generation to be the problem, when the cost of the primary or source energy available for each system is not the same at the point of generation, and still worse when primary energy costs available for each alternative also vary relatively at

different points on an electrical distribution system, any point of which might be a local generating point instead of a substation or consumer point.

4 Any cost system that can give a correct picture of competitive values for both heat and power, and when applied either to a power or heat problem will exhibit clearly the relative influence on increasing costs at successive points of efficiency between points and of the investment for equipment with disbursement incident to its use, must be based on one common unit for all. The best unit is the British thermal unit, and for it the common identical conversion factors are: 2545 B.t.u. = 1hp-hr., and 3415 B.t.u. = 1 kw-hr.

5 The first step in the cost system necessary for judging all sorts of competition in heat and power problems is the setting up of all costs in terms of cost of energy at any point and in any form, and it is most convenient to use costs in terms of cents per million B.t.u. For the conversion of electric power costs, commonly expressed in cents per kilowatt-hour, Table 1 shows cents per million B.t.u. equivalent to cents per kw-hr.

6 Considered in its broadest possible aspect, the national problem of heat and power supplies is that of discovering the cheapest way of bringing primary energy from its source to the point of use in acceptably suitable form. There are three such forms:

a Heat, to be utilized as heat

b Electricity, to be utilized as electricity

c Power, in the form of motion against resistance.

7 The common sources of primary energy are fuels: Coal, oil, and gas in the earth; and water at an elevation. To get a correct picture of the whole process, the energy costs in cents per million B.t.u. must be evaluated and periodically checked from source to point of consumption, and at every point between where there is transportation, transmission, or transformation. The more completely this is done, the more clearly it will appear just where costs have accumulated most and where, as a consequence, it is most worth while to exert effort to reduce costs. It will also appear that in this train of successive cost accumulations, other things being equal, the earlier in the train a saving is effected, the greater the ultimate effect at the end of the train, especially when the cost accumulation is the result of inefficiency or actual energy losses rather than high investments or operating disbursements. Exactly the same system and with the same units, when brought down to individual stages of a process, will clearly show the value of efficiency or energy losses, investments, and operating disbursements to control the losses in effecting either transformation of energy from one form to another or its transmission in any one form.

8 It is a principle generally applicable to every such process step that a cost at any one point in the train of transmission and

transformation from mine, well, or waterfall to the point of consumption is the cost of all the operations plus the original cost of energy at the starting point.

9 Each operation or process step adds three cost increments:

a An increment due to loss of energy

b An increment due to equipment investment, and

c An increment due to operating disbursements.

10 For a single operation starting at point A and ending at point B, the costs are formulated as follows:

Costs in cents per million B.t.u. at B (B) = Cost at A (A), plus inefficiency increment (xA), plus investment increment (I), plus operating-disbursement increment (O), or

$$B = A + xA + I + O$$

If E = efficiency from A to B = ratio of output to input energy, then

$$A + xA = \frac{A}{E} \text{ and } x = \frac{1-E}{E}$$

Therefore the inefficiency increment of cost is $(1-E)A/E$ and the total cost increments for the process are

$$B - A = \left(\frac{1-E}{E}\right)A + I + O$$

This leads to two expressions for the cost per million B.t.u. at the end in terms of cost at the beginning, the efficiency of the process, and the expenses for investment and operation. These are:

$$B = \frac{A}{E} + I + O$$

$$= A + \left(\frac{I-E}{E}\right)A + I + O$$

With this formulation it now becomes possible to establish a measure of *commercial efficiency*, E_c, for the process with an algebraic relation to its energy efficiency E. Defining commercial efficiency as the ratio of cost of input energy to cost of output energy, each in cents per million B.t.u., then,

$$E_c = \frac{A}{\frac{A}{E} + I + O}$$

$$= \frac{AE}{A + E(I+O)}$$

$$= \frac{E}{1 + E\left(\frac{I+O}{A}\right)}$$

11 A numerical example will make these relations more clear. Assume energy of fuel costing 32 cents per million B.t.u. to be transferred to energy of steam available for a turbine with an overall process efficiency of 60 per cent, and requiring an investment that adds 30 cents per million B.t.u. output, with operating disbursement adding 25 cents more.

12 Then the inefficiency increment of cost will be

$$\left(\frac{1-E}{E}\right)A = \left(\frac{1.0-0.6}{0.6}\right)\times 32 = 21.33 \text{ cents}$$

$$B = A+\left(\frac{1-E}{E}\right)A+I+O = 32+21.33+30+25 = 108.33$$

$$\text{or } B = \frac{A}{E}+I+O = \frac{32}{0.6}+30+25 = 53.33+30+25 = 108.33$$

and

$$E_c = \frac{A}{B} = \frac{32}{108.33} = 29.5 \text{ per cent}$$

$$= \frac{E}{1+E\left(\frac{I+O}{A}\right)} = \frac{0.6}{1+0.6\left(\frac{55}{32}\right)} = 29.5 \text{ per cent}$$

For this case an energy efficiency of 60 per cent yields a commercial efficiency of only 29½ per cent.

13 This method of judging processes is applicable to a series of processes constituting a system of transforming or of transmitting energy in any form. It is of special value in appraising alternatives for single processes or systems where efficiencies of processes or systems may vary considerably with equipment investment and operating-disbursement expenses. It shows directly the value of efficiency in terms of all prime variables through the approach to equality of commercial efficiency and process efficiency. That process efficiency or system efficiency is the most valuable whose commercial efficiency is most nearly equal to its own value, and of course when both are as high as possible.

14 In what follows are collected some estimates of costs for primary energy and for its power-plant transformation into electrical energy through appropriate steps, to illustrate the use of the method.

Primary Energy at the Source

15 Starting with primary energy, it may be said that water in a stream or coal, oil, and gas in the earth are worth nothing as they are. Whatever value may be fixed by appraisal must be based on costs of subsequent operations required to bring them in one form or another to the consumer — with due regard to competitive demand. For present purposes costs of energy in the

form of coal f.o.b. mines, taken at random from *The American Metal Market and Daily Iron and Steel Report* are given in Table 2.

16 The figures given in Table 3 are from quotations appearing in the *National Petroleum News* for fuel oil, kerosene, and gasoline, the normal fuel products of crude. Of these figures those most directly comparable as to use with coal are the values for fuel oil which range from 18 (minimum) to 45 (maximum) cents per million B.t.u. approximately, and which are respectively $3\frac{1}{2}$ and $3\frac{3}{4}$ times the corresponding minimum and maximum coal values.

17 Considering the fact that electrical energy generated from fuel is sold at retail to small consumers at prices ranging around

TABLE 2 COAL COSTS AT MINES IN DOLLARS PER TON AND CENTS PER MILLION B.T.U. (RUN-OF-MINE COAL)

Field	B.t.u. per lb.	High: Dollars per ton	High: Cents per million B.t.u.	Low: Dollars per ton	Low: Cents per million B.t.u.
Northern Illinois	10,981	2.75	12.52	2.25	10.24
	12,488	2.75	11.01	2.25	9.01
Central Illinois	10,514	2.25	10.70	2.00	9.51
Southern Illinois	10,981	2.75	12.52	2.50	11.38
	12,276	2.75	11.20	2.50	10.18
Indiana Fifth Vein	12,409	2.25	9.07	2.00	8.06
	11,192	2.25	10.05	2.00	8.93
Western Kentucky	11,900	2.25	9.45	1.50	6.30
	12,500	2.25	9.00	1.50	6.00
Ohio No. 6	12,560	2.05	8.16	1.95	7.76
Eastern Ohio	12,560	2.00	7.96	1.90	7.56
	12,100	2.00	8.26	1.90	7.85
Pittsburgh	13,900	2.10	7.55	1.90	6.83
Central Pennsylvania	13,600	1.75	6.43	1.60	5.88
	14,500	1.75	6.03	1.60	5.52
Connellsville (steam)	13,990	1.50	5.36	1.35	4.82
West Virginia	12,265	1.60	6.52	1.35	5.50
	15,200	1.60	5.26	1.35	4.44
West Virginia, Pocahontas	13,995	2.25	8.04	1.50	5.36
Do., New River	15,208	2.25	7.40	1.50	4.93
Do., Panhandle	14,250	1.95	6.84	1.90	6.67

10 cents per kw-hr., and at wholesale to large ones at 2 cents, there is evidently a very large cost accumulation in the whole series of processes of transportation and transformation from point of production to consumer. As 10 cents per kw-hr. is equivalent to 2898 cents per million B.t.u., the small consumer pays from 200 to 500 times as much for energy in the form of electricity at the point of consumption as it cost in the form of coal at coal-production points, and the larger consumer about one-fifth as much, or 40 to 100 times the original value. The difference between the wholesale and the retail price is a measure of the great cost of retail business, the largest single cost accumulation of the system.

18 Even these wholesale cost accumulations are large, corresponding as they do to about 1 per cent and $2\frac{1}{2}$ per cent commercial efficiency, and they focus attention on the analysis of all

the separate items of accumulation as a necessary first step in any orderly survey of means of improvement.

The Transportation or Transmission of Primary Energy

19 The first cost accumulation is that resulting from the transportation of the fuel in its original form by truck, railroad car, and boat for both coal and oil, with the special case of the pipe line for oil as an alternative. For practically all fuels the cost accumulation by rail transportation exceeds the cost at the mines, and in many cases the original cost is multiplied several times. This is especially serious when it is realized that this is the first operation in the series of processes from source of energy to consumer and that early multiplications are themselves later multiplied in succession in a sort of geometrical progression. Furthermore, freight costs will show no relation between actual cost of transportation or distance. A few figures from recent coal quotations will show the magnitude of the charges.

TABLE 3 PETROLEUM FUELS—COSTS AT REFINERY, F.O.B. TANK CAR

Fuel	Gravity, deg. Baumé	Lb. per gal.	B.t.u. per lb.	Destination	High: Cents per gal.	High: Cents per million B.t.u.	Low: Cents per gal.	Low: Cents per million B.t.u.
Fuel Oil	36–40	6.939	19,770	Pa.	6¼	45.5	6	43.6
	30–34	7.196	19,530	Pa.	5½	39.2	5¼	37.4
	38–40	6.898	19,810	Okla.	4⅛	30.0	3⅞	29.0
	32–36	7.108	19,610	Okla.	3⅝	26.0	3⅜	24.2
	24–26	7.522	19,250	Okla.	2¾	19.0	2⅝	18.2
	38–40	6.898	19,810	N. Tex.	4½	33.0	4¼	31.2
	32–36	7.108	19,610	N. Tex.	3¾	27.0	3½	25.2
	24–26	7.522	19,250	N. Tex.	3¼	22.4	3	20.7
Kerosene	45	6.661	19,840	Pa.	8	60.5	7¾	58.5
	47	6.586	19,520	Pa.	9¼	73.3	...	.....
	40–42	6.817	19,680	Okla.	5¼	39.1	5	37.2
	42–44	6.738	19,760	Okla.	6	45.1	5¾	43.2
	40–42	6.817	19,680	N. Tex.	5½	40.8	5	37.2
	41–43	6.777	19,720	N. Tex.	6	44.8	5¾	42.9
Gasoline	58 straight	6.199	20,240	Pa.	15½	124.0	15	120.0
	68 straight	5.886	20,640	Pa.	20	164.0	19½	160.0
	60–70 blend	5.976	20,520	Pa.	16	130.0	14¼	116.0
	56–58 blend	6.233	20,200	Okla.	11	87.5	10¾	85.5
	68–70 blend	5.856	20,680	Okla.	14	116.0	13¾	114.0
	52–58 blend	6.334	20,160	N. Tex.	11	86.0	10½	82.0
	68–70 blend	5.856	20,680	N. Tex.	14¼	117.0	14	115.0

20 The freight charge on coal from the New River Pocahontas field to New York is \$3 per ton for 614 miles or 0.49 cent per ton-mile, and to Boston, 846 miles, \$3.25 or 0.385 cent per ton-mile. Pennsylvania coal to New England costs \$4, and taking 600 miles as the distance the rate is 0.67 cent per ton-mile as a mean for a considerable range of distance. From Central Illinois to Chicago the rate is \$1.85, which is just about 1 cent per ton-mile. These figures show a range of from 0.3 to 1.0 cent per ton-mile, quite independent of such special cases of rates where short hauls cost more than long ones.

21 The transportation of oil in pipe lines costs less than it does in tank cars and it is probably the cheapest system of energy transmission known, but the economies are only partly reflected in the rates allowed to pipe lines as common carriers. The following figures for crude from Pogue show in all cases a differential in favor of the pipe line and are of interest relatively. From the Cushing field to Kansas, 117 miles, the rates (in 1916) were 0.311 and 0.200 cent per bbl. by rail and pipe line, respectively; to Illinois, 565 miles, 0.544 and 0.340 cent; to Port Arthur, 583 miles, 0.466 and 0.400 cent; to Indiana, 686 miles, 0.622 and 0.420 cent. These figures are equivalent to 1.87, 0.67, 0.56 and 0.63 cents per ton-mile by rail for 117, 565, 583 and 686 miles, respectively, which are higher than the coal rates. By pipe line the ton-mile rates are 1.2, 0.42, 0.48 and 0.43 cents, which are respectively 64, 63, 85 and 68 per cent of the rail rates.

TABLE 4 COAL TRANSPORTATION COSTS

(Superpower Report Estimates, 1925–1930)

Consumption Point	Transportation Cost in Dollars per Ton of 2000 lb. from		
	Clearfield	Fairmont	Pocahontas
Boston	4.45	4.68	5.40
Providence	4.59	4.81	5.15
New London	3.93	4.16	5.40
New Haven	3.81	4.18	....
Hartford	4.33	4.55	....
Northampton	4.83	5.07	....
Albany and Poughkeepsie	3.43	3.80	....
New York, Newark, and New Brunswick	3.35	3.66	....
Trenton	3.17	3.40	....
Harrisburg	2.53	2.77	....
Philadelphia, Washington, Baltimore and Wilmington	3.05	3.27	....

22 On the B.t.u. basis, using the above figures for coal (0.3 to 1.0 cent per long ton per mile) and taking a ton as 30 million B.t.u., in round numbers, the costs are 1.0 cent to 3.3 cents per million B.t.u. per 100 miles or per hundred million B.t.u.-miles.

23 In the Superpower Survey report of U. S. Geological Survey, Professional Paper No. 123, 1921, still higher figures are given for the transportation of coal in the superpower zone as shown in Table 4; these, however, include taxes, insurance, and seasonal storage cost.

Primary-Fuel Energy Cost at the Point of Use

24 As has been pointed out, coal from different mines delivered to different points suffers cost increments over the cost at mine that are always considerable — normally several times the mine cost — and that are, moreover, not proportional to distance. The effect is to more or less equalize the cost of coal energy at points of use at different distances from the mine by adding a considerable amount to the mine costs, but in general adding proportionately

more to short than to long distances. When two or more such fuels from different sources are available in the same market and are of different heating value, the tendency is to make them competitive by adjustment of price, so that the cost per million B.t.u. will be highest for the fuels most easily utilizable.

25 The same tendency exists with regard to petroleum products of different use values competing one with another — fuel oil, gas

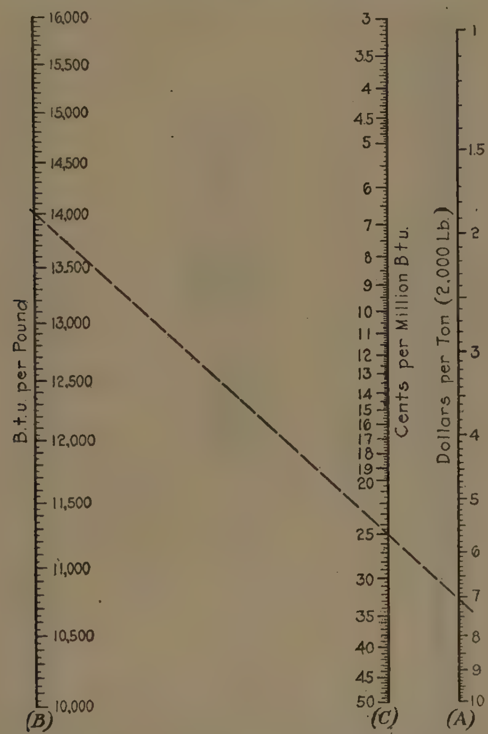

FIG. 1 COST OF ENERGY OF SOLID FUEL IN CENTS PER MILLION B.T.U.

$$\text{Cents per million B.t.u.} = \frac{\text{Dollars per ton} \times 100 \times 1{,}000{,}000}{\text{B.t.u. per lb.} \times 2{,}000} = (C) = \frac{(A)}{(B)}$$

EXAMPLE: Dollars per ton = 7; B.t.u. per lb. = 14,000

$$\therefore \text{Cents per million B.t.u.} = \frac{7 \times 100 \times 1{,}000{,}000}{14{,}000 \times 2{,}000} = 25$$

By Chart: (A) = 7; (B) = 14,000; ∴ (C) = (A)/(B) = 25 (check)

oil, kerosene, or gasoline — and also with regard to fuel oil, the less useful of this series, competing with coal. On this basis, fuel oil selling at the same cost per million B.t.u. as coal would drive coal out of use. Wherever both are used the price ratio of oil to coal will be greater than 1.0 and may rise to 2.00 or more, the higher

values permitting oil to be used in proportion as utilization costs favor the oil.

26 Some comparatively recent quotations on coal in various cities are given in Table 5, together with fuel-oil values, from which the price ratios are seen to vary from 1.25 to 3.78.

27 · These coal prices may be compared with those given in the Superpower Report as estimated for 1925-1930, taking Clearfield

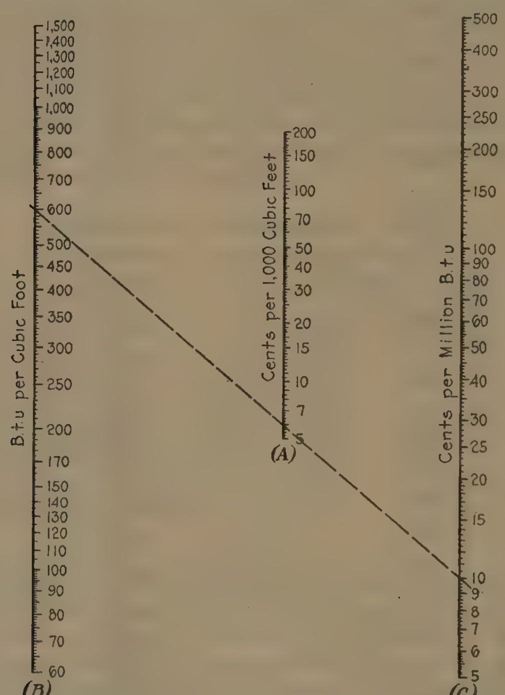

FIG. 2 COST OF ENERGY OF GASEOUS FUEL IN CENTS PER MILLION B.T.U

$$\text{Cents per million B.t.u.} = \frac{\text{Cents per 1,000 cu. ft.} \times 1{,}000{,}000}{\text{B.t.u. per cu. ft.} \times 1{,}000} = (C) = \frac{(A)}{(B)}$$

EXAMPLE: Cents per 1000 cu. ft. = 6; B.t.u. per cu. ft. = 600

$$\text{Cents per million B.t.u.} = \frac{6 \times 1{,}000{,}000}{600 \times 1000} = 10$$

By Chart: (A) = 6; (B) = 600; (C) = (A)/(B) = 10 (check)

as indicative for bituminous coals for this district, which ranges from a minimum of \$5.43 per ton at Harrisburg to \$7.73 per ton at Northampton, with intermediate rates of \$6.25 at New York and \$7.35 at Boston.

28 For retail consumption, prices are very much higher — two to three times. For a good part of the eastern district large sizes

of anthracite cost \$15 per ton, more or less, and buckwheat No. 1 \$10 and No. 3 \$7.50, which latter is to be compared with mine costs for No. 3 of about \$1.75 per ton, and with wholesale prices estimated at 63 per cent of low-volatile bituminous. Retail users

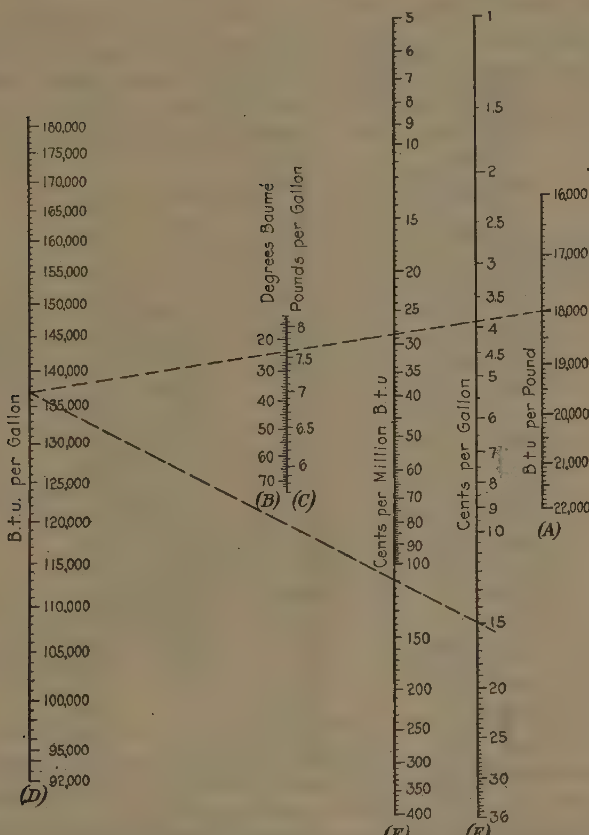

FIG. 3 COST OF ENERGY OF LIQUID FUEL IN CENTS PER MILLION B.T.U.

$$\text{Lb. per gal.} = \frac{140 \times 8.33}{130 + \text{Deg. Bé.}} = (C) = (B)$$

B.t.u. per gal. = B.t.u. per lb. × lb. per gal. = (D) = (A) × (C)

$$\text{Cents per million B.t.u.} = \frac{\text{Cents per gal.} \times 1{,}000{,}000}{\text{B.t.u. per gal.}} = (F) = \frac{(E)}{(D)}$$

EXAMPLE: Deg. Bé. = 23.5; B.t.u. per lb. = 18,000; cents per gal. = 15
Lb. per gal. = (140 × 8.33)/(130 + 23.5) = 7.6
B.t.u. per gal. = 18,000 × 7.6 = 137,000

$$\text{Cents per million B.t.u.} = \frac{15 \times 1{,}000{,}000}{137{,}000} = 110$$

By Chart: (B) = 23.5 or (C) = 7.6; (A) = 18,000; (E) = 15
First step: (D) = (A) × (C) = 137,000
Second step: (F) = (E)/(D) = 110 (check)

of heat are normally supplied with manufactured gas, kerosene, or gasoline at prices very much higher, as can be quickly checked by the conversion charts Figs. 1, 2, and 3, in terms of cents per million B.t.u. for any price per ton of coal, per cubic foot of gas,

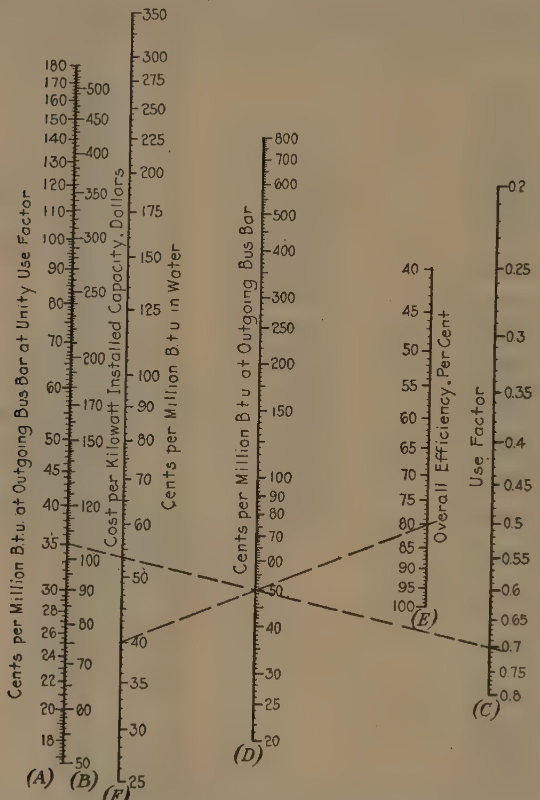

FIG. 4 COST OF ENERGY IN THE FORM OF WATER UNDER HEAD IN CENTS PER MILLION B.T.U. IDENTICAL EQUIVALENT

Cents per million B.t.u. at outgoing bus bar at unity power factor
= cost per kilowatt installed in dollars × 1,000,000 × 100 ÷ (3415 × 8760 × 10)
= (A) = (B)

Cents per million B.t.u. at outgoing bus bar
= cents per million B.t.u. at outgoing bus bar at unity power factor ÷ use factor
= (D) = (B)/(C)

Cents per million B.t.u. in water
= cents per million B.t.u. at outgoing bus bar × overall efficiency
= (F) = (D) × (E)

EXAMPLE: Cost per kw. installed = $105; use factor = 0.7; overall efficiency = 80 per cent

Cents per million B.t.u. at outgoing bus bar at unity use factor
= 105 × 1,000,000 ÷ (3415 × 8760 × 10) = 35

Cents per million B.t.u. at outgoing bus bar = 35/0.7 = 50

Cents per million B.t.u. in water = 50 × 80/100 = 40

By Chart: (B) = 105; (C) = 0.7; (E) = 80; (A) = 35; (D) = 50; (F) = 40; (check)

or per gallon of oil. To complete the comparison of costs of primary energy a chart (Fig. 4) is added for water power, giving cents per million B.t.u. for any cost per kw., site, investment, and use factor.

Preparation of Working Fluid for Power Generation

29 Before any prime mover may begin the process of transforming energy in one of its primary forms into work, there must first be a suitable working fluid carrying the energy in a form best adapted for the prime mover to work as a transformer.

TABLE 5 ENERGY COST IN FUEL DELIVERED

	Coal			Oil				
Place	Dollars per ton	B.t.u. per lb.	Cents per million B.t.u.	Cents per gal.	Lb. per gal.	B.t.u. per lb.	Cents per million B.t.u.	Ratio of costs per million B.t.u., oil to coal
Pittsburgh	3.10	13,000	12.0	6.0	7.1	19,750	43.0	3.68
Oil City	4.13	13,000	15.9	6.0	7.1	19,750	43.0	2.70
Cumberland ..	3.84	14,600	13.2	....	....	.....	....	....
Philadelphia ..	7.00	14,600	24.0	3.85	7.67	19,000	26.5	
Philadelphia ..	6.25	14,600	21.4	4.35	7.29	19,450	31.0	1.17 to 2.06
Pa. Anthracite	3.84	12,000	16.0					
	4.35	12,000	18.1					
Duluth	6.50	13,000	25.0					
	8.00	14,000	28.7	....	....	.....	39.0	1.36 to 1.56
	6.25	11,500	27.2					
Minneapolis ...	4.55	12,000	19.0	4.89	7.52	19,210	34.0	1.8 to 1.9
	4.85	13,000	18.6					
New York	7.50	14,400	26.0	4.50	7.29	19,000	32.0	1.23
	6.50	14,400	22.6	5.50	7.29	19,000	39.0	1.73
Chicago	4.20	12,750	16.5	7.25	6.98	19,500	53.0	
	4.65	12,500	18.6	6.75	6.98	19,500	50.0	
	4.85	12,500	19.4	5.06	7.52	.....	....	2.94 to 3.22
	4.45	12,200	18.2	5.06	7.38	.....	....	
	4.60	13,000	17.7	7.00	6.98	19,500	51.0	
	4.20	12,000	17.4	....	....	.....	....	
Denver	2.00	10,000	10.0	4.76	8.33	18,380	31	3.1
El Paso	5.70	12,500	22.8	5.7	7.67	19,000	39	1.71

30 When the case is that of water power, preparation of the working fluid involves the building of all of the water-collection, storage-control, and conduit system, including draft tube and tail-water disposal, for bringing water to the turbine under the maximum available head and getting it away. The cost of preparation of the working fluid in this case is the fixed charges on this site development, applied to such part of the capacity as represents the average for a year, and expressed in cents per million B.t.u.

31 When the case is that of steam power, preparation of the working fluid involves the making of superheated steam and its delivery to the turbine. In this case the cost of preparation of working fluid is the cost of making and delivering superheated steam, all charges being expressed in cents per million B.t.u. There are three items of cost increment making up this total: First, that

due to energy losses; second, that of fixed charges on the investment for necessary equipment; and third, the disbursements incident to operation and maintenance of equipment.

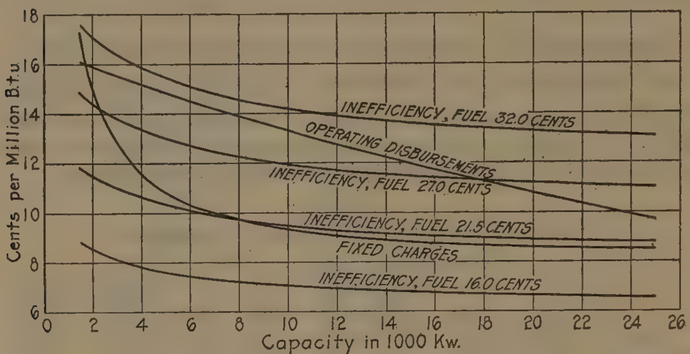

Fig. 5 Preparation of Working Fluid—Cost Increments for Steam-Turbine Plants in Cents per Million B.t.u.

(Fixed charges, 12 per cent; use factor, 40 per cent.)

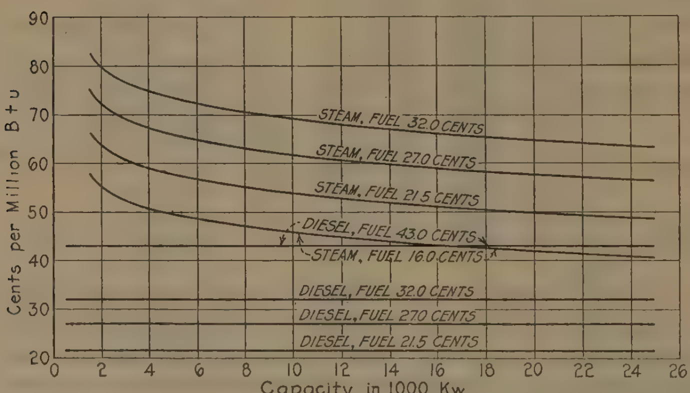

Fig. 6 Steam Turbines vs. Diesel Engines—Cost of Energy of Working Fluid in Cents per Million B.t.u.

(Fixed charges: Steam plant, 12 per cent; Diesel plant, 18 per cent; use factor, 40 per cent.)

32 Finally, for oil engines there is no cost for preparation of working fluid, because such engines take oil as delivered and atmospheric air without any preparation, and while the working fluid is actually hot gas, it is not possible to separate the costs of the two functions of converting oil and air into hot gases from that of transforming the heat of hot gases into work.

33 It is possible to estimate the costs of these two typical modes of preparing the working fluid for steam and for hydraulic turbines, and such estimates are very useful, especially in establishing the relative importance of each variable and comparing the total with that for Diesel fuel oil ready for use.

34 Water-collection costs for hydraulic turbines are all fixed charges, and these will be evaluated arbitrarily at a yearly rate of 10 per cent without any attempt being made to analyze the justification for this rate, except to point out that it is lower than the rate of 12 per cent for steam-plant equipment and 13 per cent for oil engines, which relations have been adopted as fairly representative of somewhat divergent general opinions.

TABLE 6 PREPARATION OF WORKING FLUID FOR PRIME MOVERS—POWER COSTS BY PROCESS STAGES IN CENTS PER MILLION B.T.U.

(Continuous 24-hour service. Average yearly load, 40 per cent)

A—Steam Turbines (Fixed-Charge Rate, 12 per cent)

Plant capacity, kw		1,500	2,500	5,000	10,000	15,000	20,000	25,000
Efficiency, per cent		64.50	65.65	67.50	69.33	70.20	70.68	71.00
Energy-Loss Item:								
Fuel price per million B.t.u., cents	16.0	24.81	24.37	23.70	23.07	22.74	22.64	22.53
	21.5	33.33	32.75	31.85	31.01	30.62	30.42	30.28
	27.0	41.86	41.12	40 00	38.95	38.46	38.20	38.03
	32.0	49.61	48.74	47.41	46.16	45.58	45.28	45.07
Operating disbursements		15.60	15.30	15.10	13.70	11.80	10.75	9.70
Fixed charges		17.30	13.75	10.85	9.15	8.95	8.60	8.50
Total:								
Fuel price per million B.t.u., cents	16.0	57.71	53.42	49.65	45.92	43.54	41.99	40.73
	21.5	66.23	61.80	57.80	53.86	51.37	49.77	48.48
	27.0	74.76	70.17	65.95	61.80	59.21	57.55	56.23
	32.0	82.51	77.79	73.36	69.01	66.33	64.63	63.27

B—Diesel Oil Engines (Fixed-Charge Rate, 13 per cent)

Plant capacity, kw		500	1,000	1,500	2,500	3,500
Fuel Directly Available:						
Fuel price per million B.t.u.	21.5 cents	21.5	21.5	21.5	21.5	21.5
	27.0 cents	27.0	27.0	27.0	27.0	27.0
	32.0 cents	32.0	32.0	32.0	32.0	32.0
	43.0 cents	43.0	43.0	43.0	43.0	43.0

C—Water Power (Fixed-Charge Rate, 10 per cent)

Plant capacity, kw		1,500	2,500	5,000	10,000	15,000	20,000	25,000
Total = Fixed Charges on Site:								
20 ft. Head, Site =	1.0 × Equip.	41.774	38.993	37.234	36 253	36.029	36.642	37.023
	2.0 × Equip.	83.548	77.986	74.468	72.506	72.058	73.284	74 046
	3.0 × Equip.	125.322	116.973	111.702	108.759	108.087	109.926	111.069
120 ft. Head, Site =	2.0 × Equip.	39.604	39.676	36.601	33.677	31.937	31.449	31.069
	3.0 × Equip.	59.452	59.514	54 901	50.516	47.905	47.174	46.603
	4.0 × Equip.	79.269	79.352	73.202	67.358	63.874	62.899	62.137
400 ft. Head, Site =	3.0 × Equip.	51.420	48.812	44.240	38.593	36.338	35.480	35.308
	4.0 × Equip.	68.560	65.083	58.986	51 457	48.451	47.307	47.078

35 Investment for water collection includes disbursements for land and fixed structures having a certain capacity in second-feet of water at a given head equivalent to a horsepower capacity, all of which is never used for the 8760 hours of a year. Whether the use is less than capacity because of shortage of water at times or because there is no demand for the power that could be developed at a given time, i.e., light load, is immaterial as to the evaluation

of the charges, which for present purposes will be estimated for 40 per cent of capacity over the year. The investment in dollars for any one site development does of course vary with topographical and other natural conditions, and this makes it difficult to arrive at indicative values that might be acceptable as typical of the hydro system. For present purposes, however, values for equipment, hydraulic and electrical, including building, are based

TABLE 7 PREPARATION OF WORKING FLUID FOR STEAM TURBINES

INEFFICIENCY COST INCREMENT IN CENTS PER MILLION B.T.U.

Plant capacity, kw		1,500	2,500	5,000	10,000	15,000	20,000	25,000
Fuel cost per million B.t.u.	16.0 cents.	8.81	8.37	7.70	7.07	6.74	6.64	6.53
	21.5 cents.	12.83	11.25	10.35	9.51	9.12	8.92	8.78
	27.0 cents.	14.86	14.12	13.00	11.95	11.46	11.20	11.03
	32.0 cents.	17.61	16.74	15.41	14.16	13.58	13.28	13.07

on Rushmore's curves, and site development estimated as one, two, or three times the equipment costs for 20 ft. head; two, three, and four times for 120 ft. head; and three and four times for 400 ft. head. The whole plant-investment cost per kilowatt of capacity is given in Table 13 in the usual form, with corresponding power cost in cents per kw-hr. These totals are segregated into the main process stages in Table 14, giving in cents per kw-hr. the cost of preparation of working fluid, transformation of energy of working fluid in b.hp. energy, and b.hp. energy into electrical. From

TABLE 8 PREPARATION OF WORKING FLUID FOR STEAM TURBINES

COMMERCIAL EFFICIENCY, PER CENT

Plant capacity, kw		1,500	2,500	5,000	10,000	15,000	20,000	25,000
Fuel cost per million B.t.u.	16.0 cents.	27.7	30.0	32.3	34.8	36.8	38.2	39.4
	21.5 cents.	32.5	34.8	37.3	40.0	42.0	43.2	44.5
	27.0 cents.	36.2	38.4	41.0	43.7	45.5	46.9	48.8
	32.0 cents.	38.7	40.6	43.6	46.3	48.2	49.5	50.6
THERMAL EFFICIENCY, PER CENT								
All fuel prices		64.5	66.65	67.5	69.33	70.20	70.68	71.0

this latter table the figures for the first process are converted into equivalent cost of preparation of working fluid, cents per million B.t.u. in Table 6, the last part (C) of which is concerned with the hydraulic turbine.

36 These values for cost of energy of working fluid for hydraulic turbines are directly comparable in kind with those for cost of energy of steam at the steam turbine which have been worked out for four fuel costs, namely, 16.0, 21.5, 27.0, and 32.0 cents per million B.t.u., corresponding nearly to coal at $4.50, $6.00, $7.50, and $9.00 per ton (2000 lb.). These steam-energy costs of Table 6 are worked out from the usual form of steam-power-plant investment and power costs given in Table 15, through the process cost segregation of Table 16.

37 Both of these sets of values in Table 6 for cost of preparation of working fluid for hydraulic and for steam turbines, are directly comparable with the cost of fuel oil for Diesel oil engines directly available and here taken as one-third higher than the purchase price of coal to conform to fair stable conditions in the northeast settled sections of the country.

38 To make the comparison most useful, plant capacity is made a prime variable, and seven capacities, 1500 kw. to 25,000 kw., are estimated for turbines, steam and hydraulic, with five for Diesel oil engines, air-injection, single-acting vertical, 500 kw.

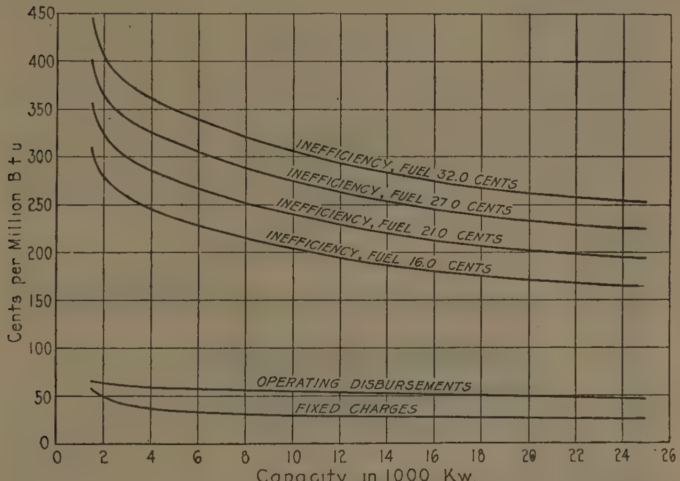

Fig. 7 Transformation of Working-Fluid Energy into B.Hp.-Energy—Cost Increments for Steam-Turbine Plants in Cents per Million B.t.u.

(Fixed charges, 12 per cent; use factor, 40 per cent.)

to 3500 kw., the maximum here being set by available catalog sizes — which are being increased. This brings out the effect of size on costs in the range where it has most effect, and also the corresponding effect of varying efficiencies which vary less in sizes beyond those used, in the case of steam, while oil-engine efficiencies remain constant and hydraulic turbines approach this more than steam. In all cases it is assumed that the load is the same, so that the average efficiencies are less than best by an amount calculated from the average unit load and number of units in service, with always one main generating unit as a spare. The other details of the plant equipment must be omitted for lack of space, but it is believed that the figures are fairly representative.

39 Referring to the figures of Table 6, it appears that the working fluid for hydraulic turbines is in general more costly by a considerable margin, with steam next, and the energy for Diesel oil engines in the form of fuel oil cheapest, even when charged at rates much higher than the one-third excess over coal that has been found to be fairly representative of competitive stability in the Northeast. The totals of cost of working fluid are shown graphically in Fig. 6 against a base of plant capacity to bring out clearly the relation of one system to the others.

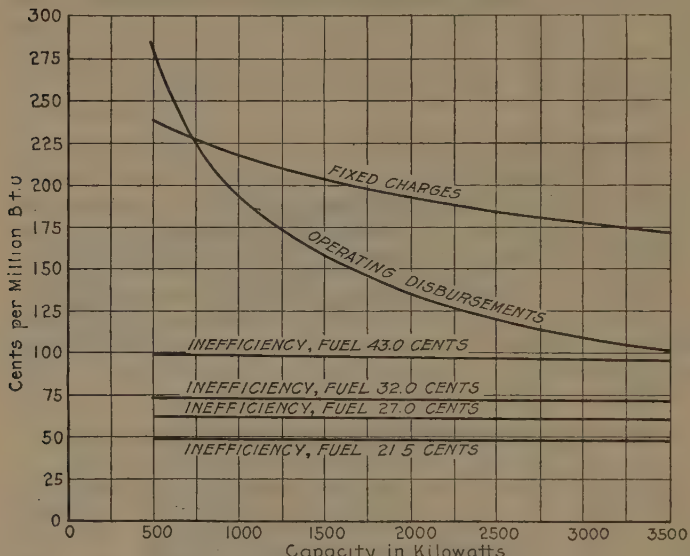

Fig. 8 Transformation of Working-Fluid Energy into B.Hp.-Energy—Cost Increments for Diesel-Engine Plants in Cents per Million B.t.u.

(Fixed charges, 13 per cent; use factor, 40 per cent.)

40 It is only in the case of steam that the preparation of working fluid involves a loss of energy with a corresponding inefficiency increment of cost in addition to the increments of investment and operating disbursement. This inefficiency increment is not shown in Table 6 directly, as the " Energy-Loss Item " is the previous cost divided by the efficiency, which is the cost added to the inefficiency increment, this latter being found by subtracting the fuel cost from the tabular value. The inefficiency cost increment thus found is as in Table 7.

41 Comparing the three cost increments plotted in Fig. 6, it appears that they are approximately equal for the most expensive

coal — $9 per ton — and for the smallest plant — 1500 kw. — but for the same fuel in larger plants the fuel increment falls less than the other two. Both the investment and operating-disbursement increments fall to almost half the value at 25,000 kw., the former exactly, the latter not quite, while the inefficiency increment falls only 20 per cent. This indicates that while higher efficiency in steam making in a 1500-kw. plant would considerably reduce the fuel increment of cost with the expensive fuel, it cannot be justified at much if any increase in investment or operating disbursements, while in a 25,000-kw. plant, material increases in either or both of these items would be justified. On the other hand, with the cheapest fuel — $4.50 coal — there would be an exception. That is, any efficiency gains must be without other expense if the net result is to be a saving in total energy cost.

42 For example: Assuming a change in boiler plant that adds 2 per cent to the efficiency, 10 per cent to investment costs, and a 5 per cent increase in operating disbursements total for steam energy, it is possible to determine readily if the change is justified.

43 Take first the smallest plant, 1500 kw., and the cheapest coal. For this condition the costs would be

	Cents per million B.t.u.
Energy-loss item	24.10
Operating disbursements	16.38
Investment fixed charges	19.03
Total	59.51

44 As the total is 59.51 cents against 57.71, it is clear that the change is not justified. Applying the same conditions to the 25,000-kw. plant, the new cost would be

	Cents per million B.t.u.
Energy-loss item	22.00
Operating disbursements	10.18
Investment fixed charges	9.35
Total	41.53

45 Here again the total, 41.53 cents, is greater than it was before, 40.73, so for this low-priced fuel (16 cents per million B.t.u.) such a change would not be justified in a plant of any size within these ranges.

46 For the high-priced fuel (32 cents per million B.t.u.), the figures for 1500 kw. would be

	Cents per million B.t.u.
Energy-loss item	48.20
Operating disbursements	16.38
Investment fixed charges	19.03
Total	83.61

47 This total, 83.61 cents, being greater than it was, 82.51, indicates again that the change is not economical. Finally, for 25,000 kw. the costs are

	Cents per million B.t.u.
Energy-loss item	44.00
Operating disbursements	10.18
Investment fixed charges	9.35
Total	63.53

48 Here the total, 63.53 cents, while larger than it was, 63.27, is much closer, so that while the change is not justified it permits the conclusion that for a somewhat larger plant with this 32-cent fuel it would be justified.

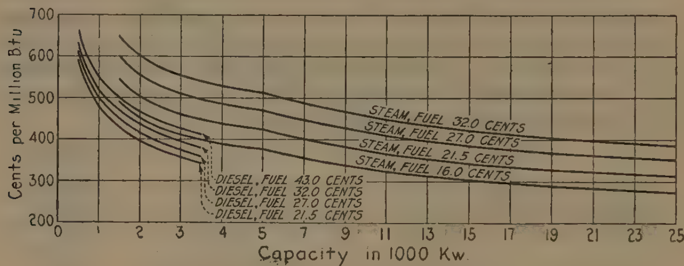

FIG. 9 STEAM TURBINES VS. DIESEL ENGINES—COST OF B.HP. ENERGY IN CENTS PER MILLION B.T.U.

(Fixed charges. Steam plant, 12 per cent; Diesel plant, 13 per cent. Use factor, 40 per cent.)

49 For the data here used the commercial efficiency may be determined by dividing the cost of input energy by the cost of output energy as in Table 8. For any change in thermal efficiency to be justified there must be an increase in commercial efficiency.

FURTHER POSSIBILITIES OF THE METHOD

50 By an extension of the same method it is possible to investigate to greater advantage two additional sets of questions of importance in connection with the generation and delivery of steam to turbines. The first of these is the distribution of the expense totals here given among the several individual successive processes involved, and for each the relative magnitude of the three increments of inefficiency, investment, and operating disbursements. This will show more precisely where economies are most likely, and by what treatment: reduction of operating disbursements or of investment, or increase of efficiency. The second of these is the probability of value without actual figures, or the exact pre-

diction of value with figures, for any one or more of the now discussed efficiency gains in steam generation and transmission to turbine, or the corresponding value of increased capacity even with decreased efficiency.

51 Taking up the first of these questions, it is clear that the overall process of steam generation, including delivery, here considered is divisible into a series of which the following will serve as an illustration:

PROCESS STAGES OF STEAM GENERATION, INCLUDING DELIVERY

FIRST	STAGE—*Process:*	Transfer of fuel from point of receipt to stoker hopper, including storage.
	Cost:	Cents per million B.t.u. in fuel at the stoker hopper.
SECOND	STAGE—*Process:*	Feeding fuel to furnace, burning fuel, supplying air, removal of flue gases, and removal of ash, including disposal.
	Cost:	Cents per million B.t.u. in hot gases in furnace available for absorption by boiler.
THIRD	STAGE—*Process:*	Feeding water to boiler, heating water, making and superheating steam by hot gases from furnace.
	Cost:	Cents per million B.t.u. in steam at the boiler nozzle.
FOURTH	STAGE—*Process:*	Steam distribution by pipe system to the turbines.
	Cost:	Cents per million B.t.u. in steam at the turbine.

52 In proportion as these costs are known, and for each of them the energy losses or efficiencies with the three cost increments (*a*) inefficiency increment, (*b*) investment increment, and (*c*) operating-disbursement increment, so can the value of every change be accurately estimated and especially the value of an efficiency change. It is possible to subdivide further whenever it is desirable to do so.

53 Auxiliaries are chargeable to any one process or series of processes in two ways, whether steam or electrically driven. The first is to charge the steam or electricity used at the cost in cents per million B.t.u. at the point of the system where taken. This is equivalent to treating this auxiliary energy the same as consumable supplies like lubricating oil. The second is to introduce efficiencies corresponding to the energy abstracted for auxiliary operation. If all costs are correctly determined, the results will be the same either way. It will be found, however, that the former is simpler and clearer.

TABLE 9 TRANSFORMATION OF ENERGY OF WORKING FLUID TO B.HP. ENERGY—POWER COSTS BY PROCESS STAGES IN CENTS PER MILLION B.T.U.

(Continuous 24-hour service. Average yearly load, 40 per cent)

	A—STEAM TURBINES (Fixed-charge rate, 12 per cent)						
Plant capacity, kw.	1,500	2,500	5,000	10,000	15,000	20,000	25,000
Efficiency, per cent	16.57	16.72	17.35	18.39	19.30	19.85	20.03
Energy-Loss Item							
Fuel price per million B.t.u. 16.0 cents	368.00	319.42	286.17	249.73	225.61	211.52	203.33
Fuel price per million B.t.u. 21.5 cents	422.40	369.53	333.14	292.91	266.18	250.72	242.02
Fuel price per million B.t.u. 27.0 cents	476.78	419.58	380.12	336.09	306.80	289.91	280.71
Fuel price per million B.t.u. 32.0 cents	526.20	465.14	422.82	375.30	343.69	325.57	315.86
Operating Disbursements	65.00	63.00	57.00	53.00	51.00	48.30	45.80
Fixed Charges	57.50	45.00	34.80	28.10	26.50	25.50	24.50
Total							
Fuel price per million B.t.u. 16.0 cents	490.50	427.42	377.97	330.83	303.11	285.32	273.63
Fuel price per million B.t.u. 21.5 cents	544.98	477.53	424.94	374.01	343.68	324.52	312.32
Fuel price per million B.t.u. 27.0 cents	599.28	527.58	471.92	417.19	384.30	368.71	351.01
Fuel price per million B.t.u. 32.0 cents	648.70	573.14	514.62	456.40	421.19	399.37	386.16
	B—DIESEL OIL ENGINES (Fixed-charge rate, 13 per cent)						
Plant capacity, kw.	......	......	500	1,000	1,500	2,500	3,500
Efficiency, per cent	......	......	30.4	30.5	30.6	30.8	30.9
Energy-Loss Item							
Fuel price per million B.t.u. 21.5 cents	......	......	70.7	70.5	70.2	69.8	69.6
Fuel price per million B.t.u. 27.0 cents	......	......	88.8	88.5	88.2	87.6	87.4
Fuel price per million B.t.u. 32.0 cents	......	......	105.3	104.9	104 6	103.9	103.5
Fuel price per million B.t.u. 43.0 cents	......	......	141.4	141.0	140.5	139.6	139.1
Operating Disbursements	......	......	285.2	192.5	157.8	119.6	101.3
Fixed Charges	......	......	238.4	216.8	203.1	183.8	171.1
Total							
Fuel price per million B.t.u. 21.5 cents	......	......	594.3	479.8	431.1	373.2	342.0
Fuel price per million B.t.u. 27.0 cents	......	......	612.4	497.8	449.1	391.0	359.8
Fuel price per million B.t.u. 32.0 cents	......	......	628.9	514.2	466.5	407.3	375.9
Fuel price per million B.t.u. 43.0 cents	......	......	665.0	550.3	501.4	443.0	411.5

C—Hydraulic Turbines (Fixed-charge rate, 10 per cent)

Plant capacity, kw.	1,500	2,500	5,000	10,000	15,000	20,000	25,000
Efficiency, per cent.	72.0	74.5	77.5	81.0	83.0	85.0	86.0
Energy-Loss Item							
20-ft. head, Site = 1.0 × Equip.	58.019	52.340	48.044	44.757	43.408	43.108	43.050
20-ft. head, Site = 2.0 × Equip.	116.038	104.680	96.514	89.514	86.816	86.216	86.100
20-ft. head, Site = 3.0 × Equip.	174.054	157.020	144.132	134.271	130.224	129.324	129.150
120-ft. head, Site = 2.0 × Equip.	55.048	53.256	47.227	41.227	38.478	36.999	36.126
120-ft. head, Site = 3.0 × Equip.	82.572	79.884	70.840	62.365	57.717	55.988	54.184
120-ft. head, Site = 4.0 × Equip.	110.096	106.513	94.454	83.154	76.956	73.999	72.252
400-ft. head, Site = 3.0 × Equip.	71.146	65.519	57.084	47.645	43.781	41.741	41.055
400-ft. head, Site = 4.0 × Equip.	95.222	87.359	76.111	63.527	58.375	55.655	54.742
Operating Disbursements	15,637	15.724	15.813	15.900	15.988	16.076	13.423
Fixed Charges, 20-ft. head	22.048	19.892	18.263	17.013	16.495	16.371	16.380
Fixed Charges, 120-ft. head	10.451	10.092	8.934	7.870	7.301	7.020	6.843
Fixed Charges, 400-ft. head	9.069	8.308	7.221	6.042	5.542	5.278	5.145
Total							
20-ft. head, Site = 1.0 × Equip.	95.704	87.956	82.120	77.670	75.891	75.555	72.853
20-ft. head, Site = 2.0 × Equip.	153.723	140.296	130.164	122.427	119.299	118.663	115.903
20-ft. head, Site = 3.0 × Equip.	211.739	192.636	178.208	167.184	162.707	161.771	158.953
120-ft. head, Site = 2.0 × Equip.	81.136	79.070	71.974	65.317	65.347	60.095	56.392
120-ft. head, Site = 3.0 × Equip.	108.660	105.698	95.587	86.135	81.006	78.594	74.450
120-ft. head, Site = 4.0 × Equip.	136.184	132.327	119.201	106.924	100.245	97.095	92.518
400-ft. head, Site = 3.0 × Equip.	96 122	89.551	80.118	69.587	65.311	63.095	59.623
400-ft. head, Site = 4.0 × Equip.	119.928	111.391	99.145	85.469	79.905	77.009	73.316

Transformation of Fluid Energy into Work (B.Hp.)

54 Considering the prime mover as an energy transformer, the cost of transformation can be determined in the same way that is used for finding the cost of energy made available for the transformation, with the increments of cost due to inefficiency, to investment, and to operating disbursements.

55 As to efficiency, the hydraulic turbine has much the highest value, the Diesel oil engine next, and the steam turbine the lowest value. The investments are otherwise related. The oil engine is highest in first cost, the steam turbine next, and the hydraulic turbine lowest, size for size. Operating disbursements are nearly the same for oil engines and for steam turbines, favoring the oil engine,

TABLE 10 TRANSFORMATION OF ENERGY OF WORKING FLUID INTO B.HP. ENERGY

STEAM TURBINE

Plant capacity, kw	1,500	2,500	5,000	10,000	15,000	20,000	25,000
	Inefficiency Increment, Cents per Million B.t.u.						
Fuel price per million B.t.u. 16.0 cents	310.29	266.00	236.52	203.81	182.07	169.53	162.60
21.5 cents	356.17	307.73	275.34	239.05	214.81	200.95	193.54
27.0 cents	402.02	349.41	314.17	274.29	247.59	232.36	224.48
32.0 cents	443.69	387.35	349.46	306.29	277.36	260.94	252.59
	Investment Increment, Cents per Million B.t.u.						
	57.50	45.00	34.80	28.10	26.50	25.30	24.50
	Operating-Disbursement Increment, Cents per Million B.t.u.						
	65.00	63.00	57.00	53.00	51.00	48.30	45.80

DIESEL OIL ENGINE

Plant capacity, kw	......	......	500	1,000	1,500	2,500	3,500
	Inefficiency Increment, Cents per Million B.t.u.						
Fuel price per million B.t.u. 21.5 cents	......	......	49.2	49.0	48.7	48.3	48.1
27.0 cents	......	......	61.8	61.5	61.2	60.6	60.4
32.0 cents	......	......	73.3	72.9	72.6	71.9	71.5
43.0 cents	......	......	98.4	98.0	97.5	96.6	96.1
	Investment Increment, Cents per Million B.t.u.						
	......	......	238.4	216.8	203.1	183.8	171.1
	Operating-Disbursement Increment, Cents per Million B.t.u.						
	......	......	285.2	192.5	157.8	119.6	101.3

TABLE 11 TRANSFORMATION OF ENERGY OF WORKING FLUID INTO B.HP. ENERGY—EFFICIENCIES

STEAM TURBINE

Plant capacity, kw.	1,500	2,500	5,000	10,000	15,000	20,000	25,000
	Commercial Efficiency, Per Cent						
Fuel price per million B.t.u. 16.0 cents	11.7	12.5	13.1	13.9	14.4	14.7	15.0
21.5 cents	12.2	13.0	13.6	14.4	15.0	15.4	15.5
27.0 cents	12.5	13.4	14.0	14.7	15.3	15.7	16.0
32.0 cents	12.8	13.6	14.2	15.2	15.7	16.2	16.4
	Thermal Efficiency, Per Cent						
	16.57	16.72	17.35	18.39	19.30	19.85	20.03

DIESEL OIL ENGINE

Plant capacity, kw.	......	......	500	1,000	1,500	2,500	3,500
	Commercial Efficiency, Per Cent						
Fuel price per million B.t.u. 21.5 cents	......	......	3.6	4.4	5.0	5.8	6.3
27.0 cents	......	......	4 3	5.3	6.0	6.9	7.5
32.0 cents	......	......	5.1	6.2	6.9	7.8	8.5
43.0 cents	......	......	6.5	7.8	8.5	9.7	10.5
	Thermal Efficiency, Per Cent						
	......	......	30.4	30.5	30.6	30.8	30.9

TABLE 12 TRANSFORMATION OF B.HP. ENERGY TO ELECTRICAL ENERGY—POWER COSTS BY PROCESS STAGES, CENTS PER MILLION B.T.U.

(Continuous 24-hour service. Average yearly load, 40 per cent)

	A—Steam Turbines (Fixed-charge rate, 12 per cent)						
Plant capacity, kw.	1,500	2,500	5,000	10,000	15,000	20,000	25,000
Efficiency, per cent.	89.0	89.5	90.0	90.5	91.0	91.5	92.0
Energy-Loss Item							
Fuel price per million B.t.u. 16.0 cents	551.12	477.56	419.97	365 56	330.01	311.82	297.43
Fuel price per million B.t.u. 21.5 cents	612.20	533.55	472.16	413.27	377.67	354.67	339.48
Fuel price per million B.t.u. 27.0 cents	677.30	589.47	524.35	460.98	422.31	397.50	381.53
Fuel price per million B.t.u. 32.0 cents	728.99	640.38	571.80	504.31	462.85	436.47	419.74
Operating Disbursements	17.60	17.60	17.60	17.60	17.60	17.60	17.60
Fixed Charges	66.50	48.75	38.10	31.15	28.50	28.35	26.70
Total							
Fuel price per million B.t.u. 16.0 cents	635.22	543.91	475.67	414.31	379.11	357.77	341.73
Fuel price per million B.t.u. 21.5 cents	696.30	599.90	527.86	462.02	423.77	400.62	383.78
Fuel price per million B.t.u. 27.0 cents	761.40	655.82	580.05	509.73	468.41	443.45	422.83
Fuel price per million B.t.u. 32.0 cents	813.00	706.73	627.50	553.06	508.95	482.42	464.04
	B—Diesel Oil Engines (Fixed-charge rate, 13 per cent)						
Plant capacity, kw.	......	......	500	1,000	1,500	2,500	3,500
Efficiency, per cent.	......	......	87.5	88.5	89.0	89.5	89.9
Energy-Loss Item							
Fuel price per million B.t.u. 21.5 cents	......	......	679.2	542.1	484.4	417.0	380.4
Fuel price per million B.t.u. 27.0 cents	......	......	699.9	562.5	504.6	436.9	400.2
Fuel price per million B.t.u. 32 0 cents	......	......	718.7	581.0	524.1	455.1	418.1
Fuel price per million B.t.u. 43.0 cents	......	......	760.0	621.8	563.4	495.0	457.7
Operating Disbursements	......	......	17.5	17.7	17.8	17.9	18.1
Fixed Charges	......	......	60.9	48.8	40.8	32.6	29.5
Total							
Fuel price per million B.t.u. 21.5 cents	......	......	757.6	608.6	543.0	467.5	428.0
Fuel price per million B.t.u. 27.0 cents	......	......	778 3	629.0	563.2	487.4	447.8
Fuel price per million B.t.u. 32.0 cents	......	......	797.1	647.5	582.7	505.6	467.5
Fuel price per million B.t.u. 43.0 cents	......	......	838.4	688.3	662.0	545.5	505.3

C—Hydraulic Turbines (Fixed-charge rate, 10 per cent)

Plant capacity, kw.	1,500	2,500	5,000	10,000	15,000	20,000	25,000
Efficiency, per cent	89.0	89.5	90.0	90.5	91.0	91.5	92.0
Energy-Loss Item							
20-ft. head, Site = 1.0 × Equip.	107.533	98.275	91.233	85.823	83.397	82.574	79.188
20-ft. head, Site = 2.0 × Equip.	177.722	156.755	144.626	135.278	131.098	129.686	125.978
20-ft. head, Site = 3.0 × Equip.	237.909	215.236	198.089	184.734	178.799	176.793	172.771
120-ft. head, Site = 2.0 × Equip.	91.164	88.346	79.971	72.207	67.876	65.678	61.513
120-ft. head, Site = 3.0 × Equip.	122.090	118.098	106.208	95.177	89.017	85.895	80.924
120-ft. head, Site = 4.0 × Equip.	153.780	147.851	132.445	118.149	110.159	106.115	100.563
400-ft. head, Site = 3.0 × Equip.	108.002	100.057	89.020	76.891	71.770	68.956	63.721
400-ft. head, Site = 4.0 × Equip.	134.750	124.459	110.161	94.441	87.807	84.163	79.691
Operating Disbursements	49.780	40.995	29.282	17.570	11.713	9.663	8.785
Fixed Charges							
20-ft. head	40.380	36.310	33.089	30.571	29.575	29.253	28.990
120-ft. head	19.180	18.448	16.281	14.202	13.118	12.533	12.181
400-ft. head	16.603	15.110	13.118	10.864	9.927	9.429	9.165
Total							
20-ft. head, Site = 1.0 × Equip.	197.693	175.580	153.604	133.964	124.685	121.490	119.942
20-ft. head, Site = 2.0 × Equip.	262.882	234.060	206.997	183.419	172.386	168.602	166.736
20-ft. head, Site = 3.0 × Equip.	328.069	292.541	260.460	232.875	220.087	215.704	213.529
120-ft. head, Site = 2.0 × Equip.	160.124	147.789	125.534	103.979	92.707	87.874	85.241
120-ft. head, Site = 3.0 × Equip.	191.050	177.541	151.771	126.949	113.848	108.091	104.869
120-ft. head, Site = 4.0 × Equip.	221.975	207.294	178.008	149.921	134.990	128.311	124.508
400-ft. head, Site = 3.0 × Equip.	174.385	156.162	131.162	105.325	93.410	88.048	85.767
400-ft. head, Site = 4.0 × Equip.	201.133	180.564	152.561	122.875	109.447	103.255	100.644

and with greater differences in small sizes, but are very much lower for hydraulic turbines.

56 The relative costs of transforming energy by the three prime movers per million B.t.u. in the form of brake-horsepower (b.hp.) energy cannot be determined without adopting as a base the cost of energy received. This leaves open two possible courses of procedure, first, setting down equal costs of energy received by each, which may or may not be possible — in general it is not, or second, using the fair probable values worked out previously from prices of primary fuel energy with investments and operating disbursements to make it available as energy of working fluid at the prime mover. The first method is purely academic and must be abandoned in favor of the second.

TABLE 13 HYDROELECTRIC POWER COSTS IN CENTS PER KILOWATT-HOUR

(Continuous 24-hour service. Average yearly load, 40 per cent; fixed-charge rate, 10 per cent)

Plant capacity, kw.	1,500	2,500	5,000	10,000	15,000	20,000	25,000
	Investment, Dollars per Kw. Capacity						
Site and Development							
20-ft. head, Site = 1.0 × Equip.	78.0	70.0	63.9	59.2	57.1	56.4	56.0
20-ft. head, Site = 2.0 × Equip.	156.0	140.0	127.8	118.4	114.2	112.8	112.0
20-ft. head, Site = 3.0 × Equip.	234.0	210.0	191.7	177.6	171.3	169.2	168.0
120-ft. head, Site = 2.0 × Equip.	74.0	71.2	62.8	55.0	50.6	48.4	47.0
120-ft. head, Site = 3.0 × Equip.	111.0	106.8	94.2	82.5	75.9	72.6	70.5
120-ft. head, Site = 4.0 × Equip.	148.0	142.4	125.6	110.0	101.2	96.8	94.0
400-ft. head, Site = 3.0 × Equip.	96.0	87.6	75.9	63.0	57.6	54.6	53.4
400-ft. head, Site = 4.0 × Equip.	128.0	116.8	101.2	84.0	76.8	72.8	71.2
Power-House Equipment, Hydraulic and Electric							
20-ft. head	78.0	70.0	63.9	59.2	57.1	56.4	56.0
120-ft. head	37.0	35.6	31.4	27.5	25.3	24.2	23.5
400-ft. head	32.0	29.2	25.3	21.0	19.2	18.2	17.8
Total							
20-ft. head, Site = 1.0 × Equip.	156.0	140.0	127.8	118.4	114.2	112.8	56.0
20-ft. head, Site = 2.0 × Equip.	234.0	210.0	191.7	177.6	171.3	169.2	112.0
20-ft. head, Site = 3.0 × Equip.	312.0	280.0	255.6	236.8	228.4	225.6	168.0
120-ft. head, Site = 2.0 × Equip.	111.0	106.8	94.2	82.5	75.9	72.6	70.5
120-ft. head, Site = 3.0 × Equip.	148.0	142.4	125.6	110.0	101.2	96.8	94.0
120-ft. head, Site = 4.0 × Equip.	185.0	178.0	157.0	137.5	126.5	121.0	117.5
400-ft. head, Site = 3.0 × Equip.	128.0	116.8	101.2	84.0	76.8	72.8	71.2
400-ft. head, Site = 4.0 × Equip.	160.0	146.0	126.5	105.0	96.0	91.0	89.0

Average Load 40%, Rate 10%							
	Fixed Charges, Cents per Kw-Hr.						
20-ft. head, Site = 1.0 × Equip.	0.445	0.400	0.365	0.338	0.326	0.322	0.320
20-ft. head, Site = 2.0 × Equip.	0.668	0.599	0.547	0.507	0.489	0.483	0.479
20-ft. head, Site = 3.0 × Equip.	0.890	0.799	0.730	0.676	0.652	0.644	0.639
120-ft. head, Site = 2.0 × Equip.	0.317	0.305	0.269	0.235	0.217	0.207	0.201
120-ft. head, Site = 3.0 × Equip.	0.422	0.406	0.358	0.314	0.289	0.276	0.268
120-ft. head, Site = 4.0 × Equip.	0.528	0.508	0.448	0.392	0.361	0.345	0.335
400-ft. head, Site = 3.0 × Equip.	0.365	0.333	0.289	0.240	0.219	0.208	0.203
400-ft. head, Site = 4.0 × Equip.	0.457	0.417	0.361	0.300	0.274	0.260	0.254
	Operating Disbursements, Cents per Kw-Hr.						
Labor and Material, Attendance and Maintenance	0.23	0.20	0.16	0.12	0.10	0.093	0.80
	Total Power Cost, Cents per Kw-Hr.						
Total for							
20-ft. head, Site = 1.0 × Equip.	0.675	0.600	0.525	0.458	0.426	0.415	0.400
20-ft. head, Site = 2.0 × Equip.	0.898	0.799	0.707	0.627	0.589	0.576	0.559
20-ft. head, Site = 3.0 × Equip.	1.120	0.999	0.890	0.796	0.752	0.737	0.719
120-ft. head, Site = 2.0 × Equip.	0.547	0.505	0.429	0.355	0.317	0.400	0.281
120-ft. head, Site = 3.0 × Equip.	0.652	0.606	0.518	0.434	0.389	0.369	0.348
120-ft. head, Site = 4.0 × Equip.	0.758	0.708	0.608	0.512	0.461	0.438	0.415
400-ft. head, Site = 3.0 × Equip.	0.595	0.533	0.444	0.360	0.319	0.301	0.283
400-ft. head, Site = 4.0 × Equip.	0.687	0.617	0.521	0.420	0.374	0.353	0.334

57 Starting with the costs of energy in cents per million B.t.u. available for the prime mover as determined in Table 6, the costs of b.hp. energy are determined in Table 9 and plotted in Fig. 9.

58 Referring to Table 9 and Fig. 9, the effect of the different efficiencies of the three prime movers, with their investment costs more or less inversely related to efficiency, is to change considerably the relative energy costs in the b.hp. form, compared to what they were in the form of energy of working fluid.

59 The most striking example of this is the case of the hydraulic turbine, where the cost of working-fluid energy is so much higher than that for the fuel-using systems. Here the efficiency of the hydraulic turbine has made b.hp. energy cost less over the whole range than for the fuel systems, though of course the low investment for prime mover has contributed materially. Too much reliance must not be placed on these figures, however, because, as noted previously, hydro-plant costs as a matter of fact are not reducible to such consistent rela-

tions as have here been introduced to get a sort of type comparison. Higher investments, especially for site and site development, which are quite likely, will change these relations. Roughly, twice the investment estimated would in general eliminate the advantage over fuel-using systems. Furthermore, the hydro plant is naturally of less competitive interest, because it cannot be placed where it will do the most good as regards the power consumer, which means that its power cost at the point of use normally carries a much higher electrical-transmission-cost increment than do the power costs of steam or Diesel oil-engine plants that are strategically located.

60 Turning to steam turbines and Diesel oil engines, the figures show that, size for size, the cost of b.hp. energy for the latter is lower than for the former, and what is especially characteristic, the Diesel-engine low costs of b.hp. energy carry down into much smaller sizes. This difference in the smaller sizes would be more apparent if

TABLE 14 HYDROELECTRIC POWER COSTS IN CENTS PER KILOWATT-HOUR—DISTRIBUTION BY PROCESS STAGES

Plant capacity, kw.	Development of Energy of Working Fluid at Prime Mover 1,500	2,500	5,000	10,000	15,000	20,000	25,000
	Fixed Charges, Cents per Kw-Hr.						
Preparation of Working Fluid							
20-ft. head, Site = 1.0 × Equip.	.2226	.1997	.1823	.1689	.1629	.1609	.1598
20-ft. head, Site = 2.0 × Equip.	.4453	.3994	.3646	.3378	.3258	.3218	.3196
20-ft. head, Site = 3.0 × Equip.	.6678	.5991	.5469	.5067	.4887	.4827	.4794
120-ft. head, Site = 2.0 × Equip.	.2112	.2032	.1792	.1569	.1444	.1381	.1341
120-ft. head, Site = 3.0 × Equip.	.3168	.3048	.2688	.2354	.2166	.2072	.2012
120-ft. head, Site = 4.0 × Equip.	.4224	.4064	.3584	.3138	.2888	.2762	.2682
400-ft. head, Site = 3.0 × Equip.	.2740	.2500	.2166	.1798	.1643	.1558	.1524
400-ft. head, Site = 4.0 × Equip.	.3653	.3333	.2888	.2397	.2192	.2077	.2032
Working Fluid to B.hp.							
20-ft. head	.0846	.0759	.0693	.0642	.0619	.0611	.0608
120-ft. head	.0401	.0385	.0339	.0297	.0274	.0262	.0254
400-ft. head	.0348	.0317	.0274	.0228	.0208	.0197	.0191
B.hp. to Electrical							
20-ft. head	.1379	.1240	.1130	.1044	.1010	.0999	.0990
120-ft. head	.0655	.0630	.0556	.0485	.0448	.0428	.0416
400-ft. head	.0567	.0516	.0448	.0371	.0339	.0322	.0314

	Operating Disbursements, Cents per Kw-Hr.						
Working fluid to b.hp.	.06	.06	.06	.06	.06	.06	.05
B.hp. to electrical	.17	.14	.10	.06	.04	.033	.03
Total	.23	.20	.16	.12	.10	.093	.08
	Total, Cents per Kw-Hr.						
Total for							
20-ft. head, Site = 1.0 × Equip.	.6750	.5996	.5246	.4575	.4258	.4149	.3996
20-ft. head, Site = 2.0 × Equip.	.8976	.7993	.7069	.6264	.5887	.5758	.5594
20-ft. head, Site = 3.0 × Equip.	1.1202	.9990	.8892	.7953	.7516	.7367	.7192
120-ft. head, Site = 2.0 × Equip.	.5468	.5047	.4287	.3551	.3166	.3001	.2811
120-ft. head, Site = 3.0 × Equip.	.6524	.6063	.5183	.4336	.3888	.3692	.3482
120-ft. head, Site = 4.0 × Equip.	.7580	.7079	.6079	.5120	.4610	.4382	.4152
400-ft. head, Site = 3.0 × Equip.	.5960	.5333	.4488	.3597	.3190	.3007	.2829
400-ft. head, Site = 4.0 × Equip.	.6875	.6166	.5210	.4196	.3739	.3526	.3437
	Efficiencies and Equivalent B.t.u. Consumption						
B.t.u. per hr. in energy of water per kw.	....	....	....	....	....	....	....
Energy of working fluid to b.hp., efficiency, per cent.	72.0	74.5	77.5	81.0	83.0	85.0	86.0
B.t.u. per hr. in b.hp. per kw.	....	....	....	....	....	....	....
B.hp. to electrical, efficiency, per cent.	89.0	89.5	90.0	90.5	91.0	91.5	92.0
B.t.u. per hr. in electrical, per kw.	3415	3415	3415	3415	3415	3415	3415
Energy of water to electrical overall efficiency, per cent.	64.0	66.7	69.7	73.4	75.5	77.8	79.0

steam-turbine plants had been estimated for sizes below the present minimum of 1500 kw. down to the 500-kw. size which is taken as the minimum for Diesel engines.

61 To see the effect of each of the three causes of cost accumulation, it is necessary to divide out the inefficiency increment from the Energy Loss Item of Table 9, by subtracting the input-energy cost from Table 6. This may then be compared most conveniently with the increments of investment and operating disbursements of Table 10 by employing Figs. 7 and 8 where they are all plotted. The overall effect of transforming energy of working fluid into b.hp. energy will be shown by the commercial efficiency compared with the efficiency of the transformation process, Table 9.

TABLE 15 STEAM-TURBINE POWER COSTS IN CENTS PER KILOWATT-HOUR

(Continuous 24-hour service. Average yearly load, 40 per cent. Fixed-charge rate, 12 per cent)

	Investment, Dollars per Kw. Capacity						
Plant capacity, kw.	1,500	2,500	5,000	10,000	15,000	20,000	25,000
Real estate, siding, etc.	6.8	6.0	4.5	2.7	2.3	2.1	2.0
Building, including stack	54.0	33.0	23.3	15.3	13.3	12.5	12.0
Boiler-room equipment	50.0	47.5	37.0	30.8	28.1	25.7	25.6
Piping, etc.	21.0	20.0	16.2	12.9	11.9	11.7	11.3
Turbo-generator units	76.0	66.0	53.0	45.0	42.1	41.1	39.5
Electrical switching apparatus, instruments	10.0	8.7	6.4	5.0	4.6	4.5	4.2
Miscellaneous equipment	9.0	8.3	6.9	5.3	4.8	4.5	4.4
Total	226.8	189.5	147.3	117.0	107.1	102.1	99.0
	Fixed Charges, Cents per Kw-Hr.						
Average load 40%; yearly rate 12%	.85	.65	.50	.40	.37	.35	.34
	Operating Disbursements (Except Fuel) Cents per Kw-Hr.						
Labor and material, attendance and maintenance.	.67	.65	.61	.54	.48	.44	.40
	Fuel, Cents per Kw.-Hr.						
B.t.u. (fuel) per kw-hr.	38,000	34,750	32,400	29,600	27,700	26,600	26,100
Fuel Price: Per mil. B.t.u. Per ton app.							
16.0 cents = $4.50	.605	.56	.52	.47	.44	.43	.42
21.5 cents = 6.00	.815	.75	.70	.63	.60	.57	.56
27.0 cents = 7.50	1.030	.95	.87	.79	.75	.72	.70
32.0 cents = 9.00	1.250	1.12	1.02	.94	.89	.85	.83
	Total Power Cost, Cents per Kw-Hr.						
Fuel Price: Per mil. B.t.u. Per ton app.							
16.0 cents = $4.50	2.13	1.86	1.63	1.41	1.29	1.22	1.16
21.5 cents = 6.00	2.30	2.05	1.81	1.57	1.41	1.36	1.30
27.0 cents = 7.50	2.55	2.25	1.98	1.73	1.60	1.51	1.44
32.0 cents = 9.00	2.77	2.42	2.13	1.88	1.74	1.64	1.57

62 In the case of the steam turbine, the outstanding feature in these figures on cost increments is the controlling value of the inefficiency item in relation to those of investment and operating disbursements. This indicates a possibility of improving commercial efficiency by increasing the efficiency of the turbine plant, even with increases in investment in turbines and their condensers and in operating disbursements.

63 Turning to the Diesel-engine figures, the striking thing is the insignificance of the fuel or inefficiency increment in comparison with the increments of investment and operating disbursements as a result of higher efficiency and lower cost of working fluid. This indicates a corresponding justification for the efforts now under way to reduce first costs of Diesel engines without adding to operating disbursements by unreliable designs, and also indicates the real importance of the features of some of the newer American designs. The labor element is, however, the real controlling item of operating disbursements. In a general comparison of the steam turbine and the Diesel-engine plant, probably the outstanding element is the fact that by reason of the two fuel-increment magnitudes the steam plant is necessarily more affected by fuel price changes than the Diesel oil engine, which latter can obviously stand much higher prices of fuel without much change in the cost of b.hp. energy as compared with steam.

64 The overall effect of the transformation of energy of working fluid into b.hp. energy is reflected in the figures given in Table 10 for commercial efficiency, or the ratio of cost of input to output energy for the two fuel systems.

65 As was the case with preparation of the working fluid, the present process of transformation into work is being attacked along many lines of possible improvement especially directed toward the steam turbine and its relations in the plant. The method of judging the value of efficiency proposals through corresponding commercial efficiencies is very useful here, as many of these plans are complicated. In some instances the result is most clearly seen in its true values by subdivision of processes such as the production of vacuum in the condenser, or by comparing one air-removal equipment for a condenser with another. More of these plans, however, require for their judging not subdivision of the transforming process but inclusion with it of all related processes — that is, those affected by the change. Among these are rise of steam pressures and superheat to high values, which add something to the cost of energy of steam at the same time they subtract another amount from transformation into work. In the same class are plans for the reheating or resuperheating of steam between stages and the bleeding of steam between stages to increase the capacity of the limiting low-pressure wheel of a turbine. This

TABLE 16 STEAM-TURBINE POWER COSTS IN CENTS PER KILOWATT-HOUR—DISTRIBUTION BY PROCESS STAGES

INVESTMENT, DOLLARS PER KW. CAPACITY

	Energy of Fuel to Energy of Working Fluid at Prime Mover						
Plant capacity, kw.	1,500	2,500	5,000	10,000	15,000	29,000	25,000
Real estate, etc.	3.40	3.00	2.25	1.35	1.15	1.05	1.00
Building	37.00	16.50	11.65	7.65	6.65	6.25	6.00
Boiler-room equipment	50.00	47.50	37.00	30.80	28.10	25.70	25.60
Piping, etc.	21.00	20.00	16.20	12.90	11.90	11.70	11.30
Miscellaneous	4.50	4.15	3.45	2.65	2.40	2.25	2.20
Total	115.90	91.15	70.55	55.35	50.20	46.95	46.10
	Energy of Working Fluid at Prime Mover to B.hp. Energy						
Real estate, etc.	1.36	1.20	.90	.54	.46	.42	.40
Building	14.80	6.60	4.66	3.06	2.66	2.50	2.40
Turbines, condenser, etc.	46.80	40.40	32.10	26.43	24.63	23.97	23.00
Miscellaneous	1.80	1.66	1.38	1.06	.96	.90	.88
Total	64.76	49.86	39.04	31.09	28.71	27.76	26.68
	B.hp. Energy to Electrical Energy						
Real estate, etc.	2.04	1.80	1.35	.81	.69	.63	.60
Building	22.20	9.90	6.99	4.59	3.99	3.75	3.60
Generators, exciters, etc.	29.20	25.60	20.90	18.58	17.48	17.13	16.50
Switching apparatus, instruments, etc.	10.00	8.70	6.40	5.00	4.60	4.50	4.20
Miscellaneous	2.70	2.49	2.70	1.59	1.44	1.35	1.32
Total	66.14	48.49	38.34	30.57	28.20	27.36	26.22
Grand Total	246.80	189.50	147.30	117.00	107.10	102.10	99.00
	FIXED CHARGES, CENTS PER KW-HR.						
Fuel energy to working fluid	.399	.313	.238	.188	.174	.160	.158
Working fluid to b.hp	.223	.171	.132	.106	.099	.095	.091
B.hp. to electrical	.228	.166	.130	.106	.097	.095	.091
Total	.85	.65	.50	.40	.37	.35	.34

Operating Disbursements, Cents per Kw-Hr. (Except Fuel)							
Fuel energy to working fluid	.36	.35	.33	.28	.23	.20	.18
Working fluid to b.hp.	.25	.24	.22	.20	.19	.18	.17
B.hp. to electrical	.06	.06	.06	.06	.06	.06	.05
Total	.67	.65	.61	.54	.48	.44	.40
Heat Consumption and Efficiency							
B.t.u. per hr. in fuel per kw	38,000	34,750	32,400	29,600	27,700	26,600	26,100
Fuel energy to working fluid, efficiency, per cent.	64.50	65.65	67.50	69.33	70.20	70.68	71.00
B.t.u. per hr. in working fluid at prime mover, kw.	24,600	22,815	21,870	20,521	19,445	18,800	18,531
Working fluid to b.hp., efficiency, per cent	16.57	16.72	17.35	18.39	19.30	19.85	20.03
B.t.u. per hr. in b.hp. per kw	3837	3815	3794	3773	3753	3732	3712
B.hp. to electrical, efficiency, per cent	89.0	89.5	90.0	90.5	91.0	91.5	92.0
B.t.u. per hr. electrical per kw	3415	3415	3415	3415	3415	3415	3415
Fuel to electrical, overall efficiency, per cent	9.0	9.83	10.54	11.61	12.33	12.84	13.08

latter case is especially complicated, as it may limit the recovery of flue waste heat by boiler feedwater, and this, in turn, is related to boiler-air preheating.

Transformation of B.Hp Energy into Electrical Energy

66 There is a prevailing impression that because the efficiency of transforming b.hp. energy into electrical energy is so very high, it is hardly worth while to give much consideration to the electrical equipment such as generators, exciters, and switching apparatus, especially as their operating disbursements are also very low. While there is some truth in this view, and while electrical generation is not concerned with the various prime movers except as speed determines size and investment-cost increments, it is also true that *the energy input to the generator is proportionately very costly* because of the accumulation of all the preceding processes, and this is affected by the cost of primary energy, which is variable. For this reason efficiency of generation must be high, especially the average efficiency over the use range of fluctuating loads. It is also true that, considering switching apparatus, the equipment expense is not only substantial in relation to generation, but the building and land items, which are included in all of these estimates, are proportionately larger.

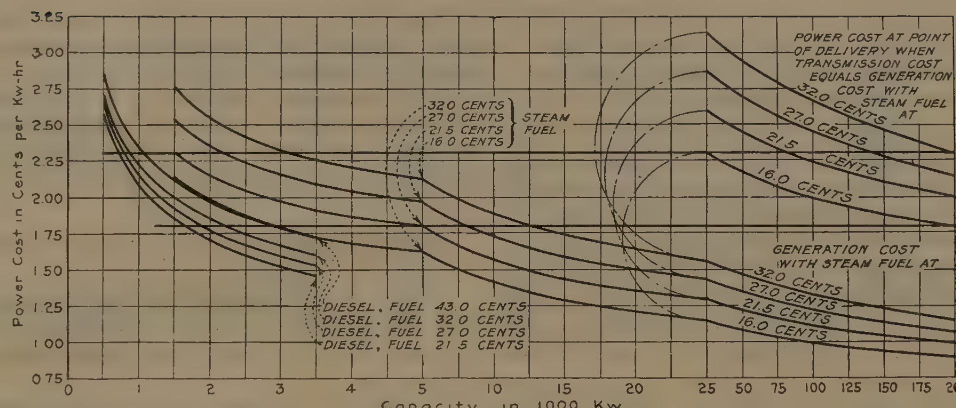

Fig. 10 Steam Turbines vs. Diesel Engines—Total Power Cost in Cents per Kw-Hr.

(Fixed charges: Steam plant, 12 per cent; Diesel plant, 13 per cent. Use factor, 40 per cent.)

67 These relations are shown in Table 12, together with the totals of cost of electrical energy in cents per million B.t.u. As they are of less interest than the other matters involved, their discussion will be omitted.

Overall Transformation of Primary Energy into Electrical Energy

68 The summation of all the cost accumulations for all the process stages required to transform primary energy into electrical energy, added to the cost of

TABLE 17 TRANSFORMATION OF ENERGY OF FUEL INTO ELECTRICAL ENERGY—OVERALL EFFICIENCIES

STEAM TURBINES

		Commercial Efficiency, Per Cent						
Plant capacity, kw...		1,500	2,500	5,000	10,000	15,000	20,000	25,000
Fuel price per million B.t.u.	16.0 cents	2.52	2.94	3.36	3.86	4.22	4.48	4.68
	21.5 cents	3.08	3.57	4.07	4.64	5.08	5.37	5.60
	27.0 cents	3.54	4.12	4.66	5.30	5.77	6.10	6 40
	32.0 cents	3.94	4.52	5.11	5.80	6.29	6.65	6.09
		Thermal Efficiency, Per Cent						
		9.0	9.83	10.54	11.61	12.33	12.84	13.08

DIESEL OIL ENGINES

		Commercial Efficiency, Per Cent						
Plant capacity, kw....		....	....	500	1,000	1,500	2,500	3,500
Fuel price per million B.t.u.	21.5 cents	....	....	2.83	3.53	3.96	4.60	5.02
	27.0 cents	....	....	3.47	4.29	4.80	5.53	6.03
	32.0 cents	....	....	4.02	4.95	5.50	6 35	6.85
	43.0 cents	....	....	5.13	6.25	6.59	7.90	8.52
		Thermal Efficiency, Per Cent						
		....	....	26.6	27.0	27.2	27.6	27.7

primary energy, gives the costs of electrical energy in cents per million B.t.u. of Table 12, which are identically equivalent to the costs, in cents per kw-hr., of Table 13, for hydraulic turbines; Table 15, for steam turbines; and Table 18, for Diesel oil engines, for plant capacities of from 500 kw. to 3500 kw. for the Diesel units, and 1500 kw. to 25,000 kw. for the two types of turbine units. The Diesel-engine limits of size used in these tables are merely those of common catalog standards and are not by any means the maximum possible.

69 These totals are plotted in Fig. 10, to which are added the values for a 200,000-kw. installation carrying similar loads; and to this in turn are added points for twice the value of the generation costs for the big plant, to represent the cost at point of delivery or use far enough away to make electrical-transmission costs equal to generation costs in a modern large high-economy station. This is so arranged in order to establish on one sheet an inclusive survey picture of the two typical systems burning fuel. The corresponding thermal and commercial efficiencies are given in Table 17.

TABLE 18 DIESEL-OIL-ENGINE POWER COSTS, CENTS PER KILOWATT-HOUR

(Continuous 24-hour service. Average yearly load, 40 per cent. Fixed-charge rate, 13 per cent)

INVESTMENT, DOLLARS PER KW. CAPACITY					
Plant capacity, kw	500	1,000	1,500	2,500	3,500
Real estate, siding, etc	7.2	5.8	4.6	3.5	2.8
Building	64.8	48.2	38.4	28.8	24.8
Generating units	193.0	184.2	176.6	163.2	153.1
Auxiliaries	9.4	7.7	7.2	6.7	6.3
Piping, tanks, etc	13.4	9.4	7.1	5.1	4.2
Electrical switching apparatus, instruments, etc.	11.7	10.0	9.2	8.3	7.8
Miscellaneous equipment	6.9	4.7	4.0	3.2	2.7
Total	306.4	270.0	247.1	218.8	201.7
FIXED CHARGES, CENTS PER KW-HR.					
Average load, 40 per cent, yearly rate, 13 per cent	1.137	1.002	.917	.812	.748
OPERATING DISBURSEMENTS (EXCEPT FUEL), CENTS PER KW-HR.					
Labor and material, attendance and maintenance	1.175	.803	.666	.517	.445
FUEL, CENTS PER KW-HR.					
B.t.u. (fuel) per kw-hr	12,830	12,640	12,540	12,400	12,300
Fuel Price Per mil. B.t.u. Per gal. app.					
21.5 cents = 3.2 cents	.276	.272	.270	.266	.264
27.0 cents = 4.0 cents	.346	.341	.339	.335	.332
32.0 cents = 4.8 cents	.410	.404	.401	.397	.393
43.0 cents = 6.4 cents	.551	.543	.539	.533	.529
TOTAL POWER COST, CENTS PER KW-HR.					
Fuel Price Per mil. B.t.u. Per gal. app.					
21.5 cents = 3.2 cents	2.59	2.08	1.86	1.60	1.46
27.0 cents = 4.0 cents	2.66	2.15	1.92	1.66	1.53
32.0 cents = 4.8 cents	2.72	2.21	1.98	1.73	1.59
43.0 cents = 6.4 cents	2.86	2.35	2.12	1.86	1.72

70 Referring to Fig. 10, it is to be noted that each curve is regular in form for each type of prime mover. The curves have different slopes so that a Diesel power-cost line will intersect a steam-turbine power-cost line at some point. A change of fuel price has the effect of displacing either curve bodily upward or downward and so changing the point of intersection. This point of intersection is the plant kilowatt capacity at which both types of prime movers would have equal electric-generating costs. It fixes the identical competition for the conditions assumed, and divides plant capacity into two zones, one between zero and the plant capacity corresponding to the intersection where Diesel engines

give cheaper power, and the other for capacities greater than that at the intersection where steam-turbine power generation is cheaper.

71 Reference to the curves also shows that their slopes are so nearly identical that the point of intersection approaches that of tangency, so that there will be a more or less extended range of plant capacities where substantially identical generating costs prevail. A comparatively slight change in any of the generating-cost items, such as fuel price, investment expense, rates of wages paid to operating force, plant loads, number of units, and unit loads, besides the various inequalities in efficiencies of various parts or the whole — and not excluding variations in management from good to very bad — any one or all of these will move a curve up or down, or somewhat change its slope. It may therefore be said that there is a band rather than a single-line curve representative of steam turbines and another band representative of Diesel oil

TABLE 19 DIESEL-OIL-ENGINE POWER COSTS IN CENTS PER KILOWATT-HOUR—DISTRIBUTION BY PROCESS STAGES

Investment, Dollars per Kw. Capacity					
Energy of Fuel at Prime Mover to B.Hp. Energy					
Plant capacity, kw	500	1,000	1,500	2,500	3,500
Real estate, etc	6.5	5.2	4.1	3.2	2.5
Building	45.0	33.0	27.0	20.1	17 3
Diesel engines	172.0	167.0	162.0	152.0	143 0
Auxiliaries	9.4	7.7	7.2	6.7	6 3
Piping, tanks, etc	13.4	9.4	7.1	5.1	4.2
Miscellaneous	4.8	3.3	2.8	2.2	1.9
Total	251.1	225.6	210.2	189.3	175.2
B.Hp. Energy to Electrical Energy					
Real estate, etc	.7	.6	.5	.3	.3
Building	19.8	15.2	11.4	8.7	7.5
Generators and exciters	21.0	17.2	14.6	11.2	10.1
Switching apparatus, instruments, etc.	11.7	10.0	9.2	8.3	7.8
Miscellaneous	2.1	1.4	1.2	1.0	.8
Total	55.3	44.4	36.9	29.5	26.5
Grand Total	306.4	270.0	247.1	218.8	201.7
Fixed Charges, Cents per Kw-Hr.					
Fuel working fluid to b.hp	.932	.837	.780	.702	.650
B.hp. to electrical	.205	.165	.137	.109	.098
Total	1.137	1.002	.917	.811	.748
Operating Disbursements, Cents per Kw-Hr. (Except Fuel)					
Fuel working fluid to b.hp	1.115	.743	.606	.456	.385
B.hp. to electrical	.060	.060	.060	.060	.060
Total	1.175	.803	.666	.516	.445
Heat Consumption and Efficiencies					
B.t.u. per hr. in fuel per kw	12,830	12,640	12,540	12,400	12,300
Fuel to b.hp., efficiency, per cent	30.4	30.5	30.6	30.8	30.9
B.t.u. per hr. in b.hp. per kw	3910	3860	3840	3820	3850
B.hp. to electrical, efficiency, per cent	87.5	88.5	89.0	89.5	89.9
B.t.u. per hr. electrical per kw	3415	3415	3415	3415	3415
Fuel to electrical, overall efficiency, per cent	26.6	27.0	27.2	27.6	27.7

engines, and that the operating costs will lie somewhere between the limits of each band, depending on the net effect of all the factors. These bands however, will also intersect, and there will be an intersection at the top and bottom, which will divide capacities into three rather than into two zones. In the first zone Diesel engines will give the cheapest power generation. The second zone is the competitive zone in which either Diesel engines or steam turbines may make cheaper power, depending on local conditions at the time. The third zone is that of bigger plants, where the steam turbine generates power most cheaply.

72 A really big plant of the recently developed steam-turbine type can generate at remarkably low cost considering past practice, but considering the fact that at the point of use power is required normally in small amounts, this low generating cost is deceptive in relation to the problem of use and widely distributed or general power service, because to be useful, a transmission charge must be added. This transmission-cost accumulation has its inefficiency increment, its investment increment, and its operating-disbursement increment, all of which make the costs at substation local-supply points high enough at some distance to neutralize the economies of the bigger station, if load and other prime conditions are similar.

73 It therefore necessarily follows that with full recognition of the many advantages of big generating stations and of electrically connecting several large generating stations, it is necessary to consider the small generating station in economic competition with a substation of such a system. No careful analysis of generating costs in small stations designed and operated as intelligently as large ones can fail to strengthen this principle. For the premises assumed in the present analysis, the conditions are very clear. Projecting a line across the curves of power costs for the small steam-turbine and Diesel-engine plants from the point of cost of a 200,000-kw. station where transmission cost equals generating cost, it intersects the steam-turbine line at about 3000 kw. when the coal costs 32 cents per million B.t.u., and the Diesel line at 1100 kw. when the fuel oil costs one-third more, or 43 cents per million B.t.u. Therefore, if at a point this far away the local requirements are below 3000 kw. and such fuel prices prevail, a local steam-turbine generating station will then give cheaper power, and if the local requirements are still less, below 1100 kw., then a Diesel oil-engine generating equipment will make cheaper power. At 3000 kw. the Diesel-plant power cost will be lower than that of the steam-turbine plant by 24 per cent. Such a station may very properly be electrically tied into a general transmission system. This makes it necessary to consider and to continue to study the economies of small-station generation, in coöperation with large-station economies and transmission costs, to keep pace in competitive economies with all the fluctuations of conditions.

TECHNICAL COMMITTEE REPORTS

IN THIS section is given a summary of the codes and standards which have been completed during 1924 and are available either in book or in pamphlet form upon application, and of preliminary or tentative drafts which have been printed in *Mechanical Engineering* during the year. Other work of the various committees in process is summarized in the Report of the Council, pages 552-557.

BOILER CODE

Revisions of the 1918 edition of the Boiler Code, which later formed the 1924 edition, and interpretations of the Code were published in *Mechanical Engineering,* February (1924), p. 103; March, p. 161; April, p. 224; June, p. 365; July, p. 427; August, p. 497; September, p. 565; and December, p. 923.

The following reports were printed in *Mechanical Engineering:*

Proposed Rules for Inspection of Material and Boilers, February, p. 100.

Report on Code for Unfired Pressure Vessels, December, p. 916.

The 1924 edition of the Boiler Code was printed in 1924. This edition contains the following sections:

Section I, Rules for Power Boilers.
Section II, Material Specifications.
Section VI, Rules for Inspection.
Appendix.

POWER TEST CODES

Four more of the new codes were issued in final pamphlet form during 1924. These are

Code on General Instructions
Test Code for Reciprocating Steam Engines
Test Code for Stationary Steam Boilers
Test Code for Internal-Combustion Engines.

During the year four of the test codes were also printed in *Mechanical Engineering,* as follows:

Test Code for Condensing Apparatus, preliminary draft, May, p. 291.

Test Code for Solid Fuels, preliminary draft, September, p. 558.

Test Code for Speed-Responsive Governors, preliminary draft, November, p. 713.

Test Code for Gas Producers, tentative draft, December, p. 910.

RESEARCH

Fluid Meters. Part 1 of the report of the Special Research Committee on Fluid Meters was issued in November, 1924. It treats of the general types of fluid meters as well as the principles and methods

involved, and gives information which may, in many cases, be applicable to various commercial meters. In this part individual makes of instruments are not discussed in detail, but are referred to only incidentally or for illustrative purposes.

Properties of Steam and Extension of Steam Table. The third annual progress report of the Special Research Committee on the Properties of Steam and Extension of Steam Table was presented at the Steam Table Session of the Annual Meeting. It consists of reports by Dr. Harvey N. Davis, R. V. Kleinschmidt, Frederick G. Keyes, Nathan S. Osborne, and H. F. Stimson, which were preprinted prior to publication in *Mechanical Engineering.*

Metal Springs. A preliminary progress report prepared by the Special Research Committee on Metal Springs was presented at the Annual Meeting in December and printed in *Mechanical Engineering,* Mid-November issue, p. 793.

Riveted Joints. A bibliography on riveted joints was prepared by Dr. A. H. Stang of the Bureau of Standards with the coöperation of the A.S.M.E. Special Research Committee on Riveted Joints. It was issued in pamphlet form in May, 1924.

STANDARDIZATION

Standardization of Shafting. The second standard developed by the Sectional Committee was published in the February, 1924, issue of *Mechanical Engineering,* p. 107. It includes standard dimensions for stock keys and standard tolerances for these dimensions.

Standardization and Unification of Screw Threads. The first report of this Sectional Committee, entitled Screw Threads for Bolts, Machine Screws, Nuts, and Commercially Tapped Holes, was approved as an American Standard by the A.E.S.C. in May of this year, and was made available to the general public in pamphlet form in July.

Standardization of Gears. The proposed dimensions of Stub-Tooth Involute Gears and Full-Depth Involute Gears were published in the September, 1924, issue of *Mechanical Engineering,* p. 572.

SAFETY CODES

The first edition of 1000 copies of the Safety Code for Mechanical Power-Transmission Apparatus, which was published in pamphlet form in December, 1923, was soon exhausted and a second edition of 6000 was printed. Five thousand of these were purchased and distributed by the National Board of Casualty and Surety Underwriters, one of the three joint sponsors for this Code.

No. 1949

NECROLOGY

CHARLES E. ANDERSON

Charles E. Anderson died suddenly on Christmas morning, 1924. Mr Anderson was born in New York City, October 1, 1877, and acquired his early education in the public schools of Brooklyn. Later, he took a correspondence course in engineering.

From 1898-1902, he was with the Varley Duplex Magnet Co., of Jersey City, N. J. He was their representative at Paris in 1900, installing and exhibiting special machinery at the Paris Exposition. In 1903, he went to the American Electric Works at Providence, R. I., where he had charge of the coil-winding departments and also designed coil-winding machines. In 1909, he became superintendent of the Willyoung Appliance Co., of Yonkers, N. Y., in 1910, was draftsman for the Western Electric Co. in New York, and from 1911 to 1915 was in the engineering department of the New York Edison Co. as a designer of power-plant work. For several years following 1915, he was designer of electrical welding machines and superintendent of construction for the Metropolitan Engineering Co., and from 1915 until his death was engineer for Thos. E. Murray, Inc.

Mr. Anderson joined the Society in 1920, and was also an associate member of the American Institute of Electrical Engineers.

WILLIAM J. BALDWIN

William J. Baldwin, known as the dean of the heating and ventilating profession in the United States, died on May 7, 1924. Mr. Baldwin was born on June 14, 1844, in Waterford, Ireland. He was educated in the primary schools of Boston and in St. Dunstan's High School, Charlottetown, P.E.I.

He began his engineering career at East Boston when he entered the employ of Donald McKay, called the "Prince of Shipbuilders." After assisting in the construction of three light-draft monitors and the reconstruction of a number of English blockade runners into United States cruisers, Mr. Baldwin went to South America as a young naval architect in Brazilian service. When he returned to America in 1868 there was very little activity in the shipbuilding line and he was forced to look to other fields. Chance led him into work connected with the development of great buildings in New York — work for which he attained a reputation not only in New York but throughout the country. He was responsible for the heating and ventilating equipment of many of the largest hospitals, public buildings, theaters, department stores and office buildings in this country.

Mr. Baldwin became a member of the Society in 1882. He was an honorary member of the American Society of Heating and Ventilating Engineers, a life member of the American Institute of Architects, a member of the American Society of Civil Engineers, a Telephone Pioneer of America, and vice-president of the engineering department of the Brooklyn Institute of Arts and Sciences. He was the author of several standard works on heating and ventilating.

BERT CHARLES BALL

Bert C. Ball, who was president of the Willamette Iron & Steel Co., Portland, Ore., until his retirement in 1922, died on January 27, 1924, after an illness of more than two years. Mr. Ball was born on June 22, 1870, in Grand Island, N. Y. He was educated in the schools of Erie, Pa., and was graduated from the Stevens Institute of Technology in 1895 with the degree of M.E.

During the Spanish-American War, Mr. Ball served in the Bureau of Engineering in the United States Navy. Following his service there, he was associated with the late William H. Corbett in New York City on consulting engineering work. In 1901 they went to Portland, Ore., and there organized the Willamette Iron & Steel Co., with Mr. Corbett as president and Mr. Ball as chief engineer. At Mr. Corbett's death, Mr. Ball became president of the company and retained that position until 1922, when he was forced to resign on account of ill health.

Mr. Ball became a junior member of the Society in 1898 and a member in 1904. He belonged to a number of the local clubs and was very active in the work of the Portland Chamber of Commerce and in many welfare and professional associations.

WALTER FRANCIS BALLINGER

Walter Francis Ballinger, head of The Ballinger Co., architects and engineers of Philadelphia and New York, died in Philadelphia on December 21, 1924, as the result of an automobile accident.

Mr. Ballinger was born in Petroleum Center, Pa., August 13, 1868, and received his education at the public school in Woodstown, N. J., at the Philadelphia Business College, from the International Correspondence School, and at the Drexel Institute. He entered the architectural offices of Geissinger & Hales in Philadelphia in 1889, and in 1895 became a member of the firm, the name being changed to Hales & Ballinger. He became the senior member of the firm on the retirement of Mr. Hales in 1901, the name again being changed to Ballinger & Perrot, which in 1920 became The Ballinger Company.

Mr. Ballinger became interested in the possibility of reinforced-concrete building construction when there were only a few such buildings in existence, and did much to aid in the development of this form of construction. He was also co-inventor of a new form of sawtooth roof construction known as "Super-Span," which eliminated a large number of the columns common to the usual type of sawtooth construction and at the same time retained the advantages of natural illumination characteristic of that type. Mr. Ballinger's professional practice included the designing and erecting of manufacturing plants, business and institutional buildings, in nearly all of which he also installed boilers, dynamos, shafting, elevators, heating, lighting and sprinkler systems.

Mr. Ballinger became a member of the Society in 1913, and was also a member of the National Fire Prevention Commission, American Society of Civil Engineers, Franklin Institute, Engineers' Club of Philadelphia, Engineers' Club of New York, and the Society of Industrial Engineers.

Mr. Ballinger was interested in organizations for civic improvement, and was a member of the Philadelphia Chamber of Commerce, Philadelphia Board of Trade, and other local associations. He was active in the work of the Methodist Episcopal Church and local charitable and religious societies. He was a Mason, and belonged to several clubs.

In 1896, Mr. Ballinger was married to Miss Bessie M. Connell, who survives him, as do also one daughter, Grace A. Ballinger, and one

son, Robert I. Ballinger. The latter is a member of the new partnership of The Ballinger Co.

WILLIAM H. BAUSCH

William H. Bausch, sales manager for the National Equipment Co., Springfield, Mass., was born in 1867 and died suddenly on August 27, 1924, at Crescent Beach, Conn., where he had been spending the summer. He was a graduate of Williston Seminary and served an apprenticeship with Munn & Bausch in Holyoke of six years. In 1886 he became manager of C. H. Bausch & Sons, machine-tool builders, and in 1896 he organized the Bausch Machine-Tool Co., of which he was president and general manager until 1904, when he became associated with the National Equipment Co. He was the designer and patentee of Bausch multiple-spindle drills, Bausch radials and Bausch boring mills. Mr. Bausch had been a member of the Society since 1904. He was a Mason and a member of the Engineering Society of Western Massachusetts.

JOHN EVERETT BELL

John Everett Bell, consulting engineer, died of pneumonia on November 27, 1924, at his country home in Quaker Street, Schenectady Co., N. Y. Mr. Bell was born in Ripley, Ohio, on May 8, 1876. He attended the Ohio State University as a special student.

In 1897, Mr. Bell went to Cripple Creek, Colo., in connection with the erection of a power house for La Bella Mill Water and Power Co. He remained there until 1899, being successively chief engineer, superintendent and manager of the company. Until 1901, Mr. Bell did consulting work, then began work for the Stirling Co. He redesigned superheaters for the Stirling boiler, and redesigned the Niclausse marine boiler. He also designed a number of stokers, including one for burning anthracite culm, which he tried out thoroughly and condemned. He was engaged in investigating and testing work in all sections of the country and during that time assisted in designing what was then known as the middle-pass Stirling superheater boiler, which was later developed into the double type of boiler known as the Class W Stirling boiler.

In 1905, Mr. Bell became assistant to E. R. Stettinius, in New York, and in 1907 went over, with the organization of the Stirling Co., to the Babcock & Wilcox Co., where he was primarily engaged in office work until 1910. After that, most of his time was taken up in development work on the outside, including tests on the Class W boilers at the Del Ray Station of the Detroit Edison Co.; investigation of the burning of natural gas, blast-furnace gas, and by-product coke-oven gas; rebaffling of Stirling boilers; stoker experiments and scientific experimenting, particularly in connection with heat-transfer rates. He became interested in the development of waste-heat boilers and planned virtually all the installations of the Babcock & Wilcox Co. until he left that firm in 1915.

In 1915, Mr. Bell started in business for himself as a consulting engineer. He continued his investigation of waste gases from cement kilns, and, in conjunction with the Edge Moor Iron Co., by whom he was retained as consulting engineer, designed and installed forty-six installations for saving and utilizing such gases. So widely known was Mr. Bell's skill and knowledge as a waste-heat engineer that inquiries came to him from all over the world.

Mr. Bell also designed the Foster marine boiler, the Foster economizer, Foster high-pressure boiler and superheaters of various forms, for the Power Specialty Co. He designed, in conjunction with E. W. Isom, vice-president of the Sinclair Refining Co., the Isom cracking

stills and various other forms of refinery apparatus. He also investigated the fuel economy of stills and stokers and modified practice in this respect. Mr. Bell was retained as consulting engineer by the Power Specialty Co. and by the Sinclair Refining Co. until his death.

In 1920, Mr. Bell became consultant for the Combustion Engineering Corporation. The corporation was just entering the field of pulverized fuel, and he was called upon to investigate the practicable possibilities of burning pulverized fuel under steam boilers. The rapid progress that followed the acceptance of his recommendations was due in a large measure to his ability. His death deprived the world of a clear-thinking engineer, thoroughly grounded in the fundamentals. The engineering profession owed much to him and lost one whom it will be difficult to replace.

Mr. Bell became a member of the Society in 1908, and also belonged to the American Iron and Steel Institute, American Association for the Advancement of Science, the Engineers' Club, Lawyers' Club, Bankers' Club, Montauk Club, and Brooklyn Chamber of Commerce, in New York City.

JOSEPH H. BLACKWOOD

Joseph H. Blackwood, a patent attorney and expert of Washington, D. C., died on July 4, 1924. Mr. Blackwood was born in Washington in August, 1862, and in early life entered upon the practice of the legal profession. His practice as a patent attorney involved validity searches relating to the various arts and sciences, such as electrical, steam and hydraulic engineering, chemistry, metallurgy and dock construction. He had been a member of the Society since 1919.

EDWIN CHAPIN BROOKS

Edwin Chapin Brooks, retired, died at Melrose, Mass., on December 13, 1924. Mr. Brooks was born December 3, 1844, at Newmarket, N. H.

Following a common-school education, and work during two summer vacations in railway shops, Mr. Brooks in 1861 became a machinist apprentice at the Erie Railway shops at Dunkirk, N. Y. From July, 1864, until January, 1865, he worked as machinist at the Atlantic Works, East Boston, and then began four years' service as third assistant engineer in the United States Navy. For about one year he was assistant engineer on steamers sailing from Detroit, and in 1870 began his work as chief engineer at the Water Works Pumping Station, Cambridge, Mass. Twenty-five years later he was made superintendent of the Water Works, and continued in that position until he retired in July, 1913.

Mr. Brooks joined the Society in 1918. For many years he was a member of the New England Water Works Association, and for some time before his death was an honorary member and a past-president of that association. He was a member of the G. A. R., and of the Naval Commandery and New England Society of Naval Engineers, and was a Mason. At the time of his death, he was a trustee of the Melrose Universalist Church.

J. GROVE BROWN

J. Grove Brown, shop superintendent of the Universal Road Machinery Co., Kingston, N. Y., died on September 16, 1924. Mr. Brown was born on December 1, 1867, in Hartford, N. Y., where he attended the public schools. Later he was graduated from Cornell University with the degree of M. E. Upon leaving college he entered the employ of the Good Road Machinery Co., Marathon, N. Y., where he designed the jaw crusher now used universally throughout the country. He was

afterward associated with the Acme Road Machinery Co., Frankfort, N. Y. For the last few years he has been identified with the Universal Road Machinery Co. Mr. Brown was not only a recognized authority on road rollers but also on oil nozzles for pressure sprayers. He became a member of the Society in 1902. He belonged to the Masonic Order and to a number of local civic organizations, in whose work he took an active interest.

RUSSELL G. BROWN

Russell G. Brown, mechanical engineer with Warren Webster & Co., New York City, died at his home in East Orange, N. J., September 25, 1924. Mr. Brown was born in Newark on October 11, 1886, and received his early education in the public schools there and in the Fawcett School of Industrial Arts. Practically all of his life was devoted to steam engineering in connection with heating, ventilating and steam-power plants. He entered this work in 1903 under John A. Serrell, engineer in charge at New York for Warren Webster & Co. and continued his studies at Cooper Union; where he received the degrees of Bachelor of Science and Mechanical Engineer. Later he was graduated from the Polytechnic Institute of Brooklyn with the degree of Mechanical Engineer. He was an instructor for several years at the Fawcett School and at the Polytechnic. He had been a member of the Society since 1921.

H. M. BYLLESBY

Henry Marison Byllesby, one of the foremost engineers in the electrical world and active head of a gigantic network of public utilities, died suddenly in Chicago, Ill., on May 1, 1924, at the age of sixty-five. He was born in Pittsburgh, Pa., on February 16, 1859. He was educated at the Western University of Pennsylvania and at Lehigh University, 1873-77, where he studied mechanical engineering. He was not graduated, leaving at the end of his Junior year. On the morning of his death he received word from the president of Lehigh, advising him that the faculty, by unanimous vote, had decided to award him a degree in mechanical engineering for his engineering accomplishments.

During his early career Colonel Byllesby was closely associated with Thomas Edison in charge of engineering problems. It was he who made all the drawings for the structure, crane, and location of boilers, engines and switchboards of the First District Pearl Street Station — a pioneer job, as it was the first steam-operated central station in the United States. He designed central stations for construction in Chile and in Canada. He installed and operated the generating plants for Edison at the Louisiana Exposition in 1884 and at the New Orleans Exposition the following year. At the age of twenty-six he became vice-president and general manager of the Westinghouse Electric Co., and a little later president of the Northwest General Electric Co. On January 1, 1902, Colonel Byllesby organized the corporation of H. M. Byllesby & Co., of which he was president and active head at the time of his death. In addition to being president of this company he was an officer and director in public-utility concerns, principally gas and electric, all over the United States.

From the first month of the World War, Colonel Byllesby devoted much time to the interests of the Entente. He was one of the few men who, from the beginning of the war, correctly visualized the tremendous issues involved. From August, 1914, until the entrance of the United States into the conflict, he was very active in platform and organization work, and, beginning April 6, 1917, he gave his entire time to the service of the Nation.

Despite his age of sixty, Colonel Byllesby served during the war as general purchasing agent of the A.E.F. and in this capacity was stationed in London. When he was honorably discharged in 1918 he was awarded the Distinguished Service Order by the British Government and later received the American Distinguished Service Medal. The latter citation stated that he " displayed great energy, a comprehensive knowledge of large business affairs and executive ability of the highest order," and that " by his broad experience, foresight and splendid ability to coöperate with the representatives of our Allies, he solved many difficult problems of fuel supply with conspicuous success and a manner which insured, at critical times, a plentiful supply of coal, both for our transport service and our troops in France, thereby rendering services of great value to the American Expeditionary Forces."

Colonel Byllesby was one of the very early members of The American Society of Mechanical Engineers, joining the organization in 1882. He was a fellow of the American Institute of Electrical Engineers and a member of the American Society of Civil Engineers, the Western Society of Engineers and the National Electric Light Association. He belonged to the Union League and University Clubs of Chicago, and to the Metropolitan, Bankers', Railroad and Lawyers' Clubs of New York.

CLARENCE P. CLARK

Clarence P. Clark, a member of the Society since 1920, died on September 28, 1924. He was born in Michigan in 1876 and educated at the English High School and Manual Training School in Chicago. As a young man he worked as a designer with the Deering Harvester Co. for a number of years, then went to Pittsburgh to the Jones & Laughlin Steel Co., where he spent the rest of his life, occupying the position of assistant to the chief engineer of their South Side plant at the time of his death. His work with this company included the construction of pumping houses, open-hearth furnaces, boiler plants, etc., and the design of a coal-loading machine and mine tipples.

ALBERT LADD COLBY

Albert Ladd Colby, consulting engineer, of Bethlehem, Pa., died in Torquay, England, from influenza on May 1, 1924, after an illness of several weeks. Mr. Colby was born in New York City on June 26, 1860, and was graduated from the Columbia University School of Mines in 1881. For several years he was an instructor in analytical chemistry at Lehigh University and in 1886 he became a chemist and metallurgical engineer for the Bethlehem Steel Co. He remained in this position until 1905, when he established himself as a consulting iron and steel metallurgical engineer, numbering among his clients the Carnegie Steel Co., the Midvale Steel Co., the United States Steel Corpn. and many others. His work took him to Europe frequently and during the World War he spent considerable time abroad for the United States Government in connection with the standardization of aircraft steels.

Mr. Colby served as one of the two jurors on metallurgy at the Paris Exposition, by appointment of the President of the United States. He was the official delegate of the United States Government to the International Congress on Methods of Testing Materials of Construction, held in Paris in 1900, and presented a paper before that Congress. He was awarded a gold medal at the Louisiana Exposition for work in making and marketing nickel steels in America. He served on the Committee for Regulating the National Bureau of Standards. These are but a few of the many honors conferred upon him in recognition of his work.

Mr. Colby had been a member of the Society since 1893. He was also a member of the American Society of Civil Engineers, the American Institute of Mining and Metallurgical Engineers, the Franklin Institute, the Iron and Steel Institute (London), and the International Association for Testing Materials (Vienna).

EDWARD LIVINGSTON COSTER

Edward L. Coster, a life member of the Society since 1896, died on April 4, 1924. Mr. Coster was born on February 28, 1870, in New York, N. Y., and was educated in that city.

His chief pleasure and occupation was in engineering as applied to the locomotive, and he made the most elaborate analyses of the strains in various parts of the locomotive mechanism that probably have ever been made. He contributed in an important way to the best literature in railroad mechanical engineering, and spent his energy in the fundamental analysis of railroad problems.

Mr. Coster was a member of the American Railway Master Mechanics' Association, the New York Railroad Club, and the Franklin Institute of Philadelphia.

ALFRED COTTON

Alfred Cotton, chief of the research department of the Heine Boiler Co., died at his home in St. Louis on April 16, 1924.

Mr. Cotton was born in Southport, England, on November 30, 1871. While serving his apprenticeship in marine engineering, he received his technical education through university-extension courses and by private tuition. His early experience was gained with Thomas Henderson at Liverpool and the Port Sunlight Soap Works of Lever Brothers, Ltd. There he designed one of the first, if not the first, double-stage centrifugal pumps, which operated with fairly high efficiency and entire satisfaction. In 1898 Mr. Cotton became assistant chief engineer for Meldrum Brothers, Ltd., working on the design and adaptation of mechanical stokers and forced draft, and collaborating in the development of destructors for generating steam by means of municipal refuse.

In 1903 he came to America, where he developed the Cotton furnace, which embodied a high-efficiency system of steam-jet forced draft. He manufactured and installed these furnaces and allied apparatus until the war, when he entered the employ of Colt's Patent Fire Arms Manufacturing Co., designing jigs, fixtures and tools for the production of the Browning machine gun. In 1918 he became chief engineer in the combustion department of the Sterling Blower Co. and in 1919 he entered the employ of the Heine Boiler Co. where he did research work in combustion and heat transfer and made notable advances in boiler design.

Mr. Cotton became a member of the Society in 1921.

WILLIS S. CRANDELL

Willis S. Crandell, general manager of the paper mills of the A.P.W Paper Co., Albany, N. Y., died on December 23, 1924. Mr. Crandell was born on September 16, 1877. He was educated at St. Austin's School. From 1896 to 1918 he was associated with the Stevens & Thompson Paper Co., where he had worked through the ranks from office boy to manager, specializing in the erecting and designing of the machinery of the plant. In 1918 he joined the A.P.W. organization. He was given a free hand on building mills for the company, and the two he built stand as monuments to his genius in that field.

Mr. Crandell became a member of the Society in 1919. He belonged to a number of the local clubs of Albany and was a member of the Masonic Order.

HENRY J. CROWLEY

Henry J. Crowley, general manager of the American Electric Power Co., Philadelphia, Pa., formerly the American Railways Co., died on October 27, 1924. Mr. Crowley was born on October 17, 1865, in Unionville, Conn. He was widely known as an electrical engineer and had been associated with the American Railways Co. since 1899, the date of its organization. His first electrical experience was with the Schuyler Electric Light Co., where he was engaged in constructing electric-lighting plants in New England, Pennsylvania and Ohio. In 1888 he became affiliated with the Thomson-Houston Electric Co., later becoming general manager of the General Electric Co. at Atlanta, Ga. After serving three years in that capacity he was transferred to the company's office in Philadelphia as railway manager.

It was in 1889 that Mr. Crowley was appointed general manager of the American Railways Co., and subsequently as a vice-president of that concern he was in charge of construction of plants of about thirty subsidiary corporations in twelve states.

Mr. Crowley became a member of the Society in 1921. He was also a fellow of the American Institute of Electrical Engineers.

HERBERT E. CUSHMAN

Herbert E. Cushman, treasurer and general manager of the Morse Twist Drill & Machine Co., New Bedford, Mass., died on January 12, 1924. Mr. Cushman was born on January 1, 1862, in Taunton, Mass., and was educated there.

He began his business life with the Taunton Locomotive Works, serving there for about a year. For the next six years he was head bookkeeper for the Williams Manufacturing Co. of Taunton. In 1887 he resigned from this position to enter the employ of the Morse Twist Drill & Machine Co., as sales agent.

In 1902, Mr. Cushman was promoted to the office of treasurer and general manager. He was a director of the First National Bank of New Bedford, the New Bedford Institution for Savings, the Firemen's Mutual Insurance Co., the Union Mutual Insurance Co. of Providence, the New Bedford Foundry & Machine Co., and the General Fire Extinguisher Co. of Providence.

Mr. Cushman became an associate of the Society in 1888. He was a member of the Masonic Order and a Knight Templar. He belonged to a number of clubs in New Bedford and was president of the local chapter of the Sons of the American Revolution and of the Old Dartmouth Historical Society. He was also a member of the Machinery Club of New York.

EDWARD DEAN

Edward Dean, general superintendent of the Century Electric Co., St. Louis, Mo., died on January 25, 1924, from injuries received in an accident. Mr. Dean was born on May 15, 1865, in Ann Arbor, Mich., and received his education there.

After serving his apprenticeship and doing some preliminary work with various firms, he became associated in 1898 with the Burroughs Adding Machine Co., St. Louis, as designer of certain of the adding-machine parts which made possible the manufacture of interchangeable units. Four years later he became superintendent of the Western Cartridge Co., East Alton, Ill., where he remained for two years, until

he became general superintendent of the Standing Forge Co., St. Louis. In 1906, Mr. Dean became general superintendent of the Century Electric Co., where he laid out new factory buildings and supervised the design of equipment, etc.

Mr. Dean became a member of the Society in 1920. He belonged also to the American Heat Treaters' Society.

J. STERLING DEANS, JR.

J. Sterling Deans, Jr., was born in Phoenixville, Pa., on July 5, 1891, and died at the Presbyterian Hospital in New York City, of pneumonia, on August 18, 1924. Mr. Deans was educated at the Hill School in Pottstown and at Yale University, receiving his Ph. B. from Sheffield Scientific School in 1912.

Following his graduation he was employed by the Pennsylvania Railroad, first in the construction department and later in the maintenance of way department. From June, 1916, to October, 1917, Mr. Deans was with the Schoellkopf Aniline & Chemical Co. in Buffalo as superintendent of the Lee Street plant. During the war he was in the United States Army, serving both at home and abroad in the Ordnance Department, first as lieutenant and then as captain. Following his discharge, he became superintendent for J. H. Byers & Son, Inc., contractors in Buffalo, which position he held at the time of his death.

He had been a member of the Society since 1919, and was also a member of the American Society of Civil Engineers and of the Saturn Club of Buffalo.

WILLIAM H. DEMING

William H. Deming was born in Ghent, N. Y., in March, 1843, and passed his boyhood there, receiving his education in the public schools. As a very young man he moved to New Haven and served an apprenticeship in the railroad shops in that city. During the Civil War he was third assistant engineer on the gunboat *Sagamore*. At the close of the war, he entered the employ of the Continental Iron Works in Brooklyn, where he served for fifty-four years as a mechanical engineer.

In October, 1923, Mr. Deming resigned and built himself a home in Waterbury, Conn., in which to spend the rest of his life. His death occurred there on September 28, 1924.

He became a member of the Society in 1897. He was a member of the Sons of the Revolution of New York State, the Society of Naval Architects and Marine Engineers, and the Masonic fraternity.

JOHN H. DIALOGUE

John H. Dialogue was born in 1863 in Camden County, New Jersey, and died on April 19, 1924. Mr. Dialogue's specialty was shipbuilding and marine engineering and he was particularly successful in building tow boats. He was employed as an engineer for the Luckenbach Steamship Company in Brooklyn. He became a member of the Society in 1921 and was also a member of the Society of Naval Architects and Marine Engineers.

WALTER H. DICKERSON

Walter H. Dickerson, president of the Industrial Waste Products Corporation of New York City, and a member of the Society since 1916, died on June 29, 1924. Mr. Dickerson was born at Newark on Decem-

ber 8, 1874 and was graduated from Stevens Institute with the degree of M. E. in 1896.

His first position after graduation was with the Atlas Manufacturing Co. in Newark, where he was a designer of special woolen machinery. Later he had experience of various kinds with Charles E. Tripler, the Canadian Rand Drill Co., the Midvale Steel Co., the American Can Co., the International Steam Pump Co. and at the West Orange plant of Thomas A. Edison, where he investigated compressed-air reheating and coal-dust combustion.

From 1903 to 1906 he was engaged in special research work in the utilization of waste products; then he designed and became supervisor of the erection of the plant of the Muskegon Extract Co., where tanning extracts and other products from waste sulphite liquor were later manufactured under processes developed by him. From there he went to the Industrial Waste Products Corporation. Mr. Dickerson was a member of the American Chemical Society, the Technical Association of the Pulp and Paper Industry, the Leather Chemists' Association, the Society of Chemical Industry, the Electro-Chemical Society and the Chemists' Club.

GEORGE H. DIMAN

George H. Diman, who was chief engineer of the Washington Mills of the American Woolen Co., at Lawrence, Mass., until his retirement in 1919, died on August 20, 1924. Mr. Diman's life was a particularly rich one in his associations. He worked for and with George H. Corliss and came to know Dean Kendall and E. D. Leavitt, who was the originator of the butt-strap boiler joint. He was also associated with John T. Henthorn and A. M. Mattice, both well known of the older power engineers.

Mr. Diman was born on September 4, 1845, in Fall River, R. I., and received the usual public grammar-school education of that city. His first employment was on the railroads, but with his bent for engineering he soon found employment with the Union Mill of Fall River. He spent seven years in the mills of Rhode Island and then went to Massachusetts as engineer for ten mills more or less obsolete as far as power plants were concerned. His work there in bringing the plants to an up-to-date basis was particularly noteworthy.

It was in 1887 that Mr. Diman became connected with the Washington Mills in Lawrence, Mass., which later became part of the American Woolen Co.'s group. He remained with the company thirty-two years, then retired on a pension. During this long period of active service he was influential in the policies of the company and made for himself an enviable reputation.

Mr. Diman became a member of the Society in 1904. He belonged to the Engineers' Blue Room Club of Boston and was a member of the Masonic Order.

GEORGE E. DOELL

George E. Doell, a member of the Society since 1920, was born in Newark, N. J., on December 17, 1877, and died on June 14, 1924. Mr. Doell received his education in a private school in Newark and served an apprenticeship in the jewelry trade in that city. A little later he attended evening school at Pratt Institute in Brooklyn, completing a two-year course in mechanical drawing and machine design in 1909. He had been with the E. W. Bliss Co. in Brooklyn for seventeen years as assistant inspector. His work included the testing and inspecting of all machines manufactured by the company, also improving and correcting their designs. Mr. Doell was a member of the American Society of Mechanical Inspectors and of the Masonic fraternity.

CHARLES E. DOWNTON, SR.

Charles E. Downton, Sr., for nearly thirty years connected with the Westinghouse Electric and Manufacturing Co. at East Pittsburgh, died suddenly on September 28, 1924. Mr. Downton was born at Covington, Ky., in 1868, and was graduated from Purdue University with a B.S. in mechanical engineering in 1891.

After several years' experience as a machinist, wireman, etc., Mr. Downton entered the employ of the Westinghouse Co. as a student apprentice. In 1896, he was placed in charge of the dynamo-testing department and in 1902 he became identified with the educational department, in charge of training courses for graduate students. When the Westinghouse Co. entered into the manufacture of war munitions, Mr. Downton was transferred to the New England Westinghouse Co. at Springfield, Mass., where he remained until the close of the war. Later he returned to the main plant of the company at East Pittsburgh, where he was identified with large electrification projects. He had been a member of the Society since 1918.

HAROLD MALCOLM DUNCAN

Harold Malcolm Duncan, managing director of the Lanston Monotype Corporation, Ltd., London, England, died on October 8, 1924. Mr. Duncan was born on October 29, 1864, in Philadelphia, Pa. He attended high school and then made special study of sciences, languages and philosophy. From 1885 to 1895 he studied journalism, editing publications devoted to the technology of printing and paper making and paying special attention to type-composing and casting machinery.

In 1896, he became connected with the Lanston Monotype Corporation in Washington, D. C. The following year, when the sale of the British rights was concluded with an English company, Mr. Duncan became technical adviser to the Board of Directors of that concern. In 1899 he was appointed managing director.

Mr. Duncan became a member of the Society in 1917. He belonged to the American Club in London and to the Caledonian Club.

ROBERT M. DYER

Robert M. Dyer who, until his resignation in 1921 was vice-president and treasurer of the Puget Sound Bridge & Dredging Co., Seattle, Wash., died on January 13, 1924. Mr. Dyer was born in 1867 in Maquoketa, Iowa, where he received his early education. Later he attended Iowa State College, from which he was graduated in 1891.

Following his graduation he entered the employ of the McCormick Harvester Co. in Chicago. From 1902 to 1905 he was connected with the Aeromotor Co., also of Chicago. At the end of that time he transferred to the coast to become the vice-president and treasurer of the Puget Sound Bridge & Dredging Co. He remained with this firm until 1921, when he retired from active business.

During his residence in Seattle, Mr. Dyer was in charge of many of the largest engineering and construction projects completed in and around the city. He engaged actively in superintending the dredging of the Government ship canal and the building of the locks at Ballard. During the war he was in charge of the shipyards operated in Seattle by his firm. He also superintended the building of several dams in the Northwest and was in charge of much of the work on the British Columbia Railroad.

Mr. Dyer became a junior member of the Society in 1892 and a member in 1904. He was a charter member of the Engineers' Club of Seattle and belonged also to a number of other local clubs.

OSCAR ELSAS

Oscar Elsas, president of the Fulton Bag and Cotton Mills, and past-chairman of the Atlanta Local Section of the Society, died in Boston on September 19, 1924, after an acute illness of four days. Mr. Elsas was born in Atlanta in September, 1871, and entered the Georgia School of Technology as a member of its first graduating class.

He left school after completing his Junior year, to enter the employ of the Fulton Bag and Cotton Mills, where he worked in all capacities. He worked in the machine and carpenter shops, designed and invented new machinery, supervised the planning and installation of machinery. supervised the erection of new buildings, and eventually took complete charge of the Atlanta plant and supervisory control of plants in New York, St. Louis, New Orleans and Dallas.

Throughout his life Mr. Elsas was a resident of Atlanta and took an active part in industrial and civic life there. He was a member of the New England Cotton Manufacturers' Association and of all textile manufacturing associations in the South. He was president of the Ingleside Country Club and a member of the Kernwood Country Club of Boston. He became a member of the Society in 1914.

ALBERT W. ERDMAN

Albert W. Erdman, manager of the small-tool department of Pratt & Whitney, Hartford, Conn., died on April 24, 1924. Mr. Erdman was born in May, 1871, in Morristown, N. J. He was graduated from Stevens Institute of Technology in 1891 with the degree of M.E. and immediately entered the employ of the American Telephone & Telegraph Co., in the engineering department of the New York office.

Seven years later he became connected with the Randolph Clowes Co., Waterbury, Conn., as mechanical engineer and general superintendent in the manufacture of sheet brass and copper tubing. Mr. Erdman resigned from this position to design and superintend the manufacture of the depression position finder for seacoast artillery and two years later became assistant mechanical superintendent in the General Electric Co., Schenectady, N. Y., and afterward engineer in charge of ordnance manufacture.

During the war he served first as major and later as lieutenant-colonel in the Inspection Division of the Ordnance Department of the Army. Upon receiving his honorable discharge, Mr. Erdman became associated with the Pratt & Whitney Co.

Mr. Erdman became a member of the Society in 1919.

EDWARD FARRAR

Edward Farrar, who, until his retirement in 1920, was consulting mechanical engineer with the General Mining and Finance Corporation, Johannesburg, Transvaal, South Africa, died on March 26, 1924. Mr. Farrar was born in March 17, 1854, in Elland, Yorkshire, England. He spent four years at the Mechanics' Institute, Huddersfield, Yorkshire, and then worked with various firms in England until 1890, when he went to South Africa to engage in general engineering in Durban, Natal.

Later he became mechanical engineer of the Denny Dalton Gold Mines, Transvaal, and then chief draftsman of the Consolidated Investment Co., Johannesburg. He was for a time mechanical engineer of the Klerksdorp Proprietary Mines, Transvaal, and then became associated with the General Mining and Financing Corporation as consulting mechanical engineer. Together with his colleague, J. N. Bulkley, he designed and superintended the battery and works of the Roodefoot

United, which is today a model of workmanship and construction in its line.

Mr. Farrar became a member of the Society in 1907. He was also a member and a past-president of the South African Institute of Mechanical Engineers. He was very much interested in civic affairs and was appointed by the Government to the Rent Board.

EDWIN FITTS

Edwin Fitts, engineer with the Detroit Stoker Co., Detroit, Mich., died on January 5, 1924. Mr. Fitts was born on August 12, 1867, in Moravia, N. Y. He was graduated from Cornell University in 1891 with the degree of M.E.

After some preliminary experience he entered the employ of the National Tube Boiler Co., New Brunswick, N. J., on construction work, where he remained until 1899, when he became connected with the Murphy Iron Works, Detroit, in construction, selling and engineering work. In 1909 Mr. Fitts accepted the position of engineer with the Detroit Stoker Co., which he held at the time of his death.

Mr. Fitts became a member of the Society in 1915.

JAMES CALVIN FITZSIMMONS

James Calvin Fitzsimmons, general sales manager of the Standard Oil Co. (California), died on January 11, 1924. Mr. Fitzsimmons was born in March, 1867, in Richford, Vt. Until 1892, he was engaged in marine engineering, but at that time he joined the Standard Oil Co. as a salesman, traveling from San Francisco. He progressed steadily upward through the company until he became general sales manager in 1919. He was to have been elected to the Board of Directors.

Mr. Fitzsimmons was essentially a pioneer. He was intensely interested in the creation of new products and new uses for existing products. He was the first to introduce the use of oil as fuel on the Pacific Coast, and played a prominent part in the development of the use of asphalt.

Mr. Fitzsimmons became a member of the Society in 1913. He belonged also to the American Society for Testing Materials, the American Petroleum Institute and the San Francisco Chamber of Commerce. He was a member of the Masonic Order and a Knight Templar.

FREDERICK C. FLADD

Frederick C. Fladd, who was associated with the E. W. Bliss Co., Brooklyn, N. Y., for over thirty-three years as traveling and consulting engineer, died on January 25, 1924. Mr. Fladd was born on December 7, 1855, in Glastonbury, Conn., where he received his early education. He served his apprenticeship as machinist with the Stiles & Parker Press Co., Middleton, Conn., then worked steadily upward through the various positions in the plant, finally becoming superintendent. In 1890, when that firm was merged with the E. W. Bliss Co., Mr. Fladd joined the latter organization as traveling and consulting engineer, a position which he held until August of 1923.

Mr. Fladd became a member of the Society in 1887. He was specially interested in music and was a member of the New York Flute Club.

MICHAEL FOGARTY

Michael Fogarty, president of Michael Fogarty, Inc., boiler makers in New York City, died in October, 1924. Mr. Fogarty was born in March, 1855, in Fall River, Mass., where he attended the public schools.

From 1868 to 1874 he worked as a rivet heater in the boiler-making department of the Fall River Line repair shops at Newport, R. I. For the next three years he was employed as boiler maker with the Taunton Locomotive Works, Taunton, Mass., and the Providence Steam Engine Company's Works, Providence, R. I. From 1877 to 1898 Mr. Fogarty was associated for varying periods with the following concerns: Delamater Iron Works, New York City, in charge of plating; the Albany Street Iron Works, New York City, as assistant foreman boiler maker; West Point Foundry, Cold-Spring-on-the-Hudson, N. Y., as superintendent of boiler shops; Samuel Booth Steam Boiler Works, New York City, as estimator; C. Cunningham & Son, Brooklyn, N. Y., as superintendent of boiler and tank works. Since 1898 Mr. Fogarty had been engaged in his own business of the manufacture of boilers.

Mr. Fogarty became a member of the Society in 1915. He belonged also to the National Metal Trades Association, the American Boiler Manufacturers' Association, the Boiler Manufacturers' Association of New York City, the Institute of Steam Boiler Inspectors, and to the Engineers' Club of New York.

WILLIAM J. FRANCKE

William J. Francke, vice-president of the Francke Co., New Brunswick, N. J., died on February 28, 1924. Mr. Francke was born on May 12, 1859, in Kentucky, but at an early age his parents moved to Waco, Tex., where he attended school. Later the family moved to New Orleans and shortly afterward Mr. Francke set out for California and settled in Santa Clara, where he took an active part in the early development of the fruit-raising industry.

For nine years prior to the Spanish-American War, Mr. Francke was engaged in engineering enterprises in Jamaica, B.W.I., where he developed for the British Government the first ice-manufacturing plant and the first electric light and power station. In 1899, he returned to the United States and business interests took him to New Brunswick, where he organized the Francke Co. for the manufacture of the Francke flexible coupling. At the time of his death he was vice-president, chief engineer and general manager of the firm.

Mr. Francke became a member of the Society in 1920. He belonged also to the American Society for Testing Materials and to a number of clubs and organizations in New Brunswick.

FRANK BUNKER GILBRETH

In the sudden death of Major Frank Bunker Gilbreth on June 14, 1924, The American Society of Mechanical Engineers sustained the loss of one of its most versatile and interesting members. With his wide experience in many lines of industry, his keenly analytical mind, his ready wit and wealth of anecdotes, Major Gilbreth made the Spring and Annual Meetings of the Society, which he seldom missed, events to be remembered by all who came in contact with him. His earnestness in debate in the professional sessions was only equaled by his entertaining reminiscences and love of fun in the more informal gatherings outside of those sessions. His passing left a gap that will be difficult to fill.

A member of the Society since 1903, Major Gilbreth had always been active in its affairs. His progressivism was labeled dangerous radicalism by some of the older members, but it is worthy of note that things that were radical when he first advocated them are commonplace today in the Society's activities. A single instance may be cited — his work to have management recognized as a major division of engineering.

Major Gilbreth was profoundly impressed with the principles enunciated by Taylor in his paper on Shop Management in 1903 before the Society. He saw almost instantly that these principles were destined to effect a revolution in industry. He predicted that Taylor would go down in history as one of the world's greatest engineers, and this at a time when a large percentage of his contemporaries regarded Taylor as a dangerous lunatic. Gilbreth joined the small but determined minority fighting to establish those principles as part of the permanent scheme of industry — even to the extent of making them the foundation stones of his own business. When the opponents of Taylor objected to further discussion of his work in the Society, Gilbreth labored long and earnestly to have the decision reversed. When apparently finally blocked he took a radical step, the formation of a society whose chief work would be the promulgation of the principles of management that had by that time become almost a religion to him. The Society to Promote the Science of Management (now the Taylor Society) was conceived and organized by him to do the work that Gilbreth thought would not be done by the A.S.M.E. Yet so hopeful was he that the latter would come to his way of thinking, that he proposed that no one not eligible to membership in the A.S.M.E. could be eligible to the new society, so that at some future time a merger could be effected if desired. In this move he rallied around him such prominent men as Henry L. Gantt, James M. Dodge, and William Kent, whose loyalty to the parent organization could not be questioned.

It was but a few years later that the A.S.M.E. formally recognized management and organized the Management Division. Major Gilbreth accepted membership on the executive committee of that division and labored as hard to make it an outstanding success as he had to bring the Society to his way of thinking ten years previous.

A great achievement to be credited to Major Gilbreth was the institution of the first International Congress on Management, held in Prague, Czechoslovakia, July 20-24, 1924, in which the A.S.M.E. had a prominent part. Much of his work had been in Europe, and since the war he had made several trips to Czechoslovakia. Impressed with the progressiveness of the Czechs, Major Gilbreth suggested to the executive committee of the Management Division the desirability of a meeting in Czechoslovakia to discuss management. He was authorized to confer informally with the Masaryk Academy in Prague, with the result that early in the year 1924 an invitation was extended through the American Engineering Council to the engineers of the United States to participate in the first international congress of its kind. Major Gilbreth was appointed a member of the Committee on American Participation, which was charged with formulating the program and securing speakers. The success of the Congress was due in no small measure to his untiring work on this committee. It and the Taylor Society will stand as monuments to his vision.

Frank Bunker Gilbreth was born on July 7, 1868, in Fairfield, Me. He became an apprentice in the building trades, supplementing his work with special courses at the Massachusetts Institute of Technology. Going into business for himself as a contractor and construction engineer, he became nationally known as a builder of big work. Several towns and industrial cities were constructed by him. At this time he began his classic work on Motion Study, and devised many methods of shortening the time of doing work by eliminating useless motions. One instance is the showing of the possibility of reducing the motions of a bricklayer from 18 to 5. It was his motion studies that first brought him in contact with Taylor, with the eventful result that he abandoned the construction field to devote all his time to management, both as a consultant and a research worker.

Major Gilbreth was probably the first to recognize the value of the motion-picture camera as an aid to industry. Conceived first as merely a more accurate method of time study, it soon became evident that as a method of research for the development of fundamentally correct methods in industry it was of the greatest possible value. Major Gilbreth at the time of his death had developed its use for the determination of the "one best way" to such a degree that the accuracy of his predictions of results was almost uncanny. It was as a Major of Engineers during the war that he developed, by means of his motion-picture-camera researches, methods of training soldiers in the operation of machine guns and similar mechanisms that greatly cut down the time necessary to put men in the field. After the war the same methods were used by him in the rehabilitation of crippled soldiers, and later for the rehabilitation of the industrially crippled.

Major Gilbreth also had done much research in the elimination of fatigue in industry, and had developed many useful inventions for this purpose. In much of his work he was assisted by his wife, Lillian Moller Gilbreth, to whom he invariably gave credit for her part. In fact, of late years all papers and results of researches have appeared as their joint effort.

In June, 1920, Major Gilbreth was awarded the degree of LL. D. by the University of Maine, for his work on waste elimination, motion study, and management, and researches in the study of fatigue. In addition to his connection with The American Society of Mechanical Engineers, he was a member of the Providence Engineering Society, the Eastern New York Engineering Society, American Academy of Political and Social Science, and an honorary member of the Society for the Promotion of Occupational Therapy.

Major Gilbreth was the author of numerous works on motion study, waste elimination, and industrial fatigue, among them being: Field System, 1908; Concrete System, 1908; Bricklaying System, 1909; Motion Study, 1911; and Primer of Scientific Management, 1911. As co-author with Mrs. Gilbreth he wrote: Time Study; Fatigue Study, 1916; Applied Motion Study, 1917; and Motion Study for the Handicapped, 1919. He also contributed many papers to the societies of which he was a member and to the technical press.

AUGUSTUS WILLIAM H. GRIEPE

Augustus William Hermann Griepe, superintendent of design of the Electric Bond & Share Co., New York, N. Y., died on April 7, 1924. Mr. Griepe was born on July 13, 1876, in Berlin, Germany. He served his apprenticeship with F. Schwabanthan & Co., engineering contractors of Berlin, and then entered the University of Charlottenburg in 1893. Upon being graduated in 1897, he entered the employ of G. Skrzivan & Co., Riga, Russia, as estimator and designer.

In 1899, he became connected with the German Garvin Machine Co. and after serving in various capacities became chief engineer of the company. He was transferred to the New York office of this concern, where he became designing engineer and estimator. Following work with several firms as erecting engineer on industrial plants, pumping and power stations, he joined the staff of the New York Edison Co. in 1906, and later engaged in experimental work for Thomas E. Murray.

In 1913, Mr. Griepe formed the Turbo Co., of which he became treasurer and manager, for the manufacture and development of a gas turbine which he had invented. His next association was with the Electric Bond & Share Co. as gas expert and chief of mechanical di-

vision, which position he held until 1912, when he was appointed superintendent of design.

Mr. Griepe became a member of the Society in 1908. He was a pioneer member of the American Aeronautical Society and was a licensed professional engineer in New York State. He had thirteen patents to his credit and was a frequent contributor to the technical press.

GUSTAV HAGLUND

Gustav Haglund was born in Brooklyn, N. Y., on June 20, 1881, and died on July 24, 1924. He was educated in the public schools of Brooklyn and had two years' training in steam and machine design at Pratt Institute. He served as cadet on a mail steamship line and secured his license as a marine engineer in England. From 1905 to 1907, Mr. Haglund was in Mexico engaged in general designing for the Guanajuato Light and Power Co. and the San Carlos Copper Co. He returned to the United States and after several years with the American Brass Co. and the Brooklyn Union Gas Co., he secured a position with the Public Service Corporation of New Jersey and was soon put in charge of the construction of their plant at Burlington. In 1917 he became associated with the Federal Shipbuilding Co. of the Emergency Fleet Corporation. At the time of his death he had returned to the Public Service Corporation of New Jersey as assistant to the chief piping designer. Mr. Haglund became a member of the Society in 1909. He belonged to the Masonic Order.

BURTON P. HALL

Burton P. Hall, assistant treasurer and manager of the Mechanical Equipment Co., New York City, died on June 22, 1924. Mr. Hall was born on April 19, 1867, in New York City. He was graduated from Stevens Institute of Technology in 1888 with the degree of M.E. and then entered the employ of the United Gas Improvement Co. of Philadelphia, as assistant engineer, where his work dealt with the adaptation of the Welsbach light to natural gas.

From 1890 to 1909, Mr. Hall was connected with the New York Steam Fitting Co., as supervising engineer, from which position he resigned to become assistant treasurer and manager of the Mechanical Equipment Co. He served as consulting engineer for the heating and power equipment of the New York Custom House, the Macy Department Store, the Whitehall Building, United States Naval Academy, the Equitable Building, and many others. He had a number of patents to his credit.

Mr. Hall became a member of the Society in 1922. He was a member of the Theta Xi and Tau Beta Pi fraternities and had served twice as mayor of Fanwood, N. J.

EDWIN A. HALL

Edwin A. Hall, assistant manager of the Standard Stoker Co., New York City, died on January 20, 1924. Mr. Hall was born on October 1, 1881, in Elmira, N. Y. He attended Hotchkiss Preparatory School and later Sheffield Scientific School at Yale University and was graduated in 1904 with the degree of Ph. B.

After preliminary service with the Dansville Merchants & Farmers Bank and the Telephone Co. at Cleveland, Ohio, he entered the employ of the Power Specialty Co., Dansville, N. Y. From 1912 to 1917, he served as assistant superintendent and then became superintendent of the plant. In 1918, he was appointed New England sales manager of

the company with headquarters in Boston, where he remained for three years, resigning to become assistant manager of the Standard Stoker Co.

Mr. Hall became an associate member of the Society in 1919.

JOHN L. HALYBURTON

John L. Halyburton, who died on April 18, 1924, was born in Philadelphia on November 20, 1863. He received his education in the public schools of that city and at the Hill School at Pottstown. At the age of seventeen he entered the employ of the Neafie and Levy Ship and Engine Building Co., where he served his apprenticeship and gained his first shop experience. Later he went to the Atlantic Refining Co. of Philadelphia, eventually becoming superintendent of construction in full charge of all mechanical engineering and civil engineering work such as the designing and erection of boilers, engine and pump houses, pipe lines, etc.

Several years later he designed the first portable electric welders ever built, for the Johnson Steel Rail Co. of Johnstown, Pa. When the New York Shipbuilding Co. was started in Camden, Mr. Halyburton had full charge of the installation of all equipment and acted as chief engineer for two years. He then went into the valve business. He invented the Halyburton Gate Valve, largely used in oil-refining industries and manufactured by the American Car & Foundry Co. in Berwick, Pa. He was engineer and sales manager of the valve department of that company for twelve years.

Mr. Halyburton had been a member of the Society since 1921. He was also a member of the Manufacturers' Club of Philadelphia and the Philadelphia Engineers' Club.

HARRY EARL HAMILTON

Harry Earl Hamilton, a junior member of the Society, died in November, 1923, at Granite City, Ill. He was born at Portland, Ore., May, 1893, and was graduated from the Oregon Agricultural College in 1916. His first position, during the summer of 1916, was with the Portland Central Heating Co., as steam fitter and stationary engineer. From October, 1916, until June, 1917, he was engaged in the inspection and laying out of toll leads for the Pacific States Telephone & Telegraph Co. at Portland, then went to Youngstown, Ohio, to a position with the Youngstown Sheet & Tube Co. In 1918, he was efficiency foreman of the coke plant for that company.

JOHN WILSON HAMILTON

John W. Hamilton, president of the Hamilton & Chambers Co., Inc., New York City, died on April 16, 1924. Mr. Hamilton was born on April 16, 1870, in Scotland. He came to this country when he was about fourteen years of age and after working with various concerns determined upon a technical education. He attended Cooper Union, where he received the degree of B.S. in 1898 and C.E. in 1900.

About 1899, Mr. Hamilton became contracting manager for the structural steel business of Milliken Brothers. On the failure of that concern he organized, in 1907, with the late H. J. Chambers, the structural steel firm of Hamilton & Chambers, New York City, specializing in exporting fabricated steel. The business was incorporated in 1916.

Mr. Hamilton became a member of the Society in 1914. He belonged also to the American Society of Civil Engineers, the American Iron and Steel Institute, the Brooklyn Engineers' Club, the Engineers' Club of New York, the American Asiatic Association and the National Geographic Society.

JOHN LYELL HARPER

John Lyell Harper, chief engineer and vice-president of the Niagara Falls Power Co., died on the morning of November 28, 1924, at the Memorial Hospital, Niagara Falls, after undergoing an operation for appendicitis four days earlier. Mr. Harper's work in the development of Niagara Falls power made him a national figure in hydroelectric engineering. He was associated with the Niagara Falls Power Co. and its predecessor, the Hydraulic Power Co., for twenty-two years, starting as assistant engineer. Under his leadership the plant grew from one of 14,000 hp. to one that today contains nearly one-half million horsepower under one roof, the largest installed capacity in any power plant in the world.

Mr. Harper was born in Harpersfield, N. Y., on September 21, 1873, and his boyhood was typical of the country lad on the farm. At twenty he was graduated from Stamford Seminary, and four years later from Cornell University with the degree of mechanical engineer. Upon his graduation he went west as a draftsman with the Oregon Improvement Co., Seattle, Wash. Four months of that work brought him a place with the Union Electric Co. of Seattle in the electrical field. In June, 1898, he was made operating and construction engineer of the Twin City Rapid Transit Co., and this brought him in contact with the work which was to make him a national figure in engineering.

In 1902, Mr. Harper became associated with the Niagara Falls Hydraulic Power & Mfg. Co., as assistant to the engineer, Wallace C. Johnson. From that time until his death his life is the romantic story of Niagara power. After two years he became chief engineer of the company. During the year 1918 the various power interests were grouped under Government direction into a new corporation taking the name of the Niagara Falls Power Co., and Mr. Harper was made its chief engineer. Upon the completion of the wartime power plant he was appointed vice-president of the company. The most famous power development in the world from the standpoint not only of engineering but of service is situated in the gorge below the falls of Niagara. It stands as a monument to the genius of John Lyell Harper.

Mr. Harper was also vice-president and chief engineer of the Harper-Taylor Co., consulting engineers. In spite of the many responsibilities with which Mr. Harper was burdened, he found time to carry on scientific investigations of the applications of electric service in the electrochemical and the electrometallurgical industries at Niagara Falls and developed and patented several electric furnaces.

He became a member of the Society in 1906. He belonged also to the American Society of Civil Engineers, the American Institute of Electrical Engineers, the American Electromechanical Society, the American Ceramic Society, the Cornell Society of Engineers, and the Engineers' Club of New York City.

GLENN B. HARRIS

Glenn B. Harris, former associate editor of *American Machinist,* died in Yonkers, N. Y., on September 29, 1924. Mr. Harris was born at Lockport, N. Y., in 1864. He was graduated from the high school in Washington, D. C., and spent a year at George Washington University. After a period devoted to the patent business, Mr. Harris was connected with the American Pneumatic Tool Co. for seven years designing and supervising the construction of machines for scaling armor plate. He then organized the Pneumatic Tool and Machinery Co., and later, the National Pneumatic Tool Co. and the Harris Pneumatic Tool Co. All of these concerns manufactured products designed by Mr. Harris, who was one of the pioneers in the pneumatic-hammer line.

During the war, he was gage engineer in the Inspection Division of the Ordnance Department of the Army in Washington.

Mr. Harris was a frequent contributor to the technical press. He had been a member of the Society since 1918.

EDWIN C. HENN

Edwin C. Henn, one of the founders of the National Acme Co., was killed on August 20, 1924, when his automobile was struck by a train at a crossing near Painesville, Ohio. His close associate, Oscar Smith, was killed in the same accident.

Edwin C. Henn was born at New Britain, Conn., on June 5, 1863, and attended the common schools of that city. The grammar school was next door to a brass foundry where his father, who was a skilful locksmith and general mechanic, had a contract on the finishing of bib cocks and barrel cocks. When work was pressing, Edwin worked in the shop until six o'clock. This work continued after school and during vacations for about five years, and by the time the boy finished his schooling, he was an experienced brass finisher.

He was employed for a short time by the Russel & Irwin Co. at this trade, but, desiring broader experience, he left New Britain at the age of eighteen, and worked at his trade in various cities in Ohio and Kentucky.

In 1883 he returned to New Britain and entered the Pratt & Cady Co. At the age of twenty-one he became a contractor of brass valves in their shop in Hartford. After about a year he left Pratt & Cady's, and organized the Standard Manufacturing Co. in partnership with several other men.

Shortly after this, the late Christopher M. Spencer of Civil War repeating-rifle fame, came to them with the designs of a new coil-feed automatic screw machine which he had invented. He gave the new concern a contract for twenty-five of these "Jumping Jacks," as they were nicknamed, and so began Mr. Henn's interest in what was to be his life's work.

About 1890, in conjunction with Reinhold Hakewessell, he invented the first successful multiple-spindle automatic, which has since become known all over the world as the "Acme Multiple." After a series of disappointments with their invention, the Standard Manufacturing Co. failed and Mr. Henn then went to work as a demonstrator for the Pratt & Whitney Co., but he and Mr. Hakewessell continued to work in the evenings. With his salary, and the scrap parts donated by Pratt & Whitney, the work was continued, and the machine perfected.

E. C. Henn finally succeeded in interesting his brother, A. W. Henn of Cleveland, in the invention and a concern was incorporated in Hartford as the Acme Machine Screw Co. A large number of Acmes were built and in 1898, W. D. B. Alexander, president of the National Screw and Tack Co. of Cleveland, was influenced to set up twenty of them at Cleveland in the manufacture of standard screw products. These were installed in a subsidiary plant at Cleveland, called the National Manufacturing Co., and this grew into The National Acme Manufacturing Co. when the Acme Machine Screw Co. was combined with it in September, 1901. In 1902, the machinery-manufacturing plant was removed from Hartford to Cleveland, and Mr. Henn became vice-president and general superintendent of the new industry, which became one of the largest in Cleveland within a few years.

He had been a member of the Society since 1913, was a member of the Cleveland Engineering Society, the Union Club, the Cleveland Athletic Club, and the Shaker Heights Club.

WALTER A. HENSON

Walter A. Henson, vice-president of the Osage Cotton Oil Co., Chattanooga, Tenn., died on January 30, 1924. Mr. Henson was born on November 30, 1878, in Louden, Tenn. He was graduated with honors from Alabama Polytechnic Institute.

His first venture in business was with the G. N. Henson Cotton Mills, an organization that controlled many cotton-seed oil mills throughout the South and Middle West. In 1902, he became associated with the American Cotton Co. as assistant operating manager. Six years later he became identified with his father's oil interests, which were reorganized into the Osage Cotton Oil Co., the second largest oil company operating in the United States. He was also president of the J. C. Francesconi Co., exporters of cotton-seed oil in New York City.

Mr. Henson became an associate of the Society in 1919. He was prominent in the civic life of Chattanooga and belonged to many of the well-known social clubs and commercial organizations of that city.

ANTHONY S. HILL

Anthony S. Hill died of apoplexy on June 24, 1924. Mr. Hill was born in Renovo, Pa., on November 20, 1868, but went to Kalamazoo, Mich., when a small boy, where he received his education.

Until 1912, he was associated with the firm of William E. Hill & Co. of Kalamazoo, manufacturers of steam specialties and saw-mill machinery, and also small steam engines. On the death of his father in 1897 he was made general manager of this firm. In this position, which he held until 1912, he directed all engineering work in addition to the general management, and brought out two patents, one on steam drag saws and one on steam dogs, both practical labor-saving machines.

From 1912 to 1915, Mr. Hill was works manager for the Allen Engineering Co. at Memphis, Tenn., where he designed a complete line of improved saw-mill machinery. He resigned from this position to become associated with the American Saw-Mill Machinery Co., soon becoming sales engineer and designer of special and heavy-duty saw-mill machinery with headquarters in New York. The last six months of his life were spent in Canton, Ohio, where he was sales engineer and manager for the Knight Manufacturing Co. During the war, Mr. Hill served as captain in the Ordnance Division of the Army.

He became an associate of the Society in 1917.

RUSSELL WALKER HIRST

Russell Walker Hirst, manager of the Boston office of the Buffalo Forge Co., died on August 10, 1924. Mr. Hirst was born on June 4, 1895, in New Bedford, Mass. He received his early education in New Bedford and later attended Worcester Polytechnic Institute, where he received the degree of B. S. in mechanical engineering in 1919.

During the war Mr. Hirst served in the United States Army as a sergeant in the Aero Squadron that was stationed in England. After receiving his honorable discharge from the Army, he entered the employ of the Buffalo Forge Co. in Buffalo, N. Y. In January, 1921, he was appointed manager of the Boston office of that concern as well as of the Carrier Air Conditioning Co., and the Buffalo Steam Pump Co.

Mr. Hirst became a junior member of the Society in 1921. He was a member of the Sigma Xi and Tau Beta Pi honorary fraternities.

HENRY J. HORSTMANN

Henry J. Horstmann, president of the F. W. Horstmann Co., Newark, N. J., died on March 20, 1924. Mr. Horstmann was born on July 18, 1866, in Newark. He attended the Newark Technical School.

From 1882 to 1886, he served his apprenticeship in the iron works of the firms of Hewes & Phillips, Newark. He continued with this firm and in 1889 was placed in charge of the drafting room. Later, he went to the Corliss Engine Works in Providence, R. I.; then, for a short time, was with the Rome St. Railroad in Rome, N. Y. In 1906, he became mechanical engineer and superintendent of the Bass Foundry & Machine Co., Fort Wayne, Ind., where he remained for about four years, until he accepted the position of mechanical engineer with the F. W. Horstmann Co., Newark.

Mr. Horstmann became a member of the Society in 1894. He belonged to the Masonic Order and was a Knight Templar.

JOHN T. HORTON

John T. Horton died on May 25, 1924, in New York City. Mr. Norton was born in 1863 in Halifax, Nova Scotia, Canada, where he was educated.

He served a five-year apprenticeship with the firm of Monroe & Wilson and then for several months gained shop experience with McDonald & Co., both firms in Halifax. From 1882 to 1884, he was in charge of repairs of a pulp mill for the Nova Scotia Wood, Pulp & Paper Co., resigning from that concern to become connected with the Continental Iron Works, Brooklyn, N. Y. In 1887, together with I. S. Hughes, he established a business in Lowville, N. Y., operating a machine shop and foundry. Later, this developed into the Lowville Iron Works, of which Mr. Horton became vice-president. In 1894, he entered the consulting field in New York City as a specialist in equipment for hoisting and handling material.

Mr. Horton became a member of the Society in 1895.

HOWARD R. HUGHES

Howard R. Hughes, president of the Hughes Tool Co., Houston, Tex., died on January 14, 1924. Mr. Hughes was born on September 9, 1869, in Lancaster, Mo. He received his education in the local schools of Iowa followed by a short course at Harvard University, class of 1897.

From 1908 to 1913, Mr. Hughes was manager of the firm of Sharp & Hughes, then for two years was president and manager of the Sharp-Hughes Tool Co. From 1915 to the time of his death, Mr. Hughes was president of the Hughes Tool Co. He was distinguished as an engineer principally because of his invention and manufacture of the rotary rock-drill bit for drilling hard rock. This bit, invented in 1909, made possible the drilling of deep oil wells and the recovery of immense quantities of oil previously inaccessible.

Mr. Hughes became an associate of the Society in 1917. He was a member of the American Institute of Mining and Metallurgical Engineers, the American Petroleum Institute, a life member of the Harvard Club of New York City, and belonged to many other social and fraternal organizations.

LESLIE A. IRVIN

Leslie A. Irvin, assistant plant manager and chief engineer of the Celite Co., Lompoc, Cal., died on September 4, 1924. Mr. Irvin was born on October 22, 1882, in Bonne Terre, Mo. He attended Washing-

ton University in St. Louis. From 1900 to 1904 he served as architectural draftsman for J. S. White and in the same capacity with Swift & Co., in East St. Louis, where in 1904 he was appointed combustion engineer. In 1905 he became connected with the Baker Iron Works, Los Angeles, Cal., as draftsman. He worked up through various positions with that concern to that of chief mechanical engineer. He resigned in 1923 to become assistant plant manager and chief engineer of the Celite Co. Mr. Irvin became a member of the Society in 1920.

VICTOR JANN

Victor Jann, district turbine engineer of the Cincinnati engineering department of the General Electric Co., died on July 2, 1924, at Schenectady, N. Y. Mr. Jann was born at Hoboken on May 29, 1873. He was educated in New York City.

In 1896, he entered the employ of the General Electric Co. at Cincinnati, spending six years in their manufacturing department. In 1902, he went to Schenectady, where he took the student course in the testing department of the company. From 1905 to 1915, he was employed in the construction department of the company, where he was assistant to the superintendent of construction and steam-turbine inspector. Here he had general supervision of outside construction work in the installation of a Curtiss steam turbine. In 1915, Mr. Jann was transferred to the Cincinnati office of the company as district turbine engineer.

Mr. Jann was a Spanish-American War veteran, and a member of the Quarter Century Club of the General Electric Co. He became a member of the Society in 1916.

HAROLD COLBERT JONES

Harold Colbert Jones, president of the Mid-West Forging Co. of Chicago, died in that city on July 18, 1924. Mr. Jones was born in Chicago in 1878. He received his education at the Chicago Manual Training School and at Cornell University, where he was graduated in 1902 with a degree in mechanical engineering. One of his early connections was with the Link-Belt Co., where he served as a draftsman. He then became associated with the Inland Steel Co., first as night superintendent of the Indiana Harbor Plant and later in charge of the Chicago Heights Works. For several years he was identified with the company's coal properties, but returned to work in their steel plants. In July, 1923, he resigned as vice-president and director of the company to become president of the Mid-West Forging Co.

Mr. Jones was largely responsible for the development of the steel-fence-post business and was also a pioneer in developing steel sectional buildings. He joined the Society in 1903. He was also a member of the American Iron and Steel Institute.

ROBERT H. KARL

Robert H. Karl, Smoke Commissioner for St. Louis, Mo., died on December 6, 1924, from injuries received in an automobile accident. Mr. Karl was born in March, 1868. He took courses with the International Correspondence School in mechanical engineering and refrigeration. He obtained his shop experience with the Columbus Machine Co., Columbus, Ohio, and then for three years was engaged in operating stationary steam plants. For four years he served as operating engineer in ice and refrigerating plants and for two years as erecting engineer for ice plants.

Mr. Karl was for a number of years associated with the Anheuser Busch Brewing Association as assistant supervising engineer engaged in erecting, constructing, improving and designing work for ice and refrigerating plants.

Mr. Karl became a member of the Society in 1914.

HENRY F. KELLEMEN

Henry F. Kellemen, of Duluth, Minn., died on January 27, 1924. Mr. Kellemen was born on May 11, 1873, in Panama. He was educated in New York schools and attended later the Rhode Island Technical School in Providence.

He served his apprenticeship with the Corliss Steam Engine Works in Providence, and then entered the employ of the Brown & Sharpe Manufacturing Co., where he gained his drawing-room experience. For two years Mr. Kellemen was connected with the Pratt & Whitney Co., Hartford, Conn., as designer, and in 1901 became assistant superintendent of the Utica Drop Forge & Tool Co.; later he became superintendent and works manager, then vice-president and general manager. At the time of his death he held the position of manager of the metals group of the Marshall Wells Co.

Mr. Kellemen became a member of the Society in 1902.

J. W. KENDRICK

J. W. Kendrick, who was chairman of the Board of the Great Northern Railroad until his resignation in January, 1924, because of ill health, died on February 16, 1924. Mr. Kendrick was born on October 14, 1853, in Worcester, Mass. He was graduated from Worcester Polytechnic Institute in 1873 with the degree of B.S

Six years later his opportunity to begin railroad work came when he joined a party organized for locating a railway in the Yellowstone Valley of Montana — the Northern Pacific Ralway. During the next twenty-three years he was engaged in the construction and operation of that railway, resigning the position of vice-president in 1901 to accept a similar position with the Santa Fe System. For ten years he was prominently identified with the rehabilitation of that property, resigning in 1911 to engage in private practice as a consulting railway expert.

Mr. Kendrick's interest in the International-Great Northern began in 1917, was renewed in 1922, and in the autumn of that year he accepted the position of chairman of the board, which he held until a month before his death.

His career embraced a careful study of various mechanical questions that are included in the scientific railroad operation of today. He was one of the first railroad executives to experiment with the use of the locomotive superheater and feedwater heater.

Mr. Kendrick became a member of the Society in 1911. He belonged also to the American Society of Civil Engineers, the American Railway Engineering Association, the American Railway Guild, the Western Society of Engineers, the New England Society of New York and to a number of clubs both of New York and Chicago.

WALTER KENNEDY

Walter Kennedy, mechanical engineer of Pittsburgh, Pa., died on July 6, 1924. Mr. Kennedy was born in November, 1851, in Poland, Ohio. He attended the Poland Union Seminary.

He served his apprenticeship with the Struthers Furnace Co., Struthers, Ohio, worked in the chemical laboratory of the Carnegie

Steel Co., and later in the Isabella Furnace chemical laboratory. For three years, Mr. Kennedy was superintendent of the Lucy Furnace Co., Pittsburgh, and for one year of the Soho Furnace Co. From 1890 to 1892, he was associated with the Jefferson Iron Works, Steubenville, Ohio, as superintendent. For the last twenty-five years Mr. Kennedy has had his own office as mechanical engineer in Pittsburgh. He joined the Society in 1914.

DAN G. KNERR

Dan G. Knerr was born at Dayton, Ohio, in September, 1875, and died there on July 19, 1924. He was graduated from the Steel High School in Dayton and served a five-year apprenticeship with the Stillwell Bierce Co. In 1899-1900, he was assistant in the engineer's office of the International Harvester Co., resigning to enter the mechanical engineer's office of the Platt Iron Works. He then began to specialize on steel-car and automatic-scale design. For some years he was with the Foos Gas Engine Co., later with the Barney and Smith Car Co., and then with the Winters-Coleman Scale Co. During the war he worked for the Dayton Ohio Production Co., making shell forgings for the United States Government. Since then Mr. Knerr has been with the International Scale Co. as chief mechanical engineer and factory manager for one of their branches. At the time of his death he was particularly interested in developing better factory management and was planning to accomplish big things in that line.

Mr. Knerr was a member of the Dayton Engineers' Club, and had been a member of the Society since 1921.

ANDREW R. K. LAUDER

Andrew R. K. Lauder, mechanical engineer of the Bloomfield Works of the General Electric Co., Bloomfield, N. J., died on January 9, 1924. Mr. Lauder was born on May 16, 1873, in Schenectady, N. Y. He was a graduate of the Union Classical Institute. From 1889 to 1918, he was connected with the General Electric Co. in their Schenectady, N. Y., plant in the capacities of assistant foreman, foreman, section head in drafting room, and designing engineer on railway-signal and industrial apparatus.

In 1918, Mr. Lauder was transferred to the Bloomfield Works as representing engineer for the Schenectady Works on industrial drum controllers. Three years later he became mechanical engineer of the plant. Mr. Lauder became a member of the Society in 1922.

ARVID M. LEVIN

Arvid M. Levin was born in Sweden in 1860 and received his M.E. in 1882 from the Royal Institute at Stockholm. After four years in his profession in Sweden he came to this country and was employed by E. D. Leavitts in Cambridge. Following this he served from one to five years with the Cleveland Shipbuilding Co., E. P. Allis & Co., the Anaconda Copper Mining Co., Carnegie Steel Co., Rarig Engineering Co., Bates Machine Co. and the Minneapolis Steel and Machine Co. For five years he was established as a consulting engineer in Chicago, specializing in the erection of manufacturing plants and mechanical works.

In 1915, Mr. Levin returned to Sweden to become teacher of mechanics at the Royal Technical High School in Stockholm. At the time of his death he had an office as a consulting engineer in Stockholm and was busy with some inventions. He is the author of Modern Gas Engines and Gas Producers. Mr. Levin became a member of the Society in 1894.

JAMES R. LOWE

James R. Lowe, sales engineer with the Henry Vogt Co., Louisville, Ky., and consulting engineer of the same city, died on March 11, 1924, of heart disease. Mr. Lowe was born on June 28, 1867, in Springfield, Tenn. He was educated in the schools of Nashville and spent one year at Vanderbilt University.

For three years he worked in the shops of the Louisville & Nashville Railroad Co., then became connected with the J. I. Case Threshing Machine Co., Racine, Wis. From 1902 to 1905, he served in the 30th and 7th United States Infantry, then entered the employ of the Kelley Springfield Road Roller Co. From 1907 to 1910, Mr. Lowe was with various western firms in refrigerating and pumping works. He spent five years with the McClary Jamison Machinery Co., Birmingham, Ala., in the design and supervision of power plants and afterwards was associated with the Matthews Electric Co. of the same city.

During the war, Mr. Lowe rendered valuable service as mechanical engineer in the Ordnance Department of the Army. After the war, for a short period he acted as Louisiana district sales agent for the Mid-West Engine Co. of Indiana. In 1921, Mr. Lowe entered the consulting-engineering field, supervising the erection and installation of ice and large waterworks plants. Because of failing health, he gave up that work to accept a position near home, and he was in the employ of the Henry Vogt Machine Co. at the time of his death.

Mr. Lowe became an associate member of the Society in 1918. He belonged to the National Association of Stationary Engineers and was a Mason and an Elk.

CHARLES FREDERICK MacGILL

Charles Frederick MacGill was born at Keeseville, N. Y., on July 20, 1859. He died in the Massachusetts Homeopathic Hospital, Boston, on July 27, 1924. In his early boyhood, his family moved to Burlington, Vt., where, after a brief common-school education, Mr. MacGill served an apprenticeship at the machinist's trade in the shops of B. F. Woodbury.

After a number of years with various concerns, he went to Schenectady, in 1889, where he had charge of one machine shop of the Westinghouse Agricultural Implement Works, and thence to Geneva, where he was superintendent and later general manager of the Dunning Boiler Works.

He returned to Schenectady, to the General Electric Co., from which he went to Peterborough, Ont., as works manager for the Canadian General Electric Co.; thence to Quebec as works manager for the Carrier-Laine Co. In 1910, he reorganized the shops of the Twin City Railway Co. and rebuilt the Duluth Inclined Railway. Later he went to St. Louis, where he superintended the construction and equipment of the Busch-Sulzer Diesel Engine Co. plant.

During the war, Mr. MacGill was in charge of the bayonet department of the Remington Arms Co. at Bridgeport and works manager for the Bullard Engineering Works in the same city.

After his retirement, he lived at Cambridge, Mass. He had been a member of the Society since 1896 and a contributor to its publications as well as to other periodicals.

His interests lay in both the experimental and human sides of his profession. He worked out many inventions, few of which he patented, was proud of his record in never having a strike among his employees, and was an advocate of the Taylor Premium Plan of wage payment.

JOHN G. D. MACK

John Givan Davis Mack, chief state engineer of the State Department of Engineering of Wisconsin, died on February 24, 1924. Mr. Mack was born on September 5, 1867, in Terre Haute, Ind. In 1887 he was graduated from the mechanical-engineering course of Rose Polytechnic Institute. He took a post-graduate course at Cornell University and received the degree of M.E. in 1888.

For the next five years he was occupied in the design of special machinery for the Peter Cartridge Co., Cincinnati, Ohio; design of machine tools for the Universal Radial Drill Co., and the Smith Silk Co. of Cincinnati; the study of plant costs for the Kester Electric Co., Terre Haute, Ind.; the study to increase efficiency of the power plant at the Baxter Court Hotel in Nashville, Tenn.

In 1893, he was appointed instructor in machine design at the University of Wisconsin and six years later was made head of the department, which position he held until 1915. In 1903 Mr. Mack was placed in charge of the valuation of rolling stock and mechanical equipment of the railroads of the state in connection with the complete valuation of all railroads by the State Board of Assessment. For several years he carried this work along in addition to his duties at the University. In 1915, the State Legislature created the State Department of Engineering, and Mr. Mack was appointed the first chief state engineer.

Mr. Mack became a member of the Society in 1890. He was a charter member of the Engineering Society of Wisconsin. He belonged also to the Masonic Order, the Kiwanis Club, the University Club, and the honorary fraternities of Tau Beta Pi, Sigma Xi, and Phi Rho Sigma.

FRANKLIN H. MARMON, JR.

Franklin H. Marmon, Jr., was born in Indianapolis, Ind., on June 4, 1899, and was killed on October 11, 1924, when his automobile, in which he was returning to Indianapolis after making a test of it at Pike's Peak, struck fresh gravel and turned over.

Mr. Marmon was graduated in 1922 from the Massachusetts Institute of Technology with the degree S.B. in mechanical engineering, and became a junior member of the Society shortly after. During the war he was an airplane mechanic at McCook Field and he continued his service in the Army during vacations as a member of the Officers' Training Corps. Mr. Marmon was employed in the sales and service department of the Nordyke and Marmon Co. in Indianapolis. He was also a member of the Society of Automotive Engineers.

ERNEST MASCALL

Ernest Mascall, a member of the Society since 1921, was born in London, England, in April, 1890, and died in April, 1924. He received his education at Sheffield University and served a three-year apprenticeship, 1906-1909, with the Taskers Engineering Co. For the next two years he was with Okey and Rollo at New Plymouth, New Zealand, and from 1911 to 1913 he was foreman of outside work for Hoiland and Gilett at Auckland, New Zealand. During the war Mr. Mascall served as engineer lieutenant of the Royal Naval Reserves, being in charge of repairs on a number of motor launches, patrol vessels and other vessels attached to his naval base. He received mention several times in despatches for effecting special repairs and for designing, opening, operating and closing the naval base. Since the war he had been in Trinidad with the Trinidad Leaseholds, Inc., which he was serving as acting chief engineer at the time of his death.

W. H. MAW

The death of William Henry Maw, LL. D., one of the founders and, up to the last, senior joint editor of *Engineering*, at his home in Kensington, England, on March 19, 1924, removed from the engineering and scientific world one of its most highly honored and deeply respected members. Dr. Maw possessed that rare combination of qualities, a truly scientific mind and a developed editorial sense. From the establishment of *Engineering* in 1866 he was continuously one of its active editors, and until within the past few years he had an independent practice as a consulting engineer, his work being largely in connection with engine and boiler construction and the design and arrangement of workshops and similar buildings.

Dr. Maw was born on December 6, 1838, at Scarborough. He received a private education and, as was the common practice of those days, became an apprentice to engineering at the locomotive works in Stratford when he was barely sixteen. Here his advancement was rapid. In less than five years he was appointed chief draftsman in the locomotive department. It was while working at Stratford that he first met Zerah Colburn and Henry Bessemer. Colburn, who had for some years been editor of *The Engineer,* proposed that they start another paper, and with the help of Bessemer and James Dredge, *Engineering* was established in 1866. In 1870, Mr. Colburn retired from his position on the paper and Mr. Maw was joined in the editorship by Mr. Dredge. After the death of Mr. Dredge in 1906, Dr. Maw continued as senior joint editor until the day of his death. The commanding influence of this paper is due in large part to the high standards which he set for it from the beginning and to his ceaseless vigor in keeping it up to these standards.

Dr. Maw's literary achievements other than those in *Engineering* consist of numerous papers and addresses delivered before scientific societies and a number of standard works of which he was joint author. His section of Mr. Colburn's book on locomotives, relating to valve gears, still remains practically a classic on the subject. In his work as a consulting engineer he took particular pleasure in arranging and laying out the printing works for several of the great daily and weekly newspapers of England, notably the *Daily Telegraph,* the *Standard,* the *Field,* the *Queen,* and others.

For many years Dr. Maw was an energetic and valuable worker for various technical institutions throughout the country. It is noteworthy that fifty-nine years elapsed between his first presidential address in 1863, before the Civil and Mechanical Engineers' Society of which he was one of the founders, and his last, before the Institution of Civil Engineers in 1922. Perhaps the institution which profited most by Dr. Maw's work was the Institution of Mechanical Engineers, to which he was elected in 1873. He served as its vice-president and president, and for thirty years continuously on its council. He did remarkable work for the British Engineering Standards Association and was a member of the Cleveland Institute of Engineers and the Royal Society of Arts.

For his work in astronomy Dr. Maw was almost as well known as for his work in engineering. He had observatories at his town house in Kensington and at his country seat at Outwood, Surrey, both of which were built to his own design. His most notable original research in astronomy was connected with double-star observations. He became a member of the Royal Astronomical Society in 1888, serving as its president for two years, 1905-1906, and was one of the founders of the British Astronomical Association. For many years he was a member of the Board of Visitors of the Greenwich Observatory and of the

Joint Solar Eclipse Committee. He was a fellow of the Royal Geographical Society and of the Royal Microscopical Society.

Dr. Maw numbered many American scientific men among his personal friends. In the days when he was an apprentice in the locomotive works at Stratford he formed his first American acquaintanceships with visitors whom he used to show through the shops. For decades after that his editorial office was a mecca for distinguished American engineers who called to introduce themselves and to thank him for what he had done for the profession, but who remained and came again to enjoy the pleasure of his companionship. He was a most approachable man with a rare sense of humor and a deep appreciation for work well done, whether it was done by the professional or by the lowliest amateur.

In his autobiography, Dr. John A. Brashear, Past-President of the Society, tells of meeting Dr. Maw in London in 1888 and subsequently. "I met no more delightful man in London than Mr. Maw," he says, and speaks of evenings spent in the house in Kensington where the observatory and 6-inch telescope became an additional bond in the growth of the friendship which existed between these two simple but unique characters.

Dr. Maw became a member of the Society in 1913.

DeCOURCY MAY

DeCourcy May, consulting engineer, of Los Angeles, Cal., died in April, 1924. Mr. May was born on July 27, 1851, in Baltimore, Md. He attended the Lycée Bonaparte, Paris, and the University of Edinburgh, Scotland. From 1870 to 1876 he was engaged in engineering work with various firms in Scotland, then came to the United States, where he entered the consulting-engineering field in Baltimore.

In 1879, he became chief draftsman for E. D. Leavitt in Cambridgeport, Mass., and two years later assistant superintendent and then superintendent of the I. P. Morris Co., Philadelphia. From 1893 to 1895, Mr. May served as assistant to the president of the Niagara Falls Power Co., later becoming chief engineer of the Cataract Construction Co., Niagara Falls, N. Y. He resigned from this position to become general manager of the Dickson Manufacturing Co., Camden, N. J., where he remained until 1900, when he was appointed general manager of the New York Shipbuilding Co., also in Camden. Later he became president of the company and then chairman of the Board. In 1916, Mr. May began his consulting work in Los Angeles.

Mr. May became a member of the Society in 1881, serving as manager from 1900 to 1903. He belonged also to the American Society of Civil Engineers and to the American Institute of Mining and Metallurgical Engineers.

HARRY C. MAY

Harry C. May was born in February, 1878, in Indianapolis and died there on September 22, 1924. He was graduated from Purdue University in 1902 with a B.S. in mechanical engineering and received his M.E. degree in 1914. Mr. May spent most of his life in the railroad business. From 1902 until 1910, he was master mechanic for the Big Four and the Louisville and Nashville roads. Later he was superintendent of motive power for the Chicago, Indianapolis and Louisville and the Lehigh Valley railways. In 1918, he became general superintendent of the Chicago, Indianapolis and Louisville, and of the Cincinnati, Indianapolis and Western roads. In 1921, he became associated with the Midwest Engine Corpn. in Indianapolis and remained with that company until his death. He joined the Society in 1918.

ELMER KINNEAR McDOWELL

Elmer Kinnear McDowell, chief engineer of the Donora Steel Works, of the American Steel & Wire Co., Donora, Pa., died on March 19, 1924. Mr. McDowell was born on March 30, 1883, in Youngsville, Pa. He was educated in the schools of Warren, Pa., and Pennsylvania State College, from which institution he received the degree of B.S. in civil engineering in 1904.

For the next six years, Mr. McDowell was identified with a number of industrial concerns in Pittsburgh until in 1910 he became associated with the Donora Steel Works of the American Steel & Wire Co. as assistant engineer. He remained with that firm until 1914, when he resigned to accept a position with the Bethlehem company. A year later he returned to the Donora Steel Works as chief engineer.

Mr. McDowell became a member of the Society in 1918. He was also a member of the American Institute of Electrical Engineers. He belonged to the Phi Kappa fraternity and to the Elks. He was the author of numerous articles published in the technical press and at the time of his death was at work on a book being published serially in *Power Plant Engineering* on Design of Short Transmission Lines. Mr. McDowell was also known as an accomplished musician.

NORMAN G. MEADE

Norman G. Meade died suddenly of heart failure on February 29, 1924. Mr. Meade was born on May 25, 1876, in Philadelphia, Pa. He was educated at the Union School and the Collegiate Institute in Jamestown, N. Y.

From 1899 to 1903, Mr. Meade served as chief electrician with the McIntosh & Seymour Corpn., Auburn, N. Y., and then became connected in the same capacity with the Corliss Engine Works in Hamilton, Ohio. From 1904 to 1907 he was industrial research engineer for the Westinghouse Electric & Manufacturing Co., and then became publicity engineer for the New York Edison Co. From 1911 to 1913, Mr. Meade was associate editor of *Power Plant Engineering*, and thereafter for three years was engaged in free-lance technical writing. At the end of that period, he accepted the position of editor of the *Southern Engineer*. Four years later he became editor of the *Electrical South*. Because of failing health, Mr. Meade was forced to resign from active business late in 1922.

Mr. Meade became an associate of the Society in 1921.

CARL J. MELLIN

Carl J. Mellin, consulting engineer of the American Locomotive Co., Schenectady, N. Y., died on October 15, 1924. Mr. Mellin was born on February 17, 1851, in Westergotland, Sweden. He was a graduate of the University of Gothenburg. For fifty years he was closely identified with naval architecture and mechanical engineering in Sweden, Scotland and America. He was also the inventor and patentee of numerous locomotive devices and improvements, among his most important works being the design of the Richmond compound locomotive. He was also instrumental in the introduction of the three-cylinder locomotive and was engaged in the designing of the three-cylinder compound locomotive when taken ill.

Mr. Mellin became a member of the Society in 1890. He belonged also to the American Society of Swedish Engineers, the Society of Engineers of Eastern New York, the American Society of Naval Engineers, the American Railroad Master Mechanics, the New York Rail-

road Club and the Locomotive Club. He had traveled extensively in Europe and was knighted and decorated by the late King Oscar of Sweden for distinguished service in the interest of engineering.

THOMAS MIRK

Thomas Mirk, president of Hunt, Mirk & Co., Inc., San Francisco, Cal., died on February 23, 1924. Mr. Mirk was born on September 3, 1866, in Falkirk, Scotland, where he was educated. He served a six-year apprenticeship with Lobnitz & Co., shipbuilders of Renfrew, Scotland, and then became assistant engineer on the S.S. *Bonny* of the British & African Steamship Co.

From 1887 to 1892, Mr. Mirk sailed between San Francisco, China and Japan, on the White Star Liner S.S. *Oceanic* as assistant engineer. He was awarded his first-class engineer's certificate by the British Government in 1891 and then served as chief engineer of construction and operation for the following companies: San Mateo Electric Railway Co., Metropolitan Electric Railway Co., Sutro Electric Railway Co., and the Independent Electric Light & Power Co., all of San Francisco.

In 1904, the firm of Hunt, Mirk & Co. was established, with Mr. Mirk as president. They were sole agents of turbines for the Westinghouse Electric & Manufacturing Co. and specialized in the erection of power plants, in which field they very soon became well known.

Mr. Mirk became a member of the Society in 1913. He was a Knight Templar and a Shriner.

JAMES LEONARD MOORE

James L. Moore, president and general manager of the Moore Steam Turbine Corporation, Wellsville, N. Y., died on April 2, 1924. Mr. Moore was born on October 24, 1875, in Cincinnati, Ark. He was graduated from the University of Kansas in 1897 with the degree of B. S. in mechanical engineering, and entered the engineering department of the Atchison, Topeka & Santa Fe Railroad.

Three years later he became connected with the Holly Manufacturing Co., Lockport, N. Y., in drafting and designing work. From 1902 to 1906, Mr. Moore was with the Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa., as designer of steam turbines, resigning at the end of that period to take charge of the design, testing and experimental work in the development and manufacture of the Kerr steam turbine.

In 1917, Mr. Moore established the Moore Steam Turbine Corporation, of which he was president at the time of his death. He was president also of the Pure Carbon Co., and director of the First National Bank, both of Wellsville, N. Y.

Mr. Moore became a member of the Society in 1911. He was a member of the Kappa Sigma fraternity and of the Old Colony Club.

JAMES JOHN MULCARE

James J. Mulcare, engineer in charge of drafting of the Schenectady Works of the General Electric Co., died on January 3, 1924. Mr. Mulcare was born on August 10, 1871, in Schenectady, N. Y. Immediately upon being graduated from high school, Mr. Mulcare entered the employ of the Chicago & Northwestern Railway Co. and served a two-year shop apprenticeship. He then returned to Schenectady, to become a draftsman with the General Electric Co.

Mr. Mulcare became a member of the Society in 1917. He belonged to the Knights of Columbus and to the Council of Maccabees.

BRUNO V. NORDBERG

Dr. Bruno V. Nordberg, for more than forty years closely associated with engineering progress and development in the power- and mining-machinery fields, died on October 30, 1924. Dr. Nordberg was born in Finland in 1857 and two years after his graduation in 1878 from the University of Helsingfors he came to this country, believing that it offered greater possibilities in his chosen field. Soon after his arrival he found employment with the E. P. Allis Co., where he had an opportunity to assist in the design of two large vertical blowing engines. The special designer who had been engaged for this work had tired of the conditions and left. Nordberg completed the job, and one of these engines is still in use for blast-furnace blowing.

Early in his career he became interested in the improvement of design to effect greater economies. He recognized the possibilities of the poppet valve when used in connection with steam engines, also the opportunity of improving the efficiency of the slide-valve steam engine, provided a better control of steam admission could be obtained. He designed a poppet-valve governor for engines of this type which permitted an economy that was considered impossible at that time. It later became an absolute necessity, in order to utilize the high steam pressures and superheats of modern steam practice. It is the type of valve used today on the Nordberg uniflow engine.

In 1886, the Bruno V. Nordberg Co. was organized to engage in manufacture of governors. The business grew by leaps and bounds, and in 1890 moved to larger quarters and changed its name to the Nordberg Manufacturing Co. Dr. Nordberg followed his natural inclinations and began the design of a line of machinery. The development of the copper country of northern Michigan soon created a demand for power and mining equipment. This was the field for which he was particularly adapted and the result was an increased line of products including steam hoists, engines, compressors, condensers, etc. In 1900 the present plant of the company was built to meet the increasing demands of the business. It covers about forty-two acres. Dr. Nordberg recognized the value of a permanent organization and most of the department heads had been in his employ for from twenty to twenty-five years at the time of his death.

Among the achievements which brought Dr. Nordberg recognition among engineers in this country and in Europe are included the large hoists and compressors for metal-mine service. In 1897 he designed the famous hoist for the Tamarack Mining Co. An unusual design followed in the compound-condensing reel hoist built for the Homestead Mining Co. His greatest achievement in this direction, however, was the building of the mammoth hoist for the Quincy Mining Co. at Hancock, Mich., which exceeds in size any ever attempted and perhaps will remain the world's largest for years to come. In addition to the above-mentioned steam hoists he designed many unusual hoists for air and electric operation. Some of the largest air compressors and Diesel engines in service in this country reflect his designing skill.

Dr. Nordberg joined the Society in 1893, and was a member of many other engineering and scientific societies. The degree of Doctor of Engineering was conferred upon him in 1923 by the University of Michigan in recognition of his skill in the design of special machinery which had been a vital factor in the copper development of the state.

FRANKLIN NOURSE

Franklin Nourse, former agent of the Lawrence Manufacturing Co., Lowell, Mass., died on August 31, 1924. Mr. Nourse was born on March 5, 1848, in Bangor, Me. He was educated in the schools of

Bangor and Boston and Harvard University, being graduated from the latter in 1870 with the degree of B.A.

Mr. Nourse entered at once upon the business of textile manufacturing and was successively connected with the Barker Mills, the Lawrence Duck Co., the York Manufacturing Co. of Saco, Me., and the Lawrence Manufacturing Co. in Lowell. He became associated with the last-named concern in 1895 as its agent and continued in its active management until his retirement as agent in 1910.

Throughout his career he was a highly successful manager of mill properties and became one of the leading figures in the textile industry of New England. He was intensely interested in the civic affairs of Lowell and gave freely of his time and service in that direction.

Mr. Nourse became a member of the Society in 1896. He belonged also to the Harvard Engineering Association, the National Cotton Manufacturers' Association, the American Academy of Political and Social Science and to a number of social clubs.

JOHN GERALD O'NEIL

John Gerald O'Neil, contracting engineer of the firm of J. G. O'Neil & Son, Chicago, Ill., died in March, 1924. Mr. O'Neil was born on April 12, 1862, in Glasgow, Scotland, and was educated at Kings College, Dublin, Ireland. He served his apprenticeship at the shops of the Clyde Steamship Co. in Scotland.

Coming to this country at the close of his apprenticeship, he entered the employ of the Davis Steam Co., Chicago, as machinist. He remained with that firm for five years and then became shop foreman with the Worthington Steam Pump Co.; later, he became erecting engineer for the same company.

Mr. O'Neil became an associate of the Society in 1904. He belonged also to the Western Society of Engineers, the Columbus Engineers' Club, and the Order of Elks.

BURTON S. ORR

Burton S. Orr was born in Riley, Kan., in 1883, and died on July 27, 1924. Mr. Orr received his B.S. in mechanical engineering from the Kansas State Agricultural College in 1907 and after a year's experience in the engineering department of Swift & Co. he returned to his alma mater as assistant in the department of mechanical engineering. From April to September, 1916, he was chief engineer for the Cole Truman Ice and Cold Storage Co. of Independence, Kan. In the fall of that year, he went to the Oregon Agricultural College as assistant professor of experimental engineering. Later, he became professor of mechanical engineering at the University of Idaho. During the war, Mr. Orr served in the Engineers' Corps and after his discharge from Army service did special inspection work on the heating plant of Government buildings for the United States Department of Agriculture. In 1919, he began work as construction foreman for the Portland Gas and Coke Co. and was master mechanic for that company at the time of his death. He became a member of the Society in 1918. He belonged to the Masonic Order.

CARL G. OSTEMAN

Carl G. Osteman died on November 11, 1924. Mr. Osteman was born on October 20, 1857, in Forsby, Sweden, where he was educated. He served his apprenticeship with the Matala Iron Works in Matala, Sweden, and remained there until 1878. In 1882, he came to the United

States and entered the employ of the Atlantic Steam Engine Works. Two years later he became connected with the S. A. Woods Machine Co., Boston, Mass., where he soon took charge of the toolroom as foreman, designing tools, jigs and special fixtures. In 1896, he resigned to become associated with the Milwaukee Sander Manufacturing Co., Green Bay, Wis., where he designed the "Columbia" sander. In 1897, he returned to the S. A. Woods Machine Co. and remained with them until the time of his death. He was designer of woodworking machinery for that company, and had charge of the drafting department.

Mr. Osteman had a number of patents to his credit. He designed the Blood turret lathe complete with tools to make Russian shrapnel, and afterward designed new tools and changed the lathe to make the French 75-mm. shells. He became a member of the Society in 1907. He belonged to the Masonic Order.

WALTER E. PARKER

Walter E. Parker, a life member of the Society, died on October 25, 1924. Mr. Parker was born in Princeton, Mass., in 1847. He received his early education in the public schools in Woonsocket, R. I., and at an early age entered the employ of the Social Mills there. Beginning as an office boy, he worked his way up to the position of superintendent, which he held for five years.

In 1881, Mr. Parker became connected with the Pacific Mills in Lawrence, Mass., as superintendent of the cotton department. In 1887, he was made agent of the cotton and worsted departments, and five years later of the print works also. In 1909, the Cocheco Division at Dover, N. H., was also put under his charge. He retired from active service on January 1, 1924. The position that the Pacific Mills holds in textile manufacturing is due largely to his guidance. Mr. Parker was known internationally as an authority on the manufacture of cotton and worsted goods and was one of the foremost cotton manufacturers of the country. He served for thirty years as trustee of Tufts College and was a leader in civic affairs in his community.

CARL H. PETERSON

Carl H. Peterson, eastern sales manager for the Standard Stoker Co., Inc., was born in Chicago in November, 1872, and died at his home in the same city on September 27, 1924. At the age of 17, Mr. Peterson entered the drafting room of the Pullman Co. In the next few years he obtained drafting experience from Swift and Co., the Chase Elevator Co., the Gates Iron Works and the Railroad Lighting and Manufacturing Co. From 1893 to 1904, he had charge of the design and construction of gas plants for the Safety Car Heating and Lighting Co. of Philadelphia. For the next two years he was with the Pressed Steel Car Co. In 1906, he became associated with the Baldwin Locomotive Works, serving as technical representative for them both in Chicago and St. Louis, until 1919, when he entered the employ of the Standard Stoker Co. Mr. Peterson became a member of the Society in 1910.

JOHN W. PLANT

John W. Plant was born on August 27, 1879, and died on August 2, 1924. He was graduated from the St. Louis Manual Training School in 1895, and after experience with the Pullman Car Co. and the Westinghouse Air Brake Co. in St. Louis, spent two years at Washington University School of Engineering. He then became assistant chief engineer for the American Steel Foundry Co. at Granite City, Mo.,

where he continued until 1904. In that year, he went to Folsom, Cal., for the Folsom Development Co., acting as chief engineer for them and for the Folsom Machine Co. In 1909, he opened an office as a general consultant in San Francisco. He was Pacific Coast representative for the Edgar Allen American Manganese Steel Co. and later for the Inland Engineering Co. of Chicago. At the time of his death he was engaged in various mining ventures. He joined the Society in 1911 and was a member of the Engineers' Club and the Union League Club of San Francisco.

WILLIAM M. RAYNOLDS

William M. Raynolds, assistant manager, of the Deane Works of the Worthington Pump & Machinery Corporation, Holyoke, Mass., died suddenly on September 30, 1924, when he was thought to be progressing favorably after a recent operation. He was born on December 28, 1864, in Thompsonville, Conn.

In January, 1884, Mr. Raynolds entered the employ of the old Deane Steam Pump Co., in Holyoke, where he worked through all the main phases of the business, finally filling the office of clerk of the corporation. In 1899, when the firm was acquired by the International Steam Pump Co., Mr. Raynolds was appointed assistant manager of the sales and engineering branches, a position that he filled most successfully, constantly broadening its activities.

Mr. Raynolds became a member of the Society in 1919. He belonged to the Engineering Society of Western Massachusetts and to a number of local clubs and civic organizations. During the war, Mr. Raynolds aided the Government by promoting special effort in the pump plant, and by devoting much of his private means to war projects.

ROBERT H. ROESEN

Robert H. Roesen, owner of the John H. Muller Co., trucking engineers of New York City, died suddenly on January 19, 1924. Mr. Roesen was born on December 26, 1890, in New York City, where he received his early education. He was graduated from Stevens Institute of Technology in 1913 with the degree of M.E., and then was employed in a special apprentice class by the American Machine and Foundry Co., Brooklyn, N. Y. Later he became assistant to the superintendent of that concern.

In 1915, he became assistant mechanical superintendent of the New York *Times*, resigning to enter the service as a lieutenant in the Engineers' Corps of the United States Army. Upon his return to civil life in 1919, Mr. Roesen became associated with the New York *Sun* as mechanical superintendent. In 1920, he took over the business of the John H. Muller Co.

Mr. Roesen became a junior member of the Society in 1914 and was advanced to the grade of associate member in 1921. He was a member of the Masonic Order and belonged also to a number of other fraternal organizations.

ROBERT FLETCHER ROGERS

Robert Fletcher Rogers, patent lawyer, died on May 17, 1923. Mr. Rogers was born at Erie, Pa., in August, 1865. He studied at Harvard and Columbia Law School. After graduation in 1886, he spent several years on the examining corps of the United States Patent Office. From 1890 until the time of his death, Mr. Rogers practiced his profession in New York City. He was closely in touch with a large variety of mechanical matters, in actual operation and testing. He became a member of the Society in 1914.

SAMUEL R. SAGUE

Samuel R. Sague, president of the J. B. Savage Co. in Cleveland, and a member of the Society since 1908, died on July 9, 1924. Mr. Sague was born in Cleveland in 1877 and after his graduation from the Cleveland Manual Training School he spent two years in the University School there. His early shop and drafting-room experience was obtained with the United States Standard Drawn Steel Co. and the Wellman-Seaver Engineering Co.

From 1900 to 1902, he was president of the Mentor Knitting Mills, designing and erecting their plants. After some years spent in general construction and contracting work and as department manager for the Wellman-Seaver-Morgan Co., Mr. Sague entered the employ of Strong, Carlisle and Hammond Co. in 1907, where he was employed in an important engineering capacity for many years. Recently he gave up this connection to become president of the J. B. Savage Co.

HANS R. SETZ

Hans R. Setz, engineer in charge of Diesel-engine development work at the Sun Shipbuiling & Dry Dock Co., died on September 6, 1924. Mr. Setz was born in Biel, Switzerland, in 1880. He received his degree in mechanical engineering from the Polytechnicum Zurich at a remarkably early age and then took a course in practical shop work at the Sulzer plant in Winterthur.

He migrated to America in 1902, where his first employment was in the steam-turbine department of the Westinghouse Electric & Machine Co. He was soon afterward transferred to the gas-engine department of that company.

Mr. Setz is perhaps best known for the line of engines which he designed and put into operation for the Fulton Iron Works of St. Louis, Mo. Many of these engines are scattered throughout the country and form impressive monuments to Mr. Setz's memory. He also spent several years as chief engineer of the Diesel-engine department of the Manitowoc Shipbuilding Co., resigning to devote his time to perfecting his designs for a double-acting two-cycle engine. In February, 1924, Mr. Setz went to the Sun Shipbuilding Co.

Mr. Setz became a member of the Society in 1912.

THOMAS CARPENTER SMITH

Thomas Carpenter Smith, a member of the Society since 1886, died on October 25, 1924. Mr. Smith was born in Kingston, Jamaica, B. W. I., on November 16, 1856. He was educated at the Royal High School of Edinburgh, Scotland, and served a three-year apprenticeship with William Thomas Henley, telegraph and submarine cable engineer and contractor. He then worked as an erector in the shops of the North British Railroad Co. at St. Margaret's, Edinburgh, and Cowlains, Glasgow, Scotland.

About 1876, he left England and came to the United States, where he was employed first in the shops of Wm. Sellers & Co., and later in the shops of the Pennsylvania Railroad at Altoona, Pa. He served for a period as superintendent of the Allegheny County Light Co., Pittsburgh, Pa., then returned to Philadelphia and opened his own office as mechanical engineer.

In 1912, Mr. Smith went to live in St. Andrews, Jamaica. He became a member of the Society in 1909, and belonged also to The Franklin Institute, the Pennsylvania Forestry Association, and the American Institute of Electrical Engineers.

MILTON PARKER SORBER

Milton Parker Sorber, staff superintendent in charge of manufacturing methods and tool equipment of the East Pittsburgh Works of the Westinghouse Electric & Manufacturing Co., died on December 11, 1924. Mr. Sorber was born on August 20, 1878, in Stoystown, Pa. In 1895, he entered the employ of the Westinghouse Co. as screw-machine operator, and advanced steadily through the ranks as foreman of the bearing department, foreman of machining and assembling departments, foreman of vehicle-motor department, superintendent of munition plant, superintendent of tools department, and finally staff superintendent. Mr. Sorber became a member of the Society in 1921.

FREDERICK C. STIMMEL

Frederick C. Stimmel, in charge of the boiler-sales and engineering departments of the Casey-Hedges Co., Chattanooga, Tenn., died on November 16, 1924. Mr. Stimmel was born on July 1, 1880, in Chattanooga. He attended high school in that city and later took correspondence-school courses in civil engineering. He was for a number of years employed as draftsman and assistant to the engineer in charge of the Chickamauga National Park. In 1899, he became connected with the Casey-Hedges Co. as draftsman, where he advanced rapidly. For sixteen years prior to his death, he was in charge of boiler sales and of the engineering department.

Mr. Stimmel became an associate member of the Society in 1915. He belonged to the Knights of Columbus and to the Elks.

TEN BROECK WILLIAMSON STINSON

Ten Broeck Williamson Stinson, consulting mechanical engineer of the Public Service Production Co., Newark, N. J., died on November 26, 1924. Mr. Stinson was born on September 24, 1889, in Malapan, N. J. He was graduated from the Stevens Institute of Technology in 1912.

Upon leaving college, he entered the engineering department of the Public Service Electric Co. as engineering assistant, later becoming assistant construction superintendent and then assistant engineer attached to the general office staff. In 1920, Mr. Stinson was appointed mechanical engineer in charge of design for extensions and improvements then being carried on. Two years later, when the Public Service Production Co. was formed, he became consulting mechanical engineer in responsible charge of all mechanical and building designs.

Mr. Stinson became an associate member of the Society in 1920. He belonged also to the National Electric Light Association and was a member of the Beta Theta Pi fraternity.

ROBERT V. STUREMAN

Robert V. Stureman, well known central-station-heating engineer of Springfield, Ill., met death in the wreck of the Twentieth Century Limited on December 9, 1923. He was born on January 1, 1893, in Battle Creek, Mich.

Mr. Stureman began his business career with the Central Illinois Light Co. in 1911. In 1915, he became superintendent of the heating department of the Springfield Gas & Electric Co., and two years later assumed in addition the duties of assistant manager. During the war, Mr. Stureman served in the Quartermaster's Corps, where he attained the rank of captain. After the war, he returned to Springfield and took

up his former duties. A year later he entered the heating contracting business, in which he has engaged at the time of his death.

Mr. Stureman became a junior member of the Society in 1917.

ALLEN PARK TOMS

Allen Park Toms, who entered the employ of the General Electric Co., Schenectady, N. Y., on October 1, 1923, as a student engineer, died on May 1, 1924, as a result of a contact with a high-potential circuit. Mr. Toms was born on March 28, 1901, in Nyack, N. Y. He was graduated from Cornell University in 1923 with the degree of B.S. in mechanical engineering.

Mr. Toms became a junior member of the Society in 1923. He belonged to the Edison Club and to the Cornell Club of Schenectady.

HENRY ROBINSON TOWNE

Henry Robinson Towne, eighth president of The American Society of Mechanical Engineers and honorary member of the Society since 1921, died at his home in New York on October 15, 1924.

Mr. Towne came of old English stock, being the direct descendant in the ninth generation from William Towne, who emigrated from Yarmouth, England, in 1640, and who settled at Salem, Mass. His father, the late John Henry Towne, was well known and highly esteemed in the city of Philadelphia, Pa., where, as a member of the firm of Merrick & Towne and of the Port Richmond Iron Works of I. P. Morris, Towne & Co., he was connected with the design and construction of important engineering work, marine engines, sugar machinery and other heavy machines.

Henry Robinson Towne was born in Philadelphia on August 28, 1844, his mother being Maria R. (Tevis) Towne. The youth was educated at private schools and at the University of Pennsylvania, followed by a course at the Sorbonne, in Paris. His practical shop training was gained in the Port Richmond Iron Works, both in the shop and in the drafting room, leading on to work in charge of erection of machinery in the navy yards of Boston, Portsmouth, and Philadelphia, this experience covering the period between 1862 and 1865, when the establishment of which his father was an active member was engaged in construction work for the Civil War activities of the Navy Department, including some of the most notable designs of Capt. John Ericsson. During 1866-1867, Mr. Towne traveled and studied engineering in Europe, under the companionship of a notable American engineer, the late Robert Briggs; one of the fruits of that association being an important investigation upon the transmission of power by belting, of which the results appeared in a paper by Briggs and Towne in the *Journal* of The Franklin Institute in January, 1868.

About the time that Mr. Towne returned from Europe to Philadelphia, there went to that city from Shelburne Falls, Mass., Linus Yale, Jr., who had for years devoted himself to the invention and production of locks. Yale was seeking two things he lacked — capital and business management — and through personal acquaintances he was brought in contact with John Henry Towne who saw the possibility of an association into which his son might well enter. Henry R. Towne perceived at once what Yale had but imperfectly grasped — the possibilities of the small-key pin-tumbler lock as a manufacturing proposition upon what has since been called the quantity-production basis — and the result of the meeting was the formation in 1868 of what was at first called the Yale Lock Co., and which in 1883 became the Yale and Towne Manufacturing Co.

Yale had devoted much of his effort to the design of elaborate and individual locks for banks and vaults, and considered his ingenious pin-tumbler lock a minor invention. Towne saw the real production advantage of this latter device, which lay in the fact that the tumbler mechanism, contained in a small cylinder separate from the bolt-work case and bulkier part of the lock, permitted thousands of locks to be made by quantity-production methods all alike except for the tumbler cylinder, so that any cylinder set to any combination might be added to any bolt case to make the completed lock. When to this segregation of the tumbler element from the bolt work was added the great advantage of the small flat key, there appeared an immediate opportunity for the conversion of the lock business from its former status as the personal trade of a locksmith to a commercial manufacturing proposition of almost unlimited possibilities.

The almost unique combination of engineering ability and technical training with executive capacity and economic vision rendered Henry R. Towne the ideal associate for the dreamy inventor, Linus Yale. Stamford, Conn., within convenient reach of New York City, and possessing excellent transportation facilities by rail and water, was selected for the seat of the new enterprise, and there, fifty-six years ago, the small building was erected which became the nucleus of the great works of the present concern.

Hardly had the new enterprise been started when it received a stunning blow in the sudden death of Linus Yale, leaving Henry R. Towne at the age of twenty-four to carry the burden alone. From that date until the year 1916, when he yielded the presidency to become chairman of the board of directors, the Yale and Towne Manufacturing Co. *was* Henry R. Towne. As a great executive, he naturally possessed the art of surrounding himself with able associates, but he was always the moving spirit, the presiding genius, and the inspiring director of that famous organization.

Mr. Towne became a member of the Society in 1882, and almost immediately the influence of his presence became evident in its councils and in its development. In 1884-1886, he became vice-president of the Society, and as such he presided at the meetings of the year 1886 because the ill health of the president, Dr. Coleman Sellers, rendered the latter unable to attend. In 1889, he was elected president of the Society, a choice which enabled most valuable services to be rendered by him to the organization to which he was so keenly devoted.

It was during his incumbency that the first memorable trip of members of the four national engineering societies was made to England and France, in connection with the Universal Exposition held in Paris in 1889. As the ranking officer among the organizations then represented, Mr. Towne became the logical and welcome leader of the party, his executive ability appearing in the coherent organization effected on shipboard before the members landed in England; and those who had the rare privilege of working with him during that memorable summer have never forgotten the inspiration of his leadership. In England the professional, social, and official events called for the exercise of most tactful dignity; while in France there was required in addition a command of the language, which can be appreciated only by those who recall his graceful and fluent response in French to the welcome extended at the reception tendered by M. Gustave Eiffel, as president of the Société des Ingénieurs Civils de France, upon the occasion of the *déjeuner* given on the platform of the great tower on the Champ de Mars.

The outstanding feature of Mr. Towne's career lay in his broad extension of the scope of the work of the engineer to include the economics of engineering, and the essential union of production and management.

At the Chicago meeting of the Society, in May, 1886, Mr. Towne presented a paper entitled The Engineer as an Economist. So unusual was this topic that many of the members were inclined to regard it as a subject unsuited for presentation to an engineering audience, and it was with misgivings that the publication committee accepted it. Today, when "Scientific Management," "Efficiency," "Quantity Production," and their several extensions are regarded as essentials in engineering, the words of this epoch-making paper may appear somewhat trite, but when it is remembered that the following passages were written nearly forty years ago, the boldness which inspired them will be better appreciated:

"The organization of productive labor must be directed and controlled by persons having not only good executive ability and possessing the practical familiarity of a mechanic or engineer with the goods produced and the processes employed, but having also, and equally, a practical knowledge of how to observe, record, analyze, and compare the essential facts in relation to wages, supplies, expense accounts, and all else that enters into or affects the economy of production and the cost of the product."

"The matter of shop management is of equal importance with that of engineering."

Referring to conditions then existing, Mr. Towne said: "The management of works is unorganized, is almost without literature, has no organ or medium for the interchange of experience, and is without association or organization of any kind."

"The remedy must not be looked for from those who are 'business men,' or clerks or accountants only; it should come from those whose training and experience has given them an understanding of both sides (the mechanical and the clerical) of the important questions involved. *It should originate from engineers!*

In 1889, at the Erie Meeting of the Society, Mr. Towne presented another paper upon the subject of gain sharing, and these two papers were the beginning of what is now recognized as the most important development in industrial engineering of the last fifty years.

Notwithstanding his intense work in the development of the great business of which he was the head, Mr. Towne found time to take part in many widely diversified activities. For five years he was the energetic president of the Merchants' Association of New York; he was most actively interested in the development of the Morris Plan Company of New York, and its president from 1914 to 1918, and chairman of the board until his death. The high responsibility of director in the Federal Reserve Bank of New York was held by him from 1914 to 1919; while among the other important directorships which Mr. Towne held may be mentioned those in the Industrial Finance Corporation, the American Dredging Co., and the Lincoln Safe Deposit Co. He was also treasurer of the National Tariff Commission Association.

In 1868, he married Cora E. White, of Philadelphia, by whom he had two sons, John Henry Towne, now secretary and treasurer of the Yale & Towne Manufacturing Co., and Frederick Tallmadge Towne, whose brilliant career as works manager of the great plant at Stamford was cut short by his lamented death in 1906. Mrs. Towne died in 1917.

Space will not permit the enumeration of Mr. Towne's many contributions to engineering literature, but mention must be made of his important memoir presented before the Society in 1906, entitled Our Weights and Measures and the Metric System, in which he presented most forcibly the objections to the compulsory introduction of the metric system into American manufacturing industries, a position which he reiterated most emphatically in one of his latest utterances.

JOHN C. TRAUTWINE, JR.

John C. Trautwine, Jr., consulting civil engineer and editor of the Trautwine handbook for civil engineers, died on July 4, 1924, at his home in Philadelphia, Pa. Mr. Trautwine was born on March 17, 1850, in Philadelphia, and was educated in the schools of that city.

At an early age he became associated with his father, who was the originator of the Trautwine handbook. In 1883, he entered the field of consulting civil engineering and at the same time took over the editorship of the handbook. He was a frequent contributor to the technical press. With Rudolph Hering he translated Ganguillet and Kutter's Flow of Waters in Rivers and Other Channels. With A. Marichal he translated Bazin's Flow Through Orifices. From 1895 to 1899, he served as chief of Philadelphia's Bureau of Water.

Mr. Trautwine became a member of the Society in 1921. He belonged also to the American Society of Civil Engineers, the Institution of Civil Engineers, and the American Water Works Association.

WILLIAM J. WALLACE

William James Wallace, engineer with the Sprague Electric Works at Bloomfield, N. J., died on September 8, 1923. Mr. Wallace was born in Schenectady, N. Y., February 4, 1878. From 1890 until 1900, he was with the General Electric Co., Schenectady, in the drafting and engineering department. From 1900 until his death, he was with the Sprague Electric Works at Bloomfield, as draftsman, designing engineer, and managing engineer in charge of design and manufacture of motors and generators. He had been a member of the Society since 1920.

LOUIS WEHNER

Louis Wehner was born in Germany in 1876 and died on September 7, 1924, in Milwaukee. He came to this country at an early age and received his education in the public schools of Philadelphia and the Massachusetts Institute of Technology. Before entering college he was with Warren Webster & Co. in Camden for a period of eight years. After college, he became draftsman for the Bucyrus Co. in South Milwaukee, and in 1909 was made their mechanical engineer. After a short period with the Bucyrus-Vulcan Co., he was sent to the Bucyrus Co.'s plant in Evansville, Ind., where he had charge of the development of the first small revolving steam shovels built by this company. From 1912 until his death, Mr. Wehner was chief engineer of the excavator department of the Pawling & Harnischfeger Co. in Milwaukee, originating and developing their line of gasoline-operated shovels, drag-line excavators and trenching machines. He became a member of the Society in 1901 and belonged also to the Society of Naval Architects and Marine Engineers.

JOSEPH J. WHITE

Joseph J. White, president of Joseph J. White, Inc., New Lisbon, N. J., died on May 4, 1924. Mr. White was born on January 28, 1846, in Springfield Township, New Jersey. He was educated in the Friend's Central School, Philadelphia, and the Philadelphia Polytechnic College.

About 1874, Mr. White became connected with the H. B. Smith Machine Co., Smithfield, N. J., where he served his shop apprenticeship. After working through various departments of the company, he was appointed superintendent of the works. In 1890, Mr. White and his brother organized the Pennsylvania Machine Co., Philadelphia, and

operated it until 1895, when Mr. White sold his interest to his brother and began to devote his time to cranberry culture.

In 1912, he incorporated his cranberry interests under the title of Joseph J. White, Inc. Mr. White was especially interested in the development of facilities for irrigating cranberries and protecting them from frosts and insects, and for this purpose he installed an elaborate system of reservoirs, dams, canals, gates, etc. He was one of the organizers, in 1895, of the Growers' Cranberry Co. for the coöperative sale of cranberries, and was president of that company for twenty-five years. He was also president of the Farmers' Trust Co., Mt. Holly, N. J.

Mr. White was one of the organizing members of the Society in 1880. He was a member of the American Association for the Advancement of Science and of the Manufacturers' Club, Philadelphia.

GUSTAV WILLIUS, JR.

Gustav Willius, Jr., who, until his retirement because of ill health, was mechanical engineer with the Robinson, Cary & Sands Co., St. Paul, Minn., died on November 19, 1924. Mr. Willius was born on July 2, 1873, in St. Paul, Minn. He was graduated from Rose Polytechnic Institute in 1897 with the degree of B. S. in electrical engineering. Shortly after his graduation, he entered the service of the Great Northern Railroad, of which in 1905 he was appointed mechanical engineer. He severed his connection with the railroad in 1908, to become a member of the firm of Robinson, Cary & Sands, engaged in the sale and installation of mechanical and electrical equipment for railways and industrial concerns. Mr. Willius was mechanical engineer and secretary of the concern. He was greatly interested in the development and use of lignite coal, and shortly before his death had been making experimental tests with pulverized lignite. He became a member of the Society in 1916.

NELSON C. WILSON

Nelson C. Wilson, mechanical engineer with the Dollar Savings & Trust Building, Pittsburgh, Pa., died on December 12, 1924. Mr. Wilson was born in Allegheny, Pa., in 1866. He received the Ph. B. degree in 1888 from the Western University of Pennsylvania and the Ph. M. in 1897. From 1888 to 1892, he was employed by the Westinghouse Machine Co., as a draftsman and as supervisor of the testing department. He then became connected with Westinghouse, Church, Kerr & Co., as salesman and engineer for their Pittsburgh office. He resigned in 1896 to enter the contracting business and for a number of years conducted a consulting-engineering business, specializing in engine and boiler testing and the design and construction of power plants.

CHARLES A. G. WINTHER

Charles A. G. Winther, president and general manager of the Chas. A. G. Winther Co., West Somerville, Mass., died on October 4, 1924. Mr. Winther was born in Halberstadt, Germany, on February 25, 1842. He was educated in the technical schools of Germany and served his apprenticeship in that country.

Mr. Winther gained his shop experience with the Nova Scotia Iron Works, Halifax, N. S., and then became designer for Brigham & Co., Boston, from 1873 to 1875. His next position was as foreman and later assistant to superintendent of the Crosby Steam Gage & Valve Co., Boston. From 1886 to 1906, Mr. Winther was associated with the Star

Brass Manufacturing Co. as superintendent. For three years he served as general superintendent of the Roe Stephens Manufacturing Co., and from 1909 to 1912 acted in the capacity of mechanical engineer and manufacturers' agent in Boston, Mass. In 1912 he organized the Chas. A. G. Winther Co., of which he held the presidency until his death.

Mr. Winther became a member of the Society in 1897. He belonged to the Masonic Order.

HUBERT A. WYNKOOP

Hubert A. Wynkoop, electrical engineer in charge of the division of electrical inspection, Bureau of Gas and Electricity of the City of New York, died on December 13, 1924. Mr. Wynkoop was born on September 20, 1866, in Yonkers, N. Y. He was graduated from Stevens Institute of Technology in 1888 with the degree of M.E.

During his career, Mr. Wynkoop was associated with the Edison General Electric Co.'s district offices at Chicago, San Francisco and Georgia, in the design of switchboards, powerhouse layout and distribution system; and with the General Electric Co., installing the Rome, Ga., street-railway system. From 1894 to 1897, he served as municipal inspector of gas and electricity for Brooklyn, N. Y., then became chief inspector and electrical engineer in charge of the Municipal Bureau of Electrical Inspection for New York City, from which position he was advanced to that which he held at the time of his death.

Mr. Wynkoop became a member of the Society in 1908. He was a fellow of the American Institute of Electrical Engineers, a member of the Municipal Engineers of the City of New York, an associate member of the Edison Pioneers, and belonged to the Sons of the Revolution and the Society of Colonial Wars.

THOMAS A. WYNNE

Thomas A. Wynne, vice-president and treasurer of the Indianapolis Light & Heat Co., Indianapolis, Ind., died on March 26, 1924. Mr. Wynne was born on August 31, 1864, in Ottawa, Canada. He was educated in the schools of Port Henry, N. Y.

Mr. Wynne served his apprenticeship with the Chicago, Milwaukee & St. Paul Railroad and then entered the employ of the Arthur Heuey Electric Co., where he remained until 1887. From 1887 to 1889, he was connected with the Jenney Electric Co., resigning to become associated with C. C. Perry in the Marmon-Perry Light Co. as superintendent of construction. In 1892, Mr. Wynne assisted Mr. Perry in the organization of a larger concern which was known as the Indianapolis Light & Power Co. In 1906, this name was changed to the Indianapolis Light & Heat Co. He has been vice-president and treasurer of that company for the past fourteen years.

During the last thirty-seven years, Mr. Wynne took an active part in the civic and industrial development of Indianapolis. He was a member of the Board of Trade and of the Chamber of Commerce of that city. He became a member of the Society in 1915.

JOHN ALLEN YATER

John Allen Yater died suddenly on October 8, 1924. Born in Cleburne, Tex., August 31, 1889, he was graduated from the Cleburne High School in 1917. For one year, he worked for the Texas Power & Light Co., and acquired so great a liking for his work that in 1918 he entered the Texas Agricultural and Mechanical College, where he re-

ceived the B.S. in M.E. in 1922. He was immediately given a position with the Santa Fe shops at Galveston, as a special apprentice, and four months later he was transferred to Cleburne. His death occurred on the last day of his apprenticeship, on his second trip on the road, which was a part of his training in the three-year special course which includes service in all departments of the shop.

Mr. Yater became a junior member of the Society in 1923.

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NOTE

1. Names of authors and discussers, also those of deceased members preceding an obituary notice, are in caps and small caps. A discusser is distinguished from the author of a paper by (*D*), placed after the subject of the discussion. Illustrations are denoted by (*I*), curves by (*C*), and tables by (*T*).
2. Titles of papers, where placed after the name of the author, and appearing in their exact form, are in italics. Papers are indexed not under their titles but under their subject-matter.
3. On pages 1361 et seq. will be found an alphabetical list of (*a*) 1924 Spring and Annual Meeting papers not included in this volume but published in *Mechanical Engineering*, or, in a few cases, in *Refrigerating Engineering;* (*b*) of papers presented before sections of the Society and published in *Mechanical Engineering;* and (*c*) of other leading articles and technical reports published in *Mechanical Engineering* during 1924.

REFERENCES TO 1924 PAPERS, ARTICLES, AND REPORTS NOT PUBLISHED IN THIS VOLUME

NOTE

Below will be found an alphabetical list of (*a*) 1924 Spring and Annual Meeting papers not included in this volume but published in *Mechanical Engineering*, or, in a few cases, in *Refrigerating Engineering;* (*b*) of papers presented before sections of the Society and published in *Mechanical Engineering;* and (*c*) of other leading articles and technical reports published in *Mechanical Engineering* during 1924.

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Airplanes, metal, design for large-scale production. *Mechanical Engineering,* vol. 46, Mid-Nov. 1924, p. 733

ALDRICH, W. H. Pulverized Fuel at Cleveland Electric Illuminating Company's Lake Shore Plant. *Mechanical Engineering,* vol. 46, Sept. 1924, p. 519

ALEXANDER, MAGNUS W. Industry's Interest in Industrial Training. *Mechanical Engineering,* vol. 47, Feb. 1925, p. 94

Alloys for die casting. *Mechanical Engineering,* vol. 46, Nov. 1924, p. 663

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ARMSTRONG, A. H. Development of the Electric Locomotive. *Mechanical Engineering,* vol. 46, Oct. 1924, p. 608

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AZBE, VICTOR J. Water-Cooling-System Efficiency. *Mechanical Engineering,* vol. 46, Mid-Nov. 1924, p. 799

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BARNARD, NORRIS CLEMENTS. Die Casting. *Mechanical Engineering,* vol. 46, Nov. 1924, p. 661

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BECKETT, C. A. Foreign Progress in Cutting Metals. *Mechanical Engineering,* vol. 46, Oct. 1924, p. 618

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BERG, ESKIL. Modern Tendencies in Steam-Turbine Power Plants. *Mechanical Engineering,* vol. 46, Oct. 1924, p. 577

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